University of Pennsylvania
The Wharton School

FNCE 100

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# PROBLEM SET #1

Fall Term 2005

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## Present Value and Term Structure

1. Given an annual interest rate of 10 percent, what is the present ($t = 0$) value of a stream of \$100 annual payments starting in one year and ending in 20 years?

2. An investment of \$1,000 earns 8 percent interest per year for three years. A second \$1,000 investment earns 1 percent for the first and second years and 22 percent the third year.

   (a) What is the average (arithmetic) of the returns on each of the investments over the three years?

   (b) Compute the terminal value of each investment. Which is larger?

   (c) What is the compound rate of interest at which the initial \$1,000 rises to the terminal value of each investment?

   (d) What is the geometric mean of the returns on each project?

   (e) How do your results in Part d compare with your results in Part c? Comment on the implications.

3. You purchase a bond for \$900 and are promised coupon payments of \$50 per year for the next 15 years and then a maturity payment of \$1000. (The coupon payments come at year end.) Your ordinary income is taxed at the 25 percent rate while your capital gains are taxed at the 10 percent rate. What is the after-tax yield to maturity on this investment? Your capital gains tax is paid when the gain is realized, i.e., when the bond matures.

4. Calculate the present value of the following stream of payments:

   \$2100 in 2 years
   \$2100 in 4 years
   \$2100 in 6 years
   ⋮ ⋮
   \$2100 continuing in this pattern forever.

   The annual rate of interest is 10 percent.

5. You need \$129,200 at the end of seventeen years. You know that the best you can do is to make equal payments into an account on which you can earn 9 percent interest compounded annually. Your first payment is to be made at the end of the first year and the final payment is to be made at the end of the 17th year.
   (a) What amount must you plan to pay annually to achieve your objective?
   (b) Instead of making annual payments, you decide to make one lump-sum payment today. To achieve your objective of \$129,200 at the end of the seventeen year period, what should this sum be? (You can still earn 9 percent interest compounded annually on your account.)
6. At a growth rate of x percent, how long does it take a sum to double?
7. A woman wants to invest enough on her 40th birthday to provide her with a \$5,000 annual pension that begins (the first payment is received) on her 60th birthday and ends (the last payment is received) on her 74th birthday. If the interest rate is 6 percent a year, what must she invest on her 40th birthday to assure herself of this pension?
8. A firm is attempting to arrange a loan from a bank to purchase some equipment. The firm has talked to four different banks and received loan terms from each. Calculate the rate of return the firm would be paying in each case:
   (a) Bank A loans \$10,000 today to the firm; the firm pays the bank \$15,385 at the end of five years.
   (b) Bank B loans \$10,000 today; the firm pays the bank \$3,500 per year at the end of each year for four years.
   (c) Bank C loans \$10,000 today; the firm pays \$2,000 per year the first three years and then \$4,000 per year for two more years (all payments at year-end).
9. A company buys a \$100,000 piece of land by paying 20 percent down and the remainder of the loan in equal \$20,000 annual payments for five years, with a \$50,000 payment in year six. What interest rate is the company paying?
10. Burley and Bright Tobacco, Ltd., manufactures and sells a popular chewing tobacco. The company receives about \$20,000 cash flow each year from the product after all expenses, including taxes. Harris Cigars has recently offered to buy the product for \$160,000. Assuming Burley and Bright's discount rate is 10 percent, should they sell the product if they think its estimated life expectancy is:

(a) 15 years?

(b) Indefinitely long?

11. What is the present value of an infinite life annuity of \$3,000 which provides the first payment in 7 years? Your opportunity cost of funds is 9%.

12. A share of stock is expected to pay a \$2 dividend in year one, a \$2.50 dividend in year two, no dividend in year three, and a \$3.50 dividend thereafter (forever). If the investor's required rate of return is 20%, find the value he places on this share of stock.

13. You can buy a note for \$12,835. If you buy it, you will receive 10 annual payments of \$2,000, the first payment to be made one year from today. What rate of return, or yield, does the note offer?

14. You take out a 3-year auto loan for \$20000 on October 1, 1995. The first of the 36 equal monthly payments of \$664.29 is made on October 31, and you continue to be current in your payments. If you decide to pay off the loan on February 29, 1996, how much will you have to pay? (Include the February 29 payment.)

15. Your parents make you the following offer: They will give you \$500 at the end of every six months for the next five years if you agree to pay them back \$500 at the end of every six months for the following ten years. Should you accept this offer if your opportunity cost is 18%, (assume this is an annual rate to be compounded semiannually)?

