A Phillips Curve with An Ss Foundation

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November, 2004

Extremely Preliminary. Please do not cite or quote.

1. Introduction

In recent years there has been considerable progress in developing structural models of inflation that are potentially useful for econometric modelling and policy evaluation. A common aspect of this approach is to start with the individual firm's price setting problem, obtain optimal decision rules, and then aggregate up behavior. The net result is a simple relation for inflation that is much in the spirit of a traditional a Phillips curve relation: inflation depends on some measure of real activity as well as expectations of the future. In addition to its forward looking nature, this relationship also differs from the traditional Phillips curve in that the coefficients are explicit functions of the primitives of the model, including the degree of price rigidity. Overall, these new Phillips curves (often grouped under the heading of the "New Keynesian") are beginning to become standard features of many modern empirical macroeconomic frameworks.

To date, these new Phillips curves reflect a pragmatic compromise between theoretical rigor and the need for empirical tractability.[1] While they evolve ultimately from optimization at the individual firm, they are typically based on time-dependent pricing strategies in which the length of the period over which prices are set is given exogenously. The alternative, of course, is state-dependent pricing, where the firm is free to adjust whenever it would like, subject to a fixed cost of price adjustment. This latter approach leads to "Ss" pricing policies which are, in general, difficult to aggregate up.[2] For this

[1] Examples include Gali and Gerler (1999, 2001), Sbordone, (2001), and Eichenbaum and Fisher (2004).

[2] See Caplin and Spulber (1985), Caplin and Leahy (1991,1997), Caballero and Engel (1991) for early analyses of dynamic Ss economies. Dotsey, King and Wolman (1999) place Ss policies within a standard dynamic stochastic general

reason, the time-dependent approach has proven to be the most popular, despite the unattractiveness of arbitrarily fixing the degree of price rigidity.

Besides tractability considerations, however, there have been two additional justifications for the time-dependent approach. First, Klenow and Kryvtsov (1999) have shown that, during the recent low inflation period in the United States, the fraction of firms that adjust their prices in any given quarter is reasonably stable, which is certainly consistent with time-dependent pricing. Second, in this spirit, it is often conjectured that time dependent models are the natural reduced forms of a state dependent framework, for economies with relatively stable inflation.

A interesting recent paper by Golosov and Lucas (GL) challenges this rationalization. As the authors point out, even if the price adjustment rates are stable (due to moderate inflation variability), there remains an important difference between state-dependence and time-dependence: under the state-dependent pricing, the firms farthest away from their target price adjust, whereas under time-dependence there is no relation between how far a firm is from its target price and whether or not its target price is changing. The authors then go on to illustrate how the state-dependent approach could lead to far greater price flexibility, and conversely weaker real effects of money, than a standard time-dependent model calibrated to have a similar degree of price stickiness at the firm level. Overall, the argument is reminiscent of the theoretical example in Caplin and Spulber, where state-dependence can turn the non-neutrality of money result from time-dependence on its head.

Because of the overall complexity their framework, Golosov and Lucas restrict attention to numerical simulation of a model that otherwise has very simple features. Among other things, they abstract from almost all interactions among firms, including real rigidities that have proved to be important in the standard approach for confronting the data. Since these interfirm interactions are the main source of persistence in time-dependent models, these simplifications make it difficult to judge in general whether state-dependence undoes the results of the conventional literature.

The purpose of our paper is to address this controversy by developing a simple Phillips curve relation that has an Ss foundation. We differ from the existing Ss literature by making assumptions that deliver simple analytical results comparable to the literature on time-dependent pricing rules. As with the standard time-dependent literature and Dotsey, King and Wolman's (DKW) state-dependent framework we focus an local approximation around the steady state. We differ from DKW by introducing idiosyncratic shocks, as in GL, and by placing restrictions on the distribution of shocks that permit an approximate analytical solution.[3] Idiosyncratic shocks are essential for matching the microeconomic evidence of Klenow and Kryvtsov who show that the average size of price adjustment is around 10% and the median time between price adjustments is less about five months at a time when the average annual inflation rate in the United States is less than 3%. The end result is a Phillips curve

equilibrium model.

[3] We borrow our distributional assumption from Danziger (1999) who solves a carefully parameterized Ss economy in closed form. We differ from Danziger in that we allow for a more flexible parameterization of the model and we linearize about steady state.

built up explicitly from state-dependent pricing at the micro level that is comparable in simplicity and tractability to the standard New Keynesian Phillips curve that arises from the time-dependent pricing

Because we restrict attention to a local approximation around a zero inflation steady state (as in time dependent literature), our analysis is limited to economies with low and stable inflation. We this cannot use our Ss framework to analyze the effect of large regime changes (which, of course is also a limitation of the time-dependent approach.) On the other hand, our framework does capture the "selection" effect of state-dependent pricing: those farthest away from target tend to adjust more frequently, a feature that need not arise in time-dependent pricing. We can thus use our model to assess quantitatively how much extra price flexibility state-dependence adds relative to time dependence. Furthermore, because our framework is tractable, we can allow for the same kinds of features that are thought key in the standard time-dependent models, including in particular real rigidities.

In section 2 we lay out the basic features of the model: a simple New Keynesian framework, but with state-dependent as opposed to time-dependent pricing. Also, firms face idiosyncratic productivity shocks which as in GL enables the model to confront the micro evidence on both the size and frequency of price adjustment. We differ from GL by assuming that at any moment in time, there is a spatial nature to the idiosyncractic shock; i.e., at any moment only a subset of the economy is hit by the turbulence from idiosyncratic shocks. This assumption turns out to be key for the aggregate dynamics of inflation, as we show. In section 3 we characterize the firm's optimal pricing policy. We make assumptions on the size of the adjustment costs that make it reasonable to consider a second-order approximation of the firm's objective function. We then turn to the key result of the paper: given certain restrictions on the idiosyncratic shocks, a firm adjusting at time t can ignore the future states of the world where an idiosyncratic shocks hit, up to a second order. Put differently, up to a second order, the firms's continuation value conditional on a idiosyncratic shock at $t + 1$ is independent of its price at t. This result greatly simplifies firm's decision problem. We proceed to derive the firm's optimal Ss policy, which includes the choice of the target and the Ss bands. We derive both the steady state and a local approximation around the steady state.

