Financial Lumpiness and Investment

Santiago Bazdresch*
Yale University
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Abstract

This paper shows that non-convex costs of financial adjustment are important for rationalizing observed investment and financing choices of firms. Using the COMPUSTAT database it first shows firm’s financing activity, like investment, can be characterized as lumpy. Financial adjustments are infrequent and a small fraction of observations account for most of the external financing activity. The paper then presents a model which shows that, in the presence of taxes and bankruptcy costs, non-convex costs of investment and financing rationalize observed firm behavior, while non-convexities on investment alone do not. It then shows other predictions of the model are verified empirically: that when issuing debt or equity firms often increase their cash holdings and that when issuing debt or equity but not both, firms ‘overadjust’, moving beyond their financial targets. The paper also shows financial non-convexities help rationalize the empirically documented convex relationship between investment and mandated investment — the investment that would take a firm to its desired capital stock in the absence of adjustment costs. Finally the paper uses the model to relate firm heterogeneity to the internal financing sensitivity of investment in the standard investment regression. In the model, a firm’s higher expected growth rate is associated with a higher coefficient on internal finance, consistent with the empirical relationship between growth and internal finance sensitivities. However, unexpected differences in the rates of growth, the result of different observations of the same stochastic growth process, do not result in internal finance sensitivities consistent with the empirical evidence suggesting a big role for expected growth rates for determining the optimal sources of investment financing.

Keywords: Corporate Finance, Investment, Non-Convex Costs of Adjustment, Trade-Off Model, Financial Frictions. JEL: E2;G3

*email: santiago.bazdresch@yale.edu; Phone: 202 - 378 - 7228; Economics Department, 28 Hillhouse Av. New Haven, CT, 06520. I am grateful to William Brainard, Eduardo Engel, George Hall, Tony Smith and Björn Brüggeman as well as seminar participants at Yale and the Federal Reserve Board for helpful comments, suggestions and advice.
1 Introduction

Recent research has shown firm investment is lumpy, characterized by large infrequent movements, rather than continuous adjustment. This fact, rationalized by the assumption that costs of adjusting capital are non-convex has proven very important in understanding individual and aggregate investment behavior. This paper looks for evidence that non-convex costs are also important for the financial behavior of firms. Using COMPUSTAT data it documents the fact that financial behavior is indeed 'lumpy', with 5% of debt issuing periods accounting for 40% of the debt issuing activity for example. Where does this lumpiness come from? Is it a result of financial non-convexities? Or is this observation the direct result of investment lumpiness?

To understand how investment and financing interact in the presence of non-convex costs of adjustment, this paper analyzes a model of optimal firm behavior, where the firm faces fixed costs of investment, of equity issuance and of debt issuance. The model shows that investment lumpiness is not sufficient to cause financial lumpiness. In the model the tradeoff between the risks of bankruptcy created by debt and the tax benefits of debt financing imply the firm would typically like to keep a capital structure aligned with its current and future profitability. As productivity and profitability evolve, the optimal composition of liabilities changes and a costless adjustment firm would indeed adjust frequently even if investment were infrequent. Therefore some friction limiting financial adjustment is necessary to rationalize the observed behavior. The model shows that fixed debt issuing and equity issuing costs are one way of achieving this result.

The paper then shows that other predictions of the model with non-convexities in investment and financing are also borne out by firm data in the COMPUSTAT database. First, the fixed costs in the model imply that firms generally will not finance incrementally from all their sources of financing, debt, equity and retained earnings, but will rather use them alternatively, either debt, equity or retained earnings, and, in fact, will often accumulate cash when issuing debt or equity. This is a markedly different implication from that of pecking order models (Myers, 1984) which suggest that the firm exhausts each type of financing progressively, not issuing debt or equity until it has no more internal financing. The paper verifies the prediction of the model empirically by comparing the average change in cash holdings of firms on debt issuing, equity issuing and non issuing periods. It documents that, on average, there is a sizable increase the cash holdings of firms when they access external markets.

Second, the model implies that firms will sometimes ‘overadjust’, their capital structures. An important question in the corporate finance literature is whether firms adjust the structure of their liabilities towards a target leverage, defined as the ratio of debt to total assets, reversing the effect of debt maturation or of changes in the value of the firm’s equity (see Welch (2004) for example). In this paper, non-convexities in the cost of adjustment functions induce the firm to take advantage of an equity or a debt issue and do a lot of it at once. Instead of issuing debt and equity to partially adjust capital
structure towards its target, firms end up overleveraged after issuing debt and underleveraged after issuing equity. To clarify empirically the extent to which adjustments are ‘over-shooting’ we estimate an equation for ‘desired’ leverage using the firm’s growth process, past and current values of sales, profits, taxes and assets as explanatory variables, and including firm fixed effects. Using the implied targets we find overshooting, in accord with the model’s predictions, in the range of 7% of the firm’s assets. This result suggests estimates of the speed of adjustment towards target capital structure derived from a partial adjustment setup, such as that in Shyam-Sunder and Myers (1999), might suffer from a mis-specification problem.

Third, the paper provides a partial rationalization for the convex relationship between investment and ‘mandated investment’ that has been documented empirically (see Caballero and Engel, 1999), but rationalized as the result of randomness in the magnitude of a fixed adjustment cost. In our model there are regions of the state space where the firm pays the fixed cost of investing, but not that of financing, and therefore invests only up to the level allowed by retained earnings. This, along with the firm’s pattern of cash accumulation, causes the relationship between average investment and mandated investment to have a convex region.

Finally, the paper discusses firm heterogeneity and the retained earnings sensitivity of investment in the context of this non-convex costs of adjustment model. In the data, low dividends to earnings which coincide with small size firms that grow quickly are associated with high investment–cash flow sensitivities, a relationship that has been interpreted to imply that small firms are financially constrained.

This paper shows that differences in the systematic rates of growth of firms subject to non-convex costs of adjustment, explain the different sensitivities found in the data. Estimating investment regressions using simulated data it shows that firms with high rates of expected growth appear more responsive to internal finance than firms with low expected growth. Our intuition for this result is that firms with high growth are better able to match investment and availability of internal finance.

The paper then shows that growth differences resulting from different observations of the same underlying growth process do not explain the different regression coefficients found empirically. It simulates the behavior of a set of firms with identical expected growth rates, and then sorts them by their average growth rates over the simulation period to show that firms that grew faster had lower internal finance sensitivities. Our intuition for this result is that firms that expect to grow accumulate cash flow sufficient to pay for their investment, while firms that simply receive unexpectedly high productivity shocks finance them as they can, with debt or equity.

These results suggest the findings by Fazzari, Hubbard, Petersen (1988) may reflect the endogenous retention of earnings for growing firms and non-convexities in the cost of adjustment rather than limited access to external finance.

After a brief review of related literature, section 2 presents the evidence of
financial lumpiness. Section 3 discusses the model which we propose to explain this fact. Section 4 describes typical moments of the simulated data, discusses the choice of parameter values for the model and presents intuition about the firm’s optimal choices. Section 5 presents the first result of the paper, that both investment and financial non-convexities are necessary to explain financial lumpiness, and discusses other model predictions for firm financing and their empirical support. Section 6 discusses the implications of the model for investment. Section 7 concludes.

