Do Tests of Capital Structure Theory Mean What They Say? *

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Abstract

In the presence of frictions firms adjust their capital structure only infrequently. As a consequence, in a dynamic economy the leverage of most firms, most of the time, is likely to differ from the “optimum” leverage at the time of readjustment. This paper explores the empirical implications of this observation. A calibrated dynamic trade-off model with adjustment costs is used to simulate firms’ capital structure paths. The results of standard cross-sectional tests on this data are found to be qualitatively and, in some cases, even quantitatively consistent with those reported in the empirical literature. In particular, the standard interpretation of some test results would lead to the rejection of the model used to generate the data. The framework can explain a number of observed puzzles related to leverage. In particular, in the simulated cross-sectional samples leverage: (a) is inversely related to profitability; (b) can be largely explained by stock returns; (c) is mean-reverting. The results suggest that, in the presence of infrequent adjustment, cross-sectional properties of economic variables in dynamics may be fundamentally different from those derived assuming that they are always at their target levels. Taken together, the results suggest a rethinking of the way capital structure tests are conducted.

Keywords: Capital structure, dynamic economy, trade-off model, simulations, asset liquidity, refinancing point, profitability, stock returns, credit spreads

JEL Classification Numbers: G12, G32
I Introduction

Recent empirical research in capital structure has focused on regularities in the cross section of leverage to discriminate between various theories of financing policy. In these tests, book and market leverage are related to profitability, book-to-market and firm size. Changes in market leverage are largely explained by changes in equity value. Past book-to-market ratios predict current capital structure. Firms seem to use debt financing too conservatively, and the leverage of stable, profitable firms appears particularly low. Even if firms have a target level of leverage, they move towards it slowly. Firms with low and high leverage react differently to external economic shocks.\footnote{See Graham (2000) on conservatism in financing decisions; Titman and Wessels (1988), Rajan and Zingales (1995), Fama and French (2002), among others, on cross-sectional determinants; Fama and French (2002), Hovakimian, Opler and Titman (2001) and Graham and Harvey (2001) on slow mean reversion of debt ratios; Baker and Wurgler (2002) on the influence of past book-to-market ratios; Welch (2004) on the influence of changes in the market value of equity on debt ratios, and Opler and Titman (1994) on reaction of highly leveraged companies to industry shocks and Korajczyk and Levy (2003) on their reaction to macro shocks.} These findings are typically evaluated in terms of the comparative statics of various capital structure models. Each of these models is supported by some evidence and challenged by other evidence. This paper attempts to understand whether our interpretation of cross-sectional tests would change if firms optimally adjust their leverage only infrequently.

The starting point is a simple but a fundamental observation: in a dynamic economy with frictions the leverage of most firms, most of the time, is likely to deviate from “optimal leverage” prescribed by models of optimal financial policy since firms adjust leverage by issuing or retiring securities infrequently, at “refinancing points”. Consequently, even if firms follow a certain model of financing, a static model may fail to explain differences between firms in a cross-section since actual and “optimal” leverage differ. It has been long recognized that deviations from optimal leverage may create problems in interpreting the results of empirical research. For example, Myers (1984, p. 578) emphasizes that “any cross-sectional test of financing behavior should specify whether firms’ debt ratios differ because they have different optimal ratios or because their actual ratios diverge from optimal ones”.

In this paper I start by constructing a model of time-consistent optimal dynamic financing in the presence of frictions and then use the model to generate dynamic paths of leverage. This cross-sectional data resembles, along a number of dimensions, data used in empirical studies. This allows me to replicate tests commonly used in such studies and ask to what extent the results are similar. My findings can be summarized as follows: (i) cross-sectional tests performed on data generated by dynamic models can produce results that are profoundly different from their predictions for corporate financing behavior at refinancing points; moreover, some results may lead to the rejection of precisely the model on which these tests are based; and (ii) even a stylized trade-off model of dynamic capital structure with adjustment costs can produce results numerically consistent with some of those observed empirically. This suggests
that we may need to rethink empirical tests in this area and also highlights the importance of
developing dynamic models of financing capable of delivering quantitative predictions.

A prerequisite for my analysis is a model that captures the dynamics of firms’ financing
behavior. Among many existing explanations of capital structure only the trade-off argument
has a fully-worked out dynamic theory that produces quantitative predictions about leverage
ratios in dynamics. In the trade-off theory firms arrive at their optimal capital structure by
balancing the corporate tax advantage to debt against bankruptcy and agency costs. Using
a trade-off model might seem regrettable because the empirical evidence for this model is, at
best, mixed. However, as I show in the paper, the data are more consistent with the trade-off
theory than is traditionally thought, and so, ex-post, using a trade-off model might seem
more justified. I take a standard state-contingent model of dynamic capital structure rooted
in a trade-off argument. While several features differentiate the model from others in the
field, the basic setup is widely used in the literature. In the model, firms are always on their
optimal capital structure path but, due to adjustment costs, they refinance only occasionally.
Small adjustment costs can lead to long waiting times and large changes in leverage, a result
consistent with the findings of Fischer, Heinkel and Zechner (1989). Firms that perform
consistently well re-leverage to exploit the tax shield of debt. Firms that perform badly face a
liquidity crisis and sell their assets to pay down debt. If their financial condition deteriorates
still further, they resort to costly equity issuance to finance their debt payments and, when
all other possibilities are exhausted, they default and ownership is transferred to debtholders.

I use the model in two ways. First, I determine the path of a firm’s optimal financing
decisions. This enables me to study the cross-section of optimal leverage at times when firms
change their leverage: I call these “refinancing points”. Naturally, when firms are at their
refinancing points, all the comparative statics predictions are in line with the predictions of
the standard trade-off theory.

In the second stage of the analysis, I perform a number of cross-sectional tests on simulated
dynamic data generated by the model. Several results stand out. First, the analysis high-
lights difficulties in interpreting the leverage-profitability relationship. In the pecking order
theory, more profitable firms reduce their dependence on costly external financing and thus
decrease their leverage. In the trade-off theory, higher profitability decreases the expected
costs of distress and allows firms to increase their tax benefits by increasing leverage. Thus,
an inverse relation between leverage and profitability, frequently found in the data and which
Myers (1993) identifies as perhaps the most pervasive empirical capital structure regularity,
represents a significant failure of the trade-off model and is considered by some writers to be
decisive in its rejection. In my model, expected profitability is positively related to leverage
at the refinancing points. However, I show that in a dynamic economy cross-sectional tests
reveal a negative relation. The intuition is simple: with infrequent adjustment, an increase in
profitability lowers leverage by increasing future profitability and thus the value of the firm. In the same vein, a decrease in profitability increases leverage. For those firms that do not refinance this results in a negative relation between leverage and profitability. Of course, in any period some firms refinance. In the simulations the subset of firms that do not refinance dominates and the cross-sectional relation between profitability and leverage is always negative. This effect is strengthened by the presence of systematic shocks in the firms’ cash flow. In a number of cases the magnitude of the coefficient is also consistent with empirical estimates.

Second, again using the model to simulate dynamic data, I replicate almost exactly the test recently conducted by Welch (2004). His main finding is that debt ratios are largely explained by past stock returns, implying that corporations do not readjust their debt levels to counteract the mechanistic effect of stock returns on leverage. This observation is important, not least because other determinants used in the literature are found to effect leverage largely through stock returns. The results of the same regression tests conducted on the simulated data are numerically very similar to those obtained by Welch, suggesting that a stylized dynamic model with small adjustment costs may be consistent with these findings.

In addition, the framework can provide an explanation for the “debt issuance mystery” (Welch, 2004), i.e. the apparent inconsistency between the passive behavior of managers in response to mechanistic changes in equity value and overall active capital structure policies of corporations. Managers are passive, since, over a one year horizon, there is almost a one-to-one relation between leverage and the implied debt ratio, a variable whose change is entirely determined by one-year stock returns. Both of these results are also observed in the model since managers decide to change the firm’s leverage based on a changes in value over a long period, a variable that is largely orthogonal to recent equity returns. Thus, in the cross-section and consistent with empirical observation, changes in outstanding debt value are contemporaneously almost independent of the changes in market value of equity.

Third, since the behavior of the cross-section in dynamics is radically different from the comparative statics properties at the refinancing points, comparing empirical findings with the theoretical properties of leverage at refinancing points can be misleading. An example is provided by the debate on possible explanations for the so-called “low leverage puzzle”. This refers to the observation that the median corporate debt to capital ratio in the U.S. over 1965–2000 averaged only 31.4% with two out of five firms having an average debt to capital ratio of less than 20%, while traditional trade-off models produce substantially higher numbers. That trade-off models imply excessively high leverage is not surprising in the light of Merton Miller’s (1977) famous remark about “horse and rabbit stew”: bankruptcy costs are simply negligible compared to the tax benefits of debt. To explain the observed low levels

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2 Estimates are based on COMPUSTAT data on the book value of debt and market value of equity. The debt to capital ratio is defined as: COMPUSTAT data items d9+d34 divided by d9+d34+d25xd199. These are unadjusted figures. Adjusted (see Rajan and Zingales (1995)) figures would be lower.
of leverage we need to understand better the factors that might offset the tax benefits. One proposed solution is to consider a dynamic framework. Studies by Goldstein, Ju and Leland (2001) and Ju et al. (2003) show that if firms are allowed to increase debt in the future, they will opt for lower leverage initially. My results suggest that average leverage over time, i.e. in “true dynamics”, tends to be larger than leverage measured at refinancing points.\(^3\) Empirical estimates of leverage should therefore be compared with model estimates in dynamics.

One possible criticism is that the findings are specific to a number of firm-specific parameters and the initial conditions of the economy. I show in extensive robustness tests that the main thrust of the results is robust to changes in the specification and parameter set.

My paper builds on several strands of previous research. First, it shares with a number of recent papers, including Leland (1998), Goldstein et al. (2001) and Ju et al. (2003), a theoretical framework in which the standard structural models of risky debt pricing are extended to incorporate dynamic financing behavior. These models follow, on the one hand, static capital structure models developed by, among others, Leland (1994) and, on the other hand, dynamic capital structure models developed by Fischer, Heinkel and Zechner (1989) whose research was, in turn, based on insights by Kane, Marcus and McDonald (1984; 1985). Fischer et al. (1989) were also the first to suggest that empirical studies of capital structure in cross-section might be more fruitful if the dependent variable were to reflect the behavior of leverage over time, e.g., its range, rather than its value at a point in time.

My model most closely resembles that of Goldstein et al. (2001). A distinct feature of the model is that firms whose value falls substantially face a prolonged period of turbulence instead of simply running up a large debt burden and then defaulting. The model thus reflects the empirical findings of Asquith, Gertner and Scharfstein (1994) according to which firms unable to service their debt obligations sell a fraction of their assets in order to pay down their debt. Asquith et al. find that more than 80% of financially distressed firms sold assets. For the sake of realism, I model asset sales as discrete. While asset sales may ease a firm’s financial position by reducing leverage and staving off the immediate threat of default, they also reduce the level of future earnings. This, combined with the financial costs that firms bear in selling assets, affects future financial flexibility and leads to a more conservative debt policy.\(^4\) A decrease in debt usage can be thought of, roughly, as a simple hedging tool. Firms whose condition continues to deteriorate have to resort to issuing equity to finance their debt

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\(^3\)For example, the benchmark case reported by Goldstein, Ju and Leland (2001) has a leverage ratio of 37%. In dynamics, however, this figure is about 45%. Ju et al. (2003) develop a model with finite maturity debt where at fixed refinancing points shareholders take optimal decisions. The three main reasons why their trade-off model produces low leverage at refinancing points are: (a) the level of asset volatility they use is chosen to match recovery rates and corporate bond spreads; (b) new debt is issued only after existing debt matures; this creates a “real option” to increase leverage in the future and reduces current leverage; (c) the default boundary is specified exogenously and leads to higher default frequency than in a model with an endogenous boundary.

\(^4\)Graham and Harvey (2001) find that firms consider financial flexibility as the most important determinant of their debt policy.
payments. Consistent with empirical research (e.g. Altinkilic and Hansen (2000)), equity issuance is costly in the model. These costs lead shareholders to default sooner and thus decrease the level of leverage at which the firm’s equityholders will find it in their interest to default and transfer their ownership rights to debtholders. Morellec (2001) also considers the effect of asset liquidity in a model of static optimal capital structure. Asset sales here differ from his case since they are conducted exclusively in financial distress at prices that reflect a discount proportional to the firm’s value at the time of sale and are conducted in discrete amounts. Acharya et al. (2002) introduce costly equity issuance in a structural model of credit spreads, but do not consider optimal leverage decisions.

The simulation approach followed in the paper resembles Berk, Green and Naik (1999) who, in a deep analysis, focus on the cross-sectional relation between a firm’s investment policy, systematic risk and expected returns. They build a non-linear dynamic model in which firms, though initially identical, become heterogeneous as a result of the endogenous evolution of the value of their assets. To investigate cross-sectional patterns and regularities in their economy they perform simulations, an approach I endeavor to replicate since my model also has strong non-linearities. Firms’ technology parameters in my model are calibrated to resemble, in a sense discussed later, the properties of samples of firms typically used in empirical studies. Then, I simulate data on firm values, leverage, etc. for dynamic economies and perform a number of cross-sectional tests similar to those performed in the empirical literature. The evolution of firms’ asset values and, thus, financing decisions are cross-sectionally dependent due to the presence of systematic shocks.


The paper proceeds as follows. Section II presents and solves the model. Section III presents the simulation procedure and replicates a number of empirical tests on data generated from the model. Section IV describes the robustness tests. Section V concludes. The appendix contains details of the simulation method.