In section 4 we characterize the complete model, focusing on a log-linear approximation of the steady state. We first derive a Phillips curve relation that is very similar in form to the New Keynesian Phillips curve. In section 6 we discuss some of the theoretical properties of our model. One important difference between our model and the standard framework is that the key primitive parameter in the equation is the Poisson arrival process for the idiosyncratic shock, as opposed to a measure of the degree of price rigidity. The reason for this distinction is that in our state-dependent framework, the frequency of price adjustment is endogenous and cannot be taken as a model primitive. Since not all firms adjust in the wake of an idiosyncratic shock, the stochastic process for the idiosyncratic shock cannot be taken as measure of the degree of price rigidity. Indeed, there is a limiting case of our model in which the idiosyncratic shock hits in every period, where monetary policy is completely neutral but the degree of micro price stickiness is consistent with the evidence. This limiting case provides a good illustration of the arguments in GL and Caplin and Spulber that state-dependence can deliver radically different results from time-dependence. On the other hand, we also show that the framework can deliver the kind of aggregative price level stickiness emphasized in the time-dependent literature and remain consistent with the microeconomic evidence on price adjustment. In this case, as we discuss, real rigidities play an important role. We illustrate all these arguments with some numerical exercises in section 6. Concluding remarks are in section 7.

2. Model: Environment

The framework we develop is a variation of a conventional New Keynesian model. The basic features include monopolistic competition, money and nominal price stickiness. For convenience, there are only consumption goods: We abstract from capital and investment.

As we noted in the introduction, we replace the assumption of time-dependent pricing with a state-dependent approach. As in Golosov and Lucas, we allow for idiosyncratic productivity shocks. This feature permits the model to match the firm level evidence on price adjustment. In contrast to GL, we allow for real rigidities which are critical to the aggregate inflation/output dynamics. In this section we lay out the basic ingredients of model. There are three types of agents: households, final goods firms and intermediates goods firms. We describe each in turn.

2.1. Households

Households consume, supply labor, hold money and hold bonds. The latter are zero in net supply. To allow for real rigidities, we assume a segmented labor market. In particular, we assume a continuum of "islands" of mass unity. On each island, there are a continuum of households of mass unity. Households can only supply labor on the island that they live. On the other hand, there is perfect consumption insurance across islands. In addition, any firm profits are redistributed lump sum to households.

Let C_t be consumption; M_t nominal money balances; P_t the nominal price index; $N_{z,t}$ labor supply on island z; $W_{z,t}$ the nominal wage on island z; $\Gamma_{z,t}$ lump sum transfers (including insurance, dividends and net taxes); B_t one period nominal discount bonds; and R^n_{t+1} the nominal interest rate from t to $t+1$. Then the objective for a representative household on island z is given by:

$$\max E_t \sum \beta^i \left\{ \log \left[C_{t+i} \cdot (\frac{M_{t+i}}{P_{t+i}})^{\nu} \right] - \frac{1}{1+\varphi} N_{z,t+i}^{1+\varphi} \right\} \tag{2.1}$$

subject to budget constraint:

$$C_t = \frac{W_{z,t}}{P_t} N_{z,t+i} + \Gamma_{z,t} - \frac{M_t - M_{t-1}}{P_t} - \frac{(1/R^n_{t+1})B_t - B_{t-1}}{P_t} \tag{2.2}$$

We index labor supply and the nominal wage by z because the island z labor market is segmented. Since there is perfect consumption insurance, there is no need to similarly index the other variables, except for lump sum transfers, which may be island-specific.

The first order necessary conditions for labor supply, consumption/saving, and money demand are given by, respectively,

$$\frac{W_{z,t}}{P_t} = \frac{N^{\varphi}_{z,t+i}}{(1/C_t)} \tag{2.3}$$

$$E_t \left\{ \beta \frac{C_t}{C_{t+1}} R^n_{t+1} \frac{P_t}{P_{t+1}} \right\} = 1 \tag{2.4}$$

$$\frac{M_t}{P_t} = \nu C_t \frac{R^n_{t+1}}{R^n_{t+1} - 1} \tag{2.5}$$

2.2. Final goods firms

Production occurs in two stages. Monopolistically competitive intermediate firms employ labor to produce input for final goods. There is a continuum of mass unity of these intermediate goods firms on each island. Final goods firms package together all the differentiated intermediate inputs to produce output. These firms are competitive and operate across all islands.

Let Y_t be output of the representative final good firm; $Y^j_{z,t}$ be input from intermediate goods producer j and island z; and $P^j_{z,t}$ be the associated nominal price. The production function for final goods is the following CES aggregate of intermediate goods:

$$Y_t = \left[\int_0^1 \int_0^1 (Y^j_{z,t})^{\frac{\varepsilon-1}{\varepsilon}} dj dz \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.6}$$

where $\varepsilon > 1$ is the price elasticity of demand for each intermediate good:

From cost minimization, the demand for each intermediate good is given by

$$Y^j_{z,t} = \left(\frac{P^j_{z,t}}{P_t} \right)^{-\varepsilon} Y_t \tag{2.7}$$

and the price index is the following CES aggregate of intermediate goods prices:

$$P_t = \left[\int_0^1 \int_0^1 [P^j_{z,t}]^{1-\varepsilon} dj dz \right]^{\frac{1}{1-\varepsilon}} \tag{2.8}$$

2.3. Intermediate goods firms

Each intermediate goods firm produces output that is the linear function of labor input:

$$Y^j_{z,t} = X^j_{z,t} \cdot N^j_{z,t} \tag{2.9}$$

Here $X_{z,t}^j$ is an idiosyncratic productivity factor for producer j and island z.

In addition, each producer faces a fixed cost of adjusting price, equal to $b_t(X_{z,t}^j)^{\varepsilon-1}$. In particular,

$$b_t = \begin{cases} b & \text{if } P_t^j \neq P_{t-1}^j \\ 0 & \text{otherwise} \end{cases} \tag{2.10}$$

We scale the adjustment cost be the factor $(X_{z,t}^j)^{\varepsilon-1}$ to keep the firm's decision problem homogenous as it size varies.[4]

The process for the productivity shock is as follows: Islands are occasionally subject to turbulence in the form of multiplicative i.i.d. productivity shocks. The arrival of the shock on island z at date t (i.e., whether firms are subject to a draw from random productivity variable at t) is perfectly correlated across all firms on the island. The realization of the draw, however, is uncorrelated across firms on the island. Let $\xi_{z,t}^j$, denote the shock to firm j on island z at date t,

$$X_{z,t}^j = \begin{cases} X_{z,t-1}^j e^{\xi_{z,t}^j} & \text{if a productivity shock occurs} \\ X_{z,t-1}^j & \text{otherwise} \end{cases} \tag{2.11}$$

We assume that the arrival of a shock on each island obeys a "truncated Poisson" process. Let n be the number of periods since last shock on island z. For $n < N$, the productivity shock arrives with probability $1-\alpha$. For $n = N$, arrival probability is equal unity. In this respect, the Poisson process is truncated if a new shock has not materialized after $N-1$ periods since the previous shock.

We assume that the random variable $\xi_{z,t}^j$ is distributed uniformly with density $1/\phi$, with

$$E\{e^{(\varepsilon-1)\xi}\} = 1$$

As will become obvious, this normalization ensures that the expected multiplicative impact of the shock on the firm's discounted profits is unity. In addition we assume that the support of $\xi_{z,t}^j$ large enough that there is some chance that a firm may either raise or lower its price in response to an idiosyncratic shock.

