Mortgage Innovation and the Foreclosure Boom

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November 9, 2009

Abstract

How much of the recent rise in foreclosures can be explained by the introduction of low downpayment, delayed amortization mortgage contracts? We present a model where heterogeneous households select from a set of possible mortgage contracts and choose whether to default on their payments given realizations of income and housing price shocks. The set of contracts consists of traditional fixed rate mortgages which require a 20% downpayment as well as nontraditional mortgages with low downpayments and delayed amortization schedules, two features which became highly popular after 2004. The mortgage market is competitive and each contract, contingent on household earnings and assets at origination as well as loan size, must earn zero expected profits. We use our model to quantify the role of mortgage innovation in the recent rise in foreclosure rates. A 20% price decline following a brief introduction of non-traditional mortgages can explain 40% of the rise of foreclosures from mid-2006 to mid-2008. If new mortgages are not introduced, the same price shock causes an increase in foreclosure rates of only 20%.

Preliminary and incomplete, comments welcome.

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1 Introduction

Between 2003 and 2006, the composition of the stock of outstanding residential mortgages in the United States changed in several important respects. The fraction of mortgages with fixed payments (FRMs) relative to all mortgages fell from 85% to under 75% (see figure 1.) At the same time, the fraction of "subprime" mortgages (mortgages issued to borrowers perceived to be high-default risks) relative to all mortgages rose from 5% to nearly 15%. Recent work (see e.g. Gerardi et al., 2009, figure 3) has revealed that many of these subprime loans are characterized by high leverage at origination and non-traditional amortization schedules. These features cause payments from the borrowers to the lender to be backloaded compared to loans with standard downpayments and standard amortization schedules. By lowering payments initially, these innovations made it possible for more households to obtain the financing necessary to purchase a house and, in other papers (e.g. Chambers, et. al. (forthcoming)) have been associated with the rise in homeownership.

Our objective is to quantify the importance of mortgage innovation for the recent flare-up in foreclosure rates. Specifically, we ask the following questions. How much of the rise in foreclosures can be attributed to innovation in mortgage contracts? What is the welfare gain associated with mortgage innovation? What types of policies can mitigate the rise in foreclosures?

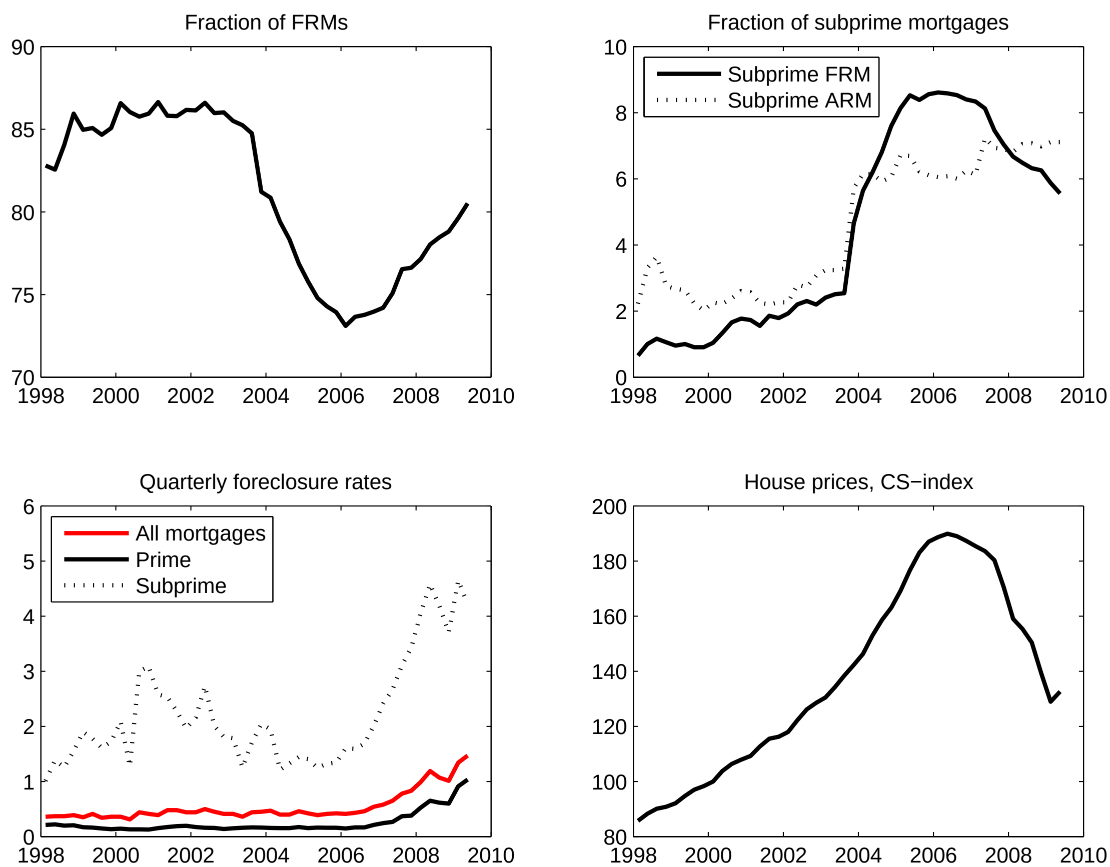
To answer these questions, we describe an economy where households value both consumption and housing services and move stochastically through several stages of life. For simplicity, agents who are young are constrained to obtain housing services from the rental market and split their remaining income between consumption and the accumulation of liquid assets. Given idiosyncratic earnings shocks, despite the fact that households begin life ex-ante identical in our model, there is an endogenous distribution of assets among the set of people who turn middle aged.

When agents become mid-aged, they have the option to purchase one of two possible quantities of housing capital: a small house or a large house. We assume they must finance house purchases via a mortgage drawn from a set of contracts with properties like those available in the United States. Standard fixed-rate mortgages (FRMs) require a 20% downpayment and fixed payments until maturity. Agents can opt instead for a mortgage with no-downpayment and delayed amortization (we will term these contracts LIP for "low initial payment"). We think of this second mortgage as capturing the backloaded nature of the mortgages that became popular after 2004 in the United States.

Mortgage holders can terminate their contract before maturity, in which case the house is immediately sold and the borrower receives any proceeds in excess of the outstanding loan principal and transaction costs. We consider a house sale to be a foreclosure if it occurs in a state where the house value is below the mortgage's balance (that is, the agent's home equity is negative) or where the agent's income realization is such that they cannot make the mortgage payment they would owe for the period.[1] In those cases, home sales are subject to

[1]Here we are assuming the default law is consistent with antideficiency (as in California for example) where

Figure 1: Recent trends in US housing



Sources: Haver analytics, National Delinquency Survey (Mortgage Bankers Association).

foreclosure costs.

Our model predicts that almost all foreclosures (99%) involve negative equity. This is because most agents with positive equity who are at a high risk of finding themselves unable to meet their mortgage payments sell before reaching that state in order to avoid foreclosure costs. On the other hand, most agents with negative equity (96%) choose to continue meeting their mortgage obligations to avoid losing their homes. Foreclosures are thus associated with a combination of negative equity and income circumstances that make meeting mortgage payments difficult. These predictions are consistent with the growing empirical literature on the determinants of foreclosure.[2]

Foreclosures are costly for lenders because of the associated transactions costs and because they occur in most cases when home equity is negative. As a result, intermediaries demand higher yields from agents whose asset and income position make foreclosure more likely. In fact, intermediaries do not issue loans to some agents because their default risk is too high or because the agents are too poor to make a downpayment. In particular, our model is consistent with the fact that agents at lower asset and income positions are less likely to become homeowners, face more expensive borrowing terms, and are more likely to default on their loan obligations.

Since high initial payments are prohibitively costly for asset and income poor agents, there is a natural role to play in our economy for mortgage innovation in the form of contracts with low initial payments. We find that in an economy calibrated to match key aspects of the US housing market prior to 2005, adding the option to issue LIP contracts causes a rise in steady state homeownership, default rates, and welfare.

In particular, we find that LIPs are necessary for asset and income poor households (those who could be interpreted as subprime) to become homeowners. At the same time, the availability of these contracts cause default rates to be higher for two complementary reasons which our environment enables us to make explicit. First, high-default risk households select into homeownership. Second, these contracts are characterized by a much slower accumulation of home equity than FRMs, which makes default in the event of a home value shock much more likely, even at equal asset and income household characteristics.

While these long-run predictions are interesting, the data in figure 1 shows that the break in the composition of the mortgage stock occurred briefly before the collapse of prices. There is also growing evidence that the fraction of high-LTV, delayed amortization mortgages in originations has dwindled to a trickle since the collapse of prices.[3]

the defaulting household is not responsible for the deficit between the proceeds from the sale of the property and the outstanding loan balance. In Section 5.7 we consider a variation in punishment following a foreclosure that resembles laws in states with recourse.

[2]See, among many other papers, Foote et al. (2008a,b), Gerardi et al. (2007), Sherlund (2008), Danis and Pennington-Cross (2005), and Deng et al. (2000).

[3]The Mortgage Bankers Association (MBA)'s mortgage origination survey suggests for instance that after falling to 50% of originations in 2005, traditional FRMs now account for 90% of originations. According to the same source, the fraction of interest-only mortgages in originations rose to nearly 20% in 2006, and has now fallen to below 5%. It is also estimated (see e.g. Harvard's "2008 State of the Nation's Housing")

We simulate this course of events using a three-stage transition experiment. Specifically, we begin in a steady state of an economy with only FRMs calibrated to match key aspects of the US economy prior to 2005. We then introduce the nonstandard mortgage option for one period, which represents two years in our calibration. In the third stage, we assume a surprise 20% collapse in home prices, remove the nonstandard mortgage option, and then let the economy transit to a new long-run steady state. This experiment causes foreclosure rates to rise by 40% during the first two years of the third stage before, and by 50% at peak. By comparison, in the data, foreclosure rates doubled between 2006 and 2008, and have now tripled. To quantify the role of mortgage innovation in this increase, we then run a similar experiment where the LIP mortgage option is not offered in the second stage. In this counterfactual, the increase in foreclosure rates caused by the price shock is only 20% on impact and 40% lower than the data at its peak. Mortgage innovation, in other words, makes the economy much more sensitive to price shocks. In addition, we find that lower downpayments account for most of the contribution of new mortgages to the increase in foreclosure rates, while delayed amortization and payment spikes play a limited role.

Our calculations are conservative in several respects. First, we assume that new mortgages only became available (and popular) over a two year period, leaving little time for these contracts to make a deep impact on the mortgage stock. In particular, in the current calibration the share of LIPs grow to only 4% of the stock of mortgages when the price shock strikes. A longer innovation stage would boost our foreclosure numbers. Second, we use a conservative 20% price drop but could have easily used 25% as well as changed the exogenous earnings process to reflect the economic downturn.

Our paper is closely related to several studies of the recent evolution of the US housing market and mortgage choice.[4] Chambers et al. (forthcoming) argue that the development of mortgages with gradually increasing payments has had a positive impact on participation in the housing market. The idea that mortgage innovation may have implications for foreclosures is taken up in Garriga and Schlagenhauf (2009). They quantify the impact of an unanticipated aggregate house price decline on default rates where there is cross-subsidization of mortgages

that subprime loans accounted for roughly 20% of originations between 2004 and 2006, up from less than 8% between 2000 and 2003. They now account for less than 5% of new mortgage issues.

[4]There are numerous other housing papers which are a bit less closely related. Campbell and Cocco (2003) study the microeconomic determinants of mortgage choice but do so in a model where all agents are homeowners by assumption, and focus their attention on the choice between adjustable rate mortgages and standard FRMs with no option for default. Rios-Rull and Sanchez-Marcos (2008) develop a model of housing choice where agents can choose to move to bigger houses over time. A different strand of the housing literature (see e.g. Gervais (2002) and Jeske and Krueger (2005) studies the macroeconomic effects of various institutional features of the mortgage industry, again where there is no possibility of default. Davis and Heathcote (2005) describe a model of housing that is consistent with the key business cycle features of residential investment. Our paper also builds on the work of Stein (1995) and Ortalo-Magné and Rady (2006) who study housing choices in overlapping generation models where downpayment requirements affect ownership decisions and house prices. Our framework shares several key features with those employed in these studies, but our primary concern is to quantify the effects of various mortgage options, particularly the option to backload payments, on foreclosure rates.

within but not **across** mortgage types (e.g. FRM or LIP). A key difference between our paper and theirs is that we consider a menu of different terms on contracts **both** within **and** across mortgage types. Effectively, Garriga and Schlagenhauf (2009) apply the equilibrium concept in Athreya (2002) while we apply the equilibrium concept in Chatterjee et al. (2007). This enables us to build a model that is consistent with the heterogeneity of foreclosure rates and mortgage terms across wealth and income categories which we document in the Survey of Consumer Finance. We present simulations that suggest that the two equilibrium concepts result in significantly different quantitative predictions.

Along this separation dimension our paper is more closely related to Guler (2008) where intermediaries offer a menu of FRMs at different possible downpayment rates without cross-subsidization or Chatterjee and Eyigungor (2009) where intermediaries offer a menu of infinite maturity interest-only mortgage contracts. Guler studies the impact of an innovation to the screening technology on default rates and Chatterjee and Eyigungor study the effect of an endogenous price drop arising out of an overbuilding shock.

Section 2 lays out the economic environment. Section 3 describes optimal behavior on the part of all agents and defines an equilibrium. Section 4 provides our calibration. Section 5 describes our steady state results, with subsections focusing on: Selection, Default, the Distribution of Interest Rates, Welfare, and Policy Experiments. Section 6 presents our main transition experiment. Section 7 concludes.

2 The Environment

We study an economic environment where time is discrete and infinite. The economy is populated by a continuum of households and by a financial intermediary. Each period a mass one of households is born. Over time, households move stochastically through four stages: young (Y), middle-aged (M), old (O) and dead. All households are born young. At the beginning of each period, young households become middle-aged with probability ρ_M, middle-age households become old with probability ρ_O, and old households die with probability ρ_D. We assume that the population size is at its unique invariant value, and that the fraction of households of each type obeys a law of large numbers.

Each period, as long as they are young or middle-aged, households receive stochastic earnings shocks denominated in terms of the unique consumption good. These shocks evolve stochastically according to a stationary transition matrix π and satisfy a law of large numbers so that there is no aggregate uncertainty. Agents begin life at an income level $y \in \{y_L, y_M, y_H\}$ drawn from the unique invariant distribution associated with π. When old, agents earn a fixed, certain amount of income denoted y^O.

Until they become old, households can save in one-period bonds that earn rate $1 + r_t \geq 0$ at date t with certainty. When old, households can buy annuities that pay rate $\frac{1+r_t}{1-\rho_D}$ in the following period provided they are alive and pay nothing otherwise. We annuitize returns in the last stage of households' life in order to rule out accidental bequests.

Households value both consumption and housing services. They order non-negative processes $\{c_t, s_t\}_{t=0}^{\infty}$ according to:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, s_t)$$

where U satisfies standard assumptions.

