The cyclical component of US asset returns*

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Preliminary (yes, still)

Abstract

We show that equity returns, the term spread, and excess returns on a broad range of assets are positively correlated with future economic growth. The common tendency for excess returns to lead the business cycle suggests a macroeconomic factor in the cyclical behavior of asset returns. We construct an exchange economy that illustrates how this might work. Its important ingredients are recursive preferences, stochastic volatility in consumption growth, and dynamic interaction between volatility and growth.

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*Preliminary and incomplete, no guarantees of accuracy or sense. We welcome comments, including references to related papers we inadvertently overlooked. We thank Stijn Van Nieuwerburgh for help with data and model. The latest version of the paper is available at:

`http://pages.stern.nyu.edu/~dbackus/GE_asset_pricing/ms/BRZ_returns_latest.pdf`.

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1 Introduction

We look at asset prices from the perspective of macroeconomists and ask: What do they tell us about the structure of the economy that generated them? We document two sets of facts that we think are worth exploring further. The first is the well-known tendency for equity prices (or their growth rates) and term spreads (differences between long- and short-term interest rates) to lead the business cycle. In US data, and to some extent in data for other countries, fluctuations in these variables are positively correlated with economic growth up to 12 months in the future. Yet in virtually all business cycle models, everything — including equity prices and term spreads — moves up and down together. Indeed, this is often treated as the defining feature of the business cycle, and therefore an attractive property of a model. The second set of facts concerns excess returns. We show that excess returns on a broad range of equity and bond portfolios also lead the cycle: high excess returns are associated with high future growth. Moreover, they display similar cyclical patterns. This property is less widely known and might even be new. It suggests that the cyclical component of excess returns is common across asset classes and has a macroeconomic origin.

If these are the facts, what do they tell us about the structure of the aggregate economy? In a general markov environment, prices and quantities are functions of the state of the economy. The facts suggest that asset prices reflect some feature of the state that is correlated with future economic growth. We construct an example that shows how this might work. To keep things simple, we consider a traditional exchange economy in which the only inputs are preferences and a stochastic process for consumption growth. The evidence points us to the consumption growth process. What we need, it seems, is a process in which predictable changes in future consumption growth show up now in equity prices and term spreads. Preferences play a role, too, since they affect the value given to expected future growth. We'd have to work out the details to see whether this line of attack delivers the goods, but it strikes us as a good starting point.

The evidence on excess returns poses more of a challenge. If we have constant variances in the consumption growth process and use common loglinear approximation methods, expected excess returns are constant by construction. We need variation in either aggregate aversion to risk (perhaps through a nonlinearity or heterogeneity across agents) or in risk itself. This point was made forcefully by Atkeson and Kehoe (2008) and we think it's a good one: you can't talk sensibly about the cyclical behavior of excess returns without cyclical variation in risk and/or risk aversion. Both paths have been taken in the literature. Without making any claim to superiority, we consider changes in risk generated by stochastic volatility in the consumption growth process. Recursive preferences allow us to assign volatility a nonzero price. Cyclical variation requires, in addition, some interaction between consumption growth (the cycle) and volatility (risk). The net result is a modest generalization of the environment of Bansal and Yaron (2004).

Here's the plan. In Sections 2 and 3, we document the cyclical behavior financial variables and economic growth. We do this with cross-correlation functions, which we regard as useful visual representations of the dynamic relations between variables. In Sections 4 and 5 we describe an exchange economy and show how it can be tuned to deliver something like the facts documented earlier. The last two sections connect our work to the literature, clean up some loose ends, and point to issues that remain unresolved.

2 Financial indicators of business cycles

In the US and elsewhere, financial variables are commonly used as indicators of future economic growth. Two of the most popular are equity prices (typically the growth rate or return of a broad-based index) and the term spread (the difference between a long-maturity interest rate and a short rate). We describe the dynamic relations between these variables and aggregate economic growth with cross-correlation functions.

Data

Our data cover the period from 1960 to the present. We use monthly series because the finer time interval allows clearer identification of leads and lags than the quarterly data commonly used in national income accounts and business cycle research. Definitions and sources are given in Appendix A, but here's a summary. Most of our financial variables come from CRSP and Ken French's web site. Bond yields refer to the end of the month — the last trading day — so the yield associated with October 2008 is that for October 31. Returns cover the whole month; the return for October 2008, for example, refers to the period September 30 to October 31. We use logarithms of gross returns because they line up more neatly with our theory.

Most of our macroeconomic variables come from FRED, the online data repository of the Federal Reserve Bank of St. Louis. They are typically time averages. Industrial production for October 2008 is an estimate of the average for that month. The same holds for our other two measures of economic growth, consumption and employment, and to measures of the price level. We compute growth rates as log-differences over the previous month ($\log x_t - \log x_{t-1}$), so that the October growth rate is the growth rate of October over September. We also use centered year-on-year growth rates ($\log x_{t+6} - \log x_{t-6}$) on occasion; this smooths out the high-frequency variation in these series without disturbing the timing. We think of these year-on-year growth rates as crude approximations to the Hodrick-Prescott filtered series often used in business cycle analysis.

Cross-correlation functions

The dynamic interrelations between financial indicators and economic growth are conveniently summarized with cross-correlation functions. If x is a financial indicator and y is a measure of economic growth, their cross-correlation function is

$$ccf_{xy}(k) \quad = \quad corr(x_t, y_{t-k}),$$

a function of the lag k. For negative values of k, this is the correlation of the indicator with future economic growth. If these correlations are nonzero, we say the indicator leads the business cycle. Similarly, positive values of k correspond to correlations of the indicator with past economic growth; nonzero values suggest a lagging indicator. Our interest is in the former: financial variables that lead economic growth and thus serve as a source of information about the future.

Consider equity returns. In Figure 1 we report the cross-correlation function for the (nominal) return on an aggregate portfolio of publicly-traded equity and the monthly growth rate of industrial production. Both series are inherently noisy — there's little persistence in either series — yet we see a modest but clear pattern. The correlations on the left show that equity returns are positively correlated with growth in industrial production up to one year in the future. The correlations are modest individually (the largest are between 0.1 and 0.2) but exhibit a clear pattern. The correlations on the right are smaller, on average, and mostly negative.

Figure 2 shows that this pattern is robust to a number of variations in measurement and methodology. In the upper right panel, we subtract inflation to generate (ex post) real returns. The picture is virtually the same. The correlations are slightly larger, but it's hard to see this in the figure. In the lower left panel, we use centered year-on-year growth in industrial production. When we average growth over 12 months, the pattern emerges even more clearly: high equity returns are associated with high economic growth several months later. This is a typical result: correlations are larger and smoother if we use year-on-year growth rates. The pattern is sharper still if we use centered annual returns (not reported). These annualized series are closer to what is done in business cycle research. We stick with month returns and growth rates because they respect the timing of the data and give us a finer (if noisier) picture of the leads and lags present in the data. Finally, in the lower right panel we use only data from 1990 on. The pattern is again similar, but the cross-correlation function is choppier with the shorter sample period.

We turn next to interest rates. In Figure 3, we show the cross-correlation function between the term spread (in this case the difference between continuously compounded nominal yields on 5-year and 1-month treasuries) and the monthly growth rate in industrial production. A large positive value for the spread indicates a steep yield curve, a small or negative value a flat or declining yield curve. Decades of research has found that steep yield curves (and large term spreads) are associated with above-average future economic growth.

We report several variations on this theme in Figure 4. The most interesting is the upper right panel, the cross-correlation function for the short rate (the 1-month yield): the panel is a mirror image of what we see with the term spread (repeated in the upper left panel). This suggests that most of what we see in the cross-correlation function for the term spread comes from the short rate. The same pattern emerges if we use the ex post real short rate (not reported). As with equity returns, the lower left panel shows that the cross-correlations are larger and smoother when we use the year-on-year growth rate of industrial production, but the overall pattern is similar. Finally, if we use data from 1990 on (lower right panel), the pattern is the same as for the whole sample.

All of these features of the data are robust to changes in our measurement of economic growth (not reported). If we replace industrial production with employment (nonfarm employment from the establishment survey) or consumption (total, real) little changes. The employment figures are the sharpest and consumption the least sharp; whether this reflects better measurement or something else is hard to say.

Most of these facts have been documented in earlier studies. Prominent examples include Ang, Piazzesi, and Wei (2006), Estrella and Hardouvelis (1991), Fama and French (1989), King and Watson (1996), Mueller (2008), Rouwenhorst (1995), and Stock and Watson (1989, 2003). Each contains an extensive set of references to related work.

3 Cyclical behavior of excess returns

If these cyclical properties of equity returns and interest rates seem familiar, a little thought raises an issue: the difference in the behavior of equity returns and the short-term interest rate. Roughly speaking, the evidence suggests that several months before an increase in economic growth, returns on equity rise and the return on the short bond falls. If we put the two together, we see that changes in economic growth are preceded (on average) by changes in the expected excess return on equity.

We see precisely this cyclical variation in excess returns in Figure 5, a plot of the cross-correlation function for the excess return on equity and the monthly growth rate of industrial production. The correlations for excess returns are slightly larger than those for returns, but the pattern is similar. Evidently most of the variation in excess returns comes from the return rather than the short rate.

This cyclical variation in equity excess returns appears in a wide range of equity portfolios, including those related to industry (49 industries based on 4-digit SIC codes), firm size (deciles based on market value), and book-to-market ratio (deciles based on the ratio of book to market values of equity). Virtually all of them exhibit the same cyclical pattern. Consider industry portfolios. Cross correlations for four examples are pictured in Figure 6. We report two industries we thought would have highly cyclical production and sales (automobiles and machinery) and two that would be less cyclical (food and drugs). Their cross-correlation functions are nevertheless quite similar. Indeed, we could say the same for virtually all 49 industry portfolios (not reported). At least to a first approximation, the cyclical behavior of these excess returns is the same.

