Mark-to-Market Accounting and Liquidity Pricing*

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Abstract

When liquidity plays an important role as in times of financial crisis, asset prices in some markets may reflect the amount of liquidity available in the market rather than the future earning power of the asset. Mark-to-market accounting is not a desirable way to assess the solvency of a financial institution in such circumstances. We show that a shock in the insurance sector can cause the current value of banks' assets to be less than the current value of their liabilities so the banks are insolvent. In contrast, if historic cost accounting is used, banks are allowed to continue and can meet all their future liabilities. Mark-to-market accounting can thus lead to contagion where none would occur with historic cost accounting.

JEL Codes: G21, G22, M41.

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1 Introduction

In recent years there has been a considerable debate on the advantages and disadvantages of moving towards a full mark-to-market accounting system for financial institutions such as banks and insurance companies. This debate has been triggered by the move of the International Accounting Standards Board (IASB) and the US Financial Accounting Standards Board (FASB) to make changes in this direction as part of an attempt to globalize accounting standards (Hansen 2004). There are two sides to the controversy in the debate. Proponents of mark-to-market accounting argue that this accounting method reflects the true (and relevant) value of the balance sheets of financial institutions. This in turn should allow investors and policy makers to better assess their risk profile and undertake more timely market discipline and corrective actions. In contrast, opponents claim that mark-to-market accounting leads to excessive and artificial volatility. As a consequence, the value of the balance sheets of financial institutions would be driven by short-term fluctuations of the market that do not reflect the value of the fundamentals and the value at maturity of assets and liabilities.

In this paper we argue that adopting a mark-to-market system to value the assets of financial institutions may not be beneficial when financial markets are illiquid. In times of financial crisis the interaction of institutions and markets can lead to situations where prices in illiquid markets do not reflect future payoffs but rather reflect the amount of cash available to buyers in the market. The level of liquidity in such markets is endogenously determined and there is liquidity pricing. When historic cost accounting is in use, this problem does not compromise the solvency of banks as it does not affect the accounting value of their assets. In contrast, when mark-to-market accounting is used, the volatility of asset prices directly affects the value of banks' assets. This can lead to contagion and force banks into insolvency even though they would be fully able to cover their commitments if they were allowed to continue until the assets mature.

The potential problems that might have arisen had Long Term Capital Management (LTCM) been allowed to go bankrupt illustrate the issue. The Federal Reserve Bank of New York justified its action of facilitating a private sector bailout of LTCM by arguing that if the fund had been liquidated many prices in illiquid markets would have fallen and this would have caused further liquidations and so on in a downward spiral. The point of our paper is to argue that using mark-to-market accounting significantly exacerbates the

problem of contagion in such circumstances compared to the use of historic cost accounting. The notion that market prices cannot be trusted to value assets in times of crisis has a long history. In his influential book, *Lombard Street,* on how central banks should respond to crises, Bagehot (1873) argued that collateral should be valued weighting panic and pre-panic prices. Our conclusion is similar in that in times of crisis market prices are not accurate measures of value.

To better understand the role of different accounting methods during crises, we present a model with a banking sector and an insurance sector based on Allen and Gale (2005a) and Allen and Carletti (2006). Banks obtain funds from depositors who can be early or late consumers in the usual way. The distinguishing feature of banks is that they have expertise in making risky loans to firms. They can invest in long and short term financial assets as well. They use the returns of the short asset to satisfy the claims of depositors withdrawing early and the returns from the loans and long asset to pay the late consumers. We focus on the case where the banks are always solvent despite the risk of their loans. The insurance companies insure a second group of firms against the possibility of their machines being damaged the following period. They collect premiums and invest them in the short asset to fund the costs of repairing the firms' machines.

In this framework there are three main elements that are necessary for contagion to occur.

- There must be a source of **systemic risk**. We show how such risk can arise optimally in the insurance sector.
- The banking and insurance sectors must both hold a long asset that can be liquidated in the market so there is the possibility of **contagion**. In our model credit risk transfer can induce the insurance companies to hold the long asset as well as the banks.
- **Liquidity pricing** of the long asset can interact with mark-to-market accounting rules to produce contagion even though with historic cost accounting there would be none.

We start by considering the operation of the banking and insurance industries separately. Conditions are identified where it is optimal for the insurance companies to insure firms when only a limited number of machines

are damaged, and go bankrupt when a large number of machines are damaged. This partial insurance is optimal if the probability of a large amount of damage is small and the return on the long asset is high so the opportunity cost of investing in the short asset is also high. The failure of insurance companies does not involve deadweight costs and does not spill over to the banking sector because the two sectors have only the short asset in common. The insurance sector though is a potential source of systemic risk in the economy.

In order for there to be contagion to the banking sector, it is necessary that both sectors hold the long asset. The insurance sector only needs to hold the short asset to pool the risk for the firms whose machines may be damaged. However, if credit risk transfer is introduced to allow the banking and insurance sectors to diversify risk, insurance companies may find it optimal to hold the long asset. This provides the potential for contagion of systemic risk from the insurance sector to the banking sector.

When insurance companies hold the long asset they must liquidate it when they go bankrupt. The market they sell the asset on will involve liquidity pricing. In order to induce some market participants to hold liquidity to purchase assets, there must be states in which asset prices are "low" so the participants can make a profit and cover the opportunity cost of holding the short asset in the other states. The low prices are determined by the endogenous amount of liquidity in the market rather than the future earning power of the asset. If historic cost accounting is in use so assets are valued at their historic cost, the low liquidation prices do not lead to contagion. Banks are not affected by the low prices. They remain solvent and can continue operating until their assets mature. In this case the credit risk transfer improves welfare. The insurance companies hold the more profitable long asset and there is no unnecessary and costly contagion when they go bankrupt.

In contrast, when mark-to-market accounting is in use so assets are priced according to market values, low prices can cause a problem of contagion from the insurance sector to the banking sector. Even if banks would be solvent if they were allowed to continue, the current market value of their assets is lower than the value of their liabilities. Banks are then declared insolvent and forced to sell their long term assets. This worsens the illiquidity problem in the market and reduces prices even further. The overall effect of this contagion is to lower welfare compared to what would happen with historic cost accounting.

This result has important implications for the debate on the optimal accounting system. In particular, it stresses the potential problems arising from the use of mark-to-market for securities traded in markets with scarce liquidity. In this sense, the accounting-induced contagion that we describe could emerge in the context of many financial institutions and markets and our results should be interpreted as one example of the phenomenon.

Our paper is related to a number of others. Plantin, Sapra, and Shin (2004) show that, while a historic cost regime can lead to some inefficiencies, mark-to-market pricing can lead to increased price volatility and suboptimal real decisions due to feedback effects. Their analysis suggests the problems with mark-to-market accounting are particularly severe when claims are long-lived, illiquid, and senior. The assets of banks and insurance companies are particularly characterized by these traits. This provides an explanation of why banks and insurance companies have been so vocal against the move to mark-to-market accounting. In the current paper an additional reason for banks and insurance companies to be disturbed by mark-to-market accounting is provided. Mark-to-market accounting can induce contagion where historic cost accounting would not.

