Exchange Rate Exposure: A Simple Model

Gordon M. Bodnar
Morris W. Ofitt Professor of International Finance
Paul H. Nitze School of Advanced International Studies
The Johns Hopkins University
1740 Massachusetts Ave NW
Washington, DC 20036
202 663 7731

and

Richard C. Marston
James R.F. Guy Professor of Finance
Wharton School
University of Pennsylvania
Philadelphia, PA 19104-6367
215 898 7626

Abstract

This study develops a model of foreign exchange exposure dependent on only three variables, the percentage of the firm’s revenues and expenses denominated in foreign currency and its profit rate. The model demonstrates that foreign exchange exposure elasticities should be largest for pure exporting and importing firms, especially those with low profit margins. Exposure elasticities should be smaller for multinational firms that match their foreign currency revenues and costs. Such operational hedges may help to explain why previous studies have found low or negligible levels of exposure when they studied the sensitivity of share prices to foreign exchange rates.

JEL Classification Code: F3

Keyword: foreign exchange exposure
1. Introduction

This paper develops a simple model to measure a firm's exposure elasticity to the exchange rate changes without the need to use stock return data. The model is that of a monopoly firm whose revenues and expenses are exposed to changes in exchange rates. It demonstrates that measured exposure elasticities are fundamentally functions of net foreign currency revenues, and profit margins. Large exposure elasticities arise in firms with large imbalances in foreign currency revenues and costs and low profit margins. In contrast, firms that develop operational hedges, in which they offset foreign currency revenues and costs, can shield themselves from the large-scale effects of exchange rate changes. In these cases exposure elasticity remains a decreasing function of the profit margin, except in the case when the proportion of revenues and costs exactly match, in which case the profit margin plays no role in the exposure elasticity.

Empirical studies of the foreign exchange rate exposure of U.S. firms (for example, Jorion (1990), Bodnar and Gentry (1993), Amihud (1994), Choi and Prasard (1995), Griffin and Stulz (2001), and Allayannis (1997)), typically find low or negligible levels of exposure for most firms, even when the firms examined have significant foreign operations. This has been considered somewhat of a puzzle. None of these studies are based explicitly on a model of firm behavior, so it is difficult to interpret their findings of low exposure in terms of economic behavior1. When calibrated for various operational conditions, this model provides a possible explanation why many of these studies have had difficulty documenting significant exposure elasticities from equity returns.

---

1 Bodnar, Dumas, and Marston (2002) provide an explicit theoretical model and they find relatively high levels of exposure. But their model is estimated for a group of Japanese firms that have been chosen because they are likely to have high levels of exposure, since their purpose is to investigate the link between pass-through and exposure behavior in firms that have high levels of exposure. Other theoretical studies of exposure include Adler and Dumas
2. Model Basics

To develop a measure of foreign exchange exposure, we need to start with an operational definition of a firm’s value. As is common in finance, we assume that the value of a firm can be expressed in terms of a stream of present and future cash flows as

\[ V = \sum_{t=1}^{\infty} \frac{CF_t}{(1 + \rho)^t} \]  

(1)

where \( CF_t \) represents the expected cash flows of the firm at time \( t \), which are equal to after-tax profits less net investment, and where \( \rho \) is the discount rate. In order to keep the model tractable so that the effects of market structure can be examined, we assume that the net investment of the firm is equal to zero and that cash flows are expected to be constant from year to year. In that case, the present value can be written

\[ V = \frac{CF}{\rho} = \frac{(1 - \tau)}{\rho} \pi \]  

(2)

where \( \tau \) is the corporate tax rate and \( \pi \) is profit before taxes. The basic measure of foreign exchange exposure is \( dV/dS \), where \( S \) is the exchange rate expressed as home currency/foreign currency. This measures the nominal value (in home currency) that is exposed to the exchange rate. With taxes and the discount rate constant, foreign exchange exposure can be expressed as:

\[ dV/dS = \left[ (1 - \tau)/\rho \right] d\pi/dS \]  

(3)

---

Exchange exposure, measured as a dollar amount, is proportional to the derivative of current profits with respect to the exchange rate. It is this latter derivative, $d\pi/dS$ that will be measured explicitly below in a more detailed setting.

3. Exchange Rate Exposure

3.1 Nominal Value Exposure

To investigate foreign exchange exposure across a variety of firms, we must adopt a model of the firm that is versatile enough to take into account different forms of exposure. Exchange rates may affect a firm through a variety of channels:

(i) a firm may produce at home for export sales,

(ii) a firm may produce with or sell imported components,

(iii) a firm may produce the same product or a different product at plants abroad.

