

Debt Dynamics

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Abstract

We develop a dynamic model with endogenous choice of leverage, distributions, and real investment in the presence of a graduated corporate income tax, individual taxes on interest and corporate distributions, costs of financial distress, and equity flotation costs. The dynamic trade-off framework allows us to explain a number of empirical findings inconsistent with the static trade-off theory. We show that: 1) there is no target leverage ratio; 2) firms can be savers or heavily levered; 3) leverage is path dependent and exhibits hysteresis; 4) leverage is decreasing in lagged liquidity; and 5) leverage varies negatively with an external finance weighted average Q ratio. In the empirical section we find that simulated model moments match data moments. Conversely, we obtain sensible estimates of key structural parameters using indirect inference.

The Miller (1977) perpetual tax shield formula has served as one of the major references for those evaluating whether taxes can explain observed financing patterns. This formula is a cornerstone of the static trade-off theory, which posits that firms weigh the tax benefits of debt against costs associated with financial distress and bankruptcy. This benchmark model has provided intuition and guidance for much of the empirical literature on corporate capital structure, which has uncovered several patterns in the data that are inconsistent with the static trade-off theory.

For example, Graham (2000) finds that, “Paradoxically, large, liquid, profitable firms with low expected distress costs use debt conservatively.” By debt “conservatism,” Graham means that firms fail to issue sufficient debt to drive their expected marginal corporate tax rate down to

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that consistent with a zero/low net benefit to debt based on the Miller formula. In yet another blow to the theory, Myers (1993) states, “The most telling evidence against the static trade-off theory is the strong inverse correlation between profitability and financial leverage... Higher profits mean more dollars for debt service and more taxable income to shield. They should mean higher target debt ratios.” Baker and Wurgler (2002) reject the trade-off theory on different grounds, stating, “The trade-off theory predicts that temporary fluctuations in the market to book ratio or any other variable should have temporary effects.” Based on finding a negative relationship between leverage and an “external finance weighted average market to book ratio” they conclude that “capital structure is the cumulative outcome of attempts to time the equity market.”

This paper shows that a dynamic trade-off model can explain these stylized facts. As such, it provides a convincing alternative to the hypotheses of non-maximizing behavior, Myers’ (1984) pecking order theory, and/or market timing. Our results also reconcile the puzzles cited above with the evidence presented by MacKie-Mason (1990) and Graham (1996a) that taxes matter. We offer a sensible interpretation of the difference between our conclusions and those in much of the rest of the literature: the latter has taken a static model and compared its predictions with data generated by firms making a sequence of dynamic financing decisions. However, corporations do not face an infinite repetition of the Miller (1977) financing problem. Consequently, his framework is an inappropriate basis for assessing whether a rational tax-based model can explain observed leverage ratios.

Accordingly, we address the seeming anomalies by solving and simulating a dynamic model of investment and financing under uncertainty, where the firm faces a realistic tax environment, small equity flotation costs, and financial distress costs. The firm maximizes its value by making two interrelated decisions: how much to invest and whether to finance this investment internally, with debt, or with external equity. The firm can either borrow or save and can be in one of three equity regimes (positive distributions, zero distributions, or equity issuance.) The firm is forward-looking, making current investment and financing decisions in anticipation of future financing needs.

The logic of our argument is as follows. Traditional formulations of the financing decision place the firm at “date zero” with no cash on hand. Such firms are at the debt versus external equity financing margin since each dollar of debt replaces a dollar of external equity. The problem with the traditional approach is that corporations do not spend their lives at date zero. Rather, they evolve in a stochastic way, finding themselves at different financing margins over time.

As an illustration, consider a firm that realized a high profit shock last period, with internal cash

exceeding desired investment. Rather than choosing between debt and external equity, this firm must choose between retention and distribution of the excess funds. Note also that each dollar of debt issued by this high liquidity firm would serve to increase the distribution to shareholders, rather than replacing external equity. As intuition would suggest, our model shows that the marginal increase in debt (reduction in saving) is more attractive when it serves as a replacement for external equity, and is less attractive when it finances an increase in distributions to shareholders. Since high liquidity firms are more likely to be at the latter financing margin, they issue less debt.

This example illustrates the pitfalls associated with the traditional static framework. The more general message to take away is that, given the importance of a corporation's endogenous financing margin, characterization of how the tax system influences the financial and investment policies of a rational firm necessitates a forward-looking dynamic framework.

We highlight the main empirical implications. First, absent any invocation of market timing or adverse selection premia, the model generates a negative relationship between leverage and lagged measures of liquidity, consistent with the evidence in Titman and Wessels (1988), Rajan and Zingales (1995), and Fama and French (2002). Second, even though the model features single-period debt, leverage exhibits hysteresis, in that firms with high *lagged* debt use more debt than otherwise identical firms. This is because firms with high lagged debt are more likely to find themselves at the debt versus external equity margin. Third, since lagged leverage is a function of the firm's history, financial policy is path dependent. Finally, the combination of path dependence and hysteresis is sufficient to generate a data series containing the main Baker and Wurgler (2002) results in a rational model without market timing or adverse selection premia.

The model is sufficiently parsimonious that it can be taken directly to data. Because of the discrete nature of the tax environment, it is impossible to generate smooth, closed-form estimating equations from the model. Therefore, we turn to simulation methods, employing the indirect inference technique in Gourieroux, Monfort, and Renault (1993) and Gourieroux and Monfort (1996). Specifically, we solve the model via value function iteration and then use this solution to generate a simulated panel of firms. Our indirect inference procedure picks parameter estimates by minimizing the distance between interesting moments from actual data and the corresponding moments from the simulated data. This procedure has an important advantage over traditional regressions: it does not suffer from simultaneity problems, since it requires none of the zero-correlation restrictions that are necessary to identify OLS and IV regressions. Rather, as in a standard GMM estimation, it merely requires at least as many moments as underlying structural parameters.

Our model is most similar to those developed by Gomes (2001) and Cooley and Quadrini (2001). The key differences between our model and that of Gomes are that we: 1) include taxation; 2) model debt issuance explicitly; and 3) allow the corporation to save. We place greater emphasis on financing since we seek to explain empirical leverage relationships, whereas Gomes focuses upon investment. Cooley and Quadrini (2001) examine industry dynamics in a model which explicitly treats the choice between debt and equity in a setting without taxes. Firms rent rather than purchase physical capital and their model imposes a cap on the equity of the firm, and hence liquid assets. This cap is rationalized by assuming the corporation earns a lower rate of return on financial investments than shareholders.

In related papers, Fischer, Heinkel, and Zechner (FHZ) (1989) and Goldstein, Ju, and Leland (GJL) (2001) formulate dynamic trade-off models with exogenous investment and distribution policies. Brennan and Schwartz (1984) and Titman and Tsyplakov (2002) endogenize investment, but maintain the assumption that free cash is distributed to shareholders. Of critical importance in understanding the contribution of our paper is that all four models hold the gross tax advantage of debt constant, independent of whether the firm is financially constrained or unconstrained.

In a recent empirical paper, Leary and Roberts (2004a) find that a modified version of the FHZ model, featuring fixed plus convex adjustment costs, can explain many of the stylized facts regarding financial timing, and can also be reconciled with the empirical findings of Baker and Wurgler (2002). Strebulaev (2004) formulates a dynamic trade-off model with adjustment costs similar to that of GJL. Simulations of his model, and indirectly of the GJL model, produce results broadly consistent with the empirical evidence in Baker and Wurgler (2002).¹

Although variants of the FHZ and GJL models enjoy some empirical support, Leary and Roberts (2004a, 2004b) present evidence directly supportive of *our* dynamic trade-off model and inconsistent with that of FHZ and GJL. In particular, they find that the gap between internal funds and anticipated capital expenditures is a key determinant of financial policy. Firms issue debt, and to a lesser extent equity, when the financing gap is large. The firm's financing gap plays no role in the FHZ and GJL models, although it is of central importance in our formulation. Consistent with our model, Leary and Roberts (2004a) also find that higher profitability is associated with significantly less external financing: equity and debt. However, the FHZ and GJL models predict that firms respond to profitability shocks by going into the capital markets and issuing more debt.

The findings in Leary and Roberts (2004a) and Strebulaev (2004) may tempt some to conclude that adjustment costs are necessary to reconcile the trade-off theory with the empirical evidence.²

Our results show that this is not true. Since our firm dynamically optimizes over leverage, payouts, and investment each and every period, it is always at a “restructuring point,” and still generates a data series consistent with the stylized facts.

Our paper is also related to the public finance literature assessing the effect of the dividend tax, with Auerbach (2000) providing a recent survey. Sinn (1991) presents a deterministic model in which the firm cannot issue debt, and must choose between internal and external equity. Auerbach (2002) presents a more satisfactory treatment of the effect of taxation on financial policy. However, his model: 1) is deterministic; 2) has no investment decision; 3) has no cost of equity issuance; 4) assumes a flat rate corporate income tax; and 5) imposes exogenous dividend and repurchase constraints.³

Another contribution of our model is that it determines optimal financial slack. Kim, Mauer, and Sherman (1998) bound corporate saving by setting an exogenous lower rate of return on corporate financial investments. Almeida, Campello, and Weisbach (2003) remove the precautionary motive for saving by imposing a finite horizon. Shyam-Sunder and Myers (1999) foreshadow our approach, arguing that, “tax or other costs of holding excess funds” may compel distributions. However, their discussion begs the following questions. First, exactly what are the “tax costs” associated with slack? Second, since pecking order theory assumes “taxes are second order,” then at what point do taxes become first order? Finally, what is the optimal amount of slack and how does it vary with tax rates and costs of external funds? Our model answers each question explicitly.

Before proceeding, it should be noted that forty years ago Modigliani and Miller (1963) articulated the need for precisely the type of model developed in this paper, stating:

The existence of a tax advantage for debt financing... does not necessarily mean that corporations should at all times seek to use the maximum possible amount of debt... For one thing, other forms of financing, notably retained earnings, may in some circumstances be cheaper still when the tax status of investors under the personal income tax is taken into account. More important, there are, as we pointed out, limitations imposed by lenders... which are not fully comprehended within the framework of static equilibrium models, either our own or those of the traditional variety.

The details of the dynamic model that Modigliani and Miller seemed to have in mind have never been worked out. Consequently, empiricists have been left with little formal guidance in

interpreting the signs and magnitudes of the regression coefficients implied by the theory. Bridging the divide between theory and data is the objective of this paper.

The remainder of the paper is organized as follows. Section I provides several simple examples that explain the main intuitive results. Section II presents the model, and sections III and IV derive the optimal financial and investment policies, respectively. Section V shows that under reasonable parameter values, the model generates regression coefficients consistent with the stylized facts. Section VI describes our data and the indirect inference procedure. Section VII concludes.

I. The Basic Argument

The following stylized examples convey the central intuition of the dynamic model. For the purpose of simplicity, this section: 1) fixes the firm’s real investment policy; 2) ignores uncertainty; and 3) assumes constant tax rates on corporate income, individual interest income, and corporate distributions, denoted τ_c , τ_i , and τ_d , respectively. These assumptions are relaxed in the model presented in Section II.