Other papers analyze the implications of mark-to-market accounting from a variety of perspectives. O'Hara (1993) focuses on the effects of market value accounting on loan maturity, and finds that this accounting system increases the interest rates for long-maturity loans, thus inducing a shift to shorter-term loans. In turn this reduces the liquidity creation function of banks and exposes borrowers to "excessive" liquidation. In a similar vein, Burkhardt and Strausz (2006) suggest that market value accounting reduces asymmetric information, thus increasing liquidity and intensifying risk-shifting problems. Finally, Freixas and Tsomocos (2004) show that market value accounting worsens the role of banks as institutions smoothing intertemporal shocks. Differently, our paper focuses on liquidity pricing to show how market value accounting can lead to contagion and so may not be desirable.

Allen and Carletti (2006) analyze how financial innovation can create contagion across sectors and lower welfare relative to the autarky solution. However, while Allen and Carletti (2006) focus on the structure of liquidity shocks hitting the banking sector as the main mechanism generating contagion, we focus here on the impact of different accounting methods and show that mark-to-market accounting can lead to contagion in situations where historic cost accounting does not.

The rest of the paper proceeds as follows. Section 2 develops a model

with a banking and an insurance sector. Section 3 considers the autarkic equilibrium where the sectors operate in isolation. Conditions are identified for systemic risk to arise in the insurance sector. Section 4 analyzes the functioning of credit risk transfer and the circumstances in which it can induce insurance companies to hold the long asset. Section 5 considers the interaction of liquidity pricing and accounting rules. In particular, it is shown that mark-to-market accounting can result in contagion even though with historic cost accounting there would be none. An example is presented in Section 6 to show that all the conditions derived in the previous sections can be simultaneously satisfied. Finally, Section 7 contains concluding remarks.

2 The model

The model is based on the analyses of crises and systemic risk in Allen and Gale (1998, 2000, 2004a-b, 2005b) and Gale (2003, 2004), and particularly in Allen and Gale (2005a) and Allen and Carletti (2006). A standard model of intermediation is extended by adding an insurance sector. The two sectors face risks that are not perfectly correlated so there is scope for diversification.

There are three dates $t = 0, 1, 2$ and a single, all-purpose good that can be used for consumption or investment at each date. The banking and insurance sectors consist of a large number of competitive institutions and their lines of business do not overlap. This is a necessary assumption, since the combination of intermediation and insurance activities in a single financial institution would eliminate the need for markets and the feasibility of mark-to-market accounting.

There are two securities, one *short* and one *long*. The short security is represented by a storage technology: one unit at date t produces one unit at date $t+1$. The long security is a simple constant-returns-to-scale investment technology that takes two periods to mature: one unit invested in the long security at date 0 produces $R > 1$ units of the good at date 2. We can think of these securities as being bonds or any other investment that is common to both banks and insurance companies. Initially we assume there is no market for liquidating the long asset at date 1.

In addition to these securities, banks and insurance companies have distinct direct investment opportunities and different liabilities. Banks can make loans to firms. Each firm borrows one unit at date 0 and invests in a risky venture that produces B units of the good at date 2 with probability β and

0 with probability $1-\beta$. There is assumed to be a limited number of such firms with total demand for loans equal to $\overline{z}$, so that they take all the surplus and give banks a repayment b ($\leq B$), as we describe more fully below. We assume throughout that there is no market for liquidating loans at date 1.

Banks raise funds from depositors, who have an endowment of one unit of the good at date 0 and none at dates 1 and 2. Depositors are uncertain about their preferences: with probability λ they are *early consumers*, who only value the good at date 1, and with probability $1-\lambda$ they are *late consumers*, who only value the good at date 2. Uncertainty about time preferences generates a preference for liquidity and a role for the intermediary as a provider of liquidity insurance. The utility of consumption is represented by a utility function $U(c)$ with the usual properties. We normalize the number of depositors to one. Since banks compete to raise deposits, they choose the contracts they offer to maximize depositors' expected utility. If they failed to do so, another bank could step in and offer a better contract to attract away all their customers.

Insurance companies sell insurance to a large number of firms, whose measure is also normalized to one. Each firm has an endowment of one unit at date 0 and owns a machine that produces A units of the good at date 2. With probability α state H is realized and a proportion α_H of machines suffers some damage at date 1. Unless repaired at a cost of $\eta < A$, they become worthless and produce nothing at date 2. With probability $1-\alpha$ state L is realized and a proportion α_L of machines suffer some damage and need to be repaired. Thus, there is aggregate risk in the insurance sector in that the fraction of machines damaged at date 1 is stochastic. Firms cannot borrow against the future income of the machines because they have no collateral and the income cannot be pledged. Instead they can buy insurance against the probability of incurring the damage at date 1 in exchange for a premium ϕ at date 0. The insurance companies collect the premiums and invest them at date 0 in order to pay the firms at date 1. The owners of the firms consume at date 2 and have a utility function $V(C)$ with the usual properties. Similarly to the banks, the insurance companies operate in competitive markets and thus maximize the expected utility of the owners of the firms. If they did not do this, another insurance company would enter and attract away all their customers.

Finally, we introduce a class of risk neutral investors who potentially provide capital to the banking and insurance sectors. Investors have a large (unbounded) amount of the good W_0 as endowment at date 0 and nothing at

dates 1 and 2. They provide capital to the intermediary through the contract $e = (e_0, e_1, e_2)$, where $e_0 \geq 0$ denotes an investor's supply of capital at date $t = 0$, and $e_t \geq 0$ denotes consumption at dates $t = 1, 2$. Although investors are risk neutral, we assume that their consumption must be non-negative at each date. Otherwise, they could absorb all risk and provide unlimited liquidity. The investors' utility function is then defined as

$$u(e_0, e_1, e_2) = \rho W_0 - \rho e_0 + e_1 + e_2,$$

where the constant ρ is the investors' opportunity cost of funds. This can represent their time preference or their alternative investment opportunities that are not available to the other agents in the model. We assume $\rho > R$ so that it is not worthwhile for investors to just invest in securities at date 0. This has two important implications. First, since investors have a large endowment at date 0 and the capital market is competitive, there will be an excess supply of capital and they will just earn their opportunity cost. Second, the fact that investors have no endowment (and non-negative consumption) at dates 1 and 2 implies that their capital must be converted into assets in order to provide risk sharing at dates 1 and 2.

All uncertainty is resolved at the beginning of date 1. Banks discover whether loans will pay off or not at date 2. Depositors learn whether they are early or late consumers. Insurance companies learn which firms have damaged assets.

3 The autarkic equilibrium

The purpose of this section is to illustrate how the sectors work in isolation. We use this as a benchmark for considering the interaction between liquidity pricing and accounting methods. The first case considered is when the banking sector and the insurance sector are autarkic and operate separately. It is initially assumed that there are no markets so that the long asset and the loans cannot be liquidated for a positive amount at date 1. Hence if a bank or insurance company goes bankrupt at date 1, the proceeds from the long asset and the loans are 0.

