

Levered Returns

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Abstract

In this paper we revisit the theoretical relation between financial leverage and stock returns in a dynamic world where both the corporate investment and financing decisions are endogenous. We find that the link between leverage and stock returns is more complex than the static textbook examples suggest and will usually depend on the investment opportunities available to the firm. In the presence of financial market imperfections leverage and investment are generally correlated so that highly levered firms are also mature firms with relatively more (safe) book assets and fewer (risky) growth opportunities. We use a quantitative version of our model to generate empirical predictions concerning the relationship between leverage and returns.

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1 Introduction

Standard finance textbooks propose a relatively straightforward link between capital structure and the expected returns on equity: increases in financial leverage directly increase the risk of the cash flows to equity holders and thus raise the required rate of return on equity. This remarkably simple idea has proved extremely powerful and has been used by countless researchers and practitioners to examine returns and measure the cost of capital across and within firms with varying capital structures.

Unfortunately, despite, or perhaps because of, its extreme clarity, this relation between leverage and returns has met with, at best, mixed empirical success. Thus, we have evidence that equity returns:

- rise with market leverage (e.g. Bhandari (1988) and Fama and French (1992))
- are insensitive or even decline with book leverage (e.g. Fama and French (1992) and George and Hwang (2007))
- are insensitive or fall with market leverage after controlling for size and book-to-market factors (e.g. Nielsen (2006) and Penman, Richardson and Tuna's (2007))

While some of these findings are often at odds with the common wisdom embedded in the standard textbook model, there is very little recent work that offers theoretical guidance to interpret them and guide future empirical research.

In this paper we begin to fill this gap by offering a new and richer view of levered returns. We suggest that the link between financial leverage and stock returns is generally complex and depends crucially on how debt is used

and on its impact on the firm's investment opportunities. Extant literature generally assumes that debt will be used to fund changes in equity, a tradition that is rooted both in the static trade off view of optimal leverage (Miller (1977)) and the Modigliani-Miller theorem decoupling the firm's investment and financing strategies.

Our analysis focuses instead on the effects of debt on the asset side of the balance sheet, as firms use debt to finance capital spending. Since this expansion naturally increases the value of assets in place to growth options it may also change the underlying business risk of the firm and thus the risk to equity holders. While these effects can be dismissed in the benchmark Modigliani-Miller setting, they become of paramount importance in the presence of financial frictions, when investment and financing strategies must be examined jointly.¹

Our theoretical results can be used to interpret the often contradictory empirical evidence about the role of leverage in determining expected returns. In a world of financial market imperfections leverage and investment are often strongly correlated. This, in turn, implies that highly levered firms are also more mature firms with (relatively safe) book assets and fewer (risky) growth opportunities. As a result, we suggest that cross-sectional studies that fail to control for the interdependence of leverage and investment decisions are unlikely to be very informative.

Clearly real life decisions by corporations will reflect both the existing textbook analysis and our new view. Nevertheless, this subtle new link between leverage and expected equity return raises serious doubts about the usefulness of the standard textbook formulas in real world applications. This

¹Complementary studies by Healy and Palepu (1990), Kaplan and Stein (1990), and Korteweg (2004) offer much evidence that changes in leverage are accompanied by significant changes in asset beta.

is particularly true when changes in the asset side of the balance sheet are important such as when making cross-sectional comparisons across firms, or when constructing the cost of capital for new projects within a firm.

We begin by constructing a simple continuous time real options model that formalizes our basic intuition and delivers closed form expressions linking equity betas and corporate decisions on investment and financing. Although stylized, the only key assumptions in this example are that debt and investment decisions are linked and, that growth options are relatively less important for large mature firms. If both assumptions are satisfied then highly levered firms will face less underlying (asset) risk and, possibly, less equity risk as well.

This simple example is very useful to develop intuition for our key insights, but it is necessarily far too stylized. Accordingly we then proceed to construct a more detailed quantitative model that inherits the key properties of our simple example, but also introduces additional features such as endogenous borrowing constraints, investment costs, and equity issues. We then use this model to show more generally how the link between expected returns and leverage arises endogenously as a result of optimal investment and financing policies of the firm and is, in general, more complex than the simple textbook formula implies.

Our quantitative model is also suitable to develop a number of empirical predictions. To accomplish this we simulate artificial panels of firms and use them as our laboratory. To test the quantitative model predictions for leverage and returns, we provide our own empirical evidence using data from the widely used CRSP/Compustat dataset. Specifically we show that simulated data from the model can successfully replicate the following stylized facts: (i) equity returns are positively related to market leverage but are insensitive

to book leverage; (ii) market leverage continues to be positively related to returns even after controlling for firm size; but, (iii) market leverage is only weakly linked to returns after controlling for book to market.

Our work is at the center of several converging lines of research. First, it builds on the growing theoretical literature that attempts to link corporate decisions to the behavior of asset returns (a partial list includes Berk, Green and Naik (1999), Gomes, Kogan, and Zhang (2003), and Carlson, Fisher and Gianmarino (2004)). From this point of view the novelty in our work is the fact that we explicitly allow for deviations of the Modigliani-Miller theorem so that corporate financing decisions will affect investment and thus asset prices.

Our paper also adds to the recent literature on dynamic models of the capital structure that attempt to link the corporate investment and leverage policies of firms (a partial list includes Hennessy and Whited (2005, 2007), Miao (2005) and Sundaresan and Wang (2006)). Here the key novelty of our work is allowing for exposure to systematic risk and our specific focus on the asset pricing implications of these models.

Our work is also related to a growing literature on dynamic quantitative models investigating the implications of firms' financing decisions on asset returns. Some recent papers along these lines include Garlappi and Yan (2007), Livdan, Sapriza and Zhang (2006), Li (2007) and Obreja (2007). Livdan, Sapriza and Zhang (2006) study the quantitative implications of firms' financing constraints and leverage in a model without default or taxes. Allowing for deviations from the Modigliani-Miller assumptions, Li (2007) focuses on the link between investment, leverage and corporate governance issues while Garlappi and Yan (2007) examine the link between distress risk and asset returns, allowing for deviations from the absolute priority rule.

Like us Obreja (2007) also investigates the link between leverage and returns but focuses instead on the role of leverage in generating the observed size and book-to-market factors in cross-sectional equity regressions. By contrast our work seeks to understand how the interaction of corporate investment and leverage decisions lead to different patterns in equity returns.

Finally, recent work by Bahmra, Kuhn, and Strebulaev (2007) and Chen (2007) also focuses on the asset pricing implications of dynamic leverage models. Both papers attempt to link optimal leverage and default decisions to the time series patterns of credit spreads and equity returns in unified setting. They both abstract from the joint investment-financing decisions and the cross-sectional implications explored in this paper.

This paper is organized as follows. Section 2 provides a simple example where we can derive in closed form the effects of endogenous leverage on expected returns. Section 3 builds on this intuition to develop our argument in a more general model where firms make joint decisions about investment, debt, and equity issues in the presence of adjustment costs to capital and leverage. Section 4 examines some of the model's quantitative implications for the cross-section returns and compares them with the empirical evidence. Finally, section 5 offers a few concluding remarks.

2 Leverage, Investment, and Returns: A Simple Example

In this section we construct a simple continuous time real options model that formalizes our basic insights and delivers closed form expressions linking expected returns and corporate decisions on investment and financing. These ideas are then integrated in the more general model developed in the next section.