1.1 Related Literature

Most research relating investment and financing has focused on whether financial restrictions are important determinants of investment. In particular a set of papers develop models comparable to the one presented here. Cummins and Nyman (2004) present a model with fixed costs of issuing debt, and show these costs are a liquidity retention motive. Cooley and Quadrini (2001) use persistent shocks and financial frictions in a model with debt, linear costs of external finance and entry and exit to rationalize the size and age dependence of firm growth and earnings volatility. Gomes (2001) constructs a general equilibrium model of investment and financing to show Q is a sufficient variable for investment even with dramatic financial constraints. Also, in a model without financial restrictions, he interprets higher $R^2$ of regressions when adding cash flow as evidence that cash flow sensitivities are there in the absence of restrictions. Cooper and Ejarque (2001) estimate a model with a convex revenue function to show that, in that context, financial constraints are not necessary to obtain a strong relationship between investment and profits and conclude that market power might be the reason for high cash flow sensitivities of investment in the data. These papers have generally confirmed the Kaplan and Zingales (1997) argument that internal finance sensitivity of investment does not have to be monotonically related to firm’s financial restrictions. however, they have not been so successful at explaining the relationship between growth and internal finance sensitivity in the data.

Also, in the context of studying firm’s capital structure, Henessy and Whited (2005) endogenize the investment decision in a financing model with taxes and bankruptcy costs but where the firm uses its debt margin to save internal resources for future investment. They then estimate the model’s parameters using simulated method of moments and show that in their simulations profitable firms are less leveraged, which they interpret as rationalizing the capital structure ‘puzzle’ posed by Myers (1988). Similarly, Froot, Scharfstein and Stein (1993) study cash holdings as a means of hedging, and show that firms will optimally hedge by accumulating cash to take advantage of investment opportunities when external finance is more costly than internal finance.

This paper also connects with the classic public finance literature of Poterba and Summers (1984) and Poterba (1987) which discuss the implications of taxes, and show that a firm’s current cost of capital is function of it’s future investment decisions and the marginal source of funds for them.
Clearly, the investment literature is one of the main sources on which we build, specially regarding the recent non-convex costs of adjustment research of Abel and Eberly (1994), Caballero and Engel (1999) and Dixit and Pindyck (1994).

This paper extends the models above by allowing for non-convexities on both investment and financing decisions, studying a rich financial structure, with debt, equity and retained earnings, risk-adjusted required rates of returns on different financial instruments and by allowing bankruptcy costs and taxes to determine an optimal financial position for the firm.

2 Evidence of Financial Lumpiness

Although corporate finance research has studied the question of how quickly firms adjust to their financial targets, no research that we are aware of has documented the fact that financial behavior is lumpy. Welch’s (2004) evidence that firms do not adjust their capital structure after large changes in the market valuation of their equity is suggestive of important non-convex costs, but he does not model firm behavior explicitly. Leary and Roberts (2004) estimate a duration model in which they show the likelihood of a financial adjustment increases over time but their estimation procedure does not explicitly allow for non-convex costs of adjustment.

Figure 1 documents the fact that debt issuance of firms is lumpy. To construct this figure we obtain the ratio of debt issuance to assets for each firm-year pair, for firms with continuous quarterly observations between 1984 and 2003. Within each firm we then rank these ratios from highest to lowest, and construct the averages over all firms by rank. The n’th bar in this figure represents the average across firms of the n’th largest debt issuance to total assets ratio. The line in this figure represents the results of performing the same ranking and averaging exercise for normal independent observations. As can be observed, in the data, a small number of periods account for most of the financial action for each firm, much more so than a normal distribution would imply. In 80% of the firm-year observations, debt issuance was less than 3% of total assets, while 5% of the periods accounted for 40% of the total debt issuance observed. The corresponding figures for a normally distributed variable normalized to represent the same gross issuance are 68% of observations below 3% of assets and 5% of observations accounting for 21% of debt issuance. The standard measure of fat-tailedness, excess kurtosis, is 151 for firm’s debt issuance.

Figure 2 presents the same graphical construction for equity issuance, investment, sales and changes in cash holdings. These graphs show that investment and equity issuance are very lumpy, while sales and changes in cash holdings are less so. This motivates our model specification with fixed costs on investment, equity issues and debt issues.

Where does the lumpiness come from? Investment and financing are two sides of the same economic unit of production and their behavior is closely linked. A natural hypothesis is that investment is the relevant variable, so that
lumpy financing is only the result of lumpy investment. But a little thought into the matter will reveal the opposite conclusion is also plausible, that in the presence of investment non-convexities alone, financing adjust more frequently in response to more variable profitability, and would therefore not be lumpy. Maybe there are important non-convexities in financing and these make investment lumpy?

Motivated by these questions and by the success of the recent non-convex cost specifications in understanding investment behavior the next section presents a model with non-convexities on investment and on financing that we use as a
laboratory to try out these hypothesis.

3 Investment and Financing Model

3.1 Overview

This paper presents a model of firm investment with an elaborate financial setting that allows us to analyze the implications of non-convex costs of adjustment. The firm solves an infinite horizon problem, although its expected life is finite due to bankruptcy. It acts on behalf of its shareholders, choosing how much to invest in capital, how much equity to issue or dividends to pay, how much long term debt to issue, how much cash to accumulate and whether to declare bankruptcy or not. Taxes, the risk of bankruptcy, and different required rates of return across financial instruments are modeled explicitly. Tax rates on corporate profits, dividend income and interest income imply debt financing is more 'tax efficient' than equity financing. However, the risk of bankruptcy, which is assumed to have real costs, limits the proportion of financing that is optimally obtained from debt. These factors are included for added realism, but also because they imply that although funds are supplied elastically by investors, the firm is always in an interior point between all-debt and all-equity financing. Finally, cash accumulation allows the firm to retain earnings for investment or debt repayment. Cash however is assumed to pay a liquidity premium implying the firm does not keep cash unless it plans to make use of it soon.

We then assume the firm cannot choose these variables freely. It pays costs of adjustment on gross investment, on debt issues or repurchases, net of maturing debt, and on equity issuance. It pays a fixed cost of undertaking positive investment, and receives a fraction of the replacement cost of capital when disinvesting. The costs of issuing or repurchasing debt and issuing equity are assumed to be fixed, but proportional to the scale of the firm.

3.2 Model Firm

Objective Function The firm’s objective is to obtain the highest returns for current shareholders, maximizing the expected present value sum of after-tax dividends, discounted with the shareholders exogenous required rate of return $r^E$. The objective function of the firm is then:

$$\max E \left[ \sum_{t=0}^{\infty} \frac{D^C_t}{(1 + r^E)^t} \right]$$

where $D^C_t$ denotes the current shareholder's dividends in period $t$.

In addition to investment, the firm chooses debt ($B_t$), cash or retained earnings ($C_t$), equity issues ($X_t$) and dividend payments ($D_t$) subject to the cash flow constraint. The possibility of having contemporaneous debt and retained earnings will allow the model to have predictions about retained earnings distinct from that of a 'debt buffer' as in Gross (1995). The firm will go bankrupt
if that is optimal for shareholders, in which case dividends are 0 from the time of bankruptcy on.