The same is true of Fama-French (1992) portfolios. Four examples are given in Figure 7: the smallest and largest firms ranked by market value and the lowest and highest ranked by book-to-market ratio. All four have positive correlations of excess returns with growth 3-12 months in the future. Also striking (but not reported) is that "difference portfolios" (the infamous "small minus big" and "high minus low") show little cyclical pattern: the cyclical behavior apparently cancels when you subtract one return from the other.

We find the common cyclical pattern in these portfolios surprising, because we know — from Fama and French (1992) and many others — that these portfolios have very different return characteristics. Their average returns, in particular, are wildly different. The cross correlation functions suggest, however, that whatever these differences are, they are unrelated to the business cycle.

If equity portfolios exhibit similar cyclical behavior, what about bonds? Bond returns are noisier than the yields we looked at in the last section, but they display a clear cyclical pattern. The four panels of Figure 8 are based on portfolios of US treasuries with different maturities: 1-6 months, 19-24 months, 55-60 months, and 55-60 months, and 115-120 months. Curiously, their cyclical behavior is similar. This is, again, despite substantial differences in their volatility and average returns. This pattern is similar to that of equities, but not identical: where equities have a correlation close to zero with current economic growth, bond returns have a noticeably negative correlation with growth 0-5 months in the future.

[Later: corporate bonds, commodities, currencies, ...]

We have seen that excess returns on a variety of equity and bond portfolios lead the business cycle: they are positively correlated with future economic growth. Cyclical variation in excess returns suggests that risk premiums vary systematically over the business cycle. The common pattern across assets suggests that a single macroeconomic factor might be able to account for all of them.

4 A theoretical exchange economy

The rest of the paper is concerned with mimicking the observed cyclical behavior of excess returns in a theoretical environment. Before diving into the mechanics, it's worth thinking a little about what we need. Consider a stationary markov environment in which quantities (consumption, for example) and asset prices (equity and bonds) at any date t are functions of a finite state vector s_t. Returns and excess returns between dates t and $t+1$ are then functions of successive states, say $r(s_t, s_{t+1})$. Variation in each of these variables thus reflects variation in the underlying state vector. The evidence of the last two sections indicates that some of this variation is positively correlated with future economic growth. Even better, the statistical work of Sargent and Sims (1978) and Stock and Watson (2005) indicates that a state vector of modest dimension (somewhere between two and seven) can account for virtually all of the variation in aggregate quantities and prices.

We illustrate this in the simplest possible macroeconomic setting: an exchange economy with a representative agent. Variation in expected excess returns is generated by changes in the conditional variance of consumption growth; stochastic volatility, in other words. Correlation with future consumption growth is produced directly, by specifying consumption growth as a process that depends on past volatility. With the exception of the interaction between consumption growth and volatility, the structure is Bansal and Yaron's (2004). We feature a loglinear approximation method adapted from Hansen, Heaton, and Li (2008).

Environment

We follow a long tradition in generating theoretical asset returns from an exchange economy in which a representative agent consumes an endowment whose growth rate follows a stationary Markov process. Our version has two essential ingredients: recursive preferences and a consumption growth process with predictable variation in consumption growth and its conditional variance.

Preferences have the now-familiar recursive structure described by Kreps and Porteus (1978), Epstein and Zin (1989), and Weil (1989). If U_t is "utility from date t on," preferences follow from the time aggregator V,

$$U_t \;=\; V[c_t, \mu_t(U_{t+1})] \;=\; \left[(1-\beta)c_t^\rho + \beta\mu_t(U_{t+1})^\rho\right]^{1/\rho}, \tag{1}$$

and (expected utility) certainty equivalent function μ,

$$\mu_t(U_{t+1}) \;=\; \left[E_t(U_{t+1}^\alpha)\right]^{1/\alpha}. \tag{2}$$

The conventional interpretation is that $\rho < 1$ captures time preference (the intertemporal elasticity of substitution is $1/(1-\rho)$) and $\alpha < 1$ captures risk aversion (the coefficient of relative risk aversion is $1-\alpha$). Additive utility is a special case with $\alpha = \rho$.

Both the time aggregator and the certainty equivalent function are homogeneous of degree one, which allows us to scale everything by current consumption and convert our problem to one in growth rates. If we define scaled utility $u_t = U_t/c_t$, equation (1) can be expressed

$$u_t \;=\; \left[(1-\beta) + \beta\mu_t(g_{t+1}u_{t+1})^\rho\right]^{1/\rho}, \tag{3}$$

where $g_{t+1} = c_{t+1}/c_t$ is the growth rate of the endowment/consumption.

With these preferences, the pricing kernel (marginal rate of substitution) is

$$\begin{aligned} m_{t+1} &= \beta(c_{t+1}/c_t)^{\rho-1}\left[U_{t+1}/\mu_t(U_{t+1})\right]^{\alpha-\rho} \\ &= \beta g_{t+1}^{\rho-1}\left[g_{t+1}u_{t+1}/\mu_t(g_{t+1}u_{t+1})\right]^{\alpha-\rho}. \end{aligned} \tag{4}$$

See Appendix B. The pricing kernel is the heart of any asset pricing model, so (4) is central to the properties of asset prices and returns. The last term is the contribution of recursive preferences. If $\alpha = \rho$, it drops out, but in general the difference between α and ρ affects how predictable changes in consumption growth and its volatility are priced.

We specify a general linear process for the logarithm of consumption growth. Let state variables x_t (a vector of arbitrary dimension) and v_t (volatility, a scalar) follow

$$\begin{aligned} x_{t+1} &= Ax_t + a(v_t - v) + v_t^{1/2} B w_{t+1} \quad (5) \\ v_{t+1} &= (1-\varphi_v)v + \varphi_v v_t + b w_{t+1}, \quad (6) \end{aligned}$$

where v is the unconditional mean of v_t, $\{w_t\} \sim \text{NID}(0, I)$, and $Bb^\top = 0$ (innovations in x_t and v_t are uncorrelated). The aggregate state is therefore $s_t = (x_t, v_t)$. Consumption growth is tied to x_t: $\log g_t = g + e^\top x_t$ for some constant vector e. This gives us flexible dynamics for x_t, and therefore consumption growth. The conditional variance of consumption growth is proportional to v_t:

$$\text{Var}_t(\log g_{t+1}) = e^\top BB^\top e\, v_t.$$

This structure also allows some interaction between the dynamics of x_t and v_t through the parameter a. If $a = 0$, consumption growth and volatility are uncorrelated. We'll see later that all of these features — a predictable component in consumption growth, stochastic volatility, and interaction between the two — are needed to account for the cyclical behavior of asset returns.

Loglinear approximation of the pricing kernel

Asset prices in this setting are functions of the state variables and returns are functions of prices. We derive loglinear approximations to equilibrium asset prices with the goal of having something that is both easy to compute and relatively transparent. We break the solution process into steps to show how it works.

Step 1 (approximate time aggregator). The starting point is equation (3), which is not loglinear unless $\rho = 0$. A first-order approximation of $\log u_t$ in $\log \mu_t$ around the point $\log \mu_t = \log \mu$ is

$$\begin{aligned} \log u_t &= \rho^{-1} \log\left[(1-\beta) + \beta \mu_t(g_{t+1}u_{t+1})^\rho\right] \\ &= \rho^{-1} \log\left[(1-\beta) + \beta e^{\rho \log \mu_t(g_{t+1}u_{t+1})}\right] \\ &\approx \kappa_0 + \kappa_1 \log \mu_t(g_{t+1}u_{t+1}), \quad (7) \end{aligned}$$

where

$$\begin{aligned}\kappa_1 &= \beta e^{\rho \log \mu}/[(1-\beta)+\beta e^{\rho \log \mu}] \\ \kappa_0 &= \rho^{-1}\log[(1-\beta)+\beta e^{\rho \log \mu}] - \kappa_1 \log \mu.\end{aligned}$$

The approximation is exact when $\rho = 0$, in which case $\kappa_0 = 0$ and $\kappa_1 = \beta$. See Hansen, Heaton, and Li (2008, Section III).

The rest of the solution follows those of many approximate solutions to dynamic programs: we guess a value function of a specific form with unknown parameters, substitute optimal decisions into the Bellman equation, and solve for the unknown parameters. In this case the decision is trivial (consume the endowment), but the rest of the solution is the same. Equation (7) serves as the (approximate) Bellman equation with κ_1 in the role of discount factor.

Step 2 (guess value function). We conjecture an approximate scaled value function

$$\log u_t = u + p_x^\top x_t + p_v v_t$$

with coefficients (p_x, p_v) to be determined.

Step 3 (compute certainty equivalent). The novel ingredient of (7) is the certainty equivalent $\mu_t(g_{t+1}u_{t+1})$. Note that

$$\begin{aligned}\log(g_{t+1}u_{t+1}) &= u + g + (e+p_x)^\top x_{t+1} + p_v v_{t+1} \\ &= u + g + (e+p_x)^\top [Ax_t + a(v_t - v) + v_t^{1/2} B w_{t+1}] + p_v[(1-\varphi_v)v + \varphi_v v_t + b w_{t+1}].\end{aligned}$$

The certainty equivalent is

$$\begin{aligned}\log \mu_t(g_{t+1}u_{t+1}) &= u + g - (e+p_x)^\top a v + (e+p_x)^\top [Ax_t + a(v_t - v)] + p_v[(1-\varphi_v)v + \varphi_v v_t] \\ &\quad + (\alpha/2)[(e+p_x)^\top BB^\top (e+p_x) v_t + p_v^2 bb^\top].\end{aligned}$$

This follows from common properties of lognormal random variables: if an arbitrary random variable $x \sim \mathrm{N}(a,b)$, then $\log E(x) = a + b/2$ and $\log \mu(x) = a + (\alpha/2)b$. The difference is

$$\begin{aligned}\log(g_{t+1}u_{t+1}) - \log \mu_t(g_{t+1}u_{t+1}) &= -(\alpha/2)\left[p_v^2 bb^\top + (e+p_x)^\top BB^\top (e+p_x) v_t\right] \\ &\quad + v_t^{1/2}(e+p_x)^\top B w_{t+1} + p_v b w_{t+1}.\end{aligned}$$

The right-hand side has two kinds of terms: innovations (the terms with w_{t+1}) and penalties for risk (those with $\alpha/2$).