Households can obtain housing services from the rental market or from the owner-occupied market. On the first market, they can rent quantity $h_1 > 0$ of housing services at unit price R_t at date t. In the period when agents move from youth to middle-age – and only in that period – agents can choose instead to purchase quantity $h \in \{h_2, h_3\}$ of housing capital for unit price q_t, where $h_3 > h_2 > h_1$.[5] We refer to this asset as a house. A house of size h initially delivers $h\theta$ of housing services every period with $\theta \geq 1$.

Homeowners face a risk that their house will devalue.[6] Specifically, every period, a fraction $\lambda > 0$ of agents who own a house of size $h = h_3$ see the quantity of capital they own fall to $h_2 > 0$. Likewise, a fraction λ of agents who own a house of size $h = h_2$ see the quantity of capital they own fall to h_1. Furthermore, houses of size h_1 generate quantity h_1 of housing services, rather than $h_1\theta$, whether owned (following a devaluation) or rented. We will interpret the devaluation shock as an idiosyncratic house price shock.[7]

There are several possible interpretations for this devaluation shock. One could think of it as a neighborhood shock which makes house in a given location less valuable. Note that while we assume that devaluation shocks satisfy a law of large numbers (the fraction of houses that devalue in each period is λ) we do not need to assume that these shocks are independent across households.[8] Alternatively, one could consider introducing more heterogeneity in houses and modeling taste shocks that render certain house types less valuable. Our devaluation shocks are a tractable way to capture the possibility of microeconomic events that affect house values and are difficult to insure against.

Since devalued houses of size h_1 provide no advantage over rental units, no agent who becomes middle-aged would strictly prefer to purchase a house of that size and all homeowners whose housing capital fall to that level are at least as well off selling their house and becoming

[5] We make the strong assumption that buying a home is a one-time-only option for computational tractability. Forcing agents who have sold their home or defaulted to become renters for the rest of their life enables us to price mortgage contracts for each possible asset-income-house size position at origination independently from rates offered to borrowers with different characteristics. If agents had the option to take another mortgage after they terminate their first one, their decisions to default – hence the intermediary's expected profits – would depend in part on what terms are offered on contracts offered at positions different from their situation when they become mid-aged. Instead of solving one fixed point problem at a time, we would need to jointly solve a high-dimensional set of fixed points.

[6] This is similar to Jeske and Krueger (2005).

[7] In the absence of such shocks, households would never find themselves with negative equity in a steady state equilibrium.

[8] In fact, independence across agents is essentially incompatible with assuming that a law of large numbers holds. See Feldman and Giles (1985).

renters as they would be if they keep their house.[9]

Owners of a house of size $h \in \{h_1, h_2, h_3\}$ bear maintenance costs δh in all periods where $\delta > 0$. Maintenance costs, denominated in terms of the consumption good, must be paid in all periods by homeowners. In that case, a house does not physically depreciate (other than through a devaluation shock), which in turn maintains the low cardinality of the housing state space. Once agents sell or foreclose their house, they are constrained to rely on the rental market for the remainder of their life. In the period in which agents become old, they must sell their house immediately and become renters for the remainder of their life. House sales due to the old age shock do not entail foreclosure costs (and hence they do not get counted in foreclosures.)

The financial intermediary holds household savings. The intermediary can store savings at exogenously given return $1 + r_t$ at date t. It can also transform the consumption good (i.e. deposits) into housing capital at a fixed rate $A > 0$. That is, it can turn quantity k into deposits into quantity Ak of housing capital at the start of any given period, or turn quantity h of housing capital into quantity $\frac{h}{A}$ of the consumption good.

Housing capital can be rented at rate R_t at date t. The intermediary incurs maintenance cost δ on each unit of housing capital rented measured in terms of the consumption good. At date t, each unit of consumption good rented thus earns net return $R_t - \delta$. The intermediary can also sell housing capital as houses to eligible households, at unit price q_t. Note that the fact that each agent's housing choice set is discrete does not impose an integer constraint on the intermediary since it deals with a continuum of households.

We assume that households that purchase a house of size $h \in \{h_2, h_3\}$ at a given date are constrained to finance this purchase with one of two possible types of mortgage contracts. The first contract (which we design to mimic the basic features of a standard fixed-rate mortgage, or FRM) requires a downpayment of size $\nu h q_t$ at date t where $\nu \in (0, 1)$ and stipulates a yield $r^{FRM,t}(a_0, y_0, h)$ that depends on the household wealth and income characteristics (a_0, y_0) at the date t of origination of the loan, and on the selected house size h. Given this yield, constant payments $m^{FRM,t}(a_0, y_0, h)$ and a principal balance schedule $\{b_n^{FRM,t}(a_0, y_0, h)\}_{n=0}^{T}$ can be computed using standard calculations, where T is the maturity of the loan.

Specifically, suppressing the initial characteristics for notational simplicity,

$$m^{FRM,t} = \frac{r^{FRM,t}}{1 - (1 + r^{FRM,t})^{-T}}(1 - \nu)hq_t$$

and, for all $n \in \{0, T - 1\}$,

$$b_{n+1}^{FRM,t} = b_n^{FRM,t}(1 + r^{FRM,t}) - m^{FRM,t},$$

where $b_0^{FRM,t} = (1 - \nu)hq$. Standard calculations show that $b_T^{FRM,t} = 0$.

[9] Arbitrage implies that the present value of renting housing services each period is the same as purchasing a depreciated house. Selling the depreciated house, however, can relax an agent's liquidity constraint.

The second LIP contract stipulates yield $r^{LIP,t}(a_0, y_0, h)$, no down-payment, constant payments $m^{LIP,t}(a_0, y_0, h) = hq_t r^{LIP,t}(a_0, y_0, h)$ that do not reduce the principal for the first $n^{LIP} < T$ periods, and fixed-payments for the following $T - n^{LIP}$ periods with a standard FRM-like balance schedule $\{b_n^{LIP,t}(a_0, y_0, h)\}_{n=n^{LIP}}^{T}$.

In other words,

$$m_n^{LIP,t} = \begin{cases} hqr^{LIP,t} & if \quad n < n^{IOM} \\ \frac{r^{LIP,t}}{1-(1+r^{LIP,t})^{-(T-n^{IOM})}} hq & if \quad n \geq n^{IOM} \end{cases}$$

and, for all $n \in \{0, T-1\}$,

$$b_{n+1}^{LIP,t} = b_n^{LIP,t}(1 + r^{LIP,t}) - m_n^{LIP,t},$$

where $b_0^{LIP,t} = hq$, and, once again, $b_T^{LIP,t} = 0$. Notice that for $n < n^{IOM}$, $b_{n+1}^{LIP,t} = b_0^{LIP,t}$ so that the principal remains unchanged for n^{IOM} periods.

Alternative mortgages, therefore, have two main characteristics: low downpayment, and delayed amortization. These are two of the salient features of the mortgages that become highly popular after 2004 in the United States (see Gerardi et al., 2007.) Naturally, delayed amortization can take many forms. Subprime mortgages, for instance, often feature balloon payments rather than interest-only periods.

Figure 2 shows typical mortgage payment schedules for both mortgage types. The chart assumes a yield of 15.75% and a loan size of 0.75, a maturity of 10 periods, and an interest-only phase of 3 periods for LIPs. Payments due on LIP mortgages jump once the interest-only phase ends, while FRM mortgages feature constant payments.

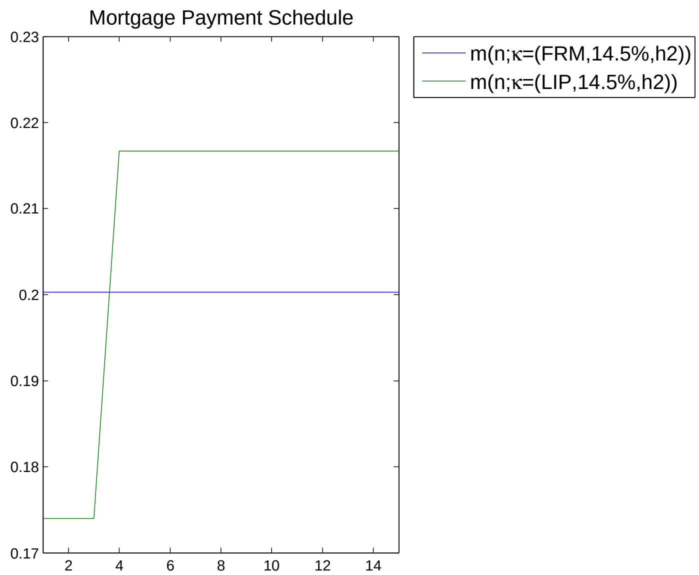
Mortgages are issued by the financial intermediary. The intermediary incurs service costs which we model as a premium $\phi > 0$ on the opportunity cost of funds loaned to the agent for housing purposes.

The household can terminate the contract at the beginning of any period, in which case the house is sold. We will consider a termination to be a foreclosure when the outstanding principal exceeds the house value or when the agent's state is such that they cannot meet their mortgage payment in the current period. In the event of foreclosure, fraction $\chi > 0$ of the house sale value is lost in transaction costs. If the mortgage's outstanding balance at the time of default is b, the intermediary collects $\min\{(1-\chi)qh, b\}$, while the household receives $\max\{(1-\chi)qh - b, 0\}$.

Agents may also choose to sell their house even when they can meet the payment and have positive equity, for instance because they are borrowing constrained in the current period. Recall also that agents sell their house when they become old. Those contract terminations, however, do not impose transaction costs on the intermediary.

The timing in each period is as follows. At the beginning of the period, agents discover whether or not they have aged, and receive a perfectly informative signal about their income draw. Middle-aged agents who own homes also observe the realization of their devaluation shock at the beginning of the period, hence the market value of their home. These agents then

Figure 2: Mortgage payments by mortgage type



decide whether to remain home-owners or to become renters either via selling their house or through foreclosure. Agents who just became middle-aged also make their home-buying and mortgage choice decisions at the beginning of the period, after all uncertainty for the period is resolved. At the end of the period, agents receive their income, mortgage payments are made, and consumption takes place.

3 Equilibrium

We will initially study equilibria in which all prices are constant. For notational simplicity, we now drop all time markers using the convention that, for a given variable x, $x_t \equiv x$ and $x_{t+1} \equiv x'$.

3.1 Agent's problem

We state the household problem recursively. In general, the household value functions will be written as $V_{age}(\omega)$ where $\omega \in \Omega_{age}$ is the state facing an agent of $age \in \{Y, M, O\}$.

3.1.1 Old agents

For old agents, the state space is $\Omega_O = I\!R_+$ with typical element $\omega \equiv a \geq 0$. The value function (that is, the expected present value of future utility) for an old agent with assets

$a \in I\!R_+$ solves

$$V_O(a) = \max_{a' \geq 0} \{U(c, h_1) + \beta(1 - \rho_D)V_O(a')\}$$

s.t.

$$c = a\frac{(1+r)}{1-\rho_D} + y^O - h_1 R - a' \geq 0$$

3.1.2 Mid-aged agents

For mid-aged agents, the state space is

$$\Omega_M = I\!R_+ \times \{y_L, y_M, y_H\} \times \{0,1\} \times \{h_1, h_2, h_3\} \times I\!N \times \{\{\{FRM, LIP\} \times I\!R_+ \times \{h_2, h_3\}\} \cup \{\emptyset\}\}$$

with typical element $\omega = (a, y, H, h, n; \kappa)$. Here, $H = 1$ denotes that the household begins the period as a homeowner, while $H = 0$ if they begin as renters. Further, $h \in \{h_1, h_2, h_3\}$ denotes the quantity of housing capital that the household owns at the start of a given period once the devaluation shock has been revealed.[10] We write $n \in \{0, 1, \ldots\}$ for the number of periods the agent has been mid-aged, hence the age of their mortgage when they have one.

The final argument, κ denotes the type of mortgage chosen by a homeowner - that is, $\kappa \equiv (\zeta, r^\zeta, h_0) \in \{FRM, LIP\} \times I\!R^+ \times \{h_2, h_3\}$ which lists the agent's mortgage and house choice when they just become mid-aged. In equilibrium, the yield on a given loan will depend on the agent's wealth-income position (a_0, y_0) and house size choice h_0 at origination. For agents who enter a period as renters, the current house size and mortgage type arguments are undefined, and so we simply let $\kappa = \emptyset$.

Working backwards, we begin with the case where the household has already made its home purchase decision (i.e. $n \geq 1$).

Case 1: $n \geq 1$

If the household enters the period as renters (i.e. $H = 0$), they must remain renters:

$$\begin{aligned} V_M(a, y, 0, h_1, n; \emptyset) &= \max_{c,a'} U(c, h_1) + \beta E_{y'|y}\left[(1 - \rho_O)V_M(a', y', 0, h_1, n+1; \emptyset) + \rho_O V_O(a')\right] \\ \text{s.t. } c + a' &= y + a(1+r) - Rh_1. \end{aligned}$$

If, on the other hand, the household owns a home (i.e. $H = 1$), they first have to decide whether to remain homeowners or to become renters. We will write $H'(\omega) = 1$ if they choose to remain home-owners and $H'(\omega) = 0$ if they become renters.

The event $H'(\omega) = 0$ entails a sale of the house of the mortgage contract. As explained in the previous section, we think of that termination as a foreclosure in two cases. First, if it is not budget feasible for the household to meet its mortgage payment $m(n; \kappa)$, that is if,

$$y + a(1+r) - m(n; \kappa) - \delta h < 0, \tag{3.1}$$

[10] We need both H and h to differentiate a renter from a homeowner whose size h_2 received a shock down to h_1.

the household is constrained to become renters. Abusing language somewhat, we call this event an *involuntary default* and in that case write $D^I(\omega) = 1$, while $D^I(\omega) = 0$ otherwise. A second form of default occurs when the household can meet their mortgage payment (i.e. (3.1) does not hold) but the household chooses nonetheless to become renters and

$$qh - b(n; \kappa) < 0, \tag{3.2}$$

i.e. home equity is negative. We call this event a *voluntary default* (the household is better off turning the house over to the intermediary in that case) and write $D^V(\omega) = 1$.

If neither (3.1) nor (3.2) holds but the household decides to sell their house and become renters, we write $S(\omega) = 1$, while $S(\omega) = 0$ otherwise. In that case, the household simply sells their house, pays their mortgage balance, and their asset position is augmented by the value of their home equity.

Note that

$$1 - H'(\omega) = S(\omega) + D^I(\omega) + D^V(\omega).$$

In other words, (S, D^I, D^V) classify a mortgage termination into three mutually exclusive events: a simple sale (in which the intermediary need not get involved), an involuntary default, or a voluntary default.