3.1 The banking sector

Since all banks are ex ante identical and compete to attract deposits, they maximize the expected utility of depositors. At date 0 banks have 1 unit of deposits and choose the amount of capital e_0 to raise from investors. Then they decide how to split the $1 + e_0$ between x units of the short asset, y units of the long asset and z of loans. Also, banks choose how much to compensate investors for their capital. Since investors are indifferent between consumption at date 1 and date 2, it is optimal to set $e_1 = 0$, invest any capital e_0 that is contributed in the long asset or loans, which have higher returns than the short asset, and make a payout e_2 to investors when loans are successful. Given this, banks' solve the following problem:

$$\text{Max } EU = \lambda U(c_1) + (1-\lambda)[\beta U(c_{2H}) + (1-\beta)U(c_{2L})] \tag{1}$$

subject to

$$c_1 = \frac{x}{\lambda}, \tag{2}$$

$$c_{2H} = \frac{yR + zb - e_2}{1-\lambda}, \tag{3}$$

$$c_{2L} = \frac{yR}{1-\lambda}, \tag{4}$$

$$x + y + z = 1 + e_0, \tag{5}$$

$$e_0\rho = \beta e_2, \tag{6}$$

$$c_1 \leq c_{2L}. \tag{7}$$

The banks' maximization problem can be explained as follows. Each bank has 1 unit of depositors with λ of them becoming early consumers and $1-\lambda$ late consumers. The first term in the objective function represents the utility $U(c_1)$ of the λ early consumers. The bank uses the entire proceeds of the short term asset to provide each of them with a level of consumption c_1 as in (2). The second term represents the $1-\lambda$ depositors who become late consumers. With probability β loans pay off B, banks receive the repayment b and have to pay e_2 to investors so that each late consumer receives consumption c_{2H} as in (3). With probability $1-\beta$ the loans pay off 0. The bank has only the return from the long asset and each late consumer gets c_{2L} as in (4). The constraint (5) is the budget constraint at date 0, while the constraint (6) is investors' participation constraint. Investors must receive an expected

payoff which makes them break even. As already mentioned, it is optimal to give them a repayment only when loans pay B and banks obtain b (which occurs with probability β) so that depositors have their lowest marginal utility of consumption. The inequality (7) is the incentive compatibility constraint. Since depositor type is unobservable there will be a run on the bank with all depositors withdrawing at date 1 if it is not satisfied.

Substituting the constraints (2)-(6) into the objective function (1), and noting that $y = 1 + e_0 - x - z$ from (5), we can reduce the number of decision variables to x, z and e_0. The amount of funds z banks invest in loans results from the equilibrium in the loan market. Given that there is a limited number of firms who want loans relative to banks, the firms obtain the surplus. This means that loans are priced so that banks are indifferent between providing loans and not providing them. Since bank capital is the marginal source of finance, the amount paid on loans in equilibrium just covers the marginal cost of equity capital and is then equal to $b = \rho/\beta < B$. At this price, banks satisfy firms' total demand for loans so that $z = \overline{z}$.

The banks' problem then reduces to choosing x and e_0 to solve the following problem:

$$\begin{aligned} \text{Max } EU &= \lambda U(\frac{x}{\lambda}) + (1-\lambda)[\beta U(\frac{(1+e_0-x-z)R + (z-e_0)(\rho/\beta)}{(1-\lambda)}) \\ &\quad +(1-\beta)U(\frac{(1+e_0-x-z)R}{(1-\lambda)})] \end{aligned}$$

subject to (7).

The solution depends on whether the constraint (7) binds or not. If it does not bind (that is, if $c_1 \leq c_{2L}$), then the first order conditions are

$$\frac{\partial EU}{\partial x} = U'(c_{1L}) - R[\beta U'(c_{2H}) + (1-\beta)U'(c_{2L})] = 0,$$

$$\frac{\partial EU}{\partial e_0} = \beta(R - \rho/\beta)U'(c_{2H}) + (1-\beta)RU'(c_{2L}) = 0, \tag{8}$$

where c_1, c_{2H} and c_{2L} are as in (2), (3) and (4), respectively.

If (7) does bind, then the bank invests an amount $x = \lambda y R/(1-\lambda)$ in the short asset such that $c_1 = c_{2L}$, while as before it chooses the amount e_0 of capital that satisfies (8).

One important issue concerns the role that capital is playing in the banking sector. Since the suppliers of capital are risk neutral they provide risk

smoothing to the depositors in the bank. The assets their capital provides pay off when the loans do not and they only receive a payment when the loans pay off. The reason that the providers of capital do not bear all the risk is that capital is costly. In other words their opportunity cost of capital is higher than the return on the long asset. If it was the same, there would be full risk sharing and depositors would consume the same amount in every state.

3.2 The insurance sector

We consider the insurance sector in isolation next. As already explained, insurance companies offer insurance to firms against the possibility that their machines are damaged at date 1 and need to be repaired at a cost η. Similarly to the banking industry, the insurance sector is competitive. Companies maximize the expected utility of the owners of the firms they insure and do not earn any profits. The insurance contract can consist of partial or full insurance. In the case of partial insurance, companies insure firms in state H and go bankrupt in state L. In the case of full insurance, firms are insured in both states and insurance companies never fail. Which contract is optimal depends on the opportunity cost of providing full insurance relative to the cost incurred in the case of bankruptcy. When the first dominates, providing partial insurance is optimal and the insurance sector is subject to systemic risk.

We start with the case of partial insurance. Companies charge a premium ϕ_p at date 0 and invest it in the short asset to have liquidity to satisfy the claims $\alpha_H \eta$ at date 1. Given the insurance sector is competitive, the companies maximize the expected utility of the owners of the firms they insure and set the premium $\phi_p = \alpha_H \eta$. Thus, firms' owners have an expected utility given by

$$EV_p = \alpha V(C_{2H}) + (1-\alpha)V(C_{2L})$$

where

$$C_{2H} = A + (1-\phi_p)R, \tag{9}$$

$$C_{2L} = \phi_p + (1-\phi_p)R. \tag{10}$$

Firms pay ϕ_p and, since there is no market for liquidating the long asset at date 1 and their owners consume only at date 2, they find it optimal to invest the remaining $1-\phi_p$ directly in the long asset and obtain the return

$(1-\phi_p)R$ in both states. Then in state H (which occurs with probability α) all damaged assets are repaired and the owners of the firms can consume the additional return A. In state L the insurance companies cannot satisfy all claims $\alpha_L\eta$ and go bankrupt. Their assets are distributed equally among the claimants so that each firm receives ϕ_p.

One way to avoid bankruptcy in state L is for the insurance companies to provide full insurance and repair the damaged assets in both states H and L. To do this, the insurance companies charge a premium $\phi_f = \alpha_L\eta \leq 1$ at date 0 and invest it in the short asset. Firms' expected utility now equals

$$EV_f = \alpha V(C_{2H}) + (1-\alpha)V(C_{2L})$$

where

$$C_{2H} = A + (1-\phi_f)R + (\phi_f - \alpha_H\eta), \tag{11}$$

$$C_{2L} = A + (1-\phi_f)R. \tag{12}$$

Differently from before, firms' owners can consume the return A from the assets at date 2 in both states and the return R from investing their remaining $(1-\phi_f)$ funds in the long asset. In state H the insurance companies use $\alpha_H\eta$ to meet their claims and, given they operate in a competitive industry, distribute the remaining $\phi_f - \alpha_H\eta$ funds to the firms. In state L they receive claims $\alpha_L\eta$ and use all their funds to satisfy them so that nothing is distributed to the firms.