Let r be the rate of return on the taxable riskless Treasury bill. Now, consider the standard “date zero” firm with no internal cash evaluating the choice between debt and external equity. Assume the firm knows marginal funds will be distributed next period. Reducing debt by one dollar increases next period’s distribution by $1 + r(1 - \tau_c)$, with the shareholder receiving the following amount after distribution taxes:⁴

$$1 + r(1 - \tau_c)(1 - \tau_d). \quad (1)$$

Now assume that each dollar raised in the equity market costs the shareholder $1 + \lambda$, where λ is interpreted as flotation costs. Reducing debt by one dollar requires the shareholder to give up $1 + \lambda$ in the current period. If the shareholder had been able to invest these funds on his own account, rather than contributing them to the firm for the purpose of debt reduction, he would have earned:

$$(1 + \lambda)[1 + r(1 - \tau_i)]. \quad (2)$$

Therefore, it is better to leave the debt outstanding when:

$$(1 + \lambda)[1 + r(1 - \tau_i)] > 1 + r(1 - \tau_c)(1 - \tau_d) \quad (3)$$

$$\Rightarrow \frac{\lambda[1 + r(1 - \tau_i)]}{r} > \tau_i - [\tau_c + \tau_d(1 - \tau_c)]. \quad (4)$$

If $\lambda = 0$, the analysis above yields the “traditional” condition on tax rates such that debt dominates external equity:

$$\tau_c > \frac{\tau_i - \tau_d}{1 - \tau_d}. \quad (5)$$

Note that Miller derives his condition for the optimality of debt finance (5) by implicitly setting up a firm at the debt versus external equity margin with non-negative distributions to shareholders in all future periods.⁵ Following Graham (2000), we temporarily choose as base-case parameters $\tau_i = 29.6\%$ and $\tau_d = 12\%$. Under these tax rates, the traditional condition (5) implies that debt should be issued so long as $\tau_c > 20\%$.

Despite the common use of condition (5) as a gauge of debt conservatism, we will show that it is only applicable if the firm has no internal funds this period and knows it will make positive distributions next period. Indeed, consider an otherwise identical firm, except that it has different expectations regarding next period’s equity regime. In particular, assume that rather than making a distribution next period, the firm anticipates issuing equity. That is, external equity represents next period’s marginal source of funds. If the firm retires a unit of debt this period, required equity issuance next period is reduced by $1 + r(1 - \tau_c)$. Next period, this saves the shareholder:

$$(1 + \lambda)[1 + r(1 - \tau_c)]. \quad (6)$$

Reducing debt by one dollar requires the shareholder to give up $1 + \lambda$ in the current period. If the shareholder had been able to invest these funds on his own account, rather than contributing them to the firm for the purpose of debt reduction, he would have earned:

$$(1 + \lambda)[1 + r(1 - \tau_i)]. \quad (7)$$

In this context, it is better to leave the debt outstanding if $\tau_c > \tau_i$. Conversely, when $\tau_c < \tau_i$, the optimal policy is to issue sufficient equity this period to retire all debt. This argument is not circular. We made no assumption regarding the source of funds this period. The firm was free to choose between debt and equity. Rather, the assumption adopted was that the firm anticipates external equity being the *marginal* source of funds *next* period. In this setting, it is optimal to delay equity issuance when the shareholder can earn a higher after-tax rate of return on savings than the corporation. Note also that under the assumed tax rates, the critical corporate tax rate needed to induce debt issuance is 29.6%, which is above the traditional trigger given in (5), which is equal to 20%. In other words, the case for debt finance is weaker when the firm anticipates issuing equity next period, rather than distributing.

The previous two examples illustrated how the choice between debt and external equity depends upon the firm's expected equity regime *next* period. The next example illustrates the importance of the firm's current financial position. In contrast to a firm needing external funds, consider a firm like Microsoft, with internal funds well in excess of the amount needed to fund the real investment program. Rather than choosing between debt and external equity, such a firm must choose between retention and distribution of excess funds.

Suppose the CFO anticipates that marginal funds will be distributed next period. If the funds are distributed today, the shareholder receives $(1 - \tau_d)$. By investing the funds on his own account, the shareholders receives the following amount next period:

$$(1 - \tau_d)[1 + r(1 - \tau_i)]. \quad (8)$$

In contrast, if the funds are retained for the purpose of corporate saving, the shareholder receives the following amount next period after distribution taxes:

$$(1 - \tau_d)[1 + r(1 - \tau_c)]. \quad (9)$$

In this context, it is better to distribute, and reduce internal saving, if $\tau_c > \tau_i$. The corporation will want to reduce saving so long as its tax rate exceeds 29.6%, which differs from the traditional trigger for the dominance of debt over external equity, which is 20% under the assumed tax rates. Intuitively, the shareholder prefers the firm to distribute the funds if he can invest at a higher after-tax rate of return than the corporation. Similar results are derived by King (1974), Auerbach (1979), and Bradford (1981).

The discussion above focused on some extreme circumstances. In reality, firms can be in three possible equity regimes: positive distributions, zero distributions, or negative distributions (equity issued). In addition, the equity regime next period should be modeled as the outcome of an optimizing decision over financing and real investment policies in light of the realized state. The model presented in the next section does so. Having said this, the simple examples provided above suggest the following insights. First, the optimal financial policy and target marginal corporate tax rate depend upon the firm's current equity regime and expectations regarding next period's equity regime. Second, optimal financial policy will exhibit path dependence, since the firm's history determines its current financing margin.

II. The Model

A. Technology and Financing

Time is discrete and the horizon infinite. Operating profits (π) depend upon capital (k) and a shock (z). The space of capital inputs is denoted $K \subseteq \mathfrak{R}_+$, with the corresponding measurable space denoted (K, \mathcal{K}) . Characteristics of the operating profit function and shock are described below.

Assumption 1. The operating profit function $\pi : K \times Z \rightarrow \mathfrak{R}_+$ is twice continuously differentiable; strictly increasing; strictly concave; and satisfies the Inada conditions:

$$\begin{aligned}\lim_{k \downarrow 0} \pi_1(k, z) &= \infty \quad \forall z \in Z, \\ \lim_{k \uparrow \infty} \pi_1(k, z) &= 0 \quad \forall z \in Z.\end{aligned}$$

Assumption 2: The profit shock takes values in a compact set $Z \equiv [z, \bar{z}]$ with Borel subsets \mathcal{Z} . The transition function Γ on (Z, \mathcal{Z}) is Markov, monotone, satisfies the Feller property, and has no atoms.⁶

Concavity of the operating profit function occurs under imperfect competition, where the firm faces a downward-sloping demand curve. Alternatively, Lucas (1978) argues that limited managerial or organizational resources result in decreasing returns. The variable z reflects shocks to demand, input prices, or productivity.

The firm has four potential sources of funds: 1) external equity; 2) current cash flow; 3) single-period debt; and 4) internal savings. The model incorporates: 1) a progressive corporate income tax; 2) personal taxes on interest income; 3) personal taxes on distributions to shareholders; 4) costs of financial distress; 5) a collateral constraint; and 6) equity flotation costs. The first four financial frictions represent the traditional ingredients of the trade-off theory, while the last two frictions add realism and tractability to the model. Equally important to note are the theories excluded. In particular, there is no notion of adjustment costs, market timing, or the rules of thumb implicit in the pecking order.

We now discuss each financial friction in detail. Smith (1977) provides detailed evidence on direct equity flotation costs. Using this data, Gomes (2001) estimates that the marginal flotation cost is 2.8%. To reflect such costs, we adopt the following assumption.

Assumption 3: For each dollar of external equity paid into the firm, there is a flotation cost $\lambda > 0$.

In Section V, we simulate the model assuming $\lambda = 2.8\%$, seeing whether a dynamic trade-off model with small flotation costs generates regression coefficients broadly consistent with the stylized facts. In Section VI, indirect inference is used to estimate λ and other parameters of interest.

The static trade-off theory posits that corporations weigh tax advantages of debt against distress costs. In order to capture this trade-off, we assume that financial distress necessitates a “fire sale” in which capital is sold at a depressed price ($s < 1$) in order to make the promised debt payment.

Assumption 4: If end-of-period internal funds are insufficient to meet debt obligations, a fire sale occurs, with capital sold for $s < 1$. Outside of financial distress, the firm may buy and sell capital for a price of one.

In support of Assumption 4, Asquith, Gertner, and Scharfstein (1994) document that asset sales are a common response to distress. The existence of fire sale costs is documented in two studies by Pulvino (1998, 1999), who finds that constrained and distressed airlines receive lower prices on the sale of aircraft than healthy airlines. In addition, distress is often a correlated event.⁷ In the event of correlated distress, it may be necessary to reallocate capital across sectors. In a study of aerospace plant closings, Ramey and Shapiro (2001) find that reallocated capital sells at a discount.

The next assumption introduces a collateral constraint.

Assumption 5: The firm may borrow and lend at the risk-free rate r before taxes. The lender imposes a collateral constraint requiring that the fire sale value of capital be sufficient to pay the loan.

Assumption 5 is made for two reasons. First, an extensive theoretical and empirical literature suggests that firms face collateral constraints.⁸ Second, Assumption 5 greatly simplifies the numerical problem solved below, eliminating the need to solve for the promised yield to maturity that would be requested by the lender when the value of liquidated assets is insufficient to cover the promised debt payment.

The endogenous state variable p' represents the face value of debt, with payment coming due next period. Positive (negative) values of p' imply the firm is borrowing (lending). The feasible set for p' is denoted $P \subseteq \mathfrak{R}$, with the corresponding measurable space denoted (P, \mathcal{P}) .

Limiting the firm to single-period debt precludes simultaneous borrowing and lending. When debt is single-period, increasing borrowing and lending in equal amounts constitutes a “neutral permutation” of the optimal policy, with interest income canceling interest expense. A natural extension of the model would be to derive optimal maturity structure, allowing the firm to borrow at long maturities while lending/borrowing at short maturities.⁹ Such a model might rationalize the observed tendency of firms to simultaneously borrow and lend. Alternative explanations for simultaneous borrowing and lending by corporations include transactional demand for cash, sinking-fund provisions in bond covenants, and banks requiring compensating deposits.

B. Taxation

Investors are homogeneous and risk neutral. The tax rate on interest is τ_i , implying investors use $r(1 - \tau_i)$ as their discount rate. Following Bradford (1981), we assume shareholders are taxed at rate τ_d on corporate distributions. The model does not impose any constraint on dividends or share repurchases. Nor is any assumption made regarding whether the corporation uses dividends or share repurchases as the method for disgoring funds. Rather, we follow Bradford in assuming there is a flat rate of tax applied to the total amount distributed. This approach allows us to characterize optimal distribution policy, as distinct from optimal dividend policy. In particular, our model pins down the total amount paid to shareholders, not the means of distribution. As such, the model is silent on the “dividend puzzle.”

In the context of the current U.S. income tax system, theory suggests that corporations *should* use share repurchases as the main vehicle for disgoring cash if the marginal shareholder is a taxable individual.¹⁰ There are three advantages of share repurchases. First, capital gains have historically enjoyed a lower statutory tax rate than dividends. Second, shareholder basis is excluded from tax, creating a tax deferral advantage. Finally, there is a tax free step-up in basis at death. In a detailed study, Green and Hollifield (2003) find that under an optimal repurchasing strategy, the effective tax rate on capital gains is only 60% of the statutory rate.¹¹

Corporate taxable income (y) is equal to operating profits less economic depreciation (which occurs at rate δ) less interest expense plus interest income:

$$y(k, p, z) \equiv \pi(k, z) - \delta k - r \left(\frac{p}{1+r} \right). \quad (10)$$

The corporate tax function is denoted g , with the marginal corporate tax rate (τ_c) satisfying:

$$\tau_c[y(k, p, z)] \equiv g_1[y(k, p, z)]. \quad (11)$$

Assumptions regarding the tax system are summarized below.