As illustrated by Danziger (1999), the uniform distribution introduces considerable tractability to the general Ss problem: The distribution of prices following an idiosyncratic shock has a simple form: uniform within the adjustment triggers and a mass at the target. As we show, this feature makes possible a reasonably simple approximation of the solution to the decision problem. It also simplifies the steady state equilibrium, as well as the local approximation around the steady state.

[4] If the economy were growing we would also have to normalize the cost of price adjustment by the real wage and aggregate output.

3. The Firm's Optimal Pricing Decision

Given the fixed cost of price adjustment, the solution to the firm's decision problem will involve an Ss-style of price adjustment. Specifically, there will be a range of inaction, where the gain in discounted earnings from adjusting is not sufficient to cover the fixed cost. The optimal policy will involve an upper trigger, a lower trigger, and a target price. The firm adjusts when its price either reaches or moves beyond either of the trigger prices.

In this section we first characterize the firms objective function. We then argue that based on a plausible assumption about the size of adjustment costs, that it is reasonable to consider a second order approximation of the objective function. Next, we show that our restriction on adjustment costs in conjunction with the uniform distribution of the shock, leads to considerable simplification of the objective, up to a second order. With this simplified objective, we characterize both the steady state and a log-linear approximation of the decision rules about the steady state.

3.1. The Firm's Objective

Period profits net of adjustment costs, Π^j_{t+i}, are given by

$$\Pi^j_{z,t+i} = \left(\frac{P^j_{z,t+i}}{P_{t+i}} - \frac{W_{z,t+i}}{P_{t+i} X_{z,t+i}} \right) Y_{t+i} - b_{t+i} (X^j_{z,t+i})^{\varepsilon - 1} \tag{3.1}$$

where b_{t+i} is defined by equation (2.10). Note that from cost minimization, the firm's real marginal cost is $W_{z,t+i}/X_{z,t+i}$.

It is convenient to define the "normalized" price, $Q^j_{z,t}$, which is the price, $P^j_{z,t}$, normalized by multiplicative impact of the idiosyncratic productivity shock on the firm's marginal cost $(1/X^j_{z,t})$:

$$Q^j_{z,t} = \frac{P^j_{z,t}}{1/X^j_{z,t}} = P^j_{z,t} X^j_{z,t} \tag{3.2}$$

There are two advantages of working with the normalized price. First, assuming that the firm's desired markup is stationary, $Q^j_{z,t}$ is stationary. In contrast $P^j_{z,t}$ is nonstationary since $X^j_{z,t}$ is nonstationary. Second, all firms that reset price in period t will wind up choosing the same normalized price, which simplifies the aggregation. Since idiosyncratic productivity differs across firms, firms will not choose the same absolute price.

Restating period profits in terms of the normalized price yields and making use the demand function the firm faces (equation (2.7)) yields

$$\Pi_{t+i} = X^{\varepsilon-1}_{t+i} Y_{t+i} (P_{t+i})^{\varepsilon - 1} \left[\left(Q^j_{z,t+i} \right)^{-\varepsilon} \left(Q^j_{z,t+i} - W_{z,t+i} \right) - b_{t+i} \right] \tag{3.3}$$

with

$$Q^j_{z,t} = \begin{cases} Q^j_{z,t-1} e^{\xi^j_{z,t}} & \text{if } P^j_{z,t} = P^j_{z,t-1} \text{ productivity shock} \\ Q^*_t & \text{if adjustment} \end{cases}$$

At this point we drop the j, z subscripts. We define the firm's value function as the maximized stream of discounted net profits, as follows:

$$V(A_t, X_t, Q_{t-1}e^{\xi_t}, W_{z,t}) = \max E_t \sum_{i=0}^{\infty} X^{\varepsilon-1}_{t+i} \beta^i [A_{t+i} Q^{-\varepsilon}_{t+i} (Q_{t+i} - W_{t+i}) - b_{t+i}] \tag{3.4}$$

with $A_t = [Y_{t+i}(P_{t+i})^{\varepsilon-1}]/C_{t+i}$.[5] Given that gross profits and adjustment costs are homogeneous in $X^{\varepsilon-1}_t$, it is convenient to define the normalized value function $v(\cdot)$:

$$V(A_t, X_t, Q_{t-1}e^{\xi_t}, W_t) = X^{\varepsilon-1}_t \cdot v(A_t, Q_{t-1}e^{\xi_t}, W_t) \tag{3.5}$$

with

$$v(A_t, Q_{t-1}e^{\xi_t}, W_t) = \max E_t \sum_{i=0}^{\infty} \left(\frac{X_{t+i}}{X_t} \right)^{(\varepsilon-1)} \beta^i [A_{t+i} Q^{-\varepsilon}_{t+i} (Q_{t+i} - W_{t+i}) - b_{t+i}]$$

To express the normalized value function in a recursive form. Let $v_n(A_t, Q_{t-1}e^{\xi_t}{}_t, W_{z,t})$ denote the value before price adjustment decision with $n \leq N$ periods before next idiosyncratic shock hits with certainty, and let $\overline{v}_n(A_t, Q_t, W_{z,t})$ denote the value after price adjustment decision with $n \leq N$ periods before next idiosyncratic shock hits with certainty. Then

$$v_n(A_t, Q_{t-1}e^{\xi_t}, W_t) = \max \left\{ \overline{v}_n(A_t, Q_{t-1}e^{\xi_t}, W_t), \max_{Q_t} \overline{v}_n(A_t, Q_t, W_t) - b \right\} \tag{3.6}$$

and for $n > 1$

$$\begin{aligned} \overline{v}_n(A_t, Q_t, W_t) \quad = \quad & A_t Q^{-\varepsilon}_t (Q_t - W_t) \\ & + \beta E_t \left\{ \alpha v_{n-1}(A_{t+1}, Q_t, W_{t+1}) + (1-\alpha) e^{(\varepsilon-1)\xi_{t+1}} v_N(A_{t+1}, Q_t e^{\xi_{t+1}}, W_{t+1}) \right\} \end{aligned} \tag{3.7}$$

or for $n = 1$

$$\overline{v}_n(A_t, Q_t, W_t) = A_t Q^{-\varepsilon}_t (Q_t - W_t) + \beta E_t \left\{ e^{(\varepsilon-1)\xi_{t+1}} v_N(A_{t+1}, Q_t e^{\xi_{t+1}}, W_{t+1}) \right\} \tag{3.8}$$

Assuming (as we do later) that aggregate shocks are not large enough to trigger an adjustment by themselves, then what complicates the firm's problem, in general, is that it must take account of the continuation value conditional on an idiosyncratic shock, $E_t \left\{ e^{(\varepsilon-1)\xi_{t+1}} v_N(\cdot) \right\}$. Absent this consideration, the choice of the target price at time t would involve just involve taking into account

[5] Here we have expressed the value function in units of utility rather than current consumption. This aids in writing the problem recursively.

discounted profits in states where the firm's price remains fixed at period t target. In this respect, the choice of the target is no more difficult than in the conventional time-dependent framework. The choice of the triggers would also simplify.