The optimal insurance scheme maximizes the expected utility of the firms' owners. Thus, partial insurance is optimal if $EV_p \geq EV_f$, which can be expressed as

$$\begin{aligned} & \alpha V\left(A + (1-\alpha_H\eta)R\right) + (1-\alpha)V\left(\alpha_H\eta + (1-\alpha_H\eta)R\right) \\ \geq\ & \alpha V\left(A + (1-\alpha_H\eta)R - (\alpha_L - \alpha_H)\eta(R-1)\right) + \\ & (1-\alpha)V\left(\alpha_H\eta + (1-\alpha_H\eta)R + A - \alpha_H\eta - (\alpha_L - \alpha_H)\eta R\right). \end{aligned} \tag{13}$$

Despite avoiding bankruptcy, full insurance may not be optimal. Insuring firms in both states requires the insurance companies to charge a higher premium ($\phi_f > \phi_p$). Thus providing full insurance implies a cost in terms of foregone return on the more profitable long asset held by the firms. When this cost is too high, providing full insurance is not optimal. With these considerations in mind, it is straightforward to see that the inequality (13) is more likely to be satisfied

- the higher is the probability α of the good state H,
- the smaller is the return of the asset A,
- the larger is the return of the long asset R, and
- the larger is the difference in the proportion of damaged assets $\alpha_H - \alpha_L$.

As a final remark note that there is no role for capital in the insurance sector so that $E_0 = 0$. The reason is that capital providers charge a premium to cover their opportunity cost ρ. Insurance companies should invest the capital provided by investors in the short asset since it is not optimal to hold any of the long asset. There are already potentially enough funds from customers to hold more of the short asset but it is not worth it. If there is a premium to be paid for the capital it is even less worth it. Capital will not be used in the insurance industry unless companies are regulated to do so.

In what follows we assume that partial insurance is optimal so that (13) is satisfied and also that the expected utility from partial insurance is greater than self-insurance and other partial strategies. This assumption ensures that there is systemic risk in the insurance sector.

4 The functioning of credit risk transfer

In the previous sections we have considered how the banking and insurance sectors operate in isolation. We have shown that the insurance sector is subject to systemic risk when partial insurance is optimal and the insurance companies go bankrupt in state L. Importantly, since the insurance companies only invest in the short asset, their failure does not affect the banking sector and banks remain solvent in all states. This may not be the case, however, if there are connections between the two sectors. For example, if banks and insurance companies hold some common assets and these assets can be liquidated at date 1, then the failure of the insurance companies could potentially propagate to the banking sector. To see when this can happen, we modify our framework in two directions. First, we consider credit risk transfer as an example of what can induce the insurance companies to invest (at least partly) in the long asset. Second, we introduce a market for liquidating the long asset at date 1. For the moment, we just assume that the long asset can be sold at a price $P \leq 1$, which depends on the state of

the world. In the next section we focus on the determination of the market price and study the interrelation between asset prices, accounting systems and contagion.

Given that the shocks affecting the two sectors are independent, we have four states of the world depending on the realizations of the variables β and α, which we can express as HH, HL, LH, and LL. The (per-capita) payoffs in each state are as follows.

Table 1

State	Probability	Bank loans	Insurance claims	Late depositors	Firms' owners
HH	$\beta \times \alpha$	B	$\alpha_H \eta$	c_{2H}	C_{2H}
HL	$\beta \times (1-\alpha)$	B	$\alpha_L \eta$	c_{2H}	C_{2L}
LH	$(1-\beta) \times \alpha$	0	$\alpha_H \eta$	c_{2L}	C_{2H}
LL	$(1-\beta) \times (1-\alpha)$	0	$\alpha_L \eta$	c_{2L}	C_{2L}

Credit risk transfer can be seen as a way to provide risk sharing between the two sectors. As Table 1 shows, late depositors have different payoffs in states HH and HL compared to states LH, and LL, and the owners of the firms also have different payoffs in states HH and LH as compared to HL and LL. This introduces the potential for risk sharing as a way to increase welfare. We consider a particularly simple form of risk transfer: the banks make a payment Z_{HL} to the insurance companies in state HL when bank loans pay off but insurance claims are high, while the insurance companies make a payment Z_{LH} to the banks in state LH when bank loans do not pay off and insurance claims are low. For simplicity, we assume that the banks' depositors obtain the surplus from the credit risk transfer. The insurance companies will compete to provide the credit risk transfer that maximizes the utility of the banks' depositors at the lowest cost to themselves. In equilibrium they will obtain their reservation utility, which is what they would receive in autarky. This credit risk transfer improves diversification, but notice that markets are still not complete.

The question is how such transfers can be implemented and what are their effects on welfare. In state HL bank loans are successful. Banks have excess funds and use them to transfer Z_{HL} to the insurance companies. Thus, the only difference relative to the autarky situation is that at date 2 in states HL and LH depositors now consume

$$c_{2HL} = \frac{yR + zb - e_2 - Z_{HL}}{1-\lambda}, \tag{14}$$

$$c_{2LH} = \frac{yR + Z_{LH}}{1-\lambda}. \tag{15}$$

The problem is more complicated for the insurance companies. In state LH the owners of the firms that insure their machines with the insurance companies have plenty of funds (equal to $A + (1-\phi_p)R$), but the insurance companies themselves do not have any. They receive $\alpha_H \eta$ in claims and use all the returns of the short asset to repair the damaged assets. In order for them to be able to make the payment Z_{LH} at date 2 to the banks they must hold extra assets. They must charge a higher premium to the firms initially and reduce the part of the endowment firms hold in long assets.

The insurance companies must then decide in which security, short or long, to invest this extra amount to be able to pay Z_{LH}. If they invest in the short asset, they need to make an initial investment $s = Z_{LH}$ to be able to make the transfer to the banks. The insurance companies can then offer to the owners of the firms an expected utility equal to

$$\begin{aligned} EV_s &= \beta\alpha V(C_{2HH}) + \beta(1-\alpha)V(C_{2HL}) + (1-\beta)\alpha V(C_{2LH}) \\ &\quad +(1-\beta)(1-\alpha)V(C_{2LL}). \end{aligned} \tag{16}$$

where

$$\begin{aligned} C_{2HH} &= A + s + (1-\phi_p - s)R, \\ C_{2HL} &= \phi_p + s + Z_{HL} + (1-\phi_p - s)R, \\ C_{2LH} &= A + s - Z_{LH} + (1-\phi_p - s)R, \\ C_{2LL} &= \phi_p + s + (1-\phi_p - s)R). \end{aligned}$$

The different terms relative to the autarkic case can be understood as follows. The insurance companies receive an initial premium $\phi_p + s$ from the firms and invest it in the short asset; and the firms invest the remaining $(1-\phi_p-s)$ in the long asset for a return $(1-\phi_p-s)R$ in each state. Additionally, in state HH (which occurs with probability $\beta\alpha$), the owners of the firms enjoy the return A of the machines and the amount s the insurance companies distribute to them. Differently, in state HL (having a probability of $\beta(1-\alpha)$) the machines are not repaired and, in addition to the return from their own investments, the owners of the firms consume what the insurance companies distribute, $\phi_p + s$ and the transfer Z_{HL} they receive from the banks. The two remaining states, LH and LL, are similar with the only difference that the insurance companies use s to make the transfer Z_{LH} to the banks in state LH and do not receive any transfer in state LL.

