Counterparty Risk in Financial Contracts: Should the Insured Worry about the Insurer?

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Abstract

We analyze the effect of counterparty risk on financial insurance contracts using the case of credit risk transfer in banking. In addition to the familiar moral hazard problem caused by the insured's ability to influence the probability of a claim, this paper uncovers a new moral hazard problem on the part of the insurer. If the insurer believes it is unlikely that a claim will be made, it is advantageous for them to invest in assets which earn higher returns, but may not be readily available if needed. We find that counterparty risk can create an incentive for the insured to reveal superior information about the risk of their "investment". In particular, a unique separating equilibrium may exist even in the absence of any signalling device. Our research is relevant to the current credit crises and suggests that regulators should be wary of risk being offloaded to other, possibly unstable parties, especially in financial markets such as that of credit derivatives.

Keywords: Counterparty Risk, Moral Hazard, Credit Derivatives, Insurance, Banking.
JEL Classification Numbers: G21, G22, D82.

1 Introduction

In this paper, we develop an agency model to analyze an insurer's optimal investment decision when failure is a possibility. We demonstrate that an insurer's investment choice may be inefficient by showing that a moral hazard problem exists on this side of the market. This insurer moral hazard problem does have an upside however, as we show that it can alleviate the possible adverse selection problem on the part of the insured.

The credit crisis of 2007/2008 has given considerable media attention to counterparty risk.[1] Many firms who issue insurance contracts (e.g. AIG, many monoline insurers[2]) have experienced nothing short of a crisis as they try to pay claims to insured parties. The question then remains: what are the incentives of those parties who insure credit risk? In addressing this, we will not attempt to incorporate all the salient features of the various insurance players and markets. Instead, we will focus on a general insurance problem using the market for credit risk transfer as our example. In taking a general approach, one of the key novelties of our paper is that we inject the insurers' asset-liability management into a standard model of insurance.

The market for risk protection is one of the most important markets available today. Figure 1 shows the growth rate in credit derivatives since 2003.[3] It is easy to see the rapid growth that these financial markets have experienced. An institution on which these markets have a particularly profound effect is the banking system. The reason is that banks were once confined to a simple borrow short and lend long strategy. However, they can now disperse credit risk through credit derivatives markets to better implement risk management policies. This in itself may be a positive development; however, two features make these markets potentially different (and dangerous) when compared to traditional insurance markets. First, the potential for unstable counterparties. In other words, potentially large credit risks are being ceded to parties such as hedge funds which may or may not be in a better position to handle them.[4] The second feature which is unique to this market is the large size of the contracts.[5] It would seem prudent then to ask the question of how stable is, and what are the incentives of the insurer? This entails a study of counterparty risk. In what is to follow, we define counterparty risk as the risk that when a claim is made, the insurer is unable to fulfil its obligations.

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[1]For a review on the causes and symptoms of the credit crises see Greenlaw et al. (2008) and Rajan (2008).

[2]Monoline insurers guarantee the timely repayment of bond principal and interest when an issuer defaults.

[3]A credit derivative, and specifically a *credit default swap* is an instrument of credit risk transfer whereby an insurer agrees to cover the losses of the insured that take place if pre-defined events happen to an underlying borrower. (In many cases, this event is the default of the underlying bond. However, some contracts include things like restructuring and ratings downgrades as triggering events.) In exchange for this protection, the insured agrees to pay an ongoing premium at fixed intervals for the life of the contract.

[4]Fitch (2006) reports that banks are the largest insured party in this market. On the insurer side, banks and hedge funds are the largest, followed by insurance companies and other financial guarantors. It should be noted that the author's of the Fitch report suspect that banks are the largest insurers, followed by hedge funds; however, they add that the data is poor and that other research reports do not support this.

[5]The two typical credit default swap contract denominations are $5 and $10 million.

Figure 1: Notional Value of Credit Derivatives (in Trillions of Dollars)[6]
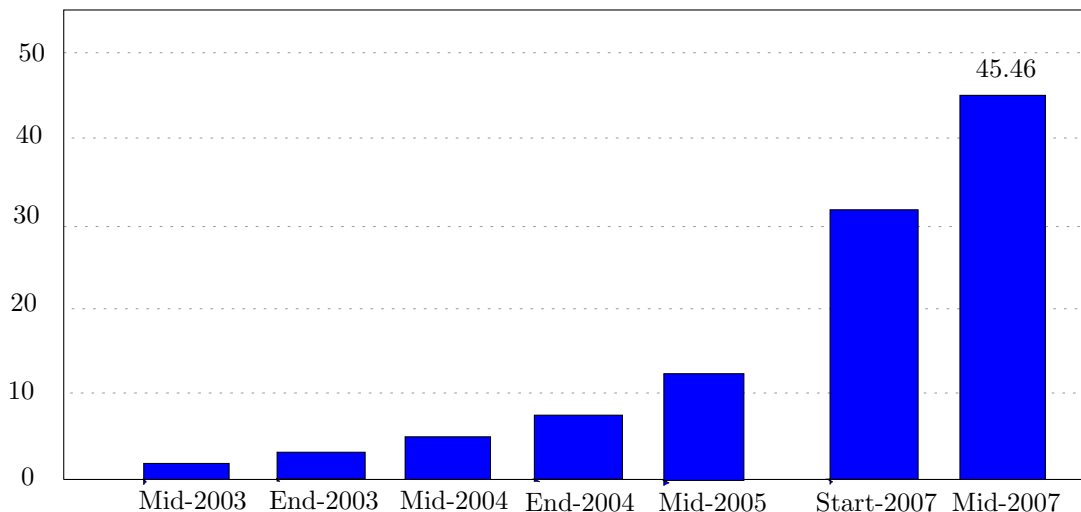
This paper arrives at two novel results. The first is that there can exist a moral hazard on the part of the insurer. We call this the *moral hazard result*. This moral hazard arises because the insurer may choose an excessively risky portfolio. The intuition behind this result is as follows. There are two key states of the world that enter into the insurer's decision problem: the first in which a claim is not made, and the second in which it is. We assume that the insurer can default in both of these states if it receives an unlucky draw. However, it can invest and influence the chances that it fails. This investment choice comes with a tradeoff: what reduces the probability of failure the most in the state in which a claim is not made, makes it more likely that the insurer will fail in the state in which it is. For example, if the insurer believes that the contract is relatively safe, it may be optimal to put capital into less liquid assets to reap higher returns, and lower the chance of failure in the state in which a claim is not made. However, assets which yield these higher returns can also be more costly to liquidate, and therefore make it more difficult to free up capital if a claim is made. The moral hazard arises because the premium is not made conditional on an observed outcome, rather it is paid upfront.[7] Therefore, there is no way to influence the insurer's investment decision by imposing penalties. We show that the resulting equilibrium is inefficient.

It is important to note that the excess risk taking result is not a direct consequence of the assumption of limited liability on the insurer. One of the first discussions concerning the connection between limited liability and excess risk taking can be found in Jensen and Meckling (1976). In that paper, a manager does not have the proper incentives to internalize the downside risk of a firm so that they take excessive gambles. In our paper however, the inefficiency arises because the insurer does not internalize the counterparty risk that it causes to the insured party. It is this fact that makes the insurer take excess risk. This is not meant to imply however, that limited liability

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[6]Note that these values are likely subject to double counting.

[7]To have a conditional premium would require a higher payment from the insured to the insurer when the insurer is able to pay than when it is not. This goes against the nature of an insurance contract: when a claim is made, the insured party does not want to pay the insurer.

does not play a role in the risk taking behavior of insurers. Rather, it does not play a major role in terms of the inefficiency we analyze.

The second result is a consequence of the moral hazard problem uncovered. Consider the adverse selection problem that may be present because of the superior information that the insured has about the underlying claim. Akerlof (1970) describes the dangers of informational asymmetries in insurance markets. In his seminal paper, it is shown how the market for good risks may break down, and one is left with insurance only being issued on the most risky of assets, or in Akerlof's terminology, lemons. The incentive that underlies this result is that the insured only wishes to obtain the lowest insurance premium. This incentive will still be present in our model; however, we uncover an opposing incentive.

In our work we show that the safer the underlying claim is perceived to be, the more severe the moral hazard problem is. Consequently, conditional on a claim being made, counterparty risk is higher for insured assets perceived by the insurer as safer. We show that truthful revelation can be optimal for the insured with a poor quality asset. In this case, the insurer will have incentives more in line with the insured, and consequently the insured is subjected to less counterparty risk. We show that this new effect, which we call the *counterparty risk effect* allows a unique separating equilibrium to be possible. This result is new in that separation can occur in the absence of a costly signalling device. After Akerlof's (1970) model showed that no separating equilibrium can exist, the literature developed the concept of signalling devices with such famous examples as education in Spence's job market signalling paper. These papers allowed the high (safe) type agents to separate themselves by performing a task which is "cheaper" for them than for the low (risky) type agents. Our paper can achieve separation by the balance between the insured's desire for the lowest insurance premium, and the desire to be exposed to the least counterparty risk. One can think of this result as adding to the *cheap talk* literature by showing an insurance problem in which costless communication can bring about separation of types.[8] We call this the *separating equilibrium result*. We discuss the starkness of this result in the robustness section 5. In particular, we consider the way in which this result would manifest itself in a richer model.

The moral hazard result holds regardless of the contract size or number of insured parties. The separating equilibrium result holds if one or both of the following two conditions are met: first, a contract is sufficiently large to affect the insurer's investment decision, and second, there is aggregate private risk shared among a pool of insured parties. The former case is plausible in some situations (e.g., as discussed above, in some financial markets single contracts can be large), however, the latter likely constitutes a wider range of cases. The former case is, however, better suited for developing the intuition behind our results, so in section 2 we model only one insured party and one insurer. In section 4, we generalize the model to the case of multiple insurers, each of which is insignificant to the insurer's investment decision. We consider the case in which the insured parties share a common component of risk (i.e., correlated risk). Martin et al. (2008) serve as an example of the importance of this type of risk in the context of the credit crises. It has

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[8] For a review of the cheap talk literature, see Farrell and Rabin (1996).

become clear that there was correlated information that inside financial institutions had on the key risks being traded (i.e., bundled mortgages). The results obtained in this section follow intuitively from the base model.

Next, in a section in which we omit for brevity (see Appendix B, section 8 which is not intended for publication), we extend the model to include a possible moral hazard problem on the part of the insured. This moral hazard arises by assuming that the insured can affect the probability that a claim is made. If we use the example of a bank insuring itself on one of its loans, the literature typically assumes that a bank possesses a proprietary monitoring technology (due to a relationship with the borrower). It is straightforward to see that if the bank is fully insured, it may not have the incentive to monitor the loan and, consequently, the probability of default could rise. This represents the classical moral hazard problem in the insurance literature. This extension shows that the new moral hazard introduced in this paper may increase the desire of the insured to monitor. This happens because counterparty risk forces the bank to internalize some of the default risk which it otherwise would not. In this section we show that with a redefinition of a payoff function, the addition of this insured moral hazard problem does not affect the results of the paper.

1.1 Related Literature

This paper contributes to two streams of literature: that of credit risk transfer and credit derivatives and that of insurance economics. The literature on credit risk transfer (CRT) is relatively small but is growing. Allen and Gale (2006) motivate a role for CRT in the banking environment while Parlour and Plantin (2007) derive conditions under which liquid CRT markets can exist. Using the same framework as Allen and Gale (2006), Allen and Carletti (2006) show how a default by an insurance company can cascade into the banking sector causing a contagion effect when the two parties are linked through CRT. Wagner and Marsh (2006) argue that setting regulatory standards that reflect the different social costs of instability in the banking and insurance sector would be welfare improving. Our paper differs from the ones above because they do not consider the agency problems of insurance contracts. As a result, they do not discuss the consequences that instability can have on the contracting environment, and how this affects the behavior of the parties involved. Duffee and Zhou (2001), Thompson (2007) and Parlour and Winton (2008) analyze informational problems in insurance contracts; however, they focus on the factors that affect the choice between sales and insurance of credit risk. These papers do not analyze counterparty risk as is done in our paper.