The model of the firm must be broad enough to capture all of these channels. The firm described below is a multinational firm, producing and selling at home and abroad, using both foreign and domestic components. Let’s define the following variables:

$X, X^*$ = sales of the domestic good at home and abroad, respectively.

$Z, Z^*$ = sales of the foreign-produced good at home and abroad, respectively. This good is produced by the domestic firm in a foreign plant.

$S$ = exchange rate (i.e., the dollar price of the Euro)

$C, C^*$ = cost functions, denominated in dollars and Euros, respectively, representing the cost of producing the domestic goods ($X + X^*$).

$K, K^*$ = cost functions, denominated in dollars and Euros, respectively, representing the cost of producing the foreign goods ($Z + Z^*$).
Combining these variables, we can state the profits of the firm as net revenues earned by selling both domestic products and foreign products:

\[
\pi = XD(X) + S X^* D^*(X^*) - C(X + X^*) - SC^*(X + X^*)
\]

\[
+ ZF(Z) + S Z^* F^*(Z^*) - K(Z + Z^*) - SK^*(Z + Z^*)
\]  

(4)

Because the firm is producing two products for two different markets, there are four first-order conditions for this firm to be a profit maximizer. Taking the derivative of profits with respect to the choice of each quantity gives us the following four relations:

\[
\frac{d\pi}{dX} = D(X) + XD_0(X) - C_0(X + X^*) - S C^*_0(X + X^*) = 0
\]  

(5a)

\[
\frac{d\pi}{dX^*} = S D^*(X^*) + S X^* D^*_0(X^*) - C_0(X + X^*) - S C^*_0(X + X^*) = 0
\]  

(5b)

\[
\frac{d\pi}{dZ} = F(Z) + ZF_0(Z) - K_0(Z + Z^*) - S K^*_0(Z + Z^*) = 0
\]  

(5c)

\[
\frac{d\pi}{dZ^*} = S F^*(Z^*) + S Z^* F^*_0(Z^*) - K_0(Z + Z^*) - S K^*_0(Z + Z^*) = 0
\]  

(5d)

In these expressions, the derivative of each function with respect to output is denoted by a subscript 0.

The exchange rate exposure of the firm is proportional to the derivative of the profit equation with respect to the exchange rate as denoted in equation (3):

\[
d\pi/dS = \left[ d\pi/dX \right] (dX / dS) + \left[ d\pi/dX^* \right] (dX^* / dS) + \left[ d\pi/dZ \right] (dZ / dS) + \left[ d\pi/dZ^* \right] (dZ^* / dS)
\]

\[
+ \{ [X^* D^*(X^*) - C^*(X + X^*)] + [Z^* F^*(Z^*) - K^*(Z + Z^*)] \}
\]  

(6)

According to this expression, there are two ways that profits adjust to changes in the exchange rate. First, output can change in response to the change in the exchange rate (these are the \( dX/dS \) (\( dX^*/dS \)) and \( dZ/dS \) (\( dZ^*/dS \)) terms on the top line of (6)). Second, profits adjust in proportion to the initial level of net revenue denominated in foreign currency (this is the effect on the second line of (6)).

5
This expression can be simplified by recognizing that the first order condition given in (5) require the response of profits to changes in the various outputs to be equal to zero at the current equilibrium. Thus if the firm was initially at a profit maximizing point, only the direct effect of the exchange rate on profits remains.\footnote{This result is discussed in detail in Marston (2001). As discussed in Marston, this result continues to hold under some but not all forms of oligopoly. For a more general discussion of the envelope theorem on which this analysis is based, see Varian (1992, pp. 490-492).} The means the expression for exposure simplifies to:

\[
\frac{d\pi}{dS} = \left\{ \left[ X^*D^*(X^*) - C^*(X + X^*) \right] + \left[ Z^*F^*(Z^*) - K^*(Z + Z^*) \right] \right\} \]  

(7)

The exchange rate exposure of the firm is equal to the initial level of net revenue denominated in foreign currency. This measure of exposure is proportional to the nominal value exposure, \(dV/dS\), as it measures the amount of current period cash flow exposed to the exchange rate. Under simple assumptions about future cash flow expectations, this exposure measure will relate to the nominal value exposure by the factor \((1-\tau)/\rho\).