2.1 Profits and Dividends

We consider the problem of value maximizing firms, indexed by the subscript i , that operate in a perfectly competitive environment. The instantaneous flow of (after tax) operating profits, Π_i , for each firm i is completely described by the expression

$$\Pi_i = (1 - \tau)X_t K_i^\alpha, 0 < \alpha < 1$$

where K_i is the productive capacity of the firm, τ is the corporate tax rate, and the variable X is an exogenous state variable that captures the state of aggregate demand (or productivity).

As usual we think of this profit function as that resulting from the determination of the optimal choices for all other (static) inputs such as labor and raw materials for example. This combination of perfect competition with decreasing returns to scale can be shown to be equivalent to that of a monopolist facing a downward sloping demand curve for its output, so that our assumptions are not too restrictive.

The state variable, X_t , is assumed to follow the stochastic process

$$dX_t/X_t = \mu dt + \sigma d\varepsilon_t$$

where we assume for simplicity that ε_t is a standard Brownian motion under a risk-neutral measure.²

2.2 Investment and Financing

A typical firm is endowed with an initial capacity K_0 and one option to expand this capacity to K_1 by purchasing additional capital in the amount $I = K_1 - K_0 > 0$. We assume that the relative price of capital goods is one

²As is well known this measure may or may not be unique depending on whether financial markets are assumed to be complete or not. At this stage however we only require the existence of one such measure.

and that there are no adjustment costs to this investment. In what follows we will say that the firm is “young” if it has not yet exercised this growth option and “mature” if this option has already been exercised.

Firms are financed with both debt and equity issues. For the purposes of our illustration we make three simplifying assumptions regarding the nature of debt financing available to firms.

- We assume that debt takes the form of a consol bond that pays a fixed coupon c_i per period for each firm i .
- As in Sundaresan and Wang (2007) we assume that new debt can only be issued to finance investment spending so that a firm is simultaneously choosing optimal investment and financing policies.
- As in Fischer, Heinkel and Zechner (1989) we assume that debt is restructured at the time of new issues. Existing debt is retired at its current market value and new debt is issued.

Given these assumptions we now denote c_i and $B(X, c_i)$ as, respectively, the flow of interest payments and the market value of debt, for a firm with productive capacity equal to K_i .

While these assumptions are convenient for the purposes of our illustration, they are not essential and are all relaxed in the more general model below. Our basic insights survive as long as at least some of the investment is financed with debt. Given the tax benefits of debt this will always be the case.

2.2.1 The Problem for Mature Firms

Given our assumptions it follows that the instantaneous dividends for the equity holders of a mature firm, with capital K_1 , are equal to

$$(1 - \tau)(X_t K_1^\alpha - c_1).$$

Given debt and its associated coupon payment, c_1 , the equity value of a mature firm, $V(X; c_1)$, satisfies the following Bellman equation

$$V(X; c_1) = (1 - \tau)(X_t K_1^\alpha - c_1) dt + (1 + r dt)^{-1} E[V(X + dX; c_1)] \quad (1)$$

Here our choice of notation, $V(X; c_1)$, emphasizes the dependence of equity value on the firm's leverage.

Equation (1) holds only as long as the firm meets its obligations to the debt holders. However, it is reasonable to assume that equity-holders will choose to close the firm and default on their debt repayments if the prospects for the firm are sufficiently bad. If equity holders have no outside options this (optimal) default occurs whenever $V(X; c_1)$ reaches zero. Formally, default occurs as soon as the value of X reaches some (endogenous) default threshold, X_{D_1} . This threshold is determined by imposing the usual value matching and smooth pasting conditions, requiring that at X_{D_1} equity value satisfies:

$$V(X_{D_1}; c_1) = 0 \quad (2)$$

$$V'(X_{D_1}; c_1) = 0 \quad (3)$$

2.2.2 The Problem for Young Firms

Young firms are similar to mature firms but, in addition, they possess an option to invest and expand their productive capacity to K_1 . For young firms the flow of operating profits (and dividends) per unit of time is then

given by the expression

$$(1 - \tau)(X_t K_0^\alpha - c_0).$$

This yields the following Bellman equation for equity value, $V(X; c_0)$:

$$V(X; c_0) = (1 - \tau)(X_t K_0^\alpha - c_0) dt + (1 + r dt)^{-1} E[V(X + dX; c_0)] \quad (4)$$

As before equation (4) holds only as long as the firm meets its obligations to the debt holders. Letting X_{D_0} denote the default threshold for a young firm we require that $V(X; c_0)$ satisfies the following boundary conditions

$$V(X_{D_0}; c_0) = 0 \quad (5)$$

$$V'(X_{D_0}; c_0) = 0 \quad (6)$$

Moreover, the equity value for a young firm also reflects the existence of a growth opportunity. Intuitively, if demand grows sufficiently, so that X is above an investment threshold, say X_I , the firm will choose to expand its productive capacity to K_1 .

Thus, at this investment threshold firm value must obey the additional boundary conditions:

$$V(X_I; c_1) + (B(X_I; c_1) - B(X_I; c_0)) - I = V(X_I; c_0) \quad (7)$$

$$V'(X_I; c_1) + (B'(X_I; c_1) - B'(X_I; c_0)) = V'(X_I; c_0) \quad (8)$$

where $(B(X_I; c_1) - B(X_I; c_0))$ denotes the value of net new debt issues at the time of investment.

2.3 Valuation

We are now ready to compute the value of equity for both young and mature firms. To compute the value of a mature firm, given a pre-determined coupon

payment, c_1 , we use Ito's Lemma in equation (1) and impose default when $X = X_{D_1}$ to solve the associated second order differential equation.

This procedure implies that the value of equity for a mature firm satisfies the expression

$$V(X; c_1) = \frac{(1 - \tau)XK_1^\alpha}{r - \mu} - \frac{(1 - \tau)c_1}{r} + A_{11}X^{v_1} \quad (9)$$

where $v_1 < 0$, and the value for the constant $A_{11} > 0$ can be obtained using the relevant boundary conditions at the default threshold.³

The first term in equation (9) is the present value of the future cash flows generated by existing assets, K_1 . From this value we must then deduct the present value of all future debt obligations, which is captured by the term $\frac{(1-\tau)c_1}{r}$. Finally, the last term shows the impact of the option to default on the value of the firm to its shareholders.

In the case of a young firm we apply Ito's Lemma to the Bellman equation (4) and solve the associated differential equation to obtain the expression

$$V(X; c_0) = \frac{(1 - \tau)XK_0^\alpha}{r - \mu} - \frac{(1 - \tau)c_0}{r} + A_{10}X^{v_1} + A_0X^{v_0} \quad (10)$$

where $v_0 > 1$ and $v_1 < 0$ are the roots of the quadratic equation

$$r = v\mu + 0.5v(v - 1)\sigma^2.$$

and $A_{10} > 0$ and $A_0 > 0$ are determined by imposing the boundary conditions at the investment and default threshold.

³In this case we obtain that

$$A_{11} = - \left(\frac{(1 - \tau)X_{D_1}K_1^\alpha}{r - \mu} - \frac{(1 - \tau)c_1}{r} \right) \left(\frac{1}{X_{D_1}^{v_1}} \right)$$

while v_1 is the negative root of the quadratic equation

$$r = v\mu + 0.5v(v - 1)\sigma^2.$$

The first three terms in equation (10) for the equity value of young firms seem identical to those of mature firms and capture, respectively the present value of the future cash flows generated by existing assets, K_0 , and future debt obligations, as well as the present value of the option to default on these obligations.