We contribute to the literature on insurance economics by raising the issue of counterparty risk which has received little attention. Henriet and Michel-Kerjan (2006) recognize that insurance contracts need not fit the traditional setup in which the insurer is the principal and the insured, the agent. The authors relax this assumption and allow the roles to change. Their paper however does not consider the possibility of counterparty risk as ours does, as they assume that neither party can fail. Plantin and Rochet (2007) study and give recommendations on the prudential regulation of insurance companies. Their work does not consider the insurance contract itself under counterparty

risk as is done in our paper. Consequently, the authors do not analyze the effects of counterparty risk on the informational problems. Instead, they conjecture an agency problem arising from a corporate governance standpoint. We analyze an agency problem driven entirely by the investment incentives of the insurer. Phillips, Cummings and Allen (1998) and Myers and Read (2001) study the capital allocation and pricing consequences of having multiple lines of insurance when there is default risk. They do not consider the investment decision nor an agency problem as is analyzed in this paper.

The paper proceeds as follows: Section 2 outlines the model and solves the insurer's problem. Section 3 determines the equilibria that can be sustained when asymmetric information is present. Furthermore, this section shows a moral hazard problem on the part of the insurer by determining that an inefficient investment choice is made. Section 4 analyzes the case of multiple insured parties, and section 6 concludes. Many of the longer proofs are relegated to the appendix in section 7. Section 8 (not for publication) extends the model to the case of the classical moral hazard problem on the insured side of the market.

2   The Model Setup

The model is in three dates indexed $t = 0, 1, 2$. There are two main agent types, multiple insured party, whom we will call banks, and multiple risk insurers, whom we will call Insuring Financial Institutions (IFIs). For expositional purposes, we will first focus on the case where there is only one bank. In section 4 we will allow multiple banks. We also assume there is an underlying borrower who has a loan with the bank. We model this party simply as a return structure. The size of the loan is normalized to 1 for simplicity. We motivate the need for insurance through an exogenous parameter (to be explained below) which makes the bank averse to risk. We assume there is no discounting; however, adding this feature will not affect our qualitative results.

2.1   The Bank

The bank is characterized by the need to shed credit (loan) risk. We use the example of a bank that faces capital regulation and must reduce its risk, or else could face a cost (which we denote by $Z \geq 0$). It is this cost that makes the bank averse to holding the risk and so finds it advantageous to shed it through insurance. This situation can be thought of as arising from an endogenous reaction to a shock to the bank's portfolio; however for simplicity, we will not model this here. There are two types of loans that a bank can insure, a safe type (S) and a risky type (R). A bank is endowed with one or the other with equal probability for simplicity. We assume that the return on either loan is $R_B > 1$ if it succeeds which happens with probability p_S (p_R) if it is safe (risky), where $1 > p_S > p_R > 0$. We assume that the return of a failed loan is zero for simplicity. The loan type is private knowledge to the bank and reflects the unique relationship between them and the underlying borrower. We assume that the loan can be costlessly monitored, so that there is no moral hazard problem in the bank-borrower relationship. In section 8.1 (not for publication) we

relax this assumption and show that introducing costly monitoring does not change the qualitative results of the paper. Note that there is nothing in the analysis to follow that requires this to be a single loan. When we interpret this as a single loan, the insurance contracts to be introduced in section 2.3 will resemble that of a *credit default swap*. In the case that this is a return on many loans, the insurance contract will closely resemble that of a *portfolio default swap* or *basket default swap*.[9]

The regulator requires the bank to insure a fixed, and equal proportion of either loan. For simplicity, the bank must insure a proportion γ of its loan, regardless of its type.[10] As will be shown in section 3, we are able to obtain a separating equilibrium without any signalling device. In standard models of insurance contracts, a costly signaling device can be the amount of insurance that the safe and risky type take on. The safe type is able to signal that it is safe by taking on less insurance (e.g., a higher deductible). In this paper, we shut down this mechanism for obtaining a separating equilibrium so that we can better understand the mechanism that counterparty risk creates. We impose the exogenous cost Z on the bank if the loan defaults and it is not insured for the appropriate amount, or if it is insured for the appropriate amount, but the counterparty is not able to fulfil a claim.[11] This two part linearity of the payoff function is what imposes the same contract size on both bank types.

In what follows, we only model the payoff to this loan for the bank; however, it can be viewed as only a portion of its total portfolio. For simplicity, we assume that the bank cannot fail. Allowing the bank to fail will not affect our qualitative results since it will not affect the insurance contract to be introduced in section 2.3. We now turn to the modelling of the IFI.

2.2 The Insuring Financial Institution

We assume that the insurer receives a random return at $t = 2$ governed by the uniform probability density function f(θ) (with corresponding distribution F(θ)) with upper (lower) bound $\overline{R}_f > 0$ ($\underline{R}_f < 0$). We assume that bankruptcy occurs when the portfolio draw is in the set $[\underline{R}_f, 0]$, however, because of limited liability, the IFI receives a payoff of zero if bankrupt.[12] Therefore the

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[9]A portfolio or basket default swap is a contract written on more than one loan. There are many different configurations of these types of contracts. For example, a first-to-default contract says that a claim can be made as soon as the first loan in the basket defaults.

[10]The assumption of a fixed amount of insurance regardless of type is not crucial. We can think of γ being solved for by the bank's own internal risk management. Therefore, we could have a differing γ depending on loan quality. What is important in this case is that the IFI is not able to perfectly infer the probability of default from γ. This assumption is justified when the counterparty does not know the exact reason the bank is insuring. To know so would require them to know everything about the bank's operations, which should be excluded as a possibility. In this enriched case, γ can be stochastic for each loan type reflecting different (private) financial situations for the bank. This topic has been addressed in the new Basel II accord which allows banks to use their own internal risk management systems in some cases to calculate needed capital holdings. One reason for this change is because of the superior information banks are thought to have on their own assets; regulators have acknowledged that the bank itself may be in the best position to evaluate their own risk.

[11]It is not crucial that Z be the same for both the situation in which the bank does not purchase insurance, and when is does but a claim cannot be fulfilled by the counterparty. We could posit different values for each situation; however, this will not affect our qualitative results.

[12]Note that in the case of monoline insurance, we can think of bankruptcy as a ratings downgrade. The monoline business is based on having a better credit rating than the client in a process called *wrapping*. Without a good rating,

expected payoff at time $t = 0$ for the IFI can be given by:

$$\Pi_{IFI}^{No\ Insurance} = \int_0^{\overline{R}_f} \theta f(\theta) d\theta. \tag{1}$$

Although the return is realized at $t = 2$, we assume for simplicity that a perfect signal is sent about the return at $t = 1$. Therefore, at $t = 1$, the random variable θ represents the portfolio value if it could be costlessly liquidated at that time. However, the IFI's portfolio is assumed to be composed of both liquid and illiquid assets. In practice, we observe financial institutions holding both liquid (e.g., t-bills, money market deposits) and illiquid (e.g., loans, some exotic options, some newer structured finance products) investments on their books.[13] Because of this, if the IFI wishes to liquidate some of its portfolio at time $t = 1$, it will be subject to a liquidity cost which we discuss below in section 2.3.

2.3 The Insurance Contract

We now introduce the means by which the bank is insured by the IFI. Because of the possible cost Z, at time $t = 0$ the bank requests an insurance contract in the amount of γ for one period of protection. Therefore, the insurance coverage is from $t = 0$ to $t = 1$. To begin, we assume that the bank contracts with one IFI who is in Bertrand competition. The IFI forms a belief b about the probability that the bank loan will default. In section 3 we will show how b is formed endogenously as an equilibrium condition of the model. In exchange for this protection, the IFI receives an insurance premium $P\gamma$, where P is the per unit price of coverage. The IFI chooses a proportion β of this premium to put in a liquid asset that, for simplicity, has a rate of return normalized to one in both $t = 1$ and $t = 2$, but can be accessed at either time period. The remaining proportion $1 - \beta$ is put in an illiquid asset with an exogenously given rate of return of $R_I > 1$ which pays out at time $t = 2$.[14] This asset can be thought of as a two period project that cannot be terminated early. It is this property that makes it illiquid. The key difference between these two assets is that the liquid asset is accessible at $t = 1$ when the underlying loan may default, whereas the illiquid asset is only available at $t = 2$.

For the remaining capital needed (net of the premium put in the liquid asset) if a claim is made, we assume that the IFI can liquidate its portfolio. Recall that the IFI's initial portfolio contains assets of possibly varying degrees of liquidity with return governed by F. To capture this, we assume that the IFI has a liquidation cost represented by the invertible function $C(\cdot)$ with $C'() > 0$, $C''() \geq 0$, and $C(0) = 0$. The weak convexity of $C(\cdot)$ implies that the IFI will choose to

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it would not be profitable for firms to insure themselves with a monoline.

[13]If another bank acts as the IFI, it is obvious that many illiquid assets are on its the books. However, this is also the very nature of many insurance companies and hedge funds businesses. In the case of insurance companies as the IFI, substantial portions of their portfolios may be in assets which cannot be liquidated easily (see Plantin and Rochet (2007)). In the case of hedge funds as the IFI, many of them specialize in trading in illiquid markets (see Brunnermeier and Pederson (2005) for example).

[14]We can think of these as two assets that are in the IFI's portfolio; however, we assume that the amount is small so that the illiquid asset and the original portfolio are uncorrelated. Adding correlation would only complicate the analysis and would not change the qualitative results.

liquidate the least costly assets first, but as more capital is required, it will be forced to liquidate illiquid assets at potentially fire sale prices.[15] $C(\cdot)$ takes as its argument the amount of capital needed from the portfolio, and returns a number that represents the actual amount that must be liquidated to achieve that amount of capital. This implies that $C(x) \geq x \ \forall x \geq 0$ so that $C'(x) \geq 1$. For example, if there is no cost of liquidation and if x_1 is required to be accessed from the portfolio, the IFI can liquidate x_1 to satisfy its capital needs. However, because liquidation may be costly in this model, the IFI must liquidate $C(x_1) = x_2 \geq x_1$ to obtain the needed x_1.

At time $t = 1$, the IFI learns a valuation of its portfolio; however, the return is not realized until $t = 2$. This could be relaxed so that the IFI receives a fuzzy signal about the return, however, this would yield no further insight into the problem. Also at $t = 1$, a claim is made if the underlying borrower defaults. If a claim is made, the IFI can liquidate its portfolio to fulfil its obligation of γ.[16] If the contract cannot be fulfilled, the IFI defaults. We assume for simplicity that if the IFI defaults, the bank receives nothing.[17] At time $t = 2$, the IFI and bank's return are realized. This setup implies that the uncertainty in the model is resolved at time $t = 1$; however, a costly liquidation problem remains from $t = 1$ to $t = 2$. Figure 1 summarizes the timing of the model.
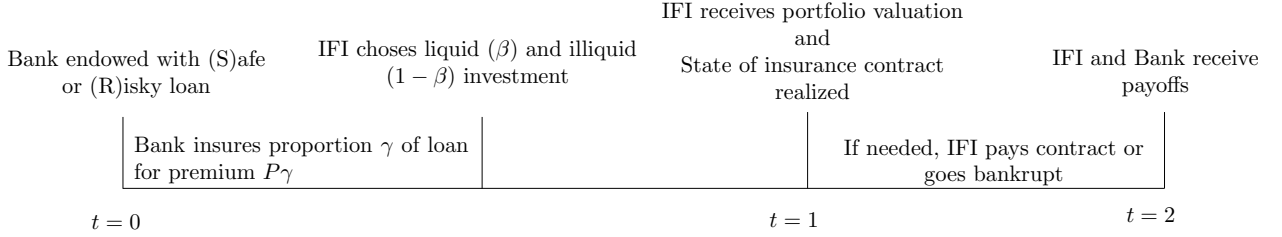

Figure 1: Timing of the Model

The expected payoff of the IFI can be written as follows.

