Theory of the Firm

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Boundaries of the Firm

- Firms are economic units that make decisions, produce, sell, etc.
- What determines the optimal size of a firm? Should two plants be organized as two independent firms or as two divisions in one firm?
- Traditional economic analysis is silent about these issues, and takes the size of the firm as given.
- Moral hazard theory, with the exception of its multitasking part, also takes the size of the firm as given.

Grossman and Hart (1986): Incomplete Contracts and Property Rights

- Some states of the world cannot be contracted upon:
    - Agents cannot think about all contingencies.
    - They cannot communicate and negotiate about all possibilities.
    - They cannot write a clear contract that courts can then enforce.
- If contracts cannot fully specify the usage of the asset in every state of the world, then who gets the right to choose?

- The owner can decide on the usage, when unspecified by contract.
    - Contracts provide specific property rights, while ownership provides **residual property rights**.
- The owner of an asset will have a stronger incentive to make asset-specific investments, knowing that he has residual property rights.
- Transferring ownership of an asset from one party to another has a benefit – encouraging investment by the acquirer – and a cost – discouraging investment by the acquired. The tradeoff generates implications for ownership structures and firm boundaries.

A Basic Model (Hart, Ch. 2)

- There are two assets, *a1* and *a2*, and two managers, *M1* and *M2*.
- *a2* in combination with *M2* can supply a unit of input to *M1*, who, in combination with *a1*, can use it to produce a unit of output and sell it on the market.
- There are two dates, 0 and 1:
  - At date 0, *M1* and *M2* make (human-capital) investments to improve productivity. Those are denoted as *i* and *e*, respectively.

- At date 1, *M1* and *M2* decide whether to conduct the transaction between them or go to the market.

- At date 0, it is too costly to write a contract on the date-1 use of the assets. The owner of an asset will have the right to choose.

- Three ownership structures are considered:
  - Non-integration: *M1* owns *a1* and *M2* owns *a2*.
  - Type-1 integration: *M1* owns *a1* and *a2*.
  - Type-2 integration: *M2* owns *a1* and *a2*.

Date-1 Payoffs and Surplus

- At date 1, *M1* receives:
  - If trade occurs between *M1* and *M2*, *M1* receives $R(i) - p$.
  - If trade does not occur, *M1* buys the input in the market and receives: $r(i, A) - \overline{p}$.
    - The revenue is different for the lack of *M2*'s human capital.
    - A denotes the assets that *M1* owns, and can be $\{a1\}$, $\{a1, a2\}$, or $\emptyset$.

- Similarly, for *M2*:
    - If trade occurs between *M1* and *M2*, *M2* receives $p - C(e)$.
    - If trade does not occur, *M2* sells the input in the market and receives: $\overline{p} - c(e, B)$.
        - *B* denotes the assets that *M2* owns: $\{a2\}$, $\{a1, a2\}$, or $\emptyset$.
- The surplus in case of trade is $R(i) - C(e)$, while in case of no trade it is $r(i, A) - c(e, B)$.
- Assuming gains from trade: For all *i* and *e* and *A* and *B*:

$$R(i) - C(e) \geq r(i, A) - c(e, B).$$

Date-1 Division of Surplus

- Ex-post, at date-1, for a given ownership structure and investments, the parties can negotiate. Hence, they choose to trade.
- It is assumed that the gains from trade $[(R - C) - (r - c)]$ are divided half-half, as in the **Nash bargaining solution**.
- The profits of *M1* and *M2* are then:

$$\pi_1 = R - p = r - \overline{p} + {}^1\!/_2\,[(R - C) - (r - c)]$$

$$= -\overline{p} + {}^1\!/_2\,(r + R - C + c),$$

$$\pi_2 = p - C = \overline{p} - c + {}^1\!/_2\,[(R - C) - (r - c)]$$

$$= \overline{p} - {}^1\!/_2\,(r - R + C + c).$$

- The price is given by: $p = \overline{p} + {}^1\!/_2\,[(R - r) - (c - C)]$.
  - Note that integration involves transformation of ownership of physical capital, but not of human capital (e.g., under type-1 integration, *M1* controls *a2*, but has no say on what *M1* does).
  - The division of surplus is independent of ownership structure.

Return on Investment

- Key assumption is that marginal return on investment is increasing in how many assets in the relationship the investor has access to.