Things work slightly differently if the insurance companies finance the transfer Z_{LH} by investing in the long asset. In this case, they charge an extra premium ℓ such that $\ell R = Z_{LH}$ and the expected utility of the owners of the firms becomes

$$EV_\ell = \beta\alpha V(C_{2HH}) + \beta(1-\alpha)V(C_{2HL}) + (1-\beta)\alpha V(C_{2LH}) + (1-\beta)(1-\alpha)V(C_{2LL})$$

where

$$C_{2HH} = A + \ell R + (1 - \phi_p - \ell)R,$$
$$C_{2HL} = \phi_p + P_{HL}\ell + (1 - \phi_p - \ell)R + Z_{HL},$$
$$C_{2LH} = A + \ell R - Z_{LH} + (1 - \phi_p - \ell)R,$$
$$C_{2LL} = \phi_p + P_{LL}\ell + (1 - \phi_p - \ell)R.$$

The terms have a similar interpretation to the case when the insurance companies finance the transfer Z_{LH} by investing in the short asset. The only difference is that now the insurance companies obtain the return R in states HH and LH on the extra premium ℓ and liquidate it for a price P_{HL} in state HL and P_{LL} in state LL. Also the owners of the firms make an initial investment of $(1 - \phi_p - \ell)$ in the long asset instead of $(1 - \phi_p - s)$.

There is then a trade-off in the implementation of the credit risk transfer for the insurance companies if P_{HL} and P_{LL} are lower than 1 (as we show in the next section). On the one hand, financing Z_{LH} with the long asset avoids the opportunity cost $s(R-1)$ that the insurance companies suffer in each state when they invest s in the short asset. On the other hand, however, investing in the long asset induces a loss when the insurance companies go bankrupt in states HL and LL and have to liquidate the long asset. Depending on which of these effects dominate, the insurance companies decide how to finance the transfer Z_{LH}. Formally, the insurance companies choose to charge an extra premium ℓ and invest it in the long asset if

$$\left.\frac{\partial EV_\ell}{\partial \ell}\right|_{\ell=0} \geq Max\left[\left.\frac{\partial EV_s}{\partial s}\right|_{s=0}, 0\right]. \tag{17}$$

In order to make this comparison we assume that the banks and insurance companies make the same transfer in expectation, that is such that

$$\beta(1-\alpha)Z_{HL} = (1-\beta)\alpha Z_{LH}. \tag{18}$$

Using this we can express $Z_{HL} = \frac{(1-\beta)\alpha}{\beta(1-\alpha)}\ell R$ and $Z_{HL} = \frac{(1-\beta)\alpha}{\beta(1-\alpha)}s$ when the insurance companies finance Z_{LH} with the long and the short asset, respectively, and show that

$$\begin{aligned}\left.\frac{\partial EV_\ell}{\partial \ell}\right|_{\ell=0} &= R[(1-\beta)\alpha[V'(\phi_p+(1-\phi_p)R) - V'(A+(1-\phi_p)R)] \\ &\quad +[\beta(1-\alpha)\frac{P_{HL}}{R} + (1-\beta)(1-\alpha)\frac{P_{LL}}{R} - (1-\alpha)]V'(\phi_p+(1-\phi_p)R)],\end{aligned}$$

$$\begin{aligned}\left.\frac{\partial EV_s}{\partial s}\right|_{s=0} &= (1-\beta)\alpha\left[V'(\phi_p+(1-\phi_p)R) - RV'(A+(1-\phi_p)R))\right] \\ &\quad -(R-1)\left[(1-\alpha)V'(\phi_p+(1-\phi_p)R) + \beta\alpha V'(A+(1-\phi_p)R)\right].\end{aligned}$$

To gain some insight into the circumstances where credit risk transfer will be used and when the insurance company will fund its claim with the short or long asset, we consider three special cases.

Case 1: $R = 1, P_{HL} = P_{LL} = 0$

In this case the long asset has no return advantage over the short asset. It has the disadvantage that nothing is received when it is liquidated as would occur, for example, if there was no market for the long asset. Now

$$\left.\frac{\partial EV_s}{\partial s}\right|_{s=0} = (1-\beta)\alpha[V'(1) - V'(A+1-\phi_p)] > 0,$$

since $A > \phi_p$, and

$$\begin{aligned}\left.\frac{\partial EV_\ell}{\partial \ell}\right|_{\ell=0} &= (1-\beta)\alpha[V'(1) - V'(A+1-\phi_p)] - (1-\alpha)V'(1) \\ &< \left.\frac{\partial EV_s}{\partial s}\right|_{s=0}.\end{aligned}$$

There will be credit risk transfer in this case and the insurance company will fund its payment with the short asset.

Case 2: $R = 1, P_{HL} = P_{LL} = 1$

Here the long asset again has no return advantage and in this case it has no liquidation disadvantage either. We obtain

$$\left.\frac{\partial EV_s}{\partial s}\right|_{s=0} = \left.\frac{\partial EV_\ell}{\partial \ell}\right|_{\ell=0} = (1-\beta)\alpha[V'(1) - V'(A+1-\phi_p)] > 0.$$

Not surprisingly credit risk transfer is beneficial and the assets are equally good at funding the insurance companies' payment.

Case 3: $R = V'(\phi_p + (1-\phi_p)R)/V'(A + (1-\phi_p)R) > 1, P_{HL} = P_{LL} = 1$

Now the long asset is at an advantage because of its higher return and it can also be liquidated. Here

$$\left.\frac{\partial EV_s}{\partial s}\right|_{s=0} = -(R-1)\left[(1-\alpha)V'(\phi_p + (1-\phi_p)R) + \beta\alpha V'(A + (1-\phi_p)R)\right] < 0,$$

so the short asset will not be used. For the long asset

$$\left.\frac{\partial EV_\ell}{\partial \ell}\right|_{\ell=0} = V'(\phi_p + (1-\phi_p)R)[(1-\beta)\alpha(R-1) + 1 - (1-\alpha)R].$$

For sufficiently large α and sufficiently small β this will be positive so it will be optimal to have credit risk transfer and the insurance companies will fund their payment with the long asset.

Thus the possibility of sharing risk between the sectors can lead the insurance company to hold the long asset even though on its own it has no need for it. We will assume that these conditions hold in what follows.

5 Liquidity pricing and accounting

In the previous sections we have analyzed the conditions where insurance companies find it optimal to offer partial insurance to the firms they insure and where credit risk transfer induces them to invest in the long asset. These elements constitute two of the important ingredients for contagion from the insurance sector to the banking sector through the market for the long asset. In this section we analyze whether the failure of the insurance companies can propagate to the banks. We show that historic cost accounting and mark-to-market accounting lead to very different outcomes.

The presence of a market for the long asset at date 1 raises the issue that somebody must supply liquidity to this market. In other words somebody must hold the short asset in order to have the funds to purchase the long asset supplied to the market in states HL and LL. If nobody held liquidity, then there would be nobody to buy and the price of the long asset would fall to zero at date 1. This can't be an equilibrium though because by holding a very small amount of the short asset somebody would be able to enter

and make a large profit. In the framework considered in this paper, the group that will supply the liquidity is the investors who provide capital to the banks. In order to be willing to hold this liquidity they must be able to recoup their opportunity cost. Since in states HH and LH when there is no liquidation of assets, they end up holding the low-return short asset throughout, they must make a significant profit in at least one of the states HL and LL when there is a positive supply of the long term asset on the market. In other words, the price of the long asset must be low in at least one of these states, and its exact level will depend on the amount of assets supplied to the market and thus in turn on the accounting method in use.

5.1 Historic cost accounting

We start with the simpler case where asset values are recorded at cost even if there is a market and asset prices exist. This illustrates the functioning of markets and the liquidity pricing in our model.