16. According to the February 7, 1983 issue of *The Sporting News*, the Kansas City Royal's designated hitter, Hal McRae (who led the major leagues in runs batted in in 1982), signed a 3 year contract in January with the following provisions: \$400,000 signing bonus, \$250,000 salary per year for 3 years, followed by 10 years of deferred payments of \$125,000 per year; plus several bonus provisions which add as much as \$75,000 per year for the 3 years of the contract. Assume that McRae has a 60 percent probability of receiving the bonuses each year (paid on Dec. 31, if earned). What is the present value of his contract in early January when he signed it? (Assume that he signed on January 1, 1983). Assume an interest rate of 6 percent per 6 months, (12.36 percent effective annual rate) and assume that salary paychecks and deferred payments are received on January 1 and July 1 of each year. Show your work. Ignore taxes.

17. Given an interest rate of 10 percent per year, what is the value in date $t = 8$ dollars of a perpetual stream of \$200 payment coming at $t = 16, 17, 18, \ldots$?

18. Due to rapid technological progress in the computer industry, Orange Computer, Inc. anticipates that its Current product line will need to be completely replaced in 3 years. For the next 3 years, Orange Computer forecasts expected net cash flow from operations of \$20,000,000 per year (received at year end). After 3 years, its manufacturing facilities will be completely obsolete. Due to its established marketing force and innovative

engineers, Orange is confident that investments of \$24,000,000 each year now and at the start of the next 3 years will result in a new generation of minicomputers which generate expected net cash flow from operations of \$50,000,000 per year in perpetuity starting at the end of year 4. If the appropriate real discount rate for Orange Computer is 10 percent, what is the value of the company today? (All dollar figures are expressed in terms of January, 1984 purchasing power. Taxes have already been incorporated in the cash flow estimates above.)

19. If earnings for 1986 were \$2.83 a share, while earnings for 1979 were \$1, what was the rate of growth in earnings between those years?

20. On December 31, Frank Ferris buys a building for \$80,000, paying 20 percent down and agreeing to pay the balance in fifteen equal annual installments that are to include principal plus 10 percent compound interest on the declining balance. What are the equal installments?

21. You have just purchased a newly issued \$1,000 five-year Vanguard Company bond at par. This bond (Bond A) pays \$60 in interest semiannually (\$120 a year). You are also negotiating the purchase of a \$1,000 six-year Vanguard Company bond that returns \$30 in semiannual interest payments and has six years remaining before it matures (Bond B).

    (a) What is the going rate of return on bonds of the risk and maturity of the Vanguard Company's bonds?

    (b) What should you be willing to pay for Bond B?

    (c) How will your answer to Part b change if Bond A pays \$40 (instead of \$60) in semiannually interest but still sells for \$1,000? (Bond B pays \$30 semiannually and \$1,000 at the end of six years.)

22. A woman, age 40, is planning a retirement pension. She wants to retire at age 65, and receive an annual benefit payment of \$20,000 for the following 15 years, i.e., until she is 80. If she wants her contributions over the next 25 years to grow at a rate of 3% per year, what must her first pension contribution (which occurs in one year) be? Assume a constant interest rate of 10%.

23. You decide to buy a home for \$100,000. You approach two savings and loan associations (S&L's) for financing. S&L #1 requires a 10% downpayment and requires monthly payments on a 20-year mortgage sufficient to earn it an effective annual yield of 12%. S&L #2 also needs a 10% downpayment, but quotes a 12% annual rate which is compounded monthly (to yield a higher effective return). What are the monthly payments on the respective mortgages?

24. The following questions relate to the article, "Software to Make Life a Bit Easier" (extracted from the *New York Times*, September 5, 1989):

In the never ending quest for computing convenience, we recently found three products especially useful. One saves money, one saves time, and the other saves aggravation.

The money saver is called A Banker's Secret Software Package ($29.95, plus $2 shipping, for IBM PC and compatibles from Good Advice Press, Box 8, Elizaville, NY 12523, ((914) 758-1460).

It is hard to exaggerate how much money this program can help the homeowner save. Consider, for instance, a couple who have just bought a home and have a 30-year, $200,000 mortgage at 11.25 percent interest.

Their monthly payment is $1,942.53. By the end of the loan, they will have paid the bank nearly $700,000. In other words, they will have paid $500,000 for the privilege of borrowing $200,000.

Their mortgage includes a prepayment option which allows them to pay off the principal (the original amount they borrowed, not the interest) at any time. Many lenders offer this feature, but few borrowers take advantage of it. The secret is that prepayments can significantly cut the overall cost of a loan.

The Banker's Secret Software Package lets the user play "what if" with the prepayment option. What if John and Mary wrote the bank a check for $2,000 each month instead of $1,942.53? They figure that they can come up with the extra $57.47 a month fairly painlessly, perhaps by saving pocket change at the end of each day.

It takes about a minute to figure out how to use the software. But the results can be startling: the prepayment will reduce the interest fees by $105,034.33, allowing them to retire their mortgage 62 months early. The loss of a tax deduction on the excess interest is more than offset by the actual savings. With the money they save, they can send a child to college. In a sense, they are investing pocket change in their mortgage and getting huge returns.

