Husbands Might Really Be That Cheap*

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Abstract

This paper explores the life insurance holdings from a general equilibrium perspective. Drawing on the data explored in Chambers, Schlagenhauf, and Young (2003), we calibrate an overlapping generations lifecycle economy with incomplete asset markets to match facts regarding the uncertainty of income and demographics. We then explore the implications for life insurance demand. We find that the aggregate amount of life insurance implies a relatively-small degree

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of actuarial-unfairness in the population. In addition, the peaks in holdings and participation found in the model are close to those in the data. However, this conformity masks some important puzzles. In particular, we find that the group that benefits the most from holding life insurance – poor households with large numbers of children – does not match well with the empirical facts.

Failure of the head of a family to insure his or her life against a sudden loss of economic value through death or disability amounts to gambling with the greatest of life's values; and the gamble is a particularly mean one because, in the case of loss, the dependent family, and not the gambler must suffer the consequences.

S. Huebner and K. Black, Jr., **Life Insurance**

1 Introduction

The life insurance market is one of the few contingent claim markets that is available to households. The size of this market is large. In terms of policy face values, the total size of this market in 1998 was 0.95 times annual GDP. Alternatively, in terms of expenditures LIMRA data reports $212 billion in total premiums paid during the 1998, and the BEA category "Expenses of Handling Life Insurance and Pension Plans" constitutes 1.4 percent of total consumption. Thus, the life insurance market provides an interesting and important laboratory for the examination of agents' consumption and risk sharing behavior. The general perception, perhaps a result of the marketing strategy of the life insurance firms, is that households are holding an insufficient amount of life insurance– the quote from the popular textbook by Hueber and Black insinuates that fact, as do commercials that assert how frequently a widow falls to poverty income levels as the result of the untimely death of their spouse.[1] A recent study by Bernheim *et.al.* (2003) examines life insurance holdings in light of financial vulnerability and find that the more financially vulnerable households seem to be under insured. I n this paper we examine whether households or subsets of households have proper life insurance positions using a dynamic general equilibrium model.

Our companion paper Chambers, Schlagenhauf, and Young (2003) established some facts about life insurance holdings in the data. Due to brevity, we cannot list exhaustively all of the facts contained in that paper, but we will list the ones we will be attempting to assess theoretically. First, we

[1]Interestingly, the life insurance industry seems to be aware of this pattern in life insurance. An advertising campaign that aired during the 2001 World Series claimed that the average widow who is under the age of 50 would use up her life insurance payment within nine months. Recently, Zick and Holden (2000) find evidence in the Survey of Income and Program Participation that widows face significant wealth declines upon the death of their spouse. See also Hurd and Wise (1989).

found in that paper that participation in life insurance is hump-shaped with a peak around age 52; the size of the peak is 80 percent participation. Second, we found that holdings peak around age 42 with a peak of $\$170,000$, just over 3 times annual average income. And third, our econometric methodology detected that single earner married couples have lower participation rates than dual earner families but hold approximately the same amount of coverage; these families are also very similar in terms of earnings, income, and wealth. Our model's job is to determine whether these observations constitute a puzzle with respect to economic theory.

For the convenience of the reader, we present in Tables 1 and 2 some facts from the distribution of wealth, income, and life insurance holdings that we discuss more completely in Chambers, Schlagenhauf, and Young (2003). We also present in Figures 1 and 2 the key distributions for life insurance – the participation rate and total holdings. We define total holdings here as the sum of term life insurance plus whole-life insurance minus the accumulated cash value of whole-life; this third component is the savings vehicle embedded in a whole-life policy and cannot be considered life insurance in the context of our model. For detailed definitions of earnings, income, and wealth, see our companion paper. It is sufficient to note here that we include both liquid and illiquid wealth in our definition.

In order to assess life insurance patterns from a theoretical perspective we construct a dynamic overlapping generations model. The decision making unit is the household, which enters a period with a demographic state comprised of age, sex, marital status, and the number of children. Households face idiosyncratic uncertainty in the hourly wage they command as well as in their demographic state. To insulate themselves against these shocks, agents can accumulate interest-bearing assets and life insurance policies and supply labor to the market. A competitive life insurance industry determines the equilibrium price of the life insurance policies. Our model is calibrated to produce a wealth and earnings distribution consistent with the data and demographic shocks that match observed transition probabilities from the Central for Disease Control and the Census Bureau.

We focus on a general equilibrium model, rather than a partial equilibrium one, because we believe that the pricing of policies may constitute an important piece of the puzzle and these prices are not specified exogenously; in reality, the life insurance industry is quite competitive. Therefore, we take seriously the notion that general equilibrium effects contribute to decisions. Unfortunately, our data does not contain the critical piece of information needed to investigate this question – the premium paid for a policy. In addition, it does not identify who the policy covers, so that the pricing data would not be perfectly informative in any case. We therefore explore different pricing schemes for the industry which range from actuarial-fairness to something less than that – a surprising finding from the theory is that the degree of actuarial-unfairness needed to reproduce the aggregate amount of life insurance held is quite small.

Furthermore, the specification of a fully-specified model allows to clearly state what is meant by "adequate life insurance." Although this term is used repeatedly in the literature – especially in Auerbach and Kotlikoff (1989, 1990, 1991), Bernheim *et.al* (2001, 2002, 2003), and Gokhale and Kotlifkoff (2003) – it is not defined in terms of a calibrated general equilibrium model. Instead, those papers use a partial equilbrium decision problem with exogenous prices and a peculiar utility function – Leontief over consumption across periods – to assess whether patterns are puzzling. We instead use more standard theory to assess the life insurance patterns.

Given our model, we make welfare calculations to determine the impact of a life insurance market. We find that aggregate welfare increases by only 0.08 percent if households have access to an actuarially-fair life insurance market. However, simulations of particular groups suggest that this increase in concentrated in the hands of the middle-aged working poor who have a large number of children. Such groups do not hold a lot of life insurance, suggesting that the mismatch identified by Bernheim *et.al.* (2001,2003) may hold up under a more complete theoretical investigation. But this observation also creates an obvious potential solution – the welfare system, which transfers resources to single mothers, may effectively act like a public life insurance market.

The paper is organized as follows. First, we present the theoretical model. Then, we calibrate the model to U.S. data. Third, we present our results in three sections – aggregate and distributional implications for life insurance, aggregate welfare, and implied time paths for widow shocks. Finally, we conclude with some suggestions for future research into life insurance.

2 The Model Economy

In this section, we describe our dynamic general equilibrium model. The decision making unit is the household, which may contain more than one individual. Households enter a period with a demographic state comprised of age, sex, size, and marital status; this state evolves stochastically over time. Within this environment, households make consumption-savings, labor-leisure, and portfolio decisions. In addition to the households, we have three other types of agents. Production firms rent capital and labor from households and produce a composite capital-consumption good. Insurance firms collect premium payments for life insurance policies and make payments to households. Finally, the government collects payroll taxes and makes social security payments to retirees.

2.1 The Demographic Structure

With the decision making unit being the household, the demographic structure of the model is rather complex as the household structure, the marital status of the household and the number of children have to be taken into account. The economy is inhabited by individuals who live a maximum of I periods and face mortality risk. The demographic structure of a household is a four-tuple that depends on age, the adult structure of the household, the marital status of the household, and the number of children in the household. Denote the age of an individual by $i \in \mathcal{I} = \{1, 2, ..., I\}$. Survival probabilities depend on age and sex.

The second element of the demographic variable is the adult structure of the household; we assume this variable can take on one of three values: $p \in \mathcal{P} = \{1, 2, 3\}$. If $p = 1$, then the household is made up of a single male. A value of $p = 2$ denotes a household comprising of a single female, while $p = 3$ denotes a household with a male and a female who are married.

The third element in the four-tuple is the marital status of the household. We define the marital status by $m \in \mathcal{M} = \{1, 2, 3, 4\}$. Four values are needed to account for various events that have an impact on the house. A value of $m = 1$ denotes a household that is composed of a single adult, either male or female, that has never been married. If $m = 2$, then the household is comprised of a single individual that has become single due to a previous divorce. If $m = 3$, the household is a single individual that has been widowed. Finally, $m = 4$ represents a married household.[2]

[2]Some gender-marital status pairs are infeasible. The only pairs that are feasible are $(p = 1, m = 1)$, $(p = 1, m = 2)$, $(p = 1, m = 3)$, $(p = 2, m = 1)$, $(p = 2, m = 2)$, $(p = 2, m = 3)$, and $(p = 3, m = 4)$.

The last element in the four-tuple denotes the number of children in the household. We denote this demographic state variable by $x \in \mathcal{X} = \{0, 1, 2, 3, 4\}$. This tells us that the household can have between zero and four children. We limit the number of children to four per household for computational reasons.[3] Single female households can bear children, but single male households cannot. We do not separately track the age of the children; rather, we assume that they age stochastically according to a process that leaves them in the household twenty years on average.

A household's demographic characteristics are then given by the four-tuple $\{i, p, m, x\}$. We will define a subset of demographic characteristics made up of the tuple $\{p, m, x\}$ as $\widehat{z}$; this subset evolves stochastically over time. We assume that the process for these demographic states is exogenous with transition probabilities denoted by $\pi_i (\widehat{z}' | \widehat{z})$; note that the transition matrix is age-dependent. To avoid excessive notation, we define the age specific transition matrices so that their rows add up to the probability of being alive in the next period. In constructing the transition matrix, a number of additional assumptions had to be made. In particular, marriage and divorce create some special problems. We assume that when a divorce occurs, the household splits into two households and economic assets are split into shares according to the sharing rule $(\rho, 1 - \rho)$ where ρ is the fraction of household wealth allocated to the male. Any children are assigned to the female. If a household happens to die off (all parents die in a given period) we assume that the children disappear as well. For marriage, we only allow individuals of the same age to marry. In addition, a male with children and a female with children can only marry if the joint number of children is less than the upper bound. This set of assumptions and our demographic structure results in a relatively sparse transition matrix.[4]

The computation of this transition matrix is described in the appendix. The basic demographics of the calibrated population are presented in Table 1. We find that 68 percent of the population is currently married and 32 percent is single. Of the single households, divorced households make up 14 percent of the population, widowed households make up 7 percent of the population,

[3] Actual data for number of children per female for 1999 indicates that the number of females with five or more children is less than 2.7 percent of females. By abstracting away from these households we are not ignoring a significant fraction of the population.

[4] The transition matrix for a specific age is $(p, m, x) \times (p, m, x)$. Out of this set of transition elements, only twenty-seven can be non-zero, plus the nonzero probability of transition into death.

and households which have never been married make up 10 percent of the population. When looking at children, we find 77 percent of households live with no kids, either because they have never had children or the children are adults and have left the household. 18 percent of households contain a single child, while households with multiple children constitute about 5 percent of the population. This distribution matches nicely with the data, suggesting our calibration procedure was successful.

2.2 The Household

2.2.1 Preferences

Household utility depends on the level of household consumption, male leisure, and female leisure. We specify the household utility function as

$$E_0 \sum_{t=1}^{I} \beta^{t-1} \frac{\left[C_t^{\mu} \left(T_m - h_{mt}\right)^{\chi(1-\mu)} \left(T_f - h_{ft} - \iota x_t\right)^{(1-\chi)(1-\mu)}\right]^{1-\sigma} - 1}{1-\sigma}$$

where C_t denotes the level of household consumption, $(T_m - h_{mt})$ represents male leisure , and $(T_f - h_{ft} - \iota x_t)$ defines female leisure. Our utility function requires some discussion. The preference ordering that is represented by this utility function assumes that there is no disagreement over future states between married individuals, which would not generally be true in the presence of differential mortality rates, wages, and leisure costs. We finesse this problem by assuming that gender has no meaning within a marriage; that is, members of a married household do not know whether they are male or female. Further, each views becoming a single male or a single female upon divorce to have the same probability (50 percent), and therefore do not disagree about the value of savings in those states.

We require that hours worked, leisure, and consumption be nonnegative for both genders. We define household consumption as

$$C_t = \left(1_{pt} + \eta x_t\right)^{\theta} c_t$$

where 1_{pt} is an indicator function that takes on the value of 1 if the state variable p is either 1 or 2 or the value 2 if p is equal to 3, (*i.e.*, the married state), x_t is the state variable indicating the number of children in the family, and (θ, η) are parameters. The parameter $\theta \in (-1, 0)$ accounts for economies of scale in consumption, while the parameter η converts children into adult equivalents. Female leisure differs from male leisure; female leisure depends on hours supplied h_f as well as a leisure cost per child captured by ιx, where $\iota \in (0, 0.245)$. In contrast, male leisure depends solely on hours supplied h_m. The remaining parameters in the utility function are the discount factor $\beta \in (0,1)$, the weight of household consumption in utility $\mu \in (0,1)$, and the Arrow-Pratt coefficient of relative risk aversion $\sigma \geq 0$.

The labor-leisure decision in our environment will not be smooth – rather, it will feature a nonconvexity in the choice set for hours. To accommodate this feature, we assume that the time endowment is 1 for each member of the household, but supplying a positive amount of labor in a given period requires a fixed time cost of 0.02 units. In addition, we restrict the labor supply decision to involve the choice of supplying zero or more than 0.15 units of time to the market, with nothing in between. We incorporated this nonconvexity into the model economy because smooth versions did not produce the wealth equality between single and dual earner families observed in the data – dual earner families had close to twice as much wealth, which is counterfactual. That is, we have the choice for hours being

$$\begin{aligned} h_m &\in \{0, [0.15, 0.98]\} \\ h_f &\in \{0, [0.15, 0.98 - \iota x]\}. \end{aligned}$$

2.2.2 Household Environment

Households live in an uncertain environment that arises from demographic factors as well as a household specific productivity shock. Each period the household receives a productivity shock $\epsilon \in \mathcal{E} = \{\epsilon_1, \epsilon_2, ..., \epsilon_E\}$.[5] In addition to the demographic state discussed above, the household begins a period with wealth $a \in \mathcal{A}$; this space will be bounded from below by the requirement that consumption be nonnegative and bounded from above by the finiteness of the individual time horizon. The state for the household is the demographic situation, the productivity shock, and the wealth position:

$$s = (a, \epsilon, p, m, x, i).$$

[5] We assume the productivity shock is household specific, meaning that both the husband and wife receive the same productivity shock. This assumption is made for computational purposes; given the strong degree of assortative matching that occurs in marriage markets, it probably is not terribly inaccurate.

Given this state, the household's sources of funds are wealth and labor earnings. Labor earnings come from the hours worked by both males and females (if of working age) or government social security payments (if retired). Let h_i denote hours worked by the household member of gender $i \in \{f, m\}$. Each unit of labor pays $w\epsilon\upsilon_i$ to the male worker and $w\epsilon\upsilon_i\phi$ to the female; w is the aggregate wage rate, ϵ is the idiosyncratic wage factor, υ_i is the age-specific earnings parameter, and $\phi \in (0, 1)$ corrects for the male-female wage gap. Let ϖ denote the social security payment, τ the payroll tax rate, and 1_ϖ an indicator of retirement. Total labor income is then given by

$$\left(1 - 1_\varpi\right)\left(1 - \tau\right) w\epsilon\upsilon_i \left(h_m + \phi h_f\right) + 1_\varpi \varpi.$$

With this level of funds, the household must consume and purchase assets. The only assets that are available are capital k and term life insurance policies l. The budget constraint for a household of age i is

$$c + k' + ql' \leq a + \left(1 - 1_\varpi\right)\left(1 - \tau\right) w\epsilon\upsilon_i \left(h_m + \phi h_f\right) + 1_\varpi \varpi \tag{1}$$

where q is the price of a life insurance policy.[6]

The next period wealth level of a household depends on the capital and life insurance choices as well the future demographic state. If the household enters the period and remains married, the future wealth level is constrained by

$$a' \leq \left(1 + r'\right)\left(k' + s'\right) \tag{2}$$

where r' is the net return of capital and s' is the accidental bequest from households who die.[7] If a divorce occurs in a household that starts the period married, the male adult in the marriage has a wealth level next period equal to

$$a' \leq \rho\left(1 + r'\right)\left(k' + s'\right) \tag{3}$$

and the female adult's next period wealth level is

$$a' \leq \left(1 - \rho\right)\left(1 + r'\right)\left(k' + s'\right) \tag{4}$$

where $\rho \in (0, 1)$ is the sharing rule. If death of a spouse occurs, the wealth evolution equation is

$$a' \leq \left(1 + r'\right)\left(k' + s'\right) + l' \tag{5}$$

[6] In our model, whole life insurance policies are equivalent to a portfolio of term life insurance policies and riskless capital.

