ESTIMATING DIVISIONAL COST OF CAPITAL
FOR INSURANCE COMPANIES

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Introduction

Two crucial issues in the financial management of insurance companies are:

- What should the rate of return on insurance companies’ equity be in a perfectly competitive market?
- What premiums should insurance companies charge?

The standard approaches to these issues are:

Insurance Capital Asset Pricing Model:

- A descriptive model of the insurance firm allows an expression for premiums charged to be derived as a function of the rate of return on equity which can be found using the CAPM.

Discounted Cash Flow Approach

- Discounts expected cash flows associated with the insurance at appropriate discount rates to give insurance premiums.
Neither of these methods is well suited to finding insurance premiums when an insurance firm has *multiple divisions*.

The problem is how the earnings on a firm's reserves should be allocated to different divisions.

In this paper a different approach is taken.

Instead of taking the structure of the insurance firm as given, a rationale for its structure is developed.

With this approach the issue of how to allocate the earnings on a firm's reserves to different divisions does not arise.
An Ideal World - A Benchmark

Two dates:

\[
\begin{array}{c|c}
0 & 1 \\
\hline
\end{array}
\]

At some point between 0 and 1 consumers face a loss \(L\) with probability \(\pi\).

There is a large number of consumers so this risk can be pooled and eliminated by an insurance company. In this case everybody bears the average loss \(EL = \pi L\) with certainty
If contracts can be costlessly enforced the following scheme could be used

\[
\begin{array}{c|c}
0 & 1 \\
\hline
\text{People sign} & \text{People that have not suffered a loss pay EL} \\
\text{insurance contracts} & \text{People that have suffered a loss receive } L - EL
\end{array}
\]

In a competitive market the insurance company has (on average) for each consumer

\[
\text{Revenue} = (1 - \pi)EL
\]

\[
\text{Costs} = \pi(L - EL) = EL - \pi EL = (1 - \pi)EL
\]

\[
\text{Profits} = 0
\]

In an ideal world of this type the insurance company does not need to be involved in financial markets.
What happens when there is insurance as above and financial markets?

0

1

Insurance

- People sign insurance contracts

Those who do not suffer a loss pay EL

Those who suffer a loss receive L - EL

Financial Markets

- People purchase optimal portfolio Z* with expected return ER*

Ex post return on portfolio is R*

Consumption

Those who do not suffer a loss consume R* - EL

Those who suffer a loss receive R* - L + L - EL = R* - EL
Consumers

- have insurance against loss L

- bear an optimal amount of investment risk

This is a useful benchmark equilibrium
The Insurance Modigliani-Miller Theorem

In practice, of course, contracts are not costlessly enforceable. It would be very difficult to make people who do not suffer a loss pay at date 1.

As a result premiums must be collected at date 0 and insurance companies must become involved in financial markets.

To illustrate one possibility suppose that insurance companies invest the premiums received at date 0 in a risk free asset with return \( r_F \). Competition ensures premiums are bid down to

\[
p_0 = \frac{EL}{1 + r_F}
\]

so insurance companies have enough to cover their liabilities but make zero profits.

What should consumers do?
They can achieve the same optimal consumption by choosing the same portfolio as before but with a reduction in their holdings of the risk free asset of $p_0$.

What is happening here is that the insurance company is simply holding the premium on behalf of the policyholders because of the enforceability problem. The policyholders adjust for this in their portfolio decisions.

What happens if the insurance company invests the premiums it receives in risky assets with a higher rate of return than the risk free asset?

Suppose it invests in a risky portfolio with expected return $E_r > r_F$ and lower bound on the return of $r_\ell$ where $0 < 1+r_\ell < 1+r_F$.

To ensure that it can meet its liabilities at date 1 it must charge a premium of

$$p_0 = \frac{EL}{1 + r_\ell} > \frac{EL}{1 + r_F}.$$
Except when the realized return is \( r_t \), the insurance company will be able to \textit{refund} part of the dividends to its policyholders.

The average effective cost of the insurance in date 0 dollars, will be

\[
c_0 = \frac{EL}{1 + Er} < \frac{EL}{1 + r_F}.
\]

The important point here is that even though the average effective premium is lowered, \textit{none of the firm's policyholders are made better off}. They will simply adjust their portfolios to offset the investments made by the insurance company.

A similar logic is possible whatever the insurance company does with the funds from the premiums provided \( r_t > 0 \).

This gives the insurance counterpart of a well-known result in corporate finance:

\textbf{The Insurance Modigliani-Miller Theorem}

If capital markets are perfect and complete and there are no taxes, an insurance company's investment strategy does not affect its policyholder's welfare.
There are a number of points here.