The program does much more than amortize loans, and part of the fun is exploring its many options. The package comes with Marc Eisenson's book "A Banker's Secret," which is helpful for anyone who has a mortgage or who is thinking of buying a home.

(a) At an annual rate of interest of 11.25 percent (compounded monthly), what is the present value of a stream of monthly payments of $1,942.53 for 30 years? Is paying "500,000 for the privilege of borrowing $200,000" a mistake?

(b) How many months of payments of $2,000 are required to have a present value $200,000? How does this compare with the article's statement that the mortgage is retired 62 months earlier?

(c) At the end of the example in the article, the author concludes that: "In a sense, they are investing pocket change in their mortgage and getting huge returns."

Ignoring the loss of tax deduction on interest, what is the annual rate of return on the additional \$57.47 put into the mortgage? How would a consideration of tax factors affect your calculation?

(d) The author states that "It is hard to exaggerate how much money this program can help the homeowner save." Do you agree? Is it worth far more than \$29.95?

25. Swiss Dun French For the Cost of Trees Felled by Napoleon

* * *

He Pledged to Pay for Damage His Army Caused in 1800:
\$73.3 Million With Interest
4-8-83
Special to *The Wall Street Journal*

BOURG-ST. PIERRE, Switzerland—Memories die hard in the Alpine village, especially the one about Napoleon's unpaid bill.

The French leader and his army passed through Bourg-St. Pierre 183 years ago on their way to Italy.

In a signed note, Napoleon promised to reimburse the community "in full" for all damage caused by his men, and for the use of local labor and mules in getting his canon over the Grand St. Bernard mountain pass.

The village carefully preserves the letter and annually revises the original bill-currently equivalent to \$19,500—to include interest. It now adds up to \$73.3 million.

"A village delegation will be presenting the bill to President (Francois) Mitterrand when he visits Switzerland next week," says Mayor Fernand Dorsaz. "It is only right that he should honor a debt of France." The French president begins a three-day visit next Thursday.

Bourg-St. Pierre, which has a population of 300, duly sent its bill to Paris after Napoleon passed through in May 1800. The village billed the French for:

- Destruction of 2,037 trees, at six francs each.
- Provision of 188 cooking pots, of which 80 never were returned.
- 3,150 logs used to roll the guns over the pass.

- Local labor, at three francs daily per man.
- Rental of mules, at six francs daily per animal.

There has never been a reply.

(a) What (annually-compounded) interest rate is Bourg-St. Pierre charging the French?

(b) What (continuously-compounded) interest rate does this correspond to?

26. This problem is intended to utilize your knowledge about the term structure of interest rates. You are given the following information about three bonds that have recently been traded.

| Bond | Price (yr 0) | Cash flows yr 1 | yr 2 | yr 3 |
|---|---|---|---|---|
| A | \$ 977.18 | 100 | 100 | 1100 |
| B | \$1026.37 | 120 | 1120 | 0 |
| C | \$ 245.93 | 100 | 100 | 100 |

(a) Set up the equations to solve for the $P_{k,0}$, $k = 1, 2, 3$, i.e. the prices of \$1 pure discount bonds at time 0 (or equivalently the discount factors, $DF_1$, $DF_2$, $DF_3$) that last for 1 pd, 2 pds, and 3 pds respectively.

(b) It is possible to create a pure discount bond of maturity 1, 2, or 3 by combining different amounts of each of the three bonds above. These amounts can be positive or negative, and can be fractions (e.g. a possible, but not correct, combination could be .5 of a bond A, 1.3 of bond B, and -.25 of bond C). Calculate exactly what amounts (weights) should be used to create:

   i. a one period pure discount bond
   ii. a two period pure discount bond
   iii. a three period pure discount bond.

(c) Solve for $P_{k,0}$, $k = 1, 2, 3$.

(d) Solve for $r_k$ $k = 1, 2, 3$, i.e. the yields to maturity on these pure discount bonds.

(e) Plot the "zero-coupon" yield curve.

(f) Using the rates in (d), calculate the NPV of a project that costs \$1217 and pays out \$840 in pd 1, \$340 in pd 2, and \$290 in pd 3. Should you take the project?

(g) Now calculate the forward rates: $f_{1,t}$, $t = 0, 1, 2$.

(h) Plot the forward rates as a function of time.

(i) You are working for a large institutional investor. Another large firm offers to lend your firm \$1 million between periods 2 and 3 at a rate of 11%. Do you accept? Explain.

(j) Someone offers your firm a chance to buy a three period zero coupon bond (bond D) with a payout (face value) of \$1000, for \$731. What is the yield to maturity (YTM) on this bond? What is the YTM on bond C (also a three period bond)? Is the bond a good deal? Why or why not?

