Asset Commonality, Debt Maturity and Systemic Risk*

Franklin Allen        Ana Babus
University of Pennsylvania Princeton University

Elena Carletti
European University Institute

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Abstract

We develop a model where financial institutions swap projects in order to diversify their individual risk. This can lead to two different asset structures. In a clustered structure groups of financial institutions hold identical portfolios and default together. In an unclustered structure defaults are more dispersed. With long term finance welfare is the same in both structures. In contrast, when short term finance is used, the network structure matters. Upon the arrival of a signal about banks’ future defaults, investors update their expectations of bank solvency. If their expectations are low, they do not roll over the debt and all institutions are early liquidated. We compare investors’ rollover decisions and welfare in the two asset structures.

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1 Introduction

Understanding the nature of systemic risk is key to understanding the occurrence and propagation of financial crises. Traditionally the term describes a situation where many (if not all) financial institutions fail as a result of a common shock or a contagion process. A typical common shock leading to systemic failures is a collapse of residential or commercial real estate values (see Reinhart and Rogoff, 2009). Contagion refers to the risk that the failure of one financial institution leads to the default of others through a domino effect in the interbank market, the payment system or though asset prices (see, for example, the survey in Allen, Babus and Carletti, 2009).

The recent developments in financial markets and the crisis that started in 2007 have highlighted the importance of another type of systemic risk related to the interconnections among financial institutions and to their funding maturity. The emergence of financial instruments in the form of credit default swaps and similar products has improved the possibility for financial institutions to diversify risk, but it has also increased the overlaps in their portfolios. Whether and how such asset commonality among banks leads to systemic risk may depend on their funding maturity structure. With short term debt, banks are informationally linked. Investors respond to the arrival of interim information in a way that depends on the shape of banks’ interconnections. With long term debt instead, interim information plays no role and the composition of asset structures does not matter for systemic risk.

In this paper we analyze the interaction between asset commonality and funding maturity in generating systemic risk through an informational channel. We develop a simple two-period model, where each bank issues debt to finance a risky project. We initially consider the case of long term debt and then that of short term debt. Projects are risky and thus banks may default at the final date. Bankruptcy is costly in that investors only recover a fraction of the bank’s project return. As project returns are independently distributed, banks have an incentive to diversify to lower their individual default probability. We model this by assuming that each bank can exchange shares of its own project
with other banks. Exchanging projects is costly as it entails a due diligence cost for each swapped project. In equilibrium, banks trade off the advantages of diversification in terms of lower default probability with the due diligence costs.

Swapping projects can generate different types of overlaps in banks’ portfolios. Banks choose the number of projects to exchange but not the asset structure that emerges in equilibrium. For ease of exposition, we focus on the case of six banks with each of them optimally exchanging projects with two other banks. This leads to two possible asset structures. In one, which we call clustered, banks are connected in two clusters of three banks each. Within each cluster all banks hold the same portfolio, but the two clusters are independent of each other. In the second, which we call unclustered, banks are connected in a circle. Each of them swaps projects only with the two neighboring banks and none of the banks holds identical portfolios.

We show that with long term debt the asset structure does not matter for welfare. The reason is that in either structure each bank’s portfolio is formed by three independently distributed projects with the same distribution of returns. The number of bank defaults and the expected costs of default are the same in the two structures and so is total welfare.

In contrast, the asset structure plays an important role in determining systemic risk and welfare when banks use short term debt. The main difference is that at the intermediate date a signal concerning banks’ future solvency is received. The signal indicates whether all banks will be solvent in the final period (good news) or whether at least one of them will default (bad news). Upon observing the signal, investors update the probability that their bank will be solvent at the final date and roll over the debt if they expect to be able to recover their opportunity cost. Rollover always occurs after a good signal is realized but not after a bad signal arrives. When rollover does not occur, all banks are forced into early liquidation. This source of systemic risk is the focus of our analysis. Investors’ rollover decisions depend on the structure of asset overlaps, the opportunity cost and the bankruptcy cost.

We show that, upon the arrival of bad news, rollover occurs less often in the clustered
than in the unclustered asset structure. When investors recover enough in bankruptcy or have a low opportunity cost, debt is rolled over in both structures. As the amount they recover decreases and their opportunity cost increases, debt is still rolled over in the unclustered network but not in the clustered one. The reason is that there is a greater information spillover in the latter as defaults are more concentrated. Upon the arrival of negative information investors infer that the conditional default probability is high and thus decide not to roll over. In the unclustered network defaults are less concentrated and the arrival of the bad signal indicates a lower probability of a rash of bank defaults. When investors obtain little after banks default because of high bankruptcy costs or have a high opportunity cost, banks are early liquidated in both structures.

The welfare properties of the two networks with short term finance depend on investors’ rollover decisions, the proceeds from early liquidation and the bankruptcy costs. When banks continue and offer investors a repayment of the same magnitude in either structure, total welfare is the same in both structures. When the debt rollover requires a higher promised repayment in the clustered than in the unclustered network, welfare is higher in the latter as it entails lower bankruptcy costs. When banks are early liquidated in the clustered structure only, the comparison of total welfare becomes ambiguous. Initially, when neither the bankruptcy costs nor the proceeds from early liquidation are too high, total welfare remains higher in the unclustered network. However, when investors recover little from bankruptcy and obtain instead large proceeds from early liquidation, welfare becomes higher in the clustered network, and remains so even when early liquidation occurs in both networks.

Our results raise the question of why banks use short term debt in the first place. We show that the optimality of short term debt depends on the asset structure and on the difference between the long and the short term rate that investors can obtain from alternative investments. The market failure in our model is that banks are unable to choose the asset structure explicitly. By choosing the efficient maturity of the debt they can improve their expected profits and welfare. However, this does not solve the problem
of the multiplicity of asset structures. Only a mechanism that would allow banks to coordinate on the architecture of their connections would solve this.

The focus of our paper is the interaction of banks’ asset structure and debt maturity in generating systemic risk. The crucial point is that the use of short term debt may lead to information contagion among financial institutions. The extent to which this happens depends on the asset structure, that is on the degree of overlaps of banks’ portfolios. In this sense, our paper is related to several strands of literature. Concerning the effects of diversification on banks’ portfolio risk, Shaffer (1994), Wagner (2010) and Ibragimov, Jaffee and Walden (2010) show that diversification is good for each bank individually, but it can lead to greater systemic risk as banks’ investments become more similar. As a consequence, it may be optimal to limit it.

Other papers analyze the rollover risk entailed in short term finance. Acharya, Gale and Yorulmazer (2010) and He and Xiong (2009) show that rollover risk can lead to market freezes and dynamic bank runs. Diamond and Rajan (2009) and Bolton, Santos and Scheinkman (2009) analyze how liquidity dry-ups can arise from the fear of fire sales or asymmetric information. All these studies use a representative bank/agent framework. By contrast, we analyze how different network structures affect the rollover risk resulting from short term finance.

Systemic risk arises in our model from the investors’ response to the arrival of interim information regarding banks’ future solvency. In this sense our paper is related to the literature on information contagion. Chen (1999) shows that sufficient negative information on the number of banks failing in the economy can generate widespread runs among depositors at other banks whose returns depend on some common factors. Dasgupta (2004) shows that linkages between banks in the form of deposit crossholdings can be a source of contagion when the arrival of negative interim information leads to coordination problems among depositors and widespread runs. Acharya and Yorumalez (2008) find that banks herd and undertake correlated investment to minimize the effect of information contagion on the expected cost of borrowing. Our paper also analyzes the systemic risk
stemming from multiple structures of asset commonality among banks, but it focuses on
the interaction with the funding maturity of financial intermediaries.

Some other papers study the extent to which banks internalize the negative external-
ities that arise from contagion. Babus (2009) proposes a model where banks share the
risk that the failure of one bank propagates through contagion to the entire system. Cas-
tiglionesi and Navarro (2010) show that an agency problem between bank shareholders
and debtholders leads to fragile financial networks. Zawadowski (2010) argue that banks
that are connected in a network of hedging contracts fail to internalize the negative effect
of their own failure. All these papers rely on a domino effect as a source of systemic risk.
In contrast, we focus on asset commonality as a source of systemic risk in the presence of
information externalities when banks use short term debt.

The rest of the paper proceeds as follows. Section 2 lays out the basic model when
banks use long term debt. Section 3 describes the equilibrium that emerges with long
term finance. Section 4 introduces short term debt. It analyzes investors’ decision to roll
over the debt in response to information about banks’ future solvency and the welfare
properties of the different network structures. Section 5 discusses a number of extensions.
Section 6 concludes.

2 The basic model with long term finance

Consider a three-date \( t = 0, 1, 2 \) economy with six banks, denoted by \( i = 1, \ldots, 6 \), and
a continuum of small, risk-neutral investors. Each bank \( i \) has access at date 0 to an
investment project that yields a stochastic return \( \theta_i = \{R_H, R_L\} \) at date 2 with probability
\( p \) and \( 1 - p \), respectively, and \( R_H > R_L > 0 \). The returns of the projects are independently
distributed across banks.

Banks raise one unit of funds each from investors at date 0 and offer them, in exchange,
a long term debt contract that specifies an interest rate \( r \) to be paid at date 2. Investors
provide finance to one bank only and are willing to do so if they expect to recover at least
their two period opportunity cost \( r^2 F < E(\theta_i) \).
We assume that $R_H > r_F^2 > R_L$ so that a bank can pay $r$ only when the project yields a high return. When the project yields a low return $R_L$, the bank defaults at date 2 and investors recover a fraction $\alpha \in [0, 1]$ of the project return. The remaining fraction $(1 - \alpha)$ is lost as bankruptcy costs. Thus, investors will finance the bank only if their participation constraint
\[ pr + (1 - p)\alpha R_L \geq r_F^2 \]
is satisfied. The first term on the left hand side represents the expected payoff to the investors when the bank repays them in full. The second term represents investors’ expected payoff when the bank defaults at date 2. The right hand side is the investors’ opportunity cost.

When the project returns $R_H$, the bank acquires the surplus $(R_H - r)$. Otherwise, it receives 0. The bank’s expected profit is then given by
\[ \pi_i = p(R_H - r). \]

Given projects are risky and returns are independently distributed, banks can reduce their default risk through diversification. We model this by assuming that each bank can exchange shares of its own project with $\ell_i$ other banks through bilateral connections. That is, bank $i$ exchanges a share of its project with bank $j$ if and only if bank $j$ exchanges a share of its project with bank $i$. When this happens, there is a link — that is, a bilateral swap of project shares— between banks $i$ and $j$ denoted as $\ell_{ij}$. Then each bank $i$ ends up with a portfolio of $1 + \ell_i$ projects with a return equal to
\[ X_i = \frac{\theta_{i1} + \theta_{i2} + ... + \theta_{i1+\ell_i}}{1 + \ell_i}. \]

Exchanging projects with other banks reduces the expected bankruptcy costs $(1 - p)(1 - \alpha)R_L$ and investors’ promised repayment $r$ but it also entails a due diligence cost $c$ per link. The idea is that banks know their own project, but they do not know those of
the other banks. Thus they need to exert costly effort to check that the projects of the other banks are bona fide as well.