Assumption 6: Investors are taxed at flat rates of $\tau_i \in (0, 1)$ on interest income and $\tau_d \in (0, 1)$ on corporate distributions. The corporate tax function $g : \mathcal{Y} \rightarrow \mathfrak{R}$ is twice differentiable; strictly increasing; strictly convex; satisfies $g(0) = 0$;

$$\begin{aligned} \lim_{y \rightarrow \infty} \tau_c(y) &\equiv \bar{\tau}_c < 1; \\ \lim_{y \rightarrow -\infty} \tau_c(y) &= 0; \\ \bar{\tau}_c &> \tau_i. \end{aligned}$$

In reality, firms with negative taxable income do not receive a check from the U.S. Treasury. Rather, losses may be carried back two years and carried forward twenty years. The convex tax schedule g is intended to capture the effects of the loss limitation provisions in a tractable way. For a careful treatment of the loss limitation rules and the implications for effective marginal tax rates, the reader is referred to Graham (1996a, 1996b).

The condition $\bar{\tau}_c > \tau_i$ is imposed for tractability, although it is not necessary. As is shown below, the condition $\bar{\tau}_c > \tau_i$ is necessary to generate bounded savings and induce distributions of excess liquidity. If the condition is not met, the model yields the prediction that the optimal policy for a corporation with excess liquidity is to save everything. We revisit this condition in Section III where the optimal financial policy is characterized.

The collateral constraint requires that the sum of after-tax cash flow plus the liquidation value of capital is at least as large as the promised debt payment:

$$p' \leq s * k'(1 - \delta) + \pi(k', \underline{z}) - g(y(k', p', \underline{z})). \quad (12)$$

If realized after-tax cash flow is insufficient to cover debt service, the firm sells the minimum amount of capital needed to make the promised payment. The random variable n denotes the number of units of capital sold in a fire sale:

$$n(k', p', z') \equiv \max \left\{ 0, \frac{p' - [\pi(k', z') - g(y(k', p', z'))]}{s} \right\}. \quad (13)$$

Since $\pi - g$ has positive support, savers never conduct fire sales.

The firm chooses k' at the start of the period, with the actual end of period capital stock, after fire sales, being stochastic. The variable $i(k, p, k', z)$ denotes the funds required to change the

capital stock to k' , given the current state (k, p, z) :

$$i(k, p, k', z) \equiv k' - [k(1 - \delta) - n(k, p, z)]. \quad (14)$$

C. The Firm's Problem

Each period, the vector (k, p, z) summarizes the state, with the firm choosing optimal investment and financial policies. Without loss of generality, attention can be confined to compact K . As in Gomes (2001), define \bar{k} as follows:

$$\pi(\bar{k}, \bar{z}) - \delta\bar{k} \equiv 0. \quad (15)$$

Under Assumption 1, \bar{k} is well defined. Since $k > \bar{k}$ is not economically profitable, let:

$$K \equiv [0, \bar{k}]. \quad (16)$$

The debt limit based on the collateral constraint (12) is increasing and concave in k' and is denoted $\bar{p}(k')$. Since k' is chosen from a compact set K , it follows that \bar{p} is bounded above. In order to ensure compactness of the set P , it is convenient to assume there is an arbitrarily low bound on p' , denoted \underline{p} . This lower bound is imposed without loss of generality, since Assumption 6 ensures bounded saving. From this analysis, it follows that the choice set $K \times P$ is non-empty, compact, and convex.

Each period, cash flow to shareholders before distribution taxes or flotation costs is equal to:

$$\max\{\pi(k, z) - g(y(k, p, z)) - p, 0\} + \frac{p'}{1+r} - i(k, p, k', z). \quad (17)$$

The first term in brackets in (17) is operating profits less corporate taxes less debt payments. When this term is negative, the lender collects all after-tax earnings, leaving equity with zero. The last two terms represent cash inflow (outflow) from new borrowing (lending) and the investment cost, respectively.

Let Φ_s and Φ_n be indicators for states in which fire sales do and do not occur, respectively. Substituting (13) and (14) into (17) and rearranging terms, the cash flow to shareholders, before flotation costs and distribution taxes, may be expressed as:

$$\frac{\pi(k, z) - g(y(k, p, z)) - p}{\Phi_n + s\Phi_s} - [k' - k(1 - \delta)] + \frac{p'}{1+r}. \quad (18)$$

From (18) it can be seen that the economic effect of fire sales is to increase the real cost per dollar of debt service in distressed states.

Letting Φ_d, Φ_i , and Φ_0 be indicators for positive distributions, equity issuance, and zero distributions, respectively, the net cash flow to shareholders is:

$$e(k, p, k', p', z) \equiv [1 + \Phi_i \lambda - \Phi_d \tau_d] \left[\frac{\pi(k, z) - g(y(k, p, z)) - p}{\Phi_n + s\Phi_s} - [k' - k(1 - \delta)] + \frac{p'}{1 + r} \right]. \quad (19)$$

The function e is continuous and strictly concave in its first two arguments. Fire sales, distribution taxes, and flotation costs generate kinks which cause the function e to be non-differentiable for states (k, p, z) such that either:

$$\pi(k, z) - g(y(k, p, z)) = p \quad (20)$$

or

$$e(k, p, k', p', z) = 0. \quad (21)$$

The objective of the manager is to maximize the discounted value of net cash flow to shareholders:

$$V_{t_0} = E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \left(\frac{1}{1 + r(1 - \tau_i)} \right)^{(t-t_0)} e_t \right\}. \quad (22)$$

The Bellman equation for this problem is:

$$V(k, p, z) = \max_{(k', p') \in K \times P} e(k, p, k', p', z) + \left[\frac{1}{1 + r(1 - \tau_i)} \right] \int V(k', p', z') \Gamma(z, dz'). \quad (23)$$

The following propositions, proved in the appendix, characterize the value function and optimal policy correspondence (h).

PROPOSITION 1: *There is a unique continuous function $V : K \times P \times Z \rightarrow \mathfrak{R}_+$ satisfying (23).*

PROPOSITION 2: *For each $z \in Z$, the equity value function $V(\cdot, \cdot, z) : K \times P \rightarrow \mathfrak{R}_+$ is strictly increasing (decreasing) in its first (second) argument and strictly concave.*

PROPOSITION 3: *The optimal policy correspondence $h(\cdot, \cdot, z) : K \times P \rightarrow K \times P$ is a continuous single-valued function.*

PROPOSITION 4: *At each (k, p, z) in the interior of $K \times P \times Z$ such that*

$$\begin{aligned} \pi(k, z) - g(y(k, p, z)) &\neq p, \\ e(k, p, k', p', z) &\neq 0, \end{aligned} \quad (24)$$

the equity value function $V(\cdot, \cdot, z)$ is continuously differentiable in its first two arguments with derivatives given by:

$$V_i(k, p, z) = e_i(k, p, k', p', z) \text{ for } i = 1, 2.$$

III. Optimal Financial Policy

This section derives the optimal financial policy holding fixed the investment program, with the next deriving the optimal investment rule in light of the firm's financial policy.

A. The Marginal Costs and Benefits of Debt

The budget constraint (19) may be restated as:

$$\begin{aligned} \frac{p'}{1+r} - \frac{e(k, p, k', p', z)}{1 + \Phi_i \lambda - \Phi_d \tau_d} &= i(k, p, k', z) - \max\{\pi(k, z) - g(y(k, p, z)) - p, 0\} \\ &= k' - k(1 - \delta) - \frac{\pi(k, z) - g(y(k, p, z)) - p}{\Phi_n + s\Phi_s}. \end{aligned} \quad (25)$$

The left side of (25) represents sources of external funds and the right side represents the *financing gap*, which is the excess of investment costs over internal funds. *Constrained (Unconstrained)* firms have positive (negative) financing gaps.

We derive the optimal financial policy holding fixed the financing gap. To do so, consider a firm at an arbitrary state (k, p, z) evaluating a candidate financing policy p' satisfying $e(k, p, k', p', z) \neq 0$. Consider a perturbation increasing p' with the funds used to finance an increase in e . Since the right side of (25) is being held fixed, the implicit function theorem implies that along the “iso-funding” line:

$$\left. \left(\frac{\partial e}{\partial p'} \right) \right|_{isofund} = \frac{1 + \Phi_i \lambda - \Phi_d \tau_d}{1 + r}. \quad (26)$$

Assuming differentiability of the value function, the total change in the right side of (23) resulting from a small increase in p' is:

$$\Delta(k, p, k', p', z) = \frac{1 + \Phi_i \lambda - \Phi_d \tau_d}{1 + r} + \left[\frac{1}{1 + r(1 - \tau_i)} \right] \int V_2(k', p', z') \Gamma(z, dz'). \quad (27)$$

Proposition 4 implies that so long as (24) holds, the value function is differentiable, with:

$$V_2(k', p', z') = -[1 + \Phi'_i \lambda - \Phi'_d \tau_d] \left[\frac{1 + r[1 - \tau_c(y(k', p', z'))]}{(1 + r)(\Phi'_n + s\Phi'_s)} \right]. \quad (28)$$

The “no atoms” condition in Assumption 2 implies that (20) occurs on a set of measure zero, so that this kink point can be disregarded in deriving the optimal policies.

Finally, we must pin down V_2 for states such that $e = 0$. It is shown below that the end-of-period equity regime hinges upon p' . High savings make it probable that positive distributions occur, while high debt is associated with equity issuance. Intermediate values of p' are associated

with zero distributions ($e = 0$). Having established concavity of the value function in Proposition 2, it follows that when $e = 0$, V_2 must be somewhere between the extremes implied by (28). Therefore, we denote the derivative of the value function in zero distribution states as:

$$\begin{aligned} V_2(k', p', z') &\equiv -[1 + \phi(k', p', z')] \left[\frac{1 + r[1 - \tau_c(y(k', p', z'))]}{(1 + r)(\Phi'_n + s\Phi'_s)} \right] \\ \phi(k', p', z') &\in (-\tau_d, \lambda). \end{aligned} \quad (29)$$

Substituting (28) and (29) into (27) and multiplying by $(1 + r)$ yields an expression for the net marginal benefit from increasing debt (reducing saving):

$$(1 + r) * \Delta(k, p, k', p', z) = MB(k, p, k', p', z) - MC(k', p', z) \quad (30)$$

$$MB(k, p, k', p', z) \equiv 1 + \Phi_i \lambda - \Phi_d \tau_d \quad (31)$$

$$MC(k', p', z) \equiv \int \frac{[1 + \Phi'_i \lambda - \Phi'_d \tau_d + \Phi'_0 \phi'] [1 + r(1 - \tau_c(y(k', p', z')))]}{[1 + r(1 - \tau_i)](\Phi'_n + s\Phi'_s)} \Gamma(z, dz'). \quad (32)$$

The term MB represents the marginal benefit to shareholders from increasing debt, reflecting either increased distributions or lower equity contributions. The term MC represents the expected discounted marginal cost of servicing the debt.

