Asymmetric Information and Intermediation Chains

Online Appendix

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April 5, 2016

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Proofs Omitted from Paper

Proof of Lemma 1: Assumption 1 implies that:

$$\left[\frac{f(x)}{1-F(x)}\right][x-c(x)] > \left[\frac{f(x')}{1-F(x')}\right][x'-c(x')]$$
(B1)

for any x > x'. We want to show that any truncated distribution on the support $[a, b] \subseteq [v_L, v_H]$ with PDF $g(x) = \frac{f(x)}{F(b) - F(a)}$ and CDF $G(x) = \frac{F(x) - F(a)}{F(b) - F(a)}$ also satisfies:

$$\left[\frac{g(x)}{1-G(x)}\right][x-c(x)] > \left[\frac{g(x')}{1-G(x')}\right][x'-c(x')].$$
(B2)

Substituting the definitions of g and G we rewrite this inequality as:

$$\begin{bmatrix} \frac{f(x)}{F(b)-F(a)} \\ \frac{F(b)-F(a)-F(x)+F(a)}{F(b)-F(a)} \end{bmatrix} [x-c(x)] > \begin{bmatrix} \frac{f(x')}{F(b)-F(a)} \\ \frac{F(b)-F(a)-F(x)}{F(b)-F(a)} \end{bmatrix} [x'-c(x')]$$

$$\Leftrightarrow \begin{bmatrix} f(x) \\ F(b)-F(x) \end{bmatrix} [x-c(x)] > \begin{bmatrix} f(x') \\ F(b)-F(x') \end{bmatrix} [x'-c(x')]$$

$$\Leftrightarrow \frac{F(b)-F(x')}{F(b)-F(x)} > \frac{f(x')}{f(x)} \begin{bmatrix} x'-c(x') \\ x-c(x) \end{bmatrix}.$$
(B3)

By Assumption 1, we know that the following inequality holds:

$$\frac{1 - F(x')}{1 - F(x)} > \frac{f(x')}{f(x)} \left[\frac{x' - c(x')}{x - c(x)} \right].$$
 (B4)

It is therefore sufficient to verify that:

$$\frac{F(b) - F(x')}{F(b) - F(x)} \ge \frac{1 - F(x')}{1 - F(x)}.$$
(B5)

Recall that F(x) > F(x') since x > x' and since f(x) is strictly positive on the support $[v_L, v_H]$. If we set z = F(b), our result simply follows from:

$$\frac{\partial}{\partial z} \left[\frac{z - F(x')}{z - F(x)} \right] = \frac{z - F(x) - [z - F(x')]}{[z - F(x)]^2} = \frac{F(x') - F(x)}{[z - F(x)]^2} < 0.$$
(B6)

Proof of Proposition 3: *Part 1.* First note that networks satisfying Assumption 2 have unique equilibria and corresponding continuation payoffs for all agents in the trading game. Now, suppose there exists a set of traders $\mathbb{S} \subseteq \mathbb{T}$ and an order-flow agreement Σ for which trade breaks down with strictly positive probability and the total surplus across all traders in \mathbb{S} is less than $\mathbb{E}[v - c(v)]$ in the equilibrium of the trading game. Further, consider that every trader in \mathbb{S} obtains an ex ante surplus, net of transfers, that is weakly positive (otherwise equilibrium conditions are immediately violated, as every trader with negative surplus strictly prefers to exit the agreement). Order-flow agreement Σ can be blocked by a coalition of traders $\mathbb{S}' \subseteq \mathbb{T}$: based on the condition stated in the proposition, there exists an order-flow agreement Σ' associated with an intermediation chain that sustains efficient trade and preserves the full surplus $\mathbb{E}[v - c(v)]$. Since the total surplus is greater under agreement Σ' and any trader not involved in Σ collects zero surplus, ex ante transfers can be chosen such that every trader in \mathbb{S}' is strictly better off.

Part 2. An intermediation chain that allows for efficient trade yields a total surplus of $\mathbb{E}[v - c(v)]$ across all traders. To prove the existence of an order-flow agreement that constitutes an equilibrium in the network-formation game and supports the efficient intermediation chain, we consider an order-flow agreement Σ that specifies a set of transfers that imply that all intermediaries involved in agreement Σ obtain zero ex ante surplus (net of transfers), and the ultimate buyer and seller split the total surplus of $\mathbb{E}[v-c(v)]$. Any coalition of traders S' that attempts to block this order-flow agreement would need to include the ultimate buyer and seller, since they are needed to generate a positive surplus from trade. A blocking order-flow agreement Σ' would thus need to make both of these ultimate traders weakly better off and at least one agent in coalition S' strictly better off, which is impossible since the ultimate buyer and seller already split the maximum surplus of $\mathbb{E}[v - c(v)]$ under agreement Σ and no intermediary would be willing to participate in the blocking order-flow agreement if promised a negative expected surplus.