The exchange of project shares creates interconnections and portfolio overlaps among banks as each of them has shares of $1 + \ell_i$ independently distributed projects in its portfolio. The collection of all interconnections can be described as a network $g$. The degree of overlaps in banks’ portfolios depends on the number $\ell_i$ of projects that each bank swaps with other banks and on the structure of connections among banks. For a given $\ell_i$ there may be multiple network structures as discussed below.

3 Long term finance

We model banks’ portfolio decisions as a network formation game. This allows us to focus on the various asset structure compositions that emerge from the swapping of projects. We first derive the participation constraint of the investors and banks’ profits when each bank $i$ has $\ell_i$ links with other banks and holds a portfolio of $1 + \ell_i$ projects. An equilibrium network structure is one where banks maximize their expected profits and do not find it worthwhile to sever or add a link.

We denote as $r = r(g, \ell_i)$ the interest rate that bank $i$ promises investors in a network structure $g$ where banks have $\ell_i$ links and $1 + \ell_i$ projects. Investors receive $r$ at date 2 when the return of bank $i$’s portfolio is $X_i \geq r$, while they receive a fraction $\alpha$ of the bank’s portfolio return when $X_i < r$. The participation constraint of the investors is then given by

$$\Pr(X_i \geq r)r + \alpha E(X_i < r) \geq r_F^2,$$

(1)

where $\Pr(X_i \geq r)$ is the probability that the bank remains solvent at date 2 and $E(X_i < r) = \sum_{x<r} x \Pr(X_i = x)$ is the bank’s expected portfolio payoff when it defaults at date 2. The equilibrium $r$ is the lowest interest rate that satisfies (1) with equality.

Banks receive the surplus $X_i - r$ whenever $X_i \geq r$ and 0 otherwise. The expected
profit of a bank $i$ in an asset structure $g$ is

$$\pi_i(g) = E(X_i \geq r) - \Pr(X_i \geq r)r - c\ell_i, \quad (2)$$

where $E(X_i \geq r) = \sum_{x \geq r} x \Pr(X_i = x)$ is the expected return of the bank’s portfolio and $\Pr(X_i \geq r)r$ is the expected repayment to investors when the bank remains solvent at date 2, and $c\ell_i$ are the total due diligence costs. Substituting the equilibrium interest rate $r$ from (1) with equality into (2), the expected profit of bank $i$ becomes

$$\pi_i(g) = E(X_i) - r^2_F - (1 - \alpha)E(X_i < r) - c\ell_i. \quad (3)$$

The bank’s expected profit is given by the expected return of its portfolio $E(X_i)$ minus the investors’ opportunity cost $r^2_F$, the expected bankruptcy costs $(1 - \alpha)E(X_i < r)$, and the total due diligence costs $c\ell_i$. As (3) shows, greater diversification involves a trade-off between lower bankruptcy costs and higher total due diligence costs.

Banks choose the number of project shares to exchange $\ell_i$ in order to maximize their expected profits. The choice of $\ell_i$ determines the (possibly multiple) equilibrium network structure(s). A network $g$ is an equilibrium if it satisfies the notion of pairwise stability introduced by Jackson and Wolinsky (1996). This is defined as follows.

**Definition 1** A network structure $g$ is pairwise stable if

(i) for any pair of banks $i$ and $j$ that are linked in the network structure $g$, neither of them has an incentive to unilaterally sever their link $\ell_{ij}$. That is, the expected profit each of them receives from deviating to the network structure $(g - \ell_{ij})$ is not larger than the expected profit that each of them obtains in the network structure $g$ ($\pi_i(g - \ell_{ij}) \leq \pi_i(g)$ and $\pi_j(g - \ell_{ij}) \leq \pi_j(g)$);

(ii) for any two banks $i$ and $j$ that are not linked in the network structure $g$, at least one of them has no incentive to form the link $\ell_{ij}$. That is, the expected profit that at least one of them receives from deviating to the network structure $(g + \ell_{ij})$ is not larger than the expected profit that it obtains in the network structure $g$ ($\pi_i(g + \ell_{ij}) \leq \pi_i(g)$ and/or
\[ \pi_j(g + \ell_{ij}) \leq \pi_j(g). \]

To make the analysis more tractable, we impose a condition to ensure that for any \( \ell_i = 0, \ldots, 5 \) the bank defaults and is unable to repay \( r \) to investors at date 2 only when all projects in its portfolio pay off \( R_L \). When this is the case, the bank’s default probability is \( \Pr(X_i < r) = (1 - p)^{1+\ell_i} \) and the probability of the bank being solvent at date 2 is \( \Pr(X_i \geq r) = 1 - (1 - p)^{1+\ell_i} \). As shown in Appendix A, a sufficient condition to ensure this is

\[
(1 - (1-p)^6) \frac{5R_L + R_H}{6} + (1-p)^6 \alpha R_L \geq r_F^2. \tag{4}
\]

Condition (4) guarantees that there exists an interest rate \( r \) in the interval \([r_F^2, \frac{\ell_i R_L + R_H}{1+\ell_i}]\) that satisfies the investors’ participation constraint (1) for any \( \ell_i = 0, \ldots, 5 \), where \( \frac{\ell_i R_L + R_H}{1+\ell_i} \) is the next smallest return realization of a bank’s portfolio after all projects return \( R_L \).

Given (4), the bank’s expected profit in (3) can be written as

\[
\pi_i(g) = E(X_i) - r_F^2 - (1 - p)^{1+\ell_i}(1 - \alpha)R_L - c\ell_i. \tag{5}
\]

It is easy to show that (5) is concave in \( \ell \) as the second derivative with respect to \( \ell \) is negative.

In what follows we will concentrate on the case where in equilibrium banks find it optimal to exchange \( \ell = 2 \) project shares and only symmetric asset structures are formed. The reason is that this is the minimum number of links such that there are multiple nontrivial asset structures. We have the following.

**Proposition 1** For any \( c \in [p(1-p)^3(1-\alpha)R_L, p(1-p)^2(1-\alpha)R_L] \) a structure \( g^* \) where all banks have \( \ell^* = 2 \) links is pairwise stable and Pareto dominates equilibria with \( \ell^* \neq 2 \).

**Proof.** See Appendix C. ■

In equilibrium banks trade off the benefit of greater diversification in terms of lower expected bankruptcy costs with higher total due diligence costs. Proposition 1 identifies the parameter space for the cost \( c \) such that this trade off is optimal at \( \ell^* = 2 \).
Banks choose the number of projects to exchange but not the asset structure so that multiple networks can emerge, for a given $\ell^*$. With $\ell^* = 2$ there are two equilibrium asset structures $g^*$ as shown in Figure 1. In the first structure, that we define as clustered ($g = C$), banks are connected in two clusters of three banks each. Within each cluster, banks hold identical portfolios but the two clusters are independent of each other. In the second structure, denoted as unclustered ($g = U$), banks are all connected in a circle. Each of them exchanges projects only with the two neighboring banks so that none of the banks holds identical portfolios. In this sense, risk is more concentrated in the clustered than in the unclustered structure.

Both networks are pairwise stable if the due diligence cost $c$ is in the interval $[p(1 - p)^3(1 - \alpha)R_L, p(1 - p)^2(1 - \alpha)R_L]$. No bank has an incentive to deviate by severing or adding a link as it obtains higher expected profit in equilibrium. Given that the bank’s expected profit function is concave in $\ell$, and that investors always recover their opportunity cost, the restriction on $c$ in Proposition 1 also guarantees that the equilibrium with $\ell^* = 2$ is the best achievable.

In either equilibrium asset structure each bank has a portfolio of $1 + \ell^* = 3$ independently distributed projects with a distribution of returns as described in Table 1. For simplicity, we assume an equal probability of a project $i$ returning $R_H$ or $R_L$, that is $p = \frac{1}{2}$. This implies that all states are equally likely. Since there are $6$ projects with two possible returns at date 2 each, there are $2^6 = 64$ states. Depending on the number of realizations of $R_L$ and $R_H$, there are $7$ possible combinations of the $6$ project returns numbered in the first column of the table. Each combination $mR_L, (6 - m)R_H$, where $0 \leq m \leq 6$, is shown in the second column, and the number of states $\binom{6}{m}$ in which it occurs is in the third column. For example, there are $\binom{6}{3} = 20$ states where the combination of projects $3R_L, 3R_H$ occurs.

The next four columns in the table show bank $i$’s portfolio return $X_i$ for each combination of the $6$ project returns. Given any $mR_L, (6 - m)R_H$, bank $i$’s portfolio returns $X_i = \frac{kR_L + (3 - k)R_H}{3}$, where $m \geq k$ and $0 \leq k \leq 3$, in $\binom{3}{k} \binom{3}{m - k}$ states. This is because for
any given \( mR_L, (6 - m)R_H \) there are \( \binom{3}{2} \) possible combinations of \( kR_L \) and \( (3 - k)R_H \) in the 3 projects of bank \( i \)'s portfolio. For each of these combinations, the remaining \( (m - k)R_L \) and \( [3 - (m - k)]R_H \) returns can be combined in \( \binom{3}{m-k} \) ways. For example, given the combination \( 3R_L, 3R_H \) of the 6 projects (that is, \( m = 3 \)), \( X_i = \frac{R_L + 2R_H}{3} \) (that is, \( k = 1 \)) realizes in \( \binom{3}{2} \binom{3}{2} = 9 \) states out of the 15 states with \( 3R_L, 3R_H \). Similarly for the remaining entries in the four columns. The final row gives the total of each column.

For example, there are 24 out of the 64 states where \( X_i = \frac{R_L + 2R_H}{3} \) occurs.

As Table 1 shows, each bank \( i \) has an identical portfolio distribution irrespective of the network structure. What matters for the banks’ portfolio returns with long term financing is only the number of projects \( \ell^* \) that each of them swaps in equilibrium, but not the resulting asset structure composition. This has direct implications for welfare. This is equal to the sum of a representative bank \( i \)'s expected profit and its investors' expected returns. Given that the investors always recover their opportunity cost, from (5) the equilibrium welfare per bank simplifies to

\[
W(g) = E(X_i) - (1 - \alpha)E(X_i < r) - 2c. 
\]

Given that each bank’s portfolio return distribution is the same in either network, all banks offer the same interest rate to investors and have the same bankruptcy probability in both structures. This gives the following result.

**Proposition 2** Total welfare is the same in the clustered and unclustered structures.

### 4 Short term finance

We now analyze the case where banks use short term finance and investors have per period opportunity cost \( r_f \). As with long term finance, we continue focusing on the clustered and unclustered structures with \( \ell^* = 2 \) and on the range \( R_L < r_f^2 < \frac{5R_L + 2R_H}{6} \) so that bankruptcy occurs only when all projects in a bank’s portfolio return \( R_L \). We show that, in contrast to the case with long term finance, the asset structure composition matters for
systemic risk and total welfare when short term finance is used.

The main difference with short term finance is that it needs to be rolled over every period. If adverse interim information arrives, investors may not roll over the debt thus forcing the bank into early liquidation. We model this by assuming that a signal about bank future solvency arrives at date 1. The signal can either indicate the good news that all banks will be solvent at date 2 ($S = G$) or the bad news that at least one bank will default ($S = B$). The idea is that investors hear of an imminent bank failure and have to infer the prospects of their own bank. For simplicity, we assume that the signal does not reveal any information about any individual bank. This ensures that as far as individual investors are concerned, all banks look alike and have an equal probability of default once the signal arrives. We consider alternative information structures in Section 5.