In the model dividends and share repurchases are treated as equivalent, ignoring their potentially different tax implications for individual shareholders. All dividends are subject to a constant rate of taxation of $\tau_D$. I assume all new equity issues are purchased by new investors and therefore constitute a ‘dilution’ of original owners’ participation in the firm. In any period, the firm’s objective is equivalently stated as:

$$V_t = \max(1 - \tau_D)D_t + \frac{E[V_{t+1}]}{1 + r^E} \left( \frac{V_t}{V_t + X_t} \right)$$

Equation 1 expresses the value of the firm to existing shareholders as the expected, after-tax, present discounted value of dividends allowing for the fact that equity issues will dilute the participation in future dividends. Current stockholders only get $V_t/(V_t + X_t)$ of the future value of the firm since they are participating with only that proportion of the firm’s capital. For the firm to be able to engage in positive equity issuance a further condition must be met, namely, that investors want to purchase this equity:

$$X_t \leq \left( \frac{X_t}{V_t + X_t} \right) \frac{E[V_{t+1}]}{1 + r^E}$$

The right hand side of the inequality in 2 is the value of the equity to the new investors buying $X_t$ of stock. The condition implies the maximum a firm can obtain from issuing new equity issues is the actuarially fair value of the new shares.

**Budget Constraint and Timing** The firm’s budget constraint every period is:

$$D_t + I_t + C_{t+1} - BI_t - X_t + (1 - \phi) Id(I_t < 0)$$

$$+ K_t \left[ \Gamma_I Id(I_t > 0) + \Gamma_X Id(X_t > 0) + \Gamma_B Id(BI_t \neq 0) \right]$$

$$= (A_t K_t)^\alpha - (r_t + \lambda) B_{t-1} - \tau_C [(A_t K_t)^\alpha - r_t B_{t-1} - \delta K_t] + (1 + r^E) C_t$$

where $Id(\cdot)$ represents the identifier function, $r_t$ stands for the rate of interest on debt outstanding at period $t$ and $BI_t = B_t - \lambda B_{t-1}$ stands for bond issues net of maturing principal.

The first two rows of equation 3 contain the firm’s decision variables, cash deposits or retained earnings, debt issues, dividends, equity issues and gross investment as well as the adjustment costs associated with the firm’s decisions. The right hand side contains the predetermined variables in period $t$, the firm’s revenue, taxes, cash holdings and interest and principal payments to bondholders.

Figure 3 details the time line of the model. Each period starts with an observation of the stochastic productivity level. The firm then decides to continue
or to go bankrupt depending on the expected return to shareholders given that productivity level. If continuation is the optimal action, the firm engages in production, meets its debt payments, pays the corresponding taxes and makes investment and financing decisions that will determine the situation of the firm in the following period.

Output and Capital The source of firm income to pay dividends is the sale of output. In this model we assume, as is standard in the literature\(^4\) (i.e. Henessy and Whited, 2005), that the firm has a convex revenue function \( R(A_t, K_t) = (A_tK_t)^{\alpha} \). The firm’s capital evolves over time because of investment (or disinvestment) and depreciation. We assume the economic rate of depreciation and the tax-related appraisal of this depreciation are equal and represent them with \( \delta \). \(^5\)

Investment \( I_t \) and depreciation then drive the evolution of capital: \( K_{t+1} = K_t(1-\delta) + I_t \). The firm invests and disinvests in response to changing ‘productivity’ level \( A_t \). this productivity level is modeled as an exogenous stochastic process that follows a geometric random walk:

\[
A_t = A_{t-1} \cdot \xi_t \\
\xi_t \sim \text{lognormal}(\mu, \sigma)
\]

We define the ‘desired level of capital’ \( K_t^* \), as a function of productivity, the cost of capital and the rate of capital depreciation, to be:

\[
K_t^* = \arg \max (A_tK_t)^{\alpha} - (r + \delta)K_t \iff K_t^* = \left( \frac{\alpha A_t^{\alpha}}{r + \delta} \right)^{\frac{1}{1-\alpha}} \quad (5)
\]

This desired level of capital corresponds to the optimal level of capital in a neo-classical, no cost of adjustment, single rate of discount model where \( A_t \) is
the expected value of productivity.

**Cash Holdings** Cash holdings \( (C_t) \) correspond to a firm’s bank deposits and are assumed to earn the risk free interest rate \( r_f \). Cash is crucial in this model. It is a buffer stock that allows the firm to accumulate retained earnings for making large, infrequent investments and avoiding frequent issues of debt or equity. This is a distinctive feature of our model. Others have proposed summarizing debt and cash in one variable and treating negative debt as cash (i.e. Bayer (2004) and Henessy and Whited (2005)), however that choice doesn’t permit an analysis of non-convex costs of debt issuance, nor can it account for internal financial resources explicitly in investment regressions.

**Debt, bankruptcy and endogenous interest rates** The crucial intuition about the firm’s optimal financial mix is that it is an interior point between all debt, all equity, and all retained earnings financing. This interior point is determined by taxes, the required rates of return on debt and equity, and the bankruptcy costs associated with debt. In the absence of costs of adjustment, the firm would always find it optimal to adjust to changes in current and future expected profitability, the main determinants of tax benefits and bankruptcy costs.

The firm issues debt with a face value of \( B_t \) and with a variable one period interest rate \( r_t \) in period \( t \). This rate of interest reflects the required rate of return of bondholders \( r^B \) the taxation of interest payments at a rate of \( \tau_P \) as well as the possibility that the firm will go bankrupt, defaulting on these obligations. We set \( r_f < r^B < r^E \) to reflect a liquidity premium on the cash holdings of the firm and a risk premium on the valuation of equity returns. Allowing for both debt and equity financing and then adjusting the valuation of debt and equity returns for their riskiness is also a non-standard feature of this model.

Equation 6 determines the interest rate paid by the firm on its outstanding debt. It reflects the fact that we assume bondholders receive the scrap value of the firm’s capital, \( \phi K_t \) in case of bankruptcy \( (\text{Solvent}_t = 0) \) and their promised payment if the firm is solvent \( (\text{Solvent}_t = 1) \).

\[
 r_t = \frac{1}{1 - \tau_P} \left[ \frac{(1 + r^B) B_t - E[\phi K_t * (1 - \text{Solvent}_t)]}{B_t E[\text{Solvent}_t]} \right]
\]  

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\]  

In the model debt is long term debt, but rather than keep track of debt issued at different times, we assume it has an exogenous rate of ‘evaporation’ or maturation of \( \lambda \). This rate is the analogous of a depreciation rate for capital. The firm’s indebtedness then declines autonomously until the firm pursues debt issuance anew.

Although in the real world bankruptcy is a complicated legal process, in this model we use a simple specification for this event. The firm goes bankrupt when its value would be negative in the absence of default. This corresponds to a situation where there is nothing the firm can do — selling capital, issuing debt or
issuing equity, that results in a positive share value. Shareholders receive nothing from the point of default on, \( D_t = 0, t > T_{BK} \), and bondholders receive \( B^P_{T_{BK}} = \psi K_{T_{BK}} \) in return, with \( 0 < \psi < 1 \), which represents the liquidation value of assets when the firm is closed down.

Besides different required rates of return and costs of bankruptcy, the firm faces a set of taxes that generally render debt a more 'tax efficient' choice of financing. The value of the firm’s revenue is affected by three taxes: as discussed above, shareholders pay taxes on dividends at a rate \( \tau_D \) so that only \((1 - \tau_D)\) of every dollar going out of the firm as payment to equity holders reaches them; debt holders also pay taxes on interest income at a rate \( \tau_P \), so that a bond with an interest rate of \( r^B \) implies the after tax payment received by the bondholder (when the firm is solvent) is \( 1 + r^B (1 - \tau_P) \); finally, the firm faces taxes on profits net of depreciation and interest payments at a rate \( \tau_C \), so that it pays \( \tau_C [(A_t K_t)^\alpha - \delta K_t - r^B B_t] \) in taxes to the government.