To see precisely how prices are formed, we first need to see how many units of the long asset are offered in the market. Let us start with the banking sector. Banks invest x units in the short asset, y in the long asset and z in loans. Given all these assets cost one per unit, under historic cost accounting they are just worth $x + y + z$. Provided

$$x + y + z > c_1, \tag{19}$$

the banks' assets are above their total liabilities at date 1 and banks remain solvent and continue operating until date 2. They do not liquidate any assets at date 1.

Assuming (19) is satisfied, the price in the market for the long asset depends on the sales of the insurance companies. In states HH and LH the insurance companies do not sell their long assets and the investors will not use their liquidity to buy any assets. The equilibrium price must then be $P_{HH} = P_{LH} = R$. The reason for this is straightforward. If $P < R$, the investors would want to buy the long asset since it would provide a higher return than the short asset between dates 1 and 2. In contrast, if $P > R$, the banks and insurance companies would sell the long asset and then hold the short asset until date 2. The only price at which both the short and the long asset will be held between dates 1 and 2, which is necessary for equilibrium in states HH and LH, is R.

In contrast, in states HL and LL the insurance companies go bankrupt and will liquidate their holdings of the long asset ℓ at a price $P_{HL} = P_{LL} = P_L$. In order for investors to supply liquidity to the market, the price P_L must be low enough to allow them to cover their opportunity cost of ρ. In equilibrium it must be the case that

$$\rho = \alpha \times 1 + (1 - \alpha) \times \frac{R}{P_L}. \tag{20}$$

The term on the left hand side is the investors' opportunity cost of capital. The first term on the right hand side is the expected payoff to holding the short asset in states HH and LH, which occur with probability α. The second term is the expected payoff from holding the short asset in states HL and LL, which occur with probability $1 - \alpha$, and using it to buy $1/P_L$ units of the long asset at date 1. Each unit of the long asset pays off R at date 2.

Solving (20) gives

$$P_L = \frac{(1 - \alpha)R}{\rho - \alpha} < 1, \tag{21}$$

since $\rho > R > 1$. As $\alpha \to 1, P_L \to 0$. The less likely is state L where the insurance companies go bankrupt, the lower the price of the long asset in that state must be. Notice that this low price is purely driven by liquidity considerations rather than the fundamentals of the asset.

Taking prices as given, the insurance companies will choose the credit risk transfer payment Z_{LH} to the banks in state LH and given our assumptions will fund it with ℓ of the long asset. The banks will choose their payment Z_{HL} to the insurance companies in state HL. In order for the market to clear at P_L in states HL and LL investors need to hold an amount of liquidity γ given by

$$\gamma = P_L \ell. \tag{22}$$

The simultaneous determination of P_L and γ is illustrated in Figure 1. As explained above, the investors' participation constraint requires that the price be given by (21). Rearranging (22) gives

$$P_L = \frac{\gamma}{\ell}.$$

This expression can be interpreted in the following way. The insurance companies are bankrupt and are forced to liquidate the long asset ℓ that they hold. The investors use their cash holdings γ to buy the long asset since

$P_L < 1 < R$. The price is the ratio of the two quantities so there is cash-in-the-market pricing. The more liquidity in the market the greater the price in states HL and LL as illustrated. The point at which this line coincides with P_L gives the market clearing amount of liquidity γ.

To sum up, when historic cost accounting is used credit risk transfer can improve welfare relative to the autarky situation. This is because credit risk transfer improves risk sharing between the two sectors and the use of historic cost accounting insulates banks' from the bankruptcy of the insurance companies. Even when P_L is quite low so that the banks would be insolvent using market prices there is no effect on their activities. This is desirable since they can fulfill all of their commitments.

5.2 Mark-to-market accounting and contagion

The crucial feature of the equilibrium with historic cost accounting is that the accounting value of the banks' assets is insensitive to the bankruptcy of the insurance companies and market prices. We now turn to the situation where mark-to-market accounting is used and analyze the mechanism through which the bankruptcy of the insurance companies can affect the accounting value of the banks' assets and lead to contagion.

The main difference compared to historic cost accounting is that the accounting value of the banks' holdings of the long asset now depends on the market price if a market exists. If no market exists, as we continue to assume for loans, the historic cost is still used. Thus, when the insurance companies sell the long asset and there is liquidity pricing, the banks' long assets are evaluated at market prices. As a result the value of assets may be below the value of liabilities at date 1. If this happens in state HL the banks will be declared bankrupt and will be forced to liquidate their assets at the low market price. In this case it will no longer be optimal for the banks to make a credit risk transfer payment to the insurance companies. We therefore focus on equilibria where there is only bankruptcy in state LL where no credit risk transfer payments are made. If P_{LL} is low enough so that

$$x + yP_{LL} + z < c_1, \tag{23}$$

the banks are declared insolvent and have to sell their long assets. The supply of the long asset on the market in state LL is then larger, as both the banks and the insurance companies need liquidity to satisfy their claims at date 2.

To see how this affects the pricing of the long asset, consider first the states, HH, HL and LH. As before, in states HH and LH neither the banks nor the insurance companies sell the long asset. In state HL the insurance companies sell the long asset while the banks do not. Differently from before, however, the equilibrium is such that there is now excess liquidity in state HL or LL as well. We focus on the case with excess liquidity in state HL. This surplus of cash means that $P_{HL} = R$ by the same argument as for P_{HH} and P_{LH} above. Thus, the price of the long asset at date 1 in these three states will be

$$P_{HH} = P_{HL} = P_{LH} = R.$$

Given this, the price P_{LL} in state LL must be such that the investors supplying liquidity to the market break even, and must satisfy

$$\rho = (1 - (1-\beta)(1-\alpha)) \times 1 + (1-\beta)(1-\alpha) \times \frac{R}{P_{LL}}. \tag{24}$$

The terms in (24) have a similar interpretation to those in (20). The left hand side is the investors' opportunity cost of capital. The first term on the right hand side is the investors' expected payoff to holding the short asset in states HH, HL and LH (which have a total probability of occurring equal to $1-(1-\beta)(1-\alpha)$). The second term is their expected payoff from using the cash in state LL (which occurs with probability $(1-\beta)(1-\alpha)$) to buy $1/P_{LL}$ units of the long asset at date 1 for a per-unit return of R at date 2. The only difference relative to (20) is that now investors hold liquidity in all states except state LL. This means that they have to make higher profits in this state to induce them to hold cash at date 0. Solving (24), we obtain

$$P_{LL} = \frac{(1-\alpha)(1-\beta)R}{\rho + \alpha\beta - \alpha - \beta} < P_L < 1.$$

Note that because there is a lower probability of the low price in state LL relative to the case with historic cost accounting, it follows that P_{LL} in (24) is lower than P_L in (21). This implies greater price volatility, in line with one of the arguments made by practitioners against marking to market. The greater volatility arises because investors hold more liquidity with historic cost accounting to absorb the assets of the bankrupt banks. This increases the price in state HL and lowers it in LL relative to historic cost accounting.

Taking prices as given, the insurance companies will choose the credit risk transfer payment Z_{LH} to the banks in state LH and will fund it with ℓ

of the long asset. The banks will choose their payment Z_{HL} to the insurance companies in state HL. In equilibrium the total supply of long asset to the market in state LL is $\ell + y$. For the the market to clear at P_{LL}, as in (24) the investors have to hold an amount γ in the short asset between dates 0 and 1 such that

$$\gamma = P_{LL}(\ell + y).$$

In order for the equilibrium described to hold, it is necessary that $\gamma = P_{LL}(\ell + y) > \ell R$ so that there is excess liquidity in state HL and $P_{HL} = R$ as explained above. If $\gamma < \ell R$ then $P_{HL} < R$ and investors make money in state HL as well as in state LL. This case can be analyzed similarly.