II The model

II.1 General assumptions

My model employs a standard contingent claims framework to analyze an individual firm and is closely based on Goldstein, Ju and Leland (2001). I consider an economy populated by $N$
firms, each of which is endowed with monopoly access to some infinitely lived project operated in continuous time. The value of each firm stems from a perpetual entitlement to the current and future income from the project (“EBIT-generating machine”). The income is divided between the net payout to claimholders and retained earnings. In common with many other models of capital structure, the Modigliani and Miller assumption that the project’s cash flows are invariant to financial policy is retained.\(^5\) Investment is financed by retained earnings; the latter are net of depreciation and result in book assets growing at a rate \(g\). The state variable in the model is the total time \(t\) net payout to claimholders, \(\delta_t\) where “claimholders” includes both insiders (equity and debt) and outsiders (government and various costs).\(^6\) The evolution of \(\delta_t\) is governed by the following process under pricing measure \(Q\): \(^7\)

\[
\frac{d\delta_t}{\delta_t} = \mu dt + \sigma dZ_t \quad \forall t \geq 0, \delta_0 > 0,
\]

where \(\mu\) and \(\sigma\) are constant parameters and \(Z_t\) is a Brownian motion defined on a filtered probability space \((\Omega, \mathcal{F}, Q, (\mathcal{F}_t)_{t \geq 0})\). Here, \(\mu\) is the risk-neutral drift and \(\sigma\) is instantaneous volatility of project’s net cash flow.

I assume that management always acts in the best interest of shareholders and, throughout the paper, I use managers and equityholders interchangeably. To avoid further complication, the default-free term structure is assumed flat with an instantaneous after-tax riskless rate \(r\) at which investors may lend and borrow freely. The marginal corporate tax rate is \(\tau_c\). The marginal personal tax rates, \(\tau_d\) on dividends and \(\tau_i\) on income, are assumed to be identical for all investors. Finally, all parameters in the model are assumed to be common knowledge.

II.2 Debt contract assumptions

All corporate debt is in the form of a perpetuity entitling debtholders to a stream of continuous coupon payments at the rate of \(c\) per annum and allowing equityholders to call the debt at any time at the face value. The main features of the debt contract are standard in the literature. The perpetuity feature is shared with numerous other models including Fischer, Heinkel and Zechner (1989), Leland (1994), and Goldstein, Ju and Leland (2001). The virtue of perpetual

\(^5\)Several papers have analyzed interactions between financing and investment policy, including joint decisions on production and capital structure (Brennan and Schwartz (1984), Mello and Parsons (1992), Mauer and Triantis (1994)) and the effects of asset substitution (Leland (1998)).

\(^6\)Another approach is to consider the dynamics of unlevered equity value, where claims of outsiders are added to the value of the firm. The “\(\delta\)-approach” has a number of methodological advantages, e.g., by eliminating the need to consider levered and unlevered assets as separately traded assets. The results of applying both approaches are, however, roughly similar. An early example of \(\delta\)-approach is by Mello and Parsons (1992).

\(^7\)Since I consider an infinite time horizon, some additional technical conditions on Girsanov measure transformation (e.g., uniform integrability) are assumed here. In addition, the existence of traded securities that span the existing set of claims is assumed. Thus, the pricing measure is unique.
debt is that it makes the problem more tractable by guaranteeing time-homogeneity of claims. Also, the debt contract is callable. As demonstrated by Goldstein et al. (2001), in a model (such as mine) with the inherent scaling feature, callability does not really matter in good times as long as it is assumed that both old and new debt have equal seniority: newly issued debt dilutes the old debt claim and in equilibrium the market price of old debt is identical to its original face value.

If the firm fails to honor a coupon payment in full, it enters restructuring. Restructuring, either a work-out or formal bankruptcy, is modelled in reduced form. The absolute priority rule is enforced and all residual rights on the project are transferred to debtholders. However, such restructuring is costly and, in the model, restructuring costs are assumed to be a fraction $\alpha$ of the value of assets on entering restructuring.

Debt contracts are assumed to be non-renegotiable, so that equity cannot default strategically, a feature modelled in several recent debt pricing models (Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997)). Additionally, debt contracts may restrict the rights of equityholders to sell the firm’s assets (Smith and Warner (1979)). This standard assumption prevents equityholders from attempting to appropriate the firm value. Essentially, all proceeds from asset sales accrue to equity only after other claims have been satisfied.

II.3 Scenarios

Figure 1 shows a number of possible paths for the firm value. Path 1 illustrates a successful firm that raises more debt to take advantage of tax deductability of interest, while paths 2 and 3 are for firms whose condition deteriorates and whose managers must take corrective action. Empirical research has shown that firms often become insolvent on a flow basis but not on a stock basis. For such firms the present value of future income exceeds their debt obligations but they experience a temporary liquidity crisis since fixed assets are a poor substitute for cash. In the model this occurs when the firm hits a “liquidity” boundary for the first time. The liquidity boundary that I model closely resembles the definition of a financially distressed firm.

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8 An alternative time-independent scenario is when the debt is continuously rolled over at a fixed interest rate. See Leland (1994) and Leland and Toft (1996) for further discussion. Introduction of finite maturity debt in the dynamic case would introduce flexibility on the part of equityholders who may be able to achieve lower adjustment costs by waiting until a bond matures rather than refinancing earlier. Ju et al. (2002) consider the case of fixed maturity, but in their model the firm must wait until the maturity date before issuing new debt. Thus, their framework does not allow equityholders to follow a dynamic debt policy by choosing the time of refinancing which is the crux of dynamic choice in my model.

9 It is assumed that debtholders are dispersed and perfectly competitive and therefore situation of “squeezes” where price of a bond speculatively rises in anticipation of a recall or purchase like the one described in Dunn and Spatt (1984) in the case of sinking fund provisions is ruled out. Also, constant interest rates are essential since otherwise decision to call will be governed by a factor not directly related to the state of the firm.

10 The model can be easily extended to take account of absolute priority violations found in empirical studies (see e.g. Franks and Torous (1989)). This is likely to lead to a lower leverage. Leland (1994) demonstrates that the impact of introducing APR violations on optimal leverage is minor at the time of readjustment.
in Asquith, Gertner and Scharfstein (1994). They classify a firm as financially distressed if either, in any two consecutive years the firm’s earnings before interest, taxes, depreciation and amortization (EBITDA) is less than its reported interest expense; or, in any one year, EBITDA is less than 80% of its interest expense. In the model, the boundary reflects an intermediate case: the firm becomes financially distressed whenever its cash flow is insufficient to cover its interest expense and thus the liquidity boundary is triggered for the first time at $T_L$ whenever 

$$
\delta_{T_L} < c \quad \text{and} \quad \delta_t \geq c \quad \text{for all} \quad t < T_L. 
$$

In most structural models of debt pricing and capital structure the mechanism that allows equityholders to avoid default is to subscribe to new equity. In practice, however, firms choose from a richer set of options. In the sample studied by Asquith et al. (1994) distressed firms restructure both bank and public debt liabilities, cut investment expenditure and sell assets.

I investigate the last of these alternatives. My motivation is driven by the observed frequency and magnitude of this activity: in the Asquith et al. (1994) sample, the majority of firms did sell assets, with 18 out of 102 companies selling over 20% of their assets. Referring to Figure 1, both firms 2 and 3 encounter financial distress at $\delta = \delta_L$ and sell a fraction of assets to decrease their debt burden. The model captures several features of asset sales that are observed in practice. First, asset sales occur in discrete amounts: when firms spin-off part of the enterprise, the transaction typically involves a significant fraction of their assets.\footnote{\label{fn:asset}Models of debt pricing also use “asset sales” or “asset liquidation” terminology, but it refers to the case of proportional asset liquidation that is equivalent in fact to the net payout ratio being positive since in these

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Possible paths of firm value.}
\end{figure}

\begin{itemize}
\item \textbf{Path 1:} Firm value increases over time, reaching a peak before declining.
\item \textbf{Path 2:} Firm value declines sharply, hitting a low point before recovering.
\item \textbf{Path 3:} Firm value remains relatively stable, with minor fluctuations.
\end{itemize}

\footnotetext[11]{Similar boundary is considered in Kim, Ramaswamy and Sundaresan (1993).}
Second, asset sales are costly: firms in financial distress realize less from asset sales than the present value of the cash flows from these assets since potential buyers are likely to be financially constrained, less well informed, lack necessary expertise; sellers are time constrained and detach their human capital from sold assets. In other words, a discount can be viewed in light of the traditional measure of liquidity (Shleifer and Vishny (1992)).

The firm sells a fraction $1 - k$ of its assets immediately upon entering financial distress and this results in a reduction of a fraction $w$ of the firm’s outstanding debt:

$$(1 - q_A)(1 - k) V_L (1 - \tau) = \frac{(1 - w)D_0}{1 - q_{RC}}.$$  

In (2) $D_0$ is the par value of debt at the time of issuance and $V_L$ is the present value of project’s future cash flows at time $T_L$. The parameter $q_A$ represents the proportional costs incurred in selling assets, and $\tau$ is the effective corporate tax rate on the asset sale.\textsuperscript{13} Thus, the left-hand side is the after-tax income received by the firm as a result of the asset sale. Equality in (2) implies that all the proceeds are used to pay down debt. The proportional adjustment costs $q_{RC}$ of issuing/retiring debt are incurred.

An asset sale reduces operating net payout to a fraction $k$ of its previous value. This is shown in Figure 1 where paths 2 and 3 have downward jumps at the time of liquidity crisis. Firm 2’s fortunes improve substantially after the liquidity crisis and it subsequently refinances. However, as path 3 demonstrates, an asset sale may provide the firm with only a temporary breathing space: if its asset value continues to decline, equityholders resort, as in earlier models, to equity issuance. A number of empirical studies have shown that issuing equity is costly (Altinkilic and Hansen (2000), Hansen (2001), Corwin (2003)). In the model the direct costs of external equity financing are proportional to the amount issued. Finally, as path 3 illustrates, a firm will default if its condition continues to worsen.

II.4 Valuation

In this section I derive the value of the firm and the conditions that determine equityholders’ decisions. The fundamental driving force of the model is the inherent conflict of interest between the different claimholders since ex-ante (prior to the issuance of debt) and ex-post (after debt has been issued) incentives of equityholders are not aligned. Debtholders foresee the future actions by equityholders and value debt accordingly.

At every date $t$ equityholders decide on their actions. As in Fischer, Heinkel and Zechner (1989), Leland (1998) and Goldstein, Ju and Leland (2001), firms whose net payout reaches models cash flows originate exclusively via asset liquidation.

\textsuperscript{13}(1 - k) V_L (1 - \tau) is the maximum price any buyer is willing to pay for these assets in the absence of frictions. I assume for simplicity that the buyer is unlevered. Note that since all firms face the same marginal tax rate, $\tau$ is also the effective tax rate of an unlevered carbon copy of the firm.
an upper threshold will choose to retire their outstanding debt at par and sell a new, larger issue to take advantage of the tax benefits to debt. Refinancing thus takes the form of a debt-for-equity swap. I call these thresholds “refinancing points”. In my framework the financial history of the firm matters: the threshold value depends on whether the firm has experienced distress in the past since future profitability and book debt are changed in that event.

The tractability of this and other similar models stems from a scaling feature or, in other words, a first order homogeneity property. A scaling feature means that the values of all thresholds and the par value of debt are scaled up by the same proportion at the first and each subsequent refinancing point. This feature is inherent in the log-normal nature of the state variable process and also only holds when costs are proportional to the value of the firm or its claims. In other words, at any refinancing point the firm is just a larger replica of itself. Therefore, I start by considering the values of equity and debt over one refinancing cycle (i.e. before the upper barrier is hit). These values, once debt is issued and before the liquidity barrier is hit, can be written as the sum of the present values of cash flows accruing to claimholders in four regimes: (i) while the firm is financially healthy, (ii) at the time the liquidity barrier is hit for the first time, (iii) on continuation after the barrier is hit and (iv) in default. Thus, the value of equity and debt in one refinancing cycle at time $t = 0$ are

$$
E^R(\delta_0) = \mathbb{E}_{\delta_0} \left[ \int_0^{T'} e^{-rs}(1 - \tau)(\delta_s - c)ds \right] + \mathbb{E}_{\delta_0} \left[ \int_{T_L}^{T''} e^{-rs}q((1 - \tau)(k\delta_s - wc) - \tau_wc1_{[\delta_s<\delta_c]})ds \right] 
$$

$$
+ \mathbb{E}_{\delta_0} \left[ e^{-rT_B} \max \left( 1 - \alpha, \int_{T_B}^{+\infty} e^{-rs}k(1 - \tau)\delta_s ds - wD_0, 0 \right) \right] \phi_{LU}^B = 0,
$$

and

$$
D^R(\delta_0) = \mathbb{E}_{\delta_0} \left[ \int_0^{T'} e^{-rs}(1 - \tau_i)c ds \right] + \mathbb{E}_{\delta_0} \left[ e^{-rT_L} |\phi_L^I = 0 \right] (1 - w)D_0 + \mathbb{E}_{\delta_0} \left[ \int_{T_L}^{T''} e^{-rs}(1 - \tau_i)wc ds \right] 
$$

$$
+ \mathbb{E}_{\delta_0} \left[ e^{-rT_B} \min \left( 1 - \alpha, \int_{T_B}^{+\infty} e^{-rs}k(1 - \tau)\delta_s ds, wD_0 \right) \right] |\phi_{LU}^B = 0
$$

where expectations, here and throughout the paper, are taken under the pricing measure $Q$, $R$ stands for one refinancing cycle, $T' = \min(T_L, T_U)$ and $T'' = \min(T_B, T_{LU})$. The functions $\phi_i^j$ take the value 0 if event $j$ occurs before event $i$, and 1 otherwise.