This setup has important implications for firm behavior. First, it introduces a 'wedge' between inside and outside funds. A firm might take advantage of an investment opportunity using retained earnings that it would not pursue if it needed external financing for it. This also implies that the path of future investment needs of a firm affect its current ‘cost of capital’. Second, note that revenues to bondholders are taxed once, with taxes on interest payments, while returns to shareholders are taxed twice, with taxes on corporate profits, and then with taxes on dividends. Therefore, debt is more 'tax-efficient' and the firm will benefit from issuing debt by taking advantage of the lower tax on bond income.

Note how an extra dollar of equity for an investment paying \( 1 + r \) dollars after depreciation, assuming only the profits are returned to shareholders implies a net value for them of: \( \frac{r(1 - \tau_D)(1 - \tau_C) - r}{(1 + r)^\alpha} \). With a required rate of returns on bonds of \( r^B / (1 - \tau_P) \), the same operation financed with debt has a net value to equity holders of \( \frac{(r - r^B)(1 - \tau_P)}{(1 - \tau_C)} \) that is, a 0 investment today and a return net of interest payments tomorrow. The bond operation return is larger than the equity operation one as long as \( r^E (1 - \tau_P) > r^B (1 - \tau_D) (1 - \tau_C) \). This non-neutrality of taxes which favors debt financing over equity financing is a feature of tax schedules common to many countries around the world.

Despite the three different tax rates considered, this is a gross simplification of the tax code faced by firms. Among the important elements of the U.S. tax code for corporations which are not part of this model are the difference between economic depreciation rate and ‘depreciation allowances’, which let the firm postpone its tax payments by assessing a fast rate of depreciation on its capital; investment tax credits, which allow a firm to reduce or postpone its tax liabilities by a proportion of its investment; and loss carry back and carry forward provisions which allow the firm to ‘use’ losses from past or future periods to reduce today’s tax liabilities.

**Non-Convex Costs of Adjustment** Non convexities are one of the distinctive features of this model and are the focus of this paper. The firm is assumed
to face non-convex costs of adjustment on its investment and external financing. It incurs a fixed cost of $KT_1 I d(I > 0)$ when making a positive gross investment. It also faces a fixed cost $KT_X I d(x > 0)$ of issuing equity and a cost of issuing or repurchasing debt $KT_R I d(BI_t \neq 0)$.

Note that these three costs are fixed in the sense that they are independent of the size of the firm’s adjustment, and should be interpreted as the cost to production and to firm profitability of the adjustment process. This interpretation is intuitive for the cost of investment, it is easy to imagine a firm plant that has to shut down for a ‘retooling’, but they can be interpreted similarly as transaction costs and underwriting fees for equity and debt. We make the assumption of costs proportional to the scale of the firm for convenience in solving the model.

The last ‘friction’ in the model is a cost of disinvestment represented by a wedge between the original and the resale prices of capital. The firm can sell capital at a fraction $\phi < 1$ of its replacement cost. This assumption is grounded on a large body of empirical evidence documenting that firm capital is highly industry specific and that the scrap value and second hand values of capital are substantially below their depreciated value or replacement cost.

4 Calibration and Optimal Firm Behavior

4.1 Model Parameters

The standard parameters we use are comparable to those in the recent literature.

- $\alpha$: Estimates of the degree of market power in the literature vary widely, with Hall (1988) reporting evidence of prices below marginal cost for a range of industries, Burnside (1996) reporting returns to scale being just under 1 and Cooper and Ejarque (2001) estimating the convexity parameter to be 0.689. A value of $\alpha$ closer to 1 has three main implications, a lower ratio of profits to capital, a higher variance of this ratio and a higher elasticity of the firm’s desired capital level to changes in profitability. In the frictionless neoclassical setting, without taxes and with a fixed cost of capital $r$, the optimal capital level is $K^* = \left(\frac{\alpha E[A^\alpha]}{(r + \delta)}\right)^{1/(1 - \alpha)}$ which then results in a ratio of revenues to capital of $(r + \delta)/\alpha$ and an elasticity of desired capital to changes in $E[A^\alpha]$ of $1/(1 - \alpha)$. For the purpose of financial decisions of firms, a value of $\alpha$ closer to 1 implies the firm tends to use more external financing, first because it has less profits to use as internal financing, and second, because its desired capital level is more volatile and it will find itself unable to finance investment out of retained earnings more often. However, it also implies the firm has a lower average leverage because lower profitability leads the firm to declare bankruptcy at lower relative levels of indebtedness.

- $\delta$: Ramey and Shapiro (2001) estimate a rate of depreciation for capital between 10% and 7% for their sample of aerospace firms, while Nadiri
Table 1: Choice of Model Parameters

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<th>Standard</th>
<th>Description</th>
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<td>$\alpha$</td>
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<td>$\delta$</td>
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<td>$\Gamma_B$</td>
<td>Fixed cost of equity issuance</td>
<td>0.01</td>
</tr>
<tr>
<td>$1-\phi$</td>
<td>Cost of bankruptcy</td>
<td>0.30</td>
</tr>
<tr>
<td>$1-\psi$</td>
<td>Linear cost of disinvestment</td>
<td>0.15</td>
</tr>
</tbody>
</table>

and Prucha (1997) estimate rates of 6% for physical capital and 12% for research and development capital in the US manufacturing sector. In the model a higher rate of depreciation results in a higher investment frequency. Also, a higher value of $\delta$ implies a higher user cost of capital because of this depreciation, and because costs of adjustment are incurred more often. We set $\delta$ at 10%.

- $r^F$, $r^E$ and $r^B$: We assume the risk adjusted required rates of return on debt and equity are 4% and 5% respectively. As the summary of parameters used in the literature above attests, there is no consensus on the right required rate of return to use in these models. Estimates of the long term rate of return on equity vary from 7% in Mehra (2003) for the very long run average stock returns, to 4.5% in Baker et al. (2005) for the expected return for the US stock market in 2005. In this paper we do not explicitly model investors risk preferences but allow for an adjustment for the higher riskiness of equity by imposing different required rates of return on these two instruments. Finally, we set the risk free rate to be 2%.

- $\tau_d, \tau_p, \tau_e$: In line with the public finance literature, we set the tax rates to reflect a tax advantage to debt financing, with $(1-\tau_p) > (1-\tau_d)(1-\tau_e)$, implying external debt financing is less expensive than external equity financing for tax reasons. We set $\tau_d = 0.15$ and $\tau_p = 0.30$ to reflect statutory
tax rates for individuals on dividends and interest income and $\tau_c = 0.20$
which is below the firm’s income tax rate, to reflect the widespread use of

**corporate tax shelters. In our model the average debt to capital ratio is**

45% comparable to the leverage ratio in the data of 33%.

- $\sigma$ and $\mu$: We set $\mu$ to reflect the average growth rate of assets of COM-
PUSTAT firms from 1983 to 2002 under the assumption that $\alpha = 0.70$.
We set $\sigma$ to imply a standard deviation of sales to capital of 15%.

- $\lambda$: In the model we set $\lambda = 0.95$ which corresponds to an average duration
of 25 years. The fact that $\delta$ and $\lambda$ differ implies that unless the firm
adjusts capital or debt, the firm’s leverage drifts higher as time passes.
Table 2 shows the parameters used recently in the literature.

- $\Gamma_I$, $\Gamma_{BI}$ and $\Gamma_X$: We set $\Gamma_I > \Gamma_X > \Gamma_{BI}$. The choice of firm financ-
ing is secondary to the choice of capital level in terms of firm profits,
so that small non-convexities on equity and debt issuance, 1/3 of those
on investment, imply much longer inaction periods on financing than on
investment.