Figure 2 shows the sequence of events in the model with short term finance. At date 0 each bank in the network structure $g = C, U$ raises one unit of funds and promises investors an interest rate $r_{01}(g)$ at date 1. Investors know the network structure, but do not know the position of any particular bank in the network. At the beginning of date 1, before investors are repaid $r_{01}(g)$, the signal $S = \{G, B\}$ arrives. With probability $q(g)$ the signal $S = G$ reveals that all banks will be solvent at date 2. With probability $1 - q(g)$ the signal $S = B$ reveals that at least one bank will default at date 2. Upon observing the signal, investors decide whether to roll the funds over for a total promised repayment of $\rho_{12}^S(g)$ at date 2 or retain $r_{01}(g)$. If rollover occurs, the bank continues till date 2. Investors receive $\rho_{12}^S(g)$ and the bank $X_i - \rho_{12}^S(g)$ if it remains solvent. Otherwise, when the bank goes bankrupt, investors receive $\alpha X_i$ and the bank 0. If rollover does not occur, the bank is forced into early liquidation at date 1. Investors receive the proceeds from early liquidation, which for simplicity we assume to be equal to $r_f$, and the bank receives 0.

The interest rate $r_{01}(g)$ promised to investors at date 0 must be such that they recover their per period opportunity cost $r_f$ at date 1. Given that investors always recover their opportunity cost at date 1, irrespective of whether the bank is continued or liquidated at
date 1, they will simply require a rate $r_{01}(g) = r_f$ at date 0.\footnote{If investors obtained only $\beta r_f$ with $\beta < 1$ as early liquidation proceeds, they would require $r_{01}(g) > r_f$ when they anticipate not rolling over the debt at date 1. This would imply higher deadweight costs and lower welfare with early liquidation, but our qualitative results would be similar.}

At date 1, after the signal $S$ is realized, investors roll over the debt if the promised repayment $\rho^S_{12}(g)$ is such that they can recover $r_{01}(g)r_f = r_f^2$ at date 2. When $S = G$ investors infer that they will always receive $\rho^G_{12}(g)$ at date 2 and thus roll over the debt for a repayment $\rho^G_{12}(g) = r_f^2$. When $S = B$, investors update the probability $\Pr(X_i \geq \rho^B_{12}(g)|B)$ that their bank will be able to repay them the promised repayment $\rho^B_{12}(g)$ at date 2. Then rollover occurs if there exists a value of $\rho^B_{12}(g)$ that satisfies investors’ date 1 participation constraint

$$\Pr(X_i \geq \rho^B_{12}(g)|B)\rho^B_{12}(g) + \alpha E(X_i < \rho^B_{12}(g)|B) \geq r_f^2.$$

The first term is the expected return to investors conditional on $S = B$ when the bank remains solvent at date 2. The second term is their expected payoff conditional on $S = B$ when the bank defaults at date 2. This is equal to a fraction $\alpha$ of the bank’s portfolio expected return $E(X_i < \rho^B_{12}(g)|B) = \sum_{x<\rho^B_{12}(g)} x \Pr(X_i = x|B)$. The equilibrium value of $\rho^B_{12}(g)$ if it exists, is the minimum promised repayment that satisfies (7) with equality and minimizes the probability of bank default conditional on $S = B$.

The expected profit of bank $i$ at date 0 depends on the realization of the signal and on the investors’ rollover decision at date 1. When rollover occurs and the bank continues at date 1, its expected profit is given by

$$\pi_i(g) = q(g) \left[ E(X_i \geq r_f^2|G) - r_f^2 \right] + (1-q(g)) \left[ E(X_i \geq \rho^B_{12}(g)|B) - \Pr(X_i \geq \rho^B_{12}(g)|B)\rho^B_{12}(g) \right] - 2c.$$ 

The first term represents the expected profit when with probability $q(g)$ the good signal $S = G$ occurs. Investors always receive $r_f^2$ at date 2 and the bank retains the expected surplus $E(X_i \geq r_f^2|G) - r_f^2$, where $E(X_i \geq r_f^2|G) = \sum_{x \geq r_f^2} x \Pr(X_i = x|G)$ is the bank’s expected portfolio return conditional on $S = G$. The second term is the expected profit when with probability $1 - q(g)$ the bad signal $S = B$ occurs. Investors obtain $\rho^B_{12}(g)$...
with probability $\Pr(X_i \geq \rho^B_{12}(g)|B)$, while the bank retains the remaining surplus $E(X_i \geq \rho^B_{12}(g)|B) - \Pr(X_i \geq \rho^B_{12}(g)|B)\rho^B_{12}(g)$, where $E(X_i \geq \rho^B_{12}(g)|B) = \sum_{x \geq \rho^B_{12}(g)} x \Pr(X_i = x|B)$ is the bank’s expected portfolio return conditional on $S = B$. The last term $2c$ is the total due diligence costs given $\ell^* = 2$.

Substituting the promised repayment $\rho^B_{12}(g)$ from (7) with equality into (8), this simplifies to

$$\pi_i(g) = E(X_i) - r^2_f - (1 - q(g))(1 - \alpha)E(X_i < \rho^B_{12}(g)|B) - 2c. \quad (9)$$

When rollover occurs at date 1, the bank’s expected profit can be expressed as in the case of long term debt by the expected return of its portfolio $E(X_i)$ minus the investors’ opportunity cost $r^2_f$, the expected bankruptcy costs $(1 - q(g))(1 - \alpha)E(X_i < \rho^B_{12}(g)|B)$, and the total due diligence costs $2c$.

When, after the realization of a bad signal, rollover does not occur, the bank is early liquidated at date 1 and receives 0. Then, its expected profit, given by

$$\pi_i(g) = q(g) \left[ E(X_i \geq r^2_f|G) - r^2_f \right] - 2c, \quad (10)$$

is positive only when with probability $q(g)$ the good signal arrives. Note that (9) and (10) imply that, in a given network structure $g$, the bank has higher expected profit when debt is rolled over at date 1 than when it is not.

### 4.1 Investors’ rollover decisions at date 1

The crucial difference between long and short term finance is that in the latter case the asset structure matters for the equilibrium interest rates, bank profits and ultimately total welfare. The reason is that the probability distribution of the signal and the associated conditional probabilities of bank default at date 2 differ in the two structures. To see this, consider first the distribution of the signal. The good signal arrives when all banks’ portfolios return at least $(2R_L + R_H)/3$ and investors can obtain the opportunity cost $r^2_f$.
at date 2. Thus, the probability of $S = G$ is

$$q(g) = \Pr(\bigcap_{i=1}^{6} X_i \geq r^2_f),$$

where $\Pr(\bigcap_{i}(X_i \geq r^2_f)) = \Pr(X_1 \geq r^2_f, X_2 \geq r^2_f, ..., X_6 \geq r^2_f)$ represents the probability that none of the six banks defaults. By contrast, the bad signal arrives when the portfolio of at least one bank returns $X_i = R_L < r^2_f$. Thus, the probability of $S = B$ is

$$1 - q(g) = \Pr(\bigcup_{i=1}^{6} X_i < r^2_f) = \Pr(\bigcup_{i=1}^{6} X_i = R_L), \tag{11}$$

where $\Pr(\bigcup_{i=1}^{6} X_i = R_L)$ is the probability that at least one of the six banks defaults.

The clustered and unclustered asset structures entail different composition of banks’ portfolios. In the former banks hold identical portfolios within each cluster. In the latter each bank shares projects with two others but no banks hold identical portfolios. This implies a different concentration of defaults in the two asset structures. In the clustered network defaults occur in groups: The 3 banks in one cluster default when all the 3 projects in their portfolios return $R_L$ or all 6 banks default when all the 6 projects in the economy give $R_L$. In the unclustered network defaults are more scattered. As banks hold diverse portfolios, each bank can fail independently of the others. When the 3 projects in one bank’s portfolio return $R_L$, only that bank defaults. As the number of projects returning $R_L$ increases, more banks also default in the unclustered network. The different concentration of defaults implies different probability distributions of the signal in the two asset structures. Formally, the probability of $S = B$ is given by

$$1 - q(C) = 2 \sum_{m=3}^{6} \frac{(6-m)}{2^6} - \frac{1}{2^6} = \frac{15}{64}, \tag{12}$$
in the clustered structure, and by

\[ 1 - q(U) = 6 \sum_{m=3}^{6} \frac{(6-3)}{2^6} - 6 \sum_{m=4}^{6} \frac{(6-4)}{2^6} + \frac{1}{2^6} = \frac{25}{64} \]  \hspace{1cm} (13)

in the unclustered structure, where as before \( m \) is the number of projects returning \( R_L \) for a given combination \( mR_L, (6 - m)R_H \) of the 6 projects in the economy (see Section B of the Appendix for a full derivation of (12) and (13)). The bad signal arrives when at least three projects forming a bank’s portfolio return \( X_i = R_L \). In the clustered structure this occurs in \( 2\binom{6-3}{6-m} \) out of the \( 2^6 = 64 \) states for any given combination \( mR_L, (6 - m)R_H \) of projects with \( m \geq 3 \). Summing up the combinations with \( m \geq 3 \) and taking into account that there is only one state where \( m = 6 \) gives (12). Similar considerations explain (13). The higher number of default states in the unclustered network \(-25\) against \( 15\) follows directly from the higher concentration of defaults when banks are clustered.

It follows that the probability of \( S = G \) is

\[ q(C) = \frac{49}{64} \text{ and } q(U) = \frac{39}{64} \]

in the clustered and unclustered asset structures, respectively, so that clearly

\[ q(C) > q(U). \]  \hspace{1cm} (14)

What matters for investors’ rollover decisions are the conditional probability distributions of banks’ portfolio returns. Tables 2 and 3 show these for the clustered and unclustered asset structures, respectively. Both tables report the conditional distributions for each combination \( mR_L, (6 - m)R_H \) of project realizations and in total. The first two columns in the tables number and describe the combinations \( mR_L, (6 - m)R_H \). The third column shows the number of states where the bad signal arrives at date 1 and at least one bank will default at date 2. The fourth set of columns shows bank \( i \)’s portfolio distribution conditional on \( S = B \). The next two sets of columns show the number of no default states
and bank $i$’s portfolio distribution conditional on $S = G$. Note that the distribution of $X_i$ conditional on $S = G$ is simply the difference between the unconditional probability distribution of $X_i$ as described in Table 1 and the conditional distribution on $S = B$.

Finally, the last row in both tables shows the total number of states where $S = B, G$ arrives out of the 64 states and the total number of states for the conditional distributions of $X_i$.