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\begin{aligned}
\Pi_{IFI} &= (1-b)\left[\int_{-P\gamma(\beta+(1-\beta)R_I)}^{\overline{R}_f} (\theta + P\gamma(\beta + (1-\beta)R_I))\, f(\theta)d\theta\right] \\
&\quad + (b)\left[\int_{C(\gamma-\beta P\gamma)}^{\overline{R}_f} (\theta - C(\gamma - \beta P\gamma) + P\gamma(1-\beta)R_I)\, f(\theta)d\theta\right]
\end{aligned} \tag{2}
$$

The first term is the expected payoff when a claim is not made, which happens with probability $1 - b$ given the IFI's beliefs. The $-P\gamma(\beta + (1-\beta)R_I)$ term in the integrand represents the benefit that engaging in these contracts can have: it reduces the probability of portfolio default when a claim is not made. We assume that $\underline{R}_f$ is sufficiently negative so that $P\gamma(\beta + (1-\beta)R_I) < |\underline{R}_f|$.

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[15]There is a growing literature on trading in illiquid markets and fire sales. See for example Subramanian and Jarrow (2001), and Brunnermeier and Pedersen (2005).

[16]In reality, the insuring institution would typically pay the full protection value, but would receive the bond of the underlying borrower in return, which may still have a recovery value. Inserting this feature into the model would only complicate the analysis without changing the qualitative results

[17]This can be relaxed to allow partial recovery of the claim, however, this would only complicate the analysis without changing the qualitative results.

Since P and β are both bounded from above,[18] it follows that this inequality is satisfied for a finite $\underline{R}_f$. This assumption ensures that the IFI cannot completely eliminate its probability of default in this state. Recall that before the IFI engaged in the insurance contract, it would be forced into insolvency when the portfolio draw was less than zero. However, if a claim is not made, it can receive a portfolio draw that is less than zero and still remain solvent (so long as the IFI's draw is greater than $-P\gamma(\beta + (1 - \beta)R_I)$). The payoff to the invested premium is also realized.

The second term is the expected payoff when a claim is made, which happens with probability b given by the IFI's beliefs. The term $C(\gamma - \beta P\gamma)$ represents the cost to the IFI of accessing the needed capital to pay a claim. Notice that the loans placed in the illiquid asset are not available if a claim is made. Furthermore, the probability of default for the IFI increases in this case. To see this, notice that before engaging in the insurance contract, the IFI defaults if its portfolio draw is $\tilde{\theta} \in [\underline{R}_f, 0]$. After the insurance contract is sold, default occurs if the draw is $\tilde{\theta} \in [\underline{R}_f, C(\gamma - \beta P\gamma) > 0]$. The invested premium is realized, however, the liquid part is used to help pay off the claim.

As stated previously, counterparty risk is defined as the risk that the IFI defaults, conditional on a claim being made. Therefore, counterparty risk is represented in the model by $\int_{\underline{R}_f}^{C(\gamma - \beta P\gamma)} f(\theta)d\theta$.

2.4 IFI Behavior

We now characterize the optimal investment choice of the IFI and the resulting market clearing price. We begin by looking at the IFI's optimal investment decision. The following lemma characterizes the optimal behavior conditional on a belief (b) and a price (P). The IFI is shown to invest more in the liquid asset if it believes a claim is more likely to be made. Let β_S^* (β_R^*) be the optimal choice of the IFI given it believes that the loan is safe (risky).

Lemma 1 *The optimal investment in the liquid asset (β^*) is weakly increasing in the belief of the probability of a claim (b). Consequently, $\beta_R^* \geq \beta_S^*$.*

Proof. *See appendix.*

It follows that the relationship in this proposition is strict when β^* attains an interior solution. Note that the implicit expression for β^* can be found in the proof to this lemma. The intuition behind this result is that when the IFI thinks a claim is likely to be made, it minimizes costly liquidation by investing in the liquid asset. Alternatively, when the IFI thinks a claim is unlikely to be made, it invests more in the illiquid asset which earns higher returns. It is easy to see that the optimal investment is conditional on a price P. We define P^* as the market clearing price. To characterize it, we use the assumption that the IFI must earn zero profit from engaging in the insurance contract. The following lemma yields both existence and uniqueness of the market clearing price P^*.

Lemma 2 *The market clearing price is unique and in the open set $(0, 1)$.*

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[18]This is true for β by construction and will be proven for P in Lemma 2.

Proof. *See appendix.*

We now analyze the properties of the market clearing price P^*. The following lemma shows that as the IFI's belief about the probability a claim increases, so too must the premium increase to compensate them for the additional risk. Let P_S^* (P_R^*) be the market clearing price given the IFI believes that the loan is safe (risky).

Lemma 3 *The market clearing price P^* is increasing in the belief of the probability of a claim (b). Consequently, $P_R^* > P_S^*$.*

Proof. *See appendix.*

The lemma yields the intuitive result that our pricing function $P(b)$ is increasing in b. We now begin a discussion on equilibrium in the model.

3 Equilibrium Beliefs

Akerlof (1970) showed how insurance contracts can be plagued by the 'lemons' problem. One underlying incentive in his model that generates this result is that the insured wishes only to minimize the premium paid. It is for this reason that high risk agents would wish to conceal their type. Subsequent literature showed how the presence of a signalling device can allow a separating equilibrium to exist. What is new in our paper is that no signalling device is needed to justify the existence of a separating equilibrium. We call the act of concealing one's type for the benefit of a lower insurance premium the *premium effect*. In this section we show that this effect may be subdued in the presence of counterparty risk. This is done by demonstrating another effect that works against the premium effect that we call the *counterparty risk effect*. The intuition of this new effect is that if high risk (risky) agents attempt to be revealed as low risk (safe), a lower insurance premium may be obtained, but the following lemma shows that counterparty risk will increase.

Lemma 4 *If b decreases, but the actual probability of a claim does not, counterparty risk rises whenever $\beta \in (0, 1]$.*

Proof. *See appendix.*

There are two factors that contribute to this result. First, Lemma 3 showed that as the perceived probability of default decreases, the premium also decreases and therefore leaves less capital available to be invested. Second, Lemma 1 showed that the IFI will put more in the illiquid asset as b decreases. Combining these two factors, the counterparty risk increases. The only case in which the counterparty risk will not rise is when the bank is already investing everything in the illiquid asset, so that as b decreases, everything is still invested in the illiquid asset.

To analyze the resulting equilibria, we employ the concept of a Perfect Bayesian Nash Equilibrium (PBE). Define $i \in \{S, R\}$ to represent the two possible bank types, and define the message $\mathcal{M} \in$

$\{S, R\}$ to represent the report that bank type i sends to the IFI. Let the bank's payoff be $\Pi(i, \mathcal{M})$ representing the profit that a type i bank receives from sending the message $\mathcal{M}$. Formally, an equilibrium in our model is defined as follows.

Definition 1 *An equilibrium is defined as a portfolio choice β, a price P, and a belief b such that:*

1. *b is consistent with Bayes' rule where possible.*

2. *Choosing P, the IFI earns zero profit with β derived according to the IFI's problem.*

3. *The bank chooses its message so as to maximize its expected profit.*

To proceed we ask: is there a separating equilibrium in which both types are revealed truthfully? The answer without counterparty risk is *no*. The reason is that without counterparty risk, it is costless for the bank with a risky loan to imitate a bank with a safe loan. However, with counterparty risk, it is possible that both types credibly reveal themselves so that separation occurs. To begin, assume that the IFI's beliefs correspond to a separating equilibrium. Therefore, if $\mathcal{M} = S$ ($\mathcal{M} = R$) then $b = 1 - p_S$ ($b = 1 - p_R$). We now write the profit for a bank with a risky loan given a truthful report ($\mathcal{M} = R$).

$$\Pi(R, R) = p_R R_B + \gamma(1 - p_R) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{\overline{R}_f} dF(\theta) - \gamma(1 - p_R) Z \int_{\underline{R}_f}^{C(\gamma - \beta_R^* P_R^* \gamma)} dF(\theta) - \gamma P_R^* \quad (3)$$

The first term represents the expected payoff to the bank when the loan does not default. The second term represents the expected payoff on the insured portion of the loan when the loan defaults and the IFI is able to pay the claim. Notice that the IFI's beliefs are such that the IFI is risky. The third term represents the expected payoff when the loan defaults and the IFI fails and so is unable to fulfil the insurance claim. The final term is the insurance premium that the bank pays to the IFI. We now state the profit of a risky bank who reports that it is safe ($\mathcal{M} = S$).

$$\Pi(R, S) = p_R R_B + \gamma(1 - p_R) \int_{C(\gamma - \beta_S^* P_S^* \gamma)}^{\overline{R}_f} dF(\theta) - \gamma(1 - p_R) Z \int_{\underline{R}_f}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta) - \gamma P_S^* \quad (4)$$

We now find the condition under which a risky bank wishes to truthfully reveal its type.

$$\Pi(R, R) \geq \Pi(R, S) \Rightarrow$$

$$\underbrace{(1 - p_R)(1 + Z) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta)}_{\text{expected saving in counterparty risk}} \geq \underbrace{P_R^* - P_S^*}_{\text{amount extra to be paid in insurance premia}} \quad (5)$$

From Lemmas 1 and 3 we know that $C(\gamma - \beta_R^* P_R^* \gamma) < C(\gamma - \beta_S^* P_S^* \gamma)$ and therefore the left hand side represents the counterparty risk that a risky bank saves by reporting truthfully. This is the *counterparty risk effect*. The right hand side represents the savings in insurance premia that

the bank would receive by misrepresenting its type. This is the *premium effect*. The inequality (5) represents the key condition for the separating equilibrium to exist. In a typical insurance problem without counterparty risk, the left hand side must be equal zero. Consequently, in the absence of counterparty risk, the risky type will always want to misrepresent its type. We now turn to a bank with a safe loan and repeat the same exercise.

$$\Pi(S, S) \geq \Pi(S, R) \Rightarrow$$

$$\underbrace{(1 - p_S)(1 + Z) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta)}_{\text{expected cost of the additional counterparty risk}} \leq \underbrace{P_R^* - P_S^*}_{\text{amount to be saved in insurance premia}} \tag{6}$$

The left hand side represents the amount of counterparty risk that the bank will save if it conceals its type. The right hand side represents the amount of insurance premia that the bank will save if it reports truthfully. Therefore, when (5) and (6) hold simultaneously, this equilibrium exists. For an example of when this can hold, take the case in which the safe loan is "very" safe. In particular, we let $p_S \rightarrow 1$ and obtain the following expressions.

$$\underbrace{0}_{\text{expected cost of the additional counterparty risk}} \leq \underbrace{P_R^* - P_S^*}_{\text{amount to be saved in insurance premia}} \tag{7}$$

$$\underbrace{(1 - p_R)(1 + Z) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta)}_{\text{expected saving in counterparty risk}} \geq \underbrace{P_R^* - P_S^*}_{\text{amount extra to be paid in insurance premia}} \tag{8}$$

Note here that $P_S \rightarrow 0$ since the probability of default of the safe loan is approaching zero. Inequality (7) is satisfied trivially, while (8) is satisfied for Z sufficiently large. Recall that Z can be interpreted as the cost of counterparty failure when a claim is made. Therefore separation can be achieved when there is a high enough 'penalty' on the bank for taking on counterparty risk. The intuition is that a larger penalty forces the bank to internalize the counterparty risk more. As a result, more information is revealed in the market. This is a sense in which counterparty risk may be beneficial to the market, since it can help alleviate the possible adverse selection problem caused by asymmetric information. Let $P_{1/2}^*$ be the market clearing price and $\beta_{1/2}^*$ be the optimal investment choice when the IFI does not update its beliefs ($b = \frac{1}{2}(2 - p_S - p_r)$). We now state the first major result of the paper.

Proposition 1 *In the absence of counterparty risk, no separating equilibrium can exist. When there is counterparty risk, the moral hazard problem allows a unique separating equilibrium to exist in which each type of bank truthfully announces its loan risk. Sufficient conditions for this include 1) the safe loan is relatively safe and 2) Z is large enough. Formally,*

1. $\left(p_s : p_s \geq 1 - \dfrac{P_R^* - P_S^*}{(1+Z) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta)} \quad \& \quad p_s > 1 - \dfrac{P_{1/2}^* - P_S^*}{(1+Z) \int_{C(\gamma - \beta_{1/2}^* P_{1/2}^* \gamma)}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta)} \right)$

2. $\left(Z : Z \geq \dfrac{P_R - P_S}{(1-p_R) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta)} - 1 \quad \& \quad Z > \dfrac{P_R^* - P_{1/2}^*}{(1-p_R) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma - \beta_{1/2}^* P_{1/2}^* \gamma)} dF(\theta)} - 1 \right).$

Proof. *See appendix.*

This proposition shows that a moral hazard problem on the part of the insurer can alleviate a possible adverse selection problem on the part of the insured. The separating equilibrium corresponds to the case in which the *premium effect* dominates for the bank with a safe loan, while the *counterparty risk effect* dominates for the bank with a risky loan. Note that there can be no separating equilibrium (different than the one above) in which the safe type reports that it is risky, and the risky type reports that it is safe.