Step 4 (solve Bellman equation). If we substitute the certainty equivalent into (3) and line up coefficients, we have

$$
\begin{aligned}
u &= \kappa_0 + \kappa_1 \left[u + g + p_v(1-\varphi_v)v + (\alpha/2)p_v^2 bb^\top \right] \\
p_x^\top &= e^\top(\kappa_1 A)(I - \kappa_1 A)^{-1} \\
p_v &= (1-\kappa_1\varphi_v)^{-1}\kappa_1 \left[(e+p_x)^\top a + (\alpha/2)(e+p_x)^\top BB^\top(e+p_x) \right].
\end{aligned}
$$

The coefficient p_x has the form

$$
p_x^\top = e^\top \left[(\kappa_1 A) + (\kappa_1 A)^2 + (\kappa_1 A)^3 + \cdots \right].
$$

We think of it as capturing the Bansal-Yaron effect: the impact of x_t on expected future consumption growth and, through this route, on future scaled utility (discounted at rate $\kappa 1$). If $A = 0$ in equation (5) (white noise consumption growth), it's zero. The volatility coefficient p_v has two components. The first (the one involving the interaction parameter a) comes from the impact of volatility v_t on future values of x_t. The second (the one involving $\alpha/2$) summarizes the impact of v_t on the conditional variance of next-period scaled utility. In practice, we compute p_x as stated, which implies

$$
(e+p_x)^\top = e^\top(I - \kappa_1 A)^{-1}.
$$

The solution for p_v follows immediately. We ignore the intercept (for now). From this point on, we take the coefficients (p_x, p_v) as given.

Step 5 (derive pricing kernel). With these inputs, the pricing kernel (4) is

$$
\begin{aligned}
\log m_{t+1} &= \log\beta + (\rho-1)\log g_{t+1} + (\alpha-\rho)\left[\log(g_{t+1}u_{t+1}) - \log\mu_t(g_{t+1}u_{t+1})\right] \\
&= \log\beta + (\rho-1)(g - e^\top av) - (\alpha-\rho)(\alpha/2)p_v^2 bb^\top \\
&\quad + (\rho-1)e^\top A x_t + [(\rho-1)e^\top a - (\alpha-\rho)(\alpha/2)(e+p_x)^\top BB^\top(e+p_x)]v_t \\
&\quad + v_t^{1/2}[(\rho-1)e + (\alpha-\rho)(e+p_x)]^\top B w_{t+1} + (\alpha-\rho)p_v b w_{t+1} \\
&= \delta_0 + \delta_x^\top x_t + \delta_v v_t + v_t^{1/2}\lambda_x^\top w_{t+1}, + \lambda_v^\top w_{t+1},
\end{aligned}
\tag{8}
$$

with the implicit definitions of $(\delta_0, \delta_x, \delta_v, \lambda_x, \lambda_v)$. You may recognize this as a close relative of so-called affine models of bond pricing. The main difference is that the parameters are not free: they're tied to preferences and the consumption growth process.

The pricing kernel illustrates the interaction of recursive preferences, predictability of consumption growth, and stochastic volatility. The state variables x_t play a role only to the extent they help to forecast future consumption growth. If they do not (in other words, when $A = 0$), they do not appear in the pricing kernel ($\delta_x = 0$). If they do, their impact is governed by the intertemporal substitution parameter ρ. Volatility v_t appears for two reasons: because it helps predict future consumption growth (the parameter a) and because it controls the conditional variance (the second term). The impact of the former is again controlled by intertemporal substitution, but the latter depends on risk aversion (through $\alpha/2$) and the departure from additive preferences (the difference $\alpha - \rho$). When either is zero, the term is also zero, and volatility affects the pricing kernel only through its impact on expected future consumption growth.

In what sense is our solution an approximation? The only relation that isn't exact — so far — is (7), which is exact when $\rho = 0$. Moreover, relative to standard methods (linearize around the deterministic steady state), uncertainty plays a central role.

Asset returns

[Despite the loglinear structure, this is still a bit of a mess. We're hoping that with some thought, the properties of the solution will be transparent.]

Given a pricing kernel m, (gross) asset returns r satisfy the pricing relation

$$E_t\left(m_{t+1} r_{t+1}\right) \quad = \quad 1.$$

An asset, for our purposes, is a claim at date t to a dividend stream $\{d_{t+j}\}$ for $j \geq 1$. We use the pricing kernel (8) to derive prices and returns for a number of common assets, whose properties can then be compared to those we documented in Sections 2 and 3.

Short rate. The dividend is one unit of the consumption good next period: $d_{t+1} = 1$. The price of this 1-period (real) bond is $q_t^1 = E_t m_{t+1}$ and the return is $r_{t+1}^1 = 1/q_t^1 = 1/E_t m_{t+1}$. Thus

$$\log r_{t+1}^1 \quad = \quad -\log q_t^1 \quad = \quad -(\delta_0 + \lambda_v^\top \lambda_v/2) - \delta_x^\top x_t - (\delta_v + \lambda_x^\top \lambda_x/2) v_t,$$

a loglinear function of the state (x_t, v_t). In practice, we'll see that the dynamics of the short rate are dominated by the volatility term. Typically, if α and $\alpha - \rho$ are negative, the short rate is decreasing in volatility. The cross-correlation function with consumption growth then depends on its interaction with volatility. In practice this feature gives the model an Atkeson-Kehoe (2008) flavor: movements in the short rate reflect variation in the conditional variance of the pricing kernel more than changes in the conditional mean.

Consumption strip. The dividend is consumption next period: $d_{t+1} = c_{t+1}$. This isn't a real asset, but it illustrates how the various ingredients interact. The price-dividend ratio is

$$q_t^s = E_t\left(m_{t+1} g_{t+1}\right)$$

and the return is $r_{t+1}^s = g_{t+1}/q_t^s$. The log growth rate is

$$\log g_{t+1} = g + e^\top x_{t+1} = g + e^\top [A x_t + a(v_t - v) + v_t^{1/2} B w_{t+1}].$$

The price is

$$\log q_t^s = (\delta_0 + g - e^\top a v + \lambda_v^\top \lambda_v/2) + (\delta_x^\top + e^\top A) x_t + [\delta_v + e^\top a + (\lambda_x^\top + e^\top B)(\lambda_x + B^\top e)/2] v_t$$

and the return is

$$\log r_{t+1}^s = -(\delta_0 + \lambda_v^\top \lambda_v/2) - \delta_x^\top x_t - [\delta_v + (\lambda_x^\top + e^\top B)(\lambda_x + B^\top e)/2] v_t + v_t^{1/2} e^\top B w_{t+1}.$$

The excess return is therefore

$$\log r_{t+1}^s - \log r_{t+1}^1 = (1/2)[\lambda_x^\top \lambda_x - (\lambda_x^\top + e^\top B)(\lambda_x + B^\top e)] v_t + v_t^{1/2} e^\top B w_{t+1}.$$

This expression give us a sense of how cross-correlations with consumption growth will work. Note that the excess return does not depend on x_t: the impact of x_t is the same on both returns, so it drops out of the excess return. This is a general result: all of the variation in the expected excess return (its conditional mean) comes from v_t. Without it, expected excess returns are constant. That means the cross-correlation function of the excess return with consumption growth is largely the cross-correlation function for volatility. Second, the innovation w_{t+1} will tend to generate a positive contemporaneous correlation, because it affects consumption growth and the return the same way. Finally, the impact of volatility depends on its relation to consumption growth. If they're independent, as they are when $a = 0$, volatility has no impact on the shape of the cross-correlation function. Evidently we need to work on both the contemporaneous correlation and the interaction of volatility and growth if we are going to reproduce the dynamic patterns we see in US data.

Consumption stream. This serves for now as equity. The dividend is $d_{t+j} = c_{t+j}$ for all $j \geq 1$; that is, equity is a claim to consumption from next period on. The return on this asset takes a particular form:

$$r_{t+1}^c = \beta^{-1} \left[g_{t+1} u_{t+1} / \mu_t(g_{t+1} u_{t+1})\right]^\rho g_{t+1}^{1-\rho}. \tag{9}$$

See Appendix B. This result depends only on the constant elasticity form of the time aggregator; it does not reflect any of the structure we've given to consumption growth or the certainty equivalent (other than linear homogeneity). Expressed in terms of state variables and innovations, the log return is

$$\begin{aligned}\log r^c_{t+1} &= -\log\beta + (1-\rho)(g - e^\top av) - (\rho\alpha/2)p_v^2 bb^\top \\ &\quad + (1-\rho)e^\top Ax_t + [(1-\rho)e^\top a - (\rho\alpha/2)(e+p_x)^\top BB^\top(e+p_x)]v_t \\ &\quad + v_t^{1/2}(e+\rho p_x)^\top Bw_{t+1} + \rho p_v b w_{t+1}.\end{aligned}$$

(You may wonder where we snuck in the loglinear approximation. The answer is that it's built into equation (7). Everything else is exact.) The excess return is

$$\begin{aligned}\log r^c_{t+1} - \log r^1_{t+1} &= (1/2)[(\alpha-\rho)^2 - \alpha^2]p_v^2 bb^\top \\ &\quad + [(\rho-1)e + (\alpha-\rho)(e+p_x)]^\top BB^\top[(\rho-1)e + (\alpha-\rho)(e+p_x)]v_t \\ &\quad - (\alpha^2/2)(e+p_x)^\top BB^\top(e+p_x)v_t \\ &\quad + v_t^{1/2}(e^\top + \rho p_x^\top)Bw_{t+1} + \rho p_v b w_{t+1}.\end{aligned}$$

Notice, again, that it does not depend on x_t: all of the variation in the conditional mean of the excess return comes from v_t. The volatility term is a little complicated, but if you consider (as we do) situations in which α is large relative to ρ, then the volatility term is dominated by

$$(\alpha^2/2)(e+p_x)^\top BB^\top(e+p_x)v_t.$$

The quadratic form is the conditional variance of next-period utility, whose impact is governed largely by risk aversion (α) squared. Thus we see that risk aversion affects not only the average excess return, but also its variation. The Bansal-Yaron term p_x also plays a role: if it's small or even negative, the impact of volatility is also small. If $p_x = 0$, the volatility term is

$$[(\alpha-1)^2 - \alpha^2/2]e^\top BB^\top e\, v_t,$$

so, again, risk aversion is central.