- $\phi$ and $\psi$: These values represent the resale price of capital when the firm
disinvests and the scrap value of capital when the firm is liquidated to
repay bondholders. We assume the discount is smaller and that there is a
real cost of bankruptcy and set $\phi = 85\%$ and $\psi = 70\%$.

<table>
<thead>
<tr>
<th>Authors</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$r^E$</th>
<th>$\phi$</th>
<th>$\sigma^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gomes (2001)</td>
<td>0.95</td>
<td>0.12</td>
<td>6.5%</td>
<td></td>
<td>0.15</td>
</tr>
<tr>
<td>Cooper and Ejarque (2001)</td>
<td>0.7</td>
<td>0.15</td>
<td>6%</td>
<td></td>
<td>0.85</td>
</tr>
<tr>
<td>Henessy and Whited (2004)</td>
<td>0.689</td>
<td>0.145</td>
<td>2.5%</td>
<td>0.75</td>
<td>0.15</td>
</tr>
<tr>
<td>Cooley and Quadrini (2001)</td>
<td>0.975</td>
<td>0.07</td>
<td>4%</td>
<td></td>
<td>0.28</td>
</tr>
<tr>
<td>Abel and Eberly (2002)</td>
<td>0.75</td>
<td>0.9</td>
<td>5%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* These values for $\sigma$ are not directly comparable to ours since our specification is a random
walk, while the values above pertain to specifications with bounded Markov chains.

### 4.2 Optimal Firm Behavior

The optimal behavior of the firm in this model is quite complex. An example of
the policy functions for leverage and capital, for given values of cash and of the
firm’s most recent revenue shock, that result from solving the firm’s problem is
shown in the appendix. To understand the behavior of the model we simulate
the model, and observe the choices of a firm that behaves according to these
optimal policies. We study first simulated moments of the data, where we have
attempted to replicate our empirical sample by simulating 3 thousand firms over a 20 year period. Then we analyze the time series of choices of a particular firm.

4.2.1 Simulated Moments

Table 3 presents simulated moments from the model.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt/Capital</td>
<td>49%</td>
</tr>
<tr>
<td>Profits/Capital</td>
<td>8.7%</td>
</tr>
<tr>
<td>Return on equity</td>
<td>4.1%</td>
</tr>
<tr>
<td>Tobin’s Q</td>
<td>2.03</td>
</tr>
<tr>
<td>Dividends/Earnings</td>
<td>82%</td>
</tr>
<tr>
<td>Debt Issue Frequency</td>
<td>8.0 years</td>
</tr>
<tr>
<td>Equity Issue Frequency</td>
<td>13.1 years</td>
</tr>
<tr>
<td>Investment Frequency</td>
<td>5 years</td>
</tr>
<tr>
<td>Debt</td>
<td>38%</td>
</tr>
<tr>
<td>Equity</td>
<td>1%</td>
</tr>
<tr>
<td>Retained Earnings</td>
<td>61%</td>
</tr>
</tbody>
</table>

4.2.2 Dynamic Behavior

I examine the typical behavior of the firm by turning the productivity shocks off and observing the firm’s choices when (unexpectedly) it receives no shocks to its profit function for a number of periods. Figure 4 shows capital, debt cash holdings and dividend payouts. As the first panel shows, the firm does not invest every period, but makes infrequent and large adjustments to its level of capital, and then lets it decline through depreciation. Debt also declines slowly through maturation of the principal.

The key intuition to obtain from this figure has to do with the firm’s earnings. The firm pays out its profits as dividends when investment is far off into the future, but keeps them as cash as investment becomes more likely.

Figure 5 presents a representative simulation of a firm in the full fledged model. The top panel describes the capital stock and the second one depicts the firm’s debt. Dark lines show the behavior of capital and debt, while the clear lines are the firm’s targets for these variables. These targets represent the firm’s optimal choices if we assumed these frictions were eliminated for one period, rather than the choices of the firm in a frictionless model.

When target capital is sufficiently high, the firm undertakes investment, generally up to that target level. In some situations however, the firm does invest but it does not reach it’s target capital. The second panel shows the debt choices of the firm along the same period and shows how the firm also adjusts debt infrequently and in large movements. The figure shows that most adjustment occasions take place contemporaneously, that is, the firm is generally making
an investment when it issues debt and vice versa. When the firm undertakes adjustment in both variables it attains its costless adjustment targets. However, there are periods in which the firm adjusts debt or capital but not both. In those situations the firm does not go all the way towards its one period costless adjustment target. The next two panels complete the picture by showing the contemporaneous movements in cash holdings, dividend payments. Two things stand out from these panels. First, the firm pays high dividends just after increasing its capital stock, and second the firm saves cash in order to invest or to reduce its outstanding debt.

5 Results

5.1 Financial Lumpiness

The model described above is able to rationalize the debt issuing pattern described in section 2. The two panels of figure 6 compare the issuing patterns predicted by the model with and without financial non-convex costs of adjustment. Each panel shows the result of simulating the behavior of 2000 firms for
16 years and then averaging observations of the ratio of debt issuance to assets by the rank of each observation in the firm’s history, as described in section 2. The first figure shows that debt issuance is lumpy in the model with investment and financial non-convexities but it is not so in the model without financial fixed costs of adjustment. The kurtosis of the simulated sample corresponding to panel a is 83, while that corresponding to panel b is 5.

This comparison shows that non-convexities are important for understanding financial firm behavior. Investment not convexities are not the source of the lumpiness found in the data. In the model, productivity shocks change the firm’s profitability, and in the absence of non-convex costs, the firm will adjust its financial mix every period, even if it does not undertake investment or disinvestment actively.

What is going on? Our intuition for this result is that the firm’s capital stock adjusts infrequently, just letting capital depreciate most of the time. However, without financial non-convexities the firm adjusts its capital structure following the firms profitability. Although investment periods are associated with large financial movements, the firm never moves away from its desired capital structure. Therefore, financial adjustments are frequent, and the large financial
adjustments associated with investment are not large enough to make financial behavior lumpy over all.

5.2 Debt, Cash and Equity: Short Term Alternation Between Financing Sources

Pecking order models of firm investment and capital structure have the strong implication that the firm will use all its financial margins in the same direction. That is to say, in these models a firm will set the marginal cost of each unit of extra financing to be equal across sources of financing, and without non-convex costs, the marginal cost is a monotone increasing function of the amount financed. Therefore firms increase financing from all margins contemporaneously, or exhaust one source of financing before going to the next. This implication is strongly at odds with the data. The model in this paper predicts firms will alternate between sources of financing.

5.2.1 Conditional Cash Increases/Decreases

Table 4 shows our model predicts the firm will occasionally access external financing and accumulate cash contemporaneously. It shows the average value of
non-negative cash accumulation for periods when the firm issues debt or equity. The first row shows that our model firm often issues equity and contemporaneously accumulates cash. The second row shows that this is the result of the non-convexities in financing, by showing that, in the context of this model the firm does not keep cash unless it faces non-convex costs of adjustment. Finally, it shows a version of the model appended with a quadratic cost of capital adjustment, implies both debt and equity issues are often accompanied by cash accumulation.