Despite these similarities the value of young firms, V_0 , differs from that of mature firms, V_1 , in two important ways. First, the equity value of young firms will depend on the (positive) value of future growth options, here captured by the term $A_0 X^{v_0}$. While in this simple example this term is entirely missing from the expression for the value of mature firms it seems nevertheless plausible to expect that the value of growth options to be relatively more important for young firms. Second, mature firms are larger ($K_1 > K_0$) and, as we will see below, precisely for that reason they are also more levered so that $c_1 > c_0$.

2.3.1 Debt Value and Coupon Payments

Before computing the value of each firm we need to determine both the market value of debt outstanding and the value of the instantaneous coupon payments, since both of these values are linked to the firm's decision to invest.

The possibility of default will naturally induce a deviation between the market and the book value of debt at any point in time. As in Leland (1994), as long as the firm does not default the market value of debt paying a per-period coupon of c_i , satisfies the Bellman equation

$$B(X; c_i) = c_i dt + (1 + r dt)^{-1} E[B(X + dX; c_i)]$$

Upon default debt holders are able to recover a fraction, $\xi > 0$, of the value of the firm upon default. Formally this leads to the following boundary

condition on debt

$$B(X_{D_i}; c_i) = \xi \frac{(1 - \tau)X_{D_i}K_i^\alpha}{r - \mu}.$$

where X_{D_i} denotes the threshold level of demand upon which firm i optimally chooses to default. Effectively this boundary condition assumes that, after accounting for transaction costs, debt holders will take over the firm and will be entitled to the entirety of its future cash flows.⁴

Given this boundary condition at default we can easily construct the expression for the market value of debt for firm i , $B(X_{D_i}; c_i)$. This is given by

$$B(X; c_i) = \frac{c_i}{r} \left(1 - \left(\frac{X}{X_{D_i}} \right)^{v_1} \right) \quad (11)$$

where $v_1 < 0$ is defined as above. Note that this expression implies that the market value of debt converges to c_i/r as X approaches infinity.

The exact value of the optimal periodic coupon payment, c_i , can now be determined by maximizing the joint value to equity and bond holders as follows

$$c_1 = \arg \max_c V(X_I; c) + B(X_I, c) \quad (12)$$

$$c_0 = \arg \max_c V(X_0; c) + B(X_0, c) \quad (13)$$

where X_0 is the (arbitrary) value of demand process X at the birth of firm when initial leverage is decided.

Note that since both $V(\cdot)$ and $B(\cdot)$ are increasing in X it follows immediately that $c_1 > c_0$ if $X_I > X_0$. Since, by definition, the young firm invests immediately at X_I it must be the case that young firms with unexercised growth options have less debt than large mature firms.

⁴Note that there is no boundary condition at the restructuring threshold since we are assuming that young firm's debt is callable at market value.

2.4 Leverage and Risk

Equity betas can be recovered by examining the sensitivity of the equity values implied by the expressions (9) and (10). In our simple example these conditional betas can be computed in closed form by examining the elasticities of the value functions with respect to X_t .⁵

We will express conditional equity betas β_{it} , for any firm, young ($i = 0$) or old ($i = 1$), in a general form as

$$\beta_{it} = 1 + \frac{(1 - \tau)c_i}{rV_{it}} + \frac{V_{it}^D}{V_{it}}(v_1 - 1) + \frac{V_{it}^G}{V_{it}}(v_0 - 1), \quad i = 0, 1 \quad (14)$$

Here we use $V_{it}^G = A_0 X^{v_0}$ to denote the value of the young firm's growth options and $V_{it}^D = A_{1i} X^{v_1}$ is the value of the default option.

The first term in this expression is common to both young and old firms and is simply the firm's revenue beta, which captures the (unlevered) riskiness of assets in place. Since operating profits are linear in the aggregate state of demand, this term is here effectively normalized to 1.

The next two components of equity risk are directly tied to leverage. Together they capture the traditional effects of leverage on returns so often emphasized in the static literature. The term, $\frac{(1-\tau)c_i}{rV_{it}}$, shows the effects on risk of levering up equity cash flows on expected returns, even in the absence of any default risk, while the third term reflects the impact of default on equity risk.⁶ Together they imply the usual positive relation between leverage and equity risk that is described in most finance textbooks.⁷

⁵More formally a corresponding conditional one-factor asset pricing model can be derived as follows. Assuming a constant factor risk premium λ , the conditional expected return on equity is obtained as

$$E_t[R_{it+1}] = r + \beta_{it}\sigma\lambda$$

where $\beta_{it} = \frac{d \log V_{it}}{d \log X_t}$

⁶Note that $\frac{(1-\tau)c_i}{r}$ is simply the value of a riskless perpetuity.

⁷Here the endogenous nature of default limits the firms downside risk ($A_{1i} > 0$). This

The novelty here however is the last term in equation (14). This term reflects the effect of growth options and depends on the relative importance of these options to the equity value of the firm. In our simple example this last term will add to the underlying risk of the young firm since $v_0 > 1$.

Although stylized, this expression for equity betas illustrates the subtle nature of the relation between leverage and risk in a world where investment and leverage decisions are linked. Equation (14) shows that while, *ceteris paribus*, financial leverage clearly increases equity risk, this simple relation holds only in a static world when leverage is already pre-determined.

In a richer dynamic setting leverage is itself endogenous and generally related to investment decisions of varying degrees of risk. And when leverage tends to be higher for mature, low growth, firms which are otherwise less risky (see Barclay, Morellec and Smith (2006) for example), simple correlations between discount rates and leverage are unlikely to be informative. Instead, equation (14) suggests that controlling for the importance of growth options is crucial when examining the relation between leverage and equity returns.

2.5 Numerical Illustration

We now illustrate and develop some of our key insights with a simple numerical example.

2.5.1 Optimal Leverage

Figure 1 shows the leverage of young and mature firms by plotting the optimal (book) leverage choice of a mature firm $B(X_T; c_1)/K_1$ for any possible level of initial leverage $B(X_0; c_0)/K_0$. The figure documents that regardless of its initial choice of book leverage, a firm will always increase its leverage when

may change however if we allow for more sophisticated default mechanisms in which the firm may be liquidated sub-optimally due to covenant violations.

exercising its growth options.

Intuitively this happens for two linked reasons: (i) a mature firm is fundamentally less risky since its value is no longer tied to that of its growth options and, as a result, (ii) the effects of a potential debt overhang on future investment that limit young firm's borrowing are no longer a concern, allowing the firm to take extra leverage.

Although our example is simple the intuition seems quite general and survives in the more realistic model below. It implies that a large mature firm will always be more levered for precisely the same reasons that reduce its underlying business risk.

Because equity returns reflect both the underlying business risk and the effects of financial leverage, it is unlikely that equity risk will be monotonic, let alone, linear, in leverage.

2.5.2 Business Cycle Effects

Figure 2 provides additional insights into the role of leverage in determining equity risk. This figure plots equity betas for both young and mature firms as a function of the state of demand, X . The dashed line shows the equity beta for mature firms, while the solid line shows the beta for the young firms.

Not surprisingly we see that expected returns rise with X for the young firms because this increases the relative importance of their growth options in total firm value. Also intuitive is the pattern for mature firms. Here risk increases as demand conditions, X , worsen since this makes it more likely that the firms will find itself in default.

Figure 2 also confirms our findings that expected returns will not, in general, be monotonic in leverage. Depending on demand conditions it is possible for low-leverage firms to be either more or less risky, as measured

by equity beta.

Another important implication of this result is that it suggests that the relationship between leverage and returns is conditional in nature: In bad times the contribution of default and cash flow risk is greater, while in good times the investment channel dominates. Thus, when default risk is small, the figures suggest that expected returns are decreasing at least in book leverage, a finding that seems consistent with the recent empirical literature.