Consider Table 2 for the clustered network first. Clearly there are no default states when $m \leq 2$ as in states 1, 2 and 3. From (12) it follows that for any $m = \{3, 4, 5\}$ the number of default states is $2(6-3)\binom{6}{m}$ out of the $\binom{6}{m}$ states where the combination $mR_L, (6-m)R_H$ is realized. For example, given $3R_L, 3R_H$ (that is, $m = 3$) the number of default states equals $2(6-3) = 2$. In each of the 2 default states, 3 banks in one cluster will default with a portfolio return of $R_L$ while the 3 other banks will remain solvent with a portfolio returning $R_H$. Thus, bank $i$’s portfolio returns $X_i = R_L$ in 1 state and $X_i = R_H$ in the other state out of the 2 default states. Similar considerations hold for the other entries.

With $m = 6$ there is clearly only one default state where all banks have $X_i = R_L$, which can be also derived from $2(6-3)\binom{6}{6} - 1$ in (12). The number of no default states when $S = G$ is simply the difference between the $\binom{6}{m}$ states where the combination $mR_L, (6-m)R_H$ is realized and the number of default states. The distribution for $X_i$ conditional on $S = G$ can be found similarly to before. For example, given $3R_L, 3R_H$, there are $\binom{6}{3} - 2(6-3)\binom{6}{6-m} = 18$ no default states where the good signal arrives. In such states, 3 banks in one cluster will have a portfolio returning $\frac{2R_L + R_H}{3}$ while the other 3 banks will have $X_i = \frac{R_L + 2R_H}{3}$. Thus, bank $i$’s portfolio returns $X_i = \frac{2R_L + R_H}{3}$ in 9 states and $X_i = \frac{R_L + 2R_H}{3}$ in the other 9 states out of the 18 no default states. Similar considerations hold for the other entries. The last row of Table 2 indicates that, for example, out of the 15 total default states, bank $i$ has portfolio return $X_i = R_L$ in 8 states; and out of the 49 states where no defaults occur its portfolio returns $X_i = \frac{2R_L + R_H}{3}$ in 21 states. Similarly for the other returns conditional on $S = B, G$.

The conditional distribution in the unclustered network as described in Table 3 is
derived similarly. The number of default states given the combination \( mR_L, (6 - m)R_H \)
follows from (13). The bad signal arrives in \( 6\binom{6 - 3}{6 - m} \) states for \( m = 3 \); in \( 6\binom{6 - 3}{6 - m} - 6\binom{6 - 4}{6 - m} \) states for \( m \in \{4, 5\} \) and in \( 6\binom{6 - 3}{6 - m} - 6\binom{6 - 4}{6 - m} + 1 \) for \( m = 6 \) out of the \( \binom{6}{m} \) states where the combination \( mR_L, (6 - m)R_H \) is realized. As before, for given default states the conditional
distribution of \( X_i \) can easily be derived. For example, given \( 3R_L, 3R_H \) (that is, \( m = 3 \))
there are \( 6\binom{6 - 3}{6 - 3} = 6 \) default states where bank \( i \)’s portfolio return is \( X_i = R_L \) or \( X_i = R_H \)
in 1 state each, and \( X_i = \frac{2R_L + R_H}{3} \) or \( X_i = \frac{R_L + 2R_H}{3} \) in 2 states each. Similarly for the
other entries conditional on the bad signal. The number of no default states is again
derived as the difference between the \( \binom{6}{m} \) states where the combination \( mR_L, (6 - m)R_H \)
is realized and the number of default states. For example, given \( 3R_L, 3R_H \), there are
\( \binom{6}{3} - 6\binom{6 - 3}{6 - 3} = 14 \) no default states where the good signal arrives. In such states, bank \( i \)’s
portfolio returns \( X_i = \frac{2R_L + R_H}{3} \) in 7 states and \( X_i = \frac{R_L + 2R_H}{3} \) in the other 7 states out of
the 14 no default states. Similar considerations hold for the other entries. The last row
in Table 3 indicates that out of the 25 total default states, bank \( i \) has portfolio return
\( X_i = R_L \) in 8 states; and out of the 39 states where no defaults occur its portfolio returns
\( X_i = \frac{2R_L + R_H}{3} \) in 13 states. Similarly for the other returns conditional on \( S = B, G \).

Comparing Tables 2 and 3, it can be seen that the conditional distributions of banks’
portfolio returns are quite different in the two asset structures. In particular, the probability
of \( X_i = R_L \) conditional on \( S = B \) in the clustered network, which is equal to \( \frac{8}{15} \),
is much higher than in the unclustered network, where it is \( \frac{8}{25} \). This also implies that
the conditional probability \( \Pr(X_i \geq \rho_{12}^B(g)|B) \) that the bank is solvent and repays \( \rho_{12}^B(g) \)
to the investors at date 2 conditional on \( S = B \) is higher in the unclustered than in the
clustered network. That is,

\[
\Pr(X_i \geq \rho_{12}^B(U)|B) > \Pr(X_i \geq \rho_{12}^B(C)|B) \tag{15}
\]

for \( \rho_{12}^B(g) \in [R_L, \frac{2R_L + R_H}{3}] \). This difference means that investors’ rollover decisions can
differ between the two asset structures. We study the clustered structure first.
Proposition 3 When the bad signal \((S = B)\) is realized in the clustered structure and \(R_H > \frac{13}{12} R_L\),

A. For \(\alpha \geq \alpha_{LOW}(C)\), investors roll over the debt for a promised repayment \(\rho_{12}^B(C) \in \left[ r_f^2, \frac{2R_L + R_H}{3} \right]\), where \(\alpha_{LOW}(C) = \frac{45r_f^2 - 7(2R_L + R_H)}{24R_L}\).

B. For \(\alpha_{MID}(C) \leq \alpha < \alpha_{LOW}(C)\), investors roll over the debt for a promised repayment \(\rho_{12}^B(C) \in \left[ \frac{2R_L + R_H}{4}, \frac{R_L + 2R_H}{3} \right]\), where \(\alpha_{MID}(C) = \frac{45r_f^2 - 4R_L - 8R_H}{3(10R_L + 3R_H)}\).

C+D. For \(\alpha < \alpha_{MID}(C)\), investors do not roll over the debt and the bank is early liquidated at date 1.

Proof. See Appendix C.

The proposition is illustrated in Figure 3, which plots investors’ rollover decisions as a function of the exogenous parameters \(\alpha\) and \(r_f^2\). The result follows immediately from the investors’ participation constraint at date 1. When the bad signal is realized, the bank continues at date 1 whenever investors can be promised a repayment that satisfies (7). Whether this is possible depends on the fraction \(\alpha\) of the bank’s portfolio return accruing to the investors when the bank defaults at date 2 and on their opportunity cost \(r_f^2\) over the two periods. When \(\alpha\) is high or \(r_f^2\) is low as in Region A in Figure 3, there exists a repayment \(\rho_{12}^B(C)\) that satisfies (7). Investors roll over the debt and the bank continues.

The promised repayment compensates the investors for the possibility that they obtain only \(\alpha X_i\) in the case of default. Given \(\alpha\) is high, \(\rho_{12}^B(C)\) does not need to be high for (7) to be satisfied. Thus, the equilibrium \(\rho_{12}^B(C)\) lies in the lowest interval of the bank’s portfolio return, \(\left[ r_f^2, \frac{2R_L + R_H}{3} \right]\). As \(\alpha\) decreases or \(r_f^2\) increases so that Region B is reached, investors still roll over the debt but require a higher promised repayment as compensation for the greater losses in the case of bank default. Thus, \(\rho_{12}^B(C)\) is higher and lies in the interval \(\left[ \frac{2R_L + R_H}{3}, \frac{R_L + 2R_H}{3} \right]\). This also implies that, conditional on the realization of the bad signal, bankruptcy occurs at date 2 not only when a bank’s portfolio pays off \(X_i = R_L\) but also when it pays \(X_i = \frac{2R_L + R_H}{3}\). As \(\alpha\) decreases or \(r_f^2\) increases further so that Regions C and D below \(\alpha_{MID}(C)\) are reached, it is no longer possible to satisfy (7) for any \(\rho_{12}^B(g) \leq R_H\). Then, investors do not roll over the debt and the bank is early liquidated at date 1.
A similar result holds for the unclustered structure.

**Proposition 4** When the bad signal \((S = B)\) is realized in the unclustered structure, \(A+B+C\). For \(\alpha \geq \alpha_{\text{LOW}}(U)\), investors roll over the debt for a promised repayment \(\rho_{12}^B(U) \in [r_f, \frac{2R_L + R_H}{3}]\), where \(\alpha_{\text{LOW}}(U) = \frac{75r_f^2 - 17(2R_L + R_H)}{24RL}\).

**D.** For \(\alpha < \alpha_{\text{LOW}}(U)\), investors do not roll over the debt and the bank is liquidated at date 1.

**Proof.** See Appendix C. ■

Proposition 4 is also illustrated in Figure 3. As in the clustered structure, investors roll over the debt when there exists a repayment \(\rho_{12}^B\) that satisfies their participation constraint (7) with equality. Whether such a repayment exists depends as before on the parameters \(\alpha\) and \(r_f^2\). When they lie in the Regions A, B and C above \(\alpha_{\text{LOW}}(U)\), the probability \(\Pr(X_i \geq \rho_{12}^B(U)|B)\) is sufficiently high to ensure that (7) is always satisfied for a repayment \(\rho_{12}^B(U)\) in the interval \([r_f, \frac{2R_L + R_H}{3}]\). However, when \(\alpha\) and \(r_f^2\) lie in Region D (7) can no longer be satisfied and the bank is early liquidated.

A comparison of Propositions 3 and 4 shows that rollover occurs for a larger and early liquidation for a smaller parameter space in the unclustered structure than in the clustered. The promised repayment is also the same or lower in the former.

**4.2 Welfare with short term finance**

We next consider welfare in the two network structures with short term finance. As with long term finance, in both structures we can focus on the total welfare per bank as defined by the sum of a representative bank \(i\)’s expected profit and its investors’ expected returns. Welfare now depends on the investors’ rollover decisions, since these affect the bank’s expected profit. Using (9) and (10), welfare is given by

\[
W(g) = E(X_i) - (1 - q(g))(1 - \alpha)E(X_i < \rho_{12}^B(g)|B) - 2c
\] (16)
when the bank is continued till date 2 and by

\[ W(g) = q(g) \left[ E(X_i \geq r_f^2 | G) \right] + (1 - q(g))r_f^2 - 2c \]  (17)

when the bank is liquidated at date 1 after the arrival of the bad signal. In (16) welfare equals the expected return of bank portfolio \( E(X_i) \) minus the expected bankruptcy costs 

\( (1 - q(g))(1 - \alpha)E(X_i \leq \rho_{12}^B(g)|B) \) and the due diligence costs \( 2c \). In contrast, in (17) welfare is given by the sum of the expected return of bank portfolio \( q(g) \left[ E(X_i \geq r_f^2 | G) \right] \)
conditional on \( S = G \) and the date 2 value of the liquidation proceeds \( (1 - q(g))r_f^2 \) minus the due diligence costs \( 2c \).

Using (16) and (17) it is easy to derive the expressions for the welfare in the two asset structures. The following then holds.

**Proposition 5** The comparison of total welfare in the two structures is as follows:

A. For \( \alpha \geq \alpha_{LOW}(C) \), total welfare is the same in the clustered and unclustered structure.

B+C1. For \( \alpha_W < \alpha < \alpha_{LOW}(C) \), total welfare is higher in the unclustered structure than in the clustered structure, where \( \alpha_W = \frac{15\rho_{12}^B - 3R_L - 4R_H}{8R_L} \).