Table 5: Estimated Cash Accumulation

<table>
<thead>
<tr>
<th>Sample Mean</th>
<th>Debt Issuing</th>
<th>Equity Issuing</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.031</td>
<td>0.066</td>
<td>0.112</td>
</tr>
</tbody>
</table>

Table 5 shows empirical estimates of firm’s change in cash holdings, for debt and equity issuance periods. To construct this table we select firm-year periods in our sample of COMPUSTAT firms in which firms report having issued debt or equity with a value of at least 3% of total assets. We then average the change in cash holdings over this periods and compare this number with the average change in cash holdings for the sample of firms. On average, firms report their cash holdings increased 4% relative to total assets during this periods. This result is a simple but convincing piece of evidence regarding non convexities in the firm’s financial cost of adjustment function.

5.2.2 ‘Overadjustment’

An interesting implication of non-convex costs has to do with the process by which firms adjust towards their targets. If these targets have a given trend, for example, in our setup, where there is positive expected growth and capital depreciates and debt matures, the fact that the firm adjusts infrequently, and that it tries to be close to its targets on average implies that it goes beyond its targets when adjusting. Since there is a positive expected growth rate, when making an investment the firm invests beyond where it would if there were no costs or no drift. Since there is a lower maturation rate than depreciation rate, the firm tends to repurchase more debt than it would without non-convexities when it repurchases debt. We call this type of behavior ‘overadjustment’ to contrast it with the typical partial adjustment notion.

In table 6 we compare two simulated samples of firm leverage, the first for
our benchmark firm, the second for a frictionless financing firm. The statistics show clearly that although the average choice of indebtedness is similar, the firm facing non-convex costs ends up with much higher leverage when it issues debt, and with much lower leverage when it issues equity.

Table 6: 'Overadjustment' to Target Leverage

<table>
<thead>
<tr>
<th></th>
<th>B/K</th>
<th>SD[B/K]</th>
<th>% change in B/K after BI ≥ 0</th>
<th>after X ≥ 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark Model</td>
<td>0.61</td>
<td>0.25</td>
<td>14.6%</td>
<td>-11%</td>
</tr>
<tr>
<td>Frictionless Finance</td>
<td>0.71</td>
<td>0.14</td>
<td>7%</td>
<td></td>
</tr>
</tbody>
</table>

The empirical evidence is consistent with this prediction of the model. On average, the difference between a firm’s leverage after debt issues and after equity issues, for firms that have taken both actions individually in our sample, is 10%. On average, firms that sometimes issue debt and sometimes equity, end up with 10 percentage points more leverage after issuing debt. The magnitude of this difference is highly suggestive of important non-convex costs of adjustment. This result is clearly consistent with the model presented above and inconsistent with partial adjustment models of capital structure.

The next part of this section provides a more careful version of the above exercise. We proceed by estimating a target leverage ratio for firms at every point in time to show evidence of overadjustment even after controlling for the firm’s factors that might lead it to issue debt or equity. We construct three variables that contain the main determinants of target leverage: potential tax savings as in Graham and Smith (1999), the firm’s expected return volatility, and a measure of bankruptcy costs.

First, we estimate an autoregressive process for the ratio of operating income to assets. I estimate a different regression for each one-digit industry and for each time period with sufficient data available. The profit prediction regressions we estimate are as follows:

\[
\hat{O}I_{i,t} = \alpha_0 + \alpha_1 OI_{i,t-1} + \alpha_2 OI_{i,t-2} + \alpha_3 S_{i,t-1} + \alpha_4 P_{i,t-2} + \alpha_5 PP_{j,t-2} + QD + \epsilon_{i,t}
\]

I predict firm returns using this equation for each industry and time period pair (j,T), using all firms i and time periods \(t < T\) with data available, and normalizing each firm variable by total firm assets. \(OI\) is a firms’ return before depreciation and interest, \(S\) stands for sales, and \(PP\) stands for the proportion of firms in an industry which report profits in a given year. \(QD\) represents quarterly dummies in the equation above. I use \(\hat{OI}\) in the predicted tax savings equation below and \(\hat{\sigma}^2_{i,t}\) as a measure of return variability. 9

**Potential Tax Savings** I construct Potential Tax Savings by estimating taxable income and applying the difference between taxation of corporate and personal income. Taxable income is constructed by taking predicted profits from
above and subtracting the observed values of depreciation, loss carry-forwards (LCF) and investment tax credits (ITC) from it. I then apply the difference between debt and equity income taxation to this taxable income to obtain Potential Tax Savings, where \( \tau_S = 1 - (1 - \tau_C)(1 - \tau_D) \).

\[
PTS_{i,t} = (\tau_S - \tau^P) \max\{(\hat{OI}_{i,t} - \delta K_{i,t} - LCF_{i,t} - ITC_{i,t}), 0\}
\]

Estimating the precise tax rates a firm faces is not a simple task. In this paper I simplify the problem by assuming at any given time that tax rates are the statutory maximum levels. Taxes on bond income are estimated to be the personal maximum rates, while taxes on stock income are estimated as the composition of maximum corporate and dividend taxes.

**Bankruptcy Costs** In this section I use a function of the fraction of Property, Plant and Equipment (PPE) in total assets as a firm-specific proxy for its bankruptcy costs parameter, as Eq. 7 describes. These particular types of assets are presumably easier to take hold of and liquidate in case of bankruptcy, or equivalently, the firm might be able to write low risk, secured debt with these assets as collateral. I construct the proxy for bankruptcy costs for a given firm as follows:

\[
BC_{i,t} = \left(1 - \frac{PPE_{i,t}}{TA_{i,t}}\right)
\]

A different hypothesis is that short term investments, or cash holdings are more valuable for bondholders, and therefore the above measure fails to capture and indeed might be negatively correlated with the main determinants of bankruptcy. However, the corporate finance literature has generally assumed the main tangible asset of the firm is its PPE, and we follow that tradition in this paper.

The equation we estimate is:

\[
L_{i,t} = \beta_0 + \beta_1 \frac{PTS_{i,t}}{TA_{i,t}} + \beta_2 \frac{\hat{\sigma}_{i,t}}{TA_{i,t}} + \beta_3 BC_{i,t} + \epsilon_{i,t}
\]

Table 7 presents the results of estimating a fixed effects regression for leverage using COMPUSTAT data from 1983 to 2002.

<table>
<thead>
<tr>
<th>PTS</th>
<th>( \sigma^2 )</th>
<th>BC</th>
<th>( R^2 ) FE</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.370</td>
<td>* -38.41 ***</td>
<td>-0.170 ***</td>
<td>N = 90100</td>
</tr>
<tr>
<td>(0.038)</td>
<td>(1.036)</td>
<td>(0.054)</td>
<td>( R^2 = 0.14 )</td>
</tr>
</tbody>
</table>

| \( R^2_{FE} \) | 0.48 |

Next, we construct the average difference between leverage and its estimated target for two groups of periods, those where the firm issues either debt or equity,
We find $L_{D_{Diff}}^E$ is -0.07 and $L_{D_{Diff}}^D$ 0.03, consistent with the model prediction of over adjustment.

### Table 8: Distance from Estimated Target Leverage

<table>
<thead>
<tr>
<th></th>
<th>Debt Issues</th>
<th>Equity Issues</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before</td>
<td>-0.02</td>
<td>-0.03</td>
</tr>
<tr>
<td>After</td>
<td>0.03</td>
<td>-0.07</td>
</tr>
</tbody>
</table>

6 Investment with Non-Convex Costs of Financing

6.1 Investment Hazard Function

Despite the success of non-convex cost specifications for modeling firm investment, empirical estimations of firm’s rate of investment as a function of the distance from their mandated or target capital are not exactly consistent with non-convex costs of adjustment. Caballero and Engel (1999) (CE), for example find a region where average investment is positive, but is only a proportion of mandated investment, while fixed costs imply the firm adjusts all the way or it does not adjust at all. In their paper, CE rationalize this fact with fixed cost of random magnitude $\omega_t$. With this stochastic costs, firms don’t have a particular point at which they adjust, rather, their decision depends on both mandated investment and the observed fixed cost. They then argue that averaging smooths over these random observations to create a ‘partial adjustment’ region when estimating the relationship between mandated and actual investment.