To sum up, differently from the case with historic cost accounting, the use of mark-to-market can generate contagion from the insurance sector to the banking sector and leads to a reduction in welfare. The investors and the insurance companies have the same levels of utility as in autarky. The banks are worse off since they go bankrupt and their assets are liquidated at a low level in state LL. However, taking prices as given the actions chosen by the insurance companies and banks are optimal. If an insurance company were not to engage in credit risk transfer it would still receive the same as in autarky. If it was to use the short asset to fund its credit risk transfer it would be strictly worse off. If a bank was to choose not to do credit risk transfer, it would still be liquidated in state LL and it would not have the benefit of the credit risk transfer. The expected utility of its depositors would fall.

The reason for the poor performance of mark-to-market accounting is that when prices are determined by liquidity rather than future payoffs they are no longer appropriate for valuing financial institutions' assets. The equilibrium prices are low to provide incentives for liquidity provision. They are not low because fundamentals are bad. This point has important implications for the debate on mark-to-market accounting versus historic accounting for financial institutions. It suggests that mark-to-market accounting can significantly increase the possibility of contagion. Historic cost accounting is superior in this respect.

5.3 Discussion

The only role of the market for the long asset at date 1 in the model is to allow the long asset to be liquidated when the insurance companies go bankrupt.

Buyers are induced to participate through low prices in some states. This is the sense in which the market is illiquid and is subject to liquidity pricing. If there were other participants in the market so that it was more liquid then the assets would be priced in a different way. For example, consider the following circumstances. In some states one group of banks has a large proportion of early consumers while the remaining group has a low proportion. In the other states the reverse is true. Overall there is no aggregate uncertainty about the proportion of early and late consumers. There is just uncertainty about which banks will have a large proportion of early consumers. As in Allen and Gale (2004b) and Allen and Carletti (2006) the banks can use the market for the long asset at date 1 to reallocate liquidity. In this case the market will be liquid. When the insurance companies go bankrupt prices will only change slightly to absorb the extra supply and this change will be insufficient to attract liquidity from the investors. Provided the market is sufficiently liquid to absorb the liquidations without large price changes the contagion effect identified in the model above will not be present.

A second important assumption of the model is that contracts are incomplete. If contracts are complete so that insurance companies' and banks' payouts can be made contingent on the state, bankruptcy can be avoided. In states HL and LL, a complete contract would allow insurance companies to provide no insurance so bankruptcy would not occur. Insurance companies would then not be forced to liquidate the long asset and there would be no contagion.

Finally, we have assumed the return on the long asset R is constant. Despite this, the price fluctuates because of liquidity pricing. If there was uncertainty in fundamentals so R was random, the problem identified would be exacerbated. The price would vary with R and this would increase volatility over and above the level with just liquidity pricing.

6 An example

In this section we present a numerical example to show that the conditions for contagion given above can be simultaneously satisfied. We assume the following values. The long asset returns $R = 1.1$, loans yield $B = 3$ with probability $\beta = 0.7$, and firms' total demand for loans is $\overline{z} = 0.3$. Depositors have utility function $U(c) = Ln(c)$ and become early consumers with probability $\lambda = 0.5$. Investors have an opportunity cost equal to $\rho = 1.15$. The

payment to banks' on their loans is $b = \rho/\beta = 1.64$ and their investment in loans in equilibrium is $z = \overline{z} = 0.3$.

Banks in autarky

Using the values of the example, we get the following solution for the bank in autarky (maximize (1) subject to (2)-(6) but ignoring (7)):

$$\begin{gathered} e_0 = 0.25; e_1 = 0; e_2 = 0.42; \\ x = 0.5; y = 0.45; z = 0.3; \\ c_1 = 1.00; c_{2H} = 1.15; c_{2L} = 1.00; \\ EU = 0.0487. \end{gathered}$$

Comparing c_1 and c_{2L} it can be seen that the constraint (7) is satisfied in this example.

The risk sharing between the depositors and the providers of capital is incomplete. The late depositors' consumption is 1.15 when the banks' loans pay off but only 1.00 when they do not. As explained in Section 3, the reason is that capital is costly. In other words the opportunity cost of capital of the providers' of capital is higher than the return on the long asset. If it was the same, there would be full risk sharing and depositors would consume the same amount in every state.

Insurance companies in autarky

To provide an example where partial insurance is optimal so that there is systemic risk in the insurance sector, we assume $A = 1.15$, $\eta = 1$, $\alpha = 0.9$, $\alpha_H = 0.5$ in state H and $\alpha_L = 1$ in state L. Finally, the utility function of the owners of the firm is $V(c) = Ln(c)$ and recall that the endowment of each firm is 1. With partial insurance we have $C_{2H} = 1.7$ and $C_{2L} = 1.05$ so that the expected utility of firms is $EV_p = 0.482$. With full insurance it is instead $C_{2H} = 1.65$ and $C_{2L} = 1.15$ so that

$$EV_f = 0.465 < EV_p = 0.482.$$

Thus despite providing higher consumption in state L full insurance is not optimal because the opportunity cost of providing it is too high. The optimal scheme is for the insurance industry to partially insure firms, charge a premium equal to $\alpha_H \eta = 0.5$ at date 0 and leave firms to invest the remaining part $1 - \alpha_H \eta = 0.5$ of their endowment in the long asset.

Credit risk transfer

We next consider credit risk transfer. Table 2 summarizes the payoffs to the banks' late depositors and the insured firms' owners in autarky.

Table 2

State	Probability	Bank loans	Insurance claims	Late depositors	Firms' owners
HH	$0.7 \times 0.9 = 0.63$	$B = 3$	$\alpha_H \phi = 0.5$	1.15	1.7
HL	$0.7 \times 0.1 = 0.07$	$B = 3$	$\alpha_L \phi = 1$	1.15	1.05
LH	$0.3 \times 0.9 = 0.27$	0	$\alpha_H \phi = 0.5$	1.00	1.7
LL	$0.3 \times 0.1 = 0.03$	0	$\alpha_L \phi = 1$	1.00	1.05

There is a market for the long asset at date 1. We initially consider what happens when there is historic cost accounting and the insurance company uses the long asset to fund its credit risk transfer. We then consider mark-to-market accounting and show that there is contagion.

Historic cost accounting

The assets of the banks are $x = 0.5; y = 0.45; z = 0.3$. If the banks' assets are evaluated at their historic cost, they are worth $x + y + z = 1.25$. This is above the total liabilities at date 1 of $c_1 = 1.00$ so the bank remains solvent irrespective of what happens to the market value of its assets.

As explained above in Section 5.1 $P_{HH} = P_{LH} = R = 1.1$. From (21)

$$P_L = \frac{(1-\alpha)R}{\rho - \alpha} = 0.44.$$

Given this value for P_L, we solve the problem under the assumption that banks retain the surplus from the credit risk transfer and the owners of the firms enjoy the same level of expected utility as in autarky. It can be shown that the optimal transfers are

$$\begin{aligned} Z_{HL} &= 0.058 \text{ in state } HL \text{ and} \\ Z_{LH} &= 0.018 \text{ in state } LH. \end{aligned}$$

Note that in doing this optimization, we keep the portfolios of the banks the same as before here and below, for ease of exposition. Strictly speaking with the transfers Z_{HL} and Z_{LH} the banks will reoptimize and have slightly different portfolios. Taking account of this change does not alter the results below.