There are also two pooling equilibria that may exist (we formalize them in the proof to Proposition 1). The first occurs when both the safe and risky bank report that they are safe. In this case, the *premium effect* dominates for both types so that the IFI does not update its prior beliefs. The second pooling equilibrium occurs when both the safe and risky bank report that they are risky. In this case, the *counterparty risk effect* dominates for both types.[19] The parameter of interest in these two pooling equilibria is Z. For example, when Z is low, the bank is subjected to little penalty for the risk that the counterparty will default when a claim is made. This could be the case if a bank engages in credit risk transfer to satisfy regulatory requirements and is not concerned with the chance that it may not be protected.[20] Specifically, Z can thought of as a proxy for the bank's aversion to counterparty risk. The less averse to that risk it is, the less it will be concerned with the welfare of its counterparty. When Z is sufficiently small (so that the second formal condition of the above proposition is not met), each type prefers a lower insurance premium versus reduced counterparty risk so that a pooling equilibrium can be supported. The other, less intuitive pooling equilibrium occurs if Z is sufficiently large (such that the first formal condition is not met) so that each type prefers less counterparty risk versus a lower premium.

We now remove a key contracting imperfection to highlight the inefficiency in the IFI's investment choice and formally prove the existence of a moral hazard problem.

3.1 Contract Inefficiency

In this section, we imagine a planning problem wherein the planner can control the investment decision of the IFI. However, we maintain the IFI's beliefs and zero profit condition. We show that

---

[19]In the separating equilibrium case, the beliefs are fully defined by Bayes' rule. In the first pooling equilibrium, any off-the-equilibrium path belief with $b > \frac{1}{2}(2 - p_S - p_r)$ if risky is reported is consistent for the IFI with the Cho-Kreps (1987) intuitive criterion. In the second pooling equilibrium, any off-the-equilibrium path belief with $b < \frac{1}{2}(2 - p_S - p_r)$ if safe is reported is consistent with the Cho-Kreps (1987) intuitive criterion.

[20]For regulatory capital, Basel II requires a counterparty to have at least as good of a credit rating as the insured. However, recent financial events such as the collapse and bailout of AIG have taught us that even those financial firms that once had good credit ratings and whom potentially act as insurers can fail.

regardless of the beliefs of the IFI, the planner can always do better than is done in equilibrium by increasing the amount of capital put in the liquid asset. Therefore, this section will show that we can get closer to a first best allocation by removing this contracting imperfection, thereby highlighting the moral hazard problem. We denote the solution to the planner's problem given any belief b as β_b^{pl}, with resulting price P_b^{pl}. The following lemma shows that the equilibrium price given the beliefs b, P_b^* must be weakly less than the planning price P_b^{pl}.

Lemma 5 *There is no price $\tilde{P} < P_b^*$ such that the IFI can earn zero profit. This implies that $P_b^* \leq P_b^{pl}$.*

Proof. It is straight-forward to see that $\Pi_{IFI}(\beta_b^*, P_b^*) = 0$ (where Π_{IFI} is defined by (2)) implies that $\Pi_{IFI}(\tilde{\beta}, \tilde{P}) \neq 0 \ \forall \ \tilde{\beta} \in [0,1]$ and for $\tilde{P} < P_b^*$.

Since Lemmas 1 and 2 show that with (β_b^*, P_b^*), zero profit is attained, it must be the case that with $\tilde{\beta} \in [0,1] \neq \beta_b^*$ and P_b^*, the IFI earns negative profits. It follows that if $\tilde{P} < P_b^*$, with $\tilde{\beta}$, the IFI must earn negative profits. Since the IFI must earn zero profits, $\tilde{P} \geq P_b^*$.

∎

We now state the second major result of the paper. The following proposition shows that the IFI chooses a β^* that is too small as compared to that of the planner's problem β^{pl} for any belief of the IFI. The proposition shows that the insurer moral hazard problem causes the level of counterparty risk in equilibrium to be strictly too high (so long as $\beta^* \in [0,1)$).

Proposition 2 *Given an equilibrium portfolio decision β^* with $\beta^* < 1$, a social planner would choose $\beta^{pl} > \beta^*$ so that the level of counterparty risk in equilibrium is too high.*

Proof. *See appendix.*

The intuition behind this result comes from two sources. First, since the social planning problem corresponds to maximizing the bank's payoff while keeping the IFI at zero profit, the bank strictly prefers to have the IFI invest more in the liquid asset. Second, the IFI must be compensated for this *individually* sub-optimal choice of β by an increase in the premium. Since from Lemma 5, P weakly increases (in the proof of Proposition 2, we show that in this case, the increase is strict), counterparty risk falls (i.e., $\int_{\underline{R}_f}^{C(\gamma - \beta P \gamma)} f(\theta) d\theta$ falls). In other words, the moral hazard problem on the part of the IFI is characterized by an inefficiency in the investment choice. The key restriction on the contracting space that yields this result is that the insurance premium is paid upfront and so the bank cannot condition its payment on an observed outcome. In the competitive equilibrium case, the bank knows that the IFI will invest too little into the liquid asset, and therefore lowers its payment accordingly. Although the limited liability, excess risk taking result of Jensen and Meckling (1968) is left unexplored, the inefficiency pointed out in our paper is not caused by managerial incentive problems. Instead, the inefficiency arises because the IFI makes its decision based only on its own risk, and not the risk that it causes to the bank. On the other hand, the social planner forces the IFI to internalize the counterparty risk it creates for others.

We now generalize the base model to the case in which there are multiple insured parties.

4 Multiple Banks

In this section, we analyze the case of multiple banks and one insurer. We assume there are a measure $M < 1$ of banks. This assumption is meant to approximate the case in which there are many banks, and the size of each individual bank's insurance contract is insignificant for the IFI's investment decision. Using an uncountably large number instead of a finite but large number of banks helps simplify the analysis greatly.[21] Each bank requests an insurance contract of size γ. At time $t = 0$, each bank receives both an aggregate and idiosyncratic shock (both private to the banks) which assigns them a probability of loan default. For simplicity, as in the case when there was only one bank, the return on the loan is R_B if it succeeds and 0 if it does not. We define the idiosyncratic shock by the random variable X and let it be uniformly distributed over $[0, M]$. The CDF can then be written as follows.

$$\Psi(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ \frac{x}{M} & \text{if } x \in (0, M) \\ 1 & \text{if } x \geq M \end{cases}$$

Next, denote the aggregate shock as q_A and let it take the following form:

$$q_A = \begin{cases} s & \text{with probability } \frac{1}{2} \\ r & \text{with probability } \frac{1}{2}, \end{cases}$$

where $0 < s < r < 1 - M$. It follows that the probability of default of bank i is $q_i = q_A + X_i$.[22] We will refer to the aggregate shock as either (s)afe or (r)isky. We write the conditional distribution's of bank types as $\mu(q_i : q_i \leq x | q_A = s) = \Psi(x - s)$ and $\mu(q_i : q_i \leq x | q_A = r) = \Psi(x - r)$. It follows that $\Psi(x - s)$ first order stochastically dominates $\Psi(x - r)$ since $\Psi(x - s) \geq \Psi(x - r) \ \forall \ x$. Note that this is in contrast to the usual definition of first order stochastic dominance which entails higher draws providing a 'better' outcome. In the case of this model, the opposite is true, since lower draws refer to a lower probability of default; a 'better' outcome.

4.1 The IFI's Problem

Because of the asymmetric information problem, the IFI does not know ex-ante whether the aggregate shock was $q_A = s$ or $q_A = r$. However, the IFI does know that the aggregate shock hits all the banks in the same way.[23] Therefore, if only a subset of the banks can successfully reveal their types, this reveals the aggregate shock for the rest of them.

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[21]An advantage to using a finite number of banks is that we could avoid any measurability issue. In the proof to Proposition 3 (which is stated at the end of this section), we detail this issue briefly and discuss how to handle it. Since the results do not depend on the continuous setup, we opt to use it for its simplification of the problem.

[22]Note that in the base model we referred to p as a probability of success, whereas here we refer to q as a probability of failure. We make this notational change because it is more intuitive in this section to have probabilities of failure when we introduce the IFI's beliefs over the measure of defaults. Of course, the simple relationship $p = 1 - q$ holds.

[23]We assume this for simplicity. We can relax the assumption that all the banks receive the same aggregate shock and allow them to receive correlated draws from a distribution.

If solvent, the IFI must pay γ to each bank whose loan defaults. In Lemma 7 we will show that there can be no separation of types within the idiosyncratic shock. Therefore, given a fixed realization of the aggregate shock, each bank pays the same premium P.[24] We assume that the IFI has the same choice as in section 2.3, so that it invests β in the liquid storage asset and $(1 - \beta)$ in the illiquid asset with return R_I. To represent the IFI's beliefs, let Y denote the measure of defaults. Furthermore, let $b(y) = prob(Y \leq y)$ be the beliefs over the measure of defaults defined over $[0, M]$. It follows that $\Psi(x - s) \geq \Psi(x - r) \; \forall \; x$ implies $b(y|q_A = s) \geq b(y|q_A = r) \; \forall \; y$. In other words, first order stochastic dominance is preserved. Since each bank insures γ, the total size of contracts insured by the IFI is: $\int_0^M \gamma d\Psi(x) = M\gamma$. The IFI's payoff can now be written as follows.

$$
\Pi_{IFI}^{MB} = \underbrace{\int_0^{\beta PM} \left[\int_{-PM\gamma(\beta + (1-\beta)R_I) + y\gamma}^{\overline{R}_f} (\theta + (\beta + (1 - \beta) R_I) PM\gamma - y\gamma) \, dF(\theta) \right] db(y)}_{\text{Term 1}}
$$
$$
+ \underbrace{\int_{\beta PM}^M \left[\int_{C(y\gamma - \beta PM\gamma)}^{\overline{R}_f} (\theta - C(y\gamma - \beta PM\gamma) + (1 - \beta)R_I PM\gamma) \, dF(\theta) \right] db(y)}_{\text{Term 2}} \quad (9)
$$

Where 'MB' denotes 'Multiple Banks'. The first term represents the case in which the IFI puts sufficient capital in the liquid asset so that there is no need to liquidate its portfolio to pay claims. This happens if a sufficiently small measure of banks make claims. Since the IFI receives $PM\gamma$ in insurance premia, it puts $\beta PM\gamma$ into the liquid asset. It follows that if less than $\beta PM\gamma$ is needed to pay claims (i.e. less than βPM banks fail), portfolio liquidation is not necessary. The second term represents the case in which the IFI must liquidate its portfolio if a claim is made. This happens if the amount they need to pay in claims is greater than $\beta PM\gamma$. $C(y\gamma - \beta PM\gamma) + \beta PM\gamma$ represents the total cost of claims, where $y\gamma - \beta PM\gamma$ is the total amount of capital the IFI needs to liquidate from its portfolio. Note that for simplicity, as in the base model, we assume that if a claim is made and the IFI defaults, the banks do not receive any payment form the IFI.

The following lemma both derives the optimal β^* and proves that counterparty risk is less severe when the IFI believes that the loans are more risky.

Lemma 6 *For a given aggregate shock, there is less counterparty risk when the IFI's beliefs put more weight on the aggregate shock being risky ($q_A = r$) as opposed to it being safe ($q_A = s$).*

Proof. *See appendix.*

The intuition for this result is similar to that of Lemma 4. If the IFI believes that the pool of loans is risky, it is optimal to invest more in the liquid asset. This happens because the expected number of claims is higher in the risky case so that the IFI wishes to prevent costly liquidation by investing more in assets that will be readily available if a claim is made.

---

[24]We assume the IFI must earn a fixed profit in equilibrium so that a market clearing price can be determined. We are not concerned with pinning down that price in this section. As in the base model, we can assume that the IFI has market power; however, it is not crucial.