(k) Someone offers to buy a bond (bond E) with the following cash flows from your firm for 1598.88, claiming that this is a fair price because it would imply the same YTM as bond C.

| **Year** | 1 | 2 | 3 |
|---|---|---|---|
| | 500 | 500 | 1000 |

Verify that the YTM on bond E is the same as on bond C. Should you go ahead and sell the bond at that price? Explain why or why not.

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Solutions to PROBLEM SET #1

Fall Term 2005

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### Present Value and Term Structure

1. $$\begin{aligned} \text{PV} &= \sum_{t=1}^{20} \tfrac{100}{(1+r)^t} = 100 \times [20 \text{ year annuity factor at } 10\%] \\ &= 100(8.536) = \$851.36 \\ &\quad \text{or, alternatively} \\ &= \tfrac{c}{r}\left[1 - \tfrac{1}{(1+r)^{20}}\right] = \tfrac{100}{0.10}\left[1 - \tfrac{1}{(1.10)^{20}}\right] = \$851.36 \end{aligned}$$

2. (a) F: $\frac{8+8+8}{3} = 8$

   S: $\frac{1+1+22}{3} = 8$

   (b) F: $1000(1.08)^3 = 1259.71$

   S: $1000(1.01)^2(1.22) = 1244.52$

   (c) $$\begin{aligned} \text{F: } 1000(1+r)^3 &= 1259.71 \\ r &= .08 \\ \text{S: } 1000(1+r)^3 &= 1244.52 \\ r &= .0756 \end{aligned}$$

   (d) F: $r = .08$

   S: $r = \sqrt[3]{(1.01)(1.01)(1.22)} - 1 = .0756$

   (e) They are the same. The annual rate of return compounded annually computed using the terminal value is the geometric mean return.

3. $$900 = \sum_{t=1}^{15} \tfrac{50(0.75)}{(1+r)^t} + \tfrac{1000}{(1+r)^{15}} - \tfrac{\$10}{(1+r)^{15}}$$
$$= \sum_{t=1}^{15} \tfrac{37.50}{(1+r)^t} + \tfrac{990}{(1+r)^{15}}$$
$r = 4.6$ percent

4. $$\text{PV} = \sum_{t=1}^{\infty} \tfrac{2100}{(1+r)^{2t}} = \sum_{t=1}^{\infty} \tfrac{2100}{(1.1)^{2t}} = \sum_{t=1}^{\infty} \tfrac{2100}{(1.21)^t} = \tfrac{2100}{0.21} = \$10,000$$

5. (a) $$\text{PV} = \tfrac{C}{r}\left[1 - \left(\tfrac{1}{1+r}\right)^{17}\right] \qquad \text{FV} = \text{PV} \cdot (1+r)^{17}$$
$$\Longrightarrow \text{FV} = \tfrac{C}{r}\left[1 - \left(\tfrac{1}{1+r}\right)^{17}\right] \cdot (1+r)^{17}$$
$$\Longrightarrow C = \frac{\text{FV}}{\frac{(1+r)^{17}}{r} - \frac{1}{r}} = \frac{129{,}200}{\frac{(1.09)^{17}}{.09} - \frac{1}{.09}} = \$3,494.38$$

(b) $$x(1.09)^{17} = 129,200 \Longrightarrow x = \tfrac{129{,}200}{(1.09)^{17}} = \$29,854.65$$

6. $$1(1+.01x)^t = 2 \Longrightarrow t(\ln(1+.01x)) = \ln(2) \Longrightarrow t = \tfrac{\ln(2)}{\ln(1+.01x)}$$

7. $$\frac{40}{-P} \quad \frac{41}{0} \quad \cdots \quad \frac{60}{+5000} \quad \frac{61}{+5000} \quad \frac{\cdots}{\cdots} \quad \frac{74}{+5000} \qquad r = .06$$
$$P = \left(\tfrac{1}{1+r}\right)^{19}\left[\tfrac{5000}{r}\left[1 - \left(\tfrac{1}{1+r}\right)^{15}\right]\right] = \$16,050$$

8. (a) $$+10,000 - \tfrac{15{,}385}{(1+r)^5} = 0 \Longrightarrow 1 + r = \left(\tfrac{15{,}385}{10{,}000}\right)^{1/5} \Longrightarrow r = 9.00\%$$

(b) $$+10,000 - \sum_{t=1}^{4} \tfrac{3500}{(1+r)^t} = 0 \Longrightarrow r = 14.96\%$$

(c) $$+10,000 - \sum_{t=1}^{3} \tfrac{2000}{(1+r)^t} - \sum_{t=4}^{5} \tfrac{4000}{(1+r)^t} = 0 \Longrightarrow r = 10.60\%$$

9. $$+(100,000 - 20,000) - \sum_{t=1}^{5} \tfrac{20{,}000}{(1+r)^t} - \tfrac{50{,}000}{(1+r)^6} = 0 \Longrightarrow r = 18.5\%$$

10. (a) $\text{PV}_0 = \sum_{t=1}^{15} \tfrac{20{,}000}{(1.1)^t} = \$152,122.$ Yes, they should sell.