C2+D. For \( \alpha < \alpha_W \), total welfare is higher in the clustered structure than in the unclustered structure.

**Proof.** See Appendix C.

Figure 4 illustrates the proposition by showing the welfare in the clustered and unclustered structures. The crucial point is that with short term finance total welfare depends on the asset structure. Which is better depends crucially on the parameters \( \alpha \) and \( r_f^2 \). As (16) shows, \( \alpha \) affects welfare when investors roll over as it determines the size of the expected bankruptcy costs in the case of bank default. As (17) shows, \( r_f^2 \) affects welfare when the bank is early liquidated as a measure of the liquidation proceeds.

In Region A, where \( \alpha \geq \alpha_{LOW}(C) \), investors roll over the debt for a promised total repayment \( \rho_{12}^B(C) \in [r_f^2, \frac{2R_L + R_H}{3}] \) in both asset structures. In either of them, banks default
when their portfolios pay off $R_L$ and make positive profits in all the other states. As with long term finance, total welfare is then the same in both networks.

In Region B, where $\alpha$ lies in between $\alpha_{MID}(C)$ and $\alpha_{LOW}(C)$, rollover occurs in both asset structures, but investors require a higher promised repayment in the interval $[\frac{2R_L+R_H}{3}, \frac{R_L+2R_H}{3}]$ in the clustered network. This implies higher expected bankruptcy costs and thus lower welfare in the clustered network as banks also default when their portfolios return $X_i = \frac{2R_L+R_H}{3}$.

In Regions C1 and C2 rollover occurs in the unclustered structure but not in the clustered one. Total welfare is then given by (16) and (17) in the unclustered and clustered networks, respectively. In the former, welfare is decreasing in the bankruptcy costs, $1 - \alpha$. In the latter, welfare is increasing with $r_f^2$ as it increases the early liquidation proceeds. As $\alpha$ falls and $r_f^2$ increases, total welfare in the unclustered network becomes equal to that in the clustered network, and it then drops below.

Finally, in Region D, where $\alpha \leq \alpha_{LOW}(U)$, banks are always early liquidated after the arrival of the bad signal so that welfare is given by (17) in both asset structures. Since, as (14) shows, the good signal occurs more often in the clustered network, the expected return $q(g) \left[ E(X_i \geq r_f^2 | G) \right]$ is higher in the clustered structure while the date 2 value of the early liquidation proceeds $(1 - q(g))r_f^2$ is higher in the unclustered structure. The first term dominates so that total welfare is greater in the clustered asset structure.

5 Extensions

In this section we consider a number of extensions of the basic model. In particular, we discuss different types of signal arriving at the interim date, banks’ choice of long term versus short term finance, and different types of coordination mechanisms in the formation of connections among banks.
5.1 Information structure

The core of our analysis is the interaction between the interim information arriving at date 1, the composition of banks’ asset structure, and the funding maturity. Interim information has been modeled as a signal indicating whether at least one bank will default at date 2. For simplicity, the signal does not reveal the identity of potentially failing banks and all investors and banks are treated alike. Investors know the network structure but do not know any bank’s position in it. Upon observing the signal, they update the conditional probability that their own bank will default at date 2. The crucial feature for our result is that the signal generates a different information partition of the states and thus different conditional probabilities of default in the two asset structures. This implies different rollover decisions and thus different welfare in the two networks with short term finance.

Any signal that generates different information partitions and leads to different conditional probabilities across asset structures will have the same qualitative effect as in our basic model. Examples are signals indicating that a particular bank, say bank 1, has gone bankrupt or that a particular real sector is more likely to fail. Both of these signals would indicate in our model that a particular project or set of projects has a higher default probability than originally believed. This would generate different information partitions on banks’ future defaults depending on the different compositions of banks’ portfolios and would thus lead to different conditional probabilities in the two networks.

An alternative signal that would not lead to differences in the two asset structures is one carrying generic information about the underlying fundamentals. An example is a signal indicating the number of projects returning $R_L$ in the economy (without specifying the identity of these projects). This would simply reveal which state of the economy or combination $mR_L, (m-6)R_H$ of projects has been realized and the consequent conditional distribution of returns. As Table 1 shows, the conditional distribution would be the same in the two asset structures, as with long term debt. This would lead to the same investor rollover decisions and welfare in the two structures. This means that in our model bank
level information about defaults or specific information on defaulting sectors is different from generic information about fundamentals. The former interacts with the composition of the asset structure in generating systemic risk, while the latter does not.

The result that information about defaults is very different from information about project outcomes holds beyond our basic model. Given any number of banks above six and of connections, the probability distribution conditional on an interim signal revealing the number of low and high return projects will be independent of the composition of banks’ asset structure. The possible combinations of project outcomes will be the same for a given number of connections irrespective of the architecture of the asset structure.

5.2 Long term versus short term finance

So far we have considered long and short term finance separately and we have shown that the latter entails rollover risk while the former does not. This raises the question as to why banks use short term finance in the first place. There are a number of theories justifying its use. Flannery (1986) and Diamond (1991) suggest that short term finance can help overcome asymmetric information problems in credit markets. Calomiris and Kahn (1991) and Diamond and Rajan (2001) argue that short term debt can play a role as a discipline device to ensure that managers behave optimally. Brunnermeier and Oehmke (2009) suggest that creditors shorten the maturity of their claims to obtain priority, leading to an excessive use of short term debt. Another important rationale for the use of short term debt is an upward sloping yield curve. Borrowing short term at low rates to finance high yielding long term assets allows significant profits to be made and this is the approach used here.

In our model the choice of the optimal maturity structure depends on the difference between the long term rate $r^2_F$ and the short term rate $r^2_f$. To see this, suppose that once the asset structure is determined, banks choose the maturity of the debt that maximizes their expected profits. With short term debt bank expected profit is given by (9) and (10) depending on the investors’ rollover decisions as described in Propositions 3 and 4.
With long term debt bank expected profit is always given by (5). Comparing the different expressions for bank expected profits with short and long term financing profits gives the following.

**Proposition 6** Let \( \tau^2_F(C) \) and \( \tau^2_F(U) \) be the set of indifference points for which bank expected profit is the same with short and long term debt in the clustered and unclustered structures, respectively. Then the optimal debt maturity structure is as follows:

1. For \( r^2_F \geq \max \{ \tau^2_F(C), \tau^2_F(U) \} \) short term debt is optimal in both structures.
2. For \( \tau^2_F(C) > r^2_F \geq \tau^2_F(U) \) short term debt is optimal in the unclustered structure and long term debt is optimal in the clustered structure.
3. For \( \tau^2_F(U) > r^2_F \geq \tau^2_F(C) \) short term debt is optimal in the clustered structure and long term debt is optimal in the unclustered structure.
4. For \( r^2_F < \min \{ \tau^2_F(C), \tau^2_F(U) \} \) long term debt is optimal in both structures.

**Proof.** See Appendix C. The expressions for the boundaries \( \tau^2_F(C) \) and \( \tau^2_F(U) \) are provided there. ■

The proposition is illustrated in Figure 5, which plots the bank’s choice of debt maturity structure as a function of the rates \( r^2_F \) and \( r^2_f \) for a given value of the fraction \( \alpha \) of the bank’s portfolio return that investors receive in case of default. The boundaries \( \tau^2_F(C) \) and \( \tau^2_F(U) \) represent the combinations of \( r^2_f \) and \( r^2_F \) such that bank expected profit is the same with short and long term debt in the clustered and unclustered structure, respectively. Both \( \tau^2_F(C) \) and \( \tau^2_F(U) \) are piecewise functions of \( r^2_f \) since bank expected profit with short term debt changes with investors’ rollover decisions. Consider, for example, \( \tau^2_F(C) \). For values of \( r^2_f \) in Region A of Proposition 3, rollover occurs for a repayment \( \rho^B_{12}(g) \in [\tau^2_f, \frac{2RL+RH}{3}] \) and the bank expected profit \( \pi_i(C) \) is given by (9). When \( r^2_f \) is in Region B, the repayment increases to \( \rho^B_{12}(g) \in [\frac{2RL+RH}{3}, \frac{RH+2RH}{3}] \). This lowers \( \pi_i(C) \) and \( \tau^2_F(C) \) jumps up. As \( r^2_f \) enters Region C+D the bank is early liquidated and \( \pi_i(C) \) is given by (10). Thus, \( \tau^2_F(C) \) jumps even higher. Similar considerations hold for \( \tau^2_F(U) \), where the expression for this depends on which region of Proposition 4 \( r^2_f \) lies in.
Proposition 6 has important implications. First, it shows that, for a given network structure, the optimality of short term debt declines with the rate $r_f^2$. The reason is that an increase in $r_f^2$ across the different regions leads investors to either require a higher repayment $\rho_{12}^B$ or force the bank into liquidation. Both of these reduce the bank expected profit in a given network, and thus the optimality of short term debt relative to long term debt. Second, Proposition 6 shows that the optimality of the short term debt depends on the asset structure. Short term debt is optimal in both networks in Region 1, but is optimal only in one network in Regions 2 and 3. The bank’s choice of the optimal debt maturity conditional on the asset structure is always efficient from a welfare perspective. The reason is that, as investors always obtain their opportunity cost, total welfare coincides with bank expected profits. Thus, the choice of the optimal debt maturity resembles the comparison of total welfare in the two networks as described in Proposition 5. When welfare is higher in the unclustered network (as in Regions B+C1 of Proposition 5), the bank’s expected profit will also be higher. This corresponds to Region 2 of Proposition 6 where short term debt is only optimal in the unclustered network. Similar considerations hold for the other regions. Finally, note that short term debt is never optimal only in the unlikely case of Region 4 where the long term rate $r_f^2$ is smaller than the short term one $r_f^2$.

5.3 What is the market failure?

An important feature of the network literature and of the equilibrium concept of Jackson and Wolinsky (1996) that we use is that banks are not able to determine the network structure. Each bank chooses individually the number of links it wishes to have taking as given the choices of the other banks. Since banks form links simultaneously, with $\ell^* = 2$ either a clustered or an unclustered structure can emerge. With long term finance the multiplicity of asset structures does not matter. However, with short term finance it does matter since systemic risk and welfare differ in the two structures. Investors are indifferent as they obtain their opportunity cost in either network, but banks would clearly prefer the
asset structure that gives them higher expected profits. The market failure in our model lies precisely in banks’ inability to choose the asset structure explicitly. Banks can choose the number of projects to swap and the efficient maturity of their debt conditional on the asset structure they are in, but not the structure itself.