Multiperiod bonds. An n-period bond is a claim to one unit of the consumption good n periods in the future: $d_{t+j} = 1$ for $j = n$, zero otherwise. Since the return on an $n+1$-period bond is $r^{n+1}_{t+1} = q^n_{t+1}/q^{n+1}_t$, the pricing relation implies that prices satisfy

$$q^{n+1}_t = E_t\left(m_{t+1} q^n_{t+1}\right).$$

We guess that prices are loglinear functions of the state:

$$\log q_t^n \;=\; \delta_0^n + \delta_x^{n\top} x_t + \delta_v^n v_t.$$

The coefficients satisfy the recursions

$$\begin{aligned}
\delta_0^{n+1} &= \delta_0 + \delta_0^n + [\delta_v^n(1-\varphi_v) - \delta_x^{n\top} a]v + (\delta_v^n b + \lambda_v^\top)(\delta_v^n b^\top + \lambda_v)/2 \\
\delta_x^{n+1\top} &= \delta_x^\top + \delta_x^{n\top} A \;=\; \delta_x^\top (I + A + \cdots + A^n) \\
\delta_v^{n+1} &= \delta_x^{n\top} a + \delta_v + \delta_v^n \varphi_v + (\delta_x^{n\top} B + \lambda_x^\top)(B^\top \delta_x^n + \lambda_x)/2.
\end{aligned}$$

Excess returns are therefore

$$\begin{aligned}
r_{t+1}^{n+1} - r_{t+1}^1 &= \log q_{t+1}^n - \log q_t^{n+1} + \log q_t^1 \\
&= (\delta_0^n - \delta_0^{n+1} - \delta_0^n) + \delta_x^{n\top} x_{t+1} + \delta_v^n v_{t+1} + (\delta_x^1 - \delta_x^{n+1})^\top x_t + (\delta_v^1 - \delta_v^{n+1}) v_t.
\end{aligned}$$

This is a little ugly, but the expressions can can be used to compute excess returns in the model, and thus their properties.

5 Cyclical behavior of theoretical excess returns

We present a numerical example to show how this works. This is illustrative: we haven't worked out all the implications for means, variances, and autocorrelations of returns. But since cross-correlations do not depend on magnitudes, it's likely we can match these features about as well as our starting point, Bansal and Yaron (2004).

Parameter values

We start with the Bansal-Yaron (2004) parameter values and vary them as needed. Details follow.

Consumption growth. Our version of (5) is a scalar ARMA(1,1) approximation to the two-component Bansal-Yaron process:

$$\log g_t - g \;=\; \varphi_g(\log g_{t-1} - g) + v_{t-1}^{1/2}(w_t - \theta w_{t-1}) + a v_{t-1}.$$

Even when $a = 0$, consumption growth has a persistent component whose magnitude is governed by $\varphi_g - \theta$. We set $\theta = \varphi_g$, which turns this process off altogether. The reason isn't apparent in what follows, but this component induces a contemporaneous correlation between interest rates and consumption growth that is inconsistent with US data. We choose a smaller value (0.95) than Bansal-Yaron, but adjust σ_g to retain their value of the unconditional variance of $\log g_t$. The autocorrelation is also similar, because consumption growth inherits some of the persistence of the volatility process.

Volatility. Mean volatility $v = 0.0080^2$, the unconditional variance of consumption growth. We use a smaller autocorrelation ($\varphi_v = 0.8$), but hold the unconditional variance of volatility constant. Otherwise the cross-correlation functions die out too slowly.

The interaction term. We set $a = 25$; if this seems large, remember v is small. This raises the first-order autocorrelation of log consumption growth from 0.0436 (the Bansal-Yaron number) to 0.0895. With these values, increases in volatility generate persistent increases in future consumption growth. It's essential that a is positive; otherwise, the correlation between volatility and future consumption growth is negative.

These parameter values imply the cross-correlation function between consumption growth and volatility reported in the top panel of Figure 9. We see a similar pattern to the cross-correlation functions for excess returns — Figure 5, for example.

Preferences. We set $\alpha = -9$ (so that the coefficient of relative risk aversion is 10) and $\rho = -1$ (so that the IES is 1/2). The former affects the magnitudes of risk premiums, but has little effect on correlations. The latter (time preference) controls the sign of the impact of volatility on excess returns on the consumption stream. If $\rho > 0$, the sign changes, as you might guess from (9). We also set $\kappa_1 = 0.997$, although in principle this should be derived from other parameters. It doesn't play an important role in any case.

Properties of excess returns

We've approached pricing two ways. The most natural from a theoretical perspective is to specify returns as functions of the expanded state $s^*_{t+1} = (s_{t+1}, s_t)$. Ditto the pricing kernel. If you take this version and substitute for s_{t+1} with its law of motion, you can express returns as functions of s_t and innovations. That's what we did above. In some ways that's more informative, but computations follow more easily from the former.

Given relations between returns (or excess returns) and the expanded state vector, we compute cross-correlations of returns from those of the state. The critical ingredient for excess returns is volatility. An example is the second panel of Figure 9: the cross-correlation function for the excess return on equity (the claim to the consumption stream) and consumption growth. It mimics, for the most part, the first panel, and the cross–correlations of Section 3. This should be no surprise, because the excess return is a function of volatility.

The exception is the contemporaneous correlation, which has a sharp positive spike at lag zero. This is a direct result of the dividend being consumption itself. In real life this isn't true: the contemporary contemporaneous correlation between consumption growth and dividend growth is small. Figure 10 shows that dividends and earnings lag industrial production, which raises an additional issue: the tendency for equity returns to lead economic growth occurs despite the tendency for its cash flows to lag.

6 Discussion

We have some work to do to nail down the details, but the numerical example shows that this kind of mechanism can replicate the shape of cross-correlation functions between excess returns and economic growth. There remain some open issues.

Risk and risk aversion. We've mimicked the cyclical behavior of excess returns in a model in which expected excess returns stem from variations in risk with constant risk aversion. We could have addressed the same issue by allowing risk aversion to vary across states, as it does in Campbell and Cochrane (1999) and Routledge and Zin (2003), or by letting the price of risk vary with the distribution of wealth across individuals, as in Lustig and Van Nieuwerburgh (2005). We have no particular reason to prefer our approach to these alternatives. Our point is simply that the data implies cyclical variation in excess returns.

A related issue is the magnitude of risk aversion used in our example. A common argument against risk aversion parameters this large is that when we extrapolate them to large risks (aggregate risks are small), the extent of risk aversion seems unreasonable. Extrapolation of this sort depends a lot on the form of risk preference, including the power form used here and expected utility in general. A natural resolution is a form of risk preference that exhibits different aversion to small and large risks. One such is disappointment aversion. Campanale, Castro, and Clementi (2006) show that the first-order risk aversion exhibited by such preferences exhibits substantial aversion to the small risks we see in the aggregate yet has modest aversion to the large risks faced by individuals. We could model that explicitly in this case, but at some cost of computational complexity. It's simpler to think of our risk preferences as a local approximation for the small risks present in this model.

An alternative is to interpret the risk aversion parameter as aversion to uncertainty about the model's structure. Barillas, Hansen, and Sargent (2008) show that a modest amount of uncertainty about the model's structure (the stochastic process for consumption, for example) can look like extreme risk aversion.

Consumption and returns. Empirical work by Canzoneri, Dumby, and Diba (2007) and Parker and Julliard (2005) shows that the contemporaneous correlation between returns and consumption growth is small, but increases as we expand the time interval. The evidence is Sections 2 and 3 is similar: since the correlation is larger with a lag of several months, it's not hard to imagine that the correlation will increase with the time interval. Our theoretical model suggestions an explanation: that the pricing kernel contains an additional factor that is correlated with consumption growth only with a lag.

Alvarez-Jermann bound. Our example has a relatively small bound. Is there enough variability in the pricing kernel with our parameter values?

Endogenous consumption. In a more complete model, variation in the conditional variance of (say) productivity shocks will generate an endogenous response from consumption. Naik (1993) and Primiceri, Schaumburg, Tambalotti (2006) are examples. It remains to be seen whether this produces the interaction we have specified for volatility and consumption growth.

Cross-section of returns. We've looked at the possibility that aggregate volatility might account for the cyclical behavior of excess returns. Koijen, Lustig, and Van Nieuwerburgh (2008) use a similar approach to account for the cross section of asset returns.

Solution method. There is a growing literature on perturbation methods, in which uncertainty doesn't appear until the second-order approximation. Collard and Juillard (2001) is an elegant example, and Van Binsbergen, Fernandez-Villaverde, Koijen, and Rubio-Ramirez (2008) show how such methods can be extended to models with recursive preferences. Our approach differs in two respects: recursive preferences lead to more complex equilibrium conditions, and we use methods similar to those in finance in which variances appear even with first-order (linear) approximations. This isn't a substitute for high-order approximations, but it allows us to generate reasonably accurate solutions without giving up the convenience of linearity.

7 Conclusions

[Later.]

A Data sources

[Later.]

B Theoretical results

The Kreps-Porteus pricing kernel

[Change to Markov environment with conditional probabilities $\pi(s_{t+1}|s_t)$.]