The first term in expression (3) is the present value of cash flows to equityholders when neither the liquidity barrier, $\delta_L$, nor the first refinancing barrier, $\delta_U$, have been reached. As residual income claimants, equityholders retain whatever is left of net income after coupons and taxes are paid. The tax rate $\tau$ takes into account both corporate and personal taxes: $\tau = 1 - (1 - \tau_c)(1 - \tau_d)$. The second term is the present value of cash flows on continuation after the liquidity barrier has been hit and until either default occurs at time $T_B$ or the second refinancing barrier, $\delta_{LU}$, is reached. The function $q(x)$ accounts for costly equity issuance and
can be written as
\[
q(x) = \begin{cases} 
1, & \text{if } k\delta_x > \omega_c \\
q_E x, & \text{otherwise}
\end{cases}
\] (5)

In addition, if corporate income, \( \delta_t \), is sufficiently small, the firm loses part of its tax shelter and this results in a lower effective tax benefit \( \tau - \tau_l \). The first and the third terms in expression (4) are the net present values of payouts to debtholders before and after a liquidity crisis, respectively. The second term reflects the amount of debt purchased when assets are sold. In default proportional costs \( \rho \) are incurred and, since absolute priority is enforced, equityholders receive either nothing or the residual after the remaining debt is repaid at its face value (the third term in (3) and the fourth in (4)).

The total value of a debt claim issued at date 0 is thus
\[
D(\delta_0) = D^R(\delta_0) + \mathbb{E}_{\delta_0} \left[ e^{-rT_U} \phi_L(0) = 0 \right] + \mathbb{E}_{\delta_0} \left[ e^{-rT_{LU}} wD_0(\phi_B^U) = 0 \right]
\] (6)

Equityholders make decisions taking into consideration what happens after refinancing occurs. The total value of all payouts to equity (except at refinancing points) is given by
\[
E^D(\delta_0) = E^R(\delta_0) + \mathbb{E}_{\delta_0} \left[ e^{-rT_U} \gamma_U E^D(\delta_0)|\phi_L^U = 0 \right] + \mathbb{E}_{\delta_0} \left[ e^{-rT_{LU}} \gamma_{LU} kE^D(\delta_0)|\phi_B^U = 0 \right]
\] (7)

and the value of all debt issues is
\[
D^D(\delta_0) = D(\delta_0) + \mathbb{E}_{\delta_0} \left[ e^{-rT_U} \gamma_U D^D(\delta_0)|\phi_L^U = 0 \right] + \mathbb{E}_{\delta_0} \left[ e^{-rT_{LU}} \gamma_{LU} kD^D(\delta_0)|\phi_B^U = 0 \right]
\] (8)

where \( \gamma_U \) and \( \gamma_{LU} \) are the proportions by which the net payout increases between two refinancing points if the liquidity barrier has, or has not been hit, respectively.

Combining these values yields the total value of the firm that equityholders maximize at time \( t = 0 \), and after scaling, at each subsequent refinancing point:
\[
F(\delta_0) = \frac{E^R(\delta_0) + (1 - q_{RC}) D(\delta_0)}{1 - \gamma_U \mathbb{E}_{\delta_0} \left[ e^{-rT_U} \phi_L(U) = 0 \right] - k\gamma_{LU} \mathbb{E}_{\delta_0} \left[ e^{-rT_{LU}} \phi_B(LU) = 0 \right]}
\] (9)

Thus, (9) states that managers maximize the sum of (i) the present value of the after-tax cash flows accruing to equity and (ii) the present value of after-tax income payments to all debt claims to be yet issued. Note that the total equity value takes into account the present value of future adjustment costs that will be incurred at future refinancing points.

Equityholders choose the coupon and barriers to maximize the ex-ante value of their claim:
\[
\mathbf{c}^* = \arg \max_{\{c, \gamma_U, \gamma_{LU}\} \in \mathbb{R}_+^3} \left[ F(\delta_0) \right]
\] (10)
An additional feature of realism in which I follow Goldstein et al. (2001) is that the firm’s financial decisions affect its net payout ratio. Empirically, higher reliance on debt leads to a larger net payout. Here, for simplicity, I assume that the net payout ratio depends linearly on the after-tax coupon rate:

$$\frac{\delta}{V_t} = a + (1 - \tau_c) \frac{c}{V_0},$$

where $V_t$ is the present value of all future net payouts at time $t$.

To characterize the default threshold, note that equityholders will balance the present value of future equity cash flows if they remain in control, with the cost of equity issuance if they subscribe now. The relevant value of equity is $E(\delta_t) = F(\delta_t) - D(\delta_t)$, where the fact that the liquidity barrier has been hit is taken into account in calculating the value of claims. It is well known that this threshold satisfies the smooth-pasting condition:

$$\frac{\partial E(\delta_t)}{\partial \delta_t} \bigg|_{\delta_t = \delta_B} = 0.\tag{12}$$

The full problem facing equityholders thus consists of solving (10) subject to (11) and (12). A closed-form solution to this problem does not exist, and so numerical procedures are used.

II.5 Comparative statics

The purpose of this subsection is to compare the properties of firms’ financial decisions at refinancing points in my model to the earlier literature. Table I summarizes the comparative statics of the main financial variables. The market leverage ratio, $ML$, is defined as the ratio of market debt value ($D(\delta_0)$) to total capital ($F(\delta_0)$),

$$ML_0 = \frac{D(\delta_0)}{F(\delta_0)}.$$

Not surprisingly, many results are similar to the comparative statics results obtained by Leland (1994) for the static case (his Table II for unprotected debt) and by Goldstein, Ju and Leland (2001) for the dynamic case (their Table 2). In particular, as expected, higher business risk, bankruptcy costs and a lower tax advantage to debt all reduce optimal leverage. A higher risk-free interest rate, contrary to the result given in Leland (1994), unambiguously reduces leverage since the higher costs of borrowing more than offset the larger tax advantage to debt. Finally, an increase in the costs of asset sales and equity issuance also lower borrowing.

The relation between the leverage ratio and the adjustment costs exhibits a reverse U-pattern. Firms with either high or low cost access to external markets optimally prefer lower leverage than those with intermediate costs. This is because firms face a trade-off between the frequency of refinancing and the amount of borrowing. Firms with low costs prefer to rebalance frequently; as costs increase, the level of the refinancing boundaries rises (note that
\( \delta_U \) and \( \delta_{UL} \) are increasing functions of \( q_{RC} \) and firms therefore borrow more initially. As costs rise further, however, debt becomes less advantageous and is replaced by equity.

Rows 2 and 3 of Table I illustrate the behavior of the default and upper refinancing boundaries. The behavior of the default boundary, including its response to changes in the risk-free rate, is very similar to that of the leverage ratio. Higher costs of bankruptcy lead to a reduction in the level of the refinancing boundaries to offset the lower amount of borrowing. Higher volatility might also be expected to lower the level of the refinancing boundaries for the same reason but it does not: unlike bankruptcy costs, higher business risk increases both the expected costs of bankruptcy and expected gain from refinancing in the future. The latter effect dominates and leads to the lower amount of borrowing.

The value of equity that managers maximize is negatively related to the tax rates on both corporate income and interest. This intuitive result is different from e.g. Fischer, Heinkel and Zechner (1989) and Leland (1994) since the state variable in their framework is the value of an unlevered firm and therefore tax benefits are accounted for as inflows of funds. The coupon level (and thus, the liquidity boundary) is negatively related to firm volatility; the difference between “investment-grade” and “junk” firms observed by Leland (1994) disappears in a dynamic model. In Leland’s world, firms with very high levels of business risk optimally commit to pay sizable coupons since they expect a dramatic improvement in their fortunes with a non-negligible probability. In a dynamic world they, instead, commit to refinancing when their fortune improves.

To complete the comparison with earlier results on comparative statics, the last row in Table I also shows the behavior of the credit spread, \( CS \), defined as \( \frac{C}{D(\delta_0)} - \frac{r}{1 - \gamma} \). A few results here merit comment. Interestingly, credit spreads are negatively related to the costs of refinancing, asset sales, equity issuance and bankruptcy. The last of these relations is also noted by Leland (1994). While higher costs make debt less attractive to creditors, they also reduce the optimal amount of borrowing and, typically, the default boundary as well. Both outcomes make debt less risky. Finally, credit spreads decrease as risk-free interest rate rises. In addition to the effect described by Longstaff and Schwartz (1995), according to which debt becomes less risky due to an increase in the risk-neutral drift of the net payout, an increase in the interest rate also lowers the riskiness of debt by reducing the optimal level of borrowing.

### III Capital Structure in a Dynamic Economy

The objective of this section is to investigate the cross-sectional properties of leverage ratios in a dynamic economy. Ultimately I am interested in building a bridge between empirical research and the empirical hypotheses that the model can deliver. The first step would be to relate the leverage ratio and other variables of interest used in empirical studies to the ones used in the model. If firms adjust their leverage only periodically, most firms most of the
time will be optimally off their optimal leverage at a refinancing point. Quite clearly, if an empiricist studies an economy generated by the model, the data would typically contain few "refinancing point" leverage ratios. To relate the model to empirical studies, it is necessary to produce within the model a cross-section of leverage ratios structurally similar to those which would have been studied by an empiricist.

That using comparative statics implications may cloud inference has been recognized for some time in studies of leverage mean-reversion and debt issuance (see e.g. Hovakimian, Opler and Titman (2001), Fama and French (2002)). If leverage deviates from its target substantially, an assertion supported empirically, then the response of firms to changes in economic conditions will not be in line with the predictions of comparative statics at refinancing points. Thus, I first study whether the cross-sectional relations in a dynamic economy different from those at refinancing point. Next I use data generated by the model to replicate a number of cross-sectional studies of capital structure that produced stylized facts. I investigate the crux of existing empirical evidence. The two questions in which I am especially interested are whether my model can produce results that are qualitatively similar to those found in empirical research, and, if so, whether the empirical estimates could have been generated by the model with reasonable probability under a feasible set of parameters.

As in Berk, Green and Naik (1999), my model is highly nonlinear in a number of important parameters and, as a result, individual dynamic leverage ratios, the main variable of interest, are difficult to obtain analytically. The complexity of dynamic effects in cross-sectional patterns of leverage means that it is impossible to identify the dynamic interaction between leverage and its determinants by performing a simple comparative statics exercise in dynamics. For example, a positive shock of a given magnitude can have different effects on firms in the same leverage group, leading to a complex interaction in the cross-section, since some firms will refinance while others will not. Similarly, high leverage can be the result of both optimally low borrowing due to high costs and unsuccessful past returns.

Therefore, as in Berk et al. (1999), I use simulation to generate artificial data from the model. Since individual leverage ratios and some commonly used regressors are observable in the simulation, I am able to replicate a number of empirical research methods. In particular, I compare the cross-sectional properties of leverage in the simulated economy with those predicted from the comparative statics of leverage at refinancing points, the focus of most current theory, and then investigate the empirical hypotheses on the issues that have been the focus of many empirical studies. These issues include the average level of leverage in the economy, the cross-sectional relation between profitability and leverage, mean reversion of leverage ratios and the impact of past stock returns on capital structure.

III.1 Running simulations
This section describes the simulation procedure. Technical details are given in Appendix A.
To start with, observe that, while only the total risk of the firm matters for pricing and capital structure decisions, economy wide shocks lead to dependencies in the evolution of the cash flow of different firms. To model such dependencies, shocks to their earnings are drawn from a distribution that has a common systematic component. Thus, cross-sectional characteristics of leverage are attributable both to firm-specific characteristics and to dependencies in the evolution of their assets. In particular, equation (1) may be rewritten as

\[
\frac{dB_t}{\delta_t} = \mu dt + \sigma I dZ^I + \beta \sigma S dZ^S \quad \forall t \geq 0, \delta_0 > 0.
\]  

(14)

Here, \( \sigma_I \) and \( \sigma_S \) are constant parameters and \( Z^I_t \) and \( Z^S_t \) are Brownian motions defined on a filtered probability space \((\Omega, \mathcal{F}, \mathbb{Q}, (\mathcal{F}_t)_{t \geq 0})\). This formulation implies that the shock to each project’s cash flow is decomposed into two components: an idiosyncratic shock that is independent of other projects \((\sigma_I dZ^I_t)\) and a systematic (market-wide) shock that affects all firms in the economy \((\sigma_S dZ^S_t)\). The parameter \( \beta \) is the systematic risk of the firm’s assets, which I will refer to as the firm’s “beta”, and systematic shocks are assumed independent from idiosyncratic shocks. The Brownian motion \( dZ \) in equation (1) is thus represented as an affine function of two independent Brownian motions, \( dZ = dZ^I + \beta dZ^S \), and

\[
\sigma = (\sigma_I^2 + \beta^2 \sigma_S^2)^{\frac{1}{2}}.
\]  

(15)

At date zero all firms in the economy are “born” and choose their optimal capital structure. The comparative statics of the system at date zero (where all firms are at their refinancing points) is thus analogous to that described in Section II.5. For the benchmark estimation I simulate 300 quarters of data for 3000 firms. To minimize the impact of the initial conditions, I drop the first 148 observations leaving a sample period of 152 quarters (37 years). I refer to the resulting dataset as one “simulated economy”. Using this resulting panel dataset I perform cross-sectional tests similar to those in the literature. The presence of a systematic shock makes cross-sectional relations dependent on the particular realization of the market-wide systematic component. Therefore I repeat the simulation and the accompanying analysis a large number (1000) of times. This allows me to study the sampling distribution for statistics of interest produced by the model in dynamics.