We next proceed to show that under plausible assumptions, that $E_t\left\{e^{(\varepsilon-1)\xi_{t+1}}v_N(\cdot)\right\}$ is independent of the firm's period t choice of the target, up to a second order approximation. The decision problem will simplify, along the lines we have just suggested.

3.2. Approximate Value Function

It is convenient to define the target and trigger in logarithmic terms. Let q_t^* denote the natural log of the target (normalized) price and let q_t^L and q_t^H be the natural logs of the upper and lower triggers. Under the Ss policy, the firm adjust to q_t^* if $\ln(Q_t) \notin [q_t^L, q_t^H]$.

Our goal now is to derive an approximate value function that leads a tractable (approximate) solution to the decision problem. To do so, we first assume that the (normalized) adjustment cost b is second order. As is well known, doing so implies that the normalized value function $v_n(\cdot)$ is always within a second order of the frictionless optimum. This in turn implies that it is reasonable to restrict attention to a second order approximation of the value function. Another standard implication is that second order adjustment costs imply that the Ss bands are within a first order of the targets (e.g. Mankiw (1985), Akerlof and Yellen (1985)). In addition, we make the assumption that the support of the idiosyncratic productivity shock is first order.[6]

There is an important additional implication of our "small" b assumption: Second order b in conjunction with the uniform distribution of the productivity shock implies that the continuation value contingent on an idiosyncratic shock, is independent of Q_t up to a second order. As a result, given that we can restrict attention to a second approximation of the objective, a firm choosing price need only take into account profits up to the next idiosyncratic shock. As we just discussed, this leads to a considerable simplification of the decision problem.

We now turn to this proposition:

Proposition 3.1. *Suppose (a) b is second order (implying $q_t^L - q_t^*, q_t^H - q_t^*$ are first-order), and (b) ϕ is first order and large enough that after a shock there is some chance that the firm will raise its price and some chance that a firm will lower its price, then the expected value of an optimal policy after an idiosyncratic shock in period $t+1$, $E\{e^{(\varepsilon-1)\xi_{t+1}}v_n(A_{t+1}, Q_t e^{\xi_{t+1}}, W_{z,t+1},)\}$ is independent of the current value of Q_t to a second order. In particular, we can show that up to a second order, the*

[6] There are several reasons for doing this. First, we will want to linearize the price index. If the productivity shock is not first order, then this will be complicated by the fact that the distribuiton of prices will not be first order. Second, if the productivity shock is not first order, the probability of non-adjustment will shrink to zero as b approaches zero, and there will be no price inertia in the limit.

firm can treat its objective as

$$\overline{v}_n(A_t, Q_t, W_t) \approx A_t Q_t^{-\varepsilon}(Q_t - W_t) + \beta E_t \left\{\alpha v_{n-1}(A_{t+1}, Q_t, W_{t+1})\right\}$$

Proof: Suppose that the firm has a current level of Q_t such that $\ln Q_t \in [q_t^L, q_t^H]$. We are interested in the expected value of an optimal policy conditional on an idiosyncratic productivity shock in period $t+1$. Also let Q_{t+1}^* denote the optimal choice of Q_{t+1} in the event of adjustment.

Consider $E\{e^{(\varepsilon-1)\xi_{t+1}} v_n(A_{t+1}, Q_t e^{\xi_{t+1}}, W_{t+1},)\}$ over the states of the world in which the idiosyncratic shock hits. Given the Ss adjustment policy:

$$\begin{aligned}
E\{e^{(\varepsilon-1)\xi_{t+1}} v_n(A_{t+1}, Q_t e^{\xi_{t+1}}, W_{t+1})\} \quad &= \quad \frac{1}{\phi}\int_{q_{t+1}^H - \ln Q_t}^{\xi^H} e^{(\varepsilon-1)\xi_{t+1}} \overline{v}_n(A_{t+1}, Q_{t+1}^*, W_{t+1}) d\xi_{t+1} \\
&\quad + \frac{1}{\phi}\int_{q_{t+1}^L - \ln Q_t}^{q_{t+1}^H - \ln Q_t} e^{(\varepsilon-1)\xi_{t+1}} \overline{v}_n(A_{t+1}, Q_t e^{\xi_{t+1}}, W_{t+1}) d\xi_{t+1} \\
&\quad + \frac{1}{\phi}\int_{\xi^L}^{q_{t+1}^L - \ln Q_t} e^{(\varepsilon-1)\xi_{t+1}} \overline{v}_n(A_{t+1}, Q_{t+1}^*, W_{t+1}) d\xi_{t+1}
\end{aligned}$$

Given the assumption on ϕ, $\xi^H > q_{t+1}^H - \ln Q_t$ and $\xi^L < q_{t+1}^L - \ln Q_t$. Rearranging yields

$$\begin{aligned}
&E\{e^{(\varepsilon-1)\xi_{t+1}} \; v_n(A_{t+1}, Q_t e^{\xi_{t+1}}, W_{t+1})\} \\
&\qquad = \quad \overline{v}_n(A_{t+1}, Q_{t+1}^*, W_{t+1}) \\
&\qquad\quad + \frac{1}{\phi}\int_{q_{t+1}^L - \ln Q_t}^{q_{t+1}^H - \ln Q_t} e^{(\varepsilon-1)\xi_{t+1}} \left[\overline{v}_n(A_{t+1}, Q_t e^{\xi_{t+1}}, W_{t+1}) - \overline{v}_n(A_{t+1}, Q_{t+1}^*, W_{t+1})\right] d\xi_{t+1}
\end{aligned}$$

A change of variable, $\Phi_{t+1} = \xi_{t+1} + \ln Q_t$, gives

$$\begin{aligned}
&E\{e^{(\varepsilon-1)\xi_{t+1}} \; v_n(A_{t+1}, Q_t e^{\xi_{t+1}}, W_{t+1})\} \\
&\qquad = \quad \overline{v}_n(A_{t+1}, Q_{t+1}^*, W_{t+1}) \\
&\qquad\quad + \frac{1}{\phi}\int_{q_{t+1}^L}^{q_{t+1}^H} e^{(\varepsilon-1)(\Phi_{t+1} - \ln Q_t)} \left[\overline{v}_n(A_{t+1}, e^{\Phi_{t+1}}, W_{t+1}) - \overline{v}_n(A_{t+1}, Q_{t+1}^*, W_{t+1})\right] d\Phi_{t+1}
\end{aligned}$$