The choice of the optimal debt maturity structure can be seen as a constrained efficient solution, given the multiplicity of the asset structures. By choosing the maturity of their debt, banks can optimize their expected profits and welfare conditional on the network structure. Ideally, any mechanism that could instead allow banks to coordinate and choose the preferred asset structure would achieve efficiency. One example of such a mechanism would be to have banks condition their linkages on the connections between all other banks in the system. This would make it possible to ensure that only efficient networks are implemented. Similarly, government regulation, centralized exchanges or clearing houses could potentially be used to ensure that only the efficient network is chosen. Clearing bank interconnections through a centralized exchange or a clearing house rather than through bilateral swaps would improve transparency concerning the structure of bank connections. The model could be extended to include this possibility by assuming that the due diligence costs for checking each counterpart remain the same. The exchange or the clearing house could implement the efficient network by allocating counterparts to correspond to the clustered or unclustered structure. However, all these solutions seem hard to implement in practice. Private coordination among banks would be difficult to achieve and sustain, particularly as the number of banks grows large. Government intervention would require the gathering of a significant amount of information from banks about their interconnections and the determination and implementation of the optimal network structure. Similarly for a mechanism operating through a centralized exchange or a clearing houses. Neither of the two organize swaps of projects in practice. Exchanges ensure the existence of a matching mechanism to pair traders’ orders rather than choosing a particular network structure. Clearing houses clear trades between counterparties when they themselves decide to participate in a certain transaction. Transforming them
into mechanisms that determine and implement the optimal asset structure by regulating and coordinating banks’ interconnections seems quite ambitious. Much more work clearly remains to be done on such policy issues.

6 Concluding remarks

Understanding asset commonality among financial institutions is important for understanding systemic risk. In this paper we have developed a model where asset commonality arises from asset swaps, and we have shown that the structure of these swaps interact with the funding maturity of financial institutions in determining systemic risk.

The asset structure matters for systemic risk and total welfare when banks use short term finance, but not when they use long term finance. The reason is that with short term finance banks are informationally linked. When adverse interim information on banks’ future solvency arrives, investors update the default probability of their own bank and decide whether to roll over the debt. This inference problem depends on the structure of bank assets. In concentrated structures defaults are more correlated than in dispersed structures. This means that a negative interim signal conveys worse information and rollover occurs less often in the former than in the latter structure. In other words, there is more systemic risk in concentrated than in dispersed asset structures. However, a dispersed asset structure does not necessarily entail higher welfare. The reason is that, exactly because defaults are less concentrated, the bad signal arrives more often in dispersed networks. Whether this leads to lower welfare in a dispersed than in a concentrated asset structure depends on the size of the bankruptcy costs and on the proceeds from early liquidation. In this sense, our results have implications for the desirability of risk concentration depending on the magnitude of the bankruptcy costs and the proceeds from early liquidation. When bankruptcy is inefficient but early liquidation is not, it is optimal to have fewer instances with more banks defaulting as in the clustered structure rather than more frequent instances with less banks defaulting as in the unclustered structure. In other cases it is better to spread out default across states as in the unclustered structure.
The key trade off between the clustered and the unclustered structures in our framework derives from the different overlap and risk concentration among banks’ portfolios in the two networks. While we have analyzed an economy where each of the six banks swap two projects, the results hold more generally. What matters is that the multiple network structures that emerge in equilibrium differ in terms of banks’ asset concentration. An increase in the number of banks in the economy would increase the multiplicity of equilibrium asset structures. Still there would exist clustered structures where banks have highly correlated portfolios and dispersed structures where banks have more diverse portfolios, as in Figure 1. As in our basic model, investors’ rollover decisions and thus welfare would then still differ in the two types of asset structures.

We model asset commonality through asset swaps. This allows us to use a standard approach based on network formation and to focus on the composition of the asset structures. However, the insights of our model hold more generally. Any mechanism leading to similar asset structures would lead to similar results. An example is banks’ lending choices. A concentrated asset structure would arise if groups of banks lend to different sectors, for instance some banks do retail mortgage lending and others do commercial mortgage lending. A dispersed—or unconcentrated—asset structure would instead arise if all banks lend to the same sectors but in different shares or in different geographical areas. In this case all banks have some assets in common but maintain distinct portfolios.

We have derived our results assuming that bankruptcy costs are constant irrespective of the number of banks defaulting. If, as in several other papers, such as Wagner (2010) and Ibragimov, Jaffee and Walden (2010), we were to assume that they were increasing in the number of defaults, the clustered structure would be less attractive but our qualitative results would be similar. The case where the bankruptcy costs are independent of the number of bank defaults is an interesting benchmark.
A Derivation of sufficiency of condition (4)

To ensure that bankruptcy only occurs when all projects in a bank’s portfolio return \( R_L \) for any \( \ell_i = 0, \ldots, 5 \), we need to show that there exists a value of \( r \) in the interval \( [r_F^{-2}, \frac{\ell_i R_L + R_H}{1 + \ell_i}] \) that satisfies the investors’ participation constraint (1). Substituting \( \Pr(X_i < r) = (1 - p)^{1 + \ell_i} \) and \( \Pr(X_i \geq r) = 1 - (1 - p)^{1 + \ell_i} \) into (1), this requires

\[
(1 - (1 - p)^{1 + \ell_i}) \frac{\ell_i R_L + R_H}{1 + \ell_i} + (1 - p)^{1 + \ell_i} \alpha R_L \geq r_F^2
\]

for any \( \ell_i = 0, \ldots, 5 \). To show that (4) is sufficient for (18) to hold, we show that the left hand side of (18) is decreasing in \( \ell_i \) for \( \ell_i = 0, \ldots, 5 \). To see this, we differentiate the left hand side of (18) with respect to \( \ell_i \) and obtain

\[
\frac{(1 - (1 - p)^{1 + \ell_i})}{1 + \ell_i} \left[ R_L - \frac{(\ell_i R_L + R_H)}{1 + \ell_i} \right] + (1 - p)^{1 + \ell_i} \log(1 - p) \left[ \alpha R_L - \frac{(\ell_i R_L + R_H)}{1 + \ell_i} \right]
\]

\[
\leq \left[ \frac{(1 - (1 - p)^{1 + \ell_i})}{1 + \ell_i} + (1 - p)^{1 + \ell_i} \log(1 - p) \right] \left[ R_L - \frac{(\ell_i R_L + R_H)}{1 + \ell_i} \right].
\]

It is sufficient that the last expression is negative for any \( \ell_i = 0, \ldots, 5 \). To see this is the case, initially consider the first term \( \frac{(1 - (1 - p)^{1 + \ell_i})}{1 + \ell_i} + (1 - p)^{1 + \ell_i} \log(1 - p) \). Its value is 0 when it is evaluated at \( p = 0 \). Differentiating it with respect to \( p \) gives

\[-(1 + \ell_i)(1 - p)\ell_i \log(1 - p) > 0\]

for any \( p \in (0, 1) \). This guarantees that the first term is positive for any \( \ell_i = 0, \ldots, 5 \). The second term is \( R_L - \frac{(\ell_i R_L + R_H)}{1 + \ell_i} < 0 \) since \( R_H > R_L \). Together, these imply that the right hand side of (19) is negative and hence also that the left hand side of (18) is decreasing in \( \ell_i \) as required. It is then sufficient to assume that (18) holds for \( \ell_i = 5 \) to ensure that it holds for any other \( \ell_i \). □

B Derivation of (12) and (13)

Recall first that banks’ portfolio composition in both the clustered and the unclustered networks are as given in Figure 1. Applying the Law for the Probability of Union of Sets\(^2\) to (11) and taking into account that bank \( i \) portfolio returns \( X_i = R_L \) when all three

\(^2\)This states that \( \Pr(\cup_{i=1}^{n} A_i) = \Sigma \Pr(A_i) - \Sigma_{ij} \Pr(A_i \cap A_j) + \Sigma_{ijk} \Pr(A_i \cap A_j \cap A_k) - \ldots + (-1)^{m}\Pr(\cap A_i) \), for any set of events \( A_i \) (see http://mathworld.wolfram.com/Probability.html).
projects in its portfolio return \( R_L \), we obtain

\[
1 - q(C) = 6 \Pr \left[ \bigcap_{i=1}^{3} (\theta_i = R_L) \right] - 6 \Pr \left[ \bigcap_{i=1}^{3} (\theta_i = R_L) \right] + 9 \Pr \left[ \bigcap_{i=1}^{6} (\theta_i = R_L) \right] \\
+ 2 \Pr \left[ \bigcap_{i=1}^{3} (\theta_i = R_L) \right] + 18 \Pr \left[ \bigcap_{i=1}^{6} (\theta_i = R_L) \right] \\
- \left( \begin{array}{c} 6 \\ 4 \end{array} \right) \Pr \left[ \bigcap_{i=1}^{6} (\theta_i = R_L) \right] + \left( \begin{array}{c} 6 \\ 5 \end{array} \right) \Pr \left[ \bigcap_{i=1}^{6} (\theta_i = R_L) \right] - \left( \begin{array}{c} 6 \\ 6 \end{array} \right) \Pr \left[ \bigcap_{i=1}^{6} (\theta_i = R_L) \right]
\]

\[= 2 \Pr \left[ \bigcap_{i=1}^{3} (\theta_i = R_L) \right] - \Pr \left[ \bigcap_{i=1}^{6} (\theta_i = R_L) \right]. \tag{20}\]

in the clustered network, and

\[
1 - q(U) = 6 \Pr \left[ \bigcap_{i=1}^{3} (\theta_i = R_L) \right] - 6 \Pr \left[ \bigcap_{i=1}^{4} (\theta_i = R_L) \right] + 6 \Pr \left[ \bigcap_{i=1}^{5} (\theta_i = R_L) \right] \\
+ 3 \Pr \left[ \bigcap_{i=1}^{6} (\theta_i = R_L) \right] + 6 \Pr \left[ \bigcap_{i=1}^{6} (\theta_i = R_L) \right] + 14 \Pr \left[ \bigcap_{i=1}^{6} (\theta_i = R_L) \right] \\
- \left( \begin{array}{c} 6 \\ 4 \end{array} \right) \Pr \left[ \bigcap_{i=1}^{6} (\theta_i = R_L) \right] + \left( \begin{array}{c} 6 \\ 5 \end{array} \right) \Pr \left[ \bigcap_{i=1}^{6} (\theta_i = R_L) \right] - \left( \begin{array}{c} 6 \\ 6 \end{array} \right) \Pr \left[ \bigcap_{i=1}^{6} (\theta_i = R_L) \right]
\]

\[= 6 \Pr \left[ \bigcap_{i=1}^{3} (\theta_i = R_L) \right] - 6 \Pr \left[ \bigcap_{i=1}^{4} (\theta_i = R_L) \right] + \Pr \left[ \bigcap_{i=1}^{6} (\theta_i = R_L) \right]. \tag{21}\]

in the unclustered network. It remains to show that

\[
\Pr \left[ \bigcap_{i=1}^{n} (\theta_i = R_L) \right] = \sum_{n=3}^{\infty} \frac{(6-n)}{26^{n}}. \tag{22}\]

for any \( n \in \{3, 4, 6\} \) where \( m \leq 6 \) is the number of projects returning \( R_L \) in the combination \( mR_L, (6 - m)R_H \). To see this, we first make use of the Law of Total Probabilities\(^3\) and

\[^3\text{This states that given } n \text{ mutually exclusive events } A_1, A_2, ..., A_n \text{ with probabilities summing to 1, then } \Pr(B) = \sum_{i=1}^{n} \Pr(B/A_i) \Pr(A_i) \text{ where } B \text{ is an arbitrary event (see http://mathworld.wolfram.com/TotalProbabilityTheorem.html).}\]
obtain

\[
\Pr \left[ \bigcap_{i=1}^{n} (\theta_i = R_L) \right] = \sum_{m=0}^{6} \Pr \left[ \bigcap_{i=1}^{n} (\theta_i = R_L) \ \bigg| \ (mR_L, (6-m)R_H) \right] \Pr(mR_L, (6-m)R_H)
\]

\[
= \sum_{m=n}^{6} \Pr \left[ \bigcap_{i=1}^{n} (\theta_i = R_L) \ \bigg| \ (mR_L, (6-m)R_H) \right] \Pr(mR_L, (6-m)R_H)
\]

once we take into account \( \Pr \left[ \bigcap_{i=1}^{n} (\theta_i = R_L) \ \bigg| \ (mR_L, (6-m)R_H) \right] = 0 \) for any \( m < n \).