[7] We employ the convention that a 'prime' on a variable denotes the value in the next period.

as the life insurance policy pays off. If a household enters as a single adult and becomes married, we have to merge the budget constraints of two single adult households. A marriage yields the wealth equation

$$a' \leq (1 + r') \left(k' + \overline{k}' + s' \right) \tag{6}$$

where $\overline{k}'$ is the average capital for single households.[8]

Both life insurance and capital holdings are restricted to be nonnegative:

$$k', l' \geq 0.$$

We do not specifically model the reasons behind our asset market restrictions. For life insurance at least, appealing to adverse selection would probably suffice as a negative position in life insurance is equivalent to a long position in an annuitized asset. For capital, however, this restriction is somewhat more troublesome. We do not wish to complicate the model further by incorporating debt constraints.

The timing of events is important. We assume that divorce and marriage occur before death; that is, demographic changes occur first and then survival is determined. Furthermore, our demographic state only includes the last change; for example, households who get married, then divorced, then remarried, then widowed, are considered widowed. Fortunately, there will be only a small number of such households in equilibrium, and we do not feel the added burden involved in tracking past states to be worthwhile. Furthermore, we lack the individual data necessary to calibrate the transition matrix to these past events.

2.3 Aggregate Technology

The production technology of this economy is given by a constant returns to scale Cobb-Douglas function

$$Y = K^{\alpha} N^{1-\alpha}$$

where $\alpha \in (0, 1)$ is capital's share of output and K and N are aggregate inputs of capital and labor, respectively. The aggregate capital stock depreciates at

[8] We should allow $\overline{k}'$ to be age-dependent. However, computing the equilibrium of this model would be infeasible as it would involve I market-clearing conditions, one for each age. With appropriate restrictions on the transition matrices our economy satisfies a mixing condition that could justify our assumption.

the rate $\delta \in [0,1]$ each period. Our assumption of constant returns to scale allows us to normalize the number of firms to one.

Given a competitive environment, the profit maximizing behavior of the representative firm yields the usual marginal conditions. That is,

$$r = \alpha K^{\alpha-1} N^{\alpha} - \delta \tag{7}$$

$$w = (1-\alpha) K^{\alpha} N^{-\alpha}. \tag{8}$$

The aggregate inputs of capital and labor depend on the decisions of the various individuals in the economy. Let Γ denote the distribution of households over the idiosyncratic states $(a, \epsilon, p, m, x, i)$ in the current period. The aggregate labor input and capital inputs are defined as

$$N = \int_{\mathcal{A}\times\mathcal{E}} \sum_{\mathcal{P}\times\mathcal{M}\times\mathcal{X}\times\mathcal{I}} \epsilon \upsilon_i \left(h_m (a, \epsilon, p, m, x, i) + \phi h_f (a, \epsilon, p, m, x, i)\right) \Gamma (da, d\epsilon, p, m, x, i)$$

and

$$K = \int_{\mathcal{A}\times\mathcal{E}} \sum_{\mathcal{P}\times\mathcal{M}\times\mathcal{X}\times\mathcal{I}} a \Gamma (da, d\epsilon, p, m, x, i).$$

2.4 The Life Insurance Firm

In this paper, we assume that the life insurance market is a perfectly competitive market. As a result, we can examine the behavior of the single firm that maximizes profits subject to a constant returns to scale technology with no input costs. The price of insurance, or the premium, will be determined by the zero profit condition in each period.

We will consider an insurance firm that offers only term life insurance; we set the term to 1 period for simplicity. The life insurance company sells policies at the price q and pays out to a household that loses a spouse. Policies have a duration of one period.[9] The price q can depend on the age and demographic characteristics of the household in general; we will restrict ourselves in this paper to study parameterized pricing schemes. An extension to investigate the properties of efficient risk-sharing in our environment is currently beyond our computational ability.

[9] We abstract from annual renewal pricing issues. Because life insurance markets are characterized by adverse selection problems which may be revealed over time, the price of renewals could differ from a first time buyer.

Life insurance only pays off if an adult household member dies; we assume that the policy covers both members. Clearly, a critical aspect in the pricing of life insurance is the expected survival rate for an individual. We will represent the probability of an age i individual surviving to age $i+1$ as $\psi_{i,p}$. The zero profit condition for a life insurance firm is

$$\int_{\mathcal{A}\times\mathcal{E}} \sum_{\mathcal{P}\times\mathcal{I}} \left(1-\psi_{i,p}\right) \frac{1}{1+r'} l'\Gamma\left(da, d\epsilon, p, m, x, i\right) = \int_{\mathcal{A}\times\mathcal{E}} \sum_{\mathcal{P}\times\mathcal{I}} q l'\Gamma\left(da, d\epsilon, p, m, x, i\right). \tag{9}$$

The right hand side of this equation measures the revenue generated from the sale of life insurance policies to households in the economy. The left hand side measures the payout due to deaths at the end of the period, appropriately discounted.

3 Stationary Equilibrium

We will use a wealth-recursive equilibrium concept for our economy and restrict ourselves to stationary steady state equilibria. Let the state of the economy be denoted by $(a, \epsilon, p, m, x, i) \in \mathcal{A}\times\mathcal{E}\times\mathcal{P}\times\mathcal{M}\times\mathcal{I}$ where $\mathcal{A}\subset\mathbb{R}_+, \mathcal{E}\subset\mathbb{R}_+, \mathcal{P}\subset\mathbb{R}_+$, $\mathcal{X}\subset\mathbb{R}_+$and $\mathcal{M}\subset\mathbb{R}_+$. For any household, define the constraint set of an age i household $\Omega_i\left(a, \epsilon, p, m, x, i\right)\subset\mathbb{R}^5_+$ as all five-tuples (c, k', l', h_m, h_f) such that the budget constraint ?? and wealth constraints ??$-$?? are satisfied as well as the nonnegativity constraints.

Let $v\left(a, \epsilon, p, m, x, i\right)$ be the value of the objective function of a household with the state vector $(a, \epsilon, p, m, x, i)$, defined recursively as

$$v\left(a, \epsilon, p, m, x, i\right) = \max_{\left(c, k', l', h_m, h_f\right)\in\Omega_i} \left\{ \begin{array}{c} U\left(\left(1_p+\eta x\right)^{\theta} c, T_m - h_m, T_f - h_f - \iota x\right) + \\ \beta E\left[v\left(a', \epsilon', p', m', x', i+1\right)|a, \epsilon, p, m, x\right] \end{array} \right\}$$

where E is the expectation operator conditional on the current state of the household. A solution to this problem is guaranteed because the objective function is continuous and the constraint correspondence is compact-valued and continuous. However, since the constraint correspondence is not convex-valued, we cannot make definitive statements about the uniqueness of the solution or the properties of the value function.

Definition 1 *A* ***stationary competitive equilibrium*** *is a collection of value functions* $v : \mathcal{A}\times\mathcal{E}\times\mathcal{P}\times\mathcal{M}\times\mathcal{I}\rightarrow\mathbb{R}_+$ *; decision rules* $k' : \mathcal{A}\times\mathcal{E}\times\mathcal{P}\times\mathcal{M}\times\mathcal{I}\rightarrow\mathbb{R}_+$,

$l' : \mathcal{A}\times\mathcal{E}\times\mathcal{P}\times\mathcal{M}\times\mathcal{I}\rightarrow\mathbb{R}_+$, $h_m : \mathcal{A}\times\mathcal{E}\times\mathcal{P}\times\mathcal{M}\times\mathcal{I}\rightarrow\mathbb{R}_+$, and $h_f : \mathcal{A}\times\mathcal{E}\times\mathcal{P}\times\mathcal{M}\times\mathcal{I}\rightarrow\mathbb{R}_+$; aggregate outcomes $\{K, N, s\}$; prices $\{q, r, w\}$; government policy variables $\{\tau, \varpi\}$; and an invariant distribution $\Gamma(a, \epsilon, p, m, x, i)$ such that

(i) given $\{w, r, q\}$, the value function v and decision rules c, k', l', h_m, and h_f solve the consumers problem;

(ii) given prices $\{w, r\}$, the aggregates $\{K, N\}$ solve the firm's profit maximization problem;

(iii) the price q is consistent with the zero-profit condition of the life insurance firm;

(iv) the goods market clears:

$$f(K, N) = \int_{\mathcal{A}\times\mathcal{E}} \sum_{\mathcal{P}\times\mathcal{M}\times\mathcal{X}\times\mathcal{I}} c(a, \epsilon, p, m, x, i)\,\Gamma(da, d\epsilon, p, m, x, i) + K' - (1-\delta)K;$$

(v) the labor market clears:

$$N = \int_{\mathcal{A}\times\mathcal{E}} \sum_{\mathcal{P}\times\mathcal{M}\times\mathcal{X}\times\mathcal{I}} \epsilon\upsilon_i\left(h_m(a, \epsilon, p, m, x, i) + \phi h_f(a, \epsilon, p, m, x, i)\right)\Gamma(da, d\epsilon, p, m, x, i);$$

(vi) the accidental bequest transfer s is equal to the aggregate wealth of households that die:

$$\begin{aligned} s \;=\; & \int_{\mathcal{A}\times\mathcal{E}} \sum_{\mathcal{P}\times\mathcal{M}\times\mathcal{X}\times\mathcal{I}} \left(1-\psi_{i,male}\right) k'\left(a, \epsilon, 1, \{1,2,3\}, x, i\right)\Gamma\left(da, d\epsilon, 1, \{1,2,3\}, x, i\right) \\ & + \int_{\mathcal{A}\times\mathcal{E}} \sum_{\mathcal{P}\times\mathcal{M}\times\mathcal{X}\times\mathcal{I}} \left(1-\psi_{i,female}\right) k'\left(a, \epsilon, 2, \{1,2,3\}, x, i\right)\Gamma\left(da, d\epsilon, 2, \{1,2,3\}, x, i\right) \\ & + \int_{\mathcal{A}\times\mathcal{E}} \sum_{\mathcal{P}\times\mathcal{M}\times\mathcal{X}\times\mathcal{I}} \left(1-\psi_{i,male}\right)\left(1-\psi_{i,female}\right) k'(a, \epsilon, 3, 4, x, i)\,\Gamma(da, d\epsilon, 3, 4, x, i); \end{aligned}$$

(vii) the retirement program is self-financing:

$$\varpi = \frac{\int_{\mathcal{A}\times\mathcal{E}} \sum_{\mathcal{P}\times\mathcal{M}\times\mathcal{X}\times\mathcal{I}} \tau\left(1-I_\varpi\right) w\epsilon\upsilon_i \begin{pmatrix} h_m(a, \epsilon, p, m, x, i) + \\ \phi h_f(a, \epsilon, p, m, x, i) \end{pmatrix}\Gamma(da, d\epsilon, p, m, x, i)}{\int_{\mathcal{A}\times\mathcal{E}} \sum_{\mathcal{P}\times\mathcal{M}\times\mathcal{X}\times\mathcal{I}} I_\varpi \Gamma(da, d\epsilon, p, m, x, i)};$$

(*viii*) *letting T be an operator which maps the set of distributions into itself, aggregation requires*

$$\Gamma'\left(a', \epsilon', p', m', x', i+1\right) = T\left(\Gamma\right)$$

and T be consistent with individual decisions.

We will restrict ourselves to equilibria which satisfy $T\left(\Gamma\right) = \Gamma$.

4 Calibration

We calibrate our model to match features in the U.S. data. Our calibration will proceed as an exercise in exactly-identified Generalized Method of Moments; we directly choose some parameters when we do not have good statistics to match from the data. As much as possible, however, we will use the equilibrium for the model to determine the appropriate values.

We select the period in our model to be one year. First we examine the preference parameters in the model. The average wealth-to-GDP ratio in the postwar period of the U.S. is about three; hence, we choose β to replicate this number. The average individual in the economy works about thirty percent of their time endowment; we use this number to set the parameter μ. From time use surveys, we note that females allocate about 2 hours per day per child for care and females conduct about two-thirds of all such care, leading us to set $\iota = 0.145$. We also select χ so as to match the ratio of the hours supplied by females to males. The 1999 Current Population Survey reports average annual hours worked for males in 1998 is 1,899 while average annual hours worked for females in the same period is 1,310. Hence, χ is chosen so that the model generates the observed ratio of 0.689. The relative wage parameter ϕ is selected to be 0.77, consistent with estimates from the 1999 CPS on the relative earnings of males and females, and we set the divorce sharing rule to $\rho = 0.5$. The other preference parameters that require specification are η , θ, and σ. We use Greenwood, Guner, and Knowles (2001) to specify the first of these parameters: $\eta = 0.3$ and $\theta = -0.5$. The last parameter, σ, is the Arrow-Pratt coefficient of relative risk aversion. Given little *a priori* consensus on the value of this parameter, we choose $\sigma = 1.5$, a value which is consistent with choices typically made in the business cycle literature.

The technology parameters that need to be specified are determined by the functional form of the aggregate production function and the capital evolution equation. The aggregate production function is assumed to have a Cobb-Douglas form, since the share of income going to capital has been essentially constant. We specify labor's share of income, $1-\alpha$, to be consistent with the long-run share of national income in the US, implying a value of $\alpha = 0.36$. The depreciation rate is specified to match the investment/GDP ratio of 0.25, taken from the same data, yielding a value of $\delta = 0.1$.

The specification of the stochastic idiosyncratic labor productivity process is extremely important because of the implications that this choice has for the eventual distribution of wealth. Storesletten, Telmer and Yaron (2001) argue that the specification of labor income or productivity process for an individual household must allow for persistent and transitory components. Based on their empirical work, we specify ϵ to evolve according to

$$\log(\epsilon') = \omega' + \varepsilon'$$

$$\omega' = \Psi\omega + v'$$

where $\varepsilon\tilde{}N(0, \sigma_\varepsilon^2)$ is the transitory component and ω is the persistent component with $v\tilde{}N(0, \sigma_v^2)$. STY estimate $\Psi = 0.935$, $\sigma_\varepsilon^2 = 0.01$, and $\sigma_v^2 = 0.061$. Fernández-Villaverde and Krueger (2000) approximate the STY process with a three state Markov chain using the Tauchen (1986) methodology – this approximation yields the productivity values $\{0.57, 0.93, 1.51\}$ and the transition matrix

$$\pi = \begin{bmatrix} 0.75 & 0.24 & 0.01 \\ 0.19 & 0.62 & 0.19 \\ 0.01 & 0.24 & 0.75 \end{bmatrix}.$$

The invariant distribution associated with this transition matrix implies that an individual will be in the low or the high productivity state just under 31 percent of the time and the middle productivity state 38 percent of the time. The age-specific component of income is estimated from earnings data in the PSID and produces a peak in earnings at real age 47.

For the horizon, we assume that the mandatory retirement age is 65 (45 in model periods) and that agents live at most 100 years (80 model periods). Pricing in the life insurance industry is done relative to an individual who lives to be 100, so this horizon seems appropriate. Furthermore, a long retirement phase mitigates the impact of the terminal age on the behavior during the working ages. We require a relatively-short period to induce the persistent demographic states that give rise to significant demand for life insurance. The transition matrix for demographic states is difficult to construct. Due to the presence of history-dependence in the probabilities of marriage, divorce, mortality, and fertility, we found that we could not analytically construct this matrix. As a result, we used a Monte Carlo approach to generate the probability of transitioning between different states. In the computational appendix we detail the procedures followed to generates the transition matrix.

The last issue we must examine is the social security system. Since we are primarily concerned with the behavior of working-age households, we choose to calibrate this system to match not benefits but rather taxes. We set $\tau = 0.153$, the average social security tax rate in the postwar US, and balance the budget by adjusting the level of benefits. Our inclusion of the government transfer program has two purposes: one, it reduces the precautionary demand for assets, which in this model would implausibly increase the demand for life insurance policies in the absence of such transfers; and two, it makes the solution of the model easier as it reduces the marginal utility of poor, retired households.[10] Without income, these households can create internodal oscillations in our approximation scheme, which is based on cubic spline interpolation. The computation of the equilibrium is outlined in the appendix.