Equipped with this notation, we can now define the value function of a homeowner (i.e. a household whose $H = 1$):

$$\begin{aligned} V_M(a, y, 1, h, n; \kappa) &= \max_{c \geq 0, a' \geq 0, (H', D^I, D^V, S) \in \{0,1\}^4} U(c, (1 - H')h_1 + H'(1_{h=h_1} + \theta 1_{\{h \neq h_1\}})h) \\ &+ (1 - H')\beta E_{y'|y}\left[(1 - \rho_O)V_M(a', y', 0, h_1, n + 1; \emptyset) + \rho_O V_O(a')\right] \\ &+ H'\beta E_{(y',h')|(y,h)}\begin{bmatrix} (1 - \rho_O)V_M(a', y', 1, h', n + 1; \kappa) \\ +\rho_O V_O(a' + \max\{qh - b(n + 1; \kappa), 0\}) \end{bmatrix} \end{aligned}$$

subject to:

$$\begin{aligned} c + a' &= y + (1 + r)(a + (1 - H')\max((1 - (D^I + D^V)\chi)qh - b(n; \kappa), 0)) \\ &\quad -H'(m(n; \kappa) + \delta h) - (1 - H')Rh_1 \\ D^I &= 1 \text{ if and only if (3.1) holds} \\ D^V &= 1 \text{ if } H' = 0 \text{ and (3.2) holds} \\ S &= 1 - H' - D^I - D^V \end{aligned}$$

There are several things to note in the statement of the household's problem. Starting with the objective, housing services (s) depend on the household's housing status, and the size of the house they occupy. Second, recall that we assumed that housing sales due to the old age shock do not entail foreclosure costs. Third, the right-hand side of the budget constraint depends on whether or not the household chooses to keep its house. When they become renters (i.e. when $H' = 0$) their asset position is increased by the value of the house net of their outstanding principal and in the event of default, net of transaction costs. Their housing expenses are the sum of mortgage and maintenance payments if they keep the house or the cost of rental otherwise. The final constraint states that selling the house without incurring default costs is only possible if the household is able to meet its mortgage obligations and has positive equity.

The house devaluation shock is part of the conditional expectation operator $E_{(y',h')|(y,h)}$ in the problem's statement. Given $h \in \{h_1, h_2, h_3\}$ and the assumptions we made on the devaluation process, next period's house value evolves according to a Markov Chain with transition matrix

$$P(h'|h) = \begin{bmatrix} 1 & 0 & 0 \\ \lambda & 1-\lambda & 0 \\ 0 & \lambda & 1-\lambda \end{bmatrix}.$$

Case 2: $n = 0$ (The agent just became mid-aged)

Agents who become mid-aged at the start of a given period must decide whether or not to buy a house, and in the event they become homeowners, what mortgage to use to finance their house purchase. Write $K(\omega_0)$ for the set of mortgage contracts available to a household that becomes mid-aged in state ω_0. The set $K(\omega_0)$ has typical element $\kappa = (\zeta, r^{\zeta}, h_0)$. The household's value function solves:

$$\begin{aligned} V_M(a,y,1,h,0;\emptyset) &= \max_{c\geq 0, a'\geq 0, H'\in\{0,1\}, \kappa\in K(\omega_0)} U(c,(1-H')h_1 + H'\theta h_0) \\ &+ (1-H')\beta E_{y'|y}\left[(1-\rho_O)V_M(a',y',0,h_1,1;\emptyset) + \rho_O V_O(a')\right] \\ &+ H'\beta E_{(y',h')|(y,h_0)}\begin{bmatrix} (1-\rho_O)V_M(a',y',1,h',1;\kappa) \\ +\rho_O V_O(a' + \max\{qh_0 - b(1;\kappa), 0\}) \end{bmatrix} \end{aligned}$$

subject to:

$$\begin{aligned} c + a' &= y + (1+r)(a - H'\nu 1_{\{\zeta=FRM\}}qh_0) \\ &\quad - H'(m(0;\kappa) + \delta h_0) - (1-H')Rh_1 \\ a &\geq H'\nu 1_{\{\zeta=FRM\}}qh_0 \end{aligned}$$

Households who choose to become homeowners ($H' = 1$) choose the contract $\kappa^* \in K(\omega_0)$ that maximizes their future expected utility. We will write $\Xi(\omega_0) = \kappa^*$ for this part of the household's choice, while $\Xi(\omega_0) = \emptyset$ if $H' = 0$. Note that included in the choice of the contract is the size of the house h_0.

3.1.3 Young agents

For young agents, the state space is $\Omega_Y = I\!R^+ \times \{y_L, y_M, y_H\}$ with typical element $\omega = (a, y)$. The value function $V_Y : \Omega_Y \mapsto I\!R$ for a young agent with assets a and income y solves

$$\begin{aligned} V_Y(a, y) &= \max_{c \geq 0, a' \geq 0} \left\{U\left(c, h_1\right) + \beta E_{y'|y}\left[(1-\rho_M)V_Y(a', y') + \rho_M V_M(a', y', 0, h_1, 0; \emptyset)\right]\right\} \\ \text{s.t. } c + a' &= y + a(1+r) - Rh_1. \end{aligned}$$

3.2 Intermediary's problem

All possible uses of loanable funds must earn the same return for the intermediary. This implies, first, that the unit price q of housing capital must equal $\frac{1}{A}$.[11] Otherwise, the intermediary would enjoy an unbounded profit opportunity turning loanable funds into houses and vice versa.

Arbitrage between renting and selling houses also requires that:

$$\begin{aligned} q &= \sum_{t=1}^{+\infty} \frac{R-\delta}{(1+r)^t} \\ \iff R &= rq + \delta. \end{aligned} \tag{3.3}$$

Note in particular that a change in q must be associated with a change in R in this environment. A bit of algebra also shows that the returns to turning a marginal unit of deposits into housing capital and renting that capital ad infinitum is the same as the returns to storing that marginal unit of deposit.

Arbitrage also requires that for all mortgages issued at a given date, the expected return on the mortgage net of expected foreclosure costs cover the opportunity cost of funds, which by assumption is the returns to storage plus the servicing premium ϕ.

To make this precise, denote the value to the intermediary of a mortgage contract κ held by a mid-aged agent in state $\omega \in \Omega_M$ by $W^\kappa(\omega)$. Again, we need to consider several cases.

- If the homeowner's mortgage is not paid off, so that $\omega = (a, y, 1, h, n; \kappa)$ with $n \in (0, T-1]$, then:

$$\begin{aligned} W^\kappa(\omega) &= \left(D^I(\omega) + D^V(\omega)\right)\min\{(1-\chi)qh, b(n;\kappa)\} + S(\omega)b(n;\kappa) \\ &\quad + \left(1 - D^I(\omega) - D^V(\omega) - S(\omega)\right)\left(\frac{m(n;\kappa)}{1+r+\phi} + E_{\omega'|\omega}\left[\frac{W^\kappa(\omega')}{1+r+\phi}\right]\right) \end{aligned}$$

- If the household just became mid-aged and her budget set is not empty so that $\omega_0 = (a_0, y_0, 0, h_1, 0)$ and, for some contract κ,

$$y_0 + \left(a_0 - \nu q h_0 \cdot 1_{\{\zeta = FRM\}}\right)(1+r) - m(0;\kappa) - \delta h_0 \;\;\geq\;\; 0,$$

[11] Specifically, the intermediary chooses k to solve $\max\ qAk - k$ which implies that $qA = 1$ must hold in equilibrium.

then

$$W^\kappa(\omega_0) = \frac{m(0;\kappa)}{1+r+\phi} + E_{\omega'|\omega_0}\left[\frac{W^\kappa(\omega')}{1+r+\phi}\right]$$

- In all other cases, $W^\kappa(\omega) = 0$.[12]

Then, the expected present discounted value of a loan contract $\kappa = (\zeta, r^\zeta, h_0)$ offered to a household that just turned mid-age with state $\omega_0 = (a_0, y_0, 1, h, 0)$ is $W^\kappa(\omega_0)$. The zero profit condition on a loan contract κ can then be written as

$$W^\kappa(\omega_0) - (1 - \nu 1_{\{\zeta = FRM\}})qh_0 = 0. \tag{3.4}$$

In equilibrium, the set $K(\omega_0)$ of mortgage contracts available to an agent who becomes mid-aged in state ω_0 is the set of contracts that satisfy condition (3.4).

3.3 Distribution of agent states

The household's problem yields decision rules for a given set of prices. In turn, these decision rules imply in the usual way transition probability functions across possible agent states. In the next section we study equilibria in which the distribution of agent states is invariant under those probability functions. This section makes this notion precise.

In our environment, the transition matrix across ages is given by:

$$\begin{bmatrix} (1-\rho_M) & \rho_M & 0 \\ 0 & (1-\rho_O) & \rho_O \\ \rho_D & 0 & 1-\rho_D \end{bmatrix}$$

since the old are immediately replaced by newly born young people. Let (n_Y, n_M, n_O) be the corresponding invariant distribution of ages. The invariant mass of agents born each period is then given by

$$\mu_0 \equiv n_O \rho_D.$$

With this notation in hand, we can define invariant distributions over possible states at each demographic stage.

3.3.1 The young

The invariant distribution μ_Y on Ω_Y solves, for all $y \in \{y_L, y_M, y_H\}$ and $A \subset I\!R_+$:

$$\mu_Y(A, y) = \mu_0 1_{\{0\in A\}}\pi^*(y) + (1-\rho_M)\int_{\omega\in\Omega_Y} 1_{\{a'_Y(\omega)\in A\}}\Pi(y|\omega)\mu_Y(d\omega)$$

[12] Specifically, this is the case when: (i) the agent just turned mid-aged and her budget set is empty; (ii) the agent is a renter; or (iii) the agent has been mid-aged for more than T periods.

where $\pi^*(y)$ is the mass of agents born with income y (in other words, π^* denotes the invariant distribution associated with our Markov process for income), $a'_Y : \Omega_Y \mapsto I\!R_+$ is the saving decision rule for young agents, and, abusing notation somewhat, $\Pi(y|\omega)$ is the likelihood of income draw $y \in \{y_L, y_M, y_H\}$ in the next period given current state $\omega \in \Omega_Y$.

3.3.2 The mid-aged

The invariant distribution for mid-aged households μ_M on Ω_M solves, for all $y \in \{y_L, y_M, y_H\}$, $A \subset I\!R_+$ and $(H, h, n; \kappa) \in \{0,1\} \times \{h_1, h_2, h_3\} \times I\!N \times \{\{\{FRM, LIP\} \times I\!R_+ \times \{h_2, h_3\}\} \cup \{\emptyset\}\}$:

$$\begin{aligned}\mu_M(A, y, H, h, n; \kappa) &= \rho_M \int_{\Omega_Y} 1_{\{(H,h,n)=(0,h_1,0)\}} 1_{\{a'_Y(\omega)\in A\}} \Pi(y|\omega)\mu_Y(d\omega) \\ &+ (1-\rho_0) \int_{\Omega_M} 1_{\{(H'(\omega)=H, n(\omega)=n-1, a'_M(\omega)\in A\}} \Pi(y|\omega) P(h|\omega)\mu_M(d\omega) \\ &\times \left\{1_{\{n(\omega)=0, \Xi(\omega)=\kappa\}} + 1_{\{n(\omega)>0, \kappa=\kappa(\omega)\}}\right\}\end{aligned}$$

where $a'_M : \Omega_M \mapsto I\!R_+$ is the optimal saving policy for mid-aged agents, $n(\omega)$ extracts the contract age argument of ω, $\kappa(\omega)$ extracts the contract type argument of ω, and $P(h|\omega)$ is the likelihood of a transition from state ω to a state where the house size is h.

The first term corresponds to agents who age from young to mid-aged, while the second integral corresponds to agents who were mid-aged in the previous period and do not get old. The indicator functions reflect the fact that agents make their mortgage choice in the first period they become mid-aged but cannot revisit that choice in subsequent periods.

3.3.3 The old

The invariant distribution μ_O on $\Omega_O \equiv I\!R_+$ solves, for all $A \subset I\!R_+$:

$$\mu_O(A) = (1-\rho_D) \int_{\Omega_O} 1_{\{a'_O(\omega)\in A\}} \mu_O(d\omega) + \rho_O \int_{\Omega_M} 1_{\{a'_M(\omega)+\max\{H'(\omega)[qh(\omega)-b(n+1,\kappa)],0\}\in A\}} \mu_M(d\omega)$$

where, for $\omega \in \Omega_M$, $h(\omega)$ extracts the house size argument of ω, while $b(n+1, \kappa)$ is the principal balance on a mortgage of type κ after $n+1$ periods. Recall that we assumed that housing sales due to the old age shock do not entail foreclosure costs.

3.4 Housing market clearing

The housing market capital clearing condition can be stated in simple terms, after some algebra. The total demand for housing (whether rented or owned) in each period is given by:

$$\int_{\Omega_Y} h_1 d\mu_Y + \int_{\Omega_O} h_1 d\mu_O + \int_{\Omega_M} h_1 1_{\{H'=0\}} d\mu_M + \int_{\Omega_M} h 1_{\{H'=1, h(\omega)=h\}} d\mu_M$$

The first two terms give the demand for housing by the young and old agents, who, by assumption, are renters. The third term is demand from mid-aged agents who choose to be renters. The last integral captures mid-aged agents who choose to be homeowners. Their use of housing capital depends on the size of the home that they own.

Similarly, the total quantity of housing available in a given period is the sum of the housing agents carry over from the past period and of the new capital produced by the intermediary. It can be stated formally as:

$$Ak + \int_{\Omega_Y} h_1 d\mu_Y \; + \int_{\Omega_O} h_1 d\mu_O + \int_{\Omega_M} h_1 1_{\{H=0\}} d\mu_M \; + \int_{\Omega_M} h 1_{\{H=1, h(\omega)=h\}} d\mu_M$$

But the laws of motion for agent states in our economy imply that:

$$\int_{\Omega_M} h 1_{\{H=1, h(\omega)=h\}} d\mu_M \; = \int_{\Omega_M} h' 1_{\{H'=1\}} P(h'|\omega) d\mu_M \tag{3.5}$$

where $P(h'|\omega)$ is the likelihood that the agent's house size will be $h' \in \{h_1, h_2, h_3\}$ in the next period given current state $\omega \in \Omega_M$.

It follows that the market for housing capital clears provided

$$\int_{\Omega_M} h 1_{\{H'=1, h(\omega)=h\}} d\mu_M \; - \int_{\Omega_M} h' 1_{\{H'=1\}} P(h'|\omega) d\mu_M \; = Ak, \tag{3.6}$$

where k is the quantity of deposits the intermediary transforms into housing capital each period.

This condition has a very intuitive interpretation. It says that in equilibrium the production of new housing capital must equal the housing capital lost to devaluation. In particular, one easily shows that, in any steady state, we must have $k > 0$. Furthermore, because $q = \frac{1}{A}$ holds in equilibrium, this condition implies that both the rental and the owner-occupied markets clear since the intermediary is willing to accommodate any allocation of total housing capital.