The pricing kernel in a representative agent model is the marginal rate of substitution between (say) consumption at date t [c_t] and consumption in state s at $t+1$ [$c_{t+1}(s)$]. Here's how that works with recursive preferences. With this notation, the certainty equivalent (2) might be expressed less compactly as

$$\mu_t(U_{t+1}) \quad = \quad \left[\sum_s \pi(s) U_{t+1}(s)^\alpha \right]^{1/\alpha},$$

where $\pi(s)$ is the conditional probability of state s and $U_{t+1}(s)$ is continuation utility. Some derivatives of (1) and (2):

$$\begin{aligned} \partial U_t / \partial c_t &= U_t^{1-\rho} (1-\beta) c_t^{\rho-1} \\ \partial U_t / \partial \mu_t(U_{t+1}) &= U_t^{1-\rho} \beta \mu_t(U_{t+1})^{\rho-1} \\ \partial \mu_t(U_{t+1}) / \partial U_{t+1}(s) &= \mu_t(U_{t+1})^{1-\alpha} \pi(s) U_{t+1}(s)^{\alpha-1}. \end{aligned}$$

The marginal rate of substitution between consumption at date t and consumption in state s at $t+1$ is

$$\begin{aligned} \frac{\partial U_t / \partial c_{t+1}(s)}{\partial U_t / \partial c_t} &= \frac{[\partial U_t / \partial \mu_t(U_{t+1})][\partial \mu_t(U_{t+1}) / \partial U_{t+1}(s)][\partial U_{t+1}(s) / \partial c_{t+1}(s)]}{\partial U_t / \partial c_t} \\ &= \pi(s)\ \beta \left(\frac{c_{t+1}(s)}{c_t} \right)^{\rho-1} \left(\frac{U_{t+1}(s)}{\mu_t(U_{t+1})} \right)^{\alpha-\rho}. \end{aligned}$$

The pricing kernel (4) is the same with the probability $\pi(s)$ left out and the state left implicit.

Equity prices and returns

We define equity at t as a claim to consumption from $t+1$ on. The return is the ratio of its value at $t+1$, measured in units of $t+1$ consumption, to the value at t, measured in units of t consumption. The value at $t+1$ is U_{t+1} expressed in c_{t+1} units:

$$
\begin{aligned}
U_{t+1}/\left(\partial U_{t+1}/\partial c_{t+1}\right) &= U_{t+1}/\left[(1-\beta)U_{t+1}^{1-\rho}c_{t+1}^{\rho-1}\right] \\
&= (1-\beta)^{-1}u_{t+1}^{\rho}c_{t+1}.
\end{aligned}
$$

The value at t is the certainty equivalent expressed in c_t units:

$$
\begin{aligned}
q_t^c c_t &= \frac{\partial U_t/\partial \mu_t(U_{t+1})}{\partial U_t/\partial c_t}\,\mu_t(U_{t+1}) \;=\; \frac{\beta\mu_t(U_{t+1})^{\rho}}{(1-\beta)c_t^{\rho}}\,c_t \\
&= \beta(1-\beta)^{-1}\mu_t(g_{t+1}u_{t+1})^{\rho}c_t.
\end{aligned}
$$

The return is the ratio:

$$
\begin{aligned}
r_{t+1}^c &= \beta^{-1}\left[u_{t+1}/\mu_t(g_{t+1}u_{t+1})\right]^{\rho} g_{t+1} \\
&= \beta^{-1}\left[g_{t+1}u_{t+1}/\mu_t(g_{t+1}u_{t+1})\right]^{\rho} g_{t+1}^{1-\rho}.
\end{aligned}
$$

Check to see if this satisfies the Euler equation:

$$
\begin{aligned}
E_t\left(m_{t+1}r_{t+1}^c\right) &= E_t\left[g_{t+1}u_{t+1}/\mu_t(g_{t+1}u_{t+1})\right]^{\alpha} \\
&= \mu_t(g_{t+1}u_{t+1})^{\alpha}/\mu_t(g_{t+1}u_{t+1})^{\alpha} \;=\; 1.
\end{aligned}
$$

[To do: Connect to Campbell-Shiller approx... show that the approx in u is equiv to that in q with the same κ_1.]

Computing cross correlations

Recall that the state is $s_t = (x_t, v_t)$ and the "expanded state" is $s_t^* = (s_t, s_{t-1})$. The latter has the law of motion

$$
s_{t+1}^* = A_* s_t^* + B_* w_{t+1},
$$

with

$$
A_s = \begin{bmatrix} A & a \\ 0 & \varphi_v \end{bmatrix}, \quad B_s = \begin{bmatrix} B \\ b \end{bmatrix}
$$

and

$$
A_* = \begin{bmatrix} A_s & 0 \\ I & 0 \end{bmatrix}, \quad B_* = \begin{bmatrix} B_s \\ 0 \end{bmatrix}.
$$

The unconditional variance is

$$
G(0) = E\left(s_t^* s_t^{*\top}\right) = A_* G(0) A_*^{\top} + B_* B_*^{\top}.
$$

We compute $G(0)$ iteratively using Hansen and Sargent's (2005) Matlab program `doublej.m`. Autocovariances follow from

$$G(k) \quad = \quad E\left(s_t^* s_{t-k}^{*\top}\right) \; = \; \begin{cases} A_*^k G(0) & k > 0 \\ G(0)(A_*^k)^\top & k < 0. \end{cases}$$

Since $G(-k) = G(k)^\top$, positive k is sufficient.

Returns and excess returns are linear functions of the expanded state: $r_t = H s_t^*$ say for a vector of returns and excess returns. Autocovariances are

$$E\left(r_t r_{t-k}^\top\right) \quad = \quad h G(k) h^\top.$$

Cross-covariances are off-diagonal elements and cross-correlations are scaled by standard deviations.

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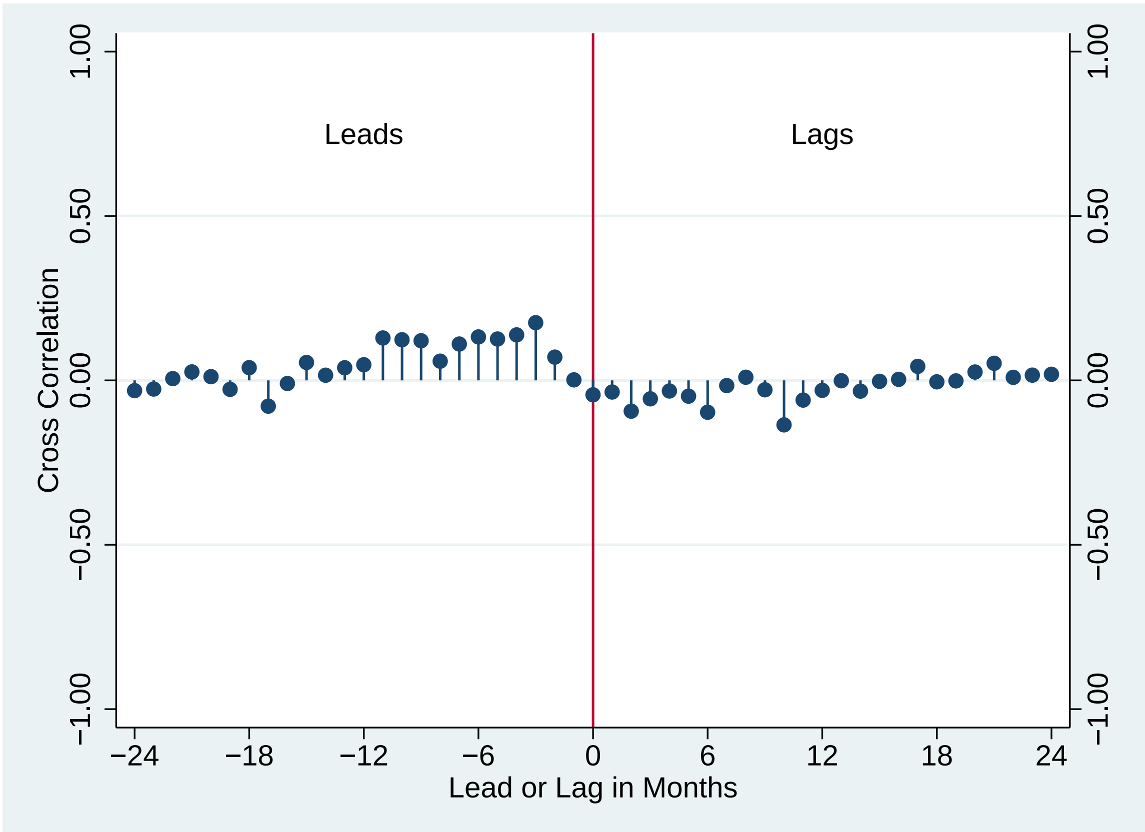
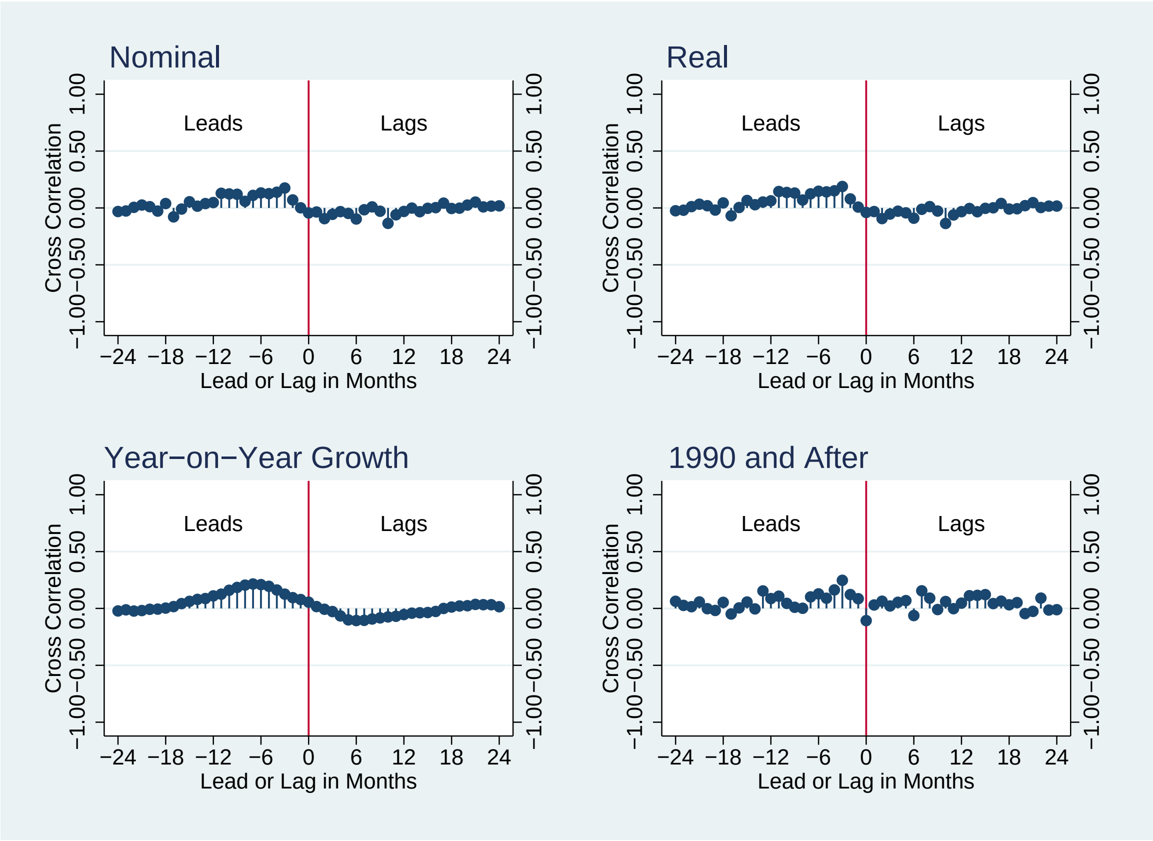
Figure 1
Cross correlations for equity returns