5 Findings

We now detail our results. This section will consist of three subsections. First, we will examine the equilibrium solution of the model under two insurance pricing strategies: an actuarially-fair premium and a pricing strategy that introduces actuarial-unfairness for the young. We define an **actuarially-fair** price scheme as a system in which the premium is set to equal the probability that one adult in the household dies in a given period; any other pricing scheme is **actuarially-unfair** because, in equilibrium, it must involve some group paying premiums that exceed this amount. The unfair case allows us to determine the sensitivity of our results to assumptions about the premiums as well as assess the imperfect loading that we suspect occurs in the real economy. In the second section, we present welfare cost computations across specifications. These calculations allow us to assess the importance of

[10]However, it does counterfactually increase the rundown of assets after retirement.

life insurance market. In the last subsection, we examine the importance of life insurance for a small subset of specific age and demographic households – poor widows with children.

To focus the paper on a narrow set of questions, we state our goals here. One, we wish to see what the model predicts for the total amount of life insurance held; in the data (see Chambers, Schlagenhauf, and Young 2003) married households have 0.65 GDPs worth of life insurance. Given that the model has no incentives for singles to hold insurance, this number is the appropriate one to use as a benchmark. Two, the participation rate of married households is 68.7 percent with single and dual earners having participation rates of 67.7 and 69.5 percent respectively; in quantitative terms, we will consider these values as being equal. Three, holdings by single and dual earners are approximately the same size. Four, the peak in participation rates lags the peak in holdings by 10 years. If the model can account for these observations, we will conclude that life insurance is not puzzling.

Identifying the pricing scheme used by the industry from the data is not easy, since there does not exist a product comparable to the theoretical object we investigate. Real-world insurance contracts contain a variety of options and renewal features that contaminate the pure insurance capacity, rendering any attempt to use them for calibration inappropriate, so we are forced to explore different schemes.

5.1 Actuarially-Fair Life Insurance

Our calibration results are presented in Table 3. As can be seen, the interest rate $r - \delta$ is around 1.1 percent per annum which is a reasonable value for risk-free government debt over the postwar US period. However, this value is about around half the average return to capital measured from NIPA data in McGrattan and Prescott (2003). Given that we have abstracted from default and aggregate risk, we do not find this to be a failure of the model. Regarding the values for $\overline{k}$ and s, the model economy finds that the amount of capital held by singles $\overline{k}$ is quite low; they hold about 13 percent of the total wealth in the economy. Finally, the accidental bequest term is small.

In Figure 3, we plot the distribution of life insurance holdings by age – we see that life insurance holdings peak around age 40, exactly where the peak occurs in the data.[11] Households older than real age 65 do not hold life

[11] The data curve here is a fitted fourth-order polynomial to the holdings of married

insurance. It is important to note that the peak of life insurance holdings occurs before the peak in earnings, which occurs at age 47 (model age 27). The reason this occurs in the model is that the peak in holdings coincides with the peak in the present value of future labor income – model households appear to insure themselves in a constant ratio relative to this number. Total life insurance holdings are 129.53 percent of GDP in the model, almost twice what the data show for married households.

The participation rate for married households in the model is 62 percent, which is a bit too low but definitely within a reasonable range. Figure 4 presents the distribution of participation rates by age – it peaks at real age 50 at a value of 95 percent, a bit too early and much too high relative to the data. Clearly the underestimate of the aggregate participation rate is due to the near-complete absence of any retirees in this market – this absence is due to a combination of high premiums and low insurance needs. Overstating the purchases of life insurance is to be expected since adverse selection problems will typically create actuarial unfairness for some members of society, particularly the young. Similarly, we expect to underpredict the participation rate since we abstract from motives that produce demand for life insurance by the elderly, in particular estate taxation and funeral costs.

Shifting to the difference between single and dual earner married households, we find that 75 percent of single earner married households have life insurance while 82 percent of dual earners hold life insurance. Retired households (who have no earners) are not participating at all, which accounts for why the aggregate participation rate can be low but these two components are high relative to the data. Furthermore, in our economy dual earner households and single earner households have almost the same amount of average wealth – 1.972 versus 1.875 – which matches what we observe in the data.

If we look at earnings, the model predicts a life insurance to earnings ratio of 2.04 for dual earner families and about 3.6 for single earners, while these groups have about the same average earnings (as in the data). In the data, these ratios are essentially equal at 3 times annual earnings. In this dimension, the model is producing an anomaly. It suggests that the groups holding life insurance in the model do not match with those from the data, because total life insurance purchases are too high in the model but the earnings ratios are too low. Apparently, in the model agents with

households according to age.

high current earnings are heavy purchasers of life insurance, which is to be expected given persistence in the earnings process, but that those agents must not be purchasers in the data.

5.2 Actuarially-Unfair Life Insurance

We now move to consider schemes which include some degree of actuarial-unfairness. We assume that the life insurance premium for a household in state $(a, \epsilon, p, m, x, i)$ is given by

$$q\left(a, \epsilon, p, m, x, i\right) = Aq + \left(1 - A\right)\psi_{i,p,m,x}, \tag{10}$$

where ψ denotes the actuarially-fair premium, q is a flat premium, and $A \in (0, 1)$. To ensure that the industry still makes zero profit, we allow q to adjust. We solve the cases

$$A \in \{0.0, 0.005, 0.008, 0.01, 0.025, 0.05, 0.075, 0.08, 0.1, 1.0\};$$

$A = 0.0$ corresponds to the actuarially-fair case above and $A = 1.0$ corresponds to an extreme case where all households pay the same premium. Examining the results from these experiments, we find that the degree of "loading" needed to generate exactly 0.65 times GDP in life insurance holdings is $A = 0.008$, which is a very small departure from perfectly-fair insurance. Figure 4 displays the relationship between A and $\frac{LI}{GDP}$; as the premia move away from actuarial-fairness we find a rapid decline in $\frac{LI}{GDP}$. This result is quite striking – small departures from actuarial-fairness torpedo the life insurance holdings in our economy. As a result, we have a strong estimate of the degree of adverse selection in the life insurance market – not much, consistent with the formal econometric exploration in McCarthy and Mitchell (2003). The case of $A = 1.0$ is a model economy in which premia are constant across the whole population – in this case, we find that only the extremely-old (above model age 75) purchase life insurance, and they hold only 4 percent of GDP. As A moves from 1.0 to 0.0, the distribution of holdings moves continuously between the extremes.

How can we be sure that this change constitutes a "small departure?" Using Table 4, we see that the flat premium required in the $A = 0.008$ case is $q = 0.0475$. Multiplying the two numbers together yields 0.00038, which is a tiny change in the intercept for the premium function; the change $A\psi_{i,p,m,x}$ yields a even smaller number for change in the slope for most of the population ($\psi_{i,p,m,x}$ is small until the retirement years). Therefore, we argue that this change is quite moderate – looking at Figure 5 we see that the total change in average premium by age (netting out the impact of demographics) is never larger than 0.0005.

We now explore the same statistics for the case $A = 0.008$ that we examined in the actuarially-fair case. The distributions of the participation rate and holdings are shown in Figures 2 and 3; relative to the actuarially-fair case, we find less purchases and these purchases have a peak that is less sharp; the addition of the Aq term to the pricing function yields higher prices for young households than they merit based on average mortality rates. The location of the peak does not change, however, suggesting that this observation is not puzzling relative to economic theory. As mentioned above, this peak coincides with the peak in the present value of future labor earnings.

The aggregate participation rate falls to 58 percent and the participation rates for single and dual earner married households fall to 69 percent and 78 percent, respectively. The dual earner rate is too high still, but the single earner rate is very close to the value from the data. Again, retirees and no-earner households are not purchasing any insurance, dragging down the aggregate number but leaving some of the components too high. In terms of life insurance to earnings ratios, the numbers for the actuarially-unfair economy are 0.98 for the dual earners and 1.74 for the single earners, far short of the ratios observed in the data for either group. We regard this feature as an anomaly since it is robust to our implementation of actuarial-unfairness.[12]

5.3 Welfare Gains

The results in the prior section implies that the aggregate welfare gains emanating from the life insurance market are likely to be small. In this section, we want to quantify these welfare gains under the two main pricing schemes: $A = 0$ and $A = 0.008$. Our preferred approach for calculating welfare gains would be to use a transitional dynamic approach, since we could make welfare statements about individuals. Unfortunately the immense computational burden of the model keeps us from using this approach. We therefore examine the welfare gains by calculating the lifetime expected welfare gains

[12] We considered quantity discounts of the sort observed in the data, but they had little impact on the economy except to sharply increase the steepness of the peaks.

associated with a newborn person asked to live either in an economy with life insurance markets or without.

We define the *ex ante* welfare of a newborn individual as:

$$W = \int_{\mathcal{E}} \sum\nolimits_{\mathcal{P}} v\left(0, \epsilon, p, 1, 0, 1\right) \pi_{\epsilon}^{inv} \pi_p. \tag{11}$$

Consistent with newborns, the value function is evaluated at age 1 and the initial asset position zero. The newborn has no children so $m = 0$. If the newborn is male, $p = 1$, while a newborn female would be characterized by $p = 2$. π_{ϵ}^{inv} denotes the invariant distribution of ϵ and π_p is the probability of being born a given gender. We compute welfare under a version of the model without operative life insurance markets; denote this welfare value by W_0. We then compute the permanent percent increase in consumption λ needed to make an individual in that world indifferent between that world and the one with operating life insurance markets. Given the utility function, this increase solves the equation

$$W_1 = \left(1 + \lambda\right)^{\mu(1-\sigma)} W_0 \tag{12}$$

where W_1 is average newborn utility in an economy with life insurance markets. λ thus measures the welfare gain associated with life insurance assets.[13] We see this statistic as a quick and dirty method of determining whether life insurance is essentially redundant in our economy.

In the middle column of Table 4, we present the computed equilibrium for the inactive life insurance model. It is clear that there is little aggregate impact on the economy – none of the equilibrium prices change out to 4 decimal places. Compared to this benchmark, we find that having access to a life insurance market that is priced actuarially-fairly yields a welfare gain of 0.08 percent of consumption, while having access to a life insurance market priced unfairly with $A = 0.008$ yields the smaller gain of 0.03 percent. Whether these gains are large or small depends on interpretation. On the one hand, they are the same order of magnitude as the calculations in Lucas (2003) for the welfare costs of consumption instability, which are universally agreed to be small. However, given that agents pay 2 percent of their consumption in life insurance premiums this number might reasonably be

[13] Note that, since we have incomplete markets, we cannot be sure that introducing additional assets will increase welfare. Such perverse outcomes are associated with very strong general equilibrium effects, which we do not have.

seen as large, depending on the intermediation costs associated with the industry. If intermediation costs account for more than 60 percent of total payments to the life insurance firms, these costs are large, since they would exceed the observed value.

The aggregate number above can be quite misleading, however, when heterogeneity is present. As mentioned above, we would prefer to compute individual-specific welfare costs based on wealth, productivity, and demographics. Such computations are impossible given the size of the model environment. However, we suspect that the welfare gains are concentrated in certain groups, in particular the poor and middle-aged widows who have large numbers of children. To explore whether our intuition is correct, in the next subsection we compute expected life paths for households who experience a death and investigate how the presence of a life insurance market affects these outcomes.

5.4 Death Shocks

Given the measured benefits to a household of having access to the life insurance market, we would like to have a more precise idea of what generates these benefits. In an attempt to identify these dimensions, we use our model to conduct a series of simulations that examine how a household is impacted by a death of a spouse over their remaining life cycle. We consider household who is impacted by a death of a wage earner when they hold and do not hold life insurance, paying particular attention to the impact of a death on the average paths for wealth, consumption, and hours worked. To conduct these experiments we choose the economy with $A = 0.008$, so that the aggregate amount of life insurance matches that in the data.

In this section, we will concern ourselves mainly with poor households. Wealthy households can self-insure effectively without having access to a life insurance market, and thus the absence of that market is of limited relevance to them. We explore the impact of being widowed when the family has limited resources during middle age, both with a small number of children and a large number (1 versus 4). Our finding here is that both groups benefit from the presence of a life insurance market and that the benefit is increasing in the number of children present.

5.4.1 Poor Households with One Child

We first consider a household with a low wealth level - less than half average wealth. Furthermore, this household is really in much poorer shape than it appears, since the adult members are 40 years old, right in the middle of prime wealth-accumulation years, and thus are very poor relative to their age cohort. Such a household cannot self-insure against the unexpected loss of a wage earner, at least not very effectively. Hence, a death in this household will likely have large ramifications for consumption-saving and labor-leisure decisions and the availability of a life insurance market may be quite important, especially relative to a wealthy household. To explore this issue, we conduct impulse response-like experiments in which we hit a household with a death shock and track the expected path of wealth, consumption, and labor supply afterwards.

Figure 7 shows the path of wealth for two identical poor households who either have access to life insurance markets or not. With life insurance, the household experiences a 150 percent increase in wealth in the current period – apparently, this household is holding a large amount of life insurance. The LI household then lets wealth decline, while the one without LI continues to accumulate wealth. More relevant for welfare are the paths for consumption and leisure, which we show in Figures 8 and 9.[14] As seen in Figure 8, consumption jumps upward by about 7 percent in the LI economy due to the large increase in wealth, and it remains higher throughout. In contrast, in the no-LI economy consumption drops immediately by about 18 percent and never completely recovers.

Turning to labor supply, Figure 9 shows that the female adult is not currently employed in the no-LI economy. After the death shock, she supplies 0.3 units of labor and this rises to a peak of about 0.34 units several years later. In the LI economy, the female adult is employed (due to lost consumption caused by the premium payments), but reduces her labor supply because of a wealth effect after the loss of her husband.

Figures 10 and 11 compare these paths to the equivalent ones that would occur if the death of the husband had not occurred for the LI economy. ¿From Figure 11 we see that widows with only one child see a large increase in their consumption relative to the case where they are not widowed. In Figure 12, we see that widows supply more labor as well; thus, their increased

[14]Given the stylized nature of our retirement system, the paths to the right of age 65 should be interpreted with extreme caution.

consumption is partly due to the life insurance payment and partly due to larger labor supply. The nonwidow family has slowly increasing labor supply for the female, reflecting the average decline in childcare costs and average rise in wages over the time period.

5.4.2 Poor Households with Four Children

The situation is somewhat different for a poor household that has 4 children, the maximum allowed for in the model. In Figure 12, we see that wealth spikes upward by 500 percent in the LI economy when the household is hit with the death of the male but declines somewhat in the no-LI economy. In Figure 13, it is clear that the household takes a major hit in consumption if life insurance is not available – with LI, consumption rises 25 percent but without it, consumption falls almost 50 percent. This experiment suggests that this group of households ought to purchase life insurance, since they are exposed to a lot of consumption risk. Furthermore, a look at Figure 14 shows that the widow must absorb a large increase in labor supply if she does not have life insurance access but only a small increase if she does.

Comparing across widows and nonwidows, we see that consumption decreases for widows with a large number of children, opposite to what occured in the one child case. Furthermore, while the increase in labor supply is proportionally much smaller here these households have a much lower effective labor endowment due to childcare costs, meaning leisure is a lot smaller. Thus, life insurance has a significant benefit because it limits the impact of the death on consumption and leisure (just compare Figures 13 and 14 with Figures 15 and 16, respectively).

6 Conclusion

Our model has examined the life insurance portfolio decisions of households in a model with a reasonable amount of demographic detail. However, some aspects of the data cannot be accounted for within our framework. For example, we observe a number of small policies being held by elderly households; these policies cluster around $5000. This not so coincidentally is the same value as the average cost of funerals in the postwar US. We suspect that the introduction of a fixed cost for funerals would generate small policy holdings for agents who otherwise hold none. Second, our model cannot account for the policy holdings of single agents. Since we abstracted from the bequest motive, single households have no incentive to purchase life insurance, as it will only pay off after they die. However, single households do purchase life insurance in the data – their participation rate is nearly 60 percent. While divorce provisions can account for some of these holdings, the bequest motive seems to be of first-order importance here. Unfortunately, extending our model to include a bequest motive is well beyond the computational technology available currently.

The results from our model suggest that the nature of the pricing system is of critical importance, since small departures from actuarial-fairness unravel the entire market. Adverse selection problems abound in the insurance market, although evidence in McCarthy and Mitchell (2003) suggest that the effects may be small in the aggregate. The reason these problems exist is that mortality rates are endogenous through the individual choices of diet, exercise, and drug use and the external impact of environment. When combined with exogenous genetic factors, one obtains unobservable heterogeneity in the mortality rates of individuals that life insurance providers must confront. Endogenizing the contracting problem then seems to be an obvious avenue for the future, even if the computational burden prohibits this approach for now.