The model in this paper provides a partial rationalization for the behavior of investment which does not rely on random fixed costs. In the model the firm occasionally finds itself willing to pay the investment cost and not willing to pay the financing fixed cost, so that investment is undertaken but it does not cover all of mandated investment. Figure 7 presents the optimal investment policy of a firm. It consists of cross sections of the investment policy function for different levels of debt assuming the firm holds cash worth 4% of the firm’s capital. The figure shows that there is a region of the state space where the firm pays the investment fixed cost but not the financing fixed costs, and therefore it invests only from current internal resources. Then another region where it issues debt, up to a limit and finally a point after which investment grows linearly with mandated investment.

Figure 8 presents the average investment behavior of the firm as a function of the difference between current and target capital. Although for any given level of debt and cash, the investment function is a step function of mandated investment like those in figure 7, the average level of investment seems to reflect
partial adjustment for a region of mandated investment, as has been documented empirically.

6.2 Cash Flow Sensitivity, Firm Heterogeneity and Financing Sources

Fazzari, Hubbard and Petersen (1988) (FHP) document the fact that firms that have higher retention ratios are associated with higher growth and higher sensitivity of investment to retained earnings. They interpret this fact as suggesting that small, fast growing firms are more financially constrained than large firms. This observation led to the development of a large literature, confirming the fact under different circumstances on one hand, and rebutting the FHP interpretation of this fact on the other. This paper studies this question by estimating internal finance sensitivities of investment simulated samples of firms from the model above. We fist show that, as expected, fixed costs of financing are able to rationalize a large coefficient on internal finance and a small coefficient on Q in investment regressions. The intuition for this is clear, the firm is more likely to invest if it can do so without paying the costs of financial adjustment in the form of debt or equity issues. Also, Q becomes less informative about
firm investment, since there is a range of values where $Q$ can be higher or lower and the firm does not change its behavior. In this sense, the paper gives some credence to the FHP hypothesis that financial frictions are the cause of these high internal finance coefficients.

However, the main point of this section is that our model can rationalize the different internal finance sensitivities found across different firms without implying some firms are more financially restricted than others. To do so we compare the result of two exercises. First we produce a large sample of firms with identical expected rates of growth and sort them into different growth classes. We also sort them into different retention rate classes to be consistent with the original FHP analysis. We then compare the estimated internal finance sensitivities across groups. We obtain a result inconsistent with the FHP facts. As table 9 shows, the faster growing firms in our sample have higher $Q$ coefficients and lower internal finance coefficients. The comparison between the two classification criteria shows they are equivalent, in the model, faster growing firms return less of their earnings, and therefore the regression coefficients are very similar.

In the following tables we present the result of estimating a linear regression of investment on average $Q$ and internal financial resources. For investment we use the ratio of gross investment to capital, consistent with FHP. Average $Q$ is the ratio of the value of equity and debt over the sum of the replacement cost of capital and accumulated cash. Internal finance ($IF_{it}$) is defined to be accumulated cash plus the periods profits gross of depreciation. With these variables we estimate the following linear relationship:
\[ \frac{I_{it}}{Kit} = \beta_0 + \beta_Q Q_{it} + \beta_{IF} IF_{it} + \epsilon_{it} \]  

(9)

Table 9: Investment Sensitivity by Groups, Benchmark Model

<table>
<thead>
<tr>
<th>Regressions by Growth Groups</th>
<th>Regressions by Retention Groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_0 )</td>
<td>0.72</td>
</tr>
<tr>
<td>( \beta_Q )</td>
<td>0.41</td>
</tr>
<tr>
<td>( \beta_{IF} )</td>
<td>-0.32</td>
</tr>
<tr>
<td>(0.02)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Group 1</td>
<td>-0.91</td>
</tr>
<tr>
<td>(0.05)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Group 2</td>
<td>-0.78</td>
</tr>
<tr>
<td>(0.06)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>Group 3</td>
<td>-0.54</td>
</tr>
<tr>
<td>(0.12)</td>
<td>(0.03)</td>
</tr>
</tbody>
</table>

The second exercise we perform consists of modeling different types of firms, with different expected rates of growth instead of simply different observed growth paths. In this case the results coincide with the FHP data. Table 10 again compares regressions across different firm groups, but in this case the groups are formed by model simulations of firms with different expected growth rates. The dependence of investment on cash flow is higher for the faster growing firms. This shows that, in the context of the model, it is systematic differences across firms that are able to explain the different internal finance coefficients of investment observed in the data.

Table 10: Investment Sensitivity by Expected Growth Rate Groups

| Regressions by Expected Growth Groups |
|-----------------------------|---|---|
| \( \beta_0 \) | -0.76 | -0.69 |
| \( \beta_Q \) | 0.36 | 0.52 |
| \( \beta_{IF} \) | -0.15 | -0.41 |
| (0.002) | (0.08) | (0.03) |
| \( \mu = 0.98 \) | -0.69 | -0.72 |
| (0.005) | (0.02) | (0.09) |
| \( \mu = 1.01 \) | 0.52 | 0.41 |
| (0.007) | (0.08) | (0.14) |
| \( \mu = 1.04 \) | -0.15 | -0.23 |
| (0.004) | (0.25) | (0.15) |

Our intuition for the negative relationship between growth rate and \( \beta_{IF} \) in our model is that firms retain earnings as cash in anticipation of investment. This implies investment is related to internal financial resources particularly when this investment is expected. This explains why investment is not related to internal financing for firms that grow unexpectedly.

The third row of table 10 shows that our model rationalizes positive coefficients on cash flow firms with high rates of growth, even as they face the same financial restrictions as the rest of the firms. This result questions the FHP
interpretation of the magnitude of internal finance coefficients in investment regressions as a measure of the degree of financial restrictions facing a firm.

We hypothesize that faster growth firms have higher internal finance sensitivity because they are less likely to end up with too much capital. They have more flexibility to invest whenever, since they plan investment in any case. Low growth firms, that face the possibility of having to disinvest in the future, are at a greater risky of paying a high cost if investing beyond what is necessary. However, this explanation would be limited to firm’s that do not use all of their earnings for investment and do not need to access external markets with great frequency.

7 Conclusion

This paper shows non-convex costs of financing and investment are important for rationalizing certain features of the financing patterns of US corporations. First it documents a high degree of financial lumpiness using COMPUSTAT data. Debt and equity adjustments are infrequent, most of the gross flows happening in a small fraction of the time periods, and sample excess kurtosis is large. Then it presents a model of firm behavior which shows that non-convexities on investment alone cannot explain this financial lumpiness but non-convexities on both investment and finance are able to rationalize it. Indeed, in the presence of taxes and bankruptcy costs, but without non-convex costs of financing, firms make financial adjustments often to reflect changes in current and future profitability, even if capital adjustments are infrequent. Furthermore, the model provides a set of predictions about firm investment and financing that are verified empirically. First, in the model, financial margins are often used as substitutes rather than complements by firms, so that firms sometimes increase their cash holdings as they issue debt, and decrease their indebtedness as they issue equity. We verify this prediction empirically by showing that when issuing debt or equity firms go past their estimated target leverage levels: when adjusting to reduce (increase) leverage they end up below (above) their estimated target. We also show that, with high statistical significance, firms on average accumulate cash when they issue debt or equity, a clear suggesting that non-convexities are important. Regarding investment, the model shows that firms’ investment as a function of the distance from ‘target capital’ is not an all or nothing function as the single constant cost of adjustment model implies, but a smooth function of this distance. The model therefore helps to rationalize the estimated investment hazard functions in Caballero and Engel (1999) without having to rely on fixed costs of adjustment of stochastic magnitude.