Note that q_{t+1}^H and q_{t+1}^L are chosen optimally in period $t+1$. They depend on the period $t+1$ state $e^{\Phi_{t+1}}$ and are independent of Q_t. The only place that Q_t enters is in the exponential term inside the integral. Now, by the assumption on the bands, $\ln Q_t$ is equal to $\ln q_t^*$ plus a first order term and, given the limits of integration Φ is equal to $\ln q_{t+1}^*$ plus a first order term. The exponential term is therefore equal to $e^{(\varepsilon-1)(\ln q_{t+1}^* - \ln q_t^*)}$ plus a first order term. The term in square brackets inside the integral is bounded by b. By the assumption on b, this term is second order. Hence:

$$\begin{aligned}
& e^{(\varepsilon-1)(\Phi-\ln Q_t)}\left[\overline{v}_N(A_{t+1},e^{\Phi_{t+1}},W_{t+1})-\overline{v}_N(A_{t+1},Q^*_{t+1},W_{t+1})\right] \\
= \; & e^{(\varepsilon-1)(\ln q^*_{t+1}-\ln q^*_t)}\left[\overline{v}_N(A_{t+1},e^{\Phi_{t+1}},W_{t+1})-\overline{v}_N(A_{t+1},Q^*_{t+1},W_{t+1})\right]+\mathcal{O}^3
\end{aligned}$$

Further, the assumption that ϕ is first order and that the range of integration is first order,

$$\begin{aligned}
& E\{e^{(\epsilon-1)\xi_{t+1}}v_n(A_{t+1},Q_t e^{\xi_{t+1}},W_{t+1})\} \\
= \; & \overline{v}_n(A_{t+1},Q^*_{t+1},W_{t+1}) \\
& +\frac{1}{\phi}\int_{q^L_{t+1}}^{q^H_{t+1}}\left[\overline{v}_n(A_{t+1},e^{\Phi_{t+1}},W_{z,t+1})-\overline{v}_n(A_{t+1},Q^*_{t+1},W_{t+1})\right]d\Phi_{t+1}+\mathcal{O}^3
\end{aligned}$$

where the integral is second order. It follows that $E\{e^{(\varepsilon-1)\xi_{t+1}}v_n(A_{t+1},Q_t e^{\xi_{t+1}},W_{t+1},)\}$ is independent of Q_t to a second order. QED

The main insight of the proposition is that in future states where the idiosyncratic shock will hit, history will be erased. The subsequent continuation value $E_t\left\{e^{(\varepsilon-1)\xi_{t+1}}v_N(A_{t+1},Q_t e^{\xi_{t+1}},W_{t+1})\right\}$ is irrelevant to current pricing decision to a second order. The intuition for this key result is as follows.

Suppose the firm is following an Ss strategy that in period $t+1$ is characterized by the following triplet : $\{q^*_{t+1},q^H_{t+1},q^L_{t+1}\}$. Now suppose that the idiosyncratic shock hits in period $t+1$. Given that the shock is log uniform, if the firm does not adjust, q_{t+1} will be uniformly distributed over (q^H_{t+1},q^L_{t+1}). If the firm does adjust, $q_{t+1}=q^*_{t+1}$. Since the triplet $\{q^*_{t+1},q^H_{t+1},q^L_{t+1}\}$ is independent of q_t (it depends only on the state at $t+1$), it follows that $v_N(A_{t+1},Q_t e^{\xi_{t+1}},W_{t+1})=v_N(A_{t+1},\exp(q_t-\xi_{t+1}),W_{t+1})=v_N(A_{t+1},\exp(\Phi_{t+1}),W_{t+1})$ is independent of q_t.

Accordingly the only way that Q_t could possibly affect $E_t\left\{e^{(\varepsilon-1)\xi_{t+1}}v_N(A_{t+1},Q_t e^{\xi_{t+1}},W_{t+1})\right\}$, is by affecting the correlation between $e^{(\varepsilon-1)\xi_{t+1}}$ and $v_N(A_{t+1},Q_t e^{\xi_{t+1}},W_{t+1})$. However, given our restrictions on b and on the size of the idiosyncratic shock, this correlation is second order and its dependence on dependence on Q_t is third order, given that Q_{t+1} is within a first order of Q^*_{t+1}.

The proposition rests on two critical assumptions. The first is that the idiosyncratic shock is uniform and has a wide enough support that both price increases and price decreases are possible. This assumption implies that the distribution of prices within the Ss bands is independent of Q_t. The second is that b is second order, this makes the correlation between the decision to change price and Q_t third order.

We now make use of Proposition 1 and the assumption that b second order to derive an explicit second order approximate of the value function from which a tractable solution to the Ss decision problem will follow. We begin by noting that in the environment we will consider, firms will only adjust price in periods where an idiosyncratic shock hits. In the steady state (absent aggregate shocks), the Ss bands are narrowest just after the idiosyncratic shock. As time passes without a new shock, the expected wait until the next shock steadily shrinks due to the truncated poisson process. As a consequence, the bands systematically widen since the fixed adjustment cost must be spread over a horizon that is steadily becoming shorter. Hence in steady state firms that do not adjust when the idiosyncratic shock hits, will not adjust until the next idiosyncratic shock. First that do adjust reset to the optimum and have even less incentive to adjust again. Outside the steady state, we assume that the aggegate shocks small enough to not trigger additional adjustment.[7]

Given that adjustment occurs only in the wake of an idiosyncratic shock, we can restrict attention to the value function, $\overline{v}_N(\cdot)$, where the aggregate shock hits with certainty the full N periods after the current shock. In addition, as we noted earlier, given that b is second order it is reasonable to consider second order approximations of $\overline{v}_N(\cdot)$.

Let Q_t^o be the optimal normalized price in the frictionless optimum (i.e. the optimum with no adjustment costs.) Then, given Proposition 1, we can restrict attention to the following second order approximations of $\overline{v}_N$ about the frictionless optimum that ignores the continuation values conditional on idiosyncratic shocks:

$$\overline{v}_N(A_t, Q_t, W_t) = \chi_1 A_t W_t^{1-\varepsilon} + \chi_2 A_t W_t^{1-\varepsilon} (\ln Q_t - \ln Q_t^o)^2 + \beta\alpha E_t \overline{v}_{N-1}(A_{t+}, Q_t, W_t) \tag{3.9}$$

where χ_1 and χ_2 are constants, with $\chi_2 = \frac{\varepsilon^{1-\varepsilon}}{(\varepsilon-1)^{-\varepsilon}}$. Note that the first order term disappears since Q_t^o is the frictionless optimum.[8] Given elasticity of demand, Q_t^o is simply the following markup μ over nominal wages, as follows:

$$Q_t^o = \mu W_t \tag{3.10}$$

with $\mu = \varepsilon/(\varepsilon - 1)$.