7 Computational Appendix

This appendix details the computational strategy used to solve the model. The appendix is divided into four parts. First, we discuss the computation of the household problem; we use backward induction along the lifetime to solve for the value function. Second, we discuss the generation of the invariant distribution over wealth, productivity, demographics, and age. Third, we discuss our method for computing market clearing prices and the solution to calibration equations. Fourth, we detail our Monte Carlo method for computing the transition matrix for the demographic states.[15]

The basic algorithm is as follows:

1. Guess a value for accidental bequests s, aggregate capital held by single individuals $\overline{k}$, the life insurance premium q, the social security benefit ϖ, and the rental rate r.

2. Solve the consumer's problem and obtain the value function v and the decision rules k', l', h_m, and h_f. This step involves building a nonlinear approximation to the value function and is described in detail below.

3. Iterate on an initial distribution of idiosyncratic states until convergence. This step assumes that the distribution of a is over only a finite number of points and redistributes mass iteratively. To conserve on computational time, we calculate the invariant distribution over stochastic states and use this information to start the iterations on the distribution of wealth.

4. Check that the values for r, s, and $\overline{k}$ agree with those in step 1, the life insurance company is earning zero profit, and the government budget balances. If not, then update and return to step 1. When calibrating the model, we add to step 1 guesses for the discount factor β, the consumption weight μ, and the relative male leisure weight χ. We then check whether our guesses imply the right values for the wealth/GDP ratio, the average hours worked, and the ratio of female to male labor supply.

[15] Fortran 95 code to solve for this equilibrium is available at http://garnet.acns.fsu.edu/~eyoung/programs. This code does not implement the parallel solution method and thus is appropriate for casual users, but runtimes are extremely long. The program's search for the equilibrium price and parameter vector also requires a significant amount of babysitting.

For the model with perfectly-loaded policies, we do not need to check the profit condition of the life insurance company, since it will earn zero profit on every state. For the intermediate cases, we assume that q adjusts to clear the market.

7.1 Solving the Household Problem

We will now discuss the solution of the household's problem. Let current wealth a lie in a finite grid $A \subset \mathbb{A}$. We must solve a two-dimensional continuous portfolio problem in (k', l'); furthermore, to complicate the problem both face short-sale constraints and the price of life insurance is small, leading to some sensitivity in the portfolios. As a result, we take the approach used in Krusell and Smith (1997) and Guvenen (2001) to solve the problem. To begin, we guess that the agent holds zero life insurance. We then find the optimum level of savings in capital by solving the Kuhn-Tucker condition

$$
\begin{aligned}
& (1_p + \eta x)^{\theta\mu(1-\sigma)} c^{\mu(1-\sigma)} (1 - h_m)^{\chi(1-\mu)(1-\sigma)} (1 - h_f - \iota x)^{(1-\chi)(1-\mu)(1-\sigma)} \times \\
& \left(\frac{\mu}{c} \left(-1 + \frac{\partial h_m}{\partial k'} w \upsilon_i \epsilon + \frac{\partial h_f}{\partial k'} \phi w \upsilon_i \epsilon \right) - \frac{\chi (1-\mu)}{1 - h_m} \frac{\partial h_m}{\partial k'} - \frac{(1-\chi)(1-\mu)}{1 - h_f - \iota x} \frac{\partial h_f}{\partial k'} \right) \\
& + \beta E \left[v_1 \left(a', \epsilon', m', i+1 \right) \right] (r + 1 - \delta) \\
\leq \; & 0
\end{aligned}
$$

where h_m and h_f solve

$$
\frac{\mu w \upsilon_j \epsilon}{a + w \upsilon_i \epsilon (h_m + \phi h_f) - k' - q l'} = \frac{\chi (1-\mu)}{1 - h_m}
$$

$$
\frac{\mu w \upsilon_i \epsilon \phi}{a + w \upsilon_i \epsilon (h_m + \phi h_f) - k' - q l'} = \frac{(1-\chi)(1-\mu)}{1 - h_f - \iota x}.
$$

If h_i fails to satisfy the lower bound 0.15, we set it to that value. Next, we let life insurance holdings be slightly positive: $l' = 0.0001$. If this increase reduces lifetime utility, the agent has zero life insurance optimally. If not, we use bisection to locate the correct value for l', increasing l' whenever the gradient at the optimal value for k' is positive and decreasing it whenever the gradient is negative. We repeat this process for zero labor supply for the female and for both members – it can be shown that the male member of a married household will never set labor supply to zero if the female supplies a positive amount.

Ignoring bequests, we assume that

$$v(\cdot,\cdot,\cdot,\cdot,I+1)=0.$$

Then, for each $i \leq I$ and using $v(\cdot,\cdot,\cdot,\cdot,i+1)$ as the value function for the next age, we can obtain the value function for this age as the solution to

$$v(a,\epsilon,p,m,i)=u\left(C^*,h_m^*,h_f^*\right)+\beta E\left[v\left(a^{*\prime},\epsilon',p',m',i+1\right)\right].$$

Cubic spline interpolation is used whenever we need to evaluate $v(\cdot)$ at points not on the grid for a.

7.2 Computing the Invariant Distribution

For the invariant distribution, the procedure outlined in Young (2002) is employed. For each idiosyncratic state and age vector (a,ϵ,p,m,i) we compute next period's wealth contingent on demographic changes. After locating $a'(a,\epsilon,p,m,i)$ in the grid using the efficient search routine **hunt.f** from Press *et.al.* (1993), we can construct the weights

$$A(a,\epsilon,p,m,i)=1-\frac{a'(a,\epsilon,p,m,i)-a_k}{a_{k+1}-a_k}$$

where

$$a' \in [a_k,a_{k+1}].$$

Now consider a point in the current distribution

$$\Gamma^n(a,\epsilon,p,m,i).$$

This mass is moved to new points according to the following process. For each set $(\epsilon,p,m,i)\times(\epsilon',p',m')$ we calculate the probability of transition; denote this value by $\rho(\epsilon,p,m,i,\epsilon',p',m')$. Mass is distributed then to the point

$$\Gamma^{n+1}(a_k,\epsilon',p',m',i+1)$$

in the fraction

$$A(a,\epsilon,p,m,i)\,\rho(\epsilon,p,m,i,\epsilon',p',m')\,\Gamma^n(a,\epsilon,p,m,i)$$

and to the point

$$\Gamma^{n+1}\left(a_{k+1}, \epsilon', p', m', i+1\right)$$

in the fraction

$$\left(1 - A\left(a, \epsilon, p, m, i\right)\right) \rho\left(\epsilon, p, m, i, \epsilon', p', m'\right) \Gamma^{n}\left(a, \epsilon, p, m, i\right).$$

Looping this process over each idiosyncratic state and age computes the new distribution. This process continues until the change in the distribution is negligible. Note that we can compute the weights and the brackets before iteration begins; since these values do not change we can store them and use them as needed without recomputing them at each step.

7.3 Solving for Market Clearing and Calibration

We now discuss how we solve for the equilibrium, given the solutions the value function and the invariant distribution. This algorithm takes the following form:

1. Take the fitness functions to be the sum of the squared deviations of the equilibrium conditions. We then attempt to solve

   $$\min_{\omega}\left\{\left\langle F\left(\omega\right), F\left(\omega\right)\right\rangle\right\}$$

   where ω is a vector of prices and parameters, F is the vector-valued function of equilibrium conditions, and $\langle\cdot\rangle$ is the inner product function. For the initial calibration this vector is of dimension 8:

   $$\left[r, p, \varpi, \overline{k}, s, \beta, \chi, \mu\right].$$

2. Set an initial population Ω which consists of n vectors ω. Given our strong priors on the values for certain variables, we do not choose this population at random. Rather, we concentrate our initial population in the region we expect solutions to lie.

3. Evaluate the fitness of each member of the initial population.

4. From the population, select n pairs with replacement. These vector-pairs will be candidates for breeding. The selection criterion weights each member by its fitness according to the rule

   $$1 - \frac{\left\langle F\left(\omega_j\right), F\left(\omega_j\right)\right\rangle}{\sum_{j=1}^{n}\left\langle F\left(\omega_j\right), F\left(\omega_j\right)\right\rangle}$$

so that more fit specimens are more likely to breed.

5. From each breeding pair we generate 1 offspring according to the BLX-α crossover routine. This routine generates a child in the following fashion. Denote the parent pair by $(\omega_i^1, \omega_i^2)_{i=1}^8$. The child is then given by

$$(h_i)_{i=1}^8$$

where $h_i \sim \mathrm{UNI}\,(c_{\min} - \alpha I, c_{\max} + \alpha I)$, $c_{\min} = \min\{\omega_i^1, \omega_i^2\}$, $c_{\max} = \max\{\omega_i^1, \omega_i^2\}$, and $I = c_{\max} - c_{\min}$. Our choice for α is 0.5, which was found to be the most efficient value by Herrera, Lozano, and Verdegay (1998) in their horse-race of genetic algorithms for an objective function most similar to ours.

6. We then introduce mutation in the children. With probability $\mu_G = 0.15 + \frac{0.33}{t}$, where t is the current generation number, we mutate a particular element of the child vector. This mutation involves 2 random numbers, r_1 and r_2, which are $\mathrm{UNI}\,(0,1)$ and 1 random number s which is $N\,(0,1)$. The element, if mutated, becomes

$$h_i = \begin{cases} h_i + s\left[1 - r_2^{\left(1-\frac{t}{T}\right)^{\delta}}\right] & \text{if } r_1 > 0.5 \\ h_i - s\left[1 - r_2^{\left(1-\frac{t}{T}\right)^{\delta}}\right] & \text{if } r_1 < 0.5 \end{cases}$$

we set $\delta = 2$ following Duffy and McNelis (2001). Note that both the rate of mutation and the size shrinks as time progresses, allowing us to zero in on potential roots.

7. Evaluate the fitness of the children.

8. From each family trio, retain the most fit member. We now are left with exactly n members of the population again.

9. Compare the most fit member of the last generation, if not selected for breeding, with the least fit member of the new generation. Keep the better of the two vectors. If the most fit member of generation $t-1$ is selected for breeding this step is not executed. This step is called **elitism** and is discussed in Arifovic (1994).

10. Return to step 4 unless the population's average fit has not changed significantly across generations.

11. After convergence, we polish the equilibrium using a multidimensional Newton-Raphson routine. This routine cannot be used to calibrate the model because the equations determining the market clearing value for r and the calibration target for β do not appear to be independent.

Note that some parameter values are not permitted; for example, μ cannot be larger than one or less than zero. In these cases the fitness of a candidate is assumed to be 10^6; that is, a large penalty function is attached to impermissible combinations. These candidates will be discarded immediately and never breed.

In our implementation of the genetic algorithm and the Newton-Raphson routine, we parallelize computation by sending each separate evaluation of $F(\omega)$ to a separate processor. For the genetic algorithm, each generation requires n evaluations for the new offspring (the parents have already been computed). For the Newton-Raphson routine each step requires 6 evaluations using one-sided numerical derivatives. We could have used the Newton-Raphson routine directly, but we found that our inability to determine a reasonable starting value seriously impacted convergence.

7.4 Monte Carlo Generation of Transition Matrix

The transition matrix for the demographic states turned out to be impossible to write down analytically. The problem is that we wish to remain faithful to the Census data on mortality, marriage, divorce, and fertility. To do so requires that the transition probabilities be dependent on the path taken to a particular state; for example, it matters for mortality of women how many children they have had, not just the number that they current have, due to the inherent health risks associated with childbirth. Also, large numbers of children typically are associated with lower income families who have higher mortality rates as well. We were not able to construct the matrix analytically as a result, since any given current demographic state could have a very large number of histories associated with it. Therefore, we chose the following Monte Carlo approach.

To begin, we draw a random $\text{UNI}(0,1)$ random variable; if below 0.495 the new household is a male, if not it is a female. We then check whether the household dies, gets married, bears children, or survives unchanged, using data from the US Census and CDC to determine age and gender specific probabilities. We truncate the number of children to 4 (which leaves out less than 2.7 percent of the population), we do not allow for multiple births within 1 year, and single males cannot have children (no adoption). In cases of divorce, the children proceed with their mother, and if the last adult in the household dies, all the children living in the household die as well. Given the data and these assumptions, we then let the household age 1 year and repeat the process until death. This procedure is repeated 60 million times; the transition matrix is then estimated using the sample probabilities. Due to sampling error (even with this gigantic number of observations), some states are rarely encountered in the simulation, which leads to some irregularities in the transition matrix used in the program.[16]

This sampling error introduced by our Monte Carlo approach to calculating the transition matrix is not innocuous. Small irregularities in the mortality rates generate large irregularities in life insurance holdings since the premium paid by an individual is tied down by their mortality rate. Thus, we are careful to generate death probabilities which match the observed data. That is, the small dip in the death probability of males around age 30 is actually observed in the data. To insure the correct probability of death, we normalize the transition matrix to the correct death probability. Each row of the matrix is divided by the simulated survival probability and then multiplied by the true survival probability. Each row contains the true survival probability and a smooth death probability is observed over the life cycle.

[16] Matlab code to generate this matrix is available at http://garnet.acns.fsu.edu/~eyoung/programs.

References

[1] American Council of Life Insurance (2000) *Life Insurance Fact Book*, American Council of Life Insurance: Washington, D.C.

[2] Arifovic, J. (1994), "Genetic Algorithm and the Cobweb Model," *Journal of Economic Dynamics and Control* **18**, pp. 3-28.

[3] Auerbach, A.J and L. J. Kotlikoff (1987), "Life Insurance of the Elderly: Its Adequacy and Determinants," in G. Burtless (ed.), *Work, Health, and Income Among the Elderly,* The Brookings Institution, Washington D. C.

[4] Auerbach, A.J and L.J. Kotlikoff (1989), "How Rational is the Purchase of Life Insurance?" NBER Working Paper No. **3063**.

[5] Auerbach, A.J. and L.J. Kotlikoff (1991a), "Life Insurance Inadequacy – Evidence from a Sample of Older Widows," NBER Working Paper No. **3765**.

[6] Auerbach, A.J., and L.J. Kotlikoff (1991b), "The Adequacy of Life Insurance Purchases," *Journal of Financial Intermediation* **1(3)**, pp. 215-41.

[7] Bernheim, B.D., S. Berstein, J. Gokhale, and L.J. Kotlikoff (2002), "Saving and Life Insurance Holdings at Boston University – A Unique Case Study," Central Bank of Chile Working Paper **161**.

[8] Bernheim, B.D., K.G. Carman, J. Gokhale, and L.J. Kotlikoff (2001), "The Mismatch Between Life Insurance Holdings and Financial Vulnerabilities: Evidence from the Survey of Consumer Finances," NBER Working Paper No. **8544**.

[9] Bernheim, B. D., L. Forni, J. Gokhale, and L.J. Kotlikoff (2003), "The Mismatch Between Life Insurance Holdings and Financial Vulnerabilities: Evidence from the Health and Retirement Survey," *American Economic Review* **93(1)**, pp. 354-65.

[10] Black, K., Jr. and H.D. Shipper (1994), *Life Insurance,* Englewood Cliffs, N.J.: Prentice Hall.

[11] Budría, S., J. Díaz-Giménez, V. Quadrini, and J.-V. Ríos-Rull (2002), "Updated Facts on the Distributions of Earnings, Income, and Wealth," Federal Reserve Bank of Minneapolis *Quarterly Review* **26(3)**, pp. 2-35.

[12] Chambers, M.S., D.E. Schlagenhauf, and E.R. Young (2003), "Are Husbands Really That Cheap? Exploring Life Insurance Holdings," manuscript, Florida State University.

[13] Díaz-Giménez, J., V. Quadrini, and J.-V. Ríos-Rull (1997), "Dimensions of Inequality: Facts on the U.S. Distributions of Earnings, Income, and Wealth," Federal Reserve Bank of Minneapolis *Quarterly Review* **21(2)**, pp. 3-21.

[14] Duffy, J. and P.D. McNelis (2001), "Approximating and Simulating the Stochastic Growth Model: Parameterized Expectations, Neural Networks, and the Genetic Algorithm," *Journal of Economic Dynamics and Control* **25**, pp. 1273-1303.