- For *M1*:

$$R'(i) > r'(i, \{a1, a2\}) \geq r'(i, \{a1\}) \geq r'(i, \emptyset),$$

  - where $R', r' > 0; R'', r'' < 0.$

- Similarly, for *M2*:

$$|C'(e)| > |c'(e, \{a1, a2\})| \geq |c'(e, \{a2\})| \geq |c'(e, \emptyset)|,$$

  - where $C', c' < 0; C'', c'' > 0.$

First-Best Investment

- If the two parties could coordinate their investment decisions, they would reach a solution that maximizes the total surplus:

$$R(i) - i - C(e) - e.$$

- Denoting the first-best solution by (i^*, e^*), we get:

$$R'(i^*) = 1,$$
$$|C'(e^*)| = 1.$$

- Yet, the parties are not able to coordinate or write a contract on their investment levels. These are **observable but not verifiable.**

Investment Choice

- *M1* and *M2* choose their investments non-cooperatively, each one maximizing his expected utility, taking the other's investment as given (Nash Equilibrium).

- *M1* chooses *i* to maximize:

$$\pi_1 - i = -\overline{p} + {}^1/_2\left(r(i, A) + R(i) - C(e) + c(e, B)\right) - i,$$

  - leading to the following first-order condition:

$${}^1/_2\left(r'(i, A) + R'(i)\right) = 1.$$

- *M2* chooses e to maximize:

$$\pi_2 - e = \overline{p} - {}^{1}\!/_{2}\left(r(i, A) - R(i) + C(e) + c(e, B)\right) - e,$$

  - leading to the following first-order condition:

$$ {}^{1}\!/_{2}\,(|c'(e, B)| + |C'(e)|) = 1.$$

- We can immediately see that the equilibrium levels of i and e are below the first-best:

  - $r'(i, A) < R'(i)$ and $|c'(e, B)| < |C'(e)|$.

Why We Don't Achieve First-Best?

- The parties do not internalize the full benefit of the investment.
  - When *M1* invests, the return increases by R'.
  - However, *M1* realizes only:
    - r': direct benefit.
    - ${}^{1}/_{2}\,(R' - r')$: share in surplus.
  - The sum of r' and ${}^{1}/_{2}\,(R' - r')$ is lower than R'. This is a reflection of a **hold-up** problem.

The Effect of Ownership

- The ownership structure determines the return a party gets on its investment in the case of no trade.

- Using subscripts 0, 1, and 2, to denote no integration, type-1 integration, and type-2 integration, respectively, we get that:

$$i^* > i_1 \geq i_0 \geq i_2,$$
$$e^* > e_2 \geq e_0 \geq e_1,$$

- Giving ownership to one party increases its investment and reduces the other party's investment.

Optimal Ownership

- Optimal ownership maximizes the total surplus.

- We choose between:

$$S_0 = R(i_0) - i_0 - C(e_0) - e_0,$$

$$S_1 = R(i_1) - i_1 - C(e_1) - e_1$$

$$S_2 = R(i_2) - i_2 - C(e_2) - e_2.$$

- We can easily imagine establishing optimal ownership if one party is unique and is offering the reservation utility to the other party.

Implications

- Overall, optimal ownership finds the balance between the effect on *M1*'s investment and the effect on *M2*'s investment.
    - In general, transferring ownership from one party to another increases one type of investment and decreases the other.
- If *M1*'s (*M2*'s) investment is inelastic, such that he chooses the same investment under all ownership structures, then type-2 (type-1) integration is optimal.

- No point of giving ownership to someone who doesn't respond to incentives.

- If *M1*'s (*M2*'s) investment is relatively unproductive, then type-2 (type-1) integration is optimal.
  - Investment being relatively unproductive can be captured by surplus decreasing to $\theta(R(i) - i)$, and θ being sufficiently small.
  - No point of giving ownership to someone whose investment is not important.

- If assets *a1* and *a2* are independent, $r'(i, \{a1, a2\}) = r'(i, \{a1\})$ and $c'(e, \{a1, a2\}) = c'(e, \{a2\})$, then non-integration is optimal.
    - There is no benefit, only cost, from shifting ownership on *a1* from *M1* to *M2*.
- If assets *a1* and *a2* are strictly complementary, $r'(i, \{a1\}) = r'(i, \{\emptyset\})$ or $c'(e, \{a2\}) = c'(e, \{\emptyset\})$, then some form of integration is optimal.
    - Once a party does not control one asset, there is no additional cost, only benefit, from taking the other asset out of his control.