3.3. Approximate Optimal Pricing Policy

Iterating forward the second order approximation of $\overline{v}_N(A_t, Q_t, W_{z,t})$ given by (3.9) yields

$$\overline{v}_N(A_t, Q_t, W_t) = E_t \sum_{i=0}^{N-1} (\alpha\beta)^i \left[\chi_1 A_{t+i} W_{t+i}^{1-\varepsilon} + \chi_2 A_{t+i} W_{t+i}^{1-\varepsilon} \left(\ln Q_t - \ln Q_{t+i}^o\right)^2\right] \tag{3.11}$$

It is now straightforward to derive the optimality conditions for the target and the two triggers. The first order necessary condition for the target is given by:

$$E_t \sum_{i=0}^{N-1} (\alpha\beta)^i \left[A_t W_{z,t}^{1-\varepsilon} (\ln Q_t^* - \ln Q_t^o)\right] = 0 \tag{3.12}$$

[7] One can think of this as taking two limits. First, choose b small enough such that the firms' obectives are approximately quadratic. Second, choose an aggregate forcing process that is small enough that it does not trigger further adjustment.

[8] This means that we do not have to worry about any of the concerns regarding second order approximations such as those raised in Woodford (2002).

The triggers in turn are given by a value matching condition that equates the gain from not adjusting to the gain from adjustment, net the adjustment cost: For $J = H, L$:

$$\overline{v}_N(A_t, Q_t^J, W_t) = \overline{v}_N(A_t, Q_t^*, W_t) - b \tag{3.13}$$

Given our quadratic approximation, the value matching condition can be restated as:

$$E_t \sum_{i=0}^{N-1} (\alpha\beta)^i \left[\chi_2 A_{t+i} W_{t+i}^{1-\varepsilon} \left(\ln Q_t^J - \ln Q_t^o\right)^2\right] = E_t \sum_{i=0}^{N-1} (\alpha\beta)^i \left[\chi_2 A_t W_t^{1-\varepsilon} \left(\ln Q_t^* - \ln Q_t^o\right)^2\right] - b \tag{3.14}$$

Since we are interested in a local approximation about the steady state, we now analyze the non-stochastic steady state as a necessary first step.

3.4. Non-stochastic Steady State

We first set the aggregate shocks at their respective means. The only disturbance in the steady state is the idiosyncratic productivity shock, which washes out in the aggregate.

It is straightforward to derive the optimal steady state target and adjustment triggers. Given that A_t and W_t are fixed, it follows from the first order conditions (3.10) and (3.12) that the steady state target price, $\bar{Q}^*$,is a constant equal to the steady state frictionless optimal price $\bar{Q}^o$, as follows:

$$\bar{Q}^* = \bar{Q}^o = \mu\bar{W} \tag{3.15}$$

The steady state triggers are pinned down by the value matching condition with A_t and W_t at their respective steady state means:

$$\overline{v}_N(\bar{A}, \bar{Q}_n^J, \bar{W}) = \overline{v}_N(\bar{A}, \bar{Q}^*, \bar{W}) - b \tag{3.16}$$

for $J = H, L$. Given the quadratic approximation of $\overline{v}_N(\cdot)$ and given $\ln \bar{Q}^* = \ln \bar{Q}^o$, we can write:

$$\sum_{i=0}^{N-1} (\alpha\beta)^i \left[\chi_2 \bar{A}\bar{W}^{1-\varepsilon} \left(\ln \bar{Q}^J - \ln \bar{Q}^o\right)^2\right] = b \tag{3.17}$$

The solution to this quadratic equation yields two steady state triggers:

$$\begin{aligned} \ln \bar{Q}^H &= \ln \bar{Q}^o + \sqrt{\frac{1-(\alpha\beta)}{1-(\alpha\beta)^N} \frac{b}{\chi_2 \bar{A}\bar{W}^{1-\varepsilon}}} \\ \ln \bar{Q}^L &= \ln \bar{Q}^o - \sqrt{\frac{1-(\alpha\beta)}{1-(\alpha\beta)^N} \frac{b}{\chi_2 \bar{A}\bar{W}^{1-\varepsilon}}} \end{aligned} \tag{3.18}$$

Note that since b is second order, the steady state bands $\bar{q}^H - \bar{q}^*$ and $\bar{q}^* - \bar{q}^L$ are first order, as we maintained earlier. In addition, as the period length N shrinks, the bands widen, also as we noted earlier.

Finally, the steady state probability of price adjustment conditional on an idiosyncratic shock is $1 - \theta = 1 - \frac{\bar{q}^H + \bar{q}^L}{\phi}$.[9] The unconditional probability of price adjustment, then is simply the product of the probability of an idiosyncratic shock times the probability of adjusting conditional on this shock: $\frac{1-a}{1-a^N} \cdot (1 - \theta)$. The average time a price is fixed, Θ, is simply the inverse of this probability:

$$\Theta = \frac{1 - \alpha^N}{(1 - \alpha) \cdot (1 - \theta)} \tag{3.19}$$

Note than, in general, the average time a price is fixed exceeds the average amount of time in between idiosyncratic shocks, $\frac{1-\alpha^N}{1-\alpha}$. This of course occurs because firms may always choose to keep their prices fixed in the event of a shock. Again, however, those firms who to not adjust in this instance will have their price within of first order of the target price.

3.5. Aggregate Shocks and Local Dynamics

We now consider shocks to A_t and W_t that are sufficiently small so as to not trigger price adjustment in periods without idiosyncratic shocks. Let $q_t^* = \ln Q_t^* - \ln \bar{Q}_t^*$ and let $w_t = \ln W_{z,t} - \ln \bar{W}$. Log-linearizing (3.12) about the steady state values of A, W, Q^0 and Q^* :

$$\begin{aligned} q_t^* &= \frac{1 - \beta\alpha}{1 - (\beta\alpha)^N} E_t \sum_{i=0}^{N-1} (\beta\alpha)^i q_{t+i}^o + \mathcal{O}^2 \\ &= \frac{1 - \beta\alpha}{1 - (\beta\alpha)^N} E_t \sum_{i=0}^{N-1} (\beta\alpha)^i w_{z,t+i} + \mathcal{O}^2 \end{aligned} \tag{3.20}$$

since $q_t^o = w_{z,t}$. As in the pure time dependent model, the target depends on a discounted stream of future values of nominal marginal cost. In the time dependent framework, however, future marginal cost in each period is weighted by the probability the price remains fixed. In our state-dependent framework, the relevant weight is the probability α^i that a new idiosyncratic shock has not arisen, which in general is a number smaller than the probability the price has stayed fixed.