3.5 Definition of a steady state equilibrium

Equipped with this notation, we may now define an equilibrium. A steady-state equilibrium is a set $K : \Omega_M \mapsto \{FRM, LIP\} \times I\!R^+ \times \{h_2, h_3\}$ of mortgages available to households conditional on any possible state upon entering mid-age, a pair of housing capital prices $(q, R) \geq (0, 0)$, a value $k > 0$ of investment in housing capital, agent value functions $V_{age} : \Omega_{age} \mapsto I\!R$ for $age \in \{Y, M, O\}$, saving policy functions $a'_{age} : \Omega_{age} \mapsto I\!R^+$, a mortgage choice policy function $\Xi : \Omega_M \mapsto K(\omega_0)$, a housing policy function $H' : \Omega_M \mapsto \{0, 1\}$, mortgage termination policy functions $D^I, D^V, S : \Omega_M \mapsto \{0, 1\}$, and distributions μ_{age} of agent states on Ω_{age} such that:

1. Household policies are optimal given all prices;

2. $q = \frac{1}{A}$;

3. The allocation of housing capital to rental and the owner-occupied market is optimal for the intermediary. That is, condition (3.3) holds;

4. The market for housing capital clears every period (i.e. (3.6) holds);

5. The intermediary expects to make zero profit on all mortgages. In other words, condition (3.4) holds for all $\omega_0 \in \Omega_M$ and all mortgages in $K(\omega_0)$;

6. The distribution of states is invariant given pricing functions and agent policies.

4 Calibration

We choose our benchmark set of parameters so that a version of our economy with only FRM mortgages matches the relevant features of the US economy prior to 2004-05. As figure 1 shows, FRMs account for around 85% of mortgages and the fraction is mostly stable between 1998 and 2005. Furthermore, evidence available from the American Housing Survey (AHS) suggests that mortgages with non-traditional amortization schedules accounted for a small fraction of the 15% of non-FRMs prior to 2005. Traditional FRMs and traditional (nominally indexed) ARMs account for 95% of all mortgages in the AHS sample before then. At the same time, data available from the Federal Housing Finance Board for fully amortizing loans show no increase in average loan-to-value ratios between 1995 and 2005. These numbers suggest that high-LTV (low downpayment), delayed amortization mortgages accounted for a small fraction of the stock of mortgages and of originations before 2005.

We will think of a model period as representing 2 years. We specify some parameters directly via their implications for certain statistics in our model. These include the parameters governing the income and demographic processes. The other parameters will be selected jointly to match a set of moments with which we want our benchmark economy to be consistent.

We set demographic parameters to $(\rho_M, \rho_0, \rho_D) = (\frac{1}{7}, \frac{1}{15}, \frac{1}{10})$ so that, on average, agents are young for 14 years starting at 20, middle-aged for 30 years, and retired for 20 years. The income process for agents in the first two stages of their life, allowing for the possibility that the process may differ across life stages, are calibrated from the Panel Study of Income Dynamics (PSID) survey. We consider households in each PSID sample whose head is between 20 and 34 years of age to be young while households between 35 and 64 years are considered to be mid-aged. Each demographic group in the 2001 and 2003 PSID surveys is then split into income terciles. The support for the income distribution is the average income in each tercile in the two surveys, after normalizing the intermediate income value for mid-aged agents to 1. This yields a support for the income distribution of young agents of $\{0.2768, 0.7771, 1.8044\}$, while the support for mid-aged agents is $\{0.3086, 1, 2.6321\}$. We assume that income in old age is 0.4. This makes retirement income 40% of median income among the mid-aged, which is consistent with standard estimates of replacement ratios.

We then equate the income transition matrix for each age group to the frequency distribution of transitions across terciles for households who appear in both the 2001 and the 2003 survey and remain in their age category. The resulting transition matrix for young agents is:

$$\begin{bmatrix} 0.7503 & 0.2007 & 0.0490 \\ 0.2180 & 0.5688 & 0.2132 \\ 0.0317 & 0.2305 & 0.7378 \end{bmatrix}$$

while, for mid-aged agents, it is:

$$\begin{bmatrix} 0.7920 & 0.2007 & 0.0490 \\ 0.2180 & 0.5688 & 0.2132 \\ 0.0561 & 0.1356 & 0.7378 \end{bmatrix}$$

The economywide cross-sectional variance of the logarithm of income implied by the resulting distribution is near 0.72, while the autocorrelation of log income is about 0.75.[13] We let the (two-year) risk-free rate be $r = 0.08$, and choose the maintenance cost (δ) to 5% to match the yearly gross rate of depreciation of housing capital, which is 2.5% annually according to Haring et al.(2007).

The terms of FRM contracts are set to mimic the features of common standard fixed-rate mortgages in the US. The down-payment ratio ν is 20% while the maturity T is 15 periods, or 30 years. The LIP contract we introduce in the second equilibrium have $n^{LIP} = 3$ and $T = 15$ so that agents make no payment toward principal for 6 years and make fixed payments for the remaining 12 contract periods (or 24 years) unless the contract is terminated before maturity.

Housing choices depend on the substitutability of consumption and housing services as well as the owner-occupied premium. We specify, for all $(c, h) > (0, 0)$,

$$U(c, s) = \psi \log c + (1 - \psi) \log s.$$

The intertemporal discount rate, likewise, plays a key role in our model by affecting asset accumulation. Preferences are fully described by (θ, ψ, β). We select these parameters in our joint calibration, to which we now turn.

We need to set the following ten remaining parameters: the owner-occupied premium (θ), households' discount rate (β), housing TFP (A), rental unit size (h_1), house sizes (h_2, h_3), the mortgage service premium (ϕ), the foreclosure cost (χ), the utility weight on consumption (ψ), and the house shock probability (λ). We select those parameters jointly to target: homeownership rates, the average ex-housing to income ratio among homeowners, the average

[13] Krueger and Perri (2005) report estimates for the cross-sectional variance of log yearly income of roughly 0.4 and for the autocorrelation of log income in the $[0.80 - 0.95]$ range. These numbers imply that log two-year income has an autocorrelation in the $[0.88 - 0.96]$ range and variance in the $[0.36 - 0.39]$ range. The details of the conversion from one-year to two-year numbers are available upon request. The difficulty is that aggregating an MA(1) process leads to an ARMA(1,1) process.

loan-to-income ratio at mortgage origination, the average ratio of rents to income in personal consumption expenditures across all households, the average rent-to-income ratio for low-income renters, the average housing spending share for homeowners, the average yields on FRMs, the average loss severity rates on foreclosed properties, the average foreclosure rates prior to the flare up, and the average market discount on foreclosed houses.

We now elaborate on our approach to measuring target values. Since our model only gives agents a one-time option to become owners when they just become mid-aged, we choose to target the ownership rate among households whose head is between 35 and 44. The Census Bureau reports that rate is roughly $\frac{2}{3}$.[14] The model's counterpart to that number is the rate of ownership among agents who have been mid-aged for three periods or fewer. This is the rate we will report throughout the paper.

The average non-housing assets to yearly income ratio we choose to target is based on Survey of Consumer Finance (SCF) data. The average ratio of non-housing assets to income[15] among homeowners whose head age is between 34 and 63 in the 2004 survey is 2.09, which corresponds to a ratio of assets to two-year worth of income of roughly 0.95.

The mortgage loan at origination $(1-\nu)hq$ for FRMs and hq for LPMs, where $h \in (h_2, h_3)$ is the initial house size. Evidence available from the American Housing Survey (AHS) suggests that prior to 2005 the ratio of this original loan size to yearly income is around 2.5 on average in the US, or 1.25 in two-year terms.

According to the evidence available from the Bureau of Economic Analysis, the ratio of housing expenditures (in imputed rent terms for owners) to overall expenditures is near 20%, and we make this our fourth target. Turning to the rent-to-income ratio for poor renters, Green and Malpezzi (1993, p11) calculate that poor households who are renters spend roughly 40% of their income on housing. On the other hand, according to the 2004 Consumer Expenditure Survey, expenditures on privately owned dwellings account for 16% of the expenditures of home-owners.

Next, we choose to target an average FRM-yield of 7.2% yearly, or 14.5% over a two-year period. This was the average contract rate on conventional, fixed rate mortgages between 1995 and 2004 according to Federal Housing Finance Board data.

The loss severity rate is the present value of all losses on a given loan as a fraction of the default date balance. As Hayre and Saraf (2008) explain, these losses are caused both by transaction and time costs associated with the foreclosure process, and by the fact that foreclosed properties tend to sell at a discount relative to other, similar properties. Using a dataset of 90,000 first-lien liquidated loans, they estimate that loss severity rates range from around 35% among recent mortgages to as much as 60% among older loans. Based on these

[14] See http://www.census.gov/hhes/www/housing/hvs/annual08/ann08ind.html, table 17.

[15] Because agents only have one asset in the model, we interpret a as net assets. Our measure of net assets do not include housing-related assets or debts, such as home equity or mortgages. Since agents are not allowed to have negative assets in our model, households who have negative non-housing assets are assumed to have zero assets in the calculation.

numbers we choose parameters so that in the event of default and on average,

$$\frac{\min\{(1-\chi)qh, b)}{b} = 0.5$$

where b is the outstanding principal at the time of default and qh is the house value. In other words, on average, the intermediary recovers 50% of the outstanding principal it is owed on defaulted loans.

We target a two-year default rate of 3% which is near the average foreclosure rate among all mortgages during the 1990s in the Mortgage Bankers Association's National Delinquency survey.

Finally, we target a market discount on foreclosed properties of 25%. We define this discount to be the average price of foreclosed properties divided by the average price of regular home sales, after conditioning on size at origination.[16] Hayre and Saraf (2008) estimate that foreclosed properties sell at a discount relative to their appraised value that ranges from 10% among properties with appraisal values over $180,000 to 45% among properties with appraisal values near $20,000. Other studies of foreclosure discounts (see Pennington-Cross, 2004, for a review) typically find discount rates near one quarter, with some exceptions.

5 Steady State Results

Our goal is to quantify the importance of contracts with back-loaded payments for the recent rise in U.S. foreclosures. To do so, in the previous section we selected parameters so that a version of our economy where LIPs are not available make steady state predictions for key statistics that match their US counterparts prior to the explosion of new mortgages around 2004-05. We then study the quantitative impact of introducing mortgages with low initial payments in such an economy. In this section we study the effects of a permanent introduction of nonstandard mortgages comparing steady state statistics across the two economies. In contrast, Section 6 studies the effect of a brief period of availability of these mortgages that ends with a collapse in house prices, and compares the features of the resulting transition experiment to the patterns displayed in figure 1.

[16] Formally,

$$\text{Foreclosure Discount} \equiv m(h_2)\frac{\frac{\int_{\{\Omega_M:h_0(\omega)=h_2\}}(D^I(\omega)+D^V(\omega))qhd\mu_M}{\int_{\{\Omega_M:h_0(\omega)=h_2\}}(D^I(\omega)+D^V(\omega))d\mu_M}}{\frac{\int_{\{\Omega_M:h_0(\omega)=h_2\}}S(\omega)qhd\mu_M}{\int_{\{\Omega_M:h_0(\omega)=h_2\}}S(\omega)d\mu_M}} + m(h_3)\frac{\frac{\int_{\{\Omega_M:h_0(\omega)=h_3\}}(D^I(\omega)+D^V(\omega))qhd\mu_M}{\int_{\{\Omega_M:h_0(\omega)=h_3\}}(D^I(\omega)+D^V(\omega))d\mu_M}}{\frac{\int_{\{\Omega_M:h_0(\omega)=h_3\}}S(\omega)qhd\mu_M}{\int_{\{\Omega_M:h_0(\omega)=h_3\}}S(\omega)d\mu_M}}$$

where h_0 is the house size at origination, while $m(h_2)$ and $m(h_3) = 1$ are the share of contract of each possible initial size in defaults, i.e., for $h_0\{h_2, h_3\}$:

$$m(h_0) = \frac{\int_{\{\Omega_M:h_0(\omega)=h_0\}}(D^I(\omega)+D^V(\omega))d\mu_M}{\int_{\{\Omega_M:h_0(\omega)=h_3\}}(D^I(\omega)+D^V(\omega))d\mu_M + \int_{\{\Omega_M:h_0(\omega)=h_3\}}(D^I(\omega)+D^V(\omega))d\mu_M}.$$

Table 1: **Benchmark parameters**

Parameter	Description	Value	Target
Parameters determined independently			
ρ_M	Fraction of young agents who become mid-aged	1/7	14 years of earnings on average prior to home purchase
ρ_O	Fraction of mid-aged agents who become old	1/15	30 years on average between home purchase and retirement
ρ_D	Fraction of old agents who die	1/10	20 years of retirement on average
r	Storage returns	0.08	2-year risk-free rate
δ	Maintenance rate	5%	Residential housing gross depreciation rate
ν	Downpayment on FRMs	0.20	Average Loan-to-Value Ratio
T	Mortgage maturity	15	30 years
n^{LIP}	Interest-only period for LIPs	3	6-years interest-only
Parameters determined jointly			
θ	Owner-occupied premium	10	Homeownership rates
λ	Housing shock probability	0.08	Foreclosure rates
A	Housing technology TFP	0.5	Average Loan-to-income ratio at origination
β	Discount rate	0.825	Average ex-housing asset-to-income ratio
ϕ	Mortgage service cost	0.04	Average mortgage yields
χ	Foreclosing costs	0.525	Loss-incidence estimates
ψ	Utility share on consumption	0.8	Average housing spending share
h_1	Size of rental unit	0.55	Rent-to-income ratio for low-income agents
h_2	Size of regular house	0.75	Owner's housing spending share
h_3	Size of luxury house	1.5	Foreclosure discount

5.1 Mortgage Innovation

The benchmark economy only has FRM mortgages available. Table 2 presents some key steady state equilibrium aggregate statistics for this benchmark environment compared to an environment in which LIPs are available.

Table 2: Steady state statistics

	Data	Benchmark	FRM +LIP
Homeownership rate	67.00	65.38	71.71
Avg. ex-housing asset/income ratio	0.95	0.91	0.89
Avg. loan to income ratio	1.25	1.20	1.30
Avg. homeowner housing expenditure share	0.20	0.19	0.19
Rents to income ratio for renters	0.40	0.37	0.37
Avg. housing spending share for homeowners	0.16	0.19	0.21
Avg. mortgage yields (FRMs, LIPs)	(14.50,NA)	(14.40,NA)	(14.32,18.55)
Loss-incidence estimates	0.50	0.59	0.60
Foreclosure rates	3.00	2.72	4.06
Foreclosure discount	0.25	0.38	0.35

The table shows that the presence of LIPs has two main consequences on steady state statistics: home-ownership rates and average default rates are much higher when LIPs are available than when they are not. When only FRMs are available, a large number of agents are unable to become homeowners because they can't afford a large downpayment.

Default rates, for their part, are higher when LIPs are present as a result of two complementary factors. First, LIPs enable agents at the bottom of the asset and income distributions to select into homeownership. These are high-default risk agents because they are more likely to find themselves unable to meet their mortgage payments at some point over the life of the contract. Second, even at equal asset/income conditions at origination, LIPs are associated with higher default rates because agents build up home equity slower than with FRMs. The next two sections make these ideas precise.

5.2 Selection

This section describes the selection of contracts and the resulting equilibrium distribution of contracts. Since we allow mortgage contracts to depend a household's earnings and asset characteristics at the time of origination, selection depends on the distribution of earnings and assets of households at the time of purchase (which in our model occurs when they turn middle aged). Conversely, making LIPs available impacts the equilibrium distribution of wealth at purchase time, since a major incentive to save in the benchmark economy with FRMs only is the need to make a downpayment on a house.

Figure 3 plots the endogenous distribution of assets among agents that just turned middle-aged. In the benchmark experiment, the upper panel shows that, quite intuitively, low income agents tend to have low assets, and vice-versa. The lower panel shows the change in the distribution when LIPs are introduced. There is a noticeable shift to the left in the distribution as many agents anticipate that they may resort to the LIP option and no-longer need to accumulate assets to meet downpayment requirements. In fact, the average level of assets of agents who just became mid-aged in the economy with LIPs is lower by 7% than its counterpart in the economy with FRMs only (0.5864 vs. 0.6283.)