The current state (k, p, z) is fixed and the financial perturbation treats k' as a constant. Therefore, the only argument in the MB function being changed is p' . As p' is increased, the MB schedule steps down from $1 + \lambda$ to $1 - \tau_d$ at a unique switch-point, denoted p'_0 :

$$e[k, p, k', p'_0, z] \equiv 0. \quad (33)$$

From (25) it follows that $p'_0/(1 + r)$ is just equal to the firm's financing gap:

$$\begin{aligned} \frac{p'_0(k, p, k', z)}{1 + r} &\equiv i(k, p, k', z) - \max\{\pi(k, z) - g(y(k, p, z)) - p, 0\} \\ &= k' - k(1 - \delta) - \frac{\pi(k, z) - g(y(k, p, z)) - p}{\Phi_n + s\Phi_s}. \end{aligned} \quad (34)$$

From (25) it follows that the sign of p'_0 depends on firm status, with:

$$\begin{aligned} \text{Unconstrained} &\Rightarrow p'_0(k, p, k', z) < 0 \\ \text{Constrained} &\Rightarrow p'_0(k, p, k', z) > 0. \end{aligned}$$

When evaluating whether to increase debt, shareholders compare the marginal benefit with the marginal cost, with the latter represented by the MC schedule. The direct cost to the corporation

of debt service is $1+r(1-\tau'_c)$, and this term appears in the numerator of (32). The term $1+r(1-\tau_i)$ in the denominator is the discount rate. The MC schedule contains two other terms affecting the shadow cost of debt service. The term $\Phi'_n + s\Phi'_s$ in the denominator implies that *the economic cost of debt service is high when there is a high probability of a fire sale*. Finally, the term $1 + \Phi'_i\lambda - \Phi'_d\tau_d + \Phi'_0\phi'$ reflects the fact that *debt service is most (least) costly for a firm that expects to be issuing equity (making a distribution) at the margin next period*. The effect of decreasing saving is analogous.

From (32) it follows that the marginal cost of debt service is increasing in the amount of debt issued:

$$\frac{\partial MC(k', p', z)}{\partial p'} > 0. \quad (35)$$

The reasoning is as follows. First, increasing p' reduces taxable income (y') in every state (z'). Therefore, the expected marginal corporate tax rate is decreasing in the amount of debt issued. Symmetrically, the expected after-tax return on corporate saving declines in the amount saved, discouraging precautionary saving. Second, raising p' increases the likelihood of a fire sale ($\Phi'_s = 1$). Finally, it is shown below that raising p' increases the likelihood of resorting to positive equity issuance next period ($\Phi'_i = 1$).

In characterizing the optimal financial policy, it will also be useful to note the limiting behavior of the MC schedule. Due to the fact that $\bar{\tau}_c > \tau_i$, firms with arbitrarily high savings will make a distribution at the margin next period. In addition, such firms converge to the maximum corporate tax rate. Therefore:¹²

$$\lim_{p' \downarrow -\infty} MC(k', p', z) = \frac{(1 - \tau_d)[1 + r(1 - \bar{\tau}_c)]}{1 + r(1 - \tau_i)} < 1 - \tau_d. \quad (36)$$

B. Graphical Exposition

[Place Figure 1 about here.]

Figure 1 depicts the optimal financial policy for three potential MC schedules. The decision-making process is similar for each schedule. To see this, assume the firm faces one of the three MC_i schedules. Now, consider a firm with $p'_0/(1+r) > H_i$. For this firm, the marginal benefit from increasing leverage is $1 + \lambda$ for debt levels less than or equal to H_i . Starting from the far left, the marginal benefit of reducing saving or increasing debt exceeds the marginal cost until H_i is

reached. Increasing debt beyond H_i is suboptimal. Since the firm chooses $p' < p'_0$, it follows that equity issuance covers the remaining financing gap ($e < 0$).

Now consider a less constrained firm facing the same schedule MC_i , with $p'_0/(1+r) < L_i$. The optimal debt issuance is equal to $L_i < H_i$. This firm issues less debt than the more constrained firm because the marginal dollar of debt goes towards a distribution rather than replacing costly external equity. The relevant marginal benefit schedule is $1 - \tau_d$, which exceeds the marginal cost to the left of L_i , but is less than the marginal cost for higher debt levels. Since debt issuance exceeds the financing gap, it follows that a positive distribution ($e > 0$) is made.

Finally, consider firms with intermediate funding needs, where:

$$\frac{p'_0(k, p, k', z)}{1+r} \in [L_i, H_i]. \quad (37)$$

For such firms, the MB schedule jumps down from $1 + \lambda$ to $1 - \tau_d$ somewhere in the interval $[L_i, H_i]$. It follows that increasing debt is optimal so long as it substitutes for external equity, but is suboptimal if it finances a higher distribution. Thus, optimal debt issuance is equal to the financing gap, implying that the distribution to equity is just equal to zero.

Summarizing the optimal policies, we have:

$$\begin{aligned} \frac{p'_0(k, p, k', z)}{1+r} > H_i &\Rightarrow \frac{p'}{1+r} = H_i, \quad e(k, p, k', p', z) < 0, \quad MC(k', p', z) = 1 + \lambda \\ \frac{p'_0(k, p, k', z)}{1+r} < L_i &\Rightarrow \frac{p'}{1+r} = L_i, \quad e(k, p, k', p', z) > 0, \quad MC(k', p', z) = 1 - \tau_d \\ \frac{p'_0(k, p, k', z)}{1+r} \in [L_i, H_i] &\Rightarrow p' = p'_0(k, p, k', z) \text{ and } e(k, p, k', p', z) = 0. \end{aligned} \quad (38)$$

This indicates that there is no target leverage ratio. Firms can be borrowers or savers under the optimal program, depending on the financing gap and position of the MC schedule.

We now turn to the optimal policies under each of the three specific MC scenarios depicted in Figure 1. Consider first the firm facing the low MC_1 schedule, where:

$$MC(k', 0, z) < 1 - \tau_d. \quad (39)$$

Referring to (32), the condition (39) is most likely to hold when the probability of making a positive distribution next period ($\Phi'_d = 1$) is high. In addition, it is easily verified that a necessary condition for (39) is:

$$\int \tau_c[y(k', 0, z')] \Gamma(z, dz') > \tau_i. \quad (40)$$

The low MC_1 scenario is most likely to hold for cash cow corporations that expect to be in the top tax bracket.¹³ Under the low MC_1 scenario, firms are heavily levered. In fact, even unconstrained firms are willing to issue debt (L_1) in order to finance higher distributions. Such behavior is a clear violation of the static pecking order.

Moving to the opposite extreme, consider the optimal financial policy when the firm faces the MC_3 schedule, where:

$$MC(k', 0, z) > 1 + \lambda. \quad (41)$$

From (32) it follows that in order for (41) to be satisfied, the probability of being in the equity issuance regime next period must be high. In addition, a necessary condition for (41) is:

$$\int \tau_c[y(k', 0, z')] \Gamma(z, dz') < \tau_i. \quad (42)$$

The high MC_3 scenario is most likely to hold for high growth firms with low taxable income. Under the high MC_3 scenario, firms avoid debt completely, with the tax disadvantage to debt at the personal level swamping the benefit of deducting interest expense at the corporate level. Firms with $p'_0/(1+r) > H_3$ exhibit a striking departure from the static pecking order. These firms simultaneously save and issue equity, despite the fact that riskless debt finance is available.

The last scenario to be considered features the intermediate MC_2 schedule satisfying:

$$1 - \tau_d < MC(k', 0, z) < 1 + \lambda. \quad (43)$$

From (32) it can be seen that this scenario is most likely to emerge when the probability of being in either the positive distribution or equity issuance regimes is not too high. In this scenario, unconstrained firms do not issue debt and do not tap external equity. Those unconstrained firms with $p'_0/(1+r) < L_2$ make positive distributions to shareholders, while those with $p'_0/(1+r) \in [L_2, 0)$ set the distribution to zero. Severely constrained firms, with $p'_0/(1+r) > H_2$ utilize a mixture of debt and external equity, with equity being the marginal source of funds. Constrained firms with $p'_0/(1+r) \in (0, H_1)$ use debt as their marginal source of funds, issuing no equity and making no distributions to shareholders.

Finally, it is interesting to note that the firm facing intermediate marginal costs of debt service follows a financial policy strikingly similar to that predicted by Myers' (1984) pecking order theory. This potential observational equivalence should be kept in mind in empirical tests pitting the dynamic trade-off theory against the pecking order.

C. Empirical Implications

Despite the fact that debt is single-period in our model, leverage is predicted to exhibit hysteresis. To see this, consider two firms with the same capital stock (k) and shock (z), with one of the firms having higher lagged debt. For a given choice of k' , the two firms face the same $MC(k', \cdot, z)$ schedule. It follows from (25) that the firm with higher lagged debt has a larger financing gap, with (38) indicating that debt issuance is weakly increasing in the financing gap. The hysteresis effect is due to the fact that, ceteris paribus, higher lagged debt (p) causes the firm to occupy the high portion of the marginal benefit schedule $(1 + \lambda)$ over a longer stretch. That is, with higher lagged debt, more debt must be issued this period before the marginal unit of debt serves to increase distributions rather than replacing external equity.

The theory offers a potential explanation for the debt conservatism of high liquidity firms, documented by Graham (2000). For high liquidity firms like Microsoft, debt issuance serves to finance higher distributions to shareholders, rather than replacing costly external equity. Since high liquidity firms occupy the lower portion of the MB schedule, debt issuance is less attractive.

It is harder to predict the implications of the model for standard OLS regressions treating leverage as the dependent variable. Positive shocks (z) result in higher lagged cash flow $(\pi - g - p)$, which lowers the financing gap. Ceteris paribus, this results in lower leverage. However, positive shocks also raise the desired capital stock, (k'), which increases the financing gap. To the extent that average Q picks up the latter effect, one would predict the coefficient on lagged measures of profitability to be negative. Given this ambiguity, in Section V we simulate the model under reasonable parameter values, pinning down the implied regression coefficients.

D. The Target Corporate Tax Rate

Using the Miller (1977) tax shield formula, Graham (2000) integrates under “net of personal tax benefit curves” to determine the target corporate tax rate. In a dynamic setting, the traditional target marginal corporate tax rate is most likely incorrect. The expected marginal corporate tax rate under the optimal dynamic policy is a complicated function of the current equity regime and expectations regarding next period’s equity regime. Proposition 5, illustrates this point:

PROPOSITION 5: *If the collateral constraint does not bind, then:*

$$e(k, p, k', p', z) < 0 \Rightarrow \int \frac{[1 + \Phi'_i \lambda - \Phi'_d \tau_d + \Phi'_0 \phi'] [1 + r(1 - \tau_c(y(k', p', z')))]}{[1 + r(1 - \tau_i)](\Phi'_n + s\Phi'_s)} \Gamma(z, dz') = 1 + \lambda.$$

$$e(k, p, k', p', z) > 0 \Rightarrow \int \frac{[1 + \Phi'_i \lambda - \Phi'_d \tau_d + \Phi'_0 \phi'] [1 + r(1 - \tau_c(y(k', p', z')))]}{[1 + r(1 - \tau_i)](\Phi'_n + s\Phi'_s)} \Gamma(z, dz') = 1 - \tau_d.$$

$$e(k, p, k', p', z) = 0 \Rightarrow 1 - \tau_d < \int \frac{[1 + \Phi'_i \lambda - \Phi'_d \tau_d + \Phi'_0 \phi'] [1 + r(1 - \tau_c(y(k', p', z')))]}{[1 + r(1 - \tau_i)](\Phi'_n + s\Phi'_s)} \Gamma(z, dz') < 1 + \lambda.$$

Clearly, the traditional ratio in (5) is a faulty basis for gauging debt conservatism or poor tax planning on the part of corporations. To take a concrete example, return to the tax rate assumptions in Section I and consider the CFO of a company like Microsoft. This company is unconstrained, has negative leverage, and is making distributions at the margin each period. Suppose also that the corporation finds itself with an expected corporate tax rate equal to 25% given its current plan. Application of the target tax rate formula in (5) suggests that the corporation should make a *larger* distribution, *reducing* the amount saved, and driving down the expected marginal tax rate to 20%.