[15] Fernández-Villaverde, J. and D. Krueger (2000), "Consumption and Saving over the Life Cycle: How Important Are Consumer Durables?" unpublished manuscript, University of Minnesota.

[16] Gokhale, J. and L.J. Kotlikoff (2002), "The Adequacy of Life Insurance," manuscript, Boston University.

[17] Greenwood, J., Guner, N., and J.A. Knowles (2000), "Women on Welfare: A Macroeconomic Analysis," *American Economic Review* **90**, pp. 383-88.

[18] Guvenen, M.F. (2001), *Essays on Limited Stock Market Participation and the Macroeconomy*, unpublished dissertation, Carnegie Mellon University.

[19] Herrera, F., M. Lozano and J.L. Verdegay (1998), "Tackling Real-coded Genetic Algorithms: Operators and Tools for Behavioral Analysis," *Artificial Intelligence Review* **12**, pp. 265-319.

[20] Hurd, M.D., and D.A. Wise (1989), "The Wealth and Poverty of Widows: Assets Before and After the Husband's Death," in D. Wise (ed.) *The Economics of Aging,* Chicago: University of Chicago Press, pp. 177-99.

[21] Krusell, P. and A.A. Smith, Jr. (1997), "Income and Wealth Heterogeneity, Portfolio Choice, and Equilibrium Asset Returns," *Macroeconomic Dynamics* **1(2)**, pp. 387-422.

[22] Lucas, R.E., Jr. (2003), "Macroeconomic Priorities," *American Economic Review* **93(1)**, pp. 1-14.

[23] McCarthy, David and Olivia S. Mitchell (2003), "International Adverse Selection in Life Insurance and Annuities," NBER Working Paper No. **9975**.

[24] McGrattan, E.R. and E.C. Prescott (2003), "Average Debt and Equity Returns: Puzzling?" Staff Report Number **313**, Federal Reserve Bank of Minneapolis.

[25] Press, W.H., S.A. Teukolsky, W.T. Vetterling, and B.P. Flannery (1993), *Numerical Recipes in FORTRAN: The Art of Scientific Computing*, Cambridge University Press.

[26] Storesletten, K., C.I. Telmer, and A. Yaron (2001), "Cyclical Dynamics in Idiosyncratic Labor-Market Risk," forthcoming, *Journal of Political Economy.*

[27] Tauchen, G. (1986), "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions," *Economics Letters* **20**, pp. 177-181.

[28] Young, E.R. (2002), "Unemployment Insurance and Capital Accumulation," manuscript, Florida State University.

[29] Zick, C.D. and K. Holden (2000), "An Assessment of the Wealth Holdings of Recent Widows," *Journals of Gerontology Series B: Psychological Sciences and Social Sciences* **55(2)**, pp. S90-7.

Table 1

Summary of Household Economic Characteristics

	Sample Size	Average Age (Head)	Average HH Size	Average Earnings	Average Income	Average Wealth
Total	4,305	48.7	2.48	42,369	52,295	283,179
By Earnings						
1st Quintile	733	61.2	1.80	-344	29,594	270,170
2nd Quintile	721	47.4	2.16	8,586	25,835	126,851
3rd Quintile	689	43.7	2.37	27,657	31,254	94,636
4th Quintile	674	44.2	2.72	48,875	48,464	153,891
5th Quintile	1488	47.1	2.99	128,366	130,689	777,760
By Income						
1st Quintile	675	51.0	1.93	4,496	7,579	48,400
2nd Quintile	635	50.9	2.01	14,494	20,062	94,688
3rd Quintile	654	47.1	2.42	29,454	33,796	128,780
4th Quintile	681	46.7	2.73	48,162	54,136	206,860
5th Quintile	1660	47.9	2.96	116,584	150,870	949,219
By Wealth						
1st Quintile	715	39.5	2.29	16,944	19,175	-4,055
2nd Quintile	637	42.5	2.40	27,635	29,486	19,286
3rd Quintile	577	50.7	2.42	35,233	39,741	73,289
4th Quintile	618	54.6	2.38	42,567	50,681	177,223
5th Quintile	1758	56.4	2.55	90,255	123,562	1,164,468
By Age						
17-29	506	25.1	2.17	26,193	26,482	30,399
30-39	764	34.8	3.10	49,174	49,897	132,517
40-49	969	44.3	2.91	62,418	66,238	273,539
50-59	867	54.1	2.23	60,218	71,608	455,020
60-0ver	1199	72.3	1.69	17,764	44,073	433,590
By Family Type						
married	2578	48.7	2.41	41,426	52,788	287,991
one worker	1343	48.8	2.45	41,136	51,826	285,233
two worker	1235	48.6	2.38	41,686	53,648	290,458
single-male NM	246	43.6	1.53	28,525	37,289	183,167
single-female NM	352	52.7	1.75	14,049	26,052	127,106
single-female widow	76	51.1	1.76	13,390	24,825	123,623

Table 2
Summary of Household Life Insurance Characteristics

	Total Life Ins.	Total Term	Total Whole	Average Holdings	Average Term	Average Whole	Insurance Participation
Total (bils $)	11,785	8,154	3,630	114,993	79,526	35,407	68.7%
By Earnings							
1st Quintile	508	269	239	24,739	13,114	11,624	57.3%
2nd Quintile	603	368	234	29,304	17,922	11,382	53.5%
3rd Quintile	1,281	1,001	279	62,303	48,691	13,612	65.3%
4th Quintile	2,661	1,988	672	129,308	96,631	32,677	81.1%
5th Quintile	6,731	4,526	2,205	332,278	223,429	108,849	88.9%
By Income							
1st Quintile	541	413.9	127.4	26,314	20,121	6,193	44.6%
2nd Quintile	833	634.4	198.7	40,478	30,825	9,653	61.6%
3rd Quintile	1,463	1,073.9	389.8	71,080	52,149	18,931	77.1%
4th Quintile	2.574	1,903.3	671.3	125,011	92,415	32,596	80.9%
5th Quintile	6,372	4,129.3	2,243.5	315,368	204,345	111,023	81.9%
By Wealth							
1st Quintile	878	766.9	111.4	42,684	37,268	5,416	44.7%
2nd Quintile	1,104	875.4	229.1	53,736	42,589	11,147	53.5%
3rd Quintile	2,125	1,749.3	375.8	103,163	84,917	18,246	65.3%
4th Quintile	2,203	1,573.7	630.2	107,146	76,506	30,640	81.1%
5th Quintile	5,473	3,189.6	2,284.1	270,425	157,580	112,845	88.9%
By Age							
17-29	960	780	179	67,218	54,634	12,585	53.4%
30-39	3,226	2,414	812	151,322	113,220	38,102	68.5%
40-49	4,028	2,855	1,172	178,712	126,685	52,027	73.6%
50-59	2,384	1,487	897	139,137	86,769	52,368	76.9%
60-over	1,186	616	569	43,549	22,640	20,910	69.6%
married	7,082	4,823	2,258	114,863	78,233	36,630	68.7%
one worker	3,441	2,392	1,049	118,208	82,161	36,048	67.7%
two worker	3,640	2,431	1,209	111,870	74,718	37,152	69.5%
single-male NM	455	331	124	73,995	53,750	20,245	59.1%
single-female NM	383	292	91	35,236	26,840	8,395	58.0%
single-female widow	73	62	11	32,021	27,127	4,894	62.3%

Table 3

Demographics of Simulated Economy

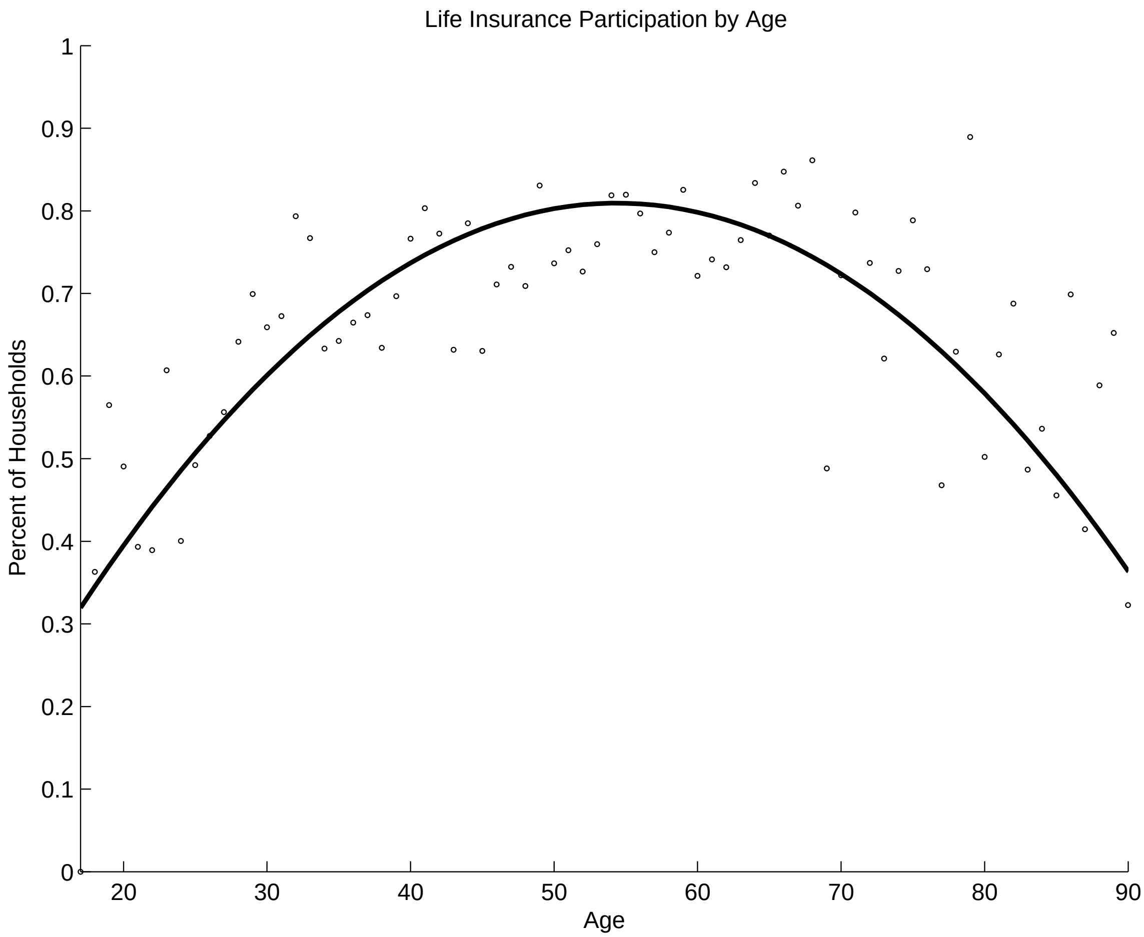
Characteristic	Percent of Population
Married	68.02
Single	31.98
Divorced	7.25
Widowed	14.49
Never Married	10.24
0 Kids	76.63
1 Kid	18.83
2 Kids	4.30
3 Kids	0.20
4 Kids	0.01

Table 4

Calibration Results

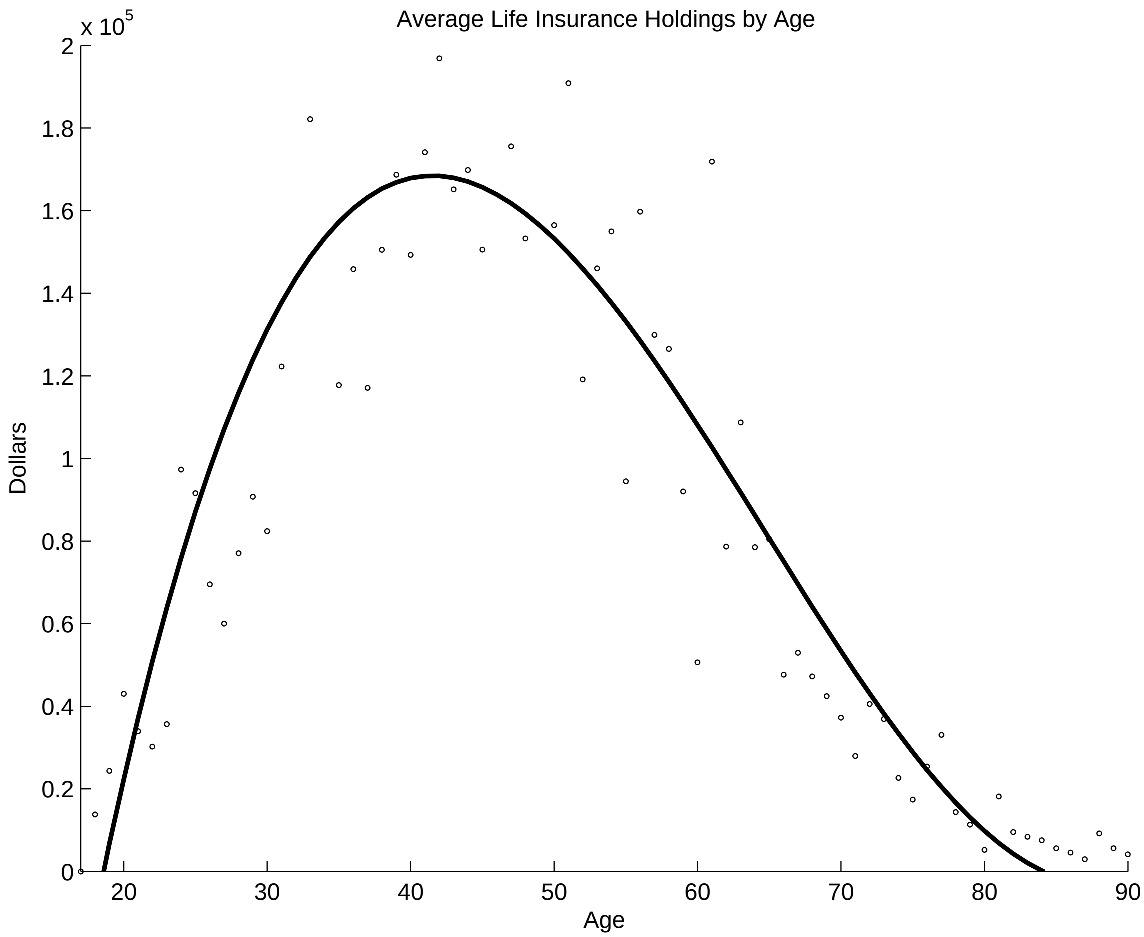
Variable	$A = 0$	$A = 0.008$	No LI
r	0.1131	0.1131	0.1131
q	NA	0.0475	NA
$\overline{k}$	0.2996	0.3001	0.2996
s	0.0055	0.0055	0.0055
ϖ	0.1674	0.1674	0.1674
β	0.9828	0.9828	0.9828
μ	0.2285	0.2285	0.2285
χ	0.5750	0.5750	0.5750

Figure 1



Life Insurance Participation by Age
Percent of Households
Age
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
20
30
40
50
60
70
80
90

Figure 2



Average Life Insurance Holdings by Age
x 10^5
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Dollars
20
30
40
50
60
70
80
90
Age