We next consider the local dynamics for the optimal triggers. Log-linearizing (3.14) about the steady state values of A, W, Q^0 and Q^* and using the definition (3.20) yields

$$q_t^H = q_t^* - \frac{1 - \beta\alpha}{1 - (\beta\alpha)^N} \frac{\bar{q}^H - \bar{q}^o}{2} E_t \sum_{i=0}^{N-1} (\alpha\beta)^i \left[a_{t+i} + (1 - \varepsilon) w_{t+i}\right] \tag{3.21}$$

[9] Since both $\bar{q}^H + \bar{q}^L$ and ϕ are first order $1 - \theta$ need not approach zero or one as b approaches zero.

$$q_t^L = q_t^* + \frac{1-\beta\alpha}{1-(\beta\alpha)^N}\frac{\bar{q}^o - \bar{q}^L}{2} E_t \sum_{i=0}^{N-1} (\alpha\beta)^i \left[a_{t+i} + (1-\varepsilon) w_{t+i}\right] \tag{3.22}$$

Note that the width of the bands $q_t^H - q_t^L$ may fluctuate. However, they widen and contract symmetrically. Thus, given the uniform distribution, the average price within bands is simply the target: $\frac{1}{2}(q_t^H + q_t^L) = q_t^*$. This result will prove useful when we next consider the local dynamics of the price level.

3.6. Price Index

We may express the price index in terms of normalized prices, as follows

$$\begin{aligned} P_t &= \left(\int P_t(z)^{1-\varepsilon} dz\right)^{\frac{1}{1-\varepsilon}} \\ &= \left(\int Q_t(z)^{1-\varepsilon} X_t(z)^{\varepsilon-1} dz\right)^{\frac{1}{1-\varepsilon}} \end{aligned} \tag{3.23}$$

Log linearization yields

$$\begin{aligned} \bar{P}^{1-\varepsilon} + \bar{P}^{1-\varepsilon}\left(\ln P_t - \ln \bar{P}\right) &= \int\int \left[\bar{P}^{1-\varepsilon} + \bar{P}^{1-\varepsilon}\left(\ln Q_{z,t}^j - \ln \bar{Q}\right)\right] X_t(z)^{\varepsilon-1} dj dz + \mathcal{O}^2 \\ \ln P_t &= \int\int \ln Q_{z,t}^j \left(X_{z,t}^j\right)^{\varepsilon-1} dj dz + \mathcal{O}^2 \\ \ln P_t &= \int\int \ln Q_{z,t}^j dj dz + \mathcal{O}^2 \end{aligned} \tag{3.24}$$

Note that in the local approximation, we can ignore any correlation between $Q_{z,t}^j$ and $X_{z,t}^j$ because the bands are first order as are the innovations in the latter.

Let $p_t = \ln P - \ln \bar{P}$. Then

$$p_t = \int\int q_{z,t}^j dj dz \tag{3.25}$$

Now consider an island z, island received an idiosyncratic shock at date $t-i$ where $i \in \{0, N-1\}$,Those who adjusted set their price equal to q_{t-i}^*Those who did not adjust remained uniformly distributed on (q_{t-i}^L, q_{t-i}^H). Given (3.21) and (3.22), the average price of the non-adjusters is also q_{t-i}^*.Hence the average price on island z is q_{t-i}^*. Given: the proportion of islands receiving the shock at t is $1-\alpha$, the proportion that received the shock at t-1 but not at t is $a(1-a)$ and so on. It follows that

$$p_t = \frac{1-\alpha}{1-\alpha^N} \sum_{i=0}^{N-1} a^i q_{t-i}^* \tag{3.26}$$

which may be expressed as

$$p_t = (1-\alpha)[q_t^* + \frac{\alpha^N}{1-\alpha^N}(q_t^* - q_{t-N}^*)] + \alpha p_{t-1} \tag{3.27}$$

4. The Complete Model

In this section we put together the complete model. We restrict attention to a log-linear approximation about the steady state. We begin with the "state-dependent" Phillips curve and then turn our attention to the rest of the model.

Manipulation of (3.20) yields the optimal reset price q_t^* as the following discounted stream of future nominal wages.

$$q_t^* = (1-\beta\alpha)E_t \sum_{i=0}^{\infty} (\beta\alpha)^i [w_{z,t+i} + \frac{(\beta\alpha)^N}{1-(\beta\alpha)^N}(w_{z,t+i} - E_t w_{z,t+N+i})] \tag{4.1}$$

Note that q_t^* depends on the island-specific wage $w_{z,t+i}$. As a step toward aggregation, we would like to derive this relation in terms of the economy-wide average wage, w_{t+i}.

Log-linearizing the household's first order condition for labor supply yields:

$$w_{z,t+i} - p_{t+i} = \varphi n_{z,t+i} + c_{t+i} \tag{4.2}$$

Averaging over this condition yields $w_t - p_{t+i} = \varphi n_{z,t+i} + c_{t+i}$, implying the following relation between the island z relative wage and the relative employment levels:

$$w_{z,t+i} - w_{t+i} = \varphi(n_{z,t+i} - n_{t+i}) \tag{4.3}$$

Making use of the demand function and the production function leads to a relationship between the relative wage and the relative price of firms who adjust at time t :

$$w_{z,t+i} = w_{t+i} - \varphi\epsilon(q_t^* - p_{t+i}) \tag{4.4}$$

Notice that $w_{z,t+i}$ depends inversely on q_t^*. Raising the price reduces output and labor demand. Since the labor market is segmented it also reduces wages on the island, thus moderating the need to raise price. As emphasized in Woodford (2003), this factor segmentation thus introduces a "real rigidity" that gives adjusting firms a motive to keep their relative prices in line with the relative prices of non-adjusting firms. This real rigidity, in turn, contributes to the overall stickiness in the movement of prices. Let Ψ_t denote the equilibrium nominal wage for a firm adjusting its price, i.e., the equilibrium value of $w_{z,t}$ (we drop the z subscript because in equilibrium Ψ_t depends only on aggregate factors).

Then combining (4.4) with (4.1) yields

$$q_t^* = (1-\beta\alpha)\{\Psi_t + \frac{(\beta\alpha)^N}{1-(\beta\alpha)^N}[\Psi_t - E_t\Psi_{t+N}]\} + \beta\alpha E_t q_{t+1}^* \tag{4.5}$$

with

$$\Psi_t - p_t = \frac{1}{1+\varphi\epsilon}(w_t - p_t) \tag{4.6}$$

In equilibrium, the real wages of adjusting firms, $\Psi_t - p_t$ moves less than one for one with the aggregate real wage, implying similarly sluggish movement in in the nominal wage Ψ_t. In this respect, the real rigidity, measured inversely by the coefficient $\frac{1}{1+\varphi\epsilon}$, dampens the adjustment of prices. In the absence of real rigidities, $\frac{1}{1+\varphi\epsilon}$ equals unity, implying Ψ_t. simply is equal to w_t.