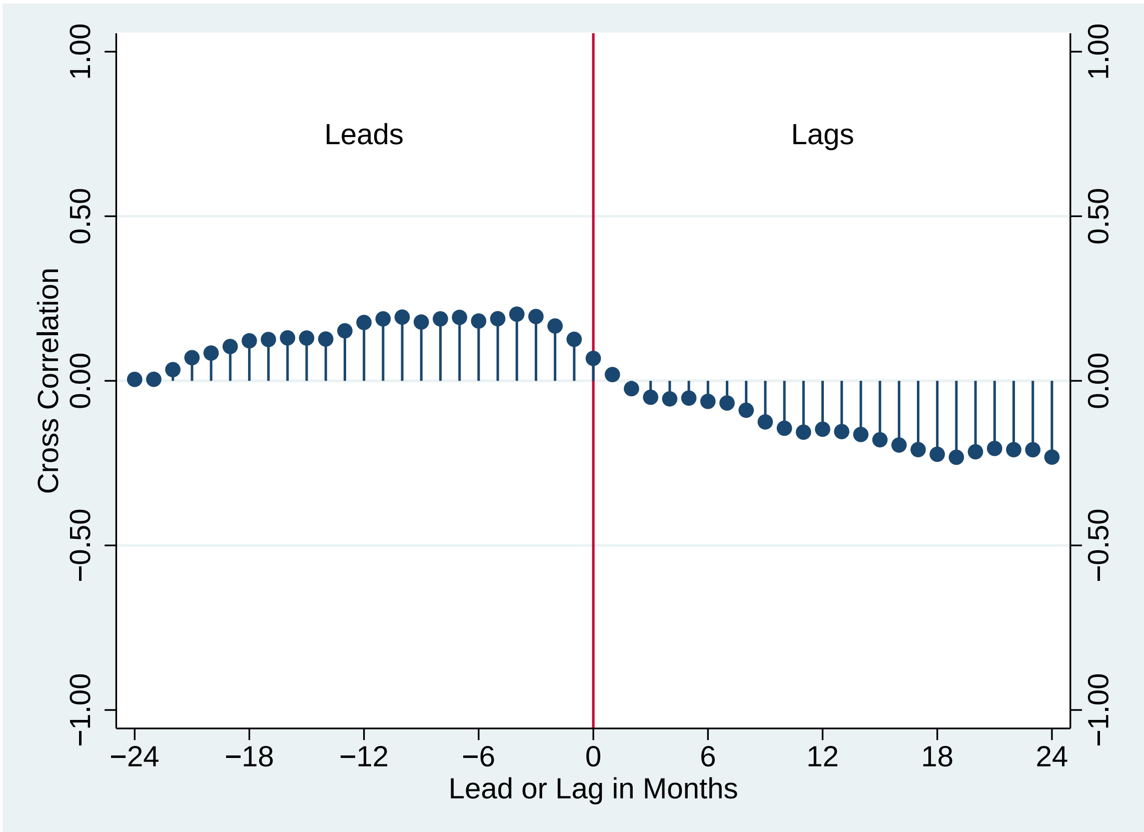
Notes. The figure depicts the cross-correlation function for the return on an aggregate equity portfolio and the monthly growth rate of industrial production. On the left side of the figure, the return leads growth, on the right side it lags. The sample period is 1960-present.

Figure 2
Cross correlations for equity returns: variations



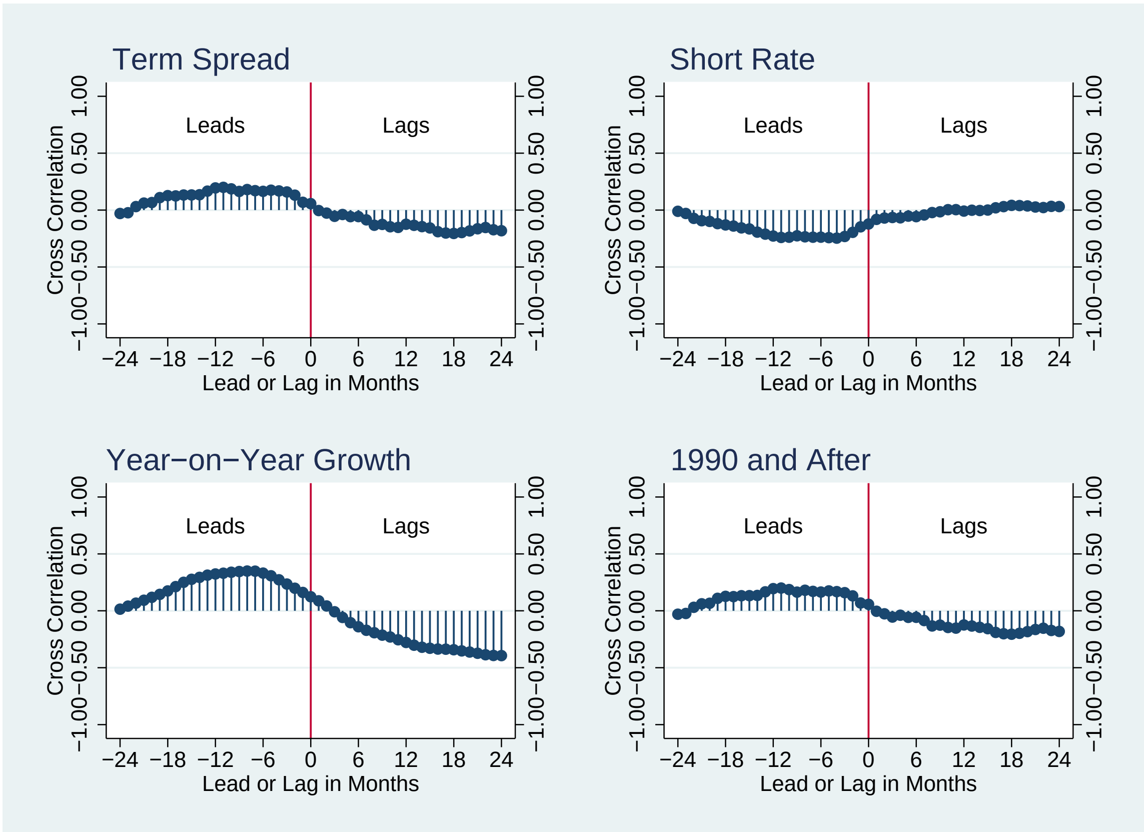
Notes. The figure depicts cross-correlation functions for the return on an aggregate equity portfolio and the growth rate of industrial production. The upper left panel is a repeat of Figure 1. The upper right panel is based on the real equity return: we subtract the monthly inflation rate from the nominal return. The bottom left panel replaces monthly growth in industrial production with centered year-on-year growth. All three use data from 1960 to the present. The lower right panel uses data from 1990 to the present.