Finally this cyclical pattern of equity risk across firms is also interesting because it shows how financial leverage can generate endogenously the kind of variation in equity returns that is often required replicate the value premium (See for example Carlson, Fisher and Gianmarino (2004)). Unlike the existing literature however, our mechanism does not rely on exogenous technological assumptions but is instead linked to the optimal capital structure of the firm.

2.5.3 Equity Risk

Finally, Figure 2 can also be used to understand the pattern of equity betas for an individual firm around its investment and financing threshold, X_I .

Because, in this example, the equity beta for a young firm is higher than that of a mature firm around the investment threshold, any firm deciding to exercise its growth option would experience a drop in equity risk. Again this happens in spite of the fact that the firm is simultaneously (and optimally) increasing its financial leverage, as documented by Figure 1.

3 The General Model

The simple example in the section 2 provides much of the intuition for our findings although at the cost of some loss of generality. The model is also too stylized to allow for a more serious quantitative investigation of its key

predictions.

In this section we embed the key ideas from our example in a more general environment that allows for more complex investment and financing strategies. Specifically, we now let firms have access to multiple investment options, while also relaxing the assumption that investment and financing must be perfectly coordinated. Firms can now issue debt (and equity) at any point in time and in any amount, subject to the natural financing constraints.

In addition we now allow for additional cross sectional firm heterogeneity in the form of firm specific shocks to both current profitability and the value of growth options. Moreover, aggregate shocks to the state of demand now impact both firm profitability and the discount rates as we no longer conduct our analysis under risk-neutral valuation.

Although this more general environment contains several additional ingredients its basic features are very similar, and our notation is, when possible, identical to that in the section 2.

3.1 Firm Problem

3.1.1 Profits and Investment

As before we begin by considering the problem of a typical value maximizing firm in a perfectly competitive environment. Time is now discrete. The flow of after tax operating profits per unit of time for each firm i is described by the expression

$$\Pi_{it} = (1 - \tau)(Z_{it}X_tK_{it}^\alpha - f), \quad 0 < \alpha < 1 \quad (15)$$

where Z_i captures a firm specific component of profits and the variables X_t and K_{it} denote, as before, the aggregate state of productivity and the book value of the firm's asset. We use $f \geq 0$ to denote a (per-period) fixed cost of production.

Both X and Z are assumed to be lognormal and obey the following laws of motion

$$\begin{aligned}\log(X_t) &= \rho_x \log(X_{t-1}) + \sigma_x \varepsilon_t \\ \log(Z_{it}) &= \rho_z \log(Z_{it-1}) + \sigma_z \eta_{it}\end{aligned}$$

and both η_i and ε are truncated (standard) normal variables.⁸ The assumption that Z_{it} is firm specific requires that

$$\begin{aligned}E\varepsilon_t \eta_{it} &= 0 \\ E\eta_{jt} \eta_{it} &= 0, \text{ for } i \neq j\end{aligned}$$

The firm is now allowed to scale operations by picking between any level of productive capacity in the set $[0, \bar{K}]$. This can be accomplished through (irreversible) investment, I_{it} , which is linked to productive capacity by the standard capital accumulation equation

$$I_{it} = K_{it+1} - (1 - \delta)K_{it} \geq 0 \tag{16}$$

where $\delta > 0$ denotes the depreciation rate of capital per unit of time.

3.1.2 Financing

Corporate investment as well as any distributions, can be financed with either the internal funds generated by operating profits or net new issues which can take the form of new debt (net of repayments) or new equity.

We now assume that debt B can take the form of a one period bond that pays a coupon c per unit of time. Thus we now allow the firm to refinance the entire value of its outstanding liabilities in every period. Formally, letting

⁸To ensure the existence of a solution to the firm's problem the shocks must be finite. We accomplish this by imposing (very large) bounds on the values of ε and η .

B_{it} denote the book value of outstanding liabilities for firm i at the beginning of period t we define the value of net new issues as

$$B_{it+1} - (1 + c_{it})B_{it}.$$

Note that now both debt and coupon payments will exhibit potentially significant time variation and will now depend on a number of firm and aggregate variables.

The firm can also raise external finance by means of seasoned equity offerings. For added realism however, we assume that these issues entail additional costs so that firms will never find it optimal to simultaneously pay dividends and issue equity. Following the existing literature we consider costs that include both fixed and variable components, which we denote by λ_0 and λ_1 , respectively.⁹ Formally, letting E_{it} denote the net payout to equity holders, total issuance costs are given by the function:

$$\Lambda(E_{it}) = (\lambda_0 - \lambda_1 \times E_{it}) \mathbb{I}_{\{E_{it} < 0\}}$$

where the indicator function implies that these costs apply only in the region where the firm is raising new equity finance so that net payout, E_{it} , is negative.

Investment, equity payout, and financing decisions must meet the following identity between uses and sources of funds

$$E_{it} + I_{it} = \Pi_{it} + \tau\delta K_{it} + B_{it+1} - (1 + (1 - \tau)c_{it})B_{it} \quad (17)$$

where again E_{it} denotes the equity payout. Note that the resource constraint (17) recognizes the tax shielding effects of both depreciated capital and interest expenditures. Distributions to shareholders, denoted D_{it} are then given

⁹See Gomes (2001) and Hennessy and Whited (2007).

as equity payout net of issuance costs:

$$D_{it} = E_{it} - \Lambda(E_{it})$$

3.1.3 Valuation

The equity value of the firm, V , is defined as the discounted sum of all future equity distributions. Here again we assume that equity-holders will choose to close the firm and default on their debt repayments if the prospects for the firm are sufficiently bad, i.e., whenever V reaches zero.

To discount future cash flows we directly parameterize the discount factor applied to future cash flows as a stochastic process given by the expression

$$\log M_{t,t+1} = \log \beta - \gamma \log(X_{t+1}/X_t)$$

with $\gamma > 0$. Although this pricing kernel is exogenous its basic properties seem plausible, most notably, the idea that the risk premium is directly related to aggregate growth in cash flows.¹⁰

The complexity of the problem is reflected in the dimensionality of the state space necessary to construct the equity value of the firm. This includes both aggregate and idiosyncratic components of demand, productive capacity, and total debt commitments, defined as

$$\hat{B}_{it} \equiv (1 + (1 - \tau)c_{it})B_{it}$$

To save on notation we henceforth use the $S_{it} = \{K_{it}, \hat{B}_{it}, X_t, Z_{it}\}$ to summarize our state space.

We can now characterize the problem facing equity holders, taking coupon payments as given. These payments will be determined endogenously below. Shareholders jointly choose investment (the next period capital stock) and

¹⁰See Berk et al (1999) for a similar application and in-depth explorations of this assumption.

financing (next period total debt commitments) strategies to maximize the equity value of each firm, which accordingly can then be computed as the solution to the following dynamic program

$$\begin{aligned}
V(S_{it}) &= \max\{0, \max_{K_{it+1}, \tilde{B}_{it+1}} \{D(S_{it}) + E[M_{t,t+1}V(S_{it+1})]\}\} \quad (18) \\
\text{s.t. } K_{it+1} &\geq (1 - \delta)K_{it}
\end{aligned}$$

where the expectation in the left hand side is taken by integrating over the conditional distributions of X and Z , Note that the first maximum captures the possibility of default at the beginning of the current period, in which case the shareholders will get nothing.¹¹ Finally, aside from the budget constraint embedded in the definition of D_{it} , the only significant constraint on this problem is the requirement that investment is irreversible.