- Complementary assets should be owned by the same party (but not under joint ownership).

- If one party's human capital is essential, e.g., for *M1*, $c'(e, \{a1, a2\}) = c'(e, \{\emptyset\})$, he should own both assets.
  - No point in giving ownership to a party when the other party is essential for the relationship.

- If both human capitals are essential, all ownership structures are equally good.
  - In this case, no party benefits from ownership without trade.

Hart and Moore (1990): Extending the Property-Rights Theory

- The Grossman-Hart model reviewed above may seem a bit special as it only talks about the incentives of managers/entrepreneurs.
- Hart and Moore (1990) consider broader implications by asking what ownership does to employees' incentives.
- The identity of the owner of the assets will affect the incentives of employees, who are linked to the assets.

Basic Setup

- The economy consists of a set $\underline{S}$ of I risk neutral individuals, and a set $\underline{A}$ of N assets $(a_1, \ldots, a_N)$.
- There are two dates. At date 0, agent i makes a human-capital investment x_i ($x = (x_1, \ldots, x_I)$) At date 1, agents produce and trade.
- The cost of investment is $C_i(x_i)$, where $C_i'(x_i) > 0$ and $C_i''(x_i) > 0$.
- Agents decide on investments non-cooperatively, and then, given investments, gains from trade are determined via bargaining.

Date-1 Coalitions and Surplus

- At date 1, agents can form coalitions to use the assets in their control.

- Coalition S of agents, controlling subset A of assets, generates value of $v(S, A|x)$.
  - Assets controlled by coalition S denoted as $\alpha(S)$.
  - Control means either that an agent in the coalition owns the assets or that agents in the coalition together have majority.

- Value is increasing in assets and agents in the coalition, so optimal value ex-post for a given x is $v\left(\underline{S}, \underline{A} \middle| x\right) \equiv V(x)$.

- Agents achieve this via negotiation.

- They split the value among them according to **Shapley values**:

$$B_i(\alpha|x) = \sum_{S|i\in S} p(S)[v(S, \alpha(S)|x) - v(S\backslash\{i\}, \alpha(S\backslash\{i\})|x)]$$

  - The logic is to compensate the agent for his marginal contribution to a coalition, and calculate an average across all coalitions.

- Here, $p(S) = \frac{(s-1)!(I-s)!}{I!}$ is the probability of ending up in coalition S with random ordering.
  - s is the number of agents in coalition S.
- $v(S, \alpha(S)|x)$ is the value achieved by the coalition when agent i is included, and $v(S\backslash\{i\}, \alpha(S\backslash\{i\})|x)$ is the value achieved when the agent is excluded.
  - From every coalition, the agent gets the difference between the two, which summarizes his marginal contribution.

Date-0 Investment

- In Coalition S, agent i's marginal return on investment is:

$$\frac{\partial v(S,A|x)}{\partial x_i} = v^i(S,A|x)$$

  - $v^i(S,A|x) \geq 0$ and $v^{ii}(S,A|x) \leq 0$.
  - $v^i(S,A|x) = 0$ if $i \notin S$.
  - $\frac{\partial v^i(S,A|x)}{\partial x_j} \geq 0$ for all $j \neq i$.
  - $v^i(S,A|x) \geq v^i(S',A'|x)$ for all $S' \subseteq S$ and $A' \subseteq A$.

Social Optimum

- The first-best solution maximizes surplus assuming that the grand coalition $\underline{S}$ will form:

$$\max_{x} W(x) \equiv V(x) - \sum_{i=1}^{I} C_i(x_i)$$

- Hence, the first order condition for all i characterizing the first best x^* is:

$$v^i\left(\underline{S}, \underline{A} \middle| x^*\right) = C_i'(x_i).$$

Investment in a Non-Cooperative Equilibrium

- Agent i chooses investment to maximize the difference between his return (based on Shapley value) and cost. This yields the following first-order condition given the equilibrium behavior of others $x^e(\alpha)$:

$$\frac{\partial B_i(\alpha|x)}{\partial x_i} = \sum_{S|i\in S} p(S)v^i\big(S, \alpha(S)\big|x^e(\alpha)\big) = C_i'\big(x_i^e(\alpha)\big)$$

- The right-hand side is clearly below $v^i\big(\underline{S}, \underline{A}\big|x\big)$ for a given x, and this reflects under-investment.