In any period each firm observes its asset value dynamics over the last quarter. If the value does not cross any boundary, the firm takes no action. It is important to stress that it is optimal, under these conditions, for the firm to remain passive. If its value crosses an upper refinancing boundary, it conducts a debt-for-equity swap re-setting the leverage ratio to the optimal level at a refinancing point, and so starting a new refinancing cycle. If the

\[\text{In the absence of the systematic shock cross-sectional relations will be nearly identical in all simulations once economies reach their steady state.}\]
liquidity boundary is hit for the first time in the current refinancing cycle, asset sales are conducted in the same period. If the firm defaults, bondholders take over the firm and it emerges in the same period as a new firm with a new optimal leverage ratio. Observe that my procedure implies a constant population of firms in the economy. This is not an important restriction since the parameters for new firms would have been drawn from the same sampling distribution as existing firms.

III.2 Choice of Parameters

This section describes how firms’ technology parameters and the economy-wide variables are calibrated to satisfy certain criteria and match a number of sample characteristics of the COM-PUSTAT and CRSP data. An important caveat is that for most of parameters of interest, there is not much empirical evidence that permits precise estimation of their sampling distribution or even their range. In addition, the model requires all parameters to be estimated as time-invariant. Overall then, the parameters used in my simulations must be regarded as ad hoc and approximate. There are two ways I deal with this problem. First, whenever possible, e.g. for tax rates, I use established empirical estimates. Second, and more importantly, I perform numerous robustness checks (see Section IV); these show that my results are not qualitatively affected by changing the parameters within a feasible range. Table II summarizes the descriptive information for the parameters described below.

III.2.1 Firm technology parameters

The present values of the net payout and book assets at date zero are identical for each firm and scaled to 100. In the model the rate of return on firm value is perfectly correlated with changes in earnings. In calibrating the standard deviation of net payout I therefore use data on securities returns. Firms differ in their systematic risk, represented by \( \beta \). A distribution of \( \beta \) is obtained by running a simple one-factor market model regression for monthly equity returns for all firms in the CRSP database having at least three years of data between 1965 and 2000 with the value-weighted CRSP index as the proxy for the market portfolio. The regression betas, \( \beta^0 \), are adjusted towards the mean (unity) using a so-called “Bloomberg” adjustment: \( \beta = 0.66\beta^0 + 0.34 \). \(^{15}\) The resulting \( \beta \) is used as an estimate of the asset beta. Systematic debt risk is assumed to have a small impact compared to systematic risk of equity.

The distribution of firms’ volatility is taken to match the parameters of the distribution of the standard deviation of rates of return on firm assets reported by Schaefer and Strebulaev (2003). \(^{16}\) The mean and standard deviation of that distribution are 0.255 and 0.10,

\(^{15}\)Beta shrinkage reflects the estimation error where larger betas are likely to be overestimated and smaller betas are likely to be underestimated. The results are unchanged by using more sophisticated methods.

\(^{16}\)Note that the Schaefer and Strebulaev’s (2003) sample is confined to firms that issue public debt. As
respectively. The standard deviation of the systematic shock, $\sigma_S$, is estimated as

$$\sigma_S = \sqrt{(1 - L_{av})^2 \sigma_E^2 + L_{av}^2 \sigma_D^2 + 2L_{av}(1 - L_{av})\sigma_{ED}}. \quad (16)$$

Here, $\sigma_E$ is the volatility of monthly returns on the CRSP value-weighted equity return index, $\sigma_D$ is the volatility of monthly returns on the 10-year T-note index over period 1965–2000 provided by CRSP, and $\sigma_{ED}$ is the covariance between equity and debt returns. Estimates of these parameters, 0.155, 0.081 and 0.023, respectively, are close to those reported by Campbell and Ammer (1993). Leverage, $L_{av}$, is computed from annual COMPUSTAT data for 1965–2000, averaging first for each year over firms and then over time. Leverage is defined as the ratio of book debt to the sum of book debt and market equity. The volatility of idiosyncratic shocks, $\sigma_I$, must be chosen to be consistent with the distribution of total risk. After considering a number of alternatives, individual shocks are assumed to have a distribution with the probability density function $f(\sigma_I) \sim a_0 + a_1\chi^2(n)$. This distribution implies that projects with both low risk and very high risk are relatively common. A positive value of $a_0$ also ensures that there will be no cash flows with negligible total risk.17

Since the proportional costs of restructuring in default, adjusting leverage, selling assets and issuing equity are all likely to be related to either the liquidity of firm assets and/or ease of access to external markets, all these costs are postulated to have a common covariance matrix. In particular, each cost, $q_x$, is drawn from the following distribution: $q_x \sim U[a_x, a_x + \frac{2}{3}(b_x - a_x)] + \frac{1}{3}(b_x - a_x)s$, where $a_x$ and $b_x$ are bounds for the value of costs and $s \sim U[0, 1]$ is the common component. This formulation implies that 20% of each costs’ value is due to the common component. This distribution is symmetric and its trapezoid probability density function implies that the values close to the boundaries are less likely to occur and the values in a range around the mean are equally likely to occur.

For the proportional cost of restructuring in default, $\alpha$, the bounds, $a_x$ and $b_x$, are assumed to be 0.03 and 0.10. Most of the empirical values reported in e.g. Weiss (1990) and Altman (1984) lie in this range. Recent evidence by Andrade and Kaplan (1998) suggests somewhat higher values. Leland (1994) uses a similarly defined cost of 0.5, Leland (1998) uses 0.25 and Goldstein, Ju and Leland (2001) use 0.05.

Fischer, Heinkel and Zechner (1989) and Goldstein et al. (2001) also define adjustment costs, $q_{RC}$, in the same way and use a value of 1%. Datta, Iskandar-Datta and Patel (1997) report total expenses of new debt issuance over 1976–1992 of 2.96% and Mikkelson and Partch (1986) find underwriting costs of 1.3% for seasoned offers. This author’s unreported calcula-

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17 The “safest” companies in the Schaefer and Strebulaev’s (2003) sample have a volatility of about 0.06.
tion using the Fixed Income Securities Database (see Davydenko and Strebulaev (2003) for a description) over the period 1980–2000 suggests that the average underwriting and management spread is about 0.05% in yield which is consistent, e.g. for a risk-free perpetuity when the risk-free rate is 5%, with a proportional cost of 1%. Note, however, that costs in this framework are proportional to the total amount of debt issued rather than to incremental amount. Therefore, I choose substantially smaller adjustment costs with range of 0.05% to 0.35% to be consistent with costs per unit of new debt issued of the order of 1%.

Proportional equity issuance costs are assumed to be distributed in the range [0.02, 0.08]. Recent empirical research has emphasized that, in initial public offerings, a simple 7% solution was used to settle underwriter costs (Hansen (2001)). The costs of seasoned equity offerings are likely to be smaller, however. Corwin (2003) reports gross spread of 5.4% and direct expenses of 1.5%. In addition, there is evidence (Altinkilic and Hansen (2000)) that equity costs derive mainly from the variable component.

The costs of asset sales in a liquidity crisis are assumed to be distributed in the range [0.05, 0.25]. These costs are admittedly enormously difficult to estimate. In one of the most elaborate empirical attempts to date, Pulvino (1998) estimates that these costs are on average around 14% for companies with an above median debt ratio. Whether companies with substantially higher debt ratios than the median (e.g. those facing a liquidity crisis) face even higher costs, as may well be the case since they have less time to search for a higher bidder, has not been empirically established. Also, it is not entirely clear to what extent the airline industry example (as analyzed by Pulvino) can be extended to other industries.

The fraction of assets that remains after an asset sale, $k$, is assumed to have a uniform distribution with support [0.6,1]. Asquith, Gertner and Scharfstein (1994) report that on average companies sell 12% of their assets.\footnote{Based on book value. For distressed companies market value estimates can be larger.} Twenty one out of seventy six companies in their sample that took visible steps to restructure their firms sell more than 20%, and, most interestingly, the median level of asset sales among these twenty one firms is 48%.

The rate of net investment growth, $g$, is assumed equal the expected growth rate of the firm’s net cash flows. This is consistent with a finite non-zero expected market-to-book ratio in an infinite horizon. It is also consistent with the fact that investment equal in magnitude to depreciation is needed to maintain the firm as a going concern. The net payout ratio increases with interest payments according to (11) and the parameter $a$ depends, ultimately, on firms’ price-earnings ratios and dividend policy. The range of its value is between 0.03 and 0.04; the value of 0.035 is also used in Goldstein et al. (2001).

When the net payout flow is very small, the firm starts losing part of its tax shelter. Since the remaining tax shelter depends on carry-forwards and carry-backs benefit provisions it is likely that firms lose a substantial part of the tax shield when current income is not sufficient
to cover interest payments. I model the partial loss offset boundary as \( \delta_\kappa = \kappa \delta_L + (1 - \kappa) \delta_B \), where \( \kappa \) is uniformly distributed on \([0.7, 0.9]\). It is assumed that, when the net payout is below \( \delta_\kappa \), \( \tau_\kappa \) is lost per dollar of full offset, where \( \tau_\kappa \) is set to be equal to 0.5. Note that this formulation assumes that full tax benefits are resumed when the firm comes out of distress.

### III.2.2 Economy-wide parameters

The corporate tax rate is assumed to be equal to the highest existing marginal tax rate, \( \tau_c = 0.35 \). To decide on marginal personal tax rates on interest income and dividend payments I follow Graham (1999; 2000). In particular, Graham (1999) estimates \( \tau_i \) as 0.351 and \( \tau_d \) as 0.122 over the period of 1980–1994. Thus, the maximum tax benefit to debt, net of personal taxes, is \( (1 - \tau_i) - (1 - \tau_c)(1 - \tau_d) = 7.8 \) cents per one dollar of debt. In estimating tax rates I ignore at least two important real world features: the variability of tax rates both across firms and across time. Introducing time-varying taxes would destroy the scaling feature of the model. Since we do not know whether marginal firm-specific tax rates are correlated with firm characteristics, I choose to deal with firm-invariant tax rates.

The after-tax risk-free interest-rate is 0.05. It is estimated as the mean three-month Treasury bill rate over the period 1965–2000, multiplied by \( (1 - \tau_i) \). Ibbotson Associates (1995) report an average annual equity risk premium of about 0.08 and expected default premium of about 0.01 for the postwar period. Using \( L_{av} \) (see above), the risk premium on the rate of return on firm assets is estimated in the region of 0.065.

### III.3 Preliminary empirical analysis

I now bring together the calibrated model with the results of comparative statics at the refinancing point and some empirical results from the literature. I use two definitions of leverage, both based on the market value of equity. The first, the market leverage ratio, has already been defined for date zero in (13) and, for any other period, it is defined analogously. Typically, however, market values of debt are not available and book values are used. I therefore introduce a second definition, the quasi-market leverage ratio, defined as the ratio of the par value of outstanding debt to the sum of this par value and the market value of equity:

\[
QML_t = \frac{D_0(\delta_t)}{D_0(\delta_t) + F(\delta_t) - D(\delta_t)}.
\]  

Typically, the difference between \( ML \) and \( QML \) is very small. For financially distressed firms, however, it can be more substantial. Intuitively, these ratios reflect to how the firm has financed itself in the past since both the par and market values of debt reflect decisions taken early in a refinancing cycle. To determine how close the firm is to financial distress, a flow measure that shows whether the firm can meet its debt payments is more relevant since firms
may encounter distress at different levels of leverage. Therefore, I also consider the interest coverage ratio which is defined as the ratio of net payout to the coupon.

Table III summarizes the cross-sectional distribution of these various measures in a dynamic economy and at the refinancing point. The average leverage ratio at the refinancing point is 0.26, compared to 0.37 in a similar model by Goldstein, Ju and Leland (2001). The two main reasons for the difference are (i) the presence in my model of additional financial constraints such as liquidity crisis costs and (ii) a lower tax advantage to debt since the tax rate on dividend income that I use is smaller.

Of more importance, however, are the descriptive statistics for dynamics. Means for dynamic statistics are estimated in a two-step procedure. First, for each simulated economy statistics are calculated for each year in the last 35 years of data. Second, statistics are averaged across years for each simulated economy and then over economies. To get a flavor of the impact of systematic shocks, for market leverage and credit spreads I also present minimum and maximum estimates over all economies. I begin by comparing the leverage statistics in the dynamic economy to those at refinancing points where the impact of the dynamic evolution of firm’s assets is ignored. What Table III shows is that leverage ratios in the dynamic cross-section are larger than at refinancing points. Leverage ratios of firms that are in distress or close to a bankruptcy typically exceed 70%, and these firms have a strong impact on the mean. Thus, it would be premature to claim that dynamic capital structure provides a simple explanation of low leverage puzzle and so care needs to be taken in using leverage at refinancing points to make any empirical claims.19

In summary, because firms at different stages in their refinancing cycle react differently to economic shocks of the same magnitude, the cross-sectional distribution of leverage as well as the other variables in Table III is different in dynamics and at refinancing point. Next I turn to comparison with empirical data on leverage. Bernanke, Campbell and Whited (1990) give the distribution of leverage for the three years 1986–88. Since they impute the market value of debt, I use the market leverage ratio from the model for comparison. Overall, the distributions are quite similar with their mean leverage ratio (0.33) close to mine (0.36). More interestingly, the right tail of my distribution mirrors theirs closely suggesting that a cross-section of leverage ratios in a dynamic economy can replicate an empirically observed distribution, while the cross-section at a refinancing point can not. Rajan and Zingales (1995) report, among other statistics, quasi-market leverage ratios. For 1991 the U.S. mean and median values are, respectively, 0.32 and 0.28, as compared with 0.37 and 0.35 in my model.20

19The Goldstein et al. (2001) model produces a dynamic average leverage ratio of 0.45. See Section IV. Also, these models are unable to deliver low leverage for a large fraction of firms in cross-section.