Figure 3

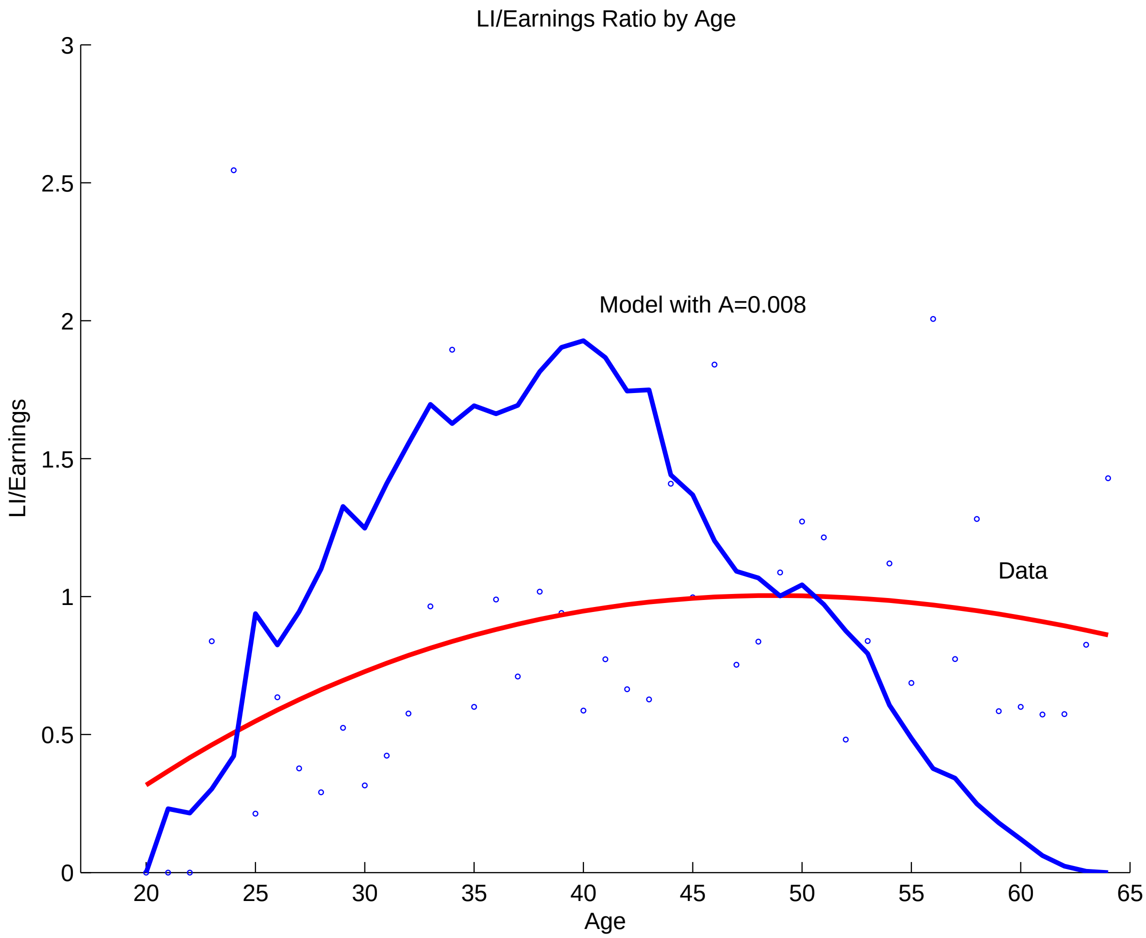
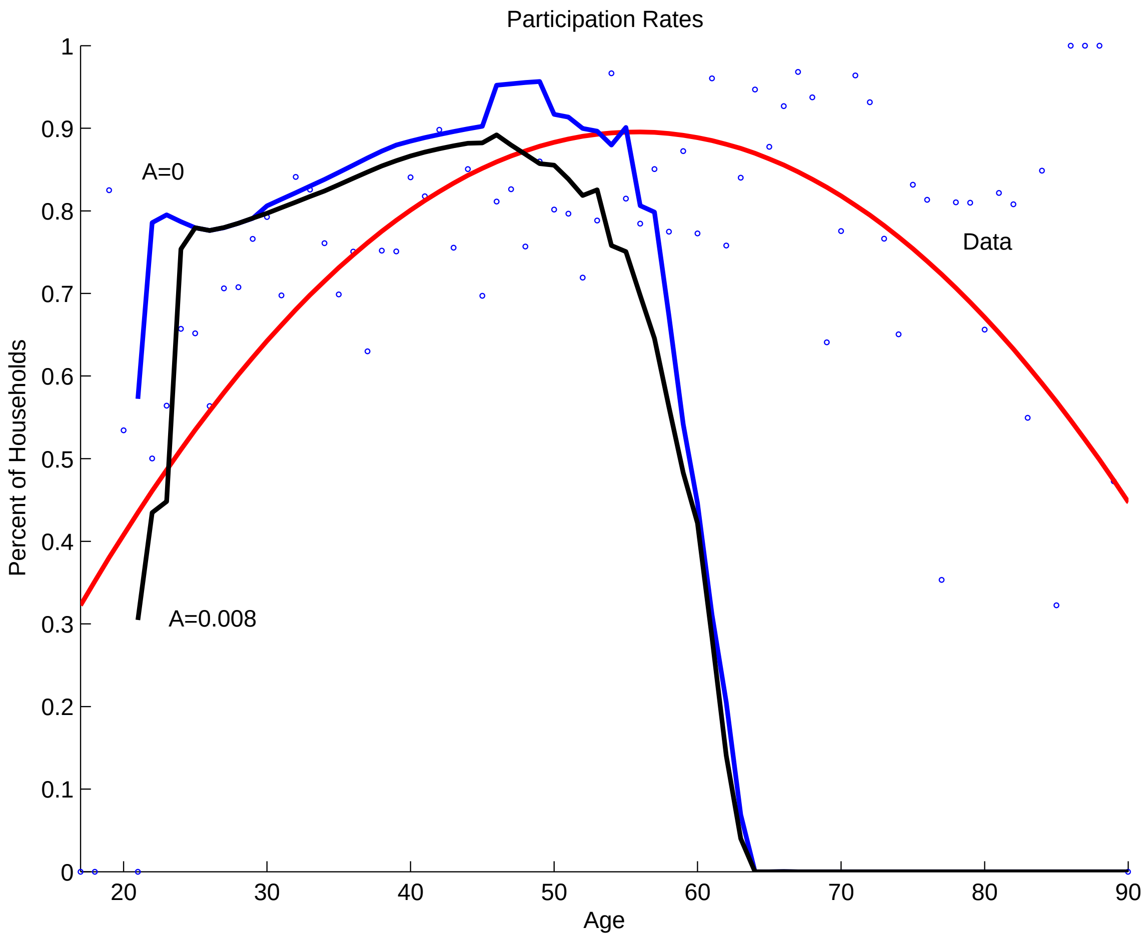

LI/Earnings Ratio by Age
Model with A=0.008
Data
LI/Earnings
Age
0
0.5
1
1.5
2
2.5
3
20
25
30
35
40
45
50
55
60
65

Figure 4



Participation Rates
A=0
A=0.008
Data
Percent of Households
Age
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
20
30
40
50
60
70
80
90

Figure 5

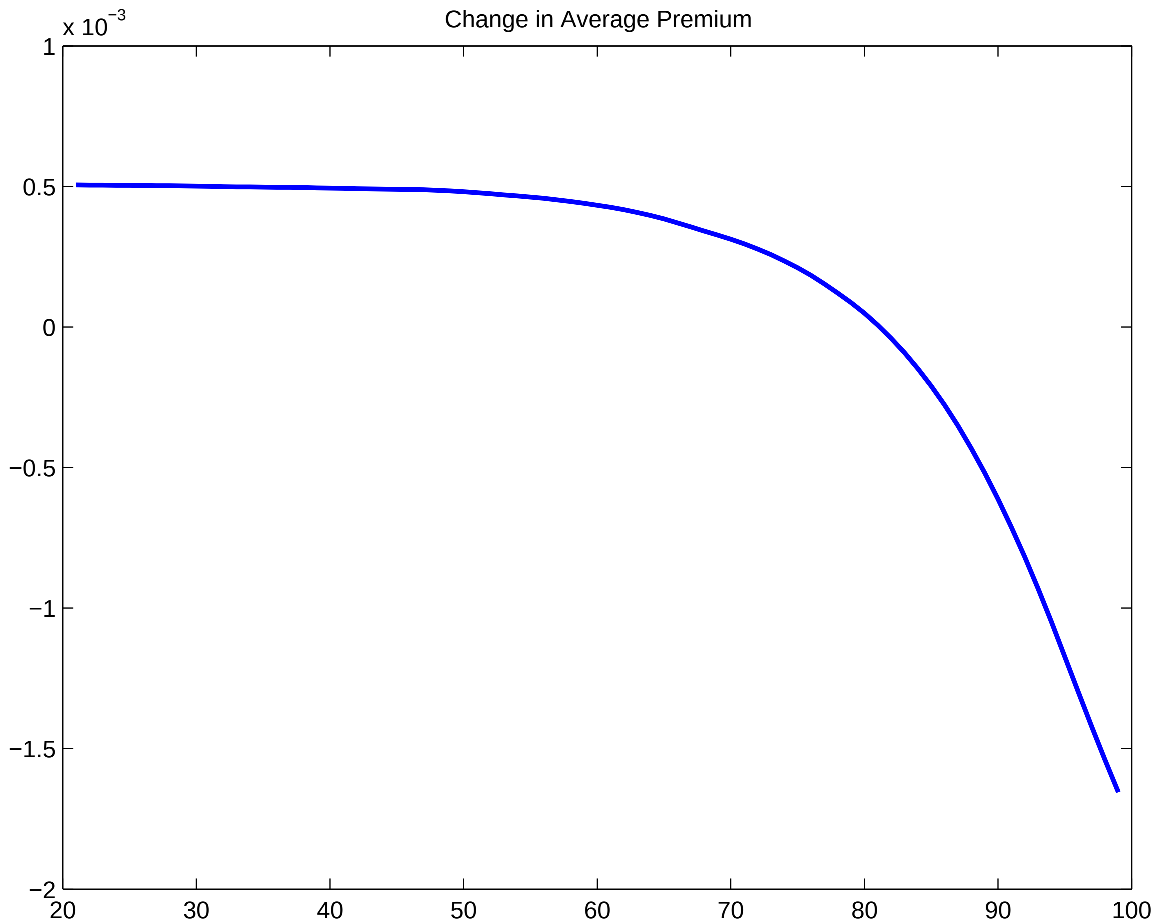

x 10^−3
Change in Average Premium
1
0.5
0
−0.5
−1
−1.5
−2
20
30
40
50
60
70
80
90
100

Figure 6

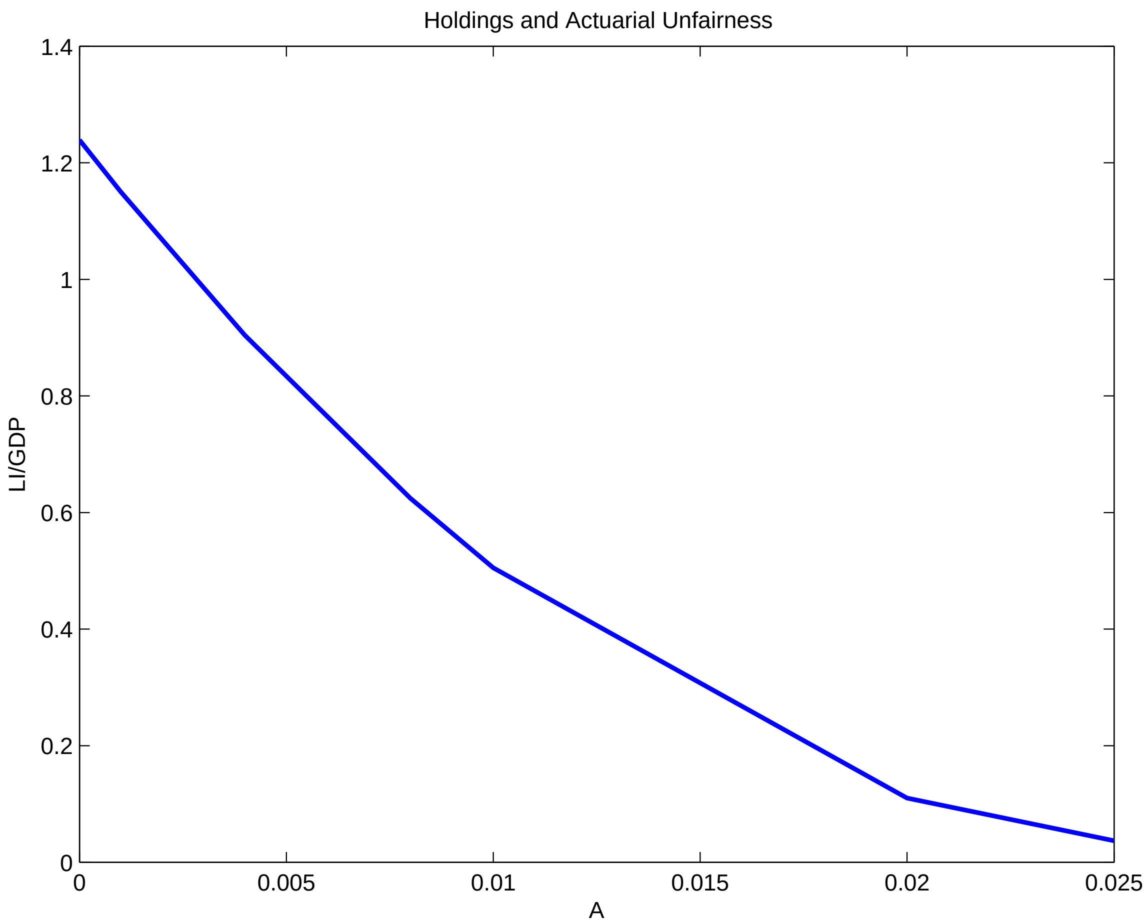

Holdings and Actuarial Unfairness
LI/GDP
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.005
0.01
0.015
0.02
0.025
A

Figure 7

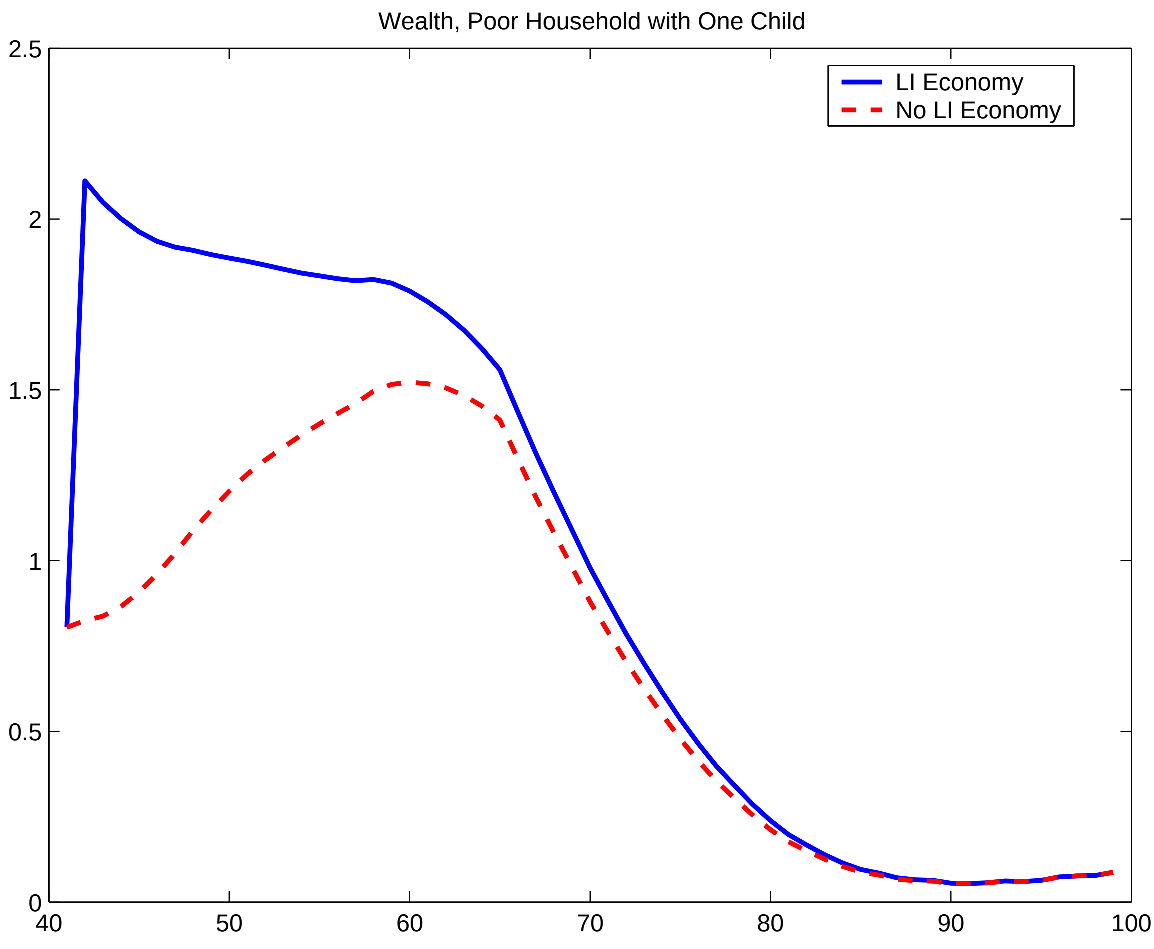

Wealth, Poor Household with One Child
2.5
2
1.5
1
0.5
0
40
50
60
70
80
90
100
LI Economy
No LI Economy