- Based on this observation, Hart and Moore show that the equilibrium vector of efforts will exhibit under-investment.
- The intuition is similar to that in Grossman and Hart (1986):
  - When deciding on his level of investment, an individual doesn't consider the full benefit, but rather only what additional benefit the investment will give him in the bargaining process.
  - Hence he ignores the externality and ends up under-investing.
- Ownership affects what agents internalize and how much they invest.

Optimal Ownership: Some Results

- When only one agent makes investment, he should own all assets.
  - As in Grossman and Hart (1986), shifting ownership from one agent to another decreases the investment of the first and increases the investment of the other.
    - An agent’s incentive to invest is affected by the assets controlled by coalitions he is part of.
  - When only one agent is investing there is no tradeoff.

- For any coalition of agents, an asset should be owned by the coalition or its complement.
  - Since incentive to invest comes from assets owned by a coalition, there is waste in leaving an asset ‘not owned’.
  - A direct implication is that not more than one agent should have veto power over an asset.
    - Otherwise, if two agents are not in the same coalition and they share control, the asset is not owned by the coalition or its complement.

- If an agent is *indispensable* to an asset, then he should own it.
  - The definition is that without agent i in the coalition, the asset has no effect on the marginal product of investment for the other members of the coalition:

$$v^j(S, A) = v^j(S, A \backslash \{a_n\}) \text{ if } i \notin S.$$

  - The asset encourages investment only when it is owned by a coalition that has agent i. To maximize such coalitions, we let the agent own the asset.
  - This shows the effect of ownership on the investments of others.

- If an agent is *dispensable* and makes no investment, he should not have any control rights.
  - The definition is that other agents' marginal product from investment is unaffected by whether the agent is in the coalition or not:

$$v^j(S, A) = v^j(S \backslash \{k\}, A) \text{ if } j \in S,\ j \neq k$$

  - Reducing the agent's ownership will not reduce others' investments by the above definition.
  - The agent himself is not investing.

- *Complementary* assets should always be controlled together.
  - Definition is that the two assets are unproductive unless they are used together:

    $$v^i(S, A\backslash\{a_m\}) = v^i(S, A\backslash\{a_n\}) = v^i(S, A\backslash\{a_m, a_n\}) \text{ if } i \in S.$$

  - The idea is that by grouping the two assets in the same ownership, we make sure that in all coalitions with one asset, the other one will be as well.
  - Otherwise, there is waste, since each asset makes a contribution only with the other one on board.

Clarifying the Role of Employees: Example

- Suppose that there are two assets a_1 and a_2.
- Each asset has a big worker (potentially employer) and a small worker (employee).
    - Big workers are denoted as m_1, m_2, and small ones as w_1, w_2.
- Suppose that small workers are only productive if they work with the assets they are linked to. We ignore synergies across workers.
- We will study the effect of ownership on small workers' incentives.

- Suppose that we consider two ownership structures: non-integration (m_1 controls a_1 and m_2 controls a_2) and type-1 integration (m_1 controls a_1 and a_2).

- The FOC for w_1 under non-integration is:

$$\frac{1}{3}v^{w1}(\{m_1, m_2\}, \{a_1, a_2\}) + \frac{1}{6}v^{w1}(\{m_1\}, \{a_1\}) = C'_{w1}(x^e_{w1})$$

While under integration it is:

$$\frac{1}{3}v^{w1}(\{m_1, m_2\}, \{a_1, a_2\}) + \frac{1}{6}v^{w1}(\{m_1\}, \{a_1, a_2\}) = C'_{w1}(x^e_{w1})$$

Recall that w_1 cares only about coalition with a_1.