20To complement the comparison, I have constructed an empirical distribution of the quasi-market debt to capital ratio on COMPUSTAT data each year over 1965–2000. The 90% and 95% percentiles of distribution are between 57 and 89%, and 62 and 92%, respectively. Footnote 2 defines debt to capital ratio.
Rajan and Zingales report a median interest coverage ratio of 2.41 (4.05) when deducting (not deducting) depreciation. For the former case Bernanke et al. report a mean value slightly above 5. Both results are similar in magnitude to the model values. The tax advantage to debt is calculated as the ratio of the difference between the current value of the firm and the after-tax value of unlevered assets to the after-tax value of unlevered assets. This ratio ranges between 0 and 10% with a mean of 5%. The gain in moving from no-leverage to the optimal dynamic leverage, accounting for personal taxation, is comparable to the results on the net tax advantage of debt estimated by Graham (2000).

A brief look at credit spreads reveals an interesting pattern: while credit spreads at the refinancing point are quite low, with a mean (median) of 95bp (80bp), in dynamics they are much larger at 149bp (97bp). Empirical estimates of average credit spreads for investment-grade rated bonds are given as 60–118bp for 1987–1996 by Elton et al. (2001) and as 109bp with median 85bp for 1996–2000 by Davydenko and Strebulaev (2003). In dynamics credit spreads may become very large indeed (the value of the 99% percentile is more than 800 bp). Since credit spreads are highly convex in the value of the firm, in dynamics higher levered and particularly distressed firms dominate the sample. This explains a substantial part of the increase in the mean credit spread. However note that, in the model, bondholders take over the firm in the case of default, albeit at a cost, and therefore extreme credit spreads that are sometimes observed in the market do not occur here.

Table IV shows that the annual default frequency is around 49 basis points; around 12% of firms restructure every year; 1.20% conduct asset sales (i.e. reach the liquidity barrier for the first time) and about 5% are in financial distress and have to resort to costly equity issuance.

As a preface to the next section, I use a simple correlation analysis to investigate whether the cross-sectional relation between leverage and its determinants differs between comparative statics at refinancing points and in dynamics. The results are in Table V. Panel A reports correlations at refinancing points, while Panel B gives the results for dynamics. One result stands out. While market leverage and credit spreads are negatively correlated at refinancing points, in dynamics the opposite result is observed. To see the former result, observe that firms that choose large leverage have on average low volatility, as well as low costs of distress and bankruptcy, and therefore low credit spreads. The direction of this relation, however, is completely at odds with empirical observations, where a positive association between leverage and credit spreads is widely reported (see e.g. Davydenko and Strebulaev (2003)). The empirically observed correlation is clearly consistent with what we observe in the model dynamics. The intuition is straightforward: firms with worst performance experience an increase in both their leverage (as the value of equity falls by more than value of debt as distress approaches) and credit spreads (as outstanding debt becomes riskier) while firms with best performance experience a decrease in their leverage (until they refinance) and, on average, credit spreads.
III.4 Cross-sectional regression analysis

III.4.1 Leverage-profitability relationship

I have established that both the distribution of leverage and the cross-sectional relation between leverage and other variables of interest may differ substantially between refinancing points and dynamics. The relation between leverage and profitability is a particularly striking example. Profitability, $\pi_t$, is defined as the ratio of earnings before interest and taxes (the sum of the net payout ($\delta_t$) and retained earnings (change in the value of book assets)) to the book value of assets in place, $A_{t-1}$:

$$\pi_t = \frac{\delta_t + \Delta A_t}{A_{t-1}}.$$  (18)

The trade-off theory predicts that a persistent increase in earnings leads firms to more extensive use of debt financing by increasing the tax advantage to debt and reducing the expected costs of distress and bankruptcy. This is reflected in a positive correlation between leverage and profitability at the refinancing point, reported in Panel A of Table V.\(^{21}\) The negative sign in Panel B shows that in dynamics this relationship can become negative even for a trade-off model. This section investigates the nature and the magnitude of this effect in the context of the empirical tests reported in the literature and uses simulated data generated by the model to gauge whether this effect is important for understanding the results of these tests. Section IV then assesses the extent to which this result is dataset- and model-dependent.

Why is the leverage-profitability relation singled out? First, as Myers (1993) has pointed out, perhaps the most pervasive empirical capital structure regularity is the inverse relation between debt usage and profitability. Indeed, the relationship is one of several widely established results in the empirical capital structure literature.\(^{22}\) More importantly, it is also one of a few, if not the only cross-sectional relation, that disentangles the trade-off model and the various theories associated with the pecking order concept, according to which, holding investment fixed, persistently higher profitability enables firms to use less leverage. This intuition holds for both static and dynamic versions of the pecking order theory. For other determinants of leverage, either the predictions of both pecking order and the trade-off theories are the same or the predictions of the various versions of the pecking order theory themselves differ.\(^{23}\) The

\(^{21}\)Note that all changes in earnings in the model are persistent and thus firms with higher profitability at date zero expect to be more profitable in the future and opt optimally for higher borrowing.

\(^{22}\)See, for example, Titman and Wessels (1988), Fama and French (2002) and Baker and Wurgler (2002). Rajan and Zingales (1995) establish that the inverse relationship holds for 6 out of 7 developed countries and Booth et al (2001) report that it also holds for most developing countries.

\(^{23}\)For example, both the pecking order and trade-off models predict that higher volatility of the firm’s cash flow is likely to lower the optimal amount of borrowing. Also, the static pecking order theory suggests that higher investment leads to higher borrowing when retained earnings are fixed, while the dynamic version predicts higher expected investment to decrease current debt so that the debt capacity is preserved for the future.
ambiguity attached to the impact of other determinants means, therefore, that a consistently negative relation found between leverage and expected profitability is interpreted as a major failure of the trade-off model.

I turn now to whether the cross-sectional leverage-profitability relation that my framework delivers, is consistent with those reported in empirical capital structure research. Recall that each simulated dataset (“economy”) consists of 3000 firms for 300 quarters and that economies differ because of a systematic shock. As described in section III.1, I simulate 1000 economies dropping the first half of the observations in each economy. For each economy I then conduct the regression tests outlined below. For each set of regressions I report mean coefficients and t-statistics over all simulated economies and for several coefficients also give the distribution.

Table VI reports the results of the first set of experiments. Column 1 reports the regression for market leverage at the refinancing point and Columns 2–4 – on simulated economies. Column 2 reports on attempts to replicate early empirical tests of capital structure (e.g. Bradley, Jarrell and Kim (1984)) by performing an OLS regression of quasi-market leverage, $QML$, at the end of the last year in each simulated economy against profitability and the constant “firm technology” parameters. Thus, the regressand and profitability are measured contemporaneously. Column 3 reports the results of the procedure that replicates the method implemented by Rajan and Zingales (1995) in which OLS regression of quasi-market leverage in year $t$ is run against four year averages of the regressors over years $(t-4) - (t-1)$, where year $t$ is the last year in each economy. Thus, the independent variables are lagged one year and then averaged over four years. Rajan and Zingales lag the regressors in order to reduce the problem of endogeneity. Since profitability is persistent and other regressors are time-invariant, it does not have any significant impact on my results. Rajan and Zingales average the explanatory variables to reduce the noise and to account for slow adjustment.

Fama and French (2002) estimate “target leverage” using a two-step procedure. They first estimate year-by-year cross-sectional regressions and then use the Fama-MacBeth methodology to estimate time-series standard errors that are not clouded by the problems encountered in both single cross-section and panel studies. The main problem with these methods stems from correlation in the regression residuals across firms and the presence of autocorrelation in the regressions coefficients. In the simulated economy, correlation in the regression residuals exist because firm values are correlated via the systematic shock and the slopes are also autocorrelated because leverage is a cumulative outcome of past idiosyncratic shocks. I follow a simple and conservative rule used by Fama and French and assume that the standard errors of the average slopes should be multiplied by a certain factor to account for autocorrelation before judging the significance of a variable. Unreported results demonstrate that average coefficient on profitability is autocorrelated and behaves like an AR(1) process with observed maximum of about 0.75 and thus (see Fama and French (2002, p. 12)) the corresponding
multiplication factor is 2.5. In other words, \( t \)-statistics are required to be around 5.0, rather than 2.0, to reject the null hypothesis. The autocorrelation of the other coefficients is of the same order. Column 4 of Table VI shows the results of running a Fama-MacBeth regression on the last 35 years of each simulated economy and then averaging these results across economies.

To summarize, each of the regressions above can be written as:

\[
QML^P = d_0 + d_1 \pi^P + d_2 \sigma + d_3 \alpha + d' x + \epsilon^P, \tag{19}
\]

where \( x \) are firm-technology parameters and \( P, P \in \{BJK,RZ,FF\} \), refers to the method. For example, \( QML^{RZ} = QML_t; \pi^{RZ} = \frac{1}{t} \sum_{m=t-1}^{t-4} \pi_m \), and so forth.

I begin by describing briefly the importance of different factors for optimal leverage at refinancing points reported in Column 1, Table VI. A 1% increase in expected profitability increases target leverage by 6.22% and a 1% increase in the firm’s business risk produces a 0.8% reduction in leverage. The effect of bankruptcy and distress costs is smaller in absolute magnitude demonstrating again that, by themselves, these costs are not sufficient to offset the tax advantage to debt in the trade-off model. Insignificance of adjustment costs is due to their non monotonic relation to leverage.

The results of Columns 2–4 are roughly similar and this is consistent with the Fama-French observation that their results are mainly supportive of previous findings. In particular, the negative correlation between leverage and profitability survives in the multivariate regression analysis. The Fama-MacBeth estimates produce negative average slopes that are more than 10 standard errors below zero.

Note the particular importance of this result: an empiricist would be likely to interpret this finding as evidence in favor of the pecking order theory and contrary to the predictions of the trade-off model. However, we know that firms in the simulated economies do indeed follow the prescription of the trade-off theory. But why, in this case, is the profitability coefficient significantly negative in dynamics? An increase in profitability effects future profitability and thus the value of the firm. But while an increase in the value of the firm always lowers leverage, it does not necessarily lead to refinancing in a world with infrequent adjustment. Note that, under the model, the target leverage for any firm is constant, and so the observed positive relation between leverage and profitability at the refinancing point is purely a cross-sectional effect. The negative relation is firm-specific since higher profitability lowers the current leverage of an individual firm, unless it refinances in that period. The negative coefficients in Table VI imply that the firm-specific effect dominates in the simulated data. The presence of systematic shock magnifies this effect.

That the presence of frictions may complicate inference has been recognized in a number of previous studies. For example, Fama and French (2002) note that their result may overstate the long-term relation between leverage and profitability by picking up transitory variation in
leverage rather than variation in target leverage. This would make it difficult to disentangle the trade-off and pecking order models since a negative coefficient may be the result of the transitory component, pecking order behavior or both. It is instructive, therefore, to look at the size of the coefficient in the simulated data to judge whether a trade-off model can give rise to values that are similar to those found empirically.

Figure 2: Profitability coefficient. Fama-French (2002) regression.

The population mean of the profitability regression coefficient is above those found by previous researchers. Profitability coefficients reported by previous studies include $-0.90$ (Fama and French (2002)), $-0.6$ (Rajan and Zingales(1995)) and $-0.61$ (Baker and Wurgler (2002)). However, my estimate of $-0.34$ for the Fama and French-type regressions is simply the population mean across all economies. To gauge the likelihood, under the model, of estimates in the range $-0.6$ to $-0.9$ I examine the distribution of the profitability coefficient. Figure 2 shows this distribution for 1000 such economies and columns 6 and 7 of Table VI report its 10% and 90% percentiles. All the coefficients are negative, though some are insignificant (taking into account the five standard error rejection threshold). The bottom line result is that under the chosen set of parameters the reported empirical estimates can be consistent with the value of the coefficient.

There are several possible ways in which this result may be qualified. First, the parameter

\[^{24}\text{Fama and French report several profitability coefficients, ranging from } -0.42 \text{ to } -0.96, \text{ since they study both book and market leverage, divide the sample of firms in the two groups – dividend payers and nonpayers – and include in some regressions a simultaneously estimated target payout ratio. The coefficient of } -0.9 \text{ is for the regression on market leverage for dividend payers not allowing for the target payout ratio.}\]
set may be unrepresentative because, for example, I do not allow for correlation between volatility and distress/bankruptcy costs. Indeed, in a number of robustness checks a coefficient substantially smaller in magnitude is obtained. In particular, smaller renegotiation costs and more widely dispersed “betas” result in a larger coefficient. For many other parameterizations changes, the result is unchanged or stronger. In a nutshell, the robustness tests suggest that, at the very least, the model is able to explain a large part of the negative relation.

Second, I use the leverage ratio based on market equity value. Fama and French (2002) argue that the profitability-leverage relation holds theoretically only for book leverage. In empirical regressions, however, the values of the slope are very similar. Therefore, while for book leverage the result is likely to hold under a broader set of conditions than for market leverage, it is unlikely that this drives the observed difference. Third, in my model as well as most dynamic models of optimal structure, the investment process is independent of the process that determines the leverage ratio. In deriving the value of book assets I make an assumption that book assets grow at a rate equal to the growth rate of the net payout under the actual distribution – the only rate under which the market-to-book ratio has a finite non-zero expected value in infinite horizon. I choose a conservative value of one for an initial market-to-book ratio since my firms may be characterized as value firms. Increasing the market-to-book ratio, however, would lead to an increase in profitability via a decrease in book assets and therefore to a decrease in the magnitude of the profitability-leverage coefficient.