3.1.4 Default and Bond Pricing

We now turn to the determination of the required coupon payments, taking into account the possibility of default by equity holders. Assuming debt is issued at par, the market value of new issues must satisfy the following condition

$$B_{it+1} = E \left[M_{t,t+1} \left((1 + c_{it+1}) B_{it+1} \mathbb{I}_{\{V_{it+1} > 0\}} + R_{it+1} (1 - \mathbb{I}_{\{V_{it+1} > 0\}}) \right) \right] \quad (19)$$

where R_{it+1} denotes the recovery on a bond in default and $\mathbb{I}_{\{V_{it+1} > 0\}}$ is an indicator function that takes the value of 1 if the firm remains active and 0 when equity chooses to default.

We follow Hennessy and Whited (2007) and specify the deadweight losses at default to consist of a fixed and a proportional component. Thus, creditors

¹¹In practice, there can be violations of the absolute priority rule, implying that shareholders in default still recover value. Garlappi and Yan (2007) analyze the asset pricing implications of such violations.

are assumed to recover a fraction of the firm's current assets and profits net of fixed liquidation costs. Formally the default payoff is equal to:

$$R_{it} = \Pi_{it} + \tau\delta K_{it} + \xi_1(1 - \delta)K_{it} - \xi_0$$

Since the equity value V_{it+1} is endogenous and itself a function of the firm's debt commitments this equation cannot be solved explicitly to determine the value of the coupon payments, c_{it} . However, using the definition of \hat{B} , we can rewrite the bond pricing equation as

$$\begin{aligned} B_{it+1} &= \frac{E \left[M_{t,t+1} \left(\frac{1}{1-\tau} \hat{B}_{it+1} \mathbb{I}_{\{V_{it+1}>0\}} + R_{it+1} (1 - \mathbb{I}_{\{V_{it+1}>0\}}) \right) \right]}{1 + \frac{\tau}{1-\tau} (E [M_{t+1} \mathbb{I}_{\{V_{it+1}>0\}}])} \\ &= \mathbf{B}(K_{it+1}, \hat{B}_{it+1}, X_t, Z_{it}) \end{aligned}$$

Given this expression and the definition of \hat{B} we can easily deduce the implied coupon payment as

$$c_{it+1} = \frac{1}{1-\tau} \left(\frac{\hat{B}_{it+1}}{B_{it+1}} - 1 \right)$$

Note that defining \hat{B} as a state variable and constructing the bond pricing schedule $\mathbf{B}(\cdot)$ offers important computational advantages. Because equity and debt values are mutually dependent (since the default condition affects the bond pricing equation) we would normally need jointly solve for both the interest rate schedule (or bond prices) and equity values. Instead our approach requires only a simple function evaluation during the value function iteration. This automatically nests the debt market equilibrium in the calculation of equity values and greatly reduces computational complexity (see Appendix A for details).

3.2 Optimal Firm Behavior

Given our assumptions, the dynamic programming problem (18) has a unique solution.¹² Since this cannot be solved in closed form we must resort to numerical methods, which are detailed in Appendix A. The solution can be characterized efficiently by optimal distribution, financing, and investment policies. We now investigate some of properties of these optimal strategies.

Our choice of parameter values, summarized in Table 1, follows closely the existing literature (e.g. Gomes (2001), Cooley and Quadrini (2001), Hennessy and Whited (2005)). The values are picked so that the model produces a cross-sectional distribution of firms that matches key unconditional moments of investment, returns, and cash flows both in the cross-section and at the aggregate level. Appendix B discusses our choices in detail.

3.2.1 Investment and Financing

Figure 3 illustrates the optimal financing and investment policies of the firm as well as their implications for equity values conditional on the aggregate level of demand. The dashed line corresponds to a high realization for the aggregate state of demand (an economic boom), the dotted line corresponds to the long-run mean of aggregate demand, while the solid line shows the results when demand is relatively weak (a recession). In all cases we set the idiosyncratic profitability shock, Z_{it} , to its mean.

The top panels, labeled “new capital stock” depicts the optimal choice of next period capital, K_{it+1} , as a function of the underlying variables. These panels neatly illustrate the interaction of financing and investment decisions, particularly for small firms. With unlimited access to external funds, the optimal choice of capacity would be independent of the current period capital

¹²The interested reader is referred to Gomes and Schmid (2008) for a proof.

stock, at least for low values of K_{it} as the irreversibility constraint only binds on disinvestment. Here however this is not the case. This is because an increase in existing K_{it} generates both higher internal cash flows and more collateral, thus alleviating the effect of financing constraints. As the picture shows this effect is particularly important for small firms.

Equally interesting is the fact that the optimal capacity choice is declining in current liabilities, B_{it} . Although reminiscent of the popular debt overhang effect this finding result is worthy of note since we explicitly allows firms to renegotiate the terms of their debt in every period.

The "new debt" panels show the optimal choice for new debt outstanding, B_{it+1} . A notable feature is the strong positive relation between current and lagged leverage, a phenomenon sometimes dubbed as 'hysteresis', and that suggests that our model is also consistent with the well documented finding that financial leverage is extremely persistent. Note that this result arises even in the absence of any of the usual suspects such as: market timing or costly debt issues. Here, persistence in leverage is due almost exclusively to the nature of investment decisions of the firm since, as we have seen above, investment and financing are closely linked.

For completeness Figure 3 also shows the behavior or the equity value of the firm. Not surprisingly these values are increasing in current assets and profitability and declining in the amount of outstanding debt.

3.2.2 Risk and Returns

Figure 4 investigates the implications of these firm decisions on various measures of risk and returns. As before the dashed line corresponds to a high realization for the aggregate state of demand (an economic boom) while the solid line shows the results when demand is relatively weak (a recession).

The bottom four panels show the effects on default probabilities and credit spreads, measured as the difference between the yield on the debt outstanding and the risk-free rate. Both credit spreads and default probabilities are countercyclical, in the sense that they are declining in the state of aggregate demand, X_t .

Interestingly we note that the model can also produce sizable credit spreads. The intuition is very similar to that in Bahmra et al (2007) and Chen (2007): What matters for credit spreads are not so much the actual default probabilities shown but the risk-adjusted default probabilities. Our parameter choices ensure that the joint variation in the pricing kernel and physical default probabilities produce large risk adjusted probabilities and thus generate significant credit spreads.

From a cross-section point of view that credit risk rises substantially when the firm is very small and leverage is high, since this scenario leads to a dramatic increase in the probability of default.

The top two panels, labeled "Beta", show the induced variation in expected equity returns. The first panel shows that controlling for both current assets and profitability, leverage increases the systematic risk to equity holders. This is precisely the result identified in traditional static models and discussed in section 2.

As before however we also find that in our more general model equity risk declines fairly quickly in firm size and is significant smaller for larger firms. The intuition is precisely the same that we identified in section 2: with decreasing returns to scale, large firms also have fewer growth options which reduces their risk.¹³

¹³Note that the presence of decreasing returns to scale effectively ties the value of growth options to current size since it ensures that the marginal value of new additions to productive capacity is always lower for large firms.

Thus to the extent that leverage and investment policies are jointly determined, the link between expected returns and leverage is likely to be more subtle than what is traditionally suggested in the literature. In fact if decreasing returns are sufficiently strong it is actually possible that the relation between returns and debt could be fairly flat or even negative.

4 Cross-Sectional Implications: Theory and Evidence

In this section we investigate some of the empirical implications of our general model in section 3 by comparing our theoretical findings with data on leverage and equity returns.