We are now in a position to present the Phillips curve. Let $\pi_t = p_t - p_{t-1}$, denote inflation. In addition, then, combining the equation for the target price (4.5) with the price index (3.27) yields

$$\pi_t = \lambda\{(\Psi_t - p_t) + \frac{\alpha^N}{1-\alpha^N}\widetilde{\Psi}_t\} + \beta E_t\pi_{t+1} \tag{4.7}$$

with

$$\widetilde{\Psi}_t \equiv \frac{1}{1-(\beta a)^N}[\beta^N(1-\alpha^N)(\Psi_t - E_t\Psi_{t+N}) + (\Psi_t - \Psi_{t-N}) + (\beta\alpha)^N(E_{t-N}\Psi_t - E_t\Psi_{t+N})] \tag{4.8}$$

and

$$\lambda = \frac{(1-\alpha)(1-\beta\alpha)}{\alpha} \tag{4.9}$$

The Phillips curve has the form of the standard New Keynesian Phillips curve based on "Calvo" time-dependent pricing (or more specifically a variation based on "truncated" Calvo pricing). A key difference, however, is that the slope coefficient λ does not depend on a direct measure of the degree of price rigidity, but rather on α, which is a measure of the infrequency of idiosyncratic shocks. As we discussed in the previous section, the degree of price rigidity depends not only on the frequency of idiosyncratic shocks, but also on the likelihood of adjustment conditional on aggregate shocks. It is thus in principle possible to have a high degree of price flexibility, but also a high average length of time over which prices are fixed. In particular, if α is small, implying that idiosyncratic shocks are frequent, then λ will be large, implying that inflation will be very sensitive to movements in wages. At the same time, however, adjustment costs and the variability of idiosyncratic shocks could be such that adjustment conditional on a shock could be relatively infrequent, implying a high degree of stickiness at the firm level. The intuition for this outcome follows directly from Golosov and Lucas, which in turn comes from Caplin and Spulber. Frequent idiosyncratic shocks give firms the option of also adjusting to aggregate shocks. Unlike the time dependent case, firms not adjusting are those that are already close to the target. Thus in general, the state time dependent formulation will yield greater flexibility than does the time dependent. How much difference this makes, however, will depend upon the entire structure of the model, including the overall parametrization as we make clear in the next section.

The rest of the model is standard. Given that there are only consumption goods and utility is logarithmic, we can log-linearize the household's intertemporal condition to obtain the following "IS" curve:

$$y_t = -(r_t^n - E_t\pi_{t+1}) + E_t y_{t+1} \tag{4.10}$$

Log-linearzing the first order condition for labor supply, averaging across households, and taking into account that consumption equals output, yields a linear relation between the aggregate real wage and output.

$$w_t - p_t = (\sigma + \varphi) y_t \tag{4.11}$$

Next, log-linearizing the first order condition for money demand and taking into account that consumption equals output yields:

$$m_t - p_t = y_t - \zeta r_{tt}^n \tag{4.12}$$

Equations (4.6), (4.7), (4.8), (4.10), (4.11) and (4.12) determine the equilibrium aggregate dynamics, conditional on a monetary policy rule (and given the definition, $p_t = \pi_t + p_{t-1}$). We consider two different kinds of policy rules: An interest rate rule:

$$i_t = \gamma\pi_t + \rho i_{t-1} + \varepsilon_t \tag{4.13}$$

and a money growth rule:

$$m_t - m_{t-1} = \zeta(m_{t-1} - m_{t-2}) + \xi_t \tag{4.14}$$

Finally, we note that conditional on the aggregate dynamics, we can determined the evolution of the triggers for the Ss bands. This will yield adjustment probabilities conditional on idiosyncratic shocks which, along the with frequency of idiosyncratic shocks , will help pin down the average time prices are fixed. From knowledge of the size of the Ss bands and the distribution of idiosyncratic shocks, we can also figure out the average size of price adjustments.

5. Properties of the Model

The properties of the model are most apparent under two special cases: $N = 1$ and $N = \infty$.

When $N = 1$, the idiosyncratic productivity shock hits each island each period. All firms receive the idiosyncratic shock. According to (3.20), $q_t^* = q_t^o$, and according to (3.26), the price index is equal to q_t^o as well. In this case, the economy is always at its frictionless optimum. Money is neutral. Neutrality holds in spite of the fact that a fraction θ of firms do not adjust their prices in each period.[10]

[10] Note that Danziger does not find neutrality in his model even though he assumes that $N = 1$. The reason is that he presents an exact analytic solution, whereas we log-linearize. The effects of money on output that Danziger finds are

What is the source of this neutrality? It is instructive to analyze it both from the perspective of a firm and from the economy as a whole. Consider first a firm that is contemplating price adjustment. It faces an expected path for the nominal wage. In a time-dependent model, the firm would set its price equal to a mark up over a weighted average of future wages where the weights represent the discounted probability that the firm has not yet had an opportunity to alter its price. The weights would be of the form $(\beta\theta)^i$. How can the state-dependent firm ignore the future path of wages and set its price as a mark up only of the current wage? The answer is that the state-dependent firm can use its future price adjustment decision to bring its costs in line with whatever price it sets today. Suppose that the wage rises in the next period. A time-dependent firm would find that its price is too low. The state-dependent firm shifts the set of productivities for which it maintains its price so that its average mark up is unchanged. The resulting distribution of mark ups is unaffected by the increase in the wage. It is important to note that this stark neutrality result depends crucially on the assumption of a uniform distribution with wide support. This assumption allows the firm to alter its adjustment triggers without altering the resulting distribution of the mark up.

From the perspective of the economy as a whole, this neutrality result is a two-sided version of the neutrality result of Caplin and Spulber. Instead of a few firms raising their prices by discrete amounts to compensate for an increase in the nominal wage, what changes is the mix of firms that raise and lower their prices. When a shock causes the nominal wage to rise, the set of firms that maintain their prices fixed changes. Some that had marginally low productivities decide to raise their prices and some that have marginally high productivities decide not to lower theirs. The result is an unchanging distribution of markups: uniform between two fixed triggers, and a fixed mass at the target.

When $N > 1$ and $\alpha > 0$, firms do not always have the option of using their price adjustment policy to compensate for movements in the nominal wage, and money is no longer neutral. When $N = \infty$, the terms associated with the truncation of the idiosyncratic shock process disappear. The the Phillips curve takes a familiar form:[11]

$$\pi_t = \lambda\{(\Psi_t - p_t)\} + \beta E_t \pi_{t+1}$$

This is exactly the standard Calvo Phillips curve, with the exception that λ depends on the arrival rate of the productivity shock α rather than the probability that prices remain unchanged, which is $\alpha + \theta(1-\alpha)$. Our Ss economy therefore looks like a time-dependent economy with a greater frequency of price adjustment. This is similar to the finding of DKW and GL that an Ss economy exhibits less inertia than a Calvo economy that is calibrated to have a similar probability of price adjustment. In our model, this increased price flexibility arises because firms use their Ss policies to bring their costs in line with their prices following an idiosyncratic shock.

second order in our framework.

[11] Although it makes the assumption that aggregate shocks do not prompt price adjustment a bit untenible, since the Ss bands no longer widen appreciably between periods.

6. Some Numerical Exercises

- Numerical Exercises (TO BE ADDED)

7. References

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