(b) $\text{PV}_0 = \sum_{t=1}^{\infty} \tfrac{20{,}000}{(1.1)^t} = \tfrac{20{,}000}{.10} = \$200,000.$ No, they should not sell.

11. $$V_0 = \tfrac{C}{r}\left(\tfrac{1}{(1+r)^6}\right) = \tfrac{3000}{.09}\left(\tfrac{1}{(1.09)^6}\right) = \$19,876$$

12. $P_0 = \frac{2}{(1.2)} + \frac{2.5}{(1.2)^2} + \frac{3.5}{.2}\left(\frac{1}{(1.2)^3}\right) = \$13.53$

13. $-12835 + \sum_{t=1}^{10} \frac{2000}{(1+r)^t} = 0 \Longrightarrow r = 9.0\%$

14. **Step 1:** calculate monthly $R$ on loan:

$$20000 - \sum_{t=1}^{36} \frac{664.29}{(1+R)^t} = 0 \Longrightarrow R = 1\% \text{ per month}$$

**Step 2:** calculate PV of payments made prior to Feb. 29, 1996:

(time 0 = Oct. 1, 1995)

$$\text{PV}_0 = \sum_{t=1}^{4} \frac{664.29}{(1.01)^t} = 2592.04$$

**Step 3:** calculate PV of amount still owed

$= 20000 - 2592.04 = 17407.96$

**Step 4:** calculate FV (2/29/96) of amount still owed

$= 17407.96 \cdot (1.01)^5 = 18295.94$

15. The PV of what you get is $\sum_{j=1}^{10} \frac{500}{(1+R)^j} = \frac{500}{R}\left[1 - \left(\frac{1}{1+R}\right)^{10}\right]$

Because the 18% is an annual rate compounded semiannually, this is equivalent to a 6 month rate of 9%.

So the PV of what you get $= \frac{500}{.09}\left[1 - \left(\frac{1}{1.09}\right)^{10}\right] = \$3209$

The PV of what you pay $= \left(\frac{1}{1+r}\right)^{10} \cdot \frac{500}{.09}\left[1 - \left(\frac{1}{1.09}\right)^{20}\right] = \$1928$

Therefore NPV $= 3209 - 1928 = 1281 > 0$, so *accept.*

16. 
$$\begin{aligned}
\text{PV} &= 400,000 + \sum_{t=0}^{5} \frac{125,000}{(1.06)^t} + \sum_{t=6}^{25} \frac{62,500}{(1.06)^t} + \sum_{s=1}^{3} \frac{\$75,000(0.60)}{(1.06)^{2s}} \\
&= 400,000 + 125,000 + 125,000(4.2124) + 62,500(12.7834 - 4.2124) + 107,417 \\
&= 400,000 + 125,000 + 526,550 + 535,687 + 107,417 \\
&= \$1,694,654
\end{aligned}$$

17. PV $(t=8)$ of $\sum_{t=16}^{\infty} \frac{200}{(1.10)^t} = \left(\frac{1}{1.10}\right)^7 \cdot \frac{200}{0.10} = \$1026.32$

18. $$\begin{aligned} \text{PV} &= \sum_{t=1}^{3} \tfrac{20{,}000{,}000-24{,}000{,}000}{(1.10)^t} - 24{,}000{,}000 + \sum_{t=4}^{\infty} \tfrac{50{,}000{,}000}{(1.10)^t} \\ &= -4{,}000{,}000(2.487) - 24{,}000{,}000 + (0.751)500{,}000{,}000 \\ &= \$341{,}550{,}000 \end{aligned}$$

19. $1(1+g)^7 = 2.83 \Longrightarrow (2.83)^{1/7} - 1 = g = 16.02\%$

20. $$\begin{aligned} \text{amount borrowed} = (80{,}000)(.8) &= 64{,}000 \\ +64{,}000 &= \sum_{t=1}^{15} \tfrac{P}{(1.1)^t} \\ \Longrightarrow P &= \$8414{,}32 \end{aligned}$$

21. Bond A: $1000 = \sum_{t=1}^{10} \frac{60}{1+R} + \frac{1000}{(1+R)^{10}}$

(a) We can see by inspection that $R$, the 6 month yield, is equal to 6%. This implies an annual yield of $(1.06)^2 - 1 = 12.36\%$.