Figure 8

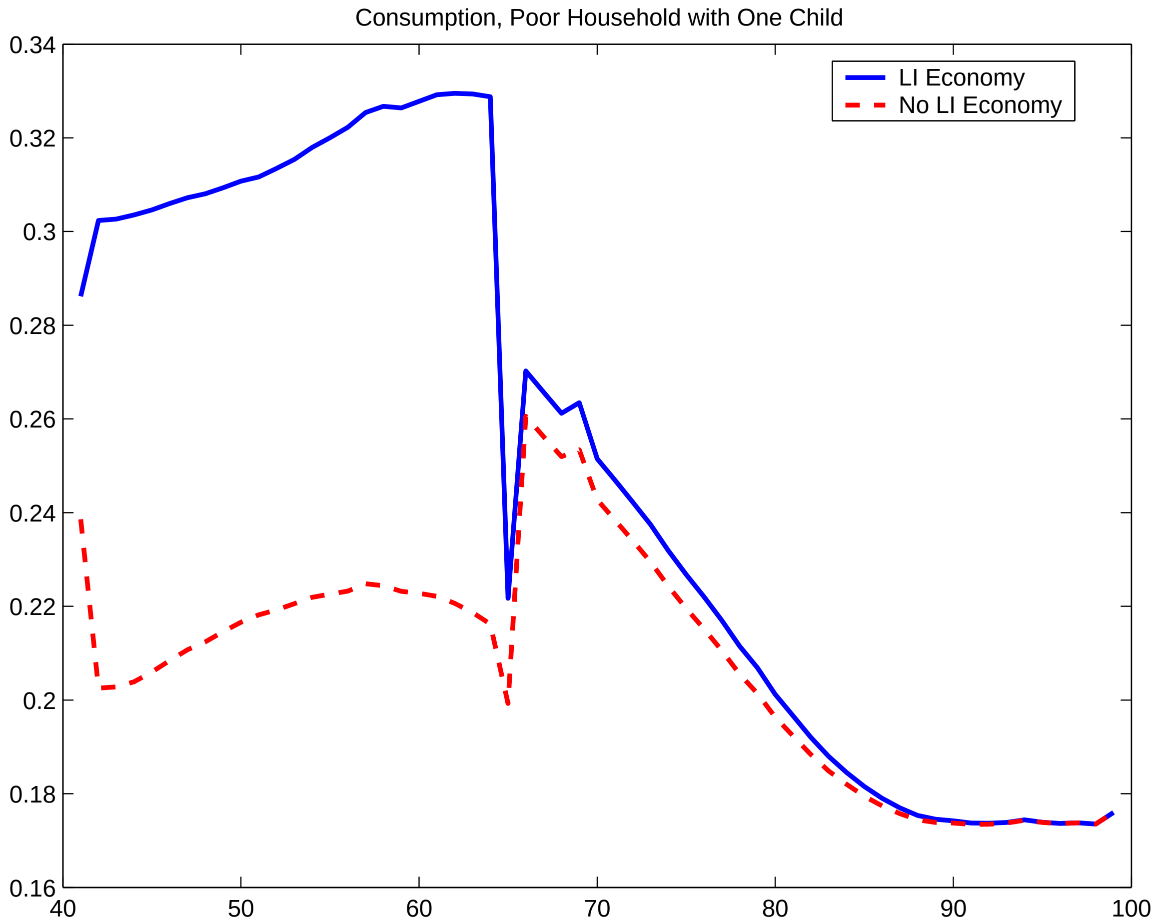
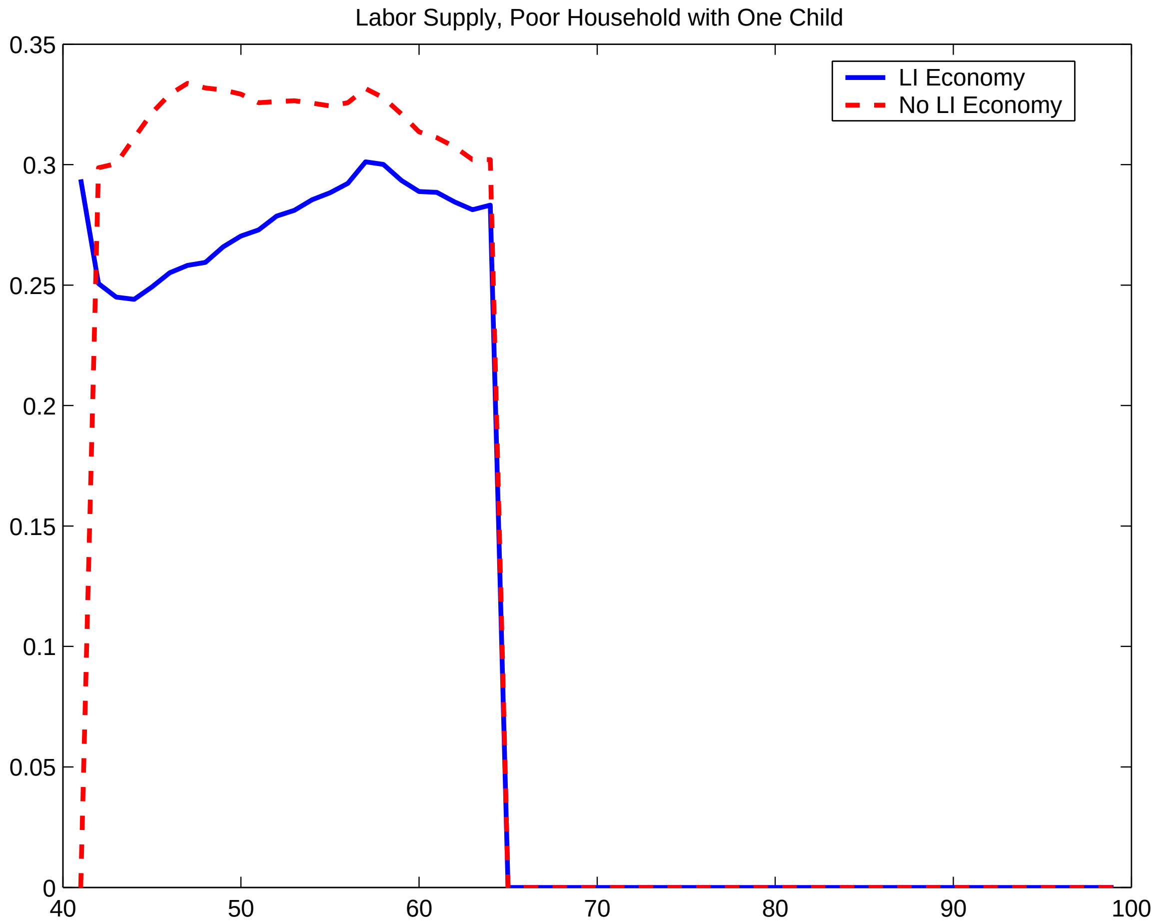
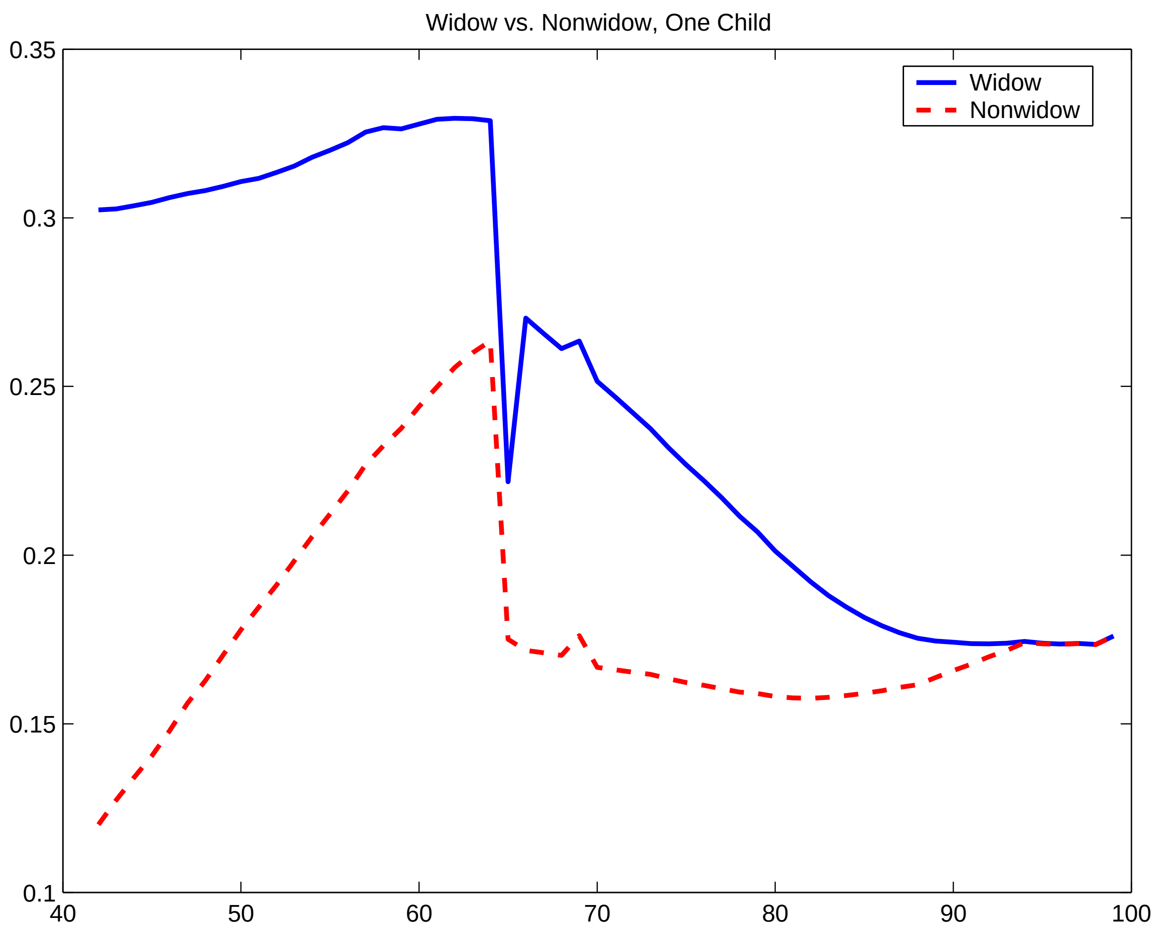

Consumption, Poor Household with One Child
0.34
0.32
0.3
0.28
0.26
0.24
0.22
0.2
0.18
0.16
40
50
60
70
80
90
100
LI Economy
No LI Economy

Figure 9



Labor Supply, Poor Household with One Child
LI Economy
No LI Economy
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
40
50
60
70
80
90
100

Figure 10



Widow vs. Nonwidow, One Child
0.35
0.3
0.25
0.2
0.15
0.1
40
50
60
70
80
90
100
Widow
Nonwidow

Figure 11

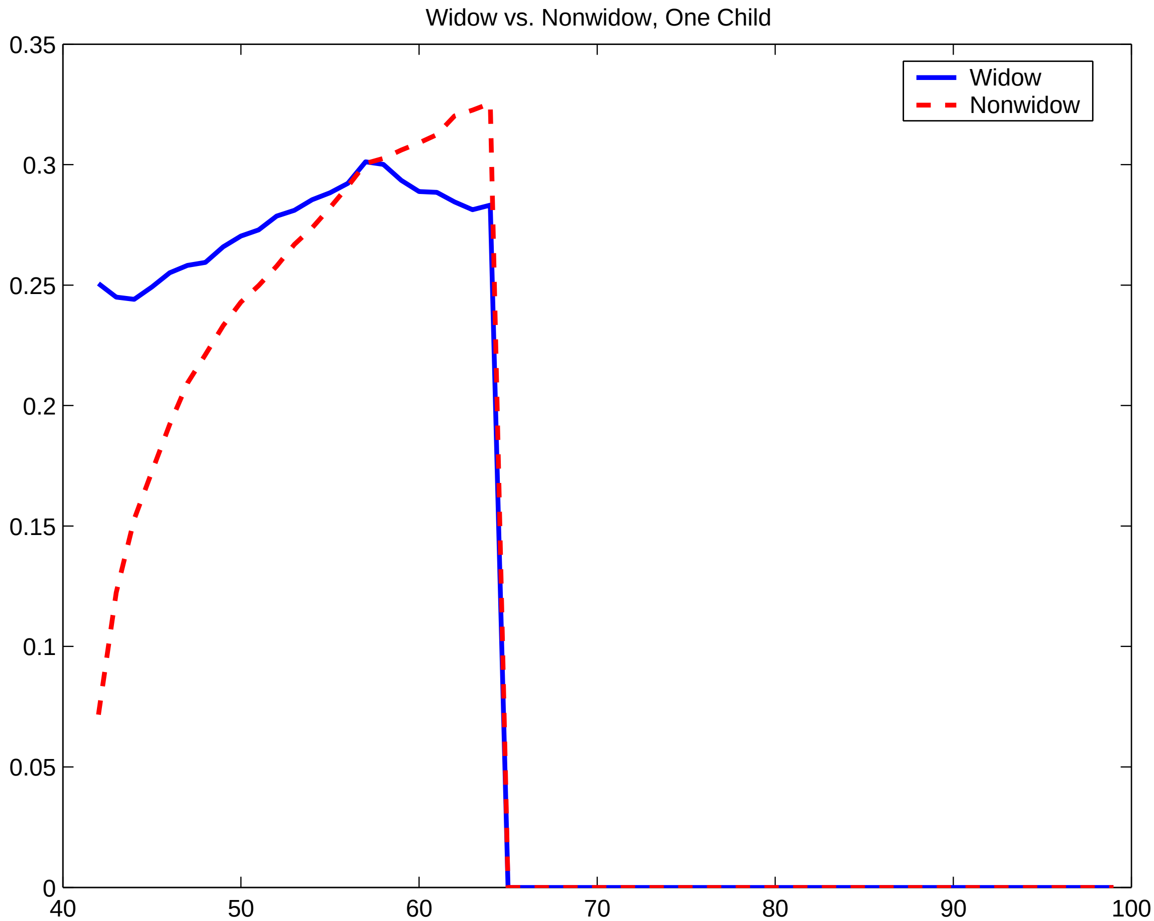

Widow vs. Nonwidow, One Child
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
40
50
60
70
80
90
100
Widow
Nonwidow