Figure 3
Cross correlations for the term spread



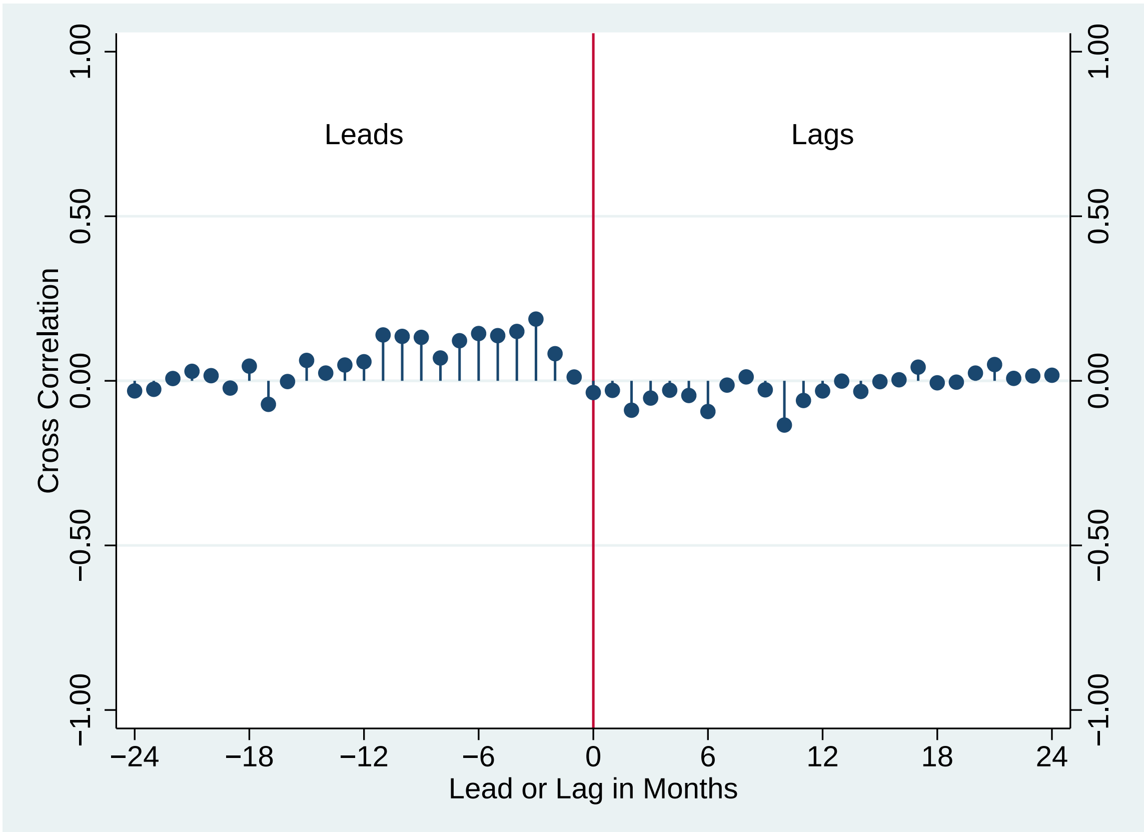
Notes. The figure depicts the cross-correlation function for the term spread (the difference between the 5-year and 1-month continuously compounded nominal yields on US treasuries) and the monthly growth rate of industrial production. The sample period is 1960-present.

Figure 4
Cross correlations for the term spread: variations



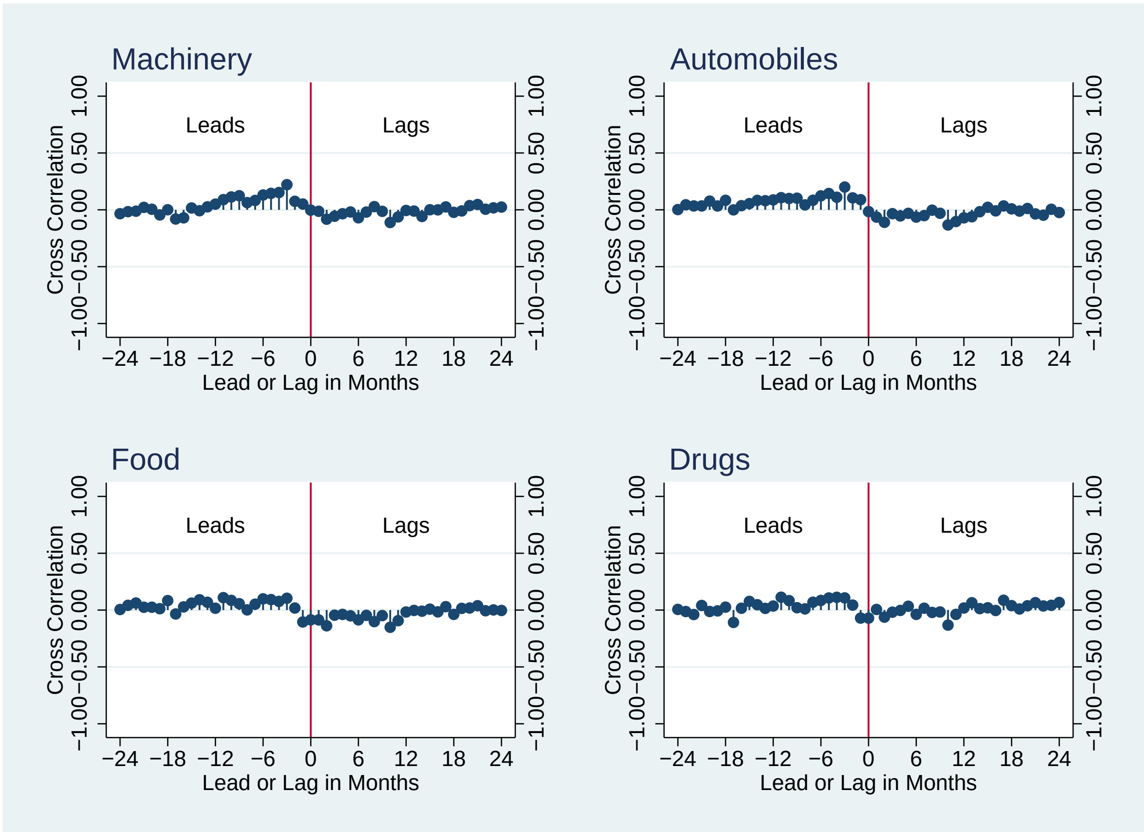
Notes. The figure depicts cross-correlation functions for interest rates and the growth rate of industrial production. The upper left panel is a repeat of Figure 3 for the term spread. The bottom left panel replaces monthly growth in industrial production with centered year-on-year growth. The upper right panel is based on the short rate (the 1-month treasury yield). All three use data from 1960 to the present. The lower right panel uses data from 1990 to the present.

Figure 5
Cross correlations for equity excess returns



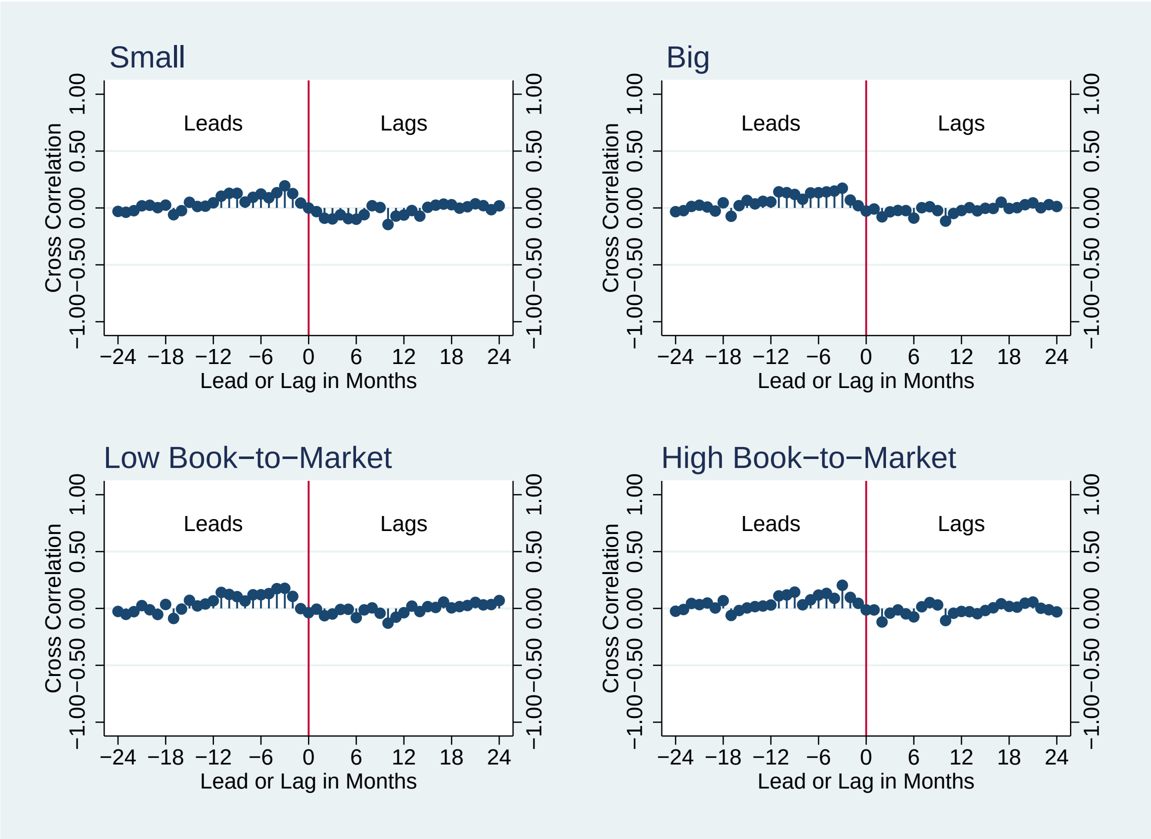
Notes. The figure depicts the cross-correlation function for the excess return on an aggregate equity portfolio and the monthly growth rate of industrial production. The sample period is 1960-present.