4.1 Basic Methodology and Definitions

We begin by constructing an artificial cross-section of firms by simulating the investment and leverage rules implied by the model. The simulation details are described in Appendix A. We then construct theoretical counterparts to the empirical measures of returns, beta, book-to-market, Q, and leverage in the widely used CRSP/Compustat dataset.

In our model the book value of assets is simply given K , while the book value of equity is $BE = K - B$. To facilitate comparisons with these studies we will henceforth use the notation $ME = V$ to denote the market value of equity. Book leverage is then measured by the ratio B/K , while book-to-market equity is defined as BE/ME . Tobin's Q is measured as the ratio of market equity plus debt over the book value of assets, K .

Because in the model V is the cum-dividend equity value, stock returns

between t and $t + k$ are defined in a straightforward fashion by the identity

$$r_{it,t+k} = \frac{V_{it+k}}{V_{it} - D_{it}}$$

4.2 Cross-Sectional Patterns in Leverage

We start by examining the model's implications for the cross-section distribution of leverage across firms. To do this we look at the popular regressions used in the empirical capital structure literature relating corporate leverage with several financial indicators (e.g. Rajan and Zingales (1995)). Specifically, we estimate the following regression equation in our simulated data-set:

$$Lev_{it} = \alpha_0 + \alpha_1 \log(sales_{it}) + \alpha_2 Q_{it} + \alpha_3 \frac{\Pi_{it}}{K_{it}}$$

where size is measured alternatively by either sales ($Z_{it}X_tK_t^\alpha$) or book assets (K_{it}) and for Lev_{it} we use both book and market leverage.

Table 3 summarize our findings which are directly comparable to those in Rajan and Zingales (1995) and other similar studies. The table confirms the positive relation between firm size and leverage. Whether measured by the level of sales or assets, an increase in firm size leads to higher levels of corporate leverage. This positive relation between leverage and firm size is intuitive and is a result of several different factors. First, and most important, the concavity of profits implying that large firms have more stable cash flows than small firms. This decreasing returns to scale assumption is the equivalent in our general model of assuming that growth options are (relatively) more important for small firms.

Our general model also introduces two additional reasons why leverage and size will be positively related: operating leverage renders small firms more risky and the fixed costs of default are also less important for large

firms. Together these assumptions ensure that leverage grows endogenously with firm size, a seemingly robust empirical finding.

Table 3 also shows that our model is able to reproduce the observed negative relationships between leverage and either profitability or Q . Both the results and their intuition are very similar to those Hennessy and Whited (2005). Both rest on the assumption that equity issues are costly. These encourage firms to save enough (alternatively to borrow prudently) to avoid the need for equity issuance. As result firms will often retire debt whenever cash flows (profits) rise or when they anticipate investment opportunities (high Q).

4.3 Leverage and Returns: Unconditional Moments

We now turn towards the relationship between leverage and returns implied by our model. To assess these implications, we compare our theoretical findings with the empirical evidence obtained from the CRSP/Compustat Merged database for the years between 1963 and 2006. Our empirical measures of returns, book-to-market, and leverage are follow the procedures in Fama and French (1992, 1993) and are described in detail in appendix C.

Table 4 is constructed by creating five value-weighted portfolios that are ranked by either book or market leverage. These portfolios are then held for 12 months following its formation and their returns are computed. The table reports the average monthly return associated with this buy-and-hold strategy, both in actual and simulated data.

The rows labeled “book-leverage” show the results of constructing portfolios that are sorted according to the book leverage of the firm, while the portfolios labeled “market-leverage” show the results of sorting on market leverage.

Although the magnitude of our numbers is a little high we see that in both cases our model conforms well with the broad patterns in the data. Specifically, we find that equity returns seem positively related to market leverage, but essentially flat on book leverage. Moreover the quantitative spread in returns induced by sorting on market leverage is also very similar to that obtained in the data. The result that book leverage is essentially unrelated to cross-sectional dispersion in returns is consistent with the inconclusive results in the empirical literature. On the other hand, market leverage, containing market capitalization in the denominator, is mechanically positively related to returns.

4.4 Size and Book-to-Market

The evidence in Table 4 is useful, but it offers little more than a crude test of the model. More interesting is to look at the role of our leverage measures when interacted with other variables. A natural benchmark is to focus on the usual suspects of firm size and the book-to-market ratio.

Table 5 looks at the relationship between market leverage and returns controlling first for either size (panels on the left) or book to market (panels on the right). The bottom row in all of these panels (labeled “All”) shows the average pattern of returns across the various portfolios and is a good way of thinking about the *conditional* relation between leverage and returns. Comparing this final row with the *unconditional* results in Table 4 provides an effective summary of the role of size and book to market in capturing the effects of leverage on returns.

In general we find that once again our model performs very well. Looking first at the left panels of Table 5 we see that leverage and returns retain a clear positive relation even after controlling for firm size. This is true both

in the model and in the data. It is also true both on average and across all of the size portfolios.

However, the two book-to-market panels on the right suggest a different story. Controlling for book to market yields only a very mild positive relation between leverage and returns. While this is more pronounced in the model, it is also significantly smaller than the unconditional link documented in Table 4.

We find these results informative in a number of important ways. First, our dynamic model of leverage and returns offers theoretical support to the common intuition that book-to-market is related to a financial distress factor, as this variable seems to capture much of the impact of leverage in returns.

Secondly, the Tables confirm our intuition that the book to market ratio is often not a very useful measure of growth options. Although not ideal, market size seems a much more useful proxy for these options. In particular, consistent with the intuition developed in our simple example, both in the model and in the data the link between leverage and returns remains apparent even after controlling for firm size.

For completeness we also include Table 6 which shows the relation between book leverage and returns controlling for either size or book-to-market. This Table confirms our earlier view that book leverage is much less informative about expected returns even after we control for size and book-to-market. Both in the data and in our model there is, at best, a very small positive link between this measure of leverage and equity returns.

5 Conclusion

In this paper we revisit the theoretical relation between financial leverage and stock returns in a dynamic world where both the corporate investment and finance decisions are endogenous. We find that in general the link between leverage and stock returns is more complex than the static textbook examples suggest and will generally depend on the investment opportunities available to the firm. In the presence of financial market imperfections leverage and investment are generally correlated so that highly levered firms are also mature firms with relatively more (safe) book assets and fewer (risky) growth opportunities. We first develop the underlying intuition qualitatively in a simple real options model, which delivers closed form expressions for firms' equity betas as functions of firm characteristics. We then construct a quantitative model incorporating the same economic mechanisms to analyze the empirical implications of our framework and test them on actual data. Our results help interpreting recent puzzling empirical evidence concerning leverage and returns and provide new insights in the economic determinants of size and book-to-market factors in equity returns. In particular, we show that the quantitative version of our model can successfully replicate the empirical relationships between leverage and returns, even after one controls for variables such as size and book to market.

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A Appendix: Computational Details

Computation of the optimal policy functions is complicated by the endogeneity of the coupon schedule on corporate debt, that is, the fact that the coupon schedule depends on firms' default probabilities, which in turn depend on their equity values. We use a two part procedure to speed up calculations.