The insurance companies find it optimal to fund their transfer with the long asset. They choose $\ell = 0.016$ to provide the necessary funds. Using (22), the amount of liquidity that the investors hold is

$$\gamma = P_L \ell = 0.007.$$

The level of utility of the banks' depositors with historic cost accounting is

$$EU^{HC} = 0.0496,$$

which is higher than the level of 0.0487 that they obtain in autarky.

The crucial feature of this equilibrium is that the accounting value of the banks' assets is insensitive to the bankruptcy of the insurance companies and market prices. The banks do not have to sell the long asset and can continue until date 2.

Mark-to-market accounting

As explained in Section 5.2, with mark-to-market accounting the form of the equilibrium is slightly different. Here

$$P_{HH} = P_{HL} = P_{LH} = R = 1.1.$$

In the remaining state LL it follows from (24) that

$$P_{LL} = \frac{(1-\alpha)(1-\beta)R}{\rho + \alpha\beta - \alpha - \beta} = 0.183.$$

Given this price, it can be shown that the optimal transfers that keep the insurance companies at their reservation level of utility and maximize the bank depositors' welfare are

$$\begin{aligned} Z_{HL} &= 0.056 \text{ in state } HL \text{ and} \\ Z_{LH} &= 0.020 \text{ in state } LH. \end{aligned}$$

The insurance companies find it optimal to fund their transfer with the long asset. They choose $\ell = 0.018$ to provide the necessary funds. In equilibrium the total supply of long asset to the market in state LL is $\ell + y = 0.018 + 0.45 = 0.468$. In order for the market to clear at $P_{LL} = 0.183$ the investors have to hold an amount γ in the short asset between dates 0 and 1 to clear the market at date 1 such that

$$\gamma = P_{LL}(\ell + y) = 0.086.$$

Since $\ell = 0.018$ we have $R\ell = 0.020 < \gamma = 0.086$ so there is excess liquidity in state HL as required for $P_{HL} = R$ above.

The low price P_{LL} provides the incentive that is needed for the investors to provide the liquidity for the market. However, it also means that the banks are forced to liquidate at date 1 in state LL. The reason is that the value of their assets is

$$x + y \times P_L + z = 0.5 + 0.45 \times 0.183 + 0.3 = 0.882,$$

and this is less than their liabilities of $c_1 = 1.00$. They therefore go bankrupt and their long assets are liquidated in the market for $0.45 \times 0.183 = 0.082$. It can then be shown that the level of utility of the banks' depositors with mark-to-market accounting is

$$EU^{MTM} = 0.0235.$$

This is clearly less than the depositors' expected utility with historic cost accounting $EU^{HC} = 0.0496$.

The example illustrates how the interaction between mark-to-market accounting and liquidity pricing can be damaging in times of crisis. There is contagion of the systemic risk that arises in the insurance sector to the banking sector. The price is low in state LL to give incentives for investors to provide liquidity to the market. It does not reflect the payoff on the asset itself. This is a constant $R = 1.1$ in all states. The banks can meet all of their commitments going forward. Nevertheless under mark-to-market accounting they are insolvent. Their premature liquidation leads to a significant loss of welfare in this example.

7 Concluding remarks

We have shown that if there is mark-to-market accounting there can be contagion which causes banks to be liquidated unnecessarily. Historic cost accounting does not suffer from this drawback.

The model presented in this paper was developed in the context of banking and insurance. It is clear that this context is not crucial for similar effects to arise. It is the interaction of incentives to provide liquidity with accounting rules that is key. This can occur in many contexts.

We have focused on one advantage of historic cost accounting compared to mark-to-market accounting. The debate is a complex one and this is

just one factor among many. If mark-to-market is adopted based on other arguments, a way of mitigating the potential for contagion is not to strictly apply this accounting methodology in times of crisis. Rather than simply declaring institutions bankrupt it may be better to wait until the episode of liquidity pricing is over.

This paper has considered the private provision of liquidity in markets and has not analyzed the role of central banks in liquidity provision. In markets with widespread participation the central bank can provide liquidity to participants and liquidity pricing will be mitigated. However, in markets with limited participation, it is likely that central banks may have problems injecting liquidity that will reach the required markets and prevent the fall in prices and contagion considered in the paper. The justification used by the Federal Reserve Bank of New York for their intervention in arranging a private sector bailout of Long Term Capital Management in 1998 explicitly used this rationale. The LTCM case was somewhat more complex than the model analyzed here as in addition to liquidity issues the future payoffs of assets were also uncertain. However, as we argued above, this uncertainty about fundamentals exacerbates the problem. Investigating the precise role of central banks in this kind of situation would be an interesting question for future research. Similarly, it would also be interesting to analyze the effects of introducing markets for other assets like loans.

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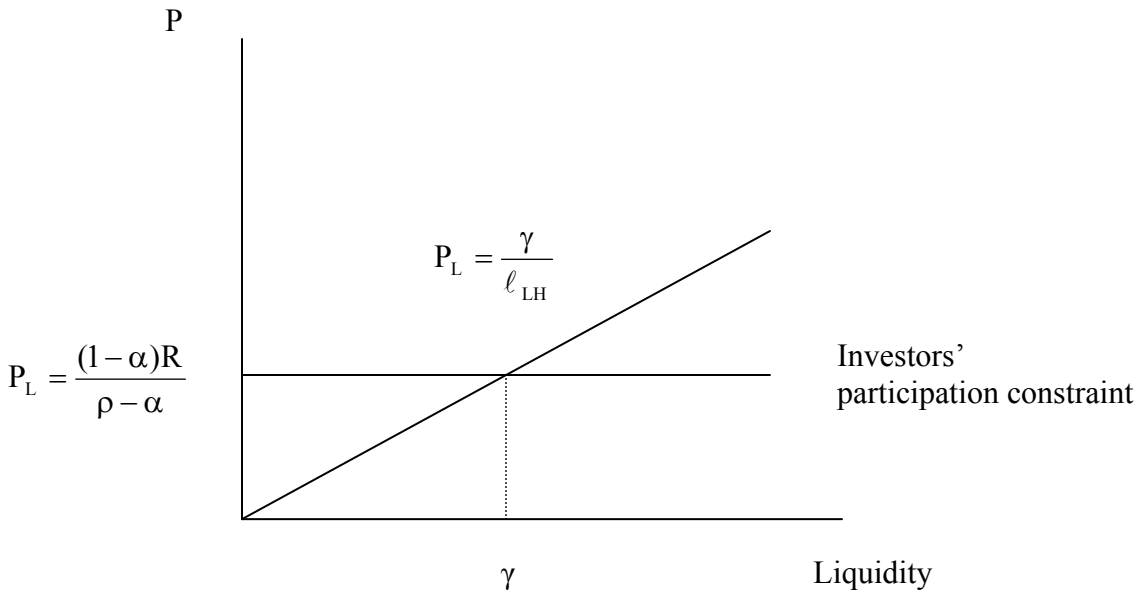

Figure 1

The determination of P_L and γ in equilibrium