All other coefficients in Table VI have the same sign in dynamics as at the refinancing points, although the magnitude of the coefficient on volatility is smaller in dynamics. Adjustment costs become significant in a dynamic economy since their increase lead to the higher level of the refinancing boundaries and thus the average waiting times between successive adjustments and the corresponding change in leverage is larger.

III.4.2 Leverage and stock returns

In a recent paper, Welch (2004) obtains empirical results that, to some extent, parallel those presented here. Welch’s main finding is that U.S. corporations do not change their capital structure to offset the mechanistic effect on leverage of changes in their stock price. As I have emphasized above, the absence of a response by the firm to these mechanistic changes in leverage may, indeed, be optimal in the presence of adjustment costs. It is instructive, therefore, to investigate to what extent the mechanistic effect observed by Welch is reflected in my dynamic economies. To this end, I replicate, again using simulated data, the regression test that he performs on the COMPUSTAT dataset (Welch (2004), Table 3). For each year $t$ I run a cross-sectional regression of the level of the market leverage ratio against the implied market debt ratio, $IDR_{t-k,t}$ in Welch’s notation, i.e. what the market leverage ratio would have been if the firm had not issued any securities between years $t - k$ and $t$, and actual
observed quasi-market leverage ratio in year $t - k$, $QML_{t-k}$ in my notation:

$$QML_t = f_0 + f_1 IDR_{t-k,t} + f_2 QML_{t-k} + \epsilon.$$  \hspace{1cm} (20)

The implied debt ratio shows the response of leverage only to changes in equity. Thus, if the coefficient $f_1$ is equal to one, firms do not readjust at all. Alternatively, a value of $f_2$ equal to one would imply that firms perfectly offset any change in equity.

Note an important difference between this regression and those replicated earlier. In the case of the cross-sectional regressions reported in Table VI, some variables of interest such as research and development expense, employed in empirical studies, cannot be included because they are not present in the model. However, regression (20) that I estimate is exactly identical to that studied by Welch. The only point of departure between my simulations and the empirical procedure followed by Welch is that the number of firms in the empirical study varies across years while in the simulations the number of firms is fixed.

![Figure 3: IDR_{t-1,t} coefficient.](image)

Figure 3: IDR_{t-1,t} coefficient.

To be precise, I compute the average of the time-series of cross-sectional regression coefficients à la Fama-MacBeth. Then, as usual, the results are averaged over many simulated economies. Table VII shows that for all four choices of $k$, between 1 and 10 years, the results appear to conform closely to those obtained by Welch. In particular, the slope of nearly one for the implied debt ratio for the one-year regression (the average slope of 1.014 in Welch and 1.028 in the model) indicates that financing behavior in the short term is almost passive, in other words, corporations do not react to changes in the value of equity by adjusting their
leverage. Figure 3 demonstrates that the coefficient over one-year horizon obtained by Welch for the implied debt ratio is well within the observed frequency of average coefficients in the model, and Table VII shows that my model produces slightly lower estimates for longer horizons. Overall, I find that my model would not reject Welch’s coefficient on the implied debt ratio over a short horizon and that the term structure patterns of the coefficients are also similar. The simulations clearly show that a model with small adjustment costs can produce results on the persistence on leverage that are consistent with those observed in reality.

There is one particular feature that deserves special attention. Welch (2004) points out that while corporations are very active in debt issuance, their motives “remain largely a mystery” given that the mechanistic effect of the change in equity value is not offset. The same apparent puzzle is observed in my framework. A coefficient close to one might be interpreted as extreme passivity on the part of shareholders in their debt decisions. However, it seems to contradict the result reported in Table IV that, on average, about 12% of firms refinance every year. In fact, my model provides a simple explanation of this “puzzle” since firm “passivity” in the Welch (2004) sense also obtains if firms issue debt quite frequently, but the contemporaneous cross-sectional covariance between new debt issues and equity returns over the chosen period is zero. And this is exactly what happens in the model since managers issue debt in response to a factor, largely orthogonal to short-term equity returns, namely long-term past stock returns. My model provides an additional insight: if the covariance between changes in outstanding debt and equity returns over $t - k$ to $t$ is weakly positive then the coefficient on $IDR_{t-k,t}$ slightly increases. For $k = 1$ year, the empirically observed covariance is indeed weakly positive explaining why the coefficient slightly exceeds one. Observe that in the model, while the equity return over the last year does not trigger debt issuance by itself, debt will be issued only if equity returns are positive (otherwise the refinancing barrier would not be reached). In addition, debt reduction due to a liquidity crisis occurs only if the last period equity return was negative. This again leads to weakly positive covariance and the same explanation of why one-year coefficient slightly exceeds one on average. It also provides an explanation why, over a long horizon, my coefficients are smaller than the Welch’s ones: in reality firms issue debt for reasons related to investment opportunities that can be positively related to changes in equity value.

It is tempting to suggest that adjustments costs are entirely responsible for the results reported by Welch (2004). However, as Welch himself point out, there are some drawbacks to this explanation: (1) direct transaction costs are small; (2) readjustment patterns are similar across firms while transaction costs are very different; and (3) firms do not seem to lack the inclination to be capital structure active, but they seem to lack the proper inclination to readjust when equity value changes. My analysis sheds light on some of these concerns. First, even small transaction costs can lead to stickiness in the firm’s debt policy. Robustness checks
in Section IV show that even taking a highly conservative estimate of transaction costs leaves
the results essentially unchanged. Second, in the model debt issuance costs are smaller than
equity issuance costs, thus the firms who reduce debt when they are in distress experience
relatively higher transaction costs. In other words, after substantially negative equity returns
firms face higher transaction costs. However, these firms are no more eager to readjust. Third,
as I have explained above, the framework accounts for both the capital structure activity and
unwillingness to readjust in response to past equity returns. At the same time, at least two
issues raised by Welch (2004) can not be addressed satisfactorily in the present framework.
First, there is no difference between small and large firms, and second, no richer set of debt
instruments is allowed that would enable corporations to lower transaction costs.

III.4.3 Changes in leverage and mean reversion

I turn next to the question of the extent to which leverage is mean-reverting in my model. Table
VIII summarizes estimates of a number of partial adjustment models where the dependent
variable in all cases is the annual change in the quasi-market leverage ratio. Columns 1 and
2 of the table follow e.g. a study by Fama and French (2002) and report the results of a two-
stage cross-sectional regression estimation. In the first stage, target leverage, $TL_t$, is estimated
using equation (19); the resulting value is then used in the regression for changes in leverage:

$$QML_t - QML_{t-1} = h_0 + h_1 TL_{t-1} + h_2 QML_{t-1} + h_3 X_{t-1} + \epsilon,$$

where $X_{t-1}$ represents other possible lagged regressors. A partial adjustment model predicts
that $h_1$ is positive and $h_2$ is negative and, furthermore, that they are equal in absolute value.
Coefficient $h_2$ measures the speed of adjustment of leverage to its target level.

Not surprisingly, we find that leverage is mean-reverting. A coefficient of -0.17 indicates
that the mean reversion of leverage is 17% per year. Fama and French (2002) report a similar
mean reversion speed of 7-10% for dividend payers and 15-18% for non dividend payers which
they refer to as “snail’s pace”. My firms may be better characterized as “crouching tigers”: most of
the time firms do nothing to the level of their book debt, but when they do make
changes it is by a large amount. Also, in line with the results reported by Fama and French
(2002), the average slopes on lagged leverage are similar in absolute value to those on target
leverage and are therefore consistent with the partial adjustment model.

Column 2 adds change in profits as an additional regressor. While the results are very
similar to those of Fama and French (2002), my interpretation is slightly different. They
suggest this result shows that short-term variation in earnings is largely absorbed by debt.
In the model that has been developed here a change in profitability that effects the leverage
ratio is due exclusively to persistence of its effect on firm value.
Columns 3 and 4 of Table VIII report estimations of regressions of the change in the leverage ratio of the type studied by Welch (2004). The regression can be written as:

\[ QML_{t} - QML_{t-k} = \beta_0 + \beta_1 (IDR_{t-k,t} - QML_{t-k}) + \beta_2 \pi_{t-k} + \beta_3 (IDR_{t-k,t} - QML_{t-k}) + \epsilon. \] (22)

The idea is that a significant coefficient on profitability, \( \pi_{t-k} \), shows that profitability incrementally explains leverage after controlling for equity returns. If the cross-term is significant, then profitability also helps to explain leverage adjustment.

The estimates in Table VIII indicate that, once stock returns are controlled for, profitability looses most of its power in explaining leverage, but is still able to account for the adjustment behavior of firms in cross-section. The latter result is similar to the finding of Welch (2004). Empirically, however, profitability is found to retain some explanatory power that can be due to its temporary component.

IV Robustness Tests

In this section I describe the results of a number of robustness tests designed to investigate the extent to which my results are sensitive to changes in parameter values and estimation procedure. They fell into two categories. First, using the benchmark dataset, I investigate whether the results are influenced by the way the sample is constructed. In particular, outliers in the simulation of the evolution of firm asset values may have an undue influence. Second, I study whether perturbing the parameters has a significant impact on the results. For each robustness test I recalculate the whole analysis but, to limit the usage of computer power, the results are calculated using 3000 firms and 50 simulated economies. Also, when needed, refinancing point solution is found for only 500 firms and so in simulations six firms are identical at refinancing point. Other features of simulation design are not changed.

The key question is whether the main results of the paper survive the robustness tests. These include: (1) the relation between the average level of leverage at refinancing points and in a dynamic economy; (2) the average slope of the leverage-profitability relationship; (3) those relating to Welch (2004) finding on capital structure and stock returns and (4) the degree of mean-reversion. To save the space, Table IX reports only a summary of the results.

The evolution of a dynamic economy leads to some outliers. While there is no measurement error in my benchmark dataset, an empiricist using the generated data of any simulated economy might be concerned that some observations dominate the results and would therefore exclude them. Following the approach used in the literature, I examine how the results are changed when: (i) the true volatility of firm cash flows is trimmed at the 5% and 95% percentile thresholds; (ii) in a dynamic economy, the time-series volatility in each year is estimated for each firm over the previous 5 years and estimates outside the 5% and 95% percentiles are
excluded; (iii) in a dynamic economy, for each year firms whose profitability lies outside the 5% and 95% percentiles are excluded; (v) firms that experience default over the previous five years are excluded. I find that none of the changes in procedure influence the main results in any significant way.

The next tests examine the dependence of the results on changes in the parameters. First, for each exogenous parameter that varies across firms, I consider five cases. For the first two, the distribution of the parameter is identical to the benchmark case except that its mean is changed: in one case increased and in the other decreased. In the remaining three cases, the parameter value is set equal across firms at (i) the upper boundary of the benchmark distribution, (ii) the lower boundary and (iii) a value equal the mean in the base case. Again I find that, qualitatively, the main results are unchanged. However, changing the volatility parameters does result in noticeable changes in the cross-sectional leverage distribution and changes in the distributions of renegotiation costs and betas effect noticeably the profitability coefficients. Under some parameter values, empirical estimates of profitability are less likely to be obtained by the model. The coefficients on the implied debt ratio are more stable.

Third, I investigate the effect of changes in macroeconomic and tax parameters. Unsurprisingly, decreasing the tax advantage to corporate debt results in lower leverage in the economy. One result not shown in the Table IX is that a decrease in $\tau_i$ from 0.35 to 0.3 lowers the average market leverage ratio from 0.32 to 0.24. The difference between the average leverage ratio in dynamics and at the refinancing point is, however, not significantly affected. Finally, I consider the effect of measurement errors. In particular, a stochastic component in the evolution of book assets and a measurement error in market leverage are introduced. Note that these measurement errors are assumed not to effect the optimal decisions by firms.

Overall, the results appear to be quite robust with respect to changes in firm-specific and environmental parameters and to changes in empirical procedure. This applies particularly to the cross-sectional results which are also the most important.

V Concluding Remarks

This paper is the first to describe a methodology for deriving the quantitative and qualitative predictions of capital structure theories in a dynamic economy with infrequent adjustment. Using a model of dynamic optimal capital structure, I generate data that structurally resembles data used in empirical studies. In this way, the method allows us to compare the predictions of a capital structure model in “true dynamics” both to the findings of the empirical literature and to the comparative statics predictions of the same model. In particular, it enables us to provide greater insight into qualitative aspects of the cross-sectional properties of leverage. The main findings of the paper are that (i) the properties of leverage in the cross-section in
“true dynamics” and in comparative statics at refinancing points differ dramatically, and (ii) the model gives rise to data that are consistent with a number of empirical results and which, using methodologies commonly employed in the literature, may lead to rejection of the model. These findings provide a clear signal of the need for further research in this area.

There are two principal directions in which the framework developed here could most usefully be extended. First, because the dynamics of financing decisions have such a profound influence on the empirical properties of the cross-section, competing theories of capital structure – beyond the trade-off theory – ought to be developed in fully dynamic form. Some first attempts have been made (Dasgupta and Sengupta (2002), for example, develop a model with moral hazard where, interestingly, dynamic interaction leads to another explanation for a positive relation between leverage and profitability), but development of alternative dynamic models that lead to quantitative predictions is still a subject for future research.

Second, a proper study of the evolution of capital structure requires a model that combines both dynamic capital structure decisions and real investment. Berk, Green and Naik (1999) may provide an excellent basis for the second of these requirements while the model developed here – may contribute to the first. The modelling approach of firm behavior in Berk et al. (1999) is both richer than mine in some areas and less rich in others: they are able to analyze a wider spectrum of questions by considering separately existing assets in place and future growth opportunities. However, their firms are in fact myopic since the fact that investment projects are assumed independent, combined with a complete lack of any financial policy, means that, in taking investment decisions, a firm does not take into account the evolution of its assets over time. Research that combines these two strands is likely to be a fruitful avenue for future research in capital structure, and more generally, corporate finance.