Then, for each combination of projects \( mR_L, (6-m)R_H \), there are \( \binom{6-n}{6-m} \) ways of selecting \( n \leq m \) projects that return \( R_L \). This implies that

\[
\Pr \left[ \bigcap_{i=1}^{n} (\theta_i = R_L) \ \bigg| \ (mR_L, (6-m)R_H) \right] = \frac{\binom{6-n}{6-m}}{6^m}.
\]

Since \( \Pr(mR_L, (6-m)R_H) = \frac{\binom{6}{m}}{2^6} \), (22) follows immediately. Using (22) in (20) and (21) gives (12) and (13).

C Proofs of Propositions

Proof of Proposition 1. Given that condition (4) implies that bankruptcy only occurs when all projects in a bank’s portfolio return \( R_L \), a bank’s expected profit (3) with \( \ell = 2 \) simplifies to

\[
\pi_i(g) = E(X_i) - r_F^2 - (1-p)^3(1-\alpha)R_L - 2c.
\]

To show pairwise stability, we first consider severing a link. Suppose that bank 1 severs the link with bank 3 so that its portfolio is now \( \frac{2}{3}\theta_1 + \frac{1}{3}\theta_2 \) and its profit is

\[
\pi_1(g - \ell_{13}) = E(X_i) - r_F^2 - (1-p)^2(1-\alpha)R_L - c.
\]

Bank 1 does not deviate if \( \pi_i(g) \geq \pi_1(g - \ell_{13}) \), which is satisfied for \( c \leq p(1-p)^2R_L \).

Suppose now that bank 1 adds a link with bank 4 so that its portfolio is now \( \frac{1}{6}\theta_1 + \frac{1}{3}\theta_2 + \frac{1}{3}\theta_3 + \frac{1}{6}\theta_4 \) and its profit is

\[
\pi_1(g + \ell_{14}) = E(X_i) - r_F^2 - (1-p)^4(1-\alpha)R_L - 3c.
\]
when bankruptcy occurs when all projects pay off $R_L$. If bankruptcy occurs more often than this, the expected profit from the deviation will be lower. Thus, it is sufficient for the deviation not to be profitable that $\pi_i(g) \geq \pi_1(g + \ell_{14})$ which requires $c \geq p(1-p)^3(1-\alpha)R_L$. Since all banks are symmetric, this shows that $\ell^* = 2$ is a pairwise stable equilibrium for the range of $c$ given in the proposition.

To see that $\ell^* = 2$ is the Pareto dominant equilibrium it is sufficient to show that the bank’s expected profit is highest in this case since the investors always obtain their opportunity cost. First note that (5) is concave in $\ell$. Combining this with the condition that $c$ lies in the range given in the proposition, it follows that a bank’s expected profit in the equilibrium with $\ell^* = 2$ is greater than in either the equilibrium with $\ell^* = 1$ or $\ell^* = 3$ or any other equilibrium. □

**Proof of Proposition 3.** We proceed in two steps. First, we find the minimum value of $\alpha$ as a function of the short term risk free rate $r^2_f$ in each interval of the bank’s portfolio return $X_i$ such that investors’ participation constraint (7) is satisfied for a feasible promised repayment $\rho^B_{12}(C)$. Second, we compare the functions representing the minimum values of $\alpha$ found in the first step to find the equilibrium value of $\rho^B_{12}(g)$.

**Step 1.** We start by determining the minimum value of $\alpha$ such that (7) is satisfied for $\rho^B_{12}(C) \in [r^2_f, \frac{2R_L + R_H}{3}]$. Substituting $\rho^B_{12}(C) = \frac{2R_L + R_H}{3}$ in (7) and using the distribution probability $\Pr(X_i = x|B)$ as in Table 3, we obtain

$$\frac{7}{15} \frac{2R_L + R_H}{3} + \frac{8}{15} R_L = r^2_f,$$

from which

$$\alpha_{LOW}(C) = \frac{45r^2_f - 7(2R_L + R_H)}{24R_L}.$$

This implies that for any $\alpha \geq \alpha_{LOW}(C)$, there exists a value of $\rho^B_{12}(C) \in [r^2_f, \frac{2R_L + R_H}{3}]$ such that investors roll over their debt. Analogously, for $\rho^B_{12}(C) \in [\frac{2R_L + R_H}{3}, R_H]$, we obtain

$$\frac{4}{15} \frac{R_L + 2R_H}{3} + \alpha \left( \frac{8}{15} R_L + \frac{3}{15} \frac{2R_L + R_H}{3} \right) = r^2_f,$$

from which

$$\alpha_{MID}(C) = \frac{45r^2_f - 4R_L - 8R_H}{3(10R_L + R_H)}.$$

Finally, for $\rho^B_{12}(C) \in [\frac{R_L + 2R_H}{3}, R_H]$ we obtain

$$\frac{1}{15} R_H + \alpha \left( \frac{8}{15} R_L + \frac{3}{15} \frac{2R_L + R_H}{3} + \frac{3}{15} \frac{R_L + 2R_H}{3} \right) = r^2_f.$$
from which
\[
\alpha_{\text{HIGH}}(C) = \frac{15r_f^2 - R_H}{11R_L + 3R_H}.
\]

The interpretation of \(\alpha_{\text{MID}}(C)\) and \(\alpha_{\text{HIGH}}(C)\) is the same as the one for \(\alpha_{\text{LOW}}(C)\).

**Step 2.** To find the equilibrium value of \(\rho_{12}^B(C)\) defined as the minimum promised repayment that satisfies (7), we now compare the functions \(\alpha_{\text{LOW}}(C)\), \(\alpha_{\text{MID}}(C)\) and \(\alpha_{\text{HIGH}}(C)\). We then obtain:

\[
\alpha_{\text{MID}}(C) - \alpha_{\text{LOW}}(C) = \frac{7R_H^2 + 20R_H R_L + 108R_L^2 - 45r_f^2(2R_L + R_H)}{24R_L(10R_L + R_H)}.
\]

We note that \(\alpha_{\text{MID}}(C) - \alpha_{\text{LOW}}(C)\) is positive for \(r_f^2 < \tau_f^2 = \frac{7R_H^2 + 20R_H R_L + 108R_L^2}{45(2R_L + R_H)} < \frac{5R_L + R_H}{6}\), and negative otherwise. Similarly, it can be shown that \(\alpha_{\text{HIGH}}(C) - \alpha_{\text{MID}}(C) > 0\) for any \(r_f^2 \in [\tau_f^2, \frac{5R_L + R_H}{6}]\) and \(R_H > \frac{13}{12} R_L\), while \(\alpha_{\text{HIGH}}(C) - \alpha_{\text{LOW}}(C) > 0\) for any \(r_f^2 \in [R_L, \tau_f^2]\). Given that in equilibrium the bank offers the minimum level of \(\rho_{12}^B(C)\) that satisfies (7), the proposition follows. □

**Proof of Proposition 4.** We proceed in two steps as in the proof of Proposition 3.

**Step 1.** We determine first the minimum value of \(\alpha\) such that (7) is satisfied for \(\rho_{12}^B(U) \in [r_f^2, \frac{2R_L + R_H}{3}]\). Substituting \(\rho_{12}^B(U) = \frac{2R_L + R_H}{3}\) in (7) and using the distribution probability \(\Pr(X_i = x|B)\) as in Table 4, we obtain

\[
\frac{17}{25} \frac{2R_L + R_H}{3} + \frac{8}{25} R_L = r_f^2,
\]

from which

\[
\alpha_{\text{LOW}}(U) = \frac{75r_f^2 - 17(2R_L + R_H)}{24R_L}.
\]

As before, this implies that for any \(\alpha \geq \alpha_{\text{LOW}}(U)\), there exists a value of \(\rho_{12}^B(U) \in [r_f^2, \frac{2R_L + R_H}{3}]\) such that investors roll over their debt. Analogously, for \(\rho_{12}^B(U) \in [\frac{2R_L + R_H}{3}, \frac{R_L + 2R_H}{3}]\) and \(\rho_{12}^B(U) \in [\frac{R_L + 2R_H}{3}, R_H]\), respectively, we obtain

\[
\frac{6}{25} \frac{R_L + 2R_H}{3} + \alpha(\frac{8}{25} R_L + \frac{112R_L + R_H}{25} \frac{3}{3}) = r_f^2
\]

from which

\[
\alpha_{\text{MID}}(U) = \frac{75r_f^2 - 6(R_L + 2R_H)}{46R_L + 11R_H};
\]

and

\[
\frac{1}{25} R_H + \alpha(\frac{8}{25} R_L + \frac{112R_L + R_H}{25} \frac{3}{3} + \frac{5}{25} R_L + \frac{2R_H}{3}) = r_f^2
\]
from which
\[ \alpha_{\text{HIGH}}(U) = \frac{25r_f^2 - RH}{17RL + 7RH}. \]

**Step 2.** We now compare the functions \( \alpha_{\text{LOW}}(U), \alpha_{\text{MID}}(U) \) and \( \alpha_{\text{HIGH}}(U) \) to find the equilibrium value of \( \rho_{12}^B(C) \). After some algebraic manipulation it is possible to see that \( \alpha_{\text{LOW}}(U) < \alpha_{\text{MID}}(U) < \alpha_{\text{HIGH}}(U) \) for any \( r_f^2 \in [RL, \frac{5RL + RH}{6}] \). Thus, the proposition follows given that the bank always offers investors the minimum total repayment that satisfies (7). \( \square \)

**Proof of Proposition 5.** The proposition follows immediately from the comparison of total welfare in the two networks in the different regions. We analyze each region in turn.

**Region A.** For \( \alpha \geq \alpha_{\text{LOW}}(C) > \alpha_{\text{LOW}}(U) \), (7) is satisfied for \( \rho_{12}^B(g) \in [r_f^2, \frac{2RL + RH}{3}] \) and investors roll over the debt in both networks. Given this, from (16) total welfare is given by
\[
W(g) = \frac{RL + RH}{2} - \frac{8}{64}(1 - \alpha)RL - 2c
\]
for \( g = U, C \) as a bank’s expected probability of default at date 2 is the same in the two structures.