3.2 Exposure in Elasticity Form

While \(dV/dS = ((1-\sigma)/\rho) d\pi/dS\) is the actual measure of cash flow exposure, the empirical literature on exposure typically measures the exposure elasticity (i.e., the percentage change in firm value (return) per percentage change in the exchange rate) rather than the nominal value exposure. The exposure elasticity, which we define as delta, \(\delta\), is given by:

\[
\delta = \frac{d \ln \pi}{d \ln S} = S \left\{ \left[ X^*D^*(X^*) - C^*(X + X^*) \right] + \left[ Z^*F^*(Z^*) - K^*(Z + Z^*) \right] \right\} / \pi 
\]

(8)
As long as the tax rate and cost of capital are assumed constant, this measure will be identical to the elasticity of the firm value with respect to the exchange rate. This expression can be simplified if we define several new variables:

\[ R = \text{total revenue} = X D(X) + S X^*D^*(X^*) + Z F(Z) + S Z^*F^*(Z^*). \]

\[ M = \text{total cost} = C(X + X^*) + S C^*(X + X^*) + K(Z + Z^*) + S K^*(Z + Z^*). \]

\[ h_1 = \text{foreign currency-denominated revenue as a percent of total revenue} = S \left[ X^*D^*(X^*) + Z^*F^*(Z^*) \right] / R \]

\[ h_2 = \text{foreign currency-denominated costs as a percent of total costs} = S \left[ C^*(X + X^*) + K^*(Z + Z^*) \right] / M \]

\[ r = \text{the profit rate (i.e., profits as a percent of total revenues)} = \pi / R \]

We can then rewrite the expression for delta in terms of \( h_1, h_2, \) and \( r \):

\[ \delta = h_1 \left( R / \pi \right) - h_2 \left( M / \pi \right) = \left( h_1 / r \right) - h_2 \left( 1 / r \right) - 1 \] (9a)

or

\[ \delta = h_1 + \left( h_1 - h_2 \right) \left( 1 / r \right) - 1 \] (9b)

In this model, delta is a function of only three variables, \( h_1, h_2, \) and \( r \). The partial derivative of delta is positive with respect to \( h_1 \), and negative with respect to \( h_2 \). These results are quite intuitive as greater foreign currency revenues lead to a greater increase in firm value in response to a depreciation of the home currency \((dS > 0)\) and greater foreign currency costs lead to a greater decrease in firm value in response to a depreciation of the home currency. The partial derivative of delta with respect to \( r \) is a negative function of the net foreign currency revenue position of the firm \((h_1 - h_2)\). The basic effect is that an increase in profit margin makes the exposure elasticity smaller in size (i.e., closer to zero). This is because an increase in profit
margin increases firm value, $V$, without changing the nominal value exposure, $dV/dS$, which is purely a function of the net foreign currency position (see (7)). As a result, the exposure elasticity, which is related to the nominal-value exposure over firm value ($(dV/dS)/V$), is smaller for more profitable firms because the increase in profit margin corresponds to a higher firm value.

4. Exposures Across Types of Firms

Despite its simplicity, the expression for delta is general enough to encompass a variety of different firms. Consider the following examples of different types of firms:

1. U.S. exporter

This firm is assumed to produce all of its output at home and to sell a fraction, $h_1$, abroad for foreign currency revenue. Moreover, all of the components needed for production are produced at home (so $h_2 = 0$). As a result, its exchange rate exposure elasticity is given by

$$\delta = h_1 / r$$

Pure exporter

For example, let's assume this firm sells forty percent its product abroad as exports for foreign currency revenues ($h_1 = 40\%$) and its profit rate is 10 \% ($r = 0.1$). Under these assumptions the firm's delta would be:

$$\delta = h_1 / r = .40/.1 = 4.0$$

A delta of 4.0 means that the firm’s profits vary four times as much in percentage terms as the exchange rate. So a 10 percent depreciation (appreciation) of the domestic currency leads to a 40 percent rise (fall) in profits for this firm. Thus a pure exporting firm is a leveraged play on exchange rate depreciation (because it has no offsetting foreign currency costs) and has a
corresponding large exposure elasticity to such an exchange rate change. This is the type of firm that is explicitly modeled in Bodnar, Dumas, and Marston (2002).

2. U.S. importer

This firm is assumed to sell nothing abroad so $h_1 = 0$, but it imports components for local production and sales from abroad so $h_2 > 0$. As a result, this firm's delta is given by

$$
\delta = -h_2 \left( \frac{1}{r} - 1 \right)
$$

Pure importer

For example, let's assume the firm imports inputs with foreign currency costs that constitute forty percent of its total costs ($h_2 = 0.4$) and its profit rate, $r$, is again 10%. In this case, the firm's delta would be:

$$
\delta = -0.4 \left( \frac{1}{0.1} - 1 \right) = -3.6
$$

Like the pure exporter, this firm has large foreign exchange exposure, but its exposure is opposite in sign to that of the exporter. It is a leveraged play on a dollar appreciation rather than on a dollar depreciation. For similar levels of foreign activity (40%) and profit margin, notice that the exposure elasticity of the importer is slightly smaller in magnitude than for the exporter. This is because the import firm is not as the leveraged a play on the exchange rate as the exporter, because the costs (the exposure for the pure importer) are smaller than the revenues (the exposure for the pure exporter) in a profitable firm.