We now give the conditions under which the IFI's beliefs $(b(y))$ are formed.

4.2 Equilibrium Beliefs

4.2.1 No Aggregate Shock

To analyze how the beliefs of the IFI are formed, consider the case where there is no aggregate shock. Since there is no aggregate uncertainty, the IFI's optimal investment choice remains the same regardless of whether it offers a pooling price or individual separating prices.[25] It follows that since an individual bank's choice will have no effect on counterparty risk, only the *premium effect* is active. It is for this reason that a separating equilibrium in the idiosyncratic shock cannot exist. To see this, assume that each bank reveals its type truthfully. Now consider the bank with the highest probability of default, call it bank M. Since it is paying the highest insurance premium, it can lie about its type without any effect on counterparty risk, and obtain a better premium, and consequently, a better payoff. The following lemma formalizes.

Lemma 7 *There can be no separating equilibrium in which the idiosyncratic shock is revealed.*

We now introduce the aggregate shock and show that separation of aggregate types can occur.

4.2.2 Aggregate and Idiosyncratic Shock

Each individual bank now receives both an aggregate and an idiosyncratic shock. We can think of this procedure as putting the banks in one of two intervals, either $[s, s + M]$ or $[r, r + M]$. We know that if one bank is able to successfully reveal its aggregate shock, then the aggregate shock is revealed for all other banks. The following proposition shows that a unique separating equilibrium can exist in this setting.

Proposition 3 *There exists a parameter range in which a unique separating equilibrium in the aggregate shock can be supported.*

Proof. *See appendix.*

Note that the parameter range that supports a unique separating equilibrium is given by (38) and (41). The insight from the above proposition follows from an individual banks ability to affect the IFI's investment choice (through the IFI's beliefs). If a bank could only reveal its own shock, its premium would be insignificant to the IFI's investment decision. However, since by successfully revealing itself, a bank also reveals the other banks, an individual's problem can have a significant effect on the IFI's investment choice. The parameter range that can support this equilibrium is similar to the case in which there was only one bank. Conditions that can support this equilibrium as unique are: Z sufficiently high, and the safe aggregate shock sufficiently low.

We now turn to a brief discussion about the robustness of the results.

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[25]To see this, note that with no aggregate risk, the IFI knows the average quality of banks and will use that to make its investment decision. Any bank claiming that they received the lowest idiosyncratic shock will not change the IFI's beliefs about the average quality.

5 Robustness

In this section we address four assumptions of the model. First, we enlarge the type space and discuss why, in addition to the separating and pooling equilibria, there can be partial information revelation. Second, we consider the assumption that the IFI is in Bertrand competition so that it must earn zero profit. As well, we discuss the implications of the bank being able to contract with multiple IFIs. Third, we detail how to relax the assumption that the IFI's portfolio is uniformly distributed. Finally, we address the assumption that there is only a liquid and an illiquid investment choice as well as the assumption that the IFI's initial portfolio is fixed.

Separating Equilibrium Result

The separating equilibrium result is no doubt stark. If we allow a bank to reveal a portion of its risk (in the case of multiple banks, a portion of their aggregate risk), we can imagine a setting in which partial information revelation is achieved. Consider three loan (or bank) types: one very risky, one moderately risky, and one safe. In this setup, a parameterization will exist in which all types wish to be revealed as safe and one in which all types wish to reveal truthfully. There are other equilibria however in which, for example, the very risky type wishes to be revealed as moderately risky. In this way, it is possible that a type wishes to disclose some but not all of its risk. This then would allow a range of equilibria to exist that fall between full separation and complete pooling in terms of information revelation. As in the model in the current paper, what would determine how much information revelation occurs is the banks aversion to counterparty risk.

Zero Profit Assumption

We can relax both the assumption that the IFI makes zero profit, and the assumption that the bank insures only with one IFI. If we allow the IFI to charge a higher price, holding all other variables constant except β, it can be shown that the IFI will increase the proportion invested in the illiquid asset. It is even possible to obtain the surprising result that counterparty risk increases when the IFI is paid more. Therefore, by simply paying a higher premium, the bank may not be able to decrease the counterparty risk. Furthermore, in the multiple bank case, each bank's premium is insignificant to the IFI's investment decision, so that paying more would have no effect on counterparty risk.

In a similar fashion, we could allow the bank to insure with multiple IFI's. In this case, each IFI would solve their problem with a smaller insurance liability (γ). It can be shown that this decreased liability will cause each IFI to act riskier than it would if there was only one of them. Therefore, counterparty risk may not decrease as much as one might expect. In extreme cases, it may not decrease at all. This result is dependant on the degree of correlation that the IFIs have with each other.

Uniform Distribution Assumption

The uniform assumption can be relaxed to a general distribution, provided that it satisfies some

conditions. For example, there must be mass in a region above and below zero. The reason for this is that according to the IFI's problem (2), there are two important states of the world: when a claim is not made (probability $1 - b$) and when it is (probability b). When a claim is not made, the key region of the return distribution is given by $[-P\gamma(\beta + (1 - \beta)R_I), 0]$. The IFI stands to get a positive payout in this region where it otherwise would not. Thus, if there is no mass in this region, the IFI will not have the incentive to invest in the illiquid asset. When a claim is made, the key region of the return distribution is given by $[0, C(\gamma - \beta P\gamma)]$. The IFI stands to receive no payoff in this region where it otherwise would have. Thus, if there is no mass in this region, the IFI will not have the incentives to invest in the liquid asset.

Investment Choice and Fixed Initial Portfolio Assumption

The fact that the IFI's investment choice is only between a liquid and an illiquid asset is not crucial. We could alter the environment and have the choice be between a risky and riskless asset and the qualitative results of the paper will still hold. In this case, the interim (illiquid) period could be dropped. The IFI then invests more in the riskless asset if it believes it is likely a claim will be made, and the risky asset otherwise.

In addition to the premium investment choice, the model assumes for simplicity that the initial portfolio is fixed. We can imagine a richer model wherein the IFI could also change its initial portfolio. The amount that the IFI would change the portfolio would depend on its size relative to the potential loses from claims. In this richer environment, as discussed above, we could include assets of varying degrees of liquidity and/or risk (as were assumed to be present in the initial portfolio). As in the simpler setup we use in our model, the decision of the IFI as to how risky or illiquid it would invest would depend on its expected loss due to claims. The results would then be qualitatively similar to the current setup.

6 Conclusion

In a setting in which insurers can fail, we construct a model to show that a new moral hazard problem can arise in insurance contracts. If the insurer suspects that the contract is safe, it puts capital into less liquid assets which earn higher returns. However, the downside of this is that when a claim is made, the insurer is less likely to be able to fulfil the contract. We show that the insurer's investment choice is inefficiently illiquid. The presence of this moral hazard is shown to allow a unique separating equilibrium to exist wherein the insured freely and credibly relays its superior information. In other words, the new moral hazard problem can alleviate the possible adverse selection problem.

The results of the base model require the contract to be large enough to affect the insurer's investment decision. We relax this assumption and allow there to be a collection of insured parties, each with a contract size that is insignificant to the insurer's investment decision. We show that our moral hazard problem still exists, and can obtain the separating equilibrium result when there is private aggregate risk.

7 Appendix

Proof Lemma 1. We wish to maximize the IFI's additional profit from the insurance contract. This can be obtained by subtracting (1) from (2). Since (1) is not a function of β it will not appear in the FOC so we drop it. Using the assumption that $f(\theta)$ is distributed uniform over the interval $[\underline{R}_f, \overline{R}_f]$, we solve for the optimal choice of β for the IFI, given b and P.

$$
\max_{\beta \in [0,1]} \quad \frac{(1-b)}{\overline{R}_f - \underline{R}_f} \left[-\frac{(P\gamma(\beta + (1-\beta)R_I))^2}{2} + \left(\overline{R}_f + P\gamma(\beta + (1-\beta)R_I) \right) P\gamma(\beta + (1-\beta)R_I) \right]
$$
$$
+ \frac{b}{\overline{R}_f - \underline{R}_f} \left[-\frac{(C(\gamma - \beta P\gamma))^2}{2} + \left(\overline{R}_f - C(\gamma - \beta P\gamma) \right) \left(-C(\gamma - \beta P\gamma) + (1-\beta)R_I P\gamma \right) \right]
$$

Using Leibniz rule to differentiate the choice variable in the integrands, we obtain the following first order equation:

$$
\begin{aligned}
0 &= \frac{bP\gamma}{\overline{R}_f - \underline{R}_f} \left[\overline{R}_f C'(\gamma - \beta P\gamma) - \overline{R}_f R_I - C'(\gamma - \beta P\gamma)C(\gamma - \beta P\gamma) \right] \\
&+ \frac{bP\gamma}{\overline{R}_f - \underline{R}_f} \left[C'(\gamma - \beta P\gamma)P\gamma(1-\beta)R_I + C(\gamma - \beta P\gamma)R_I \right] \\
&- \frac{(1-b)P\gamma}{\overline{R}_f - \underline{R}_f} \left[\overline{R}_f + (\beta + (1-\beta)R_I) \right]
\end{aligned}
\tag{10}
$$

To ensure a maximum, we take the second order condition and show the inequality that must hold.

$$
(1-b)(R_I - 1)^2 + bP\gamma \left[-\overline{R}_f C''(\gamma - \beta P\gamma) + C''(\gamma - \beta P\gamma)C(\gamma - \beta P\gamma) + C'(\gamma - \beta P\gamma)C'(\gamma - \beta P\gamma) \right]
$$
$$
+ bP\gamma \left[-C''(\gamma - \beta P\gamma)P\gamma(1-\beta)R_I - 2C'(\gamma - \beta P\gamma)R_I \right] < 0
\tag{11}
$$

Simplifying and plugging in the boundary conditions for β into the FOC (10), we now derive the optimal proportion of capital put in the liquid asset as an implicit function.

$$
\begin{cases}
\beta^* = 0 & \text{if } b \leq b^* \\
0 = b \left[C'(\gamma - \beta P\gamma) \left(\overline{R}_f + P\gamma(1-\beta)R_I - C(\gamma - \beta P\gamma) \right) - R_I \left(\overline{R}_f - C(\gamma - \beta P\gamma) \right) \right] & \\
\quad -(1-b)(R_I - 1) \left[\overline{R}_f + (\beta + (1-\beta)R_I) \right] & \text{if } b \in (b^*, b^{**}) \\
\beta^* = 1 & \text{if } b \geq b^{**}
\end{cases}
\tag{12}
$$

where $b^* = \dfrac{(R_I - 1)(\overline{R}_f + R_I)}{(R_I - 1)(\overline{R}_f + R_I) + \overline{R}_f C'(\gamma) - \overline{R}_f R_I - C'(\gamma)C(\gamma) + C'(\gamma)P\gamma R_I + C(\gamma)R_I}$,

and $b^{**} = \dfrac{(R_I - 1)(\overline{R}_f + 1)}{(R_I - 1)(\overline{R}_f + 1) + \overline{R}_f C'(\gamma - \beta P\gamma) - \overline{R}_f R_I - C'(\gamma - \beta P\gamma)C(\gamma - \beta P\gamma) + C(\gamma - \beta P\gamma)R_I}$.

We now show that the optimal proportion of capital put in the liquid asset is increasing in b by

finding $\frac{\partial \beta}{\partial b}$ from the FOC. First, define:

$$
\begin{aligned}
A &= (1-b)(R_I - 1)^2 + bP\gamma \left[-\overline{R}_f C''(\gamma - \beta P\gamma) + C''(\gamma - \beta P\gamma)C(\gamma - \beta P\gamma) \right] \\
&\quad + bP\gamma \left[C'(\gamma - \beta P\gamma)C'(\gamma - \beta P\gamma) - C''(\gamma - \beta P\gamma)P\gamma(1-\beta)R_I - 2C'(\gamma - \beta P\gamma)R_I \right] \\
&< 0
\end{aligned}
\tag{13}
$$

where the inequality (13) follows from the SOC. Assuming an interior solution and rearranging for $\frac{\partial \beta}{\partial b}$ yields the following.

$$
\begin{aligned}
\frac{\partial \beta}{\partial b} &= \frac{-b(R_I - 1)(b\overline{R}_f + (\beta + (1-\beta)R_I)) - A_1}{A} \\
&> 0
\end{aligned}
\tag{14}
$$

Where $A_1 = C'(\gamma - \beta P\gamma)\left(\overline{R}_f + P\gamma(1-\beta)R_I - C(\gamma - \beta P\gamma)\right) - R_I\left(\overline{R}_f - C(\gamma - \beta P\gamma)\right)$. For the FOC to hold, it must be the case that $A_1 > 0$. The inequality (14) then follows.