Figure 6
Cross correlations for industry portfolios



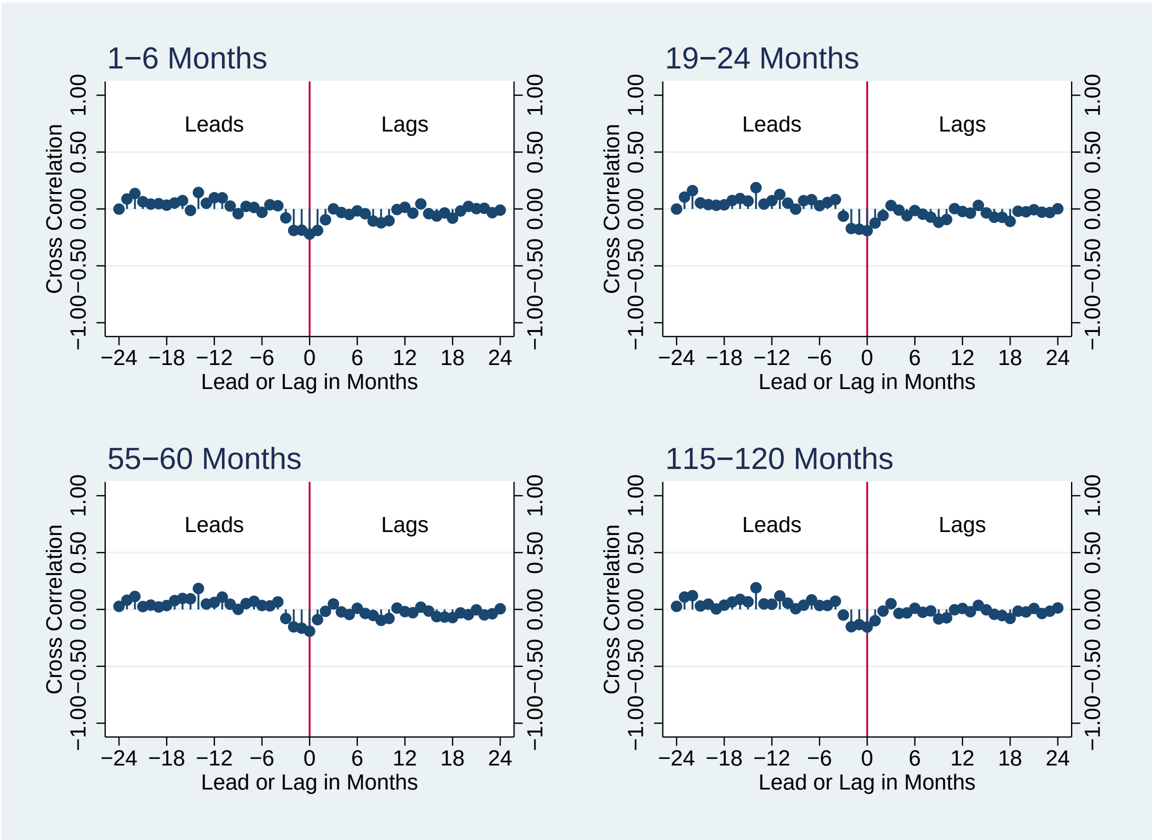
Notes. The figure depicts cross-correlation functions for excess returns on four industry portfolios and the monthly growth rate of industrial production. The sample period is 1960-present.

Figure 7
Cross correlations for size and book-to-market portfolios



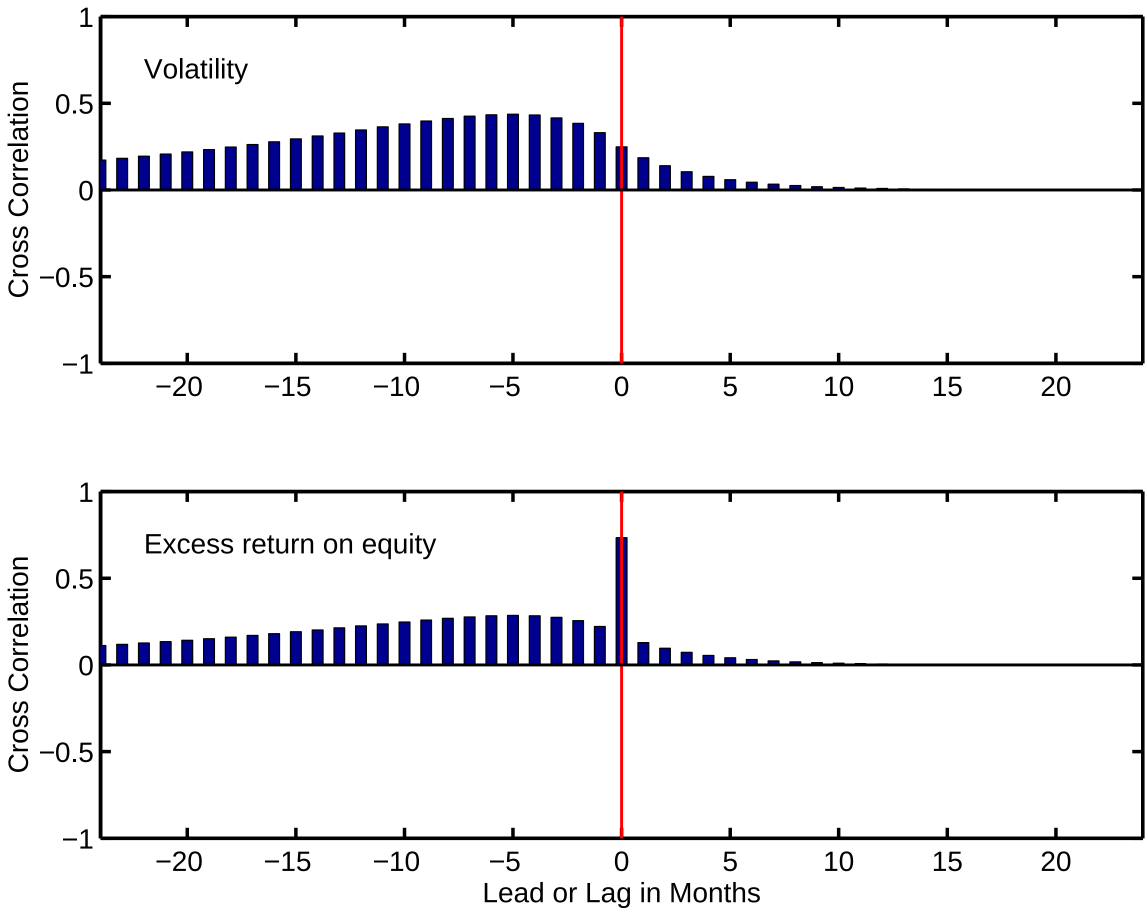
Notes. The figure depicts cross-correlation functions for excess returns on the first and tenth deciles of size and book-to-market portfolios, respectively, and the monthly growth rate of industrial production. The sample period is 1960-present.

Figure 8
Cross correlations for bond portfolios



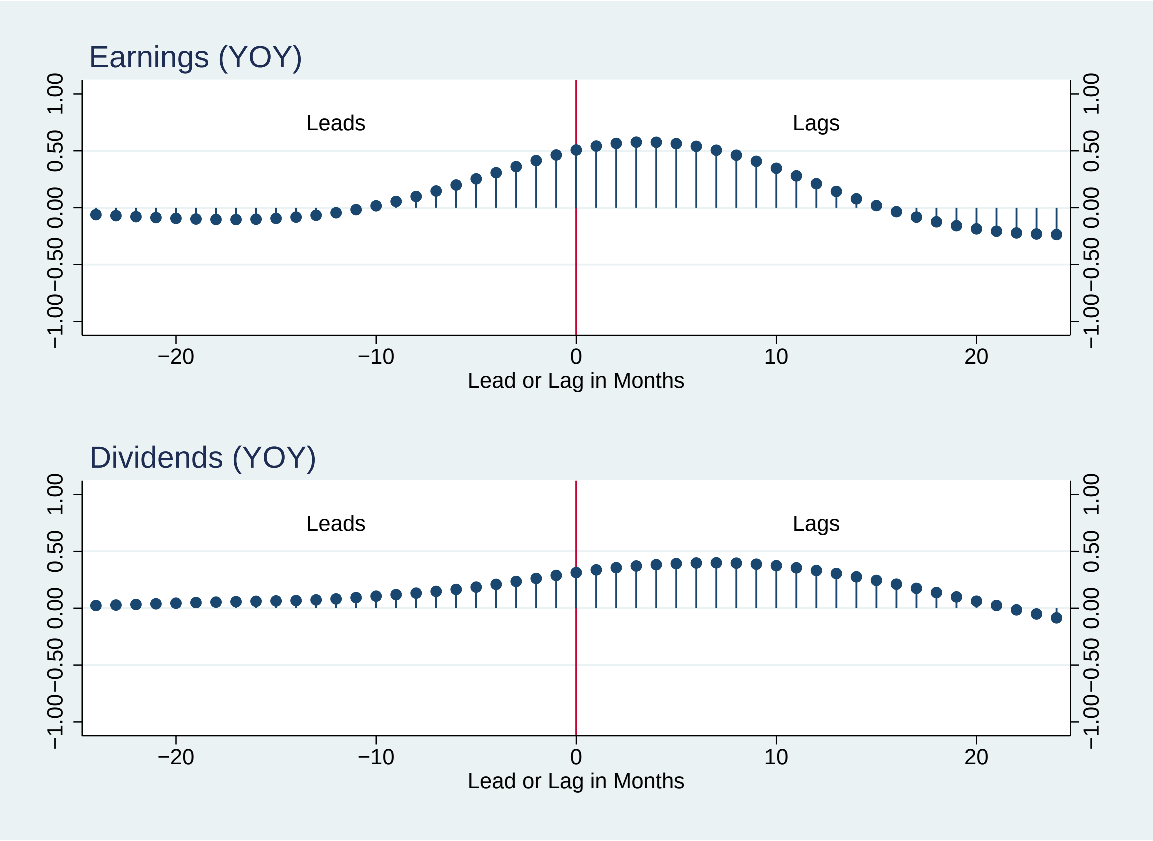
Notes. The figure depicts cross-correlation functions for the excess returns on bond portfolios of different maturities and the monthly growth rate of industrial production. The sample period is 1960-present.

Figure 9
Cross correlations for numerical example



Notes. The figure depicts cross-correlation functions for the numerical example. The top panel is the cross-correlation function for volatility and consumption growth. The bottom panel is for the excess return on "equity" (a claim to the consumption stream) and consumption growth. NB: this is based on $a = 75$ to make the graph look better.

Figure 10
Cross correlations for dividends and earnings



Notes. The figure depicts cross-correlation functions for growth rate of dividend and earnings, respectively, and industrial production.