In contrast, our model suggests that the firm in this example should actually *reduce* its distribution and *increase* savings. Intuitively, under the current plan, the firm earns a higher after-tax return than shareholders, who face a personal tax rate of 29.6% on interest income. Shareholders would therefore prefer retention of funds. To see this more formally, we may use the second optimality condition in Proposition 5 and set $\Phi'_n = \Phi'_d = 1$. In this case, the target expected marginal corporate tax rate is 29.6%, not 20%.

IV. Optimal Real Investment Policy

Consider the firm in an arbitrary state (k, p, z) evaluating an investment plan k' satisfying $e(k, p, k', p', z) \neq 0$. To pin down the optimal real investment policy, we evaluate the effect on the maximand of a small increase in k' to be financed in accordance with the optimal financial policy. Assuming differentiability of the value function, the change in the maximand is:

$$\frac{de(k, p, k', p', z)}{dk'} + \left(\frac{1}{1 + r(1 - \tau_i)} \right) \int \left[V_1(k', p', z') + \left(\frac{\partial p'}{\partial k'} \right) V_2(k', p', z') \right] \Gamma(z, dz'). \quad (44)$$

The first term in (44) represents the direct cost of investment to the shareholder in terms of the current distribution. The first term in the expectation is simply the discounted value of a unit of installed capital, with the second representing the costs associated with servicing incremental debt used to finance the project. From the firm's budget constraint, the investment funding condition may be stated as:

$$\frac{de}{dk'} = -[1 + \Phi_i\lambda - \Phi_d\tau_d] \left[1 - \left(\frac{1}{1+r} \right) \left(\frac{\partial p'}{\partial k'} \right) \right]. \quad (45)$$

From Proposition 5, we know that when the optimal financial policy entails nonzero distributions ($e \neq 0$):

$$-\frac{[1 + \Phi_i\lambda - \Phi_d\tau_d][1 + r(1 - \tau_i)]}{1 + r} = \int V_2(k', p', z') \Gamma(z, dz'). \quad (46)$$

Substituting (45) and (46) into (44), the incremental gain from increasing the capital stock is:

$$\int \left(\frac{V_1(k', p', z')}{1 + r(1 - \tau_i)} \right) \Gamma(z, dz') - [1 + \Phi_i\lambda - \Phi_d\tau_d]. \quad (47)$$

The first term in (47) represents the expected discounted value of the marginal unit of installed capital, with the second representing the marginal cost of investment, which takes into account the firm's financing margin.

The envelope condition from Proposition 4 implies that for states in which the distribution to equity is nonzero:

$$V_1(k', p', z') = [1 + \Phi'_i\lambda - \Phi'_d\tau_d] \left[\frac{\pi_1(k', z')(1 - \tau'_c) + \delta\tau'_c}{\Phi'_n + s\Phi'_s} + (1 - \delta) \right]. \quad (48)$$

Having established concavity of the value function in Proposition 2, it follows that when $e' = 0$, then V_1 lies somewhere between the extremes implied by (48). We denote the derivative of the value function in zero distribution states as:

$$\begin{aligned} V_1(k', p', z') &\equiv [1 + \widehat{\phi}(k', p', z')] \left[\frac{\pi_1(k', z')(1 - \tau'_c) + \delta\tau'_c}{\Phi'_n + s\Phi'_s} + (1 - \delta) \right] \\ \widehat{\phi}' &\in (-\tau_d, \lambda). \end{aligned} \quad (49)$$

Substituting (48) and (49) into (47) yields, the following optimality condition:

$$1 + \Phi_i\lambda - \Phi_d\tau_d = \int \left[\frac{1 + \Phi'_i\lambda - \Phi'_d\tau_d + \Phi'_0\widehat{\phi}'}{1 + r(1 - \tau_i)} \right] \left[\frac{\pi_1(k', z')(1 - \tau'_c) + \delta\tau'_c}{\Phi'_n + s\Phi'_s} + (1 - \delta) \right] \Gamma(z, dz'). \quad (50)$$

The term on the left represents the direct cash cost to equity from increasing investment, with the right representing the shadow value of installed capital. Note that the cost to equity exhibits a

downward jump at some level of capital investment, call it k'_0 , at which the distribution to equity switches from negative to positive. Some firms would then find themselves at a corner solution for capital investment, finding it profitable to increase the capital stock up to k'_0 , yet unwilling to incur the costs of external equity.

When the optimal policy entails nonzero distributions to equity, the investment rule satisfies:¹⁴

$$1 = \int \left[\frac{1 + \Phi'_i \lambda - \Phi'_d \tau_d + \Phi'_0 \widehat{\phi}'}{1 + \Phi_i \lambda - \Phi_d \tau_d} \right] \left[\frac{\pi_1(k', z')(1 - \tau'_c) + \delta \tau'_c}{[1 + r(1 - \tau_i)][\Phi'_n + s\Phi'_s]} + \frac{1 - \delta}{1 + r(1 - \tau_i)} \right] \Gamma(z, dz'). \quad (51)$$

The first bracketed term reflects the potential for shifts in equity regimes across periods. *Ceteris paribus*, investment incentives are stronger when the firm is currently in the positive distribution equity regime as opposed to the equity issuance regime. Intuitively, the incentive to invest is stronger when the funds used for investment have a low opportunity cost. For a firm that is currently making a distribution, the opportunity cost of retaining a marginal dollar is only $1 - \tau_d$. In contrast, the opportunity cost of a dollar of external equity is $1 + \lambda$.

V. Simulation

Our empirical strategy proceeds as follows. First, we present a simulation of the model based on reasonable parameter values that we glean from previous studies. Our intent is to ascertain whether our theory, with small equity flotation costs, can produce a cross section that embodies the anomalies we seek to explain. This exercise allows us to discriminate between our maximizing framework and other theories as vehicles for explaining observed phenomena. In Section VI we use our data from COMPUSTAT to estimate the basic structural parameters of the model. Since one of these parameters is λ , we will be able to estimate the magnitude of costs of external equity. Finally, we check the sensitivity of the model to variations in some of the key model parameters.

A. Design

In order to simulate our model, we need to choose a functional form for π :

$$\pi(k, z) = zk^\alpha, \quad (52)$$

where $\alpha < 1$ captures decreasing returns to scale. We assume the shock z follows an *AR*(1) process in logs:

$$\ln(z') = \rho \ln(z) + \varepsilon', \quad (53)$$

where $\varepsilon' \sim N(0, \sigma_\varepsilon^2)$. We transform (53) into a discrete-state Markov chain using the method in Tauchen (1986), letting z have 20 points of support in $\left[-3\sigma_\varepsilon / \sqrt{1 - \rho^2}, 3\sigma_\varepsilon / \sqrt{1 - \rho^2}\right]$.

The state space for (k, p, z) is discrete. The capital stock, k , lies in the set

$$\left[\bar{k}, \bar{k}(1 - \delta)^{1/2}, \bar{k}(1 - \delta), \dots, \bar{k}(1 - \delta)^{20}\right],$$

where \bar{k} is defined by (15). The state space for p is more complicated, because debt issuance is restricted by the collateral constraint (12), which in turn depends on the level of the capital stock. We specify the state space for p by choosing feasible points in the candidate set that satisfy (12) for each element of the state space for k . These state spaces for k and p appear to be sufficient for our purposes in that the optimal policy never occurs at an endpoint of the state space for k or at the lower endpoint of the state space for p .

Next we need to define the tax environment. For τ_d we use the estimate in Graham (2000) of 0.12. We set the tax rate on interest income, τ_i , equal to 0.25: a number slightly less than the Graham (2000) estimate of 0.296. To define the marginal corporate income tax schedule, we let $\mathcal{N}(y, \mu_\tau, \sigma_\tau)$ be the cumulative normal distribution function with mean μ_τ and standard deviation σ_τ . The marginal tax rate function is

$$\tau_c(y) = 0.35\mathcal{N}(y, \mu_\tau, \sigma_\tau);$$

the tax bill for positive y is given by

$$\int_0^y \tau_c(x) dx;$$

and the tax bill for negative y is given by

$$-\int_y^0 \tau_c(x) dx.$$

For our first exercise, we parameterize our model along the lines of Gomes (2001) and Cooper and Ejarque (2001). From Gomes we take $\rho = 0.62$, $\sigma_\varepsilon = 0.15$, $\delta = 0.145$, and $\lambda = 0.028$. Because Gomes uses a technological specification slightly different from ours, we turn to Cooper and Ejarque, who use an identical production function, and we set $\alpha = 0.689$. We set s equal to 0.75, a number lying within the broad range of estimates of capital resale discounts in Ramey and Shapiro (2001). Our two remaining parameters are (μ_τ, σ_τ) , which we set so that the average realized marginal corporate tax rate is 0.30. Finally, we use a real risk-free interest rate of 0.025. As will be seen below, the stylized facts we generate with this simple simulation hold up when we base it on a parameterization obtained from our simulated moments estimation.

We solve the model via iteration on the Bellman equation, which produces the value function $V(k, p, z)$ and the policy function $\{k', p'\} = h(k, p, z)$. Our model simulation proceeds by taking a random draw from the distribution of ε each period, updating the z shock, and then computing $V(k, p, z)$ and $h(k, p, z)$. For our initial exercise, we simulate the model for 1000 time periods, dropping the first 50 observations in order to allow the firm to work its way out of a possibly sub-optimal starting point.

Knowledge of h and V also allows us to compute interesting quantities such as cash flow, Tobin's Q , debt, and distributions. Specifically, we define our variables to mimic the sorts of variables used in the literature.

Ratio of investment to the “book value” of assets	$(k' - (1 - \delta)k) / k$
Ratio of cash flow to the book value of assets	$(\pi(k, z) - g[y(k, p, z)] - p) / k$
Tobin's Q	$(V(k, p, z) + p' / (1 + r)) / k'$
Ratio of debt to the “market value” of assets	$(p' / (1 + r)) / (V(k, p, z) + p' / (1 + r))$
Ratio of EBITDA to the book value of assets	$\pi(k, z) / k$
Ratio of equity issuance to the book value of assets	$e(k, p, k', p', z) / k$

Here we have scaled all variables by the book value of assets, except for debt. We adopt this convention because of the use of market leverage in Rajan and Zingales (1995) and Baker and Wurgler (2002).¹⁵

B. Results

Before presenting our simulation results we examine the properties of the simulated policy function, $\{k', p'\} = h(k, p, z)$. We start with the investment rule, where we note first that the choice of k' depends on the current level of debt. If two firms with identical (k, z) come into the current period with different stocks of outstanding debt, the one with higher debt never chooses a higher k' and usually chooses a lower k' .

The debt rule also exhibits several interesting characteristics. First, it is clear that the choice of p' not only depends on (k, z) but on the current level of p . Firms with identical (k, z) and different p almost always choose different levels of p' . Further the condition

$$\frac{\partial p'}{\partial p} \geq 0$$

always holds. As will be seen below, this hysteresis is important for generating behavior that appears to look like market timing. Second, large firms with low profit shocks tend to hold the

most cash. This result is consistent with observed “debt conservatism.” Third, the debt rule displays substantial persistence: for a firm with a given (k, p) a large shock is required for an adjustment of debt policy. Finally, the firm engages in fire sales only if it has both a small capital stock and a large negative shock, and in the simulations described and reported below, a fire sale occurs at most 0.4% of the time. This result is particularly important in that it suggests that our results are not driven by the collateral constraint.