- The pricing of insurance is arbitrary in this situation. Lower premiums do not raise consumer welfare.

- Given this companies may as well invest in the risk free asset and set

\[
p_0 = \frac{\text{EL}}{1 + r_F}.
\]

What happens if capital markets are imperfect?

- In this case the insurance company may be able to improve consumer welfare if it can ease the effect of the capital market imperfections.
The Role of Insurance Company Equityholders

So far there have been no insurance company equityholders.

What is their role?

It was assumed above that

(i) 1 + \( r_e \) > 0;

(ii) The risk faced by consumers could be perfectly pooled by the insurance company.

If either of these assumptions is violated there will be residual risk. The role of equityholders is to bear this residual risk.

How do equityholders bear residual risk?

In an ideal world where contracts are enforceable it would be possible for them to guarantee they will make up any shortfall at date 1. In fact Lloyd's of London works in a way that is rather similar to this. "Names" pledge their wealth using unlimited liability contracts.

What determines premiums and the average cost of insurance in this case?
The Market for Residual Risk

Suppose equityholders bear residual risk $\epsilon$ with mean $\mu_\epsilon$ and variance $\sigma_\epsilon^2$.

It is shown in the Appendix of the paper that the amount needed to compensate somebody for bearing this residual risk is

$$y_1 = \mu_\epsilon + \Theta \sigma_\epsilon^2$$

where $\Theta$ is the price that is paid for each unit of variance borne.

Assuming that the insurance company invests premiums in the risk free asset the average cost of insurance is then

$$c_0 = \frac{EL}{1 + r_F} + \frac{\mu_\epsilon + \Theta \sigma_\epsilon^2}{1 + r_F}.$$
What is the corresponding cost of capital, i.e. the value that liabilities should be discounted at? If $\gamma$ is used to denote the corresponding cost of capital then

$$\frac{EL}{1 + \gamma} = \frac{EL}{1 + r_F} + \frac{\mu_\epsilon + \Theta \sigma_\epsilon^2}{1 + r_F}.$$
Guaranteeing Reserves

We have been assuming a Lloyd’s type system where the wealth is guaranteed at date 1. There is again an enforcement problem - it is often difficult to get people to pay up when this is necessary.

A more typical arrangement is that equityholders put up guaranteeing reserves at date 0.

These are held by the insurance company on equityholders behalf. If there is a shortfall at date 1 they are used to cover this. If there is no shortfall they can be returned to equityholders.

The analysis of premiums is unaffected by guaranteeing reserves. This leaves the fair rate of return on equity $r_E$. Suppose the guaranteeing reserves are $G$ and these are invested in the risk free asset, then

$$r_E = \frac{(1+r_F)G + \mu_\epsilon + \Theta \sigma_\epsilon^2}{G} - 1.$$
Divisional Cost of Capital

What happens when there are multiple divisions?

The concept of residual risk can be readily applied in this case.

For example, suppose that a company has two divisions. The residual risk in division \( i (= 1, 2) \) has mean \( \mu_i \) and variance \( \sigma_i^2 \). The expected loss being insured in division \( i \) is \( EL_i \).

The average cost of the insurance would be

\[
c_{0i} = \frac{EL_i}{1 + r_F} + \frac{\mu_i + \Theta \sigma_i^2}{1 + r_F}.
\]

The fair rate of return for shareholders who put up a total \( G \) of guaranteeing reserves is

\[
r_E = \frac{(1+r_F)G + \mu_1 + \Theta \sigma_1^2 + \mu_2 + \Theta \sigma_2^2}{G} - 1.
\]

The extension to the case where there are more than two divisions is straightforward.
Concluding Remarks

Standard financial analyses of the divisional cost of capital take the structure of the insurance firm as given. As a result they are not able to adequately deal with the issue of how to allocate the earnings from surplus to different divisions.

This paper starts with the benchmark case where contracts are fully enforceable. This allows the insurance market component of companies’ activities to be separated from the financial market component.

The important concept here is the market for residual risk since it is the price of residual risk and the amount of residual risk in each division that determines the compensation insurance companies’ equityholders receive.

The important issue for future research is how the price for residual risk could be found. Two ways to do this are:

- Use data from markets where residual risk is directly guaranteed such as Lloyd’s of London.

- Estimate the excess risk premium earned by insurance company equityholders and compare with the residual risk they bear.

Once an empirical estimate for the price of residual risk has been identified, it should be relatively straightforward to apply the concepts in this paper.