(b) If interest rates in the economy are equal over time and/or the term structure is completely flat and the bonds are correctly priced, we can use the yield calculated above to price the other bond. Thus:

$$\text{PV (Bond B)} = \sum_{t=1}^{12} \frac{30}{(1.06)^t} + \frac{1000}{(1.06)^{12}} = \$748$$

(c) This would give $R = 4\%$ (6 month yield)

$$\Longrightarrow \text{PV (Bond B)} = \sum_{t=1}^{12} \frac{30}{(1.04)^t} + \frac{1000}{(1.04)^{12}} = \$906$$

22.

| 40 | 41 | 42 | … | 65 | 66 | 67 | … | 80 |
|---|---|---|---|---|---|---|---|---|
| | $-C$ | $-C(1+g)$ | $\cdots$ | $-C(1+g)^{24}$ | $+20{,}000$ | $+20{,}000$ | $\cdots$ | $+20{,}000$ |

$$0 = \sum_{t=1}^{25} \left[ -C \frac{(1+g)^{t-1}}{(1+r)^t} \right] + \sum_{t=26}^{40} \frac{20{,}000}{(1+r)^t}$$

First term is a growing annuity. This can be valued as the difference of two growing perpetuities or the growing annuity formula can be used.

$$
\begin{aligned}
&= \tfrac{-C}{r-g}\left[1-\tfrac{(1+g)^{25}}{(1+r)^{25}}\right] = -C\left(\tfrac{1}{.10-.03}\right)\left[1-\tfrac{(1.03)^{25}}{(1.10)^{25}}\right] \\
&= -C\left(\tfrac{1}{.07}\right)\left[1-\left(\tfrac{1}{1.068}\right)^{25}\right] \\
&= -C(11.525)
\end{aligned}
$$

Second term $= \left(\frac{1}{1+r}\right)^{25}\sum_{t=1}^{15}\frac{20{,}000}{(1+r)^t} = .0923(152,122) = \$14,040$

Putting this together gives:

$$
\begin{aligned}
-C(11.525) + 14,040 &= 0 \\
\Longrightarrow C = \frac{14,040}{11.525} &= \$1218
\end{aligned}
$$

Her first pension contribution is \$1218, the next is \$1218 (1.03), etc.

23. In each case, the loan is for $100,000 - .10(100,000) = \$90,000$. To calculate the monthly payments, we must therefore compute the $X$ which solves

$$90,000 = \frac{X}{1+R} + \frac{X}{(1+R)^2} + \cdots + \frac{X}{(1+R)^{240}}$$

Since there are 240 monthly payments for the 20 year mortgages. The only difference in the two S&L terms is the $R$ to be used in discounting the future payments. S&L #1 wants an effective yield of 12% and therefore uses a monthly rate of $R$ computed from:

$$(1+R)^{12} = 1.12 \qquad \text{[S\&L \#1]}$$

which implies:

$$1 + R = \sqrt[12]{1.12} = 1.0095 \Longrightarrow r_1 = 0.95\%$$

S&L #2 quotes an annual rate of 12% compounded monthly and therefore uses an $R = \frac{.12}{12} = .01$ or 1%.

The monthly payment charged by S&L #1 is therefore $Y$, the solution of:

$$90,000 = \frac{Y}{1.0095} + \frac{Y}{(1.0095)^2} + \cdots + \frac{Y}{(1.0095)^{240}}$$

$$
\begin{aligned}
&= \frac{Y}{.0095}\left[1 - \frac{1}{(1.0095)^{240}}\right] = Y\left(\frac{.8966}{.0095}\right) \\
\Longrightarrow Y &= \$953.59
\end{aligned}
$$

On the other hand, the monthly payment to S&L #2 would be $Z$ from:

$$
\begin{aligned}
90,000 &= \frac{Z}{1.01} + \frac{Z}{(1.01)^2} + \cdots + \frac{Z}{(1.01)^{240}} \\
&= \frac{Z}{.01}\left[1 - \frac{1}{(1.01)^{240}}\right] = Z\left(\frac{.9082}{.01}\right) \\
\Longrightarrow Z &= \$990.97
\end{aligned}
$$

24. (a) Using the annuity formula, the present value is \$200,000. The difference between the total amount paid and the present value reflects the time value of money.

    (b) Paying \$2,000 allows the mortgage to be retired in a little over 297 months. Since 30 years is 360 months, the mortgage can be retired 62 months earlier.

    (c) They are earning exactly 11.25% on their additional investment. Since interest is tax deductible, a more rapid repayment schedule will reduce their tax deductions resulting in a lower after tax return.

    (d) The program simply does what you can do with a calculator and the present value formulas.

25. (a) $r = \sqrt[183]{\frac{73{,}300{,}000}{19{,}500}} - 1 = 4.6\%$

    (b) Let $\tilde{r}$ = the continuously compounded rate

$$
\begin{aligned}
\$73,300,000 &= 19,500 \cdot e^{\tilde{r}t} \\
t &= 183 \\
\Longrightarrow \ln(73,300,000) &= \ln(19,500) + \tilde{r} \cdot 183 \\
\tilde{r} &= \frac{\ln(73,300,000) - \ln(19,500)}{183} = 4.5\% \quad (= \ln(1.046))
\end{aligned}
$$

26. (a) Let $P_{k,t}$ be the price at $t$ of a $k$ pd \$1 pure discount bond. We can divide bonds A, B, C each into three pure discount bonds.