[Place Table I about here.]

The results from this simulation are in Table I, where we present summary statistics on debt and investment, as well as several coefficients from regressions commonly run in the empirical capital structure literature. Note in the first row of the table that the simulated firm on average issues debt, though note in the second line that it holds cash 28.5% of the time. The static trade-off theory implies that a firm with our tax schedule should never hold cash. In contrast, such “debt conservatism” is in our model a rational decision in the face of tax incentives. This result on cash holdings also emphasizes the point made earlier that static models of capital structure are by nature inappropriate, since the firm’s financing margin can change over time. We also find that the firm does undertake on average positive investment, but that this rate of investment has a substantial standard deviation. This result underscores the idea that allowing an endogenous investment decision is crucial to understanding capital structure changes. Finally, when the firm does issue equity, the ratio of issuance to assets is on average 0.055. In calculating this figure, we have only averaged over those observations in which the firm actually does issue equity, which account for 14% of the simulated sample.

The more interesting results are the signs of the regression coefficients. First, we examine the effect of liquidity on debt, as in Table IX in Rajan and Zingales (1995). Our first measure of liquidity, cash flow, comes in with a negative coefficient in a regression of the debt to assets ratio on lagged Q and cash flow. Second, as seen in the next line of the table, this result is robust to our use of EBITDA, as in Rajan and Zingales, as a measure of liquidity. The intuition for these negative coefficients lies in the endogeneity of the firm’s equity regime. In our model, firms that have experienced high profits have lower equity regime switch points (p'_0) and therefore tend to issue less debt. Finally, we run a regression similar to the one in equation (5) in Baker and Wurgler (2002), where once again the debt to assets ratio is the left side variable. The regressors are lagged

Q , lagged EBITDA, and lagged external finance weighted Q . This latter variable is constructed as in Baker and Wurgler (2002), where we use a 20 period moving average to calculate weighted Q .¹⁶ Note that we can replicate the “market timing” result in Baker-Wurgler with a time-invariant λ only equal to 0.028 to represent flotation costs. Our result is a product not of the cumulative attempts to time the equity market, but is merely a product of debt hysteresis and the fact that firms with large productivity shocks simultaneously have high Q s and tend to finance large desired investment with equity at the margin.

VI. Simulated Moments Parameter Estimation

Our data are from the full coverage 2002 Standard and Poor’s COMPUSTAT industrial files. We select a sample by first deleting firm-year observations with missing data. Next, we delete observations in which total assets, the gross capital stock, or sales are either zero or negative. To avoid rounding errors, we delete firms whose total assets are less than two million dollars and gross capital stocks are less than one million dollars. Further, we delete observations that fail to obey standard accounting identities. Finally, we omit all firms whose primary SIC classification is between 4900 and 4999 or between 6000 and 6999, since our model is inappropriate for regulated or financial firms. We end up with an unbalanced panel of firms from 1993 to 2001 with between 592 and 1128 observations per year. We truncate our sample period below at 1993, because our tax parameters are relevant only for this period; see Graham (2000).

Structural estimation of this model faces several challenges. First, the existence of different equity regimes prevents the derivation of an Euler equation or decision rule that is a smooth function of the data. To deal with these issues, we opt for an estimation technique based on simulation of the model. Specifically, we estimate the structural parameters of the model via the indirect inference method proposed in Gourieroux, Monfort, and Renault (1993) and Gourieroux and Monfort (1996). This procedure chooses the parameters to minimize the distance between model-generated moments and the corresponding moments from actual data. Because the moments of the model-generated data depend on the structural parameters utilized, minimizing this distance will, under certain conditions discussed below, provide consistent estimates of the structural parameters. Another appealing feature of this approach is that it allows us to establish a link between our model and existing, less structural empirical evidence.

We now give a brief outline of this procedure. The goal is to estimate a vector of structural

parameters, b , by matching a set of *simulated moments*, denoted as m , with the corresponding set of actual *data moments*, denoted as M . The candidates for the moments to be matched include simple summary statistics, OLS regression coefficients, and coefficient estimates from non-linear reduced-form models.

Without loss of generality, the moments to be matched can be represented as the solution to the maximization of a criterion function

$$\hat{M}_N = \arg \max_M J_N(Y_N, M),$$

where Y_N is a data matrix of length N . For example, the sample mean of a variable, x , can be thought of as the solution to minimizing of the sum of squared errors of the regression of x on a constant. We estimate \hat{M}_N and then construct S simulated data sets based on a given parameter vector. For each of these data sets, we estimate m by maximizing an analogous criterion function

$$\hat{m}_{N'}^s(b) = \arg \max_m J_{N'}(Y_{N'}^s, m),$$

where $Y_{N'}^s$ is a simulated data matrix of length N' . Note that we express the simulated moments, $\hat{m}_{N'}^s(b)$, as explicit functions of the structural parameters, b . The indirect estimator of b is then defined as the solution to the minimization of

$$\begin{aligned} \hat{b} &= \arg \min_b \left[\hat{M}_N - \frac{1}{S} \sum_{s=1}^S \hat{m}_{N'}^s(b) \right]' \hat{W}_N \left[\hat{M}_N - \frac{1}{S} \sum_{s=1}^S \hat{m}_{N'}^s(b) \right] \\ &\equiv \arg \min_b \hat{G}'_N \hat{W}_N \hat{G}_N \end{aligned}$$

where \hat{W}_N is a positive definite matrix that converges in probability to a deterministic positive definite matrix W . In our application, a consistent estimator of W is given by $\left[N \text{var}(\hat{M}_N) \right]^{-1}$. Since our moment vector consists of both means and regression coefficients, we use the influence-function approach in Erickson and Whited (2000) to calculate this covariance matrix. Specifically, we stack the influence functions for each of our moments and then form the covariance matrix by taking the sample average of the inner product of this stack.

The indirect estimator is asymptotically normal for fixed S . Define $J \equiv \text{plim}_{N \rightarrow \infty} (J_N)$. Then

$$\sqrt{N}(\hat{b} - b) \xrightarrow{d} \mathcal{N}(0, \text{avar}(\hat{b}))$$

where

$$\text{avar}(\hat{b}) \equiv \left(1 + \frac{1}{S} \right) \left[\frac{\partial J}{\partial b \partial m'} \left(\frac{\partial J}{\partial m} \frac{\partial J'}{\partial m} \right)^{-1} \frac{\partial J}{\partial m \partial b'} \right]^{-1}. \quad (54)$$

Further, the technique provides a test of the overidentifying restrictions of the model, with

$$\frac{NS}{1+S} \hat{G}'_N \hat{W}_N \hat{G}_N$$

converging in distribution to a χ^2 with degrees of freedom equal to the dimension of M minus the dimension of b .

The success of this procedure relies on picking moments m that can identify the structural parameters b . In other words, the model must be identified. Global identification of a simulated moments estimator obtains when the expected value of the difference between the simulated moments and the data moments equal zero if and only if the structural parameters equal their true values. A sufficient condition for identification is a one-to-one mapping between the structural parameters and a subset of the data moments of the same dimension. Although our model does not yield such a closed form mapping, we take care in choosing appropriate moments to match, and we use a minimization algorithm, simulated annealing, that avoids local minima. Finally, we perform an informal check of the numerical condition for local identification. Let $\hat{m}_{bN'}^s$ be a subvector of m with the same dimension as b . Local identification implies that the Jacobian determinant, $\det(\partial \hat{m}_{N'}^s(b) / \partial b)$, is non-zero. This condition can be interpreted loosely as saying that the moments, m , are informative about the structural parameters, b ; that is, the sensitivity of m to b is high. If this were not the case, not only would $\det(\partial \hat{m}_{N'}^s(b) / \partial b)$ be near zero, but the sample counterpart to the term $\partial J / \partial b \partial m'$ in (54) would be as well—a condition that would cause the parameter standard errors to blow up.

To generate simulated data comparable to COMPUSTAT, we create $S = 6$ artificial panels, containing 10,000 *i.i.d.* firms.¹⁷ We simulate each firm for 50 time periods and then keep the last nine, where we pick the number “nine” to correspond to the time span of our COMPUSTAT sample. Dropping the first part of the series allows us to observe the firm after it has worked its way out of a possibly suboptimal starting point.

One final issue is unobserved heterogeneity in our data from COMPUSTAT. Recall that our simulations produce *i.i.d.* firms. Therefore, in order to render our simulated data comparable to our actual data we can either add heterogeneity to the simulations, or take the heterogeneity out of the actual data. We opt for the latter approach, using fixed firm and year effects in the estimation of all of our data moments.

In sum, we need to estimate eight parameters $(\alpha, s, \delta, \rho, \sigma_\varepsilon^2, \mu_\tau, \sigma_\tau, \lambda)$ by matching at least eight model-generated moments with corresponding data moments. We use ten data moments in order

to have an overidentified model. We start with five simple means: the average ratio of investment to total assets, the average ratio of operating income to assets, the frequency of equity issuance, the average ratio of net equity issuance to total assets, and the average ratio of net debt to total assets, where net debt is defined as total long-term debt less cash. Here, assets are COMPUSTAT item #6, investment is item #30, equity issuance is item #108 minus item #115, and net debt is item #9 plus item #34 minus item #1. The mean of investment will help pin down δ ; the mean of operating income will help pin down the curvature of the profit function, α ; and the means of the three financing variables will help pin down s and λ . Our next two moments capture the important features of the driving process for z . Here, we estimate a first-order panel autoregression of operating income on lagged operating income using the technique in Holtz-Eakin, Newey, and Rosen (1988). Operating income is defined as COMPUSTAT item #13 divided by item #6. The two moments that we match from this exercise are the autoregressive parameter and the shock variance.¹⁸ Our final two moments are regression coefficients commonly calculated by empirical researchers. The first is the slope coefficient from a simple regression of investment on Tobin's Q , where simulated Tobin's Q is constructed as described above and actual Tobin's Q is constructed following the appendix to Whited (1992). Our final moment comes from a regression of the net debt to assets ratio on Tobin's Q .

A. Results

[Place Table II about here.]

The results from this estimation exercise are in Tables II and III. Table II compares the actual with the simulated moments. Note here that the estimation procedure does a good job of matching all of the moments but the variance of investment and the sensitivities of debt and investment to Q . The first discrepancy can be explained by the lack of adjustment costs in our model and the presence of adjustment frictions in the real world. The second discrepancy we conjecture is a result of noise in Q in the real world and perfect measurement of Q in our simulations. The other moments appear to be matched quite well: the χ^2 test of the overidentifying restrictions reported in Table III does not produce a rejection at the 5% level, though it does produce a rejection at the 10% level.

[Place Table III about here.]

Table III contains the point estimates of the structural parameters. Our estimate of δ appears reasonable, and our estimates of ρ and σ_ε^2 are close to the corresponding parameters generated by our estimation of the autoregressive process for operating income. Our estimate of α of 0.551 is consistent with decreasing returns to scale and is comparable to the estimate of 0.51 in Cooper and Haltiwanger (2002). μ_τ and σ_τ are difficult to interpret in that they depend on the state spaces for k and p . However, our estimated values imply that the firm hits the upper statutory rate of 0.35 93% of the time. In comparison, in 2001, 90.1% of the firms in our COMPUSTAT sample had incomes high enough to qualify them for the highest marginal tax rate.¹⁹ Our estimate of s , though statistically insignificant, lies in the $[0, 1]$ interval and is in line with the estimates in Ramey and Shapiro (2001).