∎

Proof of Lemma 2.

Step 1: Existence

To find P^*, we remove the payoff the IFI receives before issuing the contract (as the zero profit condition must be implemented on only the additional payoff from the insurance contract). By rearranging (2) and subtracting $\int_0^{\overline{R}_f} \theta f(\theta)d\theta$, we find a P^* that satisfies the following:

$$
\begin{aligned}
0 &= (1-b)\left[\int_{-P\gamma(\beta+(1-\beta)R_I)}^{0} \theta f(\theta)d\theta + \int_{-P\gamma(\beta+(1-\beta)R_I)}^{\overline{R}_f} P\gamma(\beta + (1-\beta)R_I)f(\theta)d\theta \right] \\
&\quad + (b)\left[-\int_0^{C(\gamma - \beta P\gamma)} \theta f(\theta)d\theta + \int_{C(\gamma - \beta P\gamma)}^{\overline{R}_f} \left(-C(\gamma - \beta P\gamma) + P\gamma(1-\beta)R_I\right) f(\theta)d\theta \right] \\
&\quad + \int_0^{\overline{R}_f} \theta f(\theta)d\theta - \int_0^{\overline{R}_f} \theta f(\theta)d\theta
\end{aligned}
\tag{15}
$$

Consider $P^* \le 0$. In this case, the IFI earns negative profits. To see this, notice that the first term is negative (the first part is zero, while the second is negative) while the second term is also negative (both parts of it are negative). Therefore, it must be that $\Pi_{IFI}(\beta^*, P^* \le 0) < 0$. This contradicts the fact that $\Pi_{IFI}(\beta^*, P^*) = 0$ in equilibrium.

Next, consider $P^* \ge 1$, and $\beta = 1$ (not necessarily the optimal value). In this case, the first term on the right hand side of (15) is strictly positive and the second term is zero since $C(0) = 0$. Since β^* can yield no less profit than $\beta = 1$ by definition of it being an optimum, it must be that $\Pi_{IFI}(\beta^*, P^* \ge 0) > 0$. This contradicts the fact that $\Pi_{IFI}(\beta^*, P^*) = 0$ in equilibrium. Therefore, if it exists, $P^* \in (0, 1)$.

To show that P^* exists in the interval $(0, 1)$, we differentiate the right hand side of (15) (using the assumption that $f(\cdot)$ is uniformly distributed) to show that profit is strictly increasing in P.

$$
\begin{aligned}
\frac{\partial \Pi_{IFI}}{\partial P} &= \frac{b}{\overline{R}_f - \underline{R}_f} \left[\left(\overline{R}_f - C(\gamma - \beta P \gamma) \right) \left(C'(\gamma - \beta P \gamma) \beta \gamma + (1 - \beta) R_I \gamma \right) \right] \\
&\quad + \frac{b}{\overline{R}_f - \underline{R}_f} \left[C'(\gamma - \beta P \gamma) \beta \gamma (1 - \beta) R_I P \gamma \right] \\
&\quad + \frac{(1 - b)}{\overline{R}_f - \underline{R}_f} \left(\overline{R}_f + P \gamma (\beta + (1 - \beta) R_I) \right) \gamma (\beta + (1 - \beta) R_I) \\
&> 0 \hspace{7cm} (16)
\end{aligned}
$$

Where the inequality (16) follows because $\overline{R}_f \geq C(\gamma - \beta P \gamma)$. Therefore, since profit is negative when $P^* \leq 0$ and positive when $P^* \geq 1$, and since profit is a (monotonically) increasing function of P^*, profit must equate to zero within $P^* \in (0, 1)$.

Step 2: Uniqueness

Assume the following holds: $\Pi_{IFI}(\beta^*, P_1^*) = 0$. Since we have already shown that profit is a strictly increasing function of P^*, then if some price $P_2^* > P_1^*$ ($P_2^* < P_1^*$) this implies $\Pi_{IFI}(\beta^*, P_2^*) > 0$ ($\Pi_{IFI}(\beta^*, P_2^*) < 0$). Therefore, $\Pi_{IFI}(\beta^*, P_2^*) = 0$ implies $P_1^* = P_2^*$ must hold, so our price is unique.

∎

Proof of Lemma 3.

From the envelop theorem, we can ignore the effect that changes in b have on β when we evaluate the payoff at β^*. Plugging $\beta = \beta^*$ into (2) and taking the partial derivative with respect to b yields:

$$
\begin{aligned}
\left. \frac{\partial \Pi_{IFI}}{\partial b} \right|_{\beta = \beta^*} &= - \left[\int_{-P\gamma(\beta^* + (1 - \beta^*)R_I)}^{\overline{R}_f} (\theta + P\gamma(\beta^* + (1 - \beta^*)R_I)) f(\theta) d\theta \right] \\
&\quad + \left[\int_{C(\gamma - \beta^* P \gamma)}^{\overline{R}_f} (\theta - C(\gamma - \beta^* P \gamma) + P\gamma(1 - \beta^*) R_I) f(\theta) d\theta \right] \\
&< 0 \hspace{7cm} (17)
\end{aligned}
$$

The sign of $\left. \frac{\partial \Pi_{IFI}}{\partial b} \right|_{\beta = \beta^*}$ is negative because the absolute value of the first term is greater than the second. To see this, notice that the limits of integration of the first term are wider as well as the integrand is greater than that of the second term. Intuitively, as b increases it is more likely that the IFI will have to pay a claim, and more likely that it will have to liquidate its portfolio. Both of these activities are costly. Since the envelop theorem is a local condition and does not hold for large changes in b, it serves as an upper bound on the decrease in profits. It follows that an increase in b must be met with an increase in P otherwise the IFI would earn negative profit and would not participate in the market.

∎

Proof of Lemma 4. Since counterparty risk is defined as $\int_{\underline{R}_f}^{C(\gamma - \beta P \gamma)} f(\theta) d\theta$, we are interested in what happens to $C(\gamma - \beta^* P^* \gamma)$ as b changes.

We first focus on the case in which $\beta^* \in (0, 1)$. We take following partial derivative where we define $\frac{\partial \beta^*}{\partial b} \equiv \frac{\partial \beta}{\partial b}\Big|_{\beta = \beta^*}$ and $\frac{\partial P^*}{\partial b} \equiv \frac{\partial P}{\partial b}\Big|_{P = P^*}$.

$$\frac{\partial (\gamma - \beta^* P^* \gamma)}{\partial b} = -\gamma \left(\frac{\partial \beta^*}{\partial b} P^* + \beta^* \frac{\partial P^*}{\partial b} \right) \tag{18}$$

From Lemma 1 we know $\frac{\partial \beta^*}{\partial b} \geq 0$. As well, from Lemma 3 we know $\frac{\partial P^*}{\partial b} > 0$. Since $\beta^* \in (0, 1)$ and $P^* > 0$ (from Lemma 2), it follows that:

$$\frac{\partial (\gamma - \beta^* P^* \gamma)}{\partial b} < 0 \tag{19}$$

Therefore, as b increases, counterparty risk decreases when $\beta \in (0, 1)$. Next, consider the case of $\beta^* = 1$. Again, from Lemma 3 we know $\frac{\partial P^*}{\partial b} > 0$. Therefore, $\frac{\partial (\gamma - \beta^* P^* \gamma)}{\partial b} < 0$ regardless of whether $\frac{\partial \beta^*}{\partial b} = 0$ or $\frac{\partial \beta^*}{\partial b} > 0$. Thus, counterparty risk decreases when b decreases if $\beta^* = 1$.

It is obvious that if $\beta^* = 0$ there will be no change in counterparty risk by noting that $\beta^* P \gamma$ will be independent of b.

■

Proof of Proposition 1. We begin by ruling out a separating equilibrium when there is no counterparty risk, regardless of the IFI's choice. This implies $\int_{\underline{R}_f}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta) = 0$. It follows that the left hand side of (5) and (6) are both zero. Since $P_R^* - P_S^* > 0$, (5) and (6) cannot be simultaneously satisfied so that this separating equilibrium cannot exist.

We proceed by showing the conditions for which the two pooling equilibria can exist. We begin with the case in which both types wish to be revealed as safe. Let β_{OE}^* and P_{OE}^* be the result from the IFI's problem when a bank gives an off the equilibrium path report of R. The following two conditions formalize this case:

$\Pi(S, S) \geq \Pi(S, R) \Rightarrow$

$$(1 - p_S)(1 + Z) \underbrace{\int_{C(\gamma - \beta_{OE}^* P_{OE}^* \gamma)}^{C(\gamma - \beta_{1/2}^* P_{1/2}^* \gamma)} dF(\theta)}_{\text{expected cost of the additional counterparty risk}} \leq \underbrace{P_{OE}^* - P_{1/2}^*}_{\text{amount to be saved in insurance premia}} \tag{20}$$

$\Pi(R, S) \geq \Pi(R, R) \Rightarrow$

$$(1 - p_R)(1 + Z) \underbrace{\int_{C(\gamma - \beta_{OE}^* P_{OE}^* \gamma)}^{C(\gamma - \beta_{1/2}^* P_{1/2}^* \gamma)} dF(\theta)}_{\text{expected cost of the additional counterparty risk}} \leq \underbrace{P_{OE}^* - P_{1/2}^*}_{\text{amount to be saved in insurance premia}} \tag{21}$$

The binding condition (21) is satisfied for Z sufficiently small. The intuition is that if counterparty risk is not too costly, the bank would wish to obtain lowest insurance premium. In other

words, the *premium effect* dominates for both types. It follows that for this equilibrium to exist, $b > \frac{1}{2}(2 - p_S - p_R)$. Next, consider the case in which both types report that they are risky. In this case, we use the notation β^*_{OE2} and P^*_{OE2} to indicate the off the equilibrium path beliefs if a bank reports that it is safe. The conditions can be characterized as follows:

$$\Pi(S, R) \geq \Pi(S, S) \Rightarrow$$

$$\underbrace{(1 - p_S)(1 + Z) \int_{C(\gamma - \beta^*_{1/2} P^*_{1/2} \gamma)}^{C(\gamma - \beta^*_{OE2} P^*_{OE2} \gamma)} dF(\theta)}_{\text{expected saving in counterparty risk}} \geq \underbrace{P^*_{1/2} - P^*_{OE2}}_{\text{amount extra to be paid in insurance premia}} \tag{22}$$

$$\Pi(R, R) \geq \Pi(R, S) \Rightarrow$$

$$\underbrace{(1 - p_R)(1 + Z) \int_{C(\gamma - \beta^*_{1/2} P^*_{1/2})}^{C(\gamma - \beta^*_{OE2} P^*_{OE2} \gamma)} dF(\theta)}_{\text{expected saving in counterparty risk}} \geq \underbrace{P^*_{1/2} - P^*_{OE2}}_{\text{amount extra to be paid in insurance premia}} \tag{23}$$

The binding condition (22) is satisfied for Z sufficiently high. Intuitively, the bank is so averse to counterparty risk, that the *counterparty risk effect* dominates for both types. It follows that for this equilibrium to exist, $b < \frac{1}{2}(2 - p_S - p_R)$.