3. U.S. multinational

A multinational firm generally both produces and sells abroad so it will have both foreign currency revenues and foreign currency costs. Thus we will have positive values for both $h_1$ and $h_2$. For example, let's assume that this firm produces goods abroad to sell to the foreign markets.
If foreign production is local then it is likely the case that \( h_1 = h_2 \). In this case, from equation 9b this firm's delta is given by:

\[
\delta = h_1
\]

**Balanced multinational firm**

More specifically, let's assume the firm produces and sells half its output abroad. With local production, half of its revenues and half of its costs will be in foreign currency, so \( h_1 = 0.5 \) and \( h_2 = 0.5 \). In this case the firm's delta would be

\[
\delta = h_1 = 0.5
\]

By producing abroad all of the products it sells in foreign markets, the foreign currency costs have created an “operational hedge” for its foreign revenues. In this case, only its profits from foreign operations remain exposed. The exposure is the percentage of profits from overseas (this turns out to be the same percentage as the revenues and costs in foreign currency). Notice that with \( \delta = 0.5 \) this multinational firm has a much smaller exposure elasticity than either of the previous two examples despite being more international (50% of sales and costs as opposed to 40% in the previous examples). This demonstrates that operational hedging can dramatically reduce the size of the estimated exchange rate exposure elasticity relative to a pure importer or pure exporter.

Let's consider another example of a multinational firm. In this case, the multinational firm has 45% of its revenues in foreign currency \( (h_1 = 0.45) \) and 50% of its costs in foreign currency \( (h_2 = 0.5) \). Once again we will assume the profit margin to be 10% \( (r = 0.1) \). In this case the firm’s delta would be:

\[
\delta = h_1 + (h_1 - h_2) ((1/r) - 1) = 0.45 + (0.45 - 0.5)((1/0.1) - 1) = 0
\]

This multinational firm has zero exposure. We would describe it as having a natural operational hedge. With its profit margin of 10%, because it has 10% more of its costs in foreign currency
than its revenues, it is naturally hedged. As a result, despite significant foreign currency activity, we would expect that this firm would not be materially affected by changes in the exchange rate.

5. Implications for Empirical Studies

It is not difficult to see that for certain operational arrangements, a multinational firm can actually have a negative exposure, in contrast to the common assumption that multinationals gain when the dollar depreciates. This model makes clear that typical multinational firms with reasonably balanced foreign currency revenues and costs will not be likely to have large exposure elasticities, despite potentially heavy foreign activity. Multinationality has commonly been a screening mechanism for identifying samples in previous empirical studies in exposure. This model provides a possible explanation as to why these studies fail to find significant exposures. In contrast, it suggests that pure exporters or importers, especially those in low profit activities should be expected to have the largest exposure elasticities. This would suggest an completely new approach to choosing firms for empirical exposure studies.

6. Conclusion

This paper develops a simple model of foreign exchange exposure to explain why previous studies have found such low exposure in most U.S. and foreign firms. The model is that of a firm that both produces and sells at home and abroad. The derived expression for foreign exchange exposure is simple enough so that exposure can be approximated easily without share price data. It requires only three informational inputs about the firm: the percentage of revenues denominated in foreign currency, the percentage of costs denominated in foreign currency, and the profit rate for foreign sales. While proxies for the first and last data are readily available in
U.S. firms’ public disclosures, information on the percentage of costs denominated in foreign currency is typically not disclosed and is available only to insiders of the firm. As a result, this model is most directly applicable to managers or other insiders wishing to gain a quick measure of their firm's sensitivity to the exchange rate. For broader research purposes, the use of this model would require the collection of information on foreign currency costs through interrogation or survey of corporate managers.

The conclusions from this model are that for many firms, especially multinational firms with significant revenues and costs in foreign currency, foreign exchange exposure may be low because the firms match their proportion of foreign currency revenues and costs. Such operational hedging is an effective way to reduce exposure to modest levels. The firms that must contend with large foreign exchange exposure are those with unbalanced revenue or cost streams. For those firms, sizable financial hedges may be necessary to moderate their exposure.
References