**Region B.** For \( \alpha_{\text{LOW}}(C) > \alpha \geq \alpha_{\text{MID}}(C) > \alpha_{\text{LOW}}(U) \), (7) is satisfied for \( \rho_{12}^B(C) \in [\frac{2RL + RH}{3}, \frac{RL + 2RH}{3}] \) in the clustered structure and for \( \rho_{12}^B(U) \in [r_f^2, \frac{2RL + RH}{3}] \) in the unclustered structure. Investors roll over the debt in both networks but the bank default probabilities now differ in the two structures. From (16) and Table 3, total welfare in the clustered structure is given by
\[
W(C) = \frac{RL + RH}{2} - \frac{15}{64}(1 - \alpha)\left[\frac{8}{15}RL + \frac{3}{15}\frac{2RL + RH}{3}\right] - 2c,
\]
and by (23) in the unclustered structure. It follows immediately that \( W(U) > W(C) \).

**Regions C1 and C2.** For \( \alpha_{\text{MID}}(C) > \alpha \geq \alpha_{\text{LOW}}(U) \), (7) cannot be satisfied for any \( \rho_{12}^B(C) \leq X_i \) in the clustered structure, whereas it is still satisfied for \( \rho_{12}^B(U) \in [r_f^2, \frac{2RL + RH}{3}] \) in the unclustered structure. Thus, the bank is liquidated and, from (17), total welfare in the clustered structure is now equal to
\[
W(C) = \frac{49}{64}\left[\frac{21}{49}\frac{2RL + RH}{3} + \frac{21}{49}\frac{RL + 2RH}{3} + \frac{7}{49}RH\right] + \frac{15}{64}r_f^2 - 2c,
\]
whereas \( W(U) \) is still given by (23) in the unclustered structure.

Comparing \( W(C) \) and \( W(U) \) gives
\[
W(U) - W(C) = \frac{1}{64}[4RH + (3 + 8\alpha)RL - 15r_f^2].
\]
Equating this to zero and solving for $\alpha$ as a function of $r_f^2$ gives the boundary between Regions C1 and C2:

$$\alpha_W = \frac{15r_f^2 - 3RL - 4RH}{8RL}.$$  

It can be seen that $W(U) > W(C)$ for $\alpha > \alpha_W$ and $W(U) < W(C)$ for $\alpha < \alpha_W$.

Region D. For $\alpha < \alpha_{LOW}(U)$, (7) cannot be satisfied for any $\rho_{12}^B(g) \leq X_i$ so that banks are early liquidated in both structures. Total welfare is still as in (24) in the clustered structure, while, from (17), it equals

$$W(U) = \frac{39}{64} \left[ \frac{13}{39} 2RL + RH + \frac{19}{39} RL + 2RH + \frac{7}{39} RH \right] + \frac{25}{64} r_f^2 - 2c$$

in the unclustered structure. The difference between the two expressions is given by

$$W(C) - W(U) = \frac{1}{32} (2RH + 3RL + -5r_f^2),$$

which is positive for any $r_f^2 \in [RL, \frac{5RL + RH}{6}]$. □

**Proof of Proposition 6.** The proposition follows immediately from the comparison of the bank expected profits $\pi_i(g)$ and $\pi_i^{LT}$ in the two structures with short and long term debt. The expression for $\pi_i(g)$ is given by (9) or (10) depending on investors’ rollover decisions while $\pi_i^{LT}$ is always given by (5) with $\ell_i = 2$. Consider the clustered structure first and the regions for investors’ rollover decision in Proposition 3. In Region A, investors roll over the debt for a repayment $\rho_{12}^B(g) \in [r_f^2 \frac{2RL + RH}{3}, \frac{2RL + 2RH}{3}]$ so that $\pi_i(C)$ is given by (9) with $q(C) = \frac{49}{64}$ and $E(X_i < \rho_{12}^B(C)|B) = \frac{8}{15} RL$ using the conditional probabilities Pr$(X_i = x|B)$ as in Table 2. After some simplifications, we obtain

$$\pi_i(C) - \pi_i^{LT} = r_f^2 - r_f^2$$

from which $\pi_F^2(C) = r_f^2$ for all values of $r_f^2$ and $\alpha$ in Region A. In Region B investors still roll over their debt but for $\rho_{12}^B(g) \in [\frac{2RL + RH}{3}, \frac{RL + 2RH}{3}]$. The profit $\pi_i(C)$ still comes from (9) but now $E(X_i < \rho_{12}^B(C)|B) = \frac{R_L}{15} + \frac{3}{15} \frac{2RL + RH}{3}$. Then,

$$\pi_i(C) - \pi_i^{LT} = r_f^2 - r_f^2 + \frac{1}{64} (1 - \alpha) \frac{2RL + RH}{3}$$

from which $\pi_F^2 = r_f^2 + \frac{1}{64} (1 - \alpha) \frac{2RL + RH}{3}$ for all values of $r_f^2$ and $\alpha$ in Region B. In Region C+D investors no longer roll over their debt. The expression for $\pi_i(C)$ is now given by (10) with $q(C) = \frac{49}{64}$ and $E(X_i > r_f^2|G) = \frac{21}{49} \frac{2RL + RH}{3} + \frac{21}{49} \frac{RL + 2RH}{3} + \frac{7}{49} RH$ using the conditional
probabilities $\Pr(X_i = x|G)$ as in Table 2. After some simplifications, we obtain

$$\pi_i(C) - \pi_i^{LT} = r_F^2 - \frac{49}{64} r_j^2 + \frac{1}{64} (3R_L + 4R_H) + \frac{8}{64} \alpha R_L$$

from which $r_F^2 = \frac{49}{64} r_j^2 + \frac{1}{64} (3R_L + 4R_H) + \frac{8}{64} \alpha R_L$ for all values of $r_j^2$ and $\alpha$ in Regions C+D.

Consider now the unclustered structure. From Proposition 4 in Regions A+B+C investors roll over their debt and $\pi_i(U)$ is given by (9) with $q(U) = \frac{39}{64}$ and $E(X_i < \rho_{12}^B(U)|B) = \frac{8}{25} R_L$ using again the conditional probabilities $\Pr(X_i = x|B)$ as in Table 2. We then have

$$\pi_i(U) - \pi_i^{LT} = r_F^2 - r_j^2$$

from which $r_F^2(C) = r_j^2$ for all values of $r_j^2$ and $\alpha$ in Regions A+B+C. In Region D the bank is early liquidated so that $\pi_i(U)$ is given by (10) with $q(U) = \frac{39}{64}$ and $E(X_i > r_j^2|G) = \frac{13}{39} \frac{2R_L + R_H}{3} + \frac{19}{39} \frac{R_L + 2R_H}{3} + \frac{7}{39} R_H$. Then,

$$\pi_i(U) - \pi_i^{LT} = r_F^2 - \frac{39}{64} r_j^2 + \frac{1}{64} (9R_L + 8R_H) + \frac{8}{64} \alpha R_L$$

from which $r_F^2 = \frac{39}{64} r_j^2 + \frac{1}{64} (9R_L + 8R_H) + \frac{8}{64} \alpha R_L$ for all values of $r_j^2$ and $\alpha$ in Region D. The proposition follows. □
References


He, Z., and W. Xiong, 2009, Dynamic Bank Runs, working paper, University of Chicago.


Table 1: Project realizations and bank $i$’s portfolio return distribution.

<table>
<thead>
<tr>
<th>Project realizations</th>
<th>Number of states</th>
<th>Bank $i$’s return $X_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_L$</td>
<td>$\frac{2R_L+R_H}{3}$</td>
</tr>
<tr>
<td>1 6$R_H$</td>
<td>$\binom{6}{0} = 1$</td>
<td>0</td>
</tr>
<tr>
<td>2 $R_L, 5R_H$</td>
<td>$\binom{6}{1} = 6$</td>
<td>0</td>
</tr>
<tr>
<td>3 2$R_L, 4R_H$</td>
<td>$\binom{6}{2} = 15$</td>
<td>0</td>
</tr>
<tr>
<td>4 3$R_L, 3R_H$</td>
<td>$\binom{6}{3} = 20$</td>
<td>1</td>
</tr>
<tr>
<td>5 4$R_L, 2R_H$</td>
<td>$\binom{6}{4} = 15$</td>
<td>3</td>
</tr>
<tr>
<td>6 5$R_L, R_H$</td>
<td>$\binom{6}{5} = 6$</td>
<td>3</td>
</tr>
<tr>
<td>7 6$R_L$</td>
<td>$\binom{6}{6} = 1$</td>
<td>1</td>
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<tr>
<td>Total</td>
<td>64</td>
<td>8</td>
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</table>
Table 2: Conditional distribution of bank $i$’s portfolio returns in the \textit{clustered network}

<table>
<thead>
<tr>
<th>Project realizations</th>
<th>Default states $s = B$</th>
<th>Bank $i$’s return $X_i$ given $s = B$</th>
<th>No default states</th>
<th>Bank $i$’s return $X_i$ given $s = G$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$s = B$</td>
<td>$R_L$</td>
<td>$\frac{2R_L + R_H}{3}$</td>
<td>$\frac{R_L + 2R_H}{3}$</td>
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<td>1</td>
<td>6$R_H$</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>$R_L, 5R_H$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>2$R_L, 4R_H$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>3$R_L, 3R_H$</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>4$R_L, 2R_H$</td>
<td>6</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>5$R_L, R_H$</td>
<td>6</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>6$R_L$</td>
<td>1</td>
<td>1</td>
<td>0</td>
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<tr>
<td><strong>Total</strong></td>
<td>15</td>
<td>8</td>
<td>3</td>
<td>3</td>
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</table>
Table 3: Conditional distribution of bank $i$’s portfolio returns in the *unclustered* network

<table>
<thead>
<tr>
<th>Project realizations</th>
<th>Default states $s = B$</th>
<th>Bank $i$’s return $X_i$ given $s = B$</th>
<th>No default states $s = G$</th>
<th>Bank $i$’s return $X_i$ given $s = G$</th>
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<tr>
<td></td>
<td>$R_L$  $\frac{2R_L+R_H}{3}$ $\frac{R_L+2R_H}{3}$ $R_H$</td>
<td>$R_L$  $\frac{2R_L+R_H}{3}$ $\frac{R_L+2R_H}{3}$ $R_H$</td>
<td>$R_L$  $\frac{2R_L+R_H}{3}$ $\frac{R_L+2R_H}{3}$ $R_H$</td>
<td></td>
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<td>1  $6R_H$</td>
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<td>1</td>
<td>0 0 0 0 1</td>
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<tr>
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<td>0 0 0 0 0</td>
<td>6</td>
<td>0 0 3 3 3</td>
</tr>
<tr>
<td>3  $2R_L, 4R_H$</td>
<td>0</td>
<td>0 0 0 0 0</td>
<td>15</td>
<td>0 3 9 3 3</td>
</tr>
<tr>
<td>4  $3R_L, 3R_H$</td>
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<td>14</td>
<td>0 7 7 0 0</td>
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<td>5  $4R_L, 2R_H$</td>
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<td>3</td>
<td>0 3 0 0 0</td>
</tr>
<tr>
<td>6  $5R_L, R_H$</td>
<td>6</td>
<td>3 3 0 0 0</td>
<td>0</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>7  $6R_L$</td>
<td>1</td>
<td>1 0 0 0 0</td>
<td>0</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>Total</td>
<td>25</td>
<td>8 11 5 1 1</td>
<td>39</td>
<td>0 13 19 7 7</td>
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