- Part I: Specify a fairly coarse grid for the state space with $n_K^0 \times n_B^0 \times n_Z^0 \times n_X^0$ points.¹⁴ We use the Tauchen-Hussey procedure to transform the autoregressive processes for X and Z into finite Markov Chains.
 1. Given an initial guess for $c(S) = c^0(S)$ iterate on (18) until convergence. Given our assumptions this procedure has a unique fixed point.
 2. Given the computed equity value $V(S)$ and the implied default policy to construct a revised guess for the coupon $c^1(S)$ from equation (19).
 3. Compute the distance $\|c^1(S) - c^0(S)\|$. If this is small we stop, otherwise we return to step 1.
- Part II: Implement the direct computation described in the text on a finer grid with $250 \times 250 \times 10 \times 10$ points. Specifically:
 1. Use the values of $V(S)$ and $c(S)$ obtained in Part I to construct initial guess for the value function on the finer grid.
 2. Iterate the Bellman equation for equity value until convergence. Our convergence criterion is set to be 0.0001.

¹⁴We use $n_K^0 = n_B^0 = 25$ and $n_X^0 = n_Z^0 = 10$.

3. Use this to construct the market value of debt and the implied coupon value.

In principle we can use the procedure described in Part I alone and this is guaranteed to converge to the true solution.¹⁵ Computation speed however increases significantly if we use the two-step procedure. Since the algorithm described in Part II is not a contraction mapping it is important to start close enough to the actual solution which is why Part I must be used first.

Our two step approach is very robust and the accuracy of the direct computation was confirmed for a number of parameter values by simply using the method in Part I for very small tolerances and in large grids.

To construct an artificial cross-section of firms we simulate the investment, leverage and default rules implied by the model. For all simulations our artificial dataset is generated by simulating the model with 2000 firms over 1500 monthly periods and dropping the first 1000 periods. This procedure is repeated 50 times and the average results are reported.

For the leverage regressions we construct annual data by accumulating monthly profits and sales over 12 periods. We then run the regressions on the annual observations and report both the mean coefficient estimates and the average t-statistics across simulations.

B Appendix: Parameter Choices

The persistence, ρ_x , and conditional volatility, σ_x , of aggregate productivity, are set equal to 0.983 and 0.0023 which is close to the corresponding values reported in Cooley and Prescott (1995). For the persistence, ρ_z , and conditional volatility, σ_z , of firm-specific productivity, we choose values close to the

¹⁵For a formal proof see Gomes and Schmid (2008).

corresponding ones constructed by Gomes (2001) to match the cross-sectional properties of firm investment and valuation ratios.

The depreciation rate of capital, δ , is set equal to 0.01 which provides a good approximation to the average monthly rate of investment found in both macro and firm level studies. For the degree of decreasing returns to scale we use 0.65. Although probably low this number is almost identical to the estimates in Cooper and Ejarque (2003) as well as several other recent micro studies.

We set ξ_1 which is one minus the proportional cost of bankruptcy equal to 0.75, which is in line with recent empirical estimates in Hennessy and Whited (2006) as well as consistent with values traditionally used in the macroeconomics. Additionally, under the assumption that close to default the asset value of the unlevered firm is close to its book value, the number is consistent with the traditional estimates of the direct costs of bankruptcy obtained in the empirical corporate finance literature. We then choose ξ_0 , the fixed cost of bankruptcy, such that we match average market-to-book values in the economy.

The costs of equity issuance λ_0 and λ_1 are chosen similarly as in Gomes (2001). Later empirical studies (Hennessy and Whited, 2004) have confirmed that these values are good estimates.

We choose the pure time discount factor β and the pricing kernel parameter γ so that the model approximately matches the mean risk free rate and the equity premium. This implies that β equals 0.995 and γ is 15.

To assess the fit of our calibration, we report in Table 2 the implied moments generated by our parameterization for some key variables. Our calibration ensures that the simulated data matches some key statistics related to asset market data and firms' investment and financing decisions quite well.

This strengthens our confidence in the inference procedure in the paper.

C Appendix: Data Description

Our empirical analysis is based on the Industrial Annual Data from the CRSP/Compustat Merged data base. Our dataset covers the period between 1963 to 2006.

To construct our measures of book-to-market, size, book and market leverage we follow the procedures in Fama and French (1992, 1993). Total assets is measured by Compustat data item 6, book value of common equity is defined as the Compustat book value of stockholders' equity, plus balance-sheet deferred taxes and investment tax credit (if available), minus the book value of preferred stock, which is estimated using redemption, liquidation or par value (items 216, 35, 56, 10, 130). Size is price times shares outstanding. Book-to-market is the book value divided by size, market leverage is total asset value minus book value divided by total asset minus book value plus market value and book leverage is total asset minus book value divided by total assets.

Portfolios are formed on July 1st every year (t) and run through June 30th of the next year ($t + 1$) based on Compustat and CRSP data for each firm as of December of the previous year ($t - 1$). Size bins are created by sorting on NYSE stocks only and then using the break points for all NYSE, Amex and NASDAQ stocks. All other bins are of equal size. We drop all observations with negative book values. To correct for survival bias we only include stocks which are in Compustat for more than two years and restrict our sample to common stocks. For portfolio formation only firms with asset, book and size as of December of $t - 1$ are included in portfolios. We use monthly value weighted excess returns (over 30 day T-bill) that are averaged

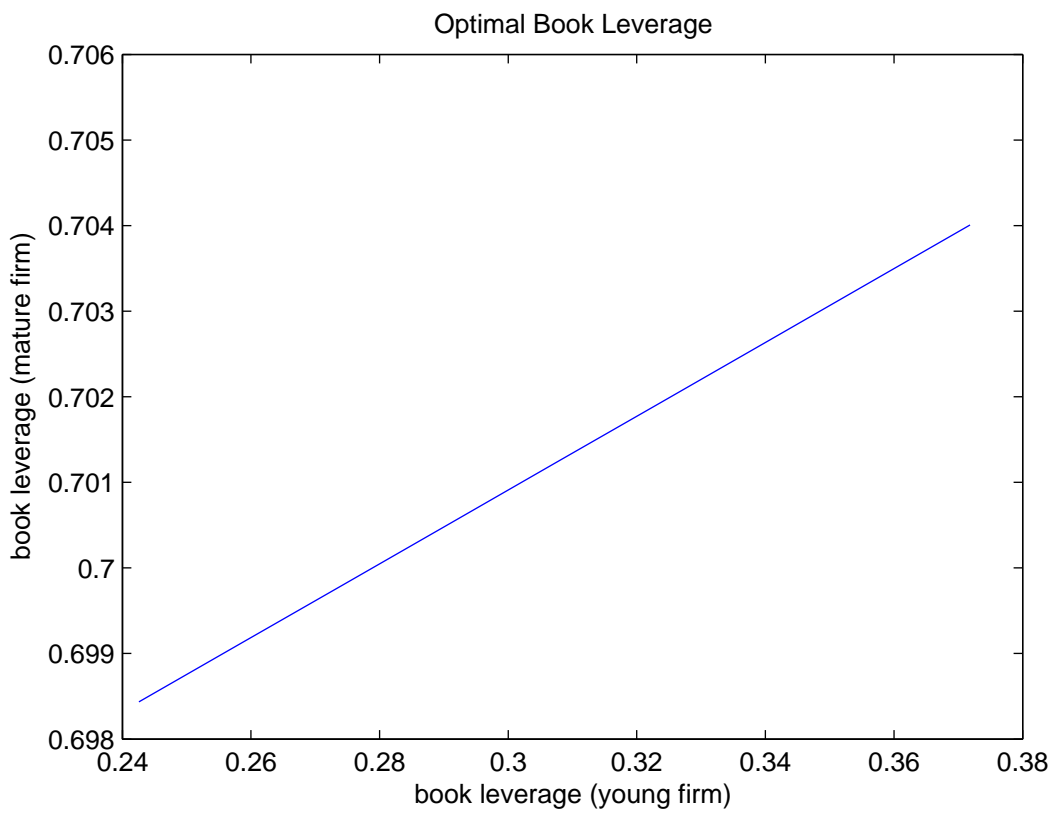
over all months and years. We included the bias correction for delisted firms suggested by Shumway (1997) and Shumway and Warther (1999).