Appendix A Details of Simulation Analysis

The process for \( \delta \) is discretized using the following approximation:

\[
\delta_t = \delta_{t-\Delta t} e^{(\mu_A - \frac{\sigma^2}{2})\Delta t + \sigma \sqrt{\Delta t} z_t},
\]

(A1)

where \( \Delta t \) is one quarter, \( z_t \) is a standard normal variable and \( \mu_A \) is the growth rate of the net payout ratio under the physical measure. The benchmark simulation is for 300 quarters and 3000 firms. Note that while I discretize the model for the purpose of simulation, firms still operate in a continuous environment. In particular, it must happen now that firms will sometimes “overshoot” over boundaries and make their financial decisions not exactly at the prescribed optimal times. Unreported robustness checks show that increasing the frequency of observations does not at all change the results.

To choose the number of observations that will be dropped to minimize the impact of initial conditions the following procedure has been implemented. I simulate the panel dataset for 3000 firms with the benchmark set of parameters. Then, I choose 24 randomly selected initial leverage ratios for each firm within the feasible range. Using the same systematic shock realization, 24 economies are simulated. The economy is defined as converged to its steady state if the difference between maximum and minimum values of average leverage across simulated economies is less than 2% for at least 10 quarters. I repeat the same analysis for 250 different systematic shock realizations. The resulting
distribution of steady state times has the 95% percentile value of 122 quarters. For a conservative estimate I add another 30 quarters. Observe that while the impact of initial firms' condition disappears, the presence of systematic shocks implies that simulated economies differ at the truncation date.

References


### Table I
**Comparative Statics of Financial Variables at the Refinancing Point**

The table gives the comparative statics at the refinancing point of the following variables: the optimal market leverage ratio ($ML$), bankruptcy boundary ($\delta_B$), restructuring boundaries ($\delta_U$ and $\delta_UL$), total firm value ($F(\delta_0)$), the coupon rate ($c$), the liquidity barrier ($\delta_L$) and credit spread ($CS$). $\tau_c$ is the corporate tax rate, $\tau_d$ is the dividend tax rate, $\tau_i$ is the interest tax rate, $r$ is the pre-tax risk-free interest rate, $\sigma$ is the volatility of the firm’s cash flow, $\alpha$ is the fraction of asset value lost in bankruptcy, $q_{RC}$ is the adjustment cost, $q_A$ is the cost of selling assets in a liquidity crisis, $q_E$ is the cost of equity issuance and $k$ is the fraction of asset value that remains after an asset sale.

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<th>$r$</th>
<th>$\sigma$</th>
<th>$\alpha$</th>
<th>$q_{RC}$</th>
<th>$q_A$</th>
<th>$q_E$</th>
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<td>$CS$</td>
<td>Invariant to $\delta$</td>
<td>$&gt; 0$</td>
<td>$&lt; 0$</td>
<td>$&lt; 0$</td>
<td>$&gt; 0$</td>
<td>$&lt; 0$</td>
<td>$&lt; 0$</td>
<td>$&lt; 0$</td>
<td>$&lt; 0$</td>
<td>$&gt; 0$</td>
</tr>
</tbody>
</table>
Table II
Parameter Values for Simulations

Listed are the values and sampling distributions chosen for all parameters required to simulate the benchmark case of the model. $RP_A$ is the asset risk premium. All other parameters are defined in the text.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Mean</th>
<th>Std.dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_0$</td>
<td>constant</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>$A_0$</td>
<td>constant</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>empirical</td>
<td>0.993</td>
<td>0.47</td>
</tr>
<tr>
<td>$\sigma_E$</td>
<td>constant</td>
<td>0.155</td>
<td></td>
</tr>
<tr>
<td>$\sigma_D$</td>
<td>constant</td>
<td>0.081</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{ED}$</td>
<td>constant</td>
<td>0.023</td>
<td></td>
</tr>
<tr>
<td>$L_{av}$</td>
<td>constant</td>
<td>0.314</td>
<td></td>
</tr>
<tr>
<td>$\sigma_I$</td>
<td>$a_0 + a_1 \chi^2(n)$</td>
<td>0.22</td>
<td>0.107</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>empirical</td>
<td>0.255</td>
<td>0.10</td>
</tr>
<tr>
<td>$q_{RC}$</td>
<td>$U[0.0005, 0.0025] + 0.001s$</td>
<td>0.002</td>
<td>0.0006</td>
</tr>
<tr>
<td>$q_E$</td>
<td>$U[0.02, 0.06] + 0.02s$</td>
<td>0.013</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$U[0.03, 0.077] + 0.023s$</td>
<td>0.06</td>
<td>0.026</td>
</tr>
<tr>
<td>$q_A$</td>
<td>$U[0.05, 0.183] + 0.067s$</td>
<td>0.15</td>
<td>0.043</td>
</tr>
<tr>
<td>$s$</td>
<td>$U[0, 1]$</td>
<td>0.5</td>
<td>0.29</td>
</tr>
<tr>
<td>$k$</td>
<td>$U[0.6, 1]$</td>
<td>0.8</td>
<td>0.116</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>$U[0.7, 0.9]$</td>
<td>0.8</td>
<td>0.058</td>
</tr>
<tr>
<td>$g$</td>
<td>constant</td>
<td>$\mu + RP_A$</td>
<td></td>
</tr>
<tr>
<td>$a$</td>
<td>$U[0.03, 0.04]$</td>
<td>0.035</td>
<td>0.003</td>
</tr>
<tr>
<td>$RP_A$</td>
<td>constant</td>
<td>0.065</td>
<td></td>
</tr>
<tr>
<td>$\tau_\kappa$</td>
<td>constant</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>constant</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>$\tau_i$</td>
<td>constant</td>
<td>0.351</td>
<td></td>
</tr>
<tr>
<td>$\tau_d$</td>
<td>constant</td>
<td>0.122</td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>constant</td>
<td>0.05</td>
<td></td>
</tr>
</tbody>
</table>
Table III
Descriptive Statistics

The table reports descriptive statistics for the following variables: market leverage (ML), quasi-market leverage (QML), interest coverage ratio (the ratio of net payout, δ, to coupon, c), tax advantage to debt (i.e., the increase in firm value if the firm moves from no-leverage to its optimal leverage ratio and is given by the formula \( F(h_i) - (1 - \tau)Y(h_i) \)), and credit spread (CS). Ref. point refers to the case when all firms are at their refinancing points. All other statistics are given for dynamics. 1000 datasets are generated, each containing 75 years of quarterly data for 3000 firms. For each dataset the statistics are first calculated for each year in the last 35 years of data and then are averaged across years. Finally, they are averaged over datasets. Min and Max give the minimum and maximum over the 1000 datasets of the annual averages.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean 1% 50% 90% 95% 99% st. dev.</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Market leverage, ML</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ref. point</td>
<td>0.26 0.04 0.27 0.40 0.43 0.50 0.10</td>
<td>3000</td>
</tr>
<tr>
<td>Average</td>
<td>0.36 0.06 0.34 0.56 0.66 0.87 0.16</td>
<td>3000</td>
</tr>
<tr>
<td>Min</td>
<td>0.30 0.06 0.29 0.46 0.53 0.76 0.13</td>
<td>3000</td>
</tr>
<tr>
<td>Max</td>
<td>0.43 0.07 0.41 0.71 0.80 0.94 0.20</td>
<td>3000</td>
</tr>
<tr>
<td><strong>Quasi-market leverage, QML</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.37 0.06 0.35 0.59 0.70 0.91 0.17</td>
<td>3000</td>
</tr>
<tr>
<td>Min</td>
<td>0.31 0.06 0.29 0.47 0.56 0.82 0.14</td>
<td>3000</td>
</tr>
<tr>
<td>Max</td>
<td>0.44 0.07 0.42 0.74 0.84 0.96 0.21</td>
<td>3000</td>
</tr>
<tr>
<td><strong>Interest coverage ratio</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ref. point</td>
<td>3.98 2.01 3.22 5.74 7.80 17.83 3.24</td>
<td>3000</td>
</tr>
<tr>
<td>Average</td>
<td>3.08 0.69 2.64 4.78 6.08 11.26 2.35</td>
<td>3000</td>
</tr>
<tr>
<td><strong>Tax advantage to debt</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ref. point</td>
<td>0.05 0.02 0.05 0.07 0.07 0.08 0.01</td>
<td>3000</td>
</tr>
<tr>
<td>Average</td>
<td>0.04 0 0.04 0.07 0.08 0.09 0.02</td>
<td>3000</td>
</tr>
<tr>
<td><strong>Credit spreads, CS</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ref. point</td>
<td>0.95 0.24 0.80 1.69 2.09 3.27 0.61</td>
<td>3000</td>
</tr>
<tr>
<td>Average</td>
<td>1.49 0.24 0.97 3.01 4.40 8.83 1.72</td>
<td>3000</td>
</tr>
<tr>
<td>Min</td>
<td>1.25 0.23 0.85 2.30 3.40 7.51 1.46</td>
<td>3000</td>
</tr>
<tr>
<td>Max</td>
<td>1.84 0.25 1.21 3.89 5.41 9.82 1.97</td>
<td>3000</td>
</tr>
</tbody>
</table>
Table IV
Frequency of Events

The table reports the frequency of various events in the generated datasets. Restructure\textsubscript{U} refers to restructuring at the upper boundary when no liquidity crisis has occurred in the current refinancing cycle and Restructure\textsubscript{UL} refers to the case where an asset sale has occurred. 1000 data sets are generated, each containing 75 years of quarterly data for 3000 firms. For each dataset frequencies are computed across the last 35 years of data and then averaged over datasets. Min, 25\%, 75\% and Max give, correspondingly, the minimum, 25\% percentile, 75\% percentile and maximum annual averages over all datasets. All frequencies are annualized and given in percentages.

<table>
<thead>
<tr>
<th></th>
<th>Default</th>
<th>Restructure\textsubscript{U}</th>
<th>Restructure\textsubscript{UL}</th>
<th>Asset Sale</th>
<th>Equity Issuance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.49</td>
<td>12.17</td>
<td>0.16</td>
<td>1.20</td>
<td>4.78</td>
</tr>
<tr>
<td>Median</td>
<td>0.46</td>
<td>11.63</td>
<td>0.14</td>
<td>1.16</td>
<td>4.52</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.24</td>
<td>3.46</td>
<td>0.09</td>
<td>0.39</td>
<td>1.94</td>
</tr>
<tr>
<td>Min</td>
<td>0.16</td>
<td>5.38</td>
<td>0.06</td>
<td>0.54</td>
<td>1.71</td>
</tr>
<tr>
<td>25% percentile</td>
<td>0.31</td>
<td>9.90</td>
<td>0.10</td>
<td>0.90</td>
<td>3.36</td>
</tr>
<tr>
<td>75% percentile</td>
<td>0.64</td>
<td>13.94</td>
<td>0.19</td>
<td>1.49</td>
<td>5.81</td>
</tr>
<tr>
<td>Max</td>
<td>1.37</td>
<td>23.78</td>
<td>0.50</td>
<td>2.33</td>
<td>11.54</td>
</tr>
</tbody>
</table>
Table V
Correlation Structure

The table reports correlations for the following model variables: the market leverage ratio ($ML$), profitability ($\pi$), credit spreads ($CS$), volatility of cash flows ($\sigma$), bankruptcy costs ($\alpha$), asset sale costs ($q_A$), adjustment costs ($q_{RC}$), equity issuance costs ($q_E$) and the fraction of assets that remain after an asset sale ($k$). Panel A gives correlations for the case when all firms are at their refinancing points and Panel B for the generated datasets (dynamics). 1000 data sets are generated, each containing 75 years of quarterly data for 3000 firms. For each dataset correlations are computed for each year across the last 35 years of data and then averaged over datasets.

<table>
<thead>
<tr>
<th></th>
<th>$ML$</th>
<th>$\pi$</th>
<th>$CS$</th>
<th>$\sigma$</th>
<th>$\alpha$</th>
<th>$q_A$</th>
<th>$k$</th>
<th>$q_{RC}$</th>
<th>$q_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Correlations at refinancing point</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$ML$</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.76</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$CS$</td>
<td>-0.62</td>
<td>-0.31</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>-0.91</td>
<td>-0.65</td>
<td>0.83</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-0.08</td>
<td>-0.08</td>
<td>-0.05</td>
<td>-0.00</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q_A$</td>
<td>-0.11</td>
<td>-0.12</td>
<td>-0.08</td>
<td>0.01</td>
<td>0.21</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k$</td>
<td>0.19</td>
<td>0.20</td>
<td>0.39</td>
<td>-0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q_{RC}$</td>
<td>-0.00</td>
<td>0.01</td>
<td>-0.03</td>
<td>0.00</td>
<td>0.21</td>
<td>0.20</td>
<td>0.00</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>$q_E$</td>
<td>-0.03</td>
<td>-0.02</td>
<td>-0.04</td>
<td>-0.01</td>
<td>0.19</td>
<td>0.19</td>
<td>-0.01</td>
<td>0.20</td>
<td>1.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$ML$</th>
<th>$\pi$</th>
<th>$CS$</th>
<th>$\sigma$</th>
<th>$\alpha$</th>
<th>$q_A$</th>
<th>$k$</th>
<th>$q_{RC}$</th>
<th>$q_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel B: Correlations in dynamic economies</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$ML$</td>
<td>1.00</td>
<td>-0.08</td>
<td>0.42</td>
<td>-0.28</td>
<td>-0.06</td>
<td>-0.09</td>
<td>0.16</td>
<td>-0.04</td>
<td>-0.04</td>
</tr>
<tr>
<td>$\pi$</td>
<td>1.00</td>
<td>-0.06</td>
<td>-0.03</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$CS$</td>
<td>1.00</td>
<td>0.64</td>
<td>-0.02</td>
<td>-0.03</td>
<td>0.16</td>
<td>-0.01</td>
<td>-0.02</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Table VI
Cross-sectional Regressions

The table reports the results of cross-sectional regressions on the level of the quasi-market leverage ratio, \( QML \). Independent variables are profitability (\( \pi \)), volatility of cash flows (\( \sigma \)), bankruptcy costs (\( \alpha \)), asset sale costs (\( q_{A} \)) and restructuring costs (\( q_{RC} \)). The Ref.Point column gives the results obtained by running the regression at the refinancing point. The BJK, RZ, FF columns report the results of regressions that replicate the empirical procedures used, respectively, by Bradley, Jarrel, and Kim (1985), Rajan and Zingales (1995) and Fama and French (2002). Coefficients and \( t \)-statistics are means over 1000 simulations. The last three columns report additional information on the FF regression: the standard deviation of coefficients and \( t \)-statistics, and the 10\% and 90\% percentile values of these coefficients across simulations.