Figure 12

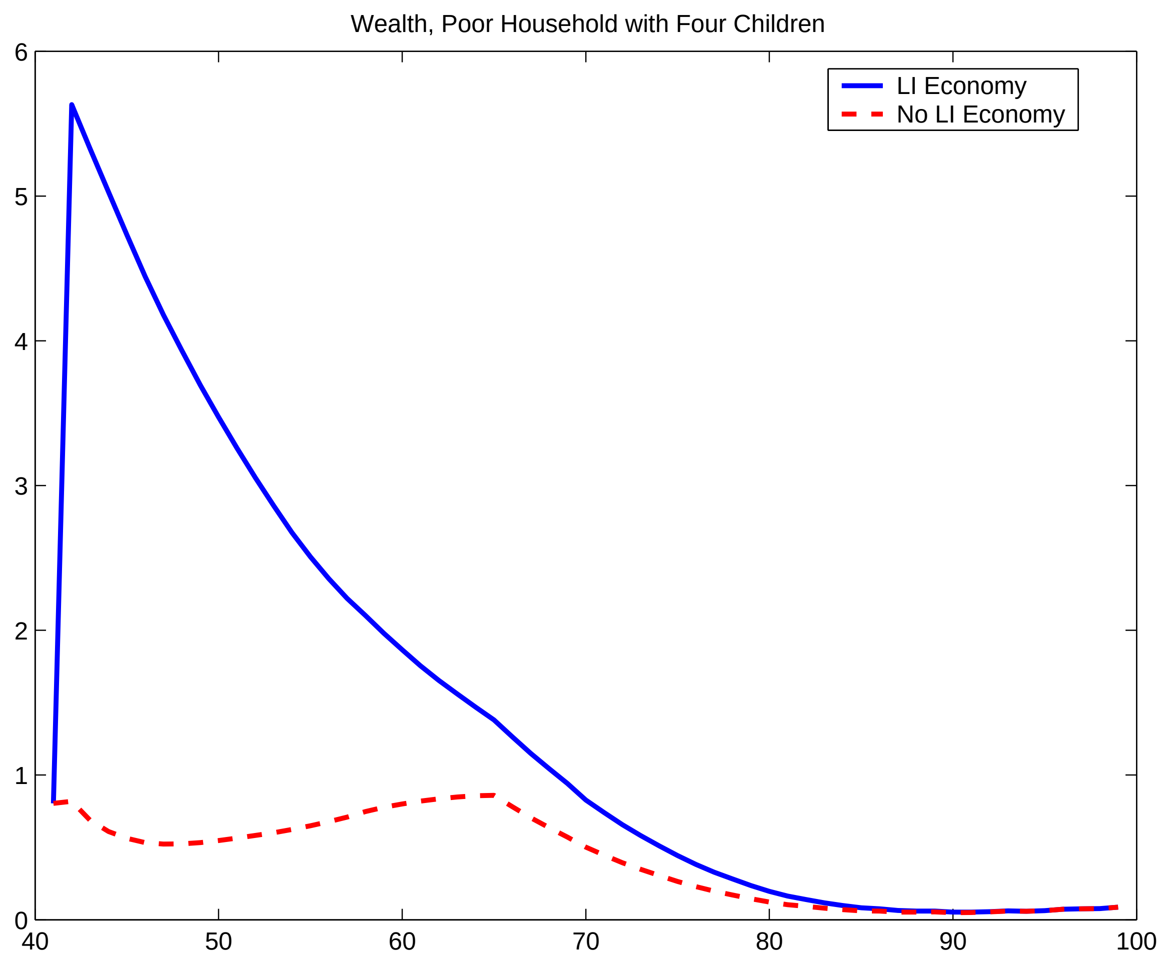

Wealth, Poor Household with Four Children
LI Economy
No LI Economy
0
1
2
3
4
5
6
40
50
60
70
80
90
100

Figure 13

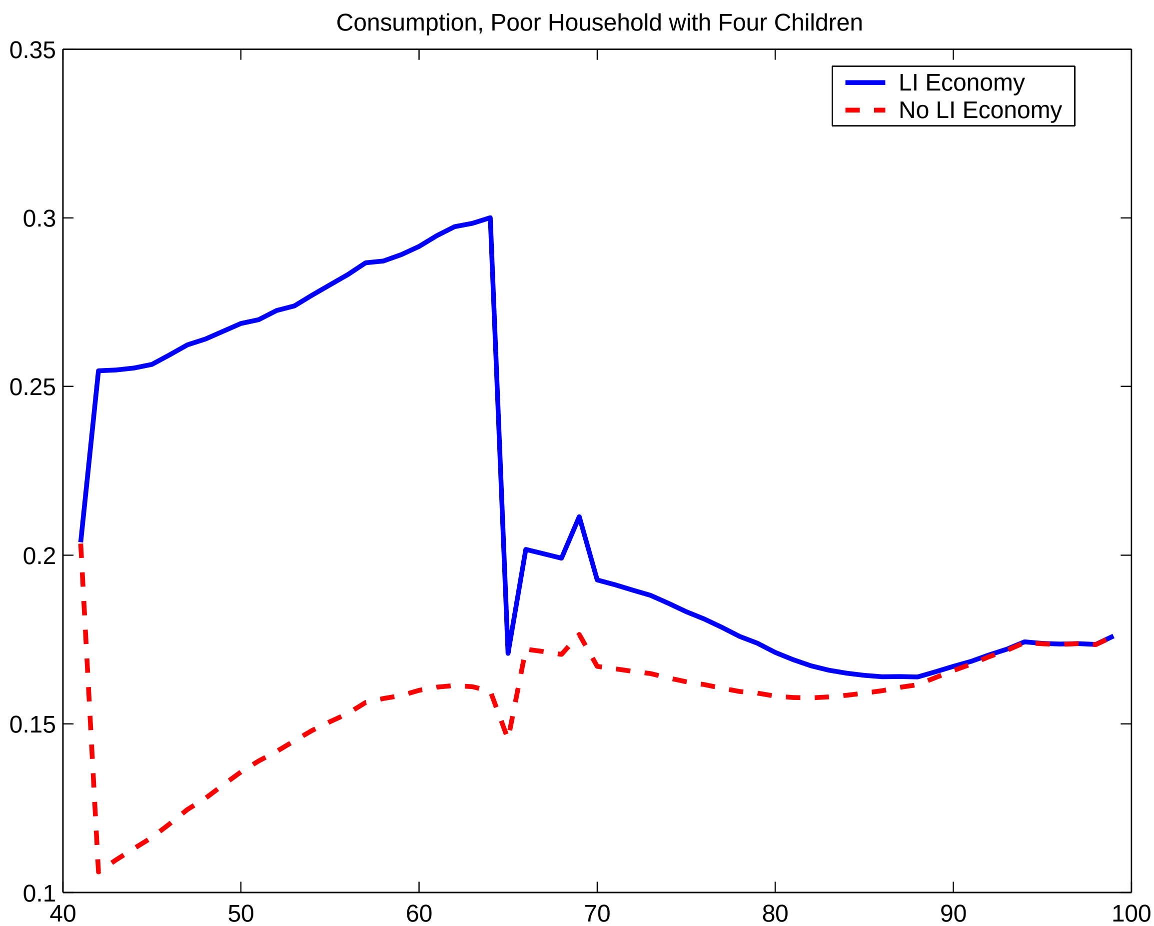
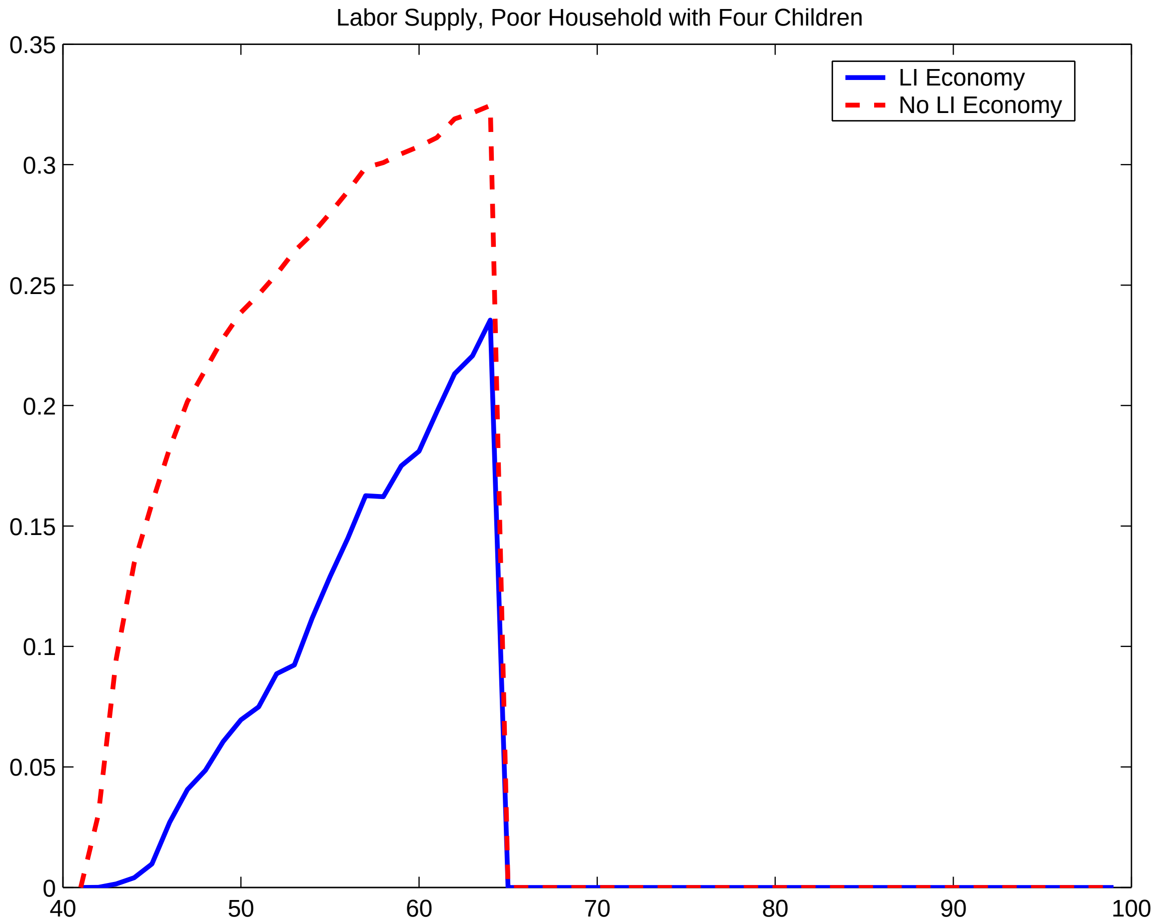
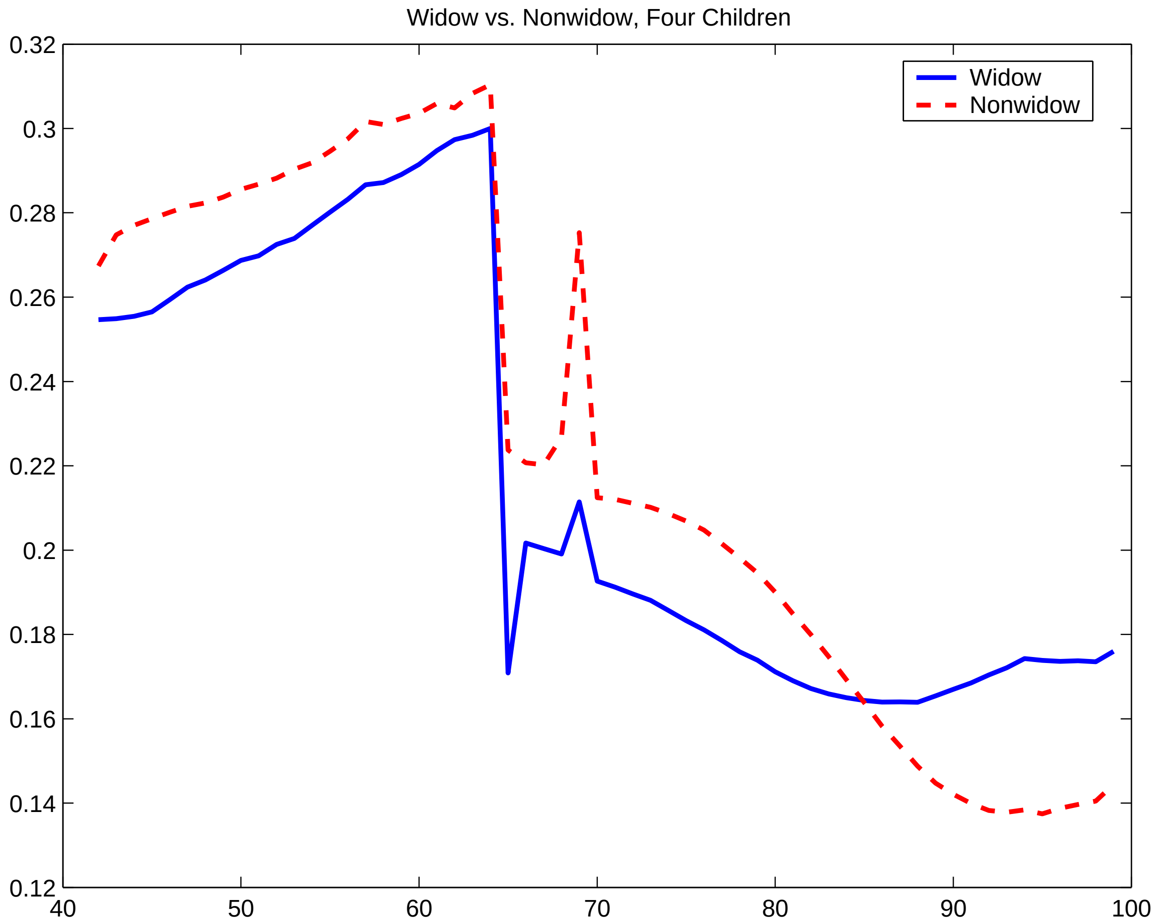

Consumption, Poor Household with Four Children
LI Economy
No LI Economy
0.35
0.3
0.25
0.2
0.15
0.1
40
50
60
70
80
90
100

Figure 14



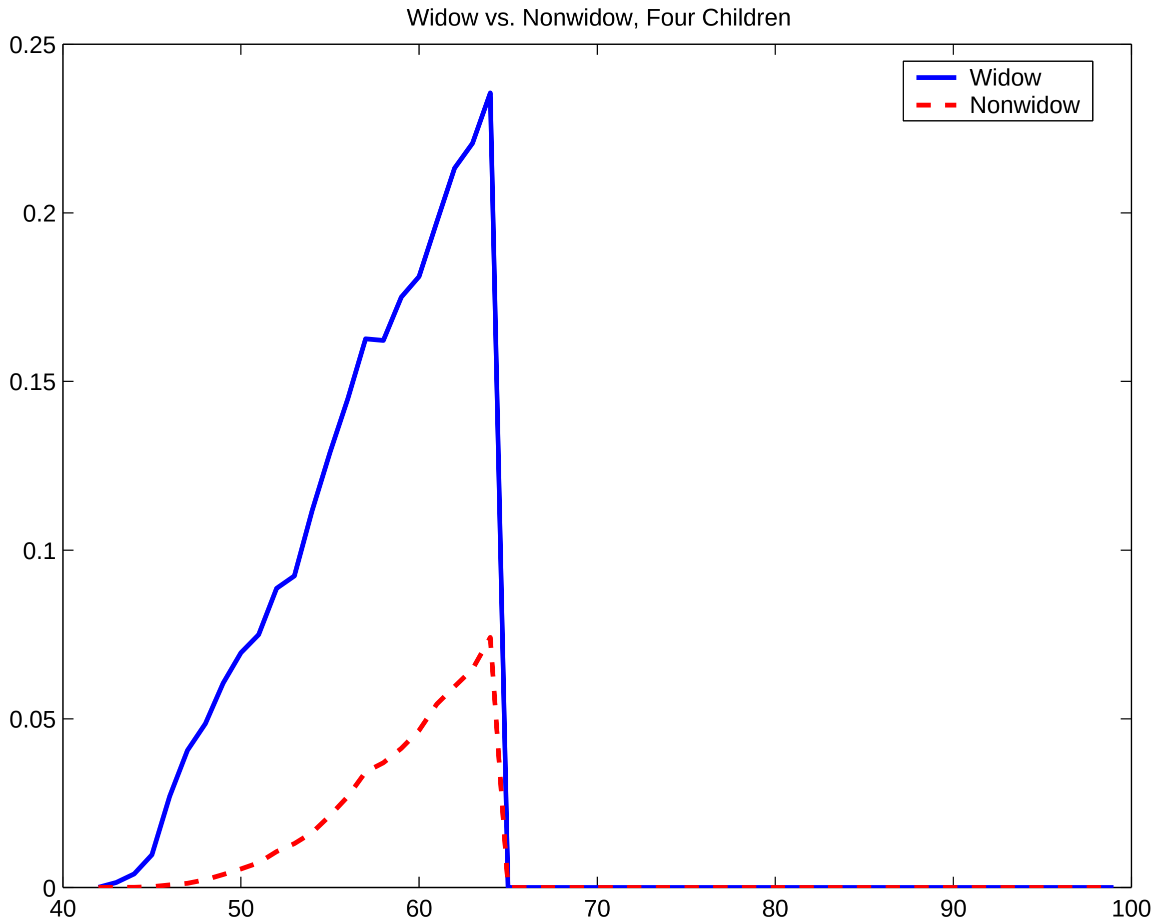
Labor Supply, Poor Household with Four Children
LI Economy
No LI Economy
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
40
50
60
70
80
90
100

Figure 15



Widow vs. Nonwidow, Four Children
Widow
Nonwidow
0.32
0.3
0.28
0.26
0.24
0.22
0.2
0.18
0.16
0.14
0.12
40
50
60
70
80
90
100

Figure 16


Widow vs. Nonwidow, Four Children
Widow
Nonwidow
0.25
0.2
0.15
0.1
0.05
0
40
50
60
70
80
90
100