Figure 3: Distribution of assets upon entering mid-age

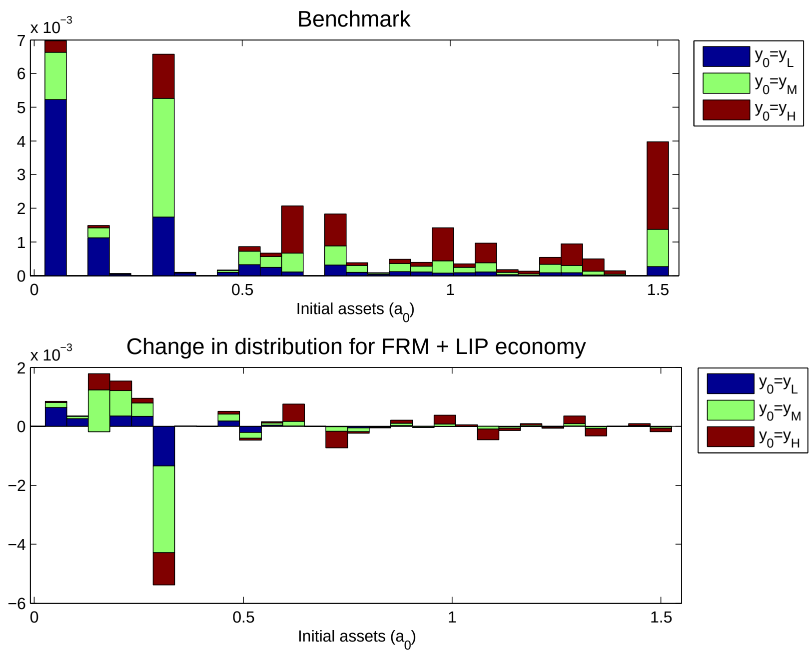


Table 3 displays contract selection patterns in steady state. It shows, first, that when LIPs are not available, many agents are constrained to rent because they cannot meet the downpayment imposed by mortgages and/or cannot make the first payment. This is true in particular of agents whose assets (a_0) are low when they become mid-aged. Introducing LIPs enables some agents at the bottom of the asset distribution to become homeowners instead of renting, as the bottom panel of the table shows. This is true, in fact, of all agents except those at the bottom of the income distribution. The table also shows that the introduction of LIPs enables agents with high-income but low assets to buy bigger houses than they would without that option. These agents can afford high mortgage payments, but their assets are too low to meet high downpayment requirements.

Figure 4 displays the relation between mortgage choices and asset and income levels.

Figure 4: Distribution of contract choice by asset and income level

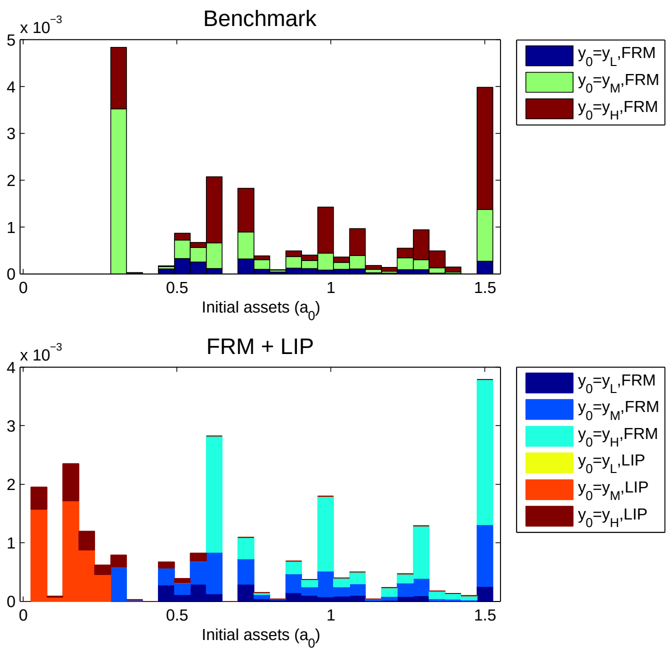


Table 3: Rent-or-own decision rules by asset and income group

Contract	Rent	LIP		FRM	
House size	h_1	h_2	h_3	h_2	h_3
Benchmark					
y_L	$a_0 < 0.36$	–	–	$0.36 \leq a_0 < 4.80$	$4.80 \leq a_0$
y_M	$a_0 < 0.30$	–	–	$0.30 \leq a_0 < 3.15$	$3.15 \leq a_0$
y_H	$a_0 < 0.30$	–	–	$0.30 \leq a_0 < 0.60$	$0.60 \leq a_0$
FRM + LIP					
y_L	$a_0 < 0.36$	–	–	$0.36 \leq a_0 < 4.80$	$4.80 \leq a_0$
y_M	–	$a_0 < 0.30$	–	$0.30 \leq a_0 < 3.15$	$3.15 \leq a_0$
y_H	–	–	$a_0 < 0.59$	–	$0.59 \leq a_0$

When the LIP option is introduced (as we go from the top to the bottom panel of the figure), agents at the bottom of asset distribution become able to purchase homes. The figure also shows that LIPs are the contract of choice for agents at the bottom of the asset distribution, whereas wealthier agents take an FRM (to take advantage of lower rates, as the next section will discuss.)

All told, the availability of LIPs cause homeownership rates to rise by giving agents more financing options. The fraction of newly mid-aged agents who enter housing markets and buy smaller houses rises from 59.74% to 70.69% when the LIP option is introduced. In addition, the fraction of agents who buy large houses rises from 40.26% to 45.33% as a result of the looser financial requirements imposed by LIPs.

Overall, LIPs turn out to be selected by roughly 29% of newly mid-aged households, and tend to be selected by households whose assets are low. The next section argues that, holding contract terms fixed, poor agents are more likely to default than other agents. In addition, it shows that LIPs, holding initial asset to income position fixed, are inherently more prone to default. Combined, these facts imply that LIP-holders account for a disproportionate share of overall default rates, and explains why default rates are higher in the economy where the LIP option is present than in the economy where only FRMs are available.

5.3 Default

We have classified default associated with two events: first, if the household can meet its mortgage payment but the net equity in their house is negative and they decide to walk away from it, we call this event a *voluntary default*; and second, if it is not budget feasible for the household to meet its mortgage payment, we call this event an *involuntary default*.

To understand the first event, the evolution of the principal balance and home equity as a function of maturity for each type of contract is displayed on figure 5. While FRM contracts feature a progressive reduction of mortgage debt and a corresponding increase in home equity, LIP contracts only begin this process after three periods. The result is a much greater risk that agents will find themselves with negative equity following a devaluation shock. The dotted lines at the bottom of the figure show home equity following a devaluation from h_2 to h_1 as a function of maturity. The shock causes equity to become much more negative at any given maturity for LIPs than for FRMs.

Figure 6 illustrates the impact of contract choices on overall default hazard rates for an agent whose initial asset-income position when she becomes mid-aged is $(a_0, y_0) = (1.07, 1)$, the median values of both arguments. If agents experience a housing devaluation shock, home equity is more likely to become negative for agents with LIPs than for agents with FRMs, which is reflected in the voluntary default pattern. Neither type of contract displays any voluntary default at these median origination characteristics. However, LIPs are selected by some agents with very low initial assets, and involuntary defaults do occur on LIPs in equilibrium, as table 4 will reveal. Sale rates, for their part, peak between periods 3 and 5 of both types of mortgage contracts. When we introduce a surprise aggregate price shock in our transition experiment, many of these sales will involve negative equity and, therefore, become foreclosures, which accounts for the fact that foreclosure rates peak a few periods after the initial shock rather than on impact.

Selection and home-equity accumulation effects imply that, in equilibrium, the frequency of default is much higher among LIP-holders than it is among FRM-holders. Table 4 provides

Figure 5: Mortgage debt and home equity by contract type

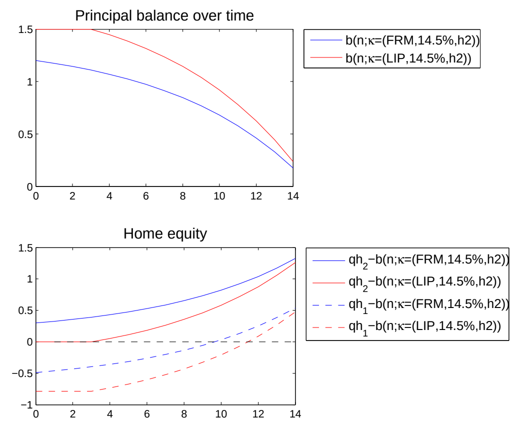
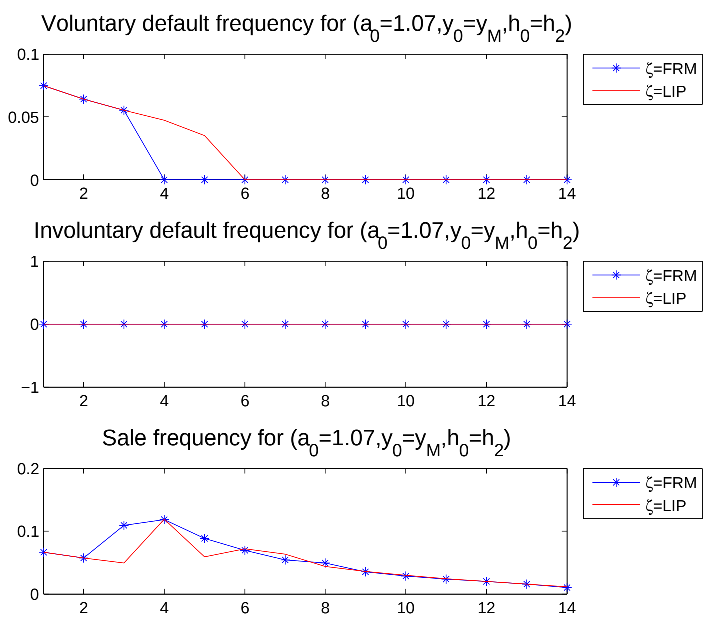


Figure 6: Default frequency patterns by contract type



a breakdown of default frequencies by contract type across experiments. Each entry gives the fraction of mortgages of each type that go into default in steady state in each of the two economies we consider.[17]

The table also shows that involuntary defaults – defaults occuring when the agent is unable

[17]In the notation we introduced in section 3.1.2, involuntary and voluntary default rates on a FRM contracts

Table 4: Default rates by mortgage type

	voluntary	involuntary
Benchmark		
FRM	2.64	0.08
FRM + LIP		
FRM	2.49	0.10
LIP	4.38	3.59

to meet current obligations – are rare in the benchmark economy, but account for around a quarter of defaults in the economy with LIPs. However, even in the second economy, the vast majority (99%) of defaults involve negative equity. Agents with recently issued LIPs who find themselves with negative equity continue to meet payments as long as they are low, and wait until the payment reset to default. Because the payment jumps up markedly in that period, a non-negligible of agents are effectively in an involuntary default situation. But negative equity plays the determinant role, even in those cases. Agents who have positive equity in their house and foresee that they may find themselves in an involuntary default situation tend to sell rather than run the risk of losing their equity to transaction costs.

Several steady state statistics illustrate this behavior. Consider for instance the set of households who, should they choose to keep their house, face a positive probability of being in an involuntary default situation in the next period. Almost 82% of these high-risk households choose to sell their house[18], while selling rates are below 6% among other mortgage holders. Conversely, among agents who choose to sell in a given period, the probability that they would be in an involuntary default situation in the next period should they choose not to sell

are given by, respectively:

$$\frac{\int_{\Omega_M} D^I(\omega) 1_{\{\zeta=FRM,n<T,H=1\}} d\mu_M(\omega)}{\int_{\Omega_M} 1_{\{\zeta=FRM,n<T,H=1\}} d\mu_M(\omega)}$$

$$\text{and} \qquad \frac{\int_{\Omega_M} D^V(\omega) 1_{\{\zeta=FRM,n<T,H=1\}} d\mu_M(\omega)}{\int_{\Omega_M} 1_{\{\zeta=FRM,n<T,H=1\}} d\mu_M(\omega)}.$$

Similar expressions give default rates for LIPs.

[18]Let $\gamma(\omega)$ be the probability of homeowners facing a positive probability of being in an involuntary default situation next period if they stay in their house. Specifically, $\gamma(\omega) = E_{\omega'|\omega,H'=1}\left[1_{\{D^I(\omega')=1\}}\right]$. The probability that a household sells its house when this probability is positive is then given by

$$\frac{\int_{\Omega_M} 1_{\{S(\omega)=1,\gamma(\omega)>0,n<T,H=1\}} d\mu_M(\omega)}{\int_{\Omega_M} 1_{\{\gamma(\omega)>0,n<T,H=1\}} d\mu_M(\omega)}$$

.

is 80%[19]. Among households who do not sell, that probability is 38%. In other words, almost all households at a risk of imminent involuntary default choose to sell. Since households with positive equity stand to lose the most by defaulting for involuntary reasons, it is not surprising that most households who do end up in a situation of involuntary default have negative equity.

While the vast majority of foreclosures involve negative home equity, many households (roughly 96%) with negative home equity choose to keep their house and continue meeting their mortgage obligations. While defaulting would entail a net worth gain for these households, they would be forced to rent a smaller housing unit and would forego the ownership premium.

These model predictions are consistent with the empirical literature on the determinants of foreclosure (see, e.g., Gerardi et al., 2007.) Available data suggest that most foreclosures involve negative equity but that, at the same time, most households with negative equity choose not to foreclose. Our model captures the fact that most foreclosures involve a combination of negative equity and adverse income shocks.

Table 5: Share of overall default rates

	voluntary	involuntary	total
Benchmark			
FRM	2.64	0.08	**2.72**
FRM + LIP			
FRM	1.82	0.07	1.89
LIP	1.19	0.98	2.17
Total	3.01	1.05	**4.06**

Since LIPs are characterized by much higher default rates than FRMs, they account for a disproportionate fraction of the overall default rate. Table 5 shows the contributions of each contract type to each type of default rate in each of the scenarios we consider.[20] The

[19]The probability is given by

$$\frac{\int_{\Omega_M} 1_{\{S(\omega)=1,\gamma(\omega)>0,n<T,H=1\}} d\mu_M(\omega)}{\int_{\Omega_M} 1_{\{S(\omega)=1,n<T,H=1\}} d\mu_M(\omega)}$$

.

