

UNIVERSITY OF PENNSYLVANIA  
The Wharton School

FNCE 100

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PROBLEM SET #4

Fall Term 2005

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CAPITAL BUDGETING (UNCERTAINTY)

1. Both Dow Chemical Company, a large natural gas user, and Superior Oil, a major natural gas producer, are thinking of investing in a natural gas well near Pittsburgh. The well that Dow is thinking of investing in is located north of Pittsburgh, while Superior Oil's would be south of Pittsburgh, but otherwise the projects are identical. Both companies have analyzed their respective investments, which would involve a negative cash flow now and positive expected cash flows in the future. These cash flows would be the same for both firms. Both companies estimate that their project would have a net present value of \$1.0 million at an 18 percent nominal discount rate and a -\$1.1 million NPV at a 22 percent discount rate. Dow has an asset beta of 1.25 and Superior Oil has an asset beta of 0.75 which is equal to the beta of natural gas production. The expected risk premium on the market is 8.0 percent and risk-free bonds are yielding 12.0 percent. Should either company proceed? Both? Why?
2. Beatrice Foods is a large conglomerate that manufactures Dannon Yogurt, Melnor Lawn Sprinklers, and Samsonite Luggage, among other items. CBS is a broadcasting and movie company whose size and debt-equity ratios are similar to those of Beatrice Foods. Beatrice and CBS both have stock betas of approximately 1.0. One would expect that the fraction of the total variability of each company's stock returns that is due to unique risk is:
  - (a) Higher for CBS than for Beatrice
  - (b) Lower for CBS than for Beatrice
  - (c) Probably similar because size and debt-equity ratios are similar.
3. Standard Oil of Indiana has a debt-equity ratio of about 0.1, and Standard Oil of Ohio has a debt-equity ratio of about 0.4. ("Indiana Standard" and "Sohio" are completely separate companies, in spite of sharing credit cards and part of their name.) Both produce most of their oil in the United States and market most of their products in the Midwest, and their business risk is approximately identical. If the beta of Indiana Standard's stock is about 0.8, what is the beta of Sohio's stock if both have zero-beta debt?

4. A firm is considering the following projects:

Project	Beta	Expected return, %
A	0.5	12
B	0.8	13
C	1.2	18
D	1.6	19

The Treasury bill rate is 8 percent and the expected market premium is 7 percent.

- (a) Which projects have a higher expected return than the firm's 15 percent cost of capital?
  - (b) Which projects should be accepted?
  - (c) Which projects would be accepted or rejected incorrectly on the basis of the cost of capital as a hurdle rate?
5. The president of SH-DH Hall Pizza and Diversified Industries, a major all-equity corporation, is ecstatic over his strategic planning department. He claims that they use the newest methods including the CAPM to make investment decisions. They first determine their common stock beta and then, using their forecast of the return on the market and the observed T-bill rate, they determine their cost of capital. This required rate of return is then applied to the proposed investments, and projects with positive NPVs are accepted. Do you share the president's enthusiasm in his managerial talents? Would you recommend a different procedure? If so, what would it be and why?
6. Assume that the average cost of capital for a firm is estimated to be 16%. Expected return on a riskless security (RF) and the market portfolio (M) are 10% and 14% respectively. The standard deviation of the return on the market is 5%. Assume that the information concerning expected internal rate of return, standard deviation and the correlation coefficient with the market of four prospective projects are as follows:

Projects	$IRR_i$	$\sigma_i$	$\rho_{im}$
1	0.12	0.55	0.065
2	0.20	0.175	1.0
3	0.20	0.25	0.2
4	0.15	0.50	0.06

- (a) Find the security market line and graph it.
- (b) Find the systematic risk ( $\beta_i$ ) of each project and plot the  $(IRR, \beta)$  combinations on your graph.

- (c) Rank the four projects according to their total risk,  $\sigma_i$ , and then their systematic risk,  $\beta_i$ . Is there a difference between the two sets of rankings? If yes, how do you explain the difference?
- (d) Compute the equilibrium expected returns of the above projects. Which projects are acceptable? (Assume the projects involve positive net outlays initially and positive net inflows for period 1 only, so that the appropriate IRR rule gives us the same answers as the NPV rule).

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Solutions to PROBLEM SET #4

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1. Dow's asset beta is irrelevant. Both projects have a *project* cost of capital of 18 percent, since natural gas production has a beta of 0.75, which is equal to Superior Oil's asset beta. For both companies, at an 18 percent discount rate, the projects have positive NPVs. Both companies should proceed.

$$2. \quad r_i = r_f + \beta_i(r_m - r_f) + \varepsilon_i$$

$$\sigma_{r_i}^2 = \beta_i^2 \cdot \sigma_{r_m}^2 + \sigma_{\varepsilon_i}^2$$

The fraction of the total variance of stock returns accounted for by unique risk is =

$$\frac{\sigma_{\varepsilon_i}^2}{\sigma_{\varepsilon_i}^2 + \beta_i^2 \sigma_{r_m}^2} = \frac{\sigma_{\varepsilon_i}^2}{\sigma_{\varepsilon_i}^2 + \sigma_{r_m}^2}$$

Since Beatrice is diversified, it is likely to have less unique risk, (lower  $\sigma_{\varepsilon_i}^2$ ).

The above fraction *increases* as  $\sigma_{\varepsilon_i}^2$  increases, so the fraction should be higher for CBS than for Beatrice. Thus *A* is the answer.