Our most interesting parameter estimate is the 0.059 figure we find for λ . This estimate is slightly higher than the 0.028 figure found by Gomes and quite close to the 0.0515 figure in Altinkilic and Hansen (2000). Recall that Titman and Wessels (1988), Rajan and Zingales (1995), and Fama and French (2002) have interpreted the negative relationship between debt and liquidity as evidence in favor of the Myers (1984) information based pecking order. Recall as well that Baker and Wurgler interpret the significance of lagged weighted Q as evidence in favor of market timing. However, in Section V we showed that the tax system in conjunction with small flotation costs are sufficient to generate these stylized facts. Therefore, their evidence is insufficient to establish the existence of an adverse selection premium. On the other hand, our structural estimate is directly informative about the existence and size of asymmetric information costs.

B. Model Comparative Statics

In this subsection we present results from simulating the model using the estimates of the structural parameters in Table III. Our model still generates negative sensitivities of leverage to cash flow, EBITDA, and lagged weighted q . In particular, we find

$$\begin{aligned} \frac{\text{Debt}}{\text{Market Assets}} &= -0.045(\text{Tobin's } Q) - 0.754 \left(\frac{\text{Cash Flow}}{\text{Book Assets}} \right) \\ \frac{\text{Debt}}{\text{Market Assets}} &= -0.198(\text{Tobin's } Q) - 1.054 \left(\frac{\text{EBITDA}}{\text{Book Assets}} \right) \\ \frac{\text{Debt}}{\text{Market Assets}} &= -0.191(\text{Tobin's } Q) - 0.841(\text{Weighted } Q) - 1.042 \left(\frac{\text{EBITDA}}{\text{Book Assets}} \right), \end{aligned}$$

where we have omitted the intercepts. The negative coefficients on cash flow and EBITDA imply that the widely observed negative relationship between profitability and debt is no anomaly. Rather,

in our model, firms that have experienced high profits have lower equity regime switch points (p'_0) and therefore tend to issue less debt. All of these negative sensitivities are of particular interest in that we did not use any of these three moments to fit the model. As such, these results can be thought of loosely as a measure of the success of the “out-of-sample” performance of the model.

To flesh out the intuition behind this result, and to gain a better understanding of the behavior of our model, we next present in Table IV summary statistics from this simulation, categorized by the firm’s financing regime. The table is divided into two panels: the top corresponds to a value of λ equal to 0.059, and the bottom to a value of λ equal to 0. Eliminating the cost of external equity is particularly interesting in that it can tell us the extent to which any of our previous results are driven by this aspect of our model.

[Place Table IV about here.]

Several features of Table IV are noteworthy. Savers never issue equity and debtors never make distributions. This fits with the intermediate MC_2 scenario. Also, whereas equity issuers are the most highly indebted, invest the most, and have the highest Q s, firms making distributions have positive savings, invest the least, and have the lowest Q s. This last result is what generates a negative coefficient on lagged weighted Q , despite the lack of market timing in our model. In other words, high productivity shocks produce both high Q s and equity issuance. This phenomenon, combined with sluggish adjustment of debt, produces the observed negative correlation between current leverage and lagged weighted Q . As seen in the bottom panel, when we set $\lambda = 0$, we find strikingly similar results, suggesting that the tax environment is a more important determinant of the firm’s optimal policies than the existence of costly external equity. The two major differences we do find occur in savings behavior and equity issuance. As λ goes to zero, equity issuance naturally rises. Further, savings falls as the corporation’s precautionary motive diminishes. It does not disappear, however, because the firm still has a motive to save when τ_c is low.

Note that under these parameters, the firm behaves in rough accordance with the rules of thumb of the static pecking order theory. However, changes in the tax regime can lead to departures. We explore these possibilities via a variety of scenarios. First, we lower the maximal statutory corporate income tax rate until it is just above the tax rate on interest income. When we do this, we find that the firm will finance solely with equity approximately 3% of the time. This violation of the static pecking order occurs for firms who are current savers and who anticipate needing equity in

the future. Second, when we lower the maximal corporate tax rate below the tax rate on interest income, we find, as predicted by the model in the high MC_3 scenario, that the firm always retains funds and only finances with equity. In other words, we find a tax-based rationale for corporate liquidity holdings. This sort of phenomenon can also occur in a firm that has, for example, high non-debt tax shields. Third, when we raise the maximal corporate income tax rate to 40%, the tax benefit of debt is sufficiently great that firms occasionally find it optimal to have issue debt and make distributions at the same time. This high corporate tax rate simulation also generates some observations in which the firm has enough internal funds to finance optimal investment, but nonetheless chooses to issue debt—a result consistent with the low MC_1 scenario.

VII. Conclusion

In recent years corporate finance economists have uncovered a variety of phenomena that appear anomalous in light of the traditional static trade-off theory. Our paper questions whether these phenomena are indeed anomalies. We do so by addressing the empirical observations from a more structural perspective. We begin by fully specifying a dynamic model of investment and finance. We show theoretically and via model simulations that: 1) there is no target leverage ratio; 2) firms can be savers or heavily levered; 3) leverage is path dependent and exhibits hysteresis; 4) leverage is decreasing in lagged cash flow and profitability; and 5) leverage varies negatively with an external finance weighted average Q ratio. We also show that taxation does not have a “second order” effect on leverage decisions. Indeed, even in the presence of a premium on external equity, we find that variations in tax parameters have more power to explain anomalies than this premium. The key differences between our theory and its predecessors are the simple ideas that firms make leverage decisions jointly with current investment decisions and that this joint decision depends strongly on the current and anticipated financing margins. We also take the theory to the data with a simulated moments estimator, finding that the cost of external equity finance is a modest 5.9%, consistent with existing evidence on underwriting fees.

Overall our theoretical and empirical results underline the importance of understanding corporate financial decisions in dynamic settings, as well as the importance of having a tight connection between theory and empirical work. Given the power of our theoretical and empirical framework to explain observed leverage phenomena, it appears likely that similar success is possible in other areas of corporate finance.

Appendix

Proof of Proposition 1

The constraint correspondence for this problem is constant-valued and hence, continuous. The set $K \times P$ is nonempty and compact. Since e is a continuous function with compact domain, it is also bounded. Therefore, Assumptions 9.4-9.7 of Stokey and Lucas (SL) (1989) are satisfied. Result follows from SL Theorem 9.6. ■

Proof of Propositions 2 and 3

To prove monotonicity, simply redefine the problem by dropping the notation p and substituting the variable $l \equiv -1 * p$. The equity value function in this case can be denoted v , and satisfies $v(k, l, z) = V(k, p, z)$. The collateral constraint demands $l \in L \equiv [-p(\bar{k}), -\underline{p}]$. Since the constraint set $K \times L$ is strictly increasing and the function e is strictly increasing in (k, l) , SL Assumptions 9.4-9.9 are satisfied. SL Theorem 9.7 implies that $v(\cdot, \cdot, z)$ is strictly increasing, from which it follows that V is strictly increasing (decreasing) in its first (second) argument.

The proof of concavity and uniqueness of the optimal policy follows SL Theorem 9.8. Assumptions 9.4-9.7 have been checked. Since $K \times P$ is convex, Assumption 9.11 is also met. Thus, we need only check that Assumption 9.10, concavity of $e(\cdot, \cdot, \cdot, z)$, is satisfied. To do so, we make use of the following result from Ziemba (1974) given as lemma A2 in his note.

LEMMA: *Let \mathbf{y} be a vector of m real variables and $x_i \equiv h_i(\mathbf{y})$ for $i = 1, \dots, n$ be real valued functions, with:*

$$f(\mathbf{y}) \equiv u[h_1(\mathbf{y}), \dots, h_n(\mathbf{y})].$$

If each h_i is weakly concave; some h_i , say h_j , is strictly concave; u is weakly concave; u is nondecreasing; and u is strictly increasing in h_j , then $f(\mathbf{y})$ is strictly concave.

To use Ziemba's lemma, note that we may write:

$$\begin{aligned} e(k, p, k', p', z) &\equiv U[G_1(k, p, z), G_2(k, k', p')] \\ U(G_1, G_2) &\equiv [1 + \Phi_i \lambda - \Phi_d \tau_d][G_1 + G_2] \\ G_1(k, p, z) &\equiv \frac{\pi(k, z) - g(y(k, p, z)) - p}{\Phi_n + s\Phi_s} \\ G_2(k, k', p') &\equiv k(1 - \delta) - k' + \frac{p'}{1 + r}. \end{aligned}$$

We know that: U is strictly increasing in both arguments; U is weakly concave; and G_2 is weakly concave. Thus, to prove concavity of e , we must prove G_1 is strictly concave. To do so, note that

we may write G_1 as follows:

$$\begin{aligned} G_1(k, p, z) &\equiv u[f(k, p, z)] \\ u(f) &\equiv \frac{f}{\Phi_n + s\Phi_s} \\ f(k, p, z) &\equiv \pi(k, z) - g(y(k, p, z)) - p. \end{aligned}$$

We know that u is strictly increasing in f and weakly concave. Thus, strict concavity of f implies strict concavity of G_1 . The Hessian matrix for $f(\cdot, \cdot, z)$ is denoted \mathcal{H} :

$$\mathcal{H} \equiv \begin{bmatrix} f_{kk} & f_{kp} \\ f_{kp} & f_{pp} \end{bmatrix}$$

The necessary conditions for strict concavity of f are satisfied with:

$$\begin{aligned} |\mathcal{H}_1| &= \pi_{11}(1 - \tau_c) - g''(y)(\pi_1 - \delta)^2 < 0 \\ |\mathcal{H}_2| &= -\left(\frac{r}{1+r}\right)^2 g''(y)\pi_{11}(1 - \tau_c) > 0. \end{aligned}$$

With the signs of the determinants pinned down by $\pi_{11} < 0$ and $g'' > 0$ from Assumptions 1 and 6, respectively. ■

Proof of Proposition 4

Follows from the Envelope Theorem of Benveniste and Scheinkman (1979), applied as in the proof Theorem 9.10 in SL (1989). ■

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Footnotes

1. See also an ambitious paper by Schurhoff (2004), who formulates a trade-off theoretic model with realization-based capital gains taxes.
2. The respective authors do not claim that such costs are *necessary*. Rather, their contention is that adding adjustment costs to the trade-off theory is *sufficient* to explain the findings in Baker and Wurgler (2002).
3. In fairness, Auerbach intends to present a simple model contrasting alternative theories.
4. This expression adopts Stiglitz' (1973) assumption that the dollar of equity injected into the firm is treated as a "return of capital" exempt from the distribution tax.
5. See Miller (1977), footnote 18.
6. The "no atoms" condition is not necessary for Propositions 1-4, since the results also hold when Z is a countable set.
7. For more on correlated defaults, see Duffie and Singleton (1999).
8. See, for example, Stiglitz and Weiss (1981), Bernanke and Gertler (1989), Whited (1992), Kiyotaki and Moore (1997), and Clementi and Hopenhayn (2002).
9. See Leland and Toft (1996) and Leland (1998) for models of optimal debt maturity.
10. Corporate shareholders may prefer dividends to repurchases due to dividend exclusion rules.
11. However, Feldstein and Summers (1979) show that the failure to index basis for inflation can create effective capital gains tax rates over 100%.
12. If contrary to Assumption 6, $\bar{\tau}_c \leq \tau_i$, then the unconstrained corporation never makes a distribution.
13. For a good example, see the Mitchell (2000) case study of UST.
14. The optimality condition for the firm with a binding debt constraint contains an extra benefit term attributable to the increase in \bar{p} .
15. It is worth noting that the results reported below change little when we normalize debt by the book value of assets.