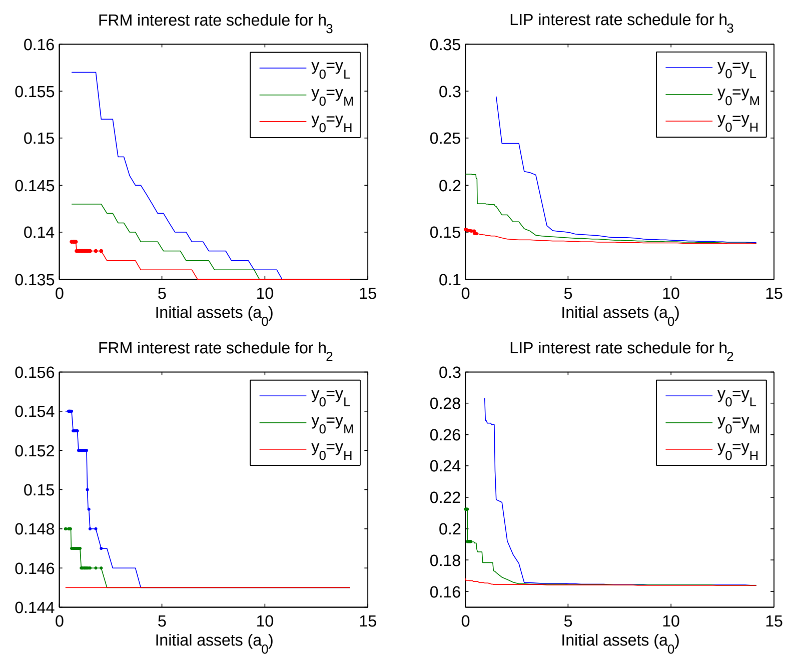
[20]For instance, the contributions of FRM contracts to involuntary default rates is given by the share of FRM mortgages in the total stock of mortgages in steady state times the rate of involuntary default on FRMs:

$$\left(\frac{\int_{\Omega_M} 1_{\{\zeta=FRM,n<T,H=1\}} d\mu_M(\omega)}{\int_{\Omega_M} 1_{\{\zeta\in\{FRM,LIP\},n<T,H=1\}} d\mu_M(\omega)} \right) \times \left(\frac{\int_{\Omega_M} D^I(\omega) 1_{\{\zeta=FRM,n<T,H=1\}} d\mu_M(\omega)}{\int_{\Omega_M} 1_{\{\zeta=FRM,n<T,H=1\}} d\mu_M(\omega)} \right).$$

table shows that LIPs account for nearly 53% of overall default rates even though they only represent 27% of all mortgages.

5.4 The Distribution of Interest rates

Figure 7: Equilibrium yield schedules for the FRM+LIP economy

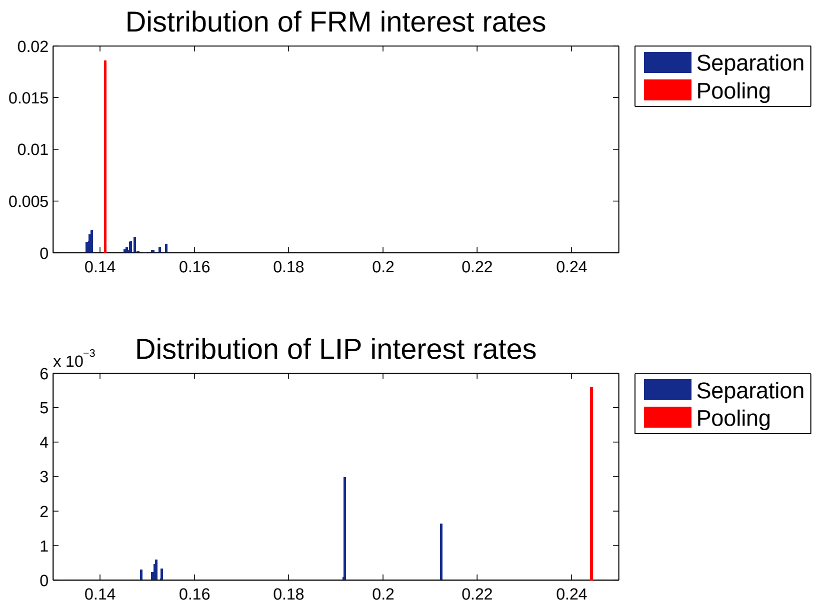


Notes: Dots show the contracts selected with positive probability in equilibrium.

A distinguishing feature of our model is that mortgage terms depend not only on mortgage types but also on the initial asset and income position of borrowers as well as the size of the loan. Figure 7 plots the menu of equilibrium FRM and LIP rate offerings agents can obtain from the intermediary when they become middle-aged, depending on the house size they opt for and their asset-income position at origination. Note that these are offerings and only some points on these menus will be selected in equilibrium.

All interest rate schedules in Figure 7 are left-truncated because agents whose income and assets are low do not get a mortgage in equilibrium. The left truncation can be thought of as an endogenous borrowing constraint associated with different borrower characteristics. The left truncation occurs for several reasons. First, asset and income poor agents cannot meet the down-payment requirement and/or mortgage payments. Second, these agents are more likely to default, hence receive less favorable borrowing terms. In some cases there is no yield

Figure 8: Distribution of equilibrium interest rates for FRM + LIP model



such that the intermediary would expect to break even on the mortgage, even when the agents have the means to finance the initial downpayment.[21]

Among agents who do receive a mortgage offer, yields fall both with assets and income. This prediction accords well with the well-documented mortgage industry practice of including overall debt-to-income ratios in their rate sheets. It is also borne out by the statistical evidence available from the Survey of Consumer Finance. Figure 7 also shows that conditional on a given asset-income position at origination, yields are higher for agents who opt for large houses than agents who opt for small houses in originations for low asset households. This prediction of our model is consistent with the well-documented fact that mortgage rates rise with borrowers' loan-to-income ratio.

When LIPs are not available (as in the benchmark case), agents face only the FRM interest rate schedule.[22] Several facts are immediately apparent. First, a glance at the vertical scale of the figure reveals that LIP rates exceed FRM rates at all possible asset-income positions. This is because LIPs entail a greater risk of default since home equity is slower to rise. The

[21] In that period (i.e. when $n = 0$), the budget set is empty when $c = a' = 0$ and

$$m(0; \kappa) > y_0 + (a_0 - vqh \cdot 1_{\{\zeta = FRM\}})(1 + r).$$

Since $m(0; \kappa)$ is strictly increasing in r^{κ}, we know there is an interest rate $\overline{r}^{\zeta}$ that depends on y_0 and a_0 such that for any $r > \overline{r}^{\zeta}$ the bank cannot break even.

[22] Yields offered on FRMs are the same in the benchmark and FRM+LIP economies. This is because the house price is unchanged and there are no externalities.

likelihood that an agent will find herself with negative equity in her home is higher when she holds a LIP than when she holds an FRM. Furthermore, LIP payments are concentrated over a lower number of periods hence become large after three contract periods, which makes involuntary default a greater possibility.

Figure 8 graphs the distribution of equilibrium interest rates by mortgage type when both mortgages are available. There is relatively little variation in FRM rates, but the distribution of LIP yields displays a lot of dispersion, and a clear bimodal pattern. Agents who take advantage of the new mortgage to become home-owners – that is, agents at the bottom of the income distribution who represent a high-risk of default – incur rates in excess of 19%. One could think of this segment of borrowers as the subprime market. On the other hand, agents who switch from FRMs to LIPs either to purchase bigger homes or because they find it optimal to delay payments receive rates not unlike those offered on FRMs, because their default risk does not rise much as a result of the change. The "no separation" red bar on figure 8 shows the yield that would prevail in equilibrium if intermediaries were not allowed to make yields contingent on asset and income, nor house size, at origination. We will discuss this equilibrium later in this section.

Table 6: Contract terms moments

	Data	Benchmark	FRM + LIP
$CV(yield)$ for FRMS	0.153	0.0184	0.0207
$CV(yield)$ for other	0.341	NA	0.2958
$\rho(yield, income)$ on FRMs	-0.12	-0.9272	-0.9857
$\rho(yield, income)$ on other	-0.18	NA	-0.9391
$\rho(yield, net\ \ worth)$ on FRMs	-0.023	-0.4666	-0.3691
$\rho(yield, net\ \ worth)$ on other	-0.141	NA	-0.5379

The correlation of yields on various contracts with assets and income is qualitatively consistent with the statistical evidence available from the Survey of Consumer Finance, as table 6 shows. To compute these moments in the data, we looked at all the mortgages issued within the two years prior to the 2004 survey. We restrict the sample to recently issued mortgages so that current income and assets are reasonable proxies for their counterparts at origination time. We also restrict our attention to households whose head age is between 30 and 45 since mortgages are only issued at the middle-age stage in our model. These data show that origination yields are negatively correlated with both net worth and income, particularly with income.[23] Restricting the sample to FRMs reduces the correlation with assets, but the

[23]We define net worth as liquid assets, CDs, stocks, bond, vehicles, primary residence, real estate investment, business interest minus housing debts, credit card, installment debts, and line of credits. This notion of net worth includes housing equity because we observe agents shortly after the mortgage origination. Housing equity, at that time, reflects mainly the down-payment made at origination by the borrower. That downpay-

correlation with income remains strong.

Notice that the model predicts less volatility in yield in the FRM sample than in the LIP sample, despite the fact that the range of FRM yields is much broader than its counterpart for LIPs. One reason for this (see the bottom panel of figure 4) is that the distribution of wealth is more highly concentrated among agents who hold an FRM than among agents who hold a LIP.

The table shows, however, that our model understates the variation in yields suggested by these data and overstates the degree to which income and yields are correlated. A key reason for both findings is that the SCF sample of both FRMs and other mortgages are characterized by much heterogeneity in maturity and initial loan-to-value ratios which we do not model and for which SCF data do not enable one to control. This heterogeneity raises the volatility of yields and reduces the correlation with asset and income for reasons which our model cannot replicate.

Given the monotonicity of rates and mortgage availability in asset and income, ownership rates are also monotonic in assets and income, as the top panel of table 3 illustrates. Overall, home ownership-rates are near 72% as the third column of table 2 shows.

5.5 Separation matters

In this section, we conduct counterfactual experiments to examine the importance of allowing intermediaries to offer mortgage contracts that separate households on these characteristics rather than offering only noncontingent or "pooling" FRM and LIP contracts (as in Garriga and Schlagenhauf (2009)). In the second equilibrium concept, the unique equilibrium mortgage rate for each mortgage type is determined by a zero expected profit condition across all households selecting into that contract (and hence the distribution of households directly affects the calculation of mortgage rates).[24] Low-risk borrowers, in such an equilibrium, subsidize high-risk borrowers and, in particular, the intermediary issues contracts on which it expects to lose money. Such cross-subsidization seems unlikely to survive in competitive environments since an intermediary can simply offer a contract with lower interest rate to households with observable high income and/or assets and skim those good customers away from the pooled contract.

There are sizeable differences in steady state statistics between the pooling and separating equilibria in an economy in which both LIPs and FRMs are available. Since LIPs enable high-default risk agents to enter, the intermediary is able to offer different yields to different borrowers in a separating equilibrium. Not surprisingly then, forcing the intermediary to offer the same yield on all contracts of the same type regardless of their asset/income position or their loan size at origination causes steady state statistics to change markedly in this case. As the last column of table 7 shows, the home-ownership rate rises significantly because

ment, in turn, was part of assets prior to the origination.

[24] A formal definition of the intermediary's net profit is provided in appendix A.3.

the intermediary now issues contracts to borrowers on whom it expects to lose money. The foreclosure rate, correspondingly, rises as well, by a full 25%.

Other noticeable changes include a huge increase in average LIP rates as the average credit quality of borrowers worsens, and perhaps surprisingly, lower incidence rates and foreclosure discounts. Loss incidence rates fall because a higher fraction of borrowers take on small houses. This, in turn, is because the high rate offered on LIPs prevents some borrowers from using them to upgrade house sizes and because most new entrants in the housing market tend to have low assets and income. Our calibration implies higher foreclosure discounts on large houses than small houses ($\frac{h_3-h_2}{h_3} > \frac{h_2-h_1}{h_2}$), and the greater fraction of small houses thus leads to lower losses for the intermediary in the event of default.

LIP rates rise so much when pooling is imposed on the intermediary that the equilibrium rate is outside of the range of the LIP rates that prevail in the separation equilibrium. This happens because the low-default risk agents who take LIPs in the separation equilibrium (in order to purchase a bigger home) now take FRMs to avoid high interest rates and because pooling enable high-default risk agents to enter the housing market. Since these high-risk agents are typically agents with low assets, they are constrained to opt for LIPs rather than FRMs. As a result of both forces, the pool of LIP borrowers is of bad credit quality in equilibrium, forcing the intermediary to charge high rates on this line of mortgage.

Table 7: The role of separation

	FRM+LIP, separating	FRM+LIP, pooling
Homeownership rate	71.71	74.28
Ex-housing asset/income ratio	0.89	1.15
Loan to income ratio	1.30	1.39
Homeowner housing expenditure share	0.19	0.18
Rents to income ratio for renters	0.37	0.37
Housing spending share for homeowners	0.21	0.23
Avg. mortgage yields (FRMs)	(14.32,18.55)	(14.11, 24.39)
Loss-incidence estimates	0.60	0.41
Foreclosure rates	4.06	5.03
Foreclosure discount	0.35	0.31

In summary, pooling terms has a large impact on steady state statistics. Therefore, not only is the pooling equilibrium inconsistent a competitive equilibrium concept and with the data (i.e. zero dispersion of mortgage rates within FRM and non-FRM contracts), but if one applies this equilibrium concept it has a big impact on the model's prediction of aggregate default probabilities and home-ownership rates. The results show in particular that the impact of introducing LIPs is much more significant when, for whatever reason, the intermediary fails to incorporate all relevant information when pricing mortgages.

5.6 Welfare

The introduction of LIPs unambiguously improve the welfare of all agents because they provide households with a new financing option without altering house prices given our CRS technology. However, the welfare consequences of innovation are bound to differ across agents. Agents whose homeownership prospects at birth are not significantly improved by the introduction of LIPs will not benefit much, while agents whose ownership prospects do rise significantly are likely to see their welfare rise markedly. This section verifies this intuition. To determine the gains, we calculate what agents would be willing to pay at birth with an income draw of y_i in the benchmark (FRM-only) economy to obtain the same welfare they can expect in an FRM+LIP economy. To calculate such consumption equivalent welfare gains under our benchmark parameterization, consider agents born with income at birth y_i where $i \in \{L, M, H\}$ and let $U^{bench}(y_i)$ and $U^{FRM+LIP}(y_i)$ denote the lifetime utility they expect at birth in the benchmark and $FRM + LIP$ economies, respectively. Denote the optimal consumption and housing service plans in the benchmark economy by $\left\{c_{t,i}^{bench}, s_{t,i}^{bench}\right\}$ for an agent born with initial income y_i. Then, let $1 + k_i$ be the multiple one has to apply to the consumption of agents born in the benchmark economy to make their welfare equal the same agents born in an FRM+LIP economy. That is, k_i solves for all $i \in \{L, M, H\}$:

$$\begin{aligned}
U^{FRM+LIP}(y_i) &= E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_{t,i}^{bench}(1+k_i), s_{t,i}^{bench})\right] \\
&= E_0\left[\sum_{t=0}^{\infty}\beta^t\left\{\ln(c_{t,i}^{bench}) + \ln(1+k_i) + \ln(s_{t,i}^{bench})\right\}\right] \\
&= U^{bench}(y_i) + \frac{\ln(1+k_i)}{(1-\beta)}
\end{aligned}$$

It follows that:

$$\begin{aligned}
(1-\beta)\left[U^{FRM+LIP}(y_i) - U^{bench}(y_i)\right] &= \ln(1+k_i) \\
\Longrightarrow k_i &= \exp\big((1-\beta)[U^{FRM+LIP}(y_i) - U^{bench}(y_i)] - 1\big)
\end{aligned}$$

We find that:

$$\begin{aligned}
k_L &= 0.035\% \\
k_M &= 0.053\% \\
k_H &= 0.175\%
\end{aligned}$$

making the average welfare gain associated with availability of the LIP option around 0.07% in consumption-equivalent terms.