3. Indiana Standard has  $\frac{D}{E} = 0.1$ ,  $\rightarrow \frac{D}{D+E} = 0.091$

Sohio has  $\frac{D}{E} = 0.4$ ,  $\rightarrow \frac{D}{D+E} = 0.286$

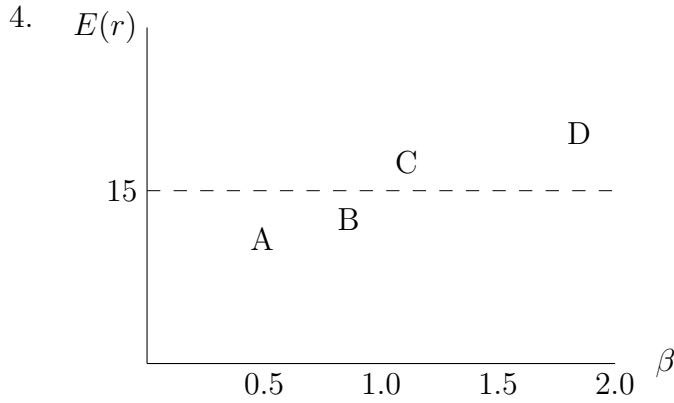
$$\text{For Indiana Standard: } \beta_{\text{IND}}^{\text{asset}} = \beta_{\text{IND}}^{\text{stock}} \left( \frac{E}{D+E} \right)_{\text{IND}} + \beta_{\text{IND}}^{\text{debt}} \left( \frac{D}{D+E} \right)_{\text{IND}}$$

$$= .8(.909) + 0(.091) = .73$$

$$\text{For Sohio: } \beta_{\text{Sohio}}^{\text{asset}} = \beta_{\text{SOH}}^{\text{stock}} \left( \frac{E}{D+E} \right)_{\text{SOH}} + \beta_{\text{SOH}}^{\text{debt}} \left( \frac{D}{D+E} \right)_{\text{SOH}}$$

$$\text{since } \beta_{\text{Sohio}}^{\text{asset}} = \beta_{\text{IND}}^{\text{asset}} \quad \text{we get : } .73 = \beta_{\text{SOH}}^{\text{stock}}(.714) + 0(.286)$$

$$\rightarrow \beta_{\text{Sohio}}^{\text{stock}} = \frac{.73}{.714} = 1.02$$



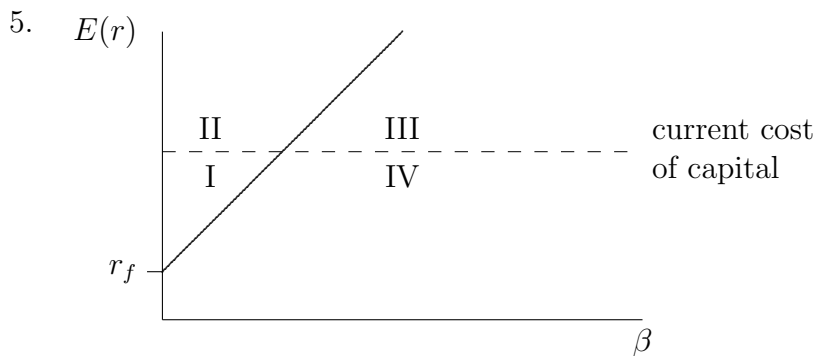
(a) Projects C and D.

(b) Given the Security Market Line (SML), accept A and C

<u>SML</u>	<u><math>E(r)</math></u>
11.5	12
13.6	13
16.4	18
19.2	19

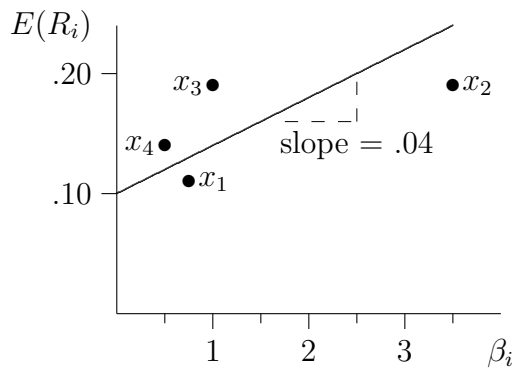
(c) Given the cost of capital (15%), accept C and D.

Note: D would have been incorrectly accepted and A incorrectly rejected.



No, because of value additivity each project should be evaluated as a separate “mini-firm.” Each of these investments has a required rate of return associated with it which is derived via the S.M.L. If the expected return on the investment is greater than the required rate, the project is accepted; if not it is rejected. The President has all the necessary information to implement such a procedure. However, SH-DH indiscriminately applies its current cost of capital to proposed investments regardless of their systematic risk. This leads to an inferior value of the firm should investments fall into regions I and III. Proposed investments in region I would be rejected even though they should be accepted, and vice versa for those in region III.

6. (a)  $E(R_i) = R_F + \beta_i[E(R_m) - R_F]$   
 $E(R_i) = .10 + \beta_i[.14 - .10]$



(b)

Project	$\beta$
1.	$(.065)(.55)(.05)/.05^2 = .715$
2.	$(1.0)(.175)(.05)/.05^2 = 3.5$
3.	$(.2)(.25)(.05)/.05^2 = 1$
4.	$(.06)(.50)(.05)/.05^2 = .6$

(c)

By $\sigma$	By $\beta$
2	4
3	1
4	3
1	2

↓ increasing order

The low correlation compensates for the high  $\sigma$ .

(d)

1:	$.10 + (.04)(.715) = .1286 > .12$	Reject
2:	$.10 + (.04)(3.5) = .24 > .20$	Reject
3:	$.10 + (.04)(1) = .14 < .20$	Accept
4:	$.10 + (.04)(.6) = .125 < .15$	Accept