Thus:
$$\begin{array}{rcrcrcrl} 977.18 &=& 100\ P_{1,0} &+& 100\ P_{2,0} &+& 1100\ P_{3,0} & \quad (A) \\ 1026.37 &=& 120\ P_{1,0} &+& 1120\ P_{2,0} &+& 0\ P_{3,0} & \quad (B) \\ 245.93 &=& 100\ P_{1,0} &+& 100\ P_{2,0} &+& 100\ P_{3,0} & \quad (C) \end{array}$$

(b) It is easiest to solve for these in reverse order, i.e. first the 3 period, then the 2 period, and then the 1 period bond.

**3 period bond:** To get a 3 pd pure discount bond, observe that bond A minus bond C gives the same cash flows as a \$1000 3 pd pure discount bond. Thus the 3 pd \$1 pure discount bond = (.001)A + (0)B + (-.001)C.

**2 period bond:** To get a 2 pd pure discount bond, observe that bond B, minus 1.2 bond C, plus 120 3 pd \$1 pure discount bonds gives the same cash flows as a \$1000 2 pd pure discount bond. Thus 2 pd \$1 pure discount bond =

$$\begin{aligned} &= (.001)B + (-.0012)C + (.120)(\text{3 pd \$1 disc. bond}) \\ &= (.001)B + (-.0012)C + (.120)[(.001)A + (0)B + (-.001)C] \\ &= (.00012)A + (.001)B + (-.00132)C \end{aligned}$$

**1 period bond:** To get a 1 pd pure discount bond, observe that bond C minus 100 2 pd \$1 pure discount bonds minus 100 3 pd \$1 pure discount bonds gives the same cash flows as a \$100 1 pd pure discount bond. Thus 1 pd \$1 pure discount bond =

$$\frac{C}{100} - \left(\frac{100}{100}\right)(\text{3 pd \$1 disc. bond}) - \left(\frac{100}{100}\right)(\text{2 pd \$1 disc. bond})$$

$$\begin{aligned} &= (.01)C - 1[(.001)A + (0)B + (-.001)C] - 1[(.00012)A \\ &\qquad + (.001)B + (-.00132)C] \\ &= (-.00112)A + (-.001)B + (.01232)C \end{aligned}$$

(c) We saw in part b that we could create a \$1 one-period pure discount bond by combining (-.0012) of bond A, (-.001) of bond B and (.01232) of bond C. This means that the price of a \$1 one-period pure discount bond would be these same fractions multiplied by the prices of bonds A, B, and C respectively. Thus:

$$\begin{aligned} P_{3,0} &= (.001)(977.18) + (0)(1026.37) + (-.001)(245.93) \\ &= .73125 \end{aligned}$$

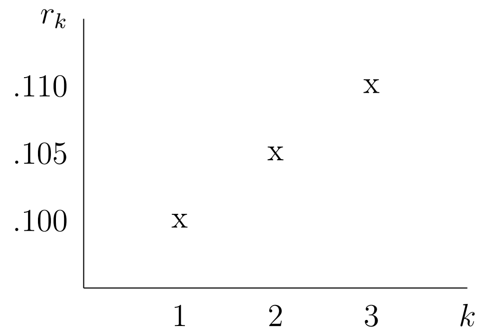
Similarly:

$$\begin{aligned} P_{2,0} &= (.00012)(977.18) + (.001)(1026.37) + (-.00132)(245.93) \\ &= .819004 \end{aligned}$$

$$\begin{aligned} P_{1,0} &= (-.00112)(977.18) + (-.001)(1026.37) + (.01232)(245.93) \\ &= .909046 \end{aligned}$$

(d)
$$\begin{array}{lllllll} P_{1,0} &=& \frac{1}{(1+r_1)} & \longrightarrow r_1 &=& \frac{1}{P_{1,0}} - 1 &=& .10 \\ P_{2,0} &=& \left(\frac{1}{1+r_2}\right)^2 & \longrightarrow r_2 &=& \left(\frac{1}{P_{2,0}}\right)^{\frac{1}{2}} - 1 &=& .105 \\ P_{3,0} &=& \left(\frac{1}{1+r_3}\right)^3 & \longrightarrow r_3 &=& \left(\frac{1}{P_{3,0}}\right)^{\frac{1}{3}} - 1 &=& .11 \end{array}$$

(e)