Agents who receive high incomes at birth obtain the highest welfare gain. This is because these agents are the most likely to opt for homeownership when they become mid-aged and they make use of LIPs either to make a downpayment if their income history turns out to limit their asset accumulation or to buy a bigger house. Somewhat surprisingly at first glance, agents born with low income prospects benefit the least from mortgage innovation. The reason for this is that in all likelihood they will remain renters their entire life. The gains are so small, in fact, that in a model where house prices respond to demand for housing, mortgage innovation is likely to have a negative impact on agents who are born poor. Mortgage innovation primarily benefits the agents who are at the margin between renting and owning or need some financial help to buy bigger houses.

5.7 Policy Experiments

So far we have maintained the assumption that, in the event of default, the borrower's liability is limited to their home. In several states – known as anti-deficiency or non-recourse states – the law does in fact preclude mortgage lenders from pursuing deficiency judgments. The list of such states varies but generally includes Arizona, California, Florida (and sometimes Texas).[25] There are other states, known as "one-action" states, that allow the holder of the claim against the household to only file one lawsuit to either obtain the foreclosed property or to sue to collect funds. The list of such states includes Nevada and New York. Even in states where deficiency judgments are legal, conventional wisdom is that the costs associated with these judgments are so high, and the expected returns are so low, that this recourse is seldom used. However, some empirical studies (e.g. Ghent and Kudlyak (2009)) find that recourse decreases the probability of default when there is a substantial likelihood that a borrower has negative home equity. In this subsection we quantify the role of the recourse assumption for equilibrium statistics.

Table 8 compares steady state statistics in our benchmark economy (which assumes anti-deficiency or non-recourse) to their counterparts in an economy where in the event of default by a borrower with assets $a \geq 0$ and house size h, the intermediary collects $\min\{(1-\chi)qh + a, b\}$, while the household retains $\max\{(1-\chi)qh + a - b, 0\}$. In other words, in the second economy, any asset the household owns at the time of default can be claimed as collateral by the lender.

As the table shows, this change in the environment greatly reduces average loss incidence rates, for obvious reasons. At the same time, this makes default much more costly for households and, as a result, foreclosure rates fall by 35%. By comparison, Ghent and Kudlyak (2009) estimate that at average borrower characteristics, the likelihood of default is 20% higher in antideficiency states than in recourse states. Note that we assume that liquid assets are collected without any transaction costs, which raises the equilibrium impact of recourse (so our results should be considered an upper bound).

An interesting aspect of this experiment is that allowing for recourse actually raises home-ownership rates. This is because mortgage rates fall when default risk decreases, making

[25]See, for instance, http://www.helocbasics.com/list-of-non-recourse-mortgage-states-and-anti-deficiency-statutes

Table 8: The role of recourse

	Benchmark (no recourse)	Full recourse
Homeownership rate	65.38	66.02
Avg. ex-housing asset/income ratio	0.91	0.90
Avg. loan to income ratio	1.20	1.21
Avg. homeowner housing expenditure share	0.19	0.19
Rents to income ratio for renters	0.37	0.37
Avg. housing spending share for homeowners	0.21	0.20
Avg. mortgage yields (FRMs, LIPs)	(14.40,NA)	(13.22,NA)
Loss-incidence estimates	0.59	0.44
Foreclosure rates	2.72	2.01
Foreclosure discount	0.37	0.35

homeownership a cheaper option ex-ante.

6 Transitional effects of mortgage innovation

The previous section shows that mortgage innovation has significant long-run effects on foreclosure rates. We will now describe a quantitative experiment designed to evaluate the role of these new mortgages in the foreclosure crisis depicted in figure 1. Figure 1 suggests that the course of events leading up to the collapse of house prices and the foreclosure crisis can be decomposed into three basic stages. Prior to 2004, the composition of the mortgage stock is stable and traditional mortgages are the dominant form of home financing. Around 2004-05, the composition of the mortgage stock changes noticeably as nontraditional mortgages start accounting for a high fraction of originations. After mid-2006, prices start collapsing and the flow of traditional mortgages begins rising once again as originations of non-traditional mortgages slow to a trickle.[26]

We will use our model to simulate this course of events and quantify the role of nontraditional mortgages using a three-stage experiment. In the first stage (the pre-2004 period), the economy is in our benchmark, FRM-only steady state. In the second stage of the experiment, we introduce the option for newly mid-aged agents to finance their house purchase with a LIP mortgage. We assume that this introduction is unanticipated by agents, but perceived as permanent once it is made. One period later, in the third stage, we hit the economy with a surprise 20% aggregate price decline and take away the LIP option. This stage is meant to approximate the state of the US housing market in 2008, a period characterized by home

[26]In the most recent Mortgage Origination Survey data, traditional FRMs account for 90% of all originations.

prices 20% below their peak and the end of the availability of non-traditional mortgages.

In the third stage, we cause home prices to fall by assuming that the productivity of the housing technology A rises. This drop in prices catches agents as a complete surprise so that, at the time of the shock, the distribution of states across agents is the one implied by the first two stages of the experiment.

Because the intermediary is also surprised by the price shock, it experiences unforeseen revenue and capital losses. A formal definition of the intermediary's net profit is provided in appendix A.3. In steady state, those profits are zero. However, the unexpected drop in prices causes default rates to rise which reduces revenues and raises foreclosure losses, causing profits to become negative until contracts written before the price shock disappear. One has to be explicit about who bears these losses. We assume that constant lump-sum taxes are imposed on all agents following the price shock in such a way as to exactly cover the intermediary's losses in present value terms. Computationally, this involves guessing a value for the constant and permanent tax, solving for the new steady state equilibrium and the transition to this new state, evaluating the present value of the intermediary's transitory losses, and updating the permanent tax level until losses and tax revenues match.[27]

Figures 9 and 10 show the outcome of this three-stage experiment. Roughly a quarter of newly mid-aged agents take advantage of the LIP option once it becomes available in the second stage, immediately raising the fraction of LIPs in the mortgage stock from 0 to 4%. The homeownership rate also rises as more agents are able to purchase homes thanks to mortgage innovation. Because we do not give homebuyers the option to default in the first period of their life and the number of originations rises in stage 2, default rates fall slightly.

Once the price shock strikes in stage 3, foreclosure rates jump up to almost 3.7%. The aggregate shock pushes a number of agents with contracts written prior to the shock (when houses were expensive) into negative equity territory. In the second period after the shock, foreclosure shocks fall slightly but they begin to rise again towards a peak of around 4% as agents who took LIPs in the second stage enter the high-payment part of their mortgages. When the shock strikes, all of these agents have negative equity, but they can continue enjoying ownership rents at a fairly low cost as long as they do not have to make large mortgage payments. As a result, many of these agents wait until high payments kick in to formally default. This provides a natural propagation mechanism.

The price shock causes foreclosure rates to rise by around 50% from start to peak, and 40% on impact. In the data displayed in figure 1, foreclosure rates more than tripled in the data to a quarterly rate of roughly 1.4%, which implies a two-year default rate of over 11%. Our three stage experiment captures about 25% of that peak increase. Between mid-2006 and mid-2008, foreclosure rates roughly doubled. Our model captures roughly 40% of this initial impact.

To quantify the importance of new mortgages in the foreclosure boom, we run a coun-

[27] See the computational appendix for details. There are obviously many possible ways to redistribute the intermediary's losses. Per capita losses are negligible in practice and barring extremely concentrated tax schemes, their exact distribution will not have large effects on the results we present.

Figure 9: Foreclosure rates during transition

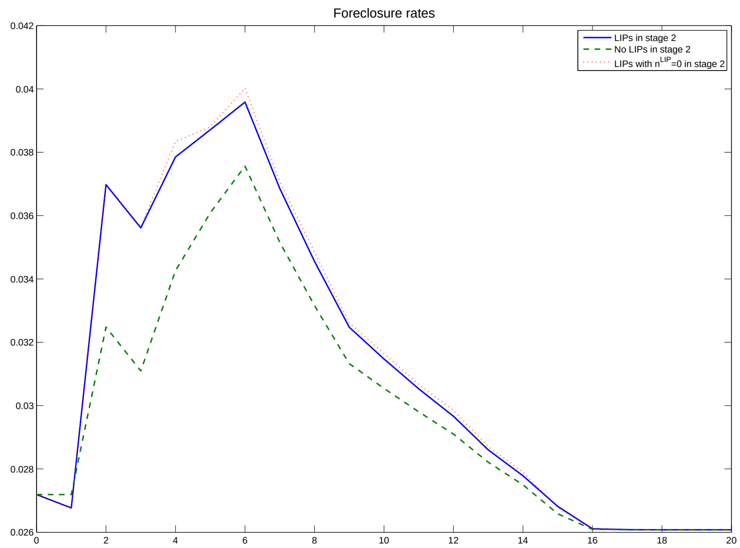


Figure 10: Fraction of LIPs in mortgage stock during transition

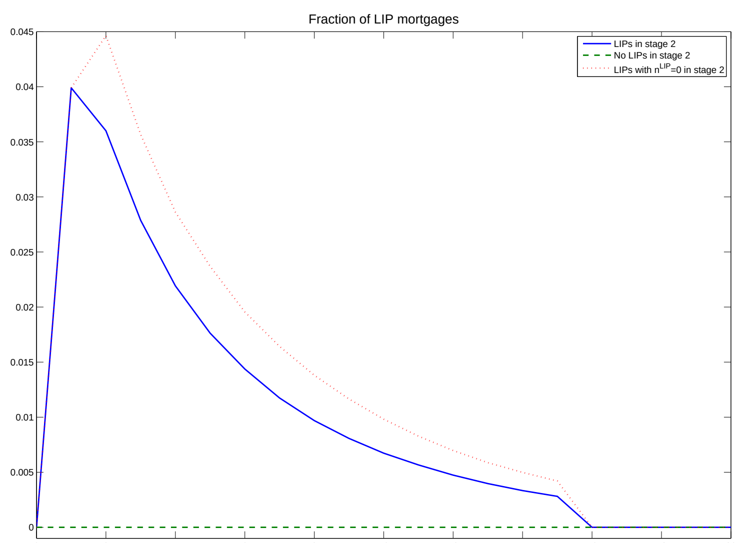


Table 9: Summary of transition results

	Data	LIPs in stage 2	No LIPs in stage 2	$n^{IOM} = 0$ in stage 2
Fraction of LIPs in originations in stage 2	*[20-35%]*	25%	0%	30%
Foreclosure increase in first two years	100%	38%	21%	38%
Foreclosure increase at peak	200%	47%	38%	49%

Notes: Our data counterpart for LIP originations is a rough estimate based on available estimates of subprime mortgage originations, and the fact that the fraction of traditional FRM mortgages in origination fell to 50% in 2005, from a peak of 85%.

terfactual experiment where in stage 2, LIPs are not offered. We shock the economy with the same price decrease in stage 3. The result is shown in the dotted lines on figure 9. The increase in foreclosure rates become noticeably smaller, because the price shocks now strike an economy where agents have more investment in their homes. Specifically, the increase on impact is only 20% when LIPs are not introduced and the foreclosure rates peak at a level that is 40% below its data counterpart.

We can also quantify the relative importance of low downpayments and delayed amortization – the two key features of our LIPs – by running a third experiment where in stage 2, LIPs feature a zero downpayment, but no IOM period. As figure 10 shows, LIPs become even more popular in stage 2 in part because, without an IOM period, the default risk on those mortgages falls as do, therefore, yields. Figure 9 shows, more importantly, that the impact on foreclosure rates of the price shock in stage 3 is very similar to what obtained in the first experiment (even somewhat higher because LIPs are more popular in stage 2), suggesting that the fact that agents with LIPs enter in their contract with zero equity is the principal factor behind their role in the foreclosure increase.

7 Conclusion

The presence of mortgages with low down-payments and delayed amortization, two features that became highly popular between 2004 and 2006, causes equilibrium default rates to become higher both in the long-run and in response to aggregate housing price shocks. These mortgages enable high-default risk agents to become home-owners, and feature a protracted accumulation of home-equity. For both reasons, these mortgages are associated with much higher default rates. In our benchmark parameterization, default frequencies are more than twice as high on LIPs than they are on FRMs, and LIPs, therefore, account for a disproportionate share of the overall foreclosure rate.

These quantitative findings have a number of implications for how one should interpret current events. Mortgage innovation serves an important purpose and can raise welfare by expanding the range of choices for a number of households. The nature of these innovations, however, does make an increase in default rates unavoidable, and significantly magnify the impact of negative aggregate housing price shocks on default rates.

Our calculations suggest, in particular, that mortgage innovation contributed significantly to the recent foreclosure crisis. Had mortgages with backloaded payments not become so popular between 2004 and 2006, the spike in foreclosures would have been noticeably smaller, even assuming the exact same collapse in house prices. One possibility we have not studied in this paper is that these new mortgages may have contributed to the price collapse directly as well, which would make their contribution to the crisis even greater. For instance, the availability of these mortgages may have led to some form of overbuilding. Their presence may also have contributed to the fragility and eventual freeze of the financial system, leading to a collapse of demand for housing, hence of housing prices. Formalizing and quantifying these ideas are promising avenues for future work, and should reinforce our main message: mortgage innovation played a very significant role in the recent foreclosure boom.

A Computational appendix

A.1 Steady State Equilibrium

1. The asset space consists of twenty equally spaced asset grid points between 0 and $\nu q h_2$, twenty equally spaced asset grid points between $\nu q h_2$ and $\nu q h_3$, twenty equally spaced asset grid points between $\nu q h_3$ and and $q h_2$, and another sixty equally spaced asset grid points from $\nu q h_2$ to wherever the asset choice decisions do not bind.

2. We use value function iteration to find $V_O(a)$ on the asset grid from which we obtain decision rules $a'_O(a)$ for old agents. The value functions are approximated by using linear interpolation.

3. Given the value functions for old agents, we use value function iteration to find $V_M(a, y, 0, \cdot)$ on the asset grid from which we obtain decision rules $a'_M(a, y, 0, \cdot)$ for mid-aged renters for each y. The value functions are approximated using linear interpolation.

4. Given the value functions for old agents and mid-aged renters, use value function iteration to find $V_M(a, y, 1, h, n > T; \kappa)$ on the asset grid from which we obtain asset choice decision rules $a'_M(a, y, 1, h, n > T; \kappa)$ and homeownership decisions $H'(a, y, 1, n > T; \kappa)$ for mid-aged homeowners who have paid off their mortgage for each (y, h). The value functions are independent of the original mortgage contract terms κ because their mortgage payments and balances are all zeros regardless of their original contracts. The value functions are approximated using linear interpolation.