16. Although seemingly arbitrary, this window length matters little for any results.
17. Michaelides and Ng (2000) find that good finite sample performance of an indirect inference estimator requires a simulated sample that is approximately ten times as large as the actual data sample.
18. As required by the Holtz-Eakin, Newey, and Rosen (1988) technique, we account for fixed effects via differencing our autoregression. For our other regressions we simply remove firm-level means from the data. We opt for this method simply because it is the method most used in the empirical literature we are trying to understand.
19. Specifically, the 2001 corporate income tax code stipulates that a firm with an income of between \$75,000 and \$100,000 had a marginal tax rate of 0.34. Although this rate is not quite at the highest statutory level, this tax schedule is almost flat after this tax bracket, and quite steep before this tax bracket. We therefore choose \$75,000 as the cutoff for the purposes of comparison with our simulated schedule.

Table I: Simple Model Simulation

Average Debt-Assets Ratio (Net of Cash)	0.095
Fraction of Observations with Positive Cash	0.285
Average Investment/Assets	0.160
Standard Deviation of Investment/Assets	0.169
Average Equity Issuance/Assets	0.055
Frequency of Equity Issuance	0.139
a_1	-0.069
a_2	-0.865
b_1	-0.298
b_2	-1.378
c_1	-0.293
c_2	-0.908
c_3	-1.359

All calculations are from a simulation of the dynamic partial-equilibrium model in Section II, which characterizes the firm's optimal choice of investment and capital structure in the face of corporate and personal taxes and costs of financial distress. The model is solved by value-function iteration and is simulated for 10,000 time periods, where the first 100 are dropped. $(a_1, a_2, b_1, b_2, c_1, c_2, c_3)$ are estimated slope coefficients in the following regressions:

$$\begin{aligned} \frac{\text{Debt}}{\text{Market Assets}} &= a_0 + a_1(\text{Tobin's Q}) + a_2 \left(\frac{\text{Cash Flow}}{\text{Book Assets}} \right) + u_a \\ \frac{\text{Debt}}{\text{Market Assets}} &= b_0 + b_1(\text{Tobin's Q}) + b_2 \left(\frac{\text{EBITDA}}{\text{Book Assets}} \right) + u_b \\ \frac{\text{Debt}}{\text{Market Assets}} &= c_0 + c_1(\text{Tobin's Q}) + c_2(\text{Weighted Q}) + c_3 \left(\frac{\text{EBITDA}}{\text{Book Assets}} \right) + u_c \end{aligned}$$

Table II: Simulated Moments Estimation: Moment Estimates

	Actual Moments	Simulated Moments
Average Investment/Assets	0.079	0.107
Variance of Investment/Assets	0.006	0.014
Average EBITDA/Assets	0.146	0.165
Average Debt-Assets Ratio (Net of Cash)	0.075	0.127
Average Equity Issuance/Assets	0.042	0.052
Frequency of Equity Issuance	0.099	0.091
Investment- q sensitivity	0.019	0.036
Debt- q sensitivity	-0.080	-0.219
Serial Correlation of Income/Assets	0.583	0.598
Standard Deviation of the shock to Incomes/Assets	0.117	0.120

Calculations are based on a sample of nonfinancial firms from the annual 2002 COMPUSTAT industrial files. The sample period is 1993 to 2001. Estimation is done with the simulated moments estimator in Gourieroux, Monfort, and Renault (1993), which chooses structural model parameters by matching the moments from a simulated panel of firms to the corresponding moments from these data. The simulated panel of firms is generated from the dynamic partial-equilibrium model in Section II, which characterizes the firm's optimal choice of investment and capital structure in the face of corporate and personal taxes and costs of financial distress. The model is solved by value-function iteration. The simulated panel contains 10,000 firms over 50 time periods, where only the last nine time periods are kept for each firm. This table reports the simulated and estimated moments.

Table III: Simulated Moments Estimation: Structural Parameter Estimates

α	δ	λ	s	σ_ε	ρ	μ_τ	σ_τ	χ^2
0.551	0.100	0.059	0.592	0.123	0.740	-12.267	9.246	4.906
(0.276)	(0.059)	(0.028)	(0.352)	(0.106)	(0.262)	(7.851)	(4.594)	(0.086)

Calculations are based on a sample of nonfinancial firms from the annual 2002 COMPUSTAT industrial files. The sample period is 1993 to 2001. Estimation is done with the simulated moments estimator in Gourieroux, Monfort, and Renault (1993), which chooses structural model parameters by matching the moments from a simulated panel of firms to the corresponding moments from these data. The simulated panel of firms is generated from the dynamic partial-equilibrium model in Section II, which characterizes the firm's optimal choice of investment and capital structure in the face of corporate and personal taxes and costs of financial distress. The model is solved by value-function iteration. The simulated panel contains 10,000 firms over 50 time periods, where only the last nine time periods are kept for each firm. This table reports the estimated structural parameters, with standard errors in parentheses. α is the curvature parameter in operating profits, zK^α ; δ is the rate of capital stock depreciation; λ is the proportional cost of external equity; s is the resale price per dollar of used capital goods; σ_ε is the standard deviation of the innovation to $\ln(z)$; ρ is the serial correlation of $\ln(z)$; and μ_τ and σ_τ are parameters that define the shape of the marginal corporate tax schedule. χ^2 is a chi-squared statistic for the test of the overidentifying restrictions. In parentheses is its p-value.

Table IV: Simulation Summary Statistics

	Saving			Debt		
	Distributions	Equity	Neither	Distributions	Equity	Neither
$\lambda = 0.059$						
Fraction	0.199	0	0.086	0	0.091	0.624
Investment/Assets	-0.032	—	-0.017	—	0.292	0.142
Debt/Assets	-0.086	—	-0.064	—	0.286	0.198
Tobin's Q	1.945	—	1.806	—	2.846	2.225
Assets	92.267	—	91.345	—	71.325	88.377
Cash Flow/Assets	0.334	—	0.298	—	0.141	0.096
Equity/Assets	0	—	0	—	0.052	0
$\lambda = 0$						
Fraction	0.137	0	0.075	0	0.147	0.641
Investment/Assets	-0.022	—	-0.012	—	0.301	0.143
Debt/Assets	-0.073	—	-0.056	—	0.254	0.143
Tobin's Q	1.721	—	1.643	—	3.095	2.505
Assets	89.456	—	85.385	—	72.753	76.768
Cash Flow/Assets	0.359	—	0.306	—	0.157	0.114
Equity/Assets	0	—	0	—	0.059	0

All calculations are from a simulation of the dynamic partial-equilibrium model in Section II, which characterizes the firm's optimal choice of investment and capital structure in the face of corporate and personal taxes and costs of financial distress. The model is solved by value-function iteration and is simulated for 10,000 time periods, where the first 100 are dropped. The parameterization for the first panel is given in Table III. The parameterization for the second panel is identical, except that λ is set to 0.

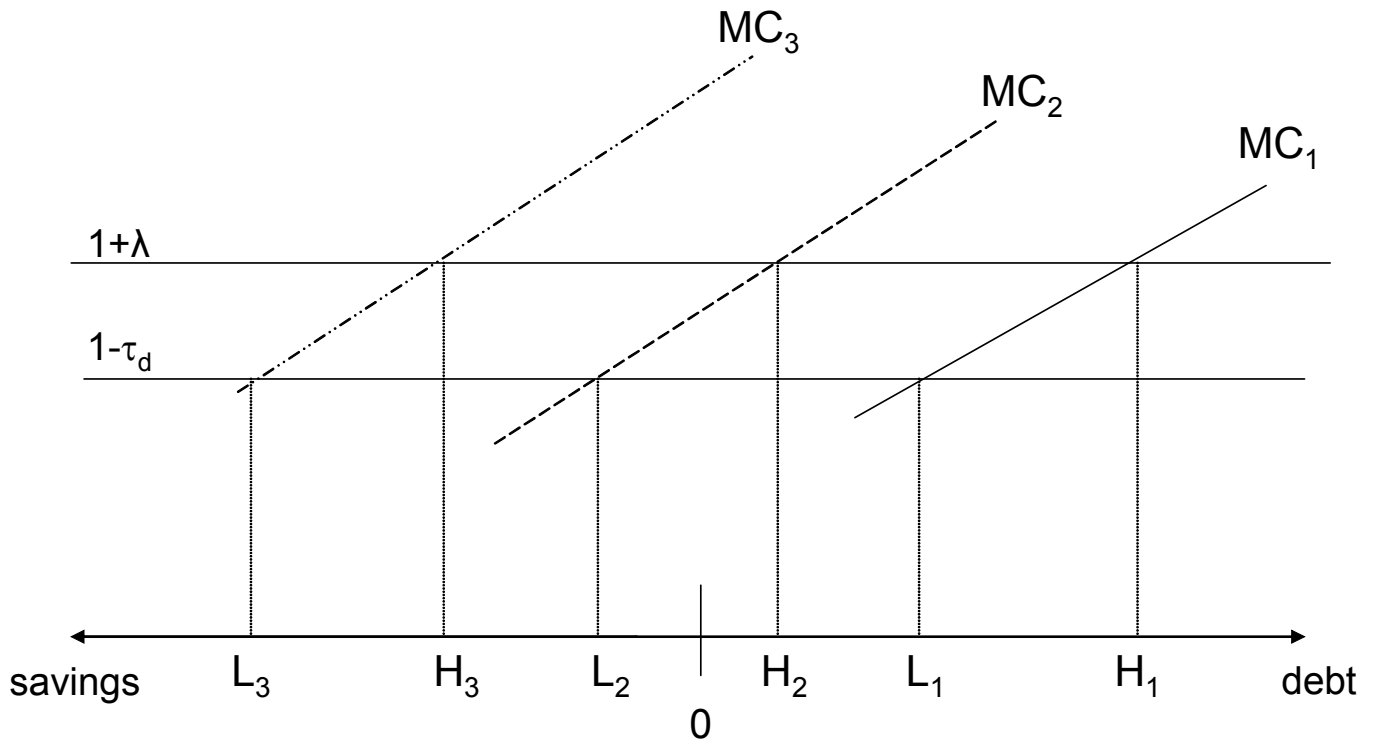


Figure 1: Optimal Financial Policy

This figure illustrates optimal financing in our model. The three MC lines represent the discounted marginal cost of debt service, taking expectations over the marginal corporate tax rate, distress costs, and the anticipated equity margin next period. At one extreme, the lowest schedule (MC_1) represents marginal debt service costs for a firm expecting a high corporate tax rate and to be making a distribution to shareholders next period. At the opposite extreme, the high schedule (MC_3) represents the marginal cost of debt service for a firm expecting a low corporate tax rate next period and anticipating a high probability of tapping external equity in that period. The MC_2 schedule represents an intermediate case. The horizontal lines represent the marginal benefit of debt issuance, which depends on the use of funds. When the increase in debt (reduction in savings) is used as a substitute for external equity, the line labeled $1 + \lambda$ measures the marginal benefit. When the increase in debt (reduction in savings) is used to finance an increase in distributions, the line labeled $1 - \tau_d$ measures the marginal benefit. The optimal financing policy equates the marginal costs and benefits of debt issuance.