<table>
<thead>
<tr>
<th></th>
<th>Ref.Point</th>
<th>BJK</th>
<th>RZ</th>
<th>FF</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>coeff</td>
<td>std</td>
<td>10%</td>
<td>90%</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.23</td>
<td>0.58</td>
<td>0.57</td>
<td>0.58</td>
<td>0.05</td>
<td>0.51</td>
<td>0.68</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(6.37)</td>
<td>(9.18)</td>
<td>(9.03)</td>
<td>(51.81)</td>
<td>(15.51)</td>
<td>(31.92)</td>
<td>(83.74)</td>
<td></td>
</tr>
<tr>
<td>( \pi )</td>
<td>6.22</td>
<td>-0.31</td>
<td>-0.17</td>
<td>-0.34</td>
<td>0.28</td>
<td>-0.92</td>
<td>-0.04</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(9.87)</td>
<td>(-1.69)</td>
<td>(-1.24)</td>
<td>(-10.36)</td>
<td>(3.81)</td>
<td>(-17.95)</td>
<td>(-5.07)</td>
<td></td>
</tr>
<tr>
<td>( \sigma )</td>
<td>-0.78</td>
<td>-0.39</td>
<td>-0.38</td>
<td>-0.39</td>
<td>0.06</td>
<td>-0.49</td>
<td>-0.29</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-28.82)</td>
<td>(-4.27)</td>
<td>(-4.22)</td>
<td>(-21.77)</td>
<td>(6.47)</td>
<td>(-34.98)</td>
<td>(-12.55)</td>
<td></td>
</tr>
<tr>
<td>( \alpha )</td>
<td>-0.44</td>
<td>-0.51</td>
<td>-0.51</td>
<td>-0.58</td>
<td>0.34</td>
<td>-1.14</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-3.02)</td>
<td>(-0.77)</td>
<td>(-0.77)</td>
<td>(-6.72)</td>
<td>(4.26)</td>
<td>(-14.85)</td>
<td>(0.18)</td>
<td></td>
</tr>
<tr>
<td>( q_{RC} )</td>
<td>3.22</td>
<td>-6.90</td>
<td>-6.88</td>
<td>-4.30</td>
<td>7.41</td>
<td>-16.65</td>
<td>6.88</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.98)</td>
<td>(-0.43)</td>
<td>(-0.43)</td>
<td>(-2.10)</td>
<td>(3.80)</td>
<td>(-8.15)</td>
<td>(3.40)</td>
<td></td>
</tr>
<tr>
<td>( q_{A} )</td>
<td>-0.11</td>
<td>-0.22</td>
<td>-0.22</td>
<td>-0.22</td>
<td>0.12</td>
<td>-0.40</td>
<td>-0.02</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.21)</td>
<td>(-0.96)</td>
<td>(-0.95)</td>
<td>(-7.13)</td>
<td>(4.38)</td>
<td>(-15.29)</td>
<td>(-0.67)</td>
<td></td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.89</td>
<td>0.09</td>
<td>0.08</td>
<td>0.09</td>
<td>0.02</td>
<td>0.06</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(1)</td>
<td>(1)</td>
<td>(35)</td>
<td>(35)</td>
<td>(35)</td>
<td>(35)</td>
<td></td>
</tr>
<tr>
<td>( N )</td>
<td>3000</td>
<td>3000</td>
<td>3000</td>
<td>3000</td>
<td>3000</td>
<td>3000</td>
<td>3000</td>
<td></td>
</tr>
</tbody>
</table>
Table VII  
Leverage and Stock Returns

The table reports the results of cross-sectional regressions on the level of the quasi-market leverage ratio, \( QML \). Independent variables are the implied debt ratio \( IDR_{t-k,t} \) and lagged quasi-market-leverage ratio \( QML_{t-k} \). Coefficients and \( t \)-statistics in Panel A are means over 1000 simulations. Row 1 of Panel B reports Welch’s (2004) estimates of the IDR coefficients. Other rows report the mean and the 5% and 95% percentiles of my estimates.

<table>
<thead>
<tr>
<th></th>
<th>Panel A</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( k ) years</td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Constant</td>
<td>0.032</td>
<td>0.086</td>
<td>0.128</td>
<td>0.198</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(15.375)</td>
<td>(26.964)</td>
<td>(34.582)</td>
<td>(42.963)</td>
<td></td>
</tr>
<tr>
<td>IDR_{t-k,t}</td>
<td>1.028</td>
<td>0.892</td>
<td>0.785</td>
<td>0.587</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(83.571)</td>
<td>(90.876)</td>
<td>(75.985)</td>
<td>(50.489)</td>
<td></td>
</tr>
<tr>
<td>QML_{t-k}</td>
<td>-0.106</td>
<td>-0.096</td>
<td>-0.087</td>
<td>-0.055</td>
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</tr>
<tr>
<td></td>
<td>(-5.773)</td>
<td>(-6.419)</td>
<td>(-6.194)</td>
<td>(-4.452)</td>
<td></td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.929</td>
<td>0.804</td>
<td>0.697</td>
<td>0.493</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(37)</td>
<td>(35)</td>
<td>(33)</td>
<td>(28)</td>
<td></td>
</tr>
<tr>
<td>( N )</td>
<td>3000</td>
<td>3000</td>
<td>3000</td>
<td>3000</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Panel B: IDR coefficients</th>
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<td></td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>10</td>
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<tr>
<td>Welch</td>
<td>1.014</td>
<td>0.944</td>
<td>0.869</td>
<td>0.708</td>
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<tr>
<td>This paper</td>
<td>1.028</td>
<td>0.892</td>
<td>0.785</td>
<td>0.587</td>
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</tr>
<tr>
<td>5%</td>
<td>0.982</td>
<td>0.841</td>
<td>0.733</td>
<td>0.536</td>
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<tr>
<td>95%</td>
<td>1.077</td>
<td>0.937</td>
<td>0.829</td>
<td>0.636</td>
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Table VIII
Cross-sectional Regressions for Leverage Changes

The table reports the results of cross-sectional regressions on changes in the quasi-market leverage ratio, $QML_t - QML_{t-1}$. Independent variables are the target quasi-market leverage ratio ($Target\ QML_{t-1}$), past leverage ($QML_{t-1}$), implied debt ratio adjustment ($IDR_{t-1,t} - QML_{t-1}$), profitability ($\pi_t$), change in profitability ($\Delta \pi_{t-1} = \pi_{t-1} - \pi_{t-2}$) and the cross-term ($\pi_{t-1} \times (IDR_{t-1,t} - QML_{t-1})$). Coefficients and $t$-statistics are means over 1000 simulations.

<table>
<thead>
<tr>
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<th>(1)</th>
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<th>(4)</th>
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<tbody>
<tr>
<td>Constant</td>
<td>0.00</td>
<td>0.01</td>
<td>0.03</td>
<td>0.12</td>
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<tr>
<td></td>
<td>(0.54)</td>
<td>(0.58)</td>
<td>(12.90)</td>
<td>(28.40)</td>
</tr>
<tr>
<td>Target $QML_{t-1}$</td>
<td>0.16</td>
<td>0.16</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>(5.49)</td>
<td>(4.98)</td>
<td></td>
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</tr>
<tr>
<td>$QML_{t-1}$</td>
<td>-0.17</td>
<td>-0.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-12.56)</td>
<td>(-12.47)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$IDR_{t-1,t} - QML_{t-1}$</td>
<td>1.02</td>
<td>0.76</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(85.33)</td>
<td>(69.77)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi_{t-1}$</td>
<td>0.02</td>
<td>0.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.38)</td>
<td>(2.23)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \pi_{t-1}$</td>
<td>-1.20</td>
<td>-1.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-7.68)</td>
<td>(-7.68)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi_{t-1} \times (IDR_{t-1,t} - QML_{t-1})$</td>
<td>0.08</td>
<td>0.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(10.78)</td>
<td>(24.25)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.10</td>
<td>0.13</td>
<td>0.79</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>(36)</td>
<td>(36)</td>
<td>(36)</td>
<td>(31)</td>
</tr>
<tr>
<td>$N$</td>
<td>3000</td>
<td>3000</td>
<td>3000</td>
<td>3000</td>
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</table>
The table summarizes the robustness tests. Column (1) reports the difference between average leverage in dynamics and at Ref. Point (See Table III). Column (2) reports the average value of the profitability coefficient in Fama-French regressions (Table VI). Column 3 reports the average value of IDR$_{t-1,t}$ (Table VII). Column 4 reports the average value of the mean reversion coefficient $f_2$ (Table VIII). The tests are as follows: 1: the distribution of $\sigma$ is trimmed at the 5% and 95% percentiles; 2: volatility is estimated from 5-year time-series of cash flow and the values outside the 5% and 95% percentiles are excluded; 3: observations with the sum of the market values of equity and debt higher than 10 are excluded; 4: observations where profitability, $\pi_{t-1}$, is outside the 5-95% range are excluded; 5: firms that experience default in the previous five years are excluded; 6: substituting market leverage, $ML$, for quasi-market leverage, $QML$; 7: the distribution of $\sigma$ is changed to have a mean of 0.15 and a st.dev. of 0.07 by changing the distribution of $\sigma_I$; 8: the value of $\sigma$ is fixed at 0.25; 9: the value of equity issuance costs, $q_E$, is fixed at 0.3; 10: the value of restructuring costs, $q_{RC}$ is fixed at 0.005; 11: the benchmark economy is as in Goldstein, Ju, and Leland (2001) (all firms are identical at Ref.Points); 12: the corporate tax rate, $\tau_c$, is 0.3; 13: the before-tax interest rate, $r$, is 0.08; 14: the standard deviation of systematic shocks, $\sigma_S$, is 0.15; 15: the standard deviation of systematic shocks, $\sigma_S$, is 0. In each test, other parameter values and empirical procedures are unchanged.

<table>
<thead>
<tr>
<th>Test</th>
<th>Description</th>
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<tr>
<td>0</td>
<td>Benchmark</td>
<td>0.10</td>
<td>-0.34</td>
<td>1.03</td>
<td>-0.17</td>
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<tr>
<td>1</td>
<td>$\sigma$, no outliers</td>
<td>0.09</td>
<td>-0.30</td>
<td>1.04</td>
<td>-0.16</td>
</tr>
<tr>
<td>2</td>
<td>est. $\sigma$, no outliers</td>
<td>0.09</td>
<td>-0.32</td>
<td>1.03</td>
<td>-0.17</td>
</tr>
<tr>
<td>3</td>
<td>$E^D + D^{RT} &gt; 10$</td>
<td>0.10</td>
<td>-0.34</td>
<td>1.03</td>
<td>-0.17</td>
</tr>
<tr>
<td>4</td>
<td>$\pi_{t-1}$, no outliers</td>
<td>0.07</td>
<td>-0.40</td>
<td>1.04</td>
<td>-0.14</td>
</tr>
<tr>
<td>5</td>
<td>no default</td>
<td>0.08</td>
<td>-0.30</td>
<td>1.03</td>
<td>-0.16</td>
</tr>
<tr>
<td>6</td>
<td>$ML$</td>
<td>0.12</td>
<td>-0.35</td>
<td>1.02</td>
<td>-0.16</td>
</tr>
<tr>
<td>7</td>
<td>$\sigma$: new distr</td>
<td>0.04</td>
<td>-0.20</td>
<td>1.02</td>
<td>-0.25</td>
</tr>
<tr>
<td>8</td>
<td>$\sigma = 0.25$</td>
<td>0.14</td>
<td>-0.39</td>
<td>1.04</td>
<td>-0.21</td>
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<tr>
<td>9</td>
<td>$q_E = 0.03$</td>
<td>0.05</td>
<td>-0.31</td>
<td>1.04</td>
<td>-0.17</td>
</tr>
<tr>
<td>10</td>
<td>$q_{RC} = 0.005$</td>
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<td>-0.22</td>
<td>1.02</td>
<td>-0.22</td>
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<tr>
<td>11</td>
<td>GJL</td>
<td>0.07</td>
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<td>1.05</td>
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</tr>
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<td>12</td>
<td>$\tau_c = 0.3$</td>
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<td>-0.21</td>
<td>1.01</td>
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<tr>
<td>13</td>
<td>$r = 0.08$</td>
<td>0.04</td>
<td>-0.2</td>
<td>1.05</td>
<td>-0.18</td>
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<tr>
<td>14</td>
<td>$\sigma_m = 0.15$</td>
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<td>-0.45</td>
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</tr>
<tr>
<td>15</td>
<td>$\sigma_m = 0$</td>
<td>0.11</td>
<td>-0.08</td>
<td>1.01</td>
<td>-0.14</td>
</tr>
</tbody>
</table>