The last part of the paper relates firm heterogeneity to the internal finance sensitivity of investment. It shows that the relationship documented by Fazzari, Hubbard and Petersen (1988) can be rationalized, in the context of the model, as the result of firm’s heterogeneous expected rates of return, along with non-convexities on investment and financing. No heterogeneity over the level of firm’s financial restrictions is necessary for this result, therefore questioning the
validity of interpreting internal finance sensitivities of investment as measures of financial restrictions. Finally the paper shows heterogeneity in firm growth that is unexpected, the result of different growth paths for firms with the same expected growth rates, does not result in estimated internal finance sensitivities like those found empirically, suggesting heterogeneity in firms expected rates of growth is important for understanding firm's investment financing decisions.
Appendix

A 'Distance from target' variables

This section shows how the model above can be expressed in more condensed form, taking advantage of the homogeneity properties of the assumed functional forms. In that way the problem’s state space is reduced from 5 to 4 dimensions and can then be solved numerically with a reasonably fine grid. The problem is rewritten in variables and a value function as a proportion of ‘target capital’.

First, define the target capital level $K^*$ as the solution to the frictionless problem with discount rate $\rho$:

$$K^* = \text{arg max}(AK)^\alpha - \delta K - rK$$

Then express the productivity coefficient in terms of target capital and eliminate actual capital from the production function:

$$A^\alpha = (K^*)^{1-\alpha} \left(\frac{\rho + \delta}{\alpha}\right)$$

Equation 3 can then be expressed using the ratio variables $z$, $\hat{z}$, $i$ and $e$:

$$\hat{V} = \hat{v} = \max \left\{ \max_{i', L', C'} \left[ (1 - \tau_D) \text{div} - x + \frac{(\xi')^{1-\alpha}}{1 + \rho} E[\hat{v}(z', L', C')] \right], 0 \right\}$$

Where the forward variables $z'$ is constructed as follows:

$$z' = (z + i)^* K^*/(K^*)^* = \frac{z + i}{\xi'(\tau_D)}$$
B Model Solution

This section describes the numerical procedure to find a solution to the problem above. This solution consists of a value function \( V \), a set of optimal policies \( H \), and a set of interest rates \( r \), consistent with each other:

1. \( V \) is the value of the firm derived from \( H \) and \( r \),
2. Interest rates reflect risk neutral pricing given \( V \) and \( H \) and,
3. \( H \) is the optimal firm policy given \( V \) and \( r \) and

I solve for an approximate solution to the firm’s problem above computationally. To set up the problem I construct a grid \( G \) in the state space of the ‘distance from targets’ problem:

\[
G_S = \{ S_t = (z_t, b_t, c_t, \xi_t) | z_t \in \{z_1, \ldots, z_n\}, b_t \in \{b_1, \ldots, b_n\}, c_t \in \{c_1, \ldots, c_n\}, \xi_t \in \{\xi_1, \ldots, \xi_n\} \}; \text{ and one in the control space:}
\]

\[
G_C = \{ C_t = (z_t, b_t, c_t) | (z_t, b_t, c_t, \xi_t) \in S_t \}\.
\]

Then I define an initial interest rate \( r_0 \) and an external equity flow function \( E(S_t, C_t | r_i) : G_S \times G_C \rightarrow \mathbb{R} \), which denotes current income as a function of state and choice variables, given an bond pricing schedule \( r \).

The Bellman equation is then:

\[
\hat{V}(S_t) = \max_{C_t \in G_C} \left\{ \max_{S_t'} E(S_t', C_t) + \beta E[\xi' \hat{V}(S_t')] \right\},
\]

(10)

where \( S_t' = (C_t, \xi') \), \( \hat{V} \) coincides with the definition above, and the expectation is over the productivity shock \( \xi' \):

\[
E[\xi' \hat{V}(S_t')] = \sum_{\xi' \in \xi_1, \ldots, \xi_n} \xi V((\hat{C}_t, \xi)) r(\xi = \xi')
\]

Equation 10 defines a functional \( V(V, r_i) \) which produces a limit value function \( V^*(r_i) \), the fixed point of the functional given \( r_i \), that is \( V^*(r_i), r_i = V^*(r_i) \). The model however, has bond prices being a function of the firm’s expected value, and therefore, once I obtain \( V^*(r_i) \), I construct \( r_{i+1}(V^*(r_i)) \) following equation 6.
C Policy Functions

Figure 9: Leverage Policy

Figure 10: Capital Policy
References


Notes

1Caballero and Engel (1999) show the changing elasticity of aggregate investment to underlying fundamentals can be traced to the changing distribution of firm’s distances from desired capital stock, in a model with stochastic fixed costs of adjustment. Caballero and Leahy (1996) show traditional investment equations break down in the presence of non-convex costs.

2This graph is the financing analogous of figure 1 in Doms and Dunne (1998) which is constructed from manufacturing investment data.

3The normal distribution is a good benchmark. In a model with normal shocks to productivity and no costs of adjustment or other frictions, capital and debt would both always be proportional to each other and to productivity making them normally distributed as well.

4We can think of this specification reflecting a firm that uses capital $K_t$, labor $N_t$ and technology $\hat{A}_t$ to produce $Y_t = f(\hat{A}_t, K_t, N_t)$ units of output and then sells each unit at a price of $P(Y_t)$. Assuming a constant returns to scale Cobb-Douglas production function $f(\hat{A}_t, K_t, N_t) = \hat{A}_t K_t^\beta N_t^{1-\beta}$ and letting the firm adjust labor freely, output can be written in terms of capital as $f(A_t, K_t) = K_t A_t$, where

$$A_t = \hat{A}_t^{\frac{1-\beta}{\beta}} \left( \frac{\beta - 1}{W_t} \right)^{\frac{1-\beta}{\beta}}$$

and $W_t$ is the wage rate. Finally, assuming the firm faces an isoelastic demand function with price elasticity of $-\eta$ we obtain a convex revenue function $R(A_t, K_t, N_t) = Y_t \cdot P(Y_t) = (A_t K_t)^\alpha$, with $\alpha = (\eta - 1)/\eta$.

5Although other economists have explored capital depreciation issues by allowing for different depreciation rates for different capital vintages or by distinguishing between depreciation and capital obsolescence, these issues are not part of the present model. A discussion of depreciation in the context of the US income and product accounts appears in Jorgenson (1996)

6This novel feature of our model is consistent with the fact that long term debt issues often impose recovery fund requirements that imply the firm must allocate funds for principal repayment. Kwan and Carrelton (2004) document that this is particularly the case for private issues.

7Ramey and Shapiro (2001) document high capital specificity and discounts of between 40% and 85% on the sale of displaced capital for the aerospace industry.

8This is a generic feature of our model. As Cummins and Nyman (1999) show, introducing a cost of external financing is in principle sufficient to make
the firm hedge by retaining earnings in the form of cash. In our model without non-convex costs there is no extra cost of external financing.

9Details of regression estimates for a group of firms are presented in Appendix B