Table 1: : **Parameter Values**

Parameter Values	
α	0.65
β	0.995
δ	0.01
γ	15
τ	0.2
λ_0	0.01
λ_1	0.025
ξ_0	0.1
ξ_1	0.75
f	0.01
ρ_x	0.983
σ_x	0.0023
ρ_z	0.92
σ_z	0.15

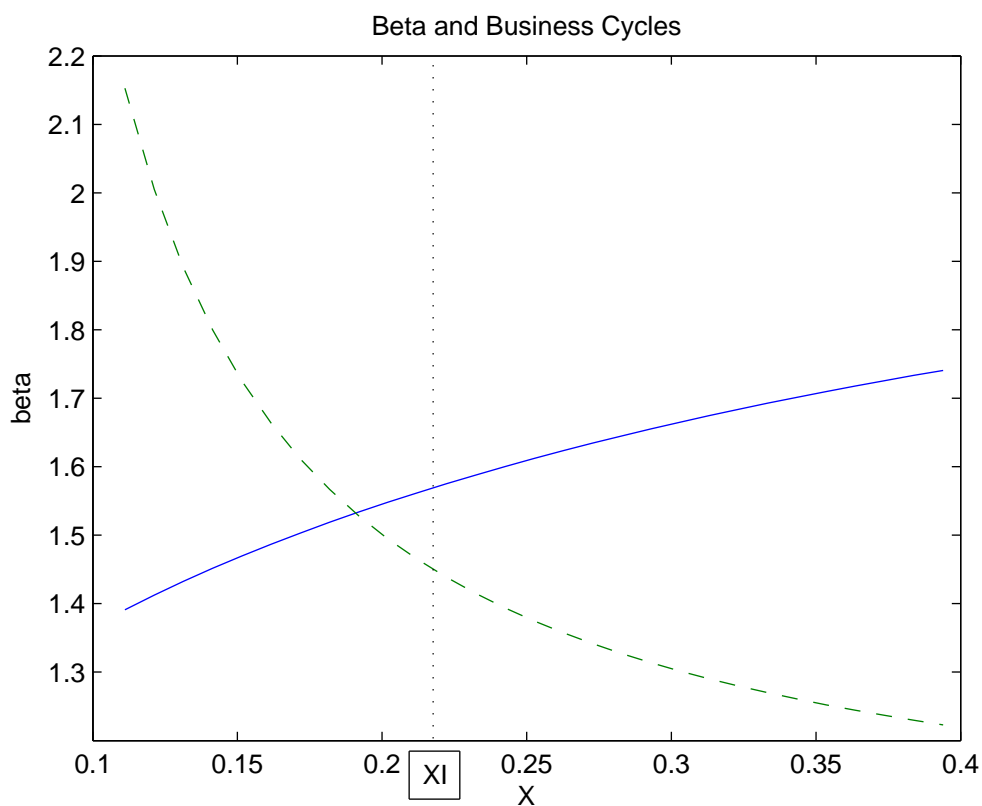
This table reports parameter choices for our general model. The model is calibrated to match annual data both at the macro level and in the cross-section. The persistence, ρ_x , and conditional volatility, σ_x , of aggregate productivity, are set close to the corresponding values reported in Cooley and Prescott (1995). The persistence, ρ_z , and conditional volatility, σ_z , of firm-specific productivity, are close to the corresponding ones constructed by Gomes (2001) to match the cross-sectional properties of firm investment and valuation ratios. The parameter δ is equal to the depreciation rate of capital and is set to approximate the average monthly investment rate. Equity issuance costs are set to values similar to those measured by Hennessy and Whited (2007). For the degree of decreasing returns to scale, α we use the evidence in Cooper and Ejarque (2003). Finally the pricing kernel parameters β and γ are chosen to match the risk free rate and the average equity premium.

Figure 1: : **Beta and Leverage**



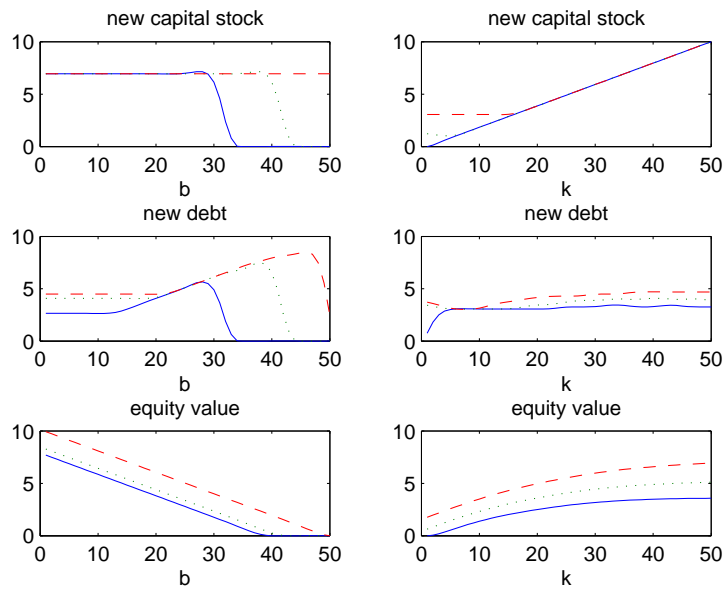
This figure depicts the optimal book leverage choice of an investing firm, $B(x_I; c_1)/K_1$, as a function of its initial leverage, $B(x_0; c_0)/K_0$. Parameter values in the example are $r = 0.05, \mu = 0.0, \tau = 0.2, \sigma = 0.2, \alpha = 0.65, K_0 = 1, K_1 = 5$. The recovery rate on debt is set to $\xi = 0.0$ to generate realistic leverage ratios at issuance.

Figure 2: : Beta and Business Cycles



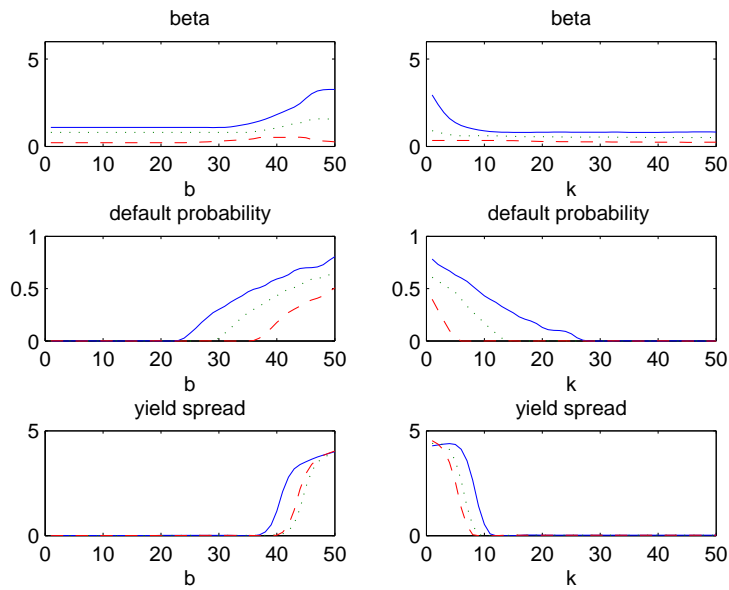
This figure presents betas