We now show that the separating equilibrium defined by (5) and (6) can be unique. Combining (5) and (6) we obtain the following condition for when the separating equilibrium exists:

$$\frac{P_R - P_S}{(1 - p_R) \int_{C(\gamma - \beta^*_R P^*_R \gamma)}^{C(\gamma - \beta^*_S P^*_S \gamma)} dF(\theta)} \leq 1 + Z \leq \frac{P^*_R - P^*_S}{(1 - p_S) \int_{C(\gamma - \beta^*_R P^*_R \gamma)}^{C(\gamma - \beta^*_S P^*_S \gamma)} dF(\theta)} \tag{24}$$

Turning to the pooling equilibria, we use extreme off the equilibrium path beliefs to illuminate the result (which is valid for the general belief as well). Let $OE = R$ and $OE2 = S$. The condition under which the pooling equilibrium cannot exist (i.e., when (22) and (23) are not satisfied) can be written as:

$$\frac{P^*_R - P^*_{1/2}}{(1 - p_R) \int_{C(\gamma - \beta^*_R P^*_R \gamma)}^{C(\gamma - \beta^*_{1/2} P^*_{1/2} \gamma)} dF(\theta)} < 1 + Z < \frac{P^*_{1/2} - P^*_S}{(1 - p_S) \int_{C(\gamma - \beta^*_{1/2} P^*_{1/2} \gamma)}^{C(\gamma - \beta^*_S P^*_S \gamma)} dF(\theta)} \tag{25}$$

It follows that if (24) and (25) are satisfied, the separating equilibrium exists and is unique.[26] To see that these conditions can be simultaneously satisfied, let $p_S \to 1$ so that the right hand side of both (24) and (25) are satisfied. It follows that if Z is sufficiently large, the left hand side of these two inequalities can be satisfied yielding a unique separating equilibrium.

∎

---

[26]Note that the separating equilibrium is unique since any other possible separating equilibrium would only differ in terms of off the equilibrium path beliefs.

Proof of Proposition 2. The proof proceeds in 3 steps. Step 1 derives the first order condition for the planning problem. Step 2 assumes the equilibrium solution and derives an expression for $\frac{\partial P}{\partial \beta}$ from the IFI's zero profit condition. Step 3 shows that β^{pl} and P^{pl} must be greater than in the equilibrium case when $\beta^* < 1$. Since we need not specify a belief for this proof, it follows that the result holds regardless if there is separation or pooling of banks.

Step 1

The profit for the bank (bk) can written as follows (note here we leave the bank's loan type as $j \in \{S, R\}$ as the proof is valid for both the safe and risky type).

$$
\Pi_{bk} = p_j R_B \gamma + \gamma(1 - p_j) \int_{C(\gamma - \beta P \gamma)}^{\overline{R}_f} dF(\theta) - \gamma(1 - p_j) Z \int_{\underline{R}_f}^{C(\gamma - \beta P \gamma)} dF(\theta) - \gamma P
$$

In the planners case, P^{pl} is now endogenous and determined by $\Pi_{IFI}(\beta^{pl}, P^{pl}) = 0$ (where Π_{IFI} is defined by (2)). Using the uniform assumption on F yields the following first order condition.

$$
\frac{\partial P}{\partial \beta} = \gamma C'(\gamma - \beta P \gamma) \left(P + \frac{\partial P}{\partial \beta} \beta \right)(1 - p_j)(1 + Z) \tag{26}
$$

The left hand side represents the marginal cost of increasing β, while the right hand side represents the marginal benefit of doing so.

Step 2

We show that if $\beta^{pl} = \beta^*$, then (26) cannot hold. We find an expression for $\left. \frac{\partial P}{\partial \beta} \right|_{\beta = \beta^*, P = P^*}$ by implicitly differentiating the equation $\Pi_{IFI}(\beta^*, P^*) = 0$.

$$
\begin{aligned}
0 &= (1 - b) \left[\int_{-P^* \gamma(\beta^* + (1 - \beta^*) R_I)}^{\overline{R}_f} (\theta + P^* \gamma(\beta^* + (1 - \beta^*) R_I)) f(\theta) d\theta \right] \\
&\quad + (b) \left[\int_{C(\gamma - \beta^* P^* \gamma)}^{\overline{R}_f} (\theta - C(\gamma - \beta^* P^* \gamma) + P^* \gamma(1 - \beta^*) R_I) f(\theta) d\theta \right]
\end{aligned}
$$

we implicitly differentiate this equation to find $\left. \frac{\partial P}{\partial \beta} \right|_{\beta = \beta^*, P = P^*}$ (recall $f(\cdot)$ is normally distributed).

$$
\begin{aligned}
A_2 \left. \frac{\partial P}{\partial \beta} \right|_{\beta = \beta^*, P = P^*} &= -\frac{bP\gamma}{\overline{R}_f - \underline{R}_f} \left[\overline{R}_f C'(\gamma - \beta P \gamma) - \overline{R}_f R_I - C'(\gamma - \beta P \gamma) C(\gamma - \beta P \gamma) \right] \\
&\quad - \frac{bP\gamma}{\overline{R}_f - \underline{R}_f} \left[C'(\gamma - \beta P \gamma) P \gamma(1 - \beta) R_I + C(\gamma - \beta P \gamma) R_I \right] \\
&\quad + (1 - b) \frac{P\gamma}{\overline{R}_f - \underline{R}_f} \left[\overline{R}_f + (\beta + (1 - \beta) R_I) \right] \tag{27}
\end{aligned}
$$

Where we define:

$$
\begin{aligned}
A_2 &= \frac{b}{\overline{R}_f - \underline{R}_f}\left[-\overline{R}_f C'(\gamma - \beta P \gamma)\beta\gamma + \overline{R}_f(1-\beta)R_I\gamma - C'(\gamma - \beta P\gamma)C(\gamma - \beta P\gamma)\beta\gamma\right] \\
&\quad + \frac{b}{\overline{R}_f - \underline{R}_f}\left[C'(\gamma - \beta P\gamma)\beta\gamma(1-\beta)R_I P\gamma - C(\gamma - \beta P\gamma)(1-\beta)R_I\gamma\right] \\
&\quad + \frac{(1-b)}{\overline{R}_f - \underline{R}_f}\left[\gamma(\beta + (1-\beta)R_I)\left(\overline{R}_f + P\gamma(\beta + (1-\beta)R_I)\right)\right].
\end{aligned}
\tag{28}
$$

It follows that $\left.\frac{\partial P}{\partial \beta}\right|_{\beta=\beta^*, P=P^*} = 0$ since the right hand side of (27) is the FOC derived in Lemma 1 and must equate to 0 at the optimum, β^*.

Step 3

Substituting $\left.\frac{\partial P}{\partial \beta}\right|_{\beta=\beta^*, P=P^*} = 0$ into (26) yields:

$$
0 = \gamma C'(\gamma - \beta^* P^* \gamma)(P^*)(1-p_j)(1+Z),
\tag{29}
$$

which cannot hold since $\gamma > 0$, $(1-p_j) > 0$ and $Z > 0$. Therefore, $\beta^{pl} \neq \beta^*$ and $P^{pl} \neq P^*$. To satisfy (26), it must be the case that $\beta^{pl} > \beta^*$, and from Lemma 5 it follows that $P^{pl} \geq P^*$. However, if $\beta^{pl} > \beta^*$, then $P^{pl} > P^*$. This implies that the following must hold.

$$
\int_0^{C(\gamma - \beta^{pl} P^{pl} \gamma)} f(\theta)d\theta < \int_0^{C(\gamma - \beta^* P^* \gamma)} f(\theta)d\theta
\tag{30}
$$

Therefore, there is strictly less counterparty risk in the planners case than in the equilibrium case. It is obvious that if $\beta^* = 1$, it is not possible for the planner to invest any more in the liquid asset. This is the case in which the IFI is already investing everything in the liquid asset. ∎

Proof of Lemma 6. Our problem can be written as follows (recalling that $f(\cdot)$ is assumed to be uniformly distributed).

$$
\begin{aligned}
\max_{\beta} \quad & \frac{1}{\overline{R}_f - \underline{R}_f}\left\{\int_0^{\beta PM}\left[\frac{(\overline{R}_f)^2}{2} - \frac{(PM\gamma(\beta + (1-\beta)R_I) - y\gamma)^2}{2}\right]db(y) + \int_{\beta PM}^{M}\left[\frac{(\overline{R}_f)^2}{2} - \frac{(C(y\gamma - \beta PM\gamma))^2}{2}\right]db(y)\right\} \\
&\quad + \frac{1}{\overline{R}_f - \underline{R}_f}\int_0^{\beta PM}\left[\left(\overline{R}_f + (PM\gamma(\beta + (1-\beta)R_I) - y\gamma)\right)\left(PM\gamma(\beta + (1-\beta)R_I) - y\gamma\right)\right]db(y) \\
&\quad + \frac{1}{\overline{R}_f - \underline{R}_f}\int_{\beta PM}^{M}\left[\left(\overline{R}_f - C(y\gamma - \beta PM\gamma)\right)\left(-C(y\gamma - \beta PM\gamma) + (1-\beta)R_I PM\gamma\right)\right]db(y)
\end{aligned}
$$

Taking the FOC yields the following.

$$
\begin{aligned}
0 \;=\;& \frac{PM\gamma}{\overline{R}_f - \underline{R}_f} \int_0^{\beta^* PM} \left[-\overline{R}_f (R_I - 1) - (PM\gamma(\beta^* + (1-\beta^*)R_I) - y\gamma)(R_I - 1) \right] db(y) \\
& + \frac{PM}{\overline{R}_f - \underline{R}_f} \left[\frac{(\overline{R}_f)^2}{2} + \overline{R}_f (1-\beta^*) R_I PM\gamma + \frac{1}{2} (PM\gamma(1-\beta^*)R_I)^2 \right] \\
& + \frac{PM\gamma}{\overline{R}_f - \underline{R}_f} \int_{\beta^* PM}^M \left[\overline{R}_f C'(y\gamma - \beta^* PM\gamma) - \overline{R}_f R_I - C'(y\gamma - \beta^* PM\gamma)C(y\gamma - \beta^* PM\gamma) \right] db(y) \\
& + \frac{PM\gamma}{\overline{R}_f - \underline{R}_f} \int_{\beta^* PM}^M \left[C'(y\gamma - \beta^* PM\gamma)(1-\beta^*)R_I PM\gamma + C(y\gamma - \beta^* PM\gamma)R_I \right] db(y) \\
& - \frac{PM}{\overline{R}_f - \underline{R}_f} \left[\frac{(\overline{R}_f)^2}{2} - \frac{(C(0))^2}{2} + \left(\overline{R}_f - C(0)\right)\left(-C(0) + (1-\beta^*)R_I PM\gamma\right) \right] \tag{31}
\end{aligned}
$$

Recalling $C(0) = 0$ we simplify the above.

$$
\begin{aligned}
0 \;=\;& -\int_0^{\beta^* PM} \left[\overline{R}_f(R_I - 1) + (PM\gamma(\beta^* + (1-\beta^*)R_I) - y\gamma)(R_I - 1) \right] db(y) \\
& + \int_{\beta^* PM}^M \left[\left(C'(y\gamma - \beta^* PM\gamma) - R_I \right) \left(\overline{R}_f - C(y\gamma - \beta^* PM\gamma) \right) \right] db(y) \\
& + \int_{\beta^* PM}^M \left[C'(y\gamma - \beta^* PM\gamma)(1-\beta^*)PM\gamma \right] db(y) \\
& + \frac{1}{2} \left(PM\gamma(1-\beta^*)R_I \right)^2 \tag{32}
\end{aligned}
$$

The SOC (which we omit for brevity) implies that the right hand side of (32) is decreasing in β^* so that our problem achieves a maximum. Define two belief distributions $b_1(y)$ and $b_2(y)$ such that $b_1(y) \geq b_2(y)$ $\forall y$. As well, let $(\beta_1^*, b_1(y))$ solve the first order condition (32). Intuitively, by moving from $b_1(y)$ to $b_2(y)$, mass shifts from the interval $[0, \beta^* PM]$ to $[\beta^* PM, M]$. Formally:

$$
\int_0^{\beta^* PM} db_1(y) \;>\; \int_0^{\beta^* PM} db_2(y) \tag{33}
$$

$$
\int_{\beta^* PM}^M db_1(y) \;<\; \int_{\beta^* PM}^M db_2(y). \tag{34}
$$

We wish to show that by moving from $b_1(y)$ to $b_2(y)$, β^* increases. We do this in two cases and show the result is the same in both. First consider the case in which $R_I \geq C'(y\gamma - \beta^* PM\gamma)$. In this case, the first term on the right hand side of (32) is negative while the second two are positive. This implies that as mass shifts from the first term to the second and third terms, the right hand side of (32). Since this equation is decreasing in β^* (by the SOC), β^* must increase as desired. Next, consider the case in which $C'(y\gamma - \beta^* PM\gamma) \geq R_I$. Rearranging the FOC (32) we obtain the

following.

$$
\begin{aligned}
0 \;=\; & -\int_{0}^{\beta^{*}PM}\left[(PM\gamma\,(\beta^{*}+(1-\beta^{*})R_{I})-y\gamma)(R_{I}-1)\right]db(y) \\
& +\int_{\beta^{*}PM}^{M}\left[\overline{R}_{f}\left(C'(y\gamma-\beta^{*}PM\gamma)-1\right)+C(y\gamma-\beta^{*}PM\gamma)\left(R_{I}-C'(y\gamma-\beta^{*}PM\gamma)\right)\right]db(y) \\
& +\int_{\beta^{*}PM}^{M}\left[C'(y\gamma-\beta^{*}PM\gamma)(1-\beta^{*})PM\gamma)\right]db(y) \\
& +\frac{1}{2}\left(PM\gamma(1-\beta^{*})R_{I}\right)^{2}-\overline{R}_{f}(R_{I}-1)
\end{aligned}
\tag{35}
$$

It follows that the first term on the right hand side of 35 is trivially negative, while the second term is positive because since $C'(y\gamma-\beta^{*}PM\gamma)>1$ (recall that $C(x)\geq x$ was assumed so that $C'(x)\geq 1$) and $R_{I}\geq C'(y\gamma-\beta^{*}PM\gamma)$. The third term is trivially positive. The results from the first case then hold in this case, with an increase in risk being met by an increase in the amount of investment in the liquid asset. It follows that the riskier are the beliefs about the distribution of loans that the IFI insures, the more that the IFI invests in the liquid asset.