5. For every pair of h_0 and (a_0, y_0), if a household does not have enough assets to make the downpayment, $\alpha q h_o$, no FRM contract will be offered. Set an initial guess of

mortgage interest rate for each contract, given the value functions for old agent, mid-aged renters, and mid-aged homeowners with one less period of mortgage payments to make $V_M(a, y, 1, h, n = t + 1; \kappa)$, solve for $V_M(a, y, 1, h, n = t; \kappa)$ by backward induction for each $(y, h, t = \{1, ..., T\})$. For each path of possible realization of incomes and housing capital given κ, keep track the household decisions along the path. Calculate the present value according to the decision rules from each path and the probability of this path being realized. If this present value is not equal to the initial loan size, update the interest rate and repeat this step. Otherwise, the equilibrium interest rate is found. The value functions are approximated using linear interpolation.

6. Given the value functions for old and mid-aged agents, use value function iteration to find $V_Y(a, y)$ on the asset grid from which we obtain decision rules $a'_Y(a, y)$ and contract selection decisions $(\zeta(a, y), h_0)$ on mortgage terms and initial housing capital. Because of the potential discontinuity caused by the downpayment requirements, the value functions for young agents are solved by grid search.

7. Solve for the equilibrium stationary distribution μ given the implied law of motion.

A.2 Transition Dynamics

1. Solve for the initial steady state equilibrium with price q^o using the algorithm above with zero lump-sum tax.

2. Start the initial guess of lump-sum tax $\tau_{i=1} = 0$. Solve for the final steady state with a new house price q^n with the lump-sum tax implemented.

3. Solve the optimization problems for homeowners who have purchased the house before the unanticipated house price shock occurs by backward induction. If a household chooses to sell its home, they sell at the new price q^n. If a household chooses to remain a homeowner, they have to follow the original mortgage terms (if they have not paid off their mortgage debt).

    - If the agent is a homeowner but it is not budget feasible for her to make her mortgage payment $m^o(n; \kappa)$, which he obtained before the unanticipated price shock, or:

    $$y + a(1 + r) - m^o(n; \kappa) - \delta h - \tau_i < 0, \tag{A.1}$$

    then the value function solves:

    $$V_M^o(a, y, 1, h, n; \kappa) = \max_{c,a'} U(c, h_1) + \beta E_{y'|y}\left[(1 - \rho_O)V_M^n(a', y', 0, \ldots) + \rho_O V_O^n(a')\right]$$
    $$\text{s.t. } c + a' = y + a(1 + r) + \max\left\{(1 - \chi)q^n h - b^o(n; \kappa), 0\right\} - R^n h_1 - \tau_i.$$

- If it is budget feasible for a homeowner to make her mortgage payment, then if the household chooses to sell its house and start to rent (so that $H' = 0$), define the value function by

$$V_M^{o,H'=0}(a, y, 1, h, n; \kappa) = \max_{c,a'} U(c, h_1) + \beta E_{y'|y}\left[(1-\rho_O)V_M^n(a', y', 0, \ldots) + \rho_O V_O^n(a')\right]$$
$$\text{s.t. } c + a' = y + a(1+r) + \max\{q^n h - b^o(n;\kappa), 0\} - R^n h_1 - \tau_i.$$

- If the agent is able to meet her current mortgage payment and chooses to keep her house ($H' = 1$), define the value function by

$$V_M^{o,H'=1}(a, y, 1, h, n; \kappa) = \max_{c,a'} U\left(c, h\left[1_{\{h=h_1\}} + (1 - 1_{\{h=h_1\}})\theta\right]\right)$$
$$+\beta E_{(y',h')|(y,h)}\left[\begin{array}{c} (1-\rho_O)V_M^o(a', y', 1, h', n+1; \kappa) \\ +\rho_O V_O^n(a' + \max\{q^n h - b^o(n+1;\kappa), 0\}) \end{array}\right]$$
$$\text{s.t. } c + a' = y + a(1+r) - m^o(n;\kappa) - \delta h - \tau_i.$$

4. Select a large integer N to be the number of periods during the transition. In the first period of transition, start the economy with the initial steady state distribution. Starting from the second period along the transition path, apply the decision rules to solve for the distribution one period ahead. For renters, use the decision rules solved for the final steady state. For homeowners, if they purchase the house before the transition starts, use the optimization problems solved in the previous step. If they purchase the house after the transition starts, use the decision rules in the final steady state. Young agents who turn mid-aged during the transition purchase houses at the new price q^n. Continue to solve for the distribution in every period of the transition path.

5. Given the decision rules and distribution along the transition path, calculate the discounted present value of the net profits for the financial intermediary over the transition path. Update the lump-sum tax τ_{i+1} such that $\frac{\tau}{r}$ is equal to the discounted present value of the net profits. Return to step 3 and repeat using τ_{i+1} until the discounted present value of the net profits equals the discounted present value of the lump-sum tax. Let the present discounted value of the net profits of intermediaries be $\Pi(\tau)$, where the net profits per period is defined as in (A.6). It depends on τ because households decisions vary with τ which in turn affects the net profits.

$$\Pi(\tau) = \frac{\tau}{r}$$

6. Check if the distribution converges to the final steady state in N periods. If not, increase N and repeat all the steps above.

A.3 Intermediary profits on mortgage activity

This appendix derives a net profit expression for the intermediary from our recursive formulation of the intermediary's problem in section 6. For simplicity but without any loss of generality we do so in the case where $T = 2$. Since breaking down default by type is irrelevant for these calculations, we will also write D for $D^I + D^V$ throughout the derivation. Finally, once again without any loss of generality, we will focus on the economy with FRMs only, and drop mortgage type superscripts (κ) everywhere. Finally we will write $n(\omega)$ for the mortgage period associated with state $\omega \in \Omega_M$. In particular, for newly born agents, $n(\omega) = 0$, while for agents in the second period of their mid-age, $n(\omega) = 1$.

Consider then an agent in state ω at origination with initial loan size b_0 and initial house size $h > 0$, with mortgage yield r^Y. Then:

$$W(\omega) = \frac{m}{1+r+\phi}$$
$$+ \int \left\{ D(\omega') \frac{\max((1-\chi)qh(\omega'), b(1))}{1+r+\phi} + S(\omega') \frac{b(1)}{1+r+\phi} + (1 - D(\omega') - S(\omega')) \frac{m}{(1+r+\phi)^2} \right\} P(\omega'|\omega) d\omega', \tag{A.2}$$

where, per standard fixed mortgage payment algebra:

$$m = b_0(1+r^Y) - b_1 \tag{A.3}$$
$$\text{and } m = b_1(1+r^Y) \tag{A.4}$$

Note that to economize on notation we do not make explicit the fact that b_0, b_1 and r^Y are functions of the agent's state. In this context:

$$\text{Net expected profits on agent of type } \omega \text{ at origination } \equiv W(\omega) - b_0. \tag{A.5}$$

Plugging expression (A.2,A.3,A.4) into (A.5) gives:

$$W(\omega) - b_0 = \frac{m}{1+r+\phi} +$$
$$\int_{\omega'} \left\{ D(\omega') \frac{\max((1-\chi)qh(\omega'), b(1))}{1+r+\phi} + S(\omega') \frac{b(1)}{1+r+\phi} + (1 - D(\omega') - S(\omega')) \frac{m}{(1+r+\phi)^2} \right\} P(\omega'|\omega) d\omega' \quad - \quad b_0$$
$$= \frac{b_0(1+r^Y) - b_1}{1+r+\phi} +$$
$$\int_{\omega'} \left\{ D(\omega') \frac{\max((1-\chi)qh(\omega'), b(1))}{1+r+\phi} + S(\omega') \frac{b(1)}{1+r+\phi} + (1 - D(\omega') - S(\omega')) \frac{b_1(1+r^Y)}{(1+r+\phi)^2} \right\} P(\omega'|\omega) d\omega' \quad - \quad b_0$$
$$= \frac{b_0(r^Y - (r+\phi))}{1+r+\phi} +$$
$$\int_{\omega'} \left\{ D(\omega') \frac{\max((1-\chi)qh(\omega'), b(1)) - b(1)}{1+r+\phi} + (1 - D(\omega') - S(\omega')) \frac{b_1(r^Y - (r+\phi))}{(1+r+\phi)^2} \right\} P(\omega'|\omega) d\omega'$$

The last equality uses the fact that $b(1) = b(1)[D(\omega') + S(\omega') + (1 - D(\omega') - S(\omega'))]$ for all ω'. Integrating it over all possible ω such that $n(\omega) = 0$ yields:

$$\begin{aligned}\text{Aggregate intermediary profits on mortgages in steady state } &\equiv \\ \int b(\omega)\left(1 - D^I(\omega) - D^V(\omega) - S(\omega)\right)\frac{(r^\zeta(\omega) - (r+\phi))}{(1+r+\phi)^{n(\omega)+1}}d\mu_M(\omega) \\ -\int (b(\omega) - \min\{(1-\chi)qh(\omega), b(\omega)\})\left(D^I(\omega) + D^V(\omega)\right)\frac{(1+r+\phi)}{(1+r+\phi)^{n(\omega)+1}}d\mu_M(\omega)\end{aligned} \tag{A.6}$$

after observing, first, that $D(\omega) + S(\omega) = 0$ when $n(\omega) = 0$ since we do not allow agents to sell or default in the first period of the mortgage and, second, that for any integrable function $g : \Omega_m \mapsto I\!R$,

$$\int_{\Omega_M}\left(\int_{\{\omega' \in \Omega_M | n(\omega')=1\}} g(\omega')P(\omega'|\omega)d\omega'\right)d\mu_M(\omega) = \int_{\{\omega' \in \Omega_M | n(\omega')=1\}} g(\omega')d\mu_M(\omega').$$

This last expression says that the mass of agents who reach a given node is the probability of reaching that node from a given ω at origination. In other words, integrating the expected present value expression over all possible origination state amounts to computing a cross-sectional average in steady state. Expression A.6 thus gives the intermediary's aggregate profits on its mortgage activities in steady state. While the argument in this appendix has assumed $T = 2$, it extends unchanged to the general case.

The first integral in the profit expression gives the net return on active mortgages that are not terminated in the current period, while the second term is the cost (direct capital losses and opportunity cost) associated with the capital lost in the event of foreclosure.

Bibliography

Athreya, K., "Welfare Implications of the Bankruptcy Reform Act of 1999," *Journal of Monetary Economics*, 49, 2002.

Campbell, J. Y., and Cocco, J. F., "Household Risk Management And Optimal Mortgage Choice," *The Quarterly Journal of Economics*, 118(4), pp 1449-1494, 2003.

Chambers, M. , C. Garriga, and D. Schlagenhauf,"Equilibrium Mortgage Choice and Housing Tenure Decisions with Refinancing," forthcoming *Review of Economic Dynamics*.

Chatterjee, S., D. Corbae, M. Nakajima, and V. Rios-Rull, "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default," *Econometrica*, 75, pp 1525-1589, 2007.

Chatterjee, S. and B. Eyigungor, "Foreclosures and House Price Dynamics: A Quantitative Analysis of the Mortgage Crisis and the Foreclosure Prevention Policy," Federal Reserve Bank of Philadelphia Working Paper 09-22, 2009.

Conesa, J. C., and Krueger, D., "Social Security Reform with Heterogeneous Agents," *Review of Economic Dynamics*, 2(4), pp 757-795, 1999.

Danis, M. A. and A. N. Pennington-Cross, "The Delinquency of Subprime Mort- gages," unpublished paper, 2008.

Davis, M. A., and Heathcote, J., "Housing and the Business Cycle," *International Economic Review* 46, pp 751.784, 2005.

Deng, Y., J. Quigley, and R. V. Order, "Mortgage terminations, heterogeneity and the exercise of mortgage options," *Econometrica* 68, 275?07, 2000.

Feldman, M. and C. Gilles "Law of Large Numbers" *Journal of Economic Theory* 17, 1985.

Foote, C., K. Gerardi, L. Goette, and P. S. Willen "Reducing Foreclosures," *Journal of Housing Economics* 17, Federal Reserve Bank of Boston Work- ing Paper 09-2, 2009.

Foote, C., K. Gerardi, L. Goette, and P. S. Willen "Just the facts: An initial analysis of subprimes role in the housing crisis," *Journal of Housing Economics* 17, 2008a.

Foote, C., K. Gerardi, and P. Willen, "Negative equity and foreclosure: Theory and evidence," *Journal of Urban Economics* 6, 234?45, 2008b.

Garriga and D. Schlagenhauf, "Home Equity, Foreclosures, and Bail-outs," Unpublished manuscript, 2009.

Ghent, A. C. and M. Kudlyak, “Recourse and Residential Mortgage Default: Theory and Evidence from U.S. States,” Federal Reserve Bank of Richmond Working Paper 09-10, 2009.

Gerardi, K., A. Shapiro, and P. Willen, “Subprime outcomes: Risky mortgages, homeownership experiences, and foreclosures,” Federal Reserve Bank of Boston Work- ing Paper 07-15, 2007.

Gerardi, K., A. Lehnert, S. M. Sherland and P. Willen, “Making Sense of the Subprime Crisis ,” Federal Reserve Bank of Boston Public Policy Discussion Paper 07-15, 09-1.

Gervais, M., “Housing Taxation and Capital Accumulation,” *Journal of Monetary Economics*, 49, pp 1461-1489, 2002.

Green, R. K. and S. Malpezzi, “US Housing Markets and Housing Policy”, The Urban Institute Press, Washington D.C., 1993.

Guler, B., “Innovations in Information Technology and the Mortgage Market,” University of Texas at Austin mimeo, 2008.

Harding J. P., Rosenthal, S. S., and C.F. Sirmans“Depreciation of housing capital, maintenance, and house price inflation: Estimates from a repeat sales model,” *Journal of Urban Economics* 61(2), pp 193-217, 2007.

Hayre L. S., and M. Saraf, “A Loss Severity Model for Residential Mortgages”, *Journal of Fixed Income*, Fall, pp 5-31, 2008.

Jeske, K. and D. Krueger, “Housing and the Macroeconomy: The Role of Implicit Guarantees for Government-Sponsored Enterprises,” Working Paper, Federal Reserve Bank of Atlanta, 2005.

Kahn, J., “What Drives Housing Prices?,” Federal Reserve Bank of New York Staff Report No. 345, 2008.

Ortalo-Magné, F. and S. Rady, “Housing Market Dynamics: On the Contribution of Income Shocks and Credit Constraints,” *Review of Economic Studies*, 73(2), pp 459-485, 2006.

Pennington-Cross, A., “The Value of Foreclosed Property” *Journal of Real Estate Research* (forthcoming).

Perri, F. and D. Krueger, “Does Income Inequality Lead to Consumption Inequality? Evidence and Theory,” *Review of Economic Studies*, 73(1), pp. 163-193, 2005.

Ríos-Rull, J.V., and V. Sanchez-Marcos, “An Aggregate Economy with Different House Sizes,” *Journal of the European Economic Association*, 6 (2-3), pp 705-714, 2008.

Sherlund, S, “The past, present, and future of subprime mortgages,” Finance and Economics Discussion Series 2008-63, Federal Reserve Board, 2008.

Stein, J. C., “Prices and Trading Volume in the Housing Market: A Model with Down-Payment Effects”, *Quarterly Journal of Economics*, 110, pp 37906, 1995.

Steuerle, C. E., C. Spiro, and A. Carasso, “Measuring replacement rates at retirement,” *Straight Talk on Social Security and Retirement Policy*, 24 (2000), Washington, DC: The Urban Institute.