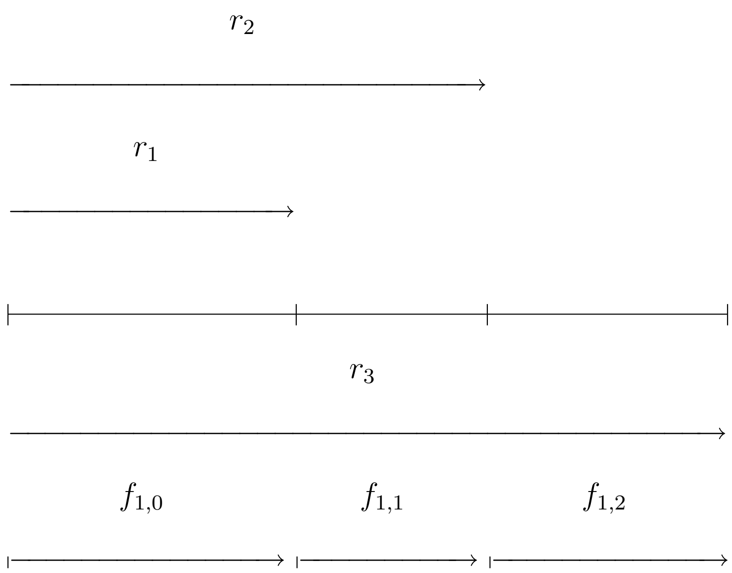

(f)

| Period | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| | −1217 | +840 | +340 | +290 |

$$\begin{aligned} \text{NPV}_0 &= -1217 + \frac{840}{1+r_1} + \frac{340}{(1+r_2)^2} + \frac{290}{(1+r_3)^3} \\ &= -1217 + 840(P_{1,0}) + 340(P_{2,0}) + 290(P_{3,0}) \\ &= \$37.12 \end{aligned}$$

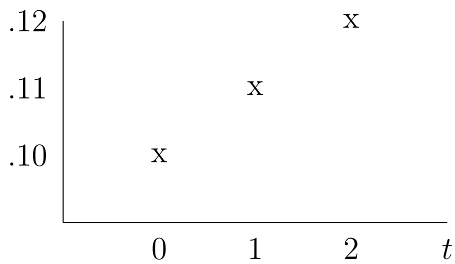
$\text{NPV}_0 > 0$, So take project.

(g)




$$
\begin{aligned}
f_{1,0} &= r_1 = .10 \\
1 + r_2 &= [(1+f_{1,0})(1+f_{1,1})]^{\frac{1}{2}} &&\longrightarrow\ f_{1,1} = .11 \\
1 + r_3 &= [(1+f_{1,0})(1+f_{1,1})(1+f_{1,2})]^{\frac{1}{3}} &&\longrightarrow\ f_{1,2} = .12
\end{aligned}
$$

(h)




(i) The forward rate implied by bonds A, B, and C between periods 2 and 3 is 12%. If you have the opportunity to borrow money between periods 2 and 3 at 11% you should definitely accept.

(j) Bond D:

| 0 | 1 | 2 | 3 |
|---|---|---|---|
| –731 | 0 | 0 | +1000 |

$$
\begin{aligned}
\text{YTM (D)}: \quad \frac{1000}{(1+y)^3} = 731 \longrightarrow 1 + y &= \left(\frac{1000}{731}\right)^{\frac{1}{3}} \\
\longrightarrow y &= .11
\end{aligned}
$$

$$
\begin{aligned}
\text{YTM (C)}: \quad \frac{100}{1+y} + \frac{100}{(1+y)^2} + \frac{100}{(1+y)^3} &= 245.93 \\
\longrightarrow y &= .1064
\end{aligned}
$$

Bond D has a higher YTM than bond C, but it is *not* a better deal. Using the

NPV rule:

$$\text{NPV (bond D)} = \frac{1000}{(1+r_3)^3} - 731 = 0$$

i.e., it is fairly priced relative to bond A, B, C.

(k) bond E: $\frac{500}{1.1064} + \frac{500}{(1.1064)^2} + \frac{1000}{(1.1064)^3} - 1598.88 = 0$

But using NPV rule, price of bond E should equal:

$$\begin{aligned} 500\ P_{1,0} + 500\ P_{2,0} + 1000\ P_{3,0} &= \tfrac{500}{1+r_1} + \tfrac{500}{(1+r_2)^2} + \tfrac{500}{(1+r_3)^3} \\ &= 500(.909046) + 500(.819004) + 1000(.73125) \\ &= \$1595.28 \end{aligned}$$

If someone offers to pay more than this, you should go ahead and sell bond E to them. You know that your firm can effectively create bond E. (500 + 1 pd \$1 pure discount bond + 500 + 2 pd \$1 pure discount bond + 1000 + 3 pd \$1 pure discount bond) or

$$\begin{aligned} & 500 \cdot [(-.00112)A + (-.001)B + (.01232)C] \\ & \quad + 500 \cdot [(.00012)A + (.001)B + (-.00132)C] \\ & \quad + 1000 \cdot [(.001)A + 0(B) + (-.001)C] \\ & = (.5)A + (0)B + (4.5)C \end{aligned}$$

I.e., If you take 1/2 of bond A and 4 1/2 of bond B, you get bond E. This costs 1/2(977.18) + 4.5(245.93) = \$1595.28 to create.