To proceed, we use a similar result to that of Lemma 3. It is straight forward to see that when the belief of defaults is higher (as in the risky case), so must the price of the contracts be higher (this can be shown in the same way that Lemma 3 was proved by showing that the profit function is decreasing in the amount of risk in the loans). Next we find what happens to counterparty risk. What is different about the case of multiple banks is that counterparty risk is defined relative to the number of banks that default: $\int_{\beta PM}^{M}\int_{\underline{R}_{f}}^{C(y\gamma-\beta PM\gamma)}dF(\theta)db(y)$.

In the case in which the IFI puts more weight on the loans being risky ($q_{A}=r$), β^{*} and P^{*} increase, so that $C(\gamma-\beta P\gamma)$ decreases. Furthermore, since from the point of view of the banks, the probability of a claim does not change, counterparty risk decreases as compared to the case in which the IFI puts more weight on the loans being safe ($q_{A}=s$).

∎

Proof of Proposition 3. The proof proceeds in 3 steps. Steps 1 and 2 determine when the pooling equilibria cannot exist. In particular, we use beliefs of the IFI for which banks have the greatest incentive to pool. In step 1 we assume that all banks report that they received the aggregate shock $q_{A}=s$ and find a condition wherein some bank (or measure of banks) that received the aggregate shock $q_{A}=r$ wish to reveal it truthfully.[27] In the second step we repeat a similar exercise to determine when a risky pooling equilibrium does not exist. Step 3 determines when a unique separating equilibrium can exist. We use beliefs such that the banks have the least incentive to separate. In this step we assume separating beliefs for the IFI and find the condition wherein both bank types do not wish to misrepresent their aggregate type.

---

[27]Note that there are other ways of arriving at a pooling equilibrium, for example, some banks of the same type report differently than others. These can arise when the IFI's beliefs are such that no new information is gleaned from the reports. Since these yield the same outcome, we will focus only on the cases described.

Step 1

Consider all banks reporting $q_A = s$, regardless of the aggregate shock. Now consider the incentive of banks that receive the aggregate shock $q_A = r$. Given that all banks report that they are safe, we see if there is any incentive for deviation (i.e., some bank to send the message $q_A = r$). If every bank reports that it received the safe aggregate shock, the IFI does not update its beliefs. However, if some bank or measure of banks[28] deviate and send the message $q_A = r$, then the IFI employs off the equilibrium path beliefs (OE) about the aggregate shock. We assume that the deviating bank(s) are believed to have received the highest idiosyncratic shock(s).[29] It is easy to show that the bank with the greatest incentive to be revealed as risky is the one with the highest idiosyncratic shock, which we denote as bank M. Denote the probability of default of the loan of this bank as q_M^r. If this bank does not deviate, it pays $P^{\frac{1}{2}}$, alongside all other banks. If this bank does deviate, it pays P_M^{OE}, while the rest of the banks pay P^{OE}, corresponding to the average quality of a risky bank. Next, we denote the optimal investment choice of the IFI in the pooling (deviating) case by $\beta^{1/2}$ (β^{OE}). Finally, we let $D^{1/2}$ (D^{OE}) represent the probability that upon a claim being made in the pooling (deviating) case, the IFI fails and so cannot pay. It follows that $D^{1/2} = \int_{\beta^{1/2}P^{1/2}M}^{M} \int_{\underline{R}_f}^{C(y\gamma - \beta^{1/2}P^{1/2}M\gamma)} dF(\theta)db(y)$ and $D^{OE} = \int_{\beta^{OE}P^{OE}M}^{M} \int_{\underline{R}_f}^{C(y\gamma - \beta^{OE}P^{OE}M\gamma)} dF(\theta)db(y)$. The condition under which this pooling equilibrium cannot exist is given as follows.

$$(1 - q_M^r)R_B + q_M^r\gamma(1 - D^{OE}) - q_M^r\gamma D^{OE}Z - \gamma P_M^{OE} \geq (1 - q_M^r)R_B + q_M^r\gamma(1 - D^{1/2}) - q_M^r\gamma D^{1/2}Z - \gamma P^{1/2}$$
$$\Rightarrow q_M^r\left(D^{1/2} - D^{OE}\right)(1 + Z) \geq P_M^{OE} - P^{1/2} \quad (36)$$

Step 2

Consider all banks reporting $q_A = r$, regardless of the aggregate shock. Now consider the incentive of banks that receive the aggregate shock $q_A = s$. We find the condition under which a bank would like to reveal that it is safe ($q_A = s$). Let the beliefs of the IFI be that if one (or some positive measure) of banks report that they are safe, then there are off the equilibrium path beliefs (OE2) about the aggregate shock. Furthermore, those reporting that they are safe are believed to have received the lowest idiosyncratic shock. We know that the bank with the greatest incentive to be revealed as safe is the one with the lowest idiosyncratic shock, call it bank 0. Denote the probability of default for the loan of this bank as q_0^s and the individual price if they reveal themselves as safe as P_M^{OE2}. In this case, let the price paid by there other banks be P^{OE2} and β^{OE2} be the optimal investment choice of the IFI. It follows that counterparty risk in this case can be defined by: $D^{OE2} = \int_{\beta^{OE2}P^{OE2}M}^{M} \int_{\underline{R}_f}^{C(y\gamma - \beta^{OE2}P^{OE2}M\gamma)} dF(\theta)db(y)$. The condition under which this pooling equilibrium cannot exist is given as follows.

$$(1 - q_0^s)R_B + q_0^s\gamma(1 - D^{OE2}) - q_0^s\gamma D^sZ - \gamma P_M^{OE2} \geq (1 - q_0^s)R_B + q_0^s\gamma(1 - D^{1/2}) - q_0^s\gamma D^{1/2}Z - \gamma P^{1/2}$$
$$\Rightarrow q_0^s\left(D^{OE2} - D^{1/2}\right)(1 + Z) \leq P^{1/2} - P_M^{OE2}$$
$$(37)$$

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[28]There is a technical issue that would not arise if there was a finite number of banks. In the traditional Riemann sense of measurability using the concept of point-wise convergence almost everywhere, a bank of measure zero cannot change the IFI's beliefs. There are two ways to rectify this. The first way is to employ the pettis-integral as in Uhlig (1994). The second way is to imagine a small but positive measure of banks deviating.

[29]This belief about the idiosyncratic shock gives the banks the greatest incentive to pool. In this way, we can rule out pooling if, under these conditions, there is still a bank that wishes to deviate.

Therefore (36) and (37) are simultaneously satisfied when:

$$\frac{P^{1/2} - P_M^{OE2}}{q_0^s(D^{OE2} - D^{1/2})} \geq 1 + Z \geq \frac{P_M^{OE} - P^{1/2}}{q_M^r(D^{1/2} - D^{OE})}. \tag{38}$$

Where $q_0^s < q_M^r$. To see that (38) can hold, consider the limit as q_0^s approaches zero with Z sufficiently large.

Step 3

We now find the conditions under which the separating equilibrium exists. Consider the following beliefs of the IFI: if all banks report that they are safe, then they are believed to be safe, if all report risky, then they are believed to be risky. Imagine the aggregate shock was $p_A = s$ and all banks report truthfully. Consider the bank with the highest idiosyncratic shock, bank M (we use this bank because they have the greatest incentive to deviate). If it reports $p_A = s$ then the price it receives is P^s and the counterparty risk it is exposed to is D^s (alongside the other banks). If it deviates and reports $p_A = r$ then the counterparty risk is D^{OE3} and the price is P_0^{OE3}, where $OE3$ represents the off the equilibrium path beliefs of the IFI. Since we are trying to determine conditions under which a separating equilibrium exists, a deviating bank is believed to have received the best idiosyncratic shock. The condition under which the bank would report truthfully is given by the following.

$$(1 - q_M^s)R_B + q_M^s\gamma(1 - D^s) - q_M^s\gamma D^s Z - \gamma P^s \geq (1 - q_M^s)R_B + q_M^s\gamma(1 - D^{OE3}) - q_M^s\gamma D^{OE3}Z - \gamma P_0^{OE3}$$
$$\Rightarrow P_0^{OE3} - P^s \geq q_M^s \left(D^s - D^{OE3} \right)(1 + Z) \tag{39}$$

Next, imagine the aggregate shock was $p_A = r$ and all banks report that $p_A = r$. Consider the bank with the lowest idiosyncratic shock, bank 0. If it reports $p_A = r$ then the price it receives is P^r and the counterparty risk it is exposed to is D^r (alongside the other banks). If it deviates and reports $p_A = s$ then the counterparty risk is D^{OE4} and the price is P_0^{OE4}, where $OE4$ are the off the equilibrium path beliefs in this case. The condition under which the bank would reveal truthfully is given by the following.

$$(1 - q_0^r)R_B + q_0^r\gamma(1 - D^r) - q_0^r\gamma D^r Z - \gamma P^r \geq (1 - q_0^r)R_B + q_0^r\gamma(1 - D^{OE4}) - q_0^r\gamma D^{OE4}Z - \gamma P_0^{OE4}$$
$$\Rightarrow q_0^r \left(D^{OE4} - D^r \right)(1 + Z) \geq P^r - P_0^{OE4} \tag{40}$$

It follows that (39) and (40) are simultaneously satisfied when:

$$\frac{P_0^{OE3} - P^s}{q_M^s \left(D^s - D^{OE3} \right)} \geq 1 + Z \geq \frac{P^r - P_0^{OE4}}{q_0^r \left(D^{OE4} - D^r \right)} \tag{41}$$

To show that there can exist a unique separating equilibrium we first need to establish that if there is a separating equilibrium, it must be the one outlined by (41). Given that any separating equilibrium must have the same prices and same level of counterparty risk,[30] they will only differ

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[30]To see this, notice that in any separating equilibrium, the safe and risky types are revealed, so that the risk the IFI faces is known. Therefore, given the fixed profit assumption on the IFI, they must charge the same price and

by off the equilibrium path beliefs. Therefore, the separating equilibrium above represents the only possible separating equilibrium. It follows that the separating equilibrium exists and is unique when both (38) and (41) are satisfied. To see that this is possible, we can define specific off the equilibrium path beliefs. Let $OE = r$, $OE2 = s$, $OE3 = r$ and $OE4 = s$. Now consider q_M^s and q_0^s sufficiently small so that the left hand sides of (38) and (41) are satisfied. It follows that for Z sufficiently large, the right hand sides of (38) and (41) can be satisfied.

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make the same investment decision (conditional on a type) regardless of the off the equilibrium path beliefs (provided these beliefs sustain the equilibrium).

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