STATISTICAL REVIEW

A. Population Statistics

Suppose there are N different states of the world. Let $P(s)$ be the probability we are in state s. If we are in state s, the return on stock A is $R_A(s)$. Then, we define:

(a) Expected Return (Mean Return):

$$E[R_A] = \sum_{s=1}^{N} P(s)\, R_A(s)$$

(b) Variance of the Return:

$$\mathrm{Var}(R_A) = \sum_{s=1}^{N} P(s)\,[R_A(s) - E[R_A]]^2$$

(c) Standard Deviation:

$$SD(R_A) = \sqrt{\mathrm{Var}(R_A)}$$

(d) Covariance:

$$\mathrm{Cov}(R_A, R_B) = \sum_{s=1}^{N} P(s)\,[R_A(s) - E[R_A]] \cdot [R_B(s) - E[R_B]]$$

(e) Correlation:

$$\mathrm{Corr}(R_A, R_B) = \frac{\mathrm{Cov}(R_A, R_B)}{SD(R_A) \cdot SD(R_B)}$$

$$-1 \le \rho \le +1$$

B. Sample Statistics

When calculating the population statistics we knew all possible outcomes. Suppose we have T years of data for stocks A and B. Let $R_{A,t}$ be the return of stock A in year t.

Define:

(a) Sample **Mean Return:**

[Arithmetic Mean]

$$\bar{R}_A = \frac{1}{T} \sum_{t=1}^{T} R_{A,t}$$

(b) **Sample Variance:**

$$\sigma_A^2 = \frac{1}{T-1} \sum_{t=1}^{T} [R_{At} - \bar{R}_A]^2$$

(c) **Sample Standard Deviation:**

$$\sigma_A = \sqrt{\sigma_A^2}$$

(d) Sample **Covariance:**

$$\sigma_{AB} = \frac{1}{T-1} \sum_{t=1}^{T} [R_{A,t} - \bar{R}_A]\,[R_{B,t} - \bar{R}_B]$$

(e) Sample Correlation:

$$\rho_{AB} = \frac{\sigma_{AB}}{\sigma_A \bullet \sigma_B}$$

Portfolios of Two Securities

Suppose you decide to invest in two securities, putting a proportion x of your money in security A and the remaining $1 - x$ of your money in security B.

Then, the return for this portfolio if state s occurs at the end of the year is

$$R_P(s) = x R_A(s) + (1 - x) R_B(s)$$

Using expected return formula, we find that the expected return for this portfolio is given by

$$\begin{aligned}
E(R_p) &= \sum_{s=1}^{N} P(s) R_P(s) \\
&= \sum_{s=1}^{N} P(s)\, x R_A(s) + (1 - x) R_B(s) \\
&= x \sum_{s=1}^{N} P(s) R_A(s) + (1 - x) \sum_{s=1}^{N} P(s) R_B(s) \\
&= xE(R_A) + (1 - x) E(R_B)
\end{aligned}$$

Thus, the expected return for a portfolio of two securities is just the weighted average of the expected returns for the two securities in the portfolio.

Similarly, using variance formula, we find that the variance of the return for this portfolio is given by

$$\mathrm{Var}(R_p)=\sum_{s=1}^{N} P(s)\left[R_P(s)-E(R_P)\right]^2$$

$$=\sum_{s=1}^{N} P(s)\Big(x\left[R_A(s)-E(R_A)\right]$$

$$+(1-x)\left[R_B(s)-E(R_B)\right]\Big)^2$$

$$=x^2\sum_{s=1}^{N} P(s)\left[R_A(s)-E(R_A)\right]^2$$

$$+(1-x)^2\sum_{s=1}^{N} P(s)\left[R_B(s)-E(R_B)\right]^2$$

$$+2x(1-x)\sum_{s=1}^{N} P(s)\left[R_A(s)-E(R_A)\right]\left[R_B(s)-E(R_B)\right]$$

$$=x^2\,\mathrm{var}(R_A)+2x(1-x)\,\mathrm{cov}(R_A,R_B)+(1-x)^2\,\mathrm{var}(R_B)$$

Therefore, the standard deviation of our portfolio will be

$$SD(R_p)=\sqrt{\mathrm{Var}(R_p)}$$