- The FOC for w_2 under non-integration is:

$$\frac{1}{3}v^{w2}(\{m_1, m_2\}, \{a_1, a_2\}) + \frac{1}{6}v^{w2}(\{m_2\}, \{a_2\}) = C'_{w2}(x^e_{w2})$$

While under integration it is:

$$\frac{1}{3}v^{w2}(\{m_1, m_2\}, \{a_1, a_2\}) + \frac{1}{6}v^{w2}(\{m_1\}, \{a_1, a_2\}) = C'_{w2}(x^e_{w2})$$

- Comparison:
    - Type-1 integration is good for w_1, while for w_2 the effect is ambiguous.

- For w_1, the productivity of investment is determined only by coalitions with a_1, which is controlled by m_1. When m_1 also controls a_2, w_1 is getting a boost to productivity when matched with m_1.
  - This effect represents better **coordination**.
- For w_2, the better coordination is also present under integration, since whenever he is matched with a_2 he gets a boost to productivity from the presence of a_1.
- But, w_2 is losing some connection to m_2, which might be costly.

Internal Capital Markets

- A different angle on the question of the boundaries of the firm comes from analyzing the optimality of **internal capital markets**.
- Internal capital markets develop when a firm has multiple divisions, potentially in different industries, and transfers resources across divisions. This is what happens in conglomerates.
- The question is what is the benefit from putting various (potentially unrelated) divisions under the same ownership.

Stein (1997)

- Stein (1997) develops a model, where divisions are constrained in their ability to raise external financing for their projects due to an **agency problem**.

- Having an internal capital market can help mitigating the problem, as the headquarters can raise the financing and allocate them more efficiently across divisions.

- Stein analyzes when this is optimal and sheds light on the trade offs in choosing the size and scope of the internal capital market.

Basic Setup

- A project started by a *founder* requires a *manager* and a *financier*.
- The amount of investment in the project can be 1 or 2.
- There are two states of the world B and G.
- In state B, investment of 1 yields y_1, and investment of 2 yields y_2, where:

$$1 < y_1 < y_2 < 2.$$

  Hence, in this state, the optimal investment is 1.

- In state *G*, investment of 1 yields θy_1, and investment of 2 yields θy_2, where $\theta > 1$, and:

$$\theta(y_2 - y_1) > 1.$$

Hence, in this state, the optimal investment is 2.

- The ex-ante probability of state *G* (*B*) is *p* (*1*-*p*).
- The realization is known only to managers, but there is a problem with their incentive to tell the truth as they also receive a non-verifiable private benefit, which is a proportion *s* of gross return.

Credit Rationing

- Eliciting information from managers will be costly when s is sufficiently large. Then, financiers and founders will have to make decisions without knowing the realization.

- If the amount financed is 1, the expected return is

$$\big(p\theta + (1-p)\big)y_1 - 1.$$

- If the amount financed is 2, the expected return is

$$\big(p\theta + (1-p)\big)y_2 - 2.$$

- Then, when p is sufficiently small, investing 1 is more desirable, and this creates **credit rationing**:
    - Credit is rationed in good states of the world because of the lack of ability to convey information.
- As a side, note that compensation contracts could be designed to elicit information.
- However, the cost of eliciting information is $(1-p)s(y_2-y_1)$, while the benefit is $p(\theta(y_2-y_1)-1)$, and hence this is not desirable when s is sufficiently large.

Corporate Headquarters and the Internal Capital Market

- Suppose that a few projects are grouped together and headquarters raises financing. A few assumptions about corporate headquarters:
  - It can acquire information about projects' prospects.
  - It has no financial resources of its own.
  - It can capture a fraction ϕ of private benefits at the cost of diluting incentives, so that cash flows fall by a fraction $k<1$.
  - It has the authority to redistribute resources across projects.

The Role of Headquarters in a Two-Project Example

- There is no role for headquarters with only one project.
  - There will be reduction in cash flows due to reduced incentives, and no better information revelation since headquarters have the incentives of managers to misreport.
- Suppose there are two uncorrelated projects as described above.
- Suppose that headquarters can perfectly tell their states.
- Suppose that the overall credit constraint is not eased, so that headquarters can raise only 2 for the two projects.

- The potential benefit from the headquarters is its ability to reallocate resources from a project in a bad state to a project in a good state. This is beneficial as long as:

$$\theta(y_2 - y_1) > y_1$$

- Headquarters will have an incentive to do this to maximize private benefits.

- Additional efficiency comes from the headquarters' broader span of control which allows it to derive private benefits from several projects simultenously.

- Summarizing the trade off:
  - The expected net output under external market is:

$$EM = 2\big(y_1\big(p\theta + (1-p)\big) - 1\big)$$

  - The expected net output under internal market is:

$$IM = 2(1-p)^2 k y_1 + 2p^2 k\theta y_1 + 2p(1-p)k\theta y_2 - 2$$

  - By moving to an internal capital market, we sacrifice efficiency at a factor of k, but in situations where the projects are in different states, we get better allocation of resources.

Noisy Information and Scope

- A question that often comes up is what is the optimal scope of an internal capital market: How correlated the different divisions should be.

- Stein provides an argument for focus:

  - When information is noisy, headquarters might make mistakes in allocating resources.

  - When the projects are close to each other, noise tends to be correlated, and then relative rankings are not harmed.

Adjusting the Assumptions

- For each project, headquarters observes information that is either H (high) or L (low).

- The informativeness of the signal is captured by q:

$$prob\left(H^i/G^i\right) = prob\left(L^i/B^i\right) = q, \text{ where } 1/2 < q < 1.$$

- A false low (high) signal in one project makes a false low (high) signal in the other project more likely. This is captured by the parameter α which summarizes the degree of correlation:

$$prob\left(L^i/G^i, G^j, L^j\right) = (1-q)(1+\alpha) > prob\left(L^i/G^i\right) = (1-q)$$

$$prob\left(H^i/B^i, B^j, H^j\right) = (1-q)(1+\alpha) > prob\left(H^i/B^i\right) = (1-q)$$

- Note that the probability of observing a false (low) signal in what project does not change the probability of observing a false (high) signal in the other project.

- Now, there are 16 possible realizations (2 signals and 2 states for each project).

- Payoffs and probabilities are shown in the following table:

	Outcome/Signal Configuration	Probability	Payoff: External Market	Payoff: Internal Market
1.	GGHH	$p^2(q^2 + \alpha(1 - q)^2)$	$2\theta y_1$	$2\theta y_1$
2.	GBHH	$p(1 - p)q(1 - q)$	$y_1(1 + \theta)$	$y_1(1 + \theta)$
3.	BGHH	$(1 - p)p(1 - q)q$	$y_1(1 + \theta)$	$y_1(1 + \theta)$
4.	BBHH	$(1 - p)^2(1 - q)^2(1 + \alpha)$	$2y_1$	$2y_1$
5.	GGHL	$p^2(q(1 - q) - \alpha(1 - q)^2)$	$2\theta y_1$	θy_2
6.	GBHL	$p(1 - p)q^2$	$y_1(1 + \theta)$	θy_2
7.	BGHL	$(1 - p)p(1 - q)^2$	$y_1(1 + \theta)$	y_2
8.	BBHL	$(1 - p)^2(q(1 - q) - \alpha(1 - q)^2)$	$2y_1$	y_2
9.	GGLH	$p^2(q(1 - q) - \alpha(1 - q)^2)$	$2\theta y_1$	θy_2
10.	GBLH	$p(1 - p)(1 - q)^2$	$y_1(1 + \theta)$	y_2
11.	BGLH	$(1 - p)pq^2$	$y_1(1 + \theta)$	θy_2
12.	BBLH	$(1 - p)^2(q(1 - q) - \alpha(1 - q)^2)$	$2y_1$	y_2
13.	GGLL	$p^2(1 - q)^2(1 + \alpha)$	$2\theta y_1$	$2\theta y_1$
14.	GBLL	$p(1 - p)(1 - q)q$	$y_1(1 + \theta)$	$y_1(1 + \theta)$
15.	BGLL	$(1 - p)pq(1 - q)$	$y_1(1 + \theta)$	$y_1(1 + \theta)$
16.	BBLL	$(1 - p)^2(q^2 + \alpha(1 - q)^2)$	$2y_1$	$2y_1$

- The result is that the benefit from an internal capital market increases in the degree of focus α and that this effect strengthens when q is smaller.
- The intuition comes from the fact that increasing focus increases the likelihood of configurations like GGHH and BBHH, and lowers the likelihood of configurations like GGHL and BBHL.
- This is good because there is no harm in a configuration like BBHH, as it causes no adverse implications for resource allocation. On the other hand, there is harm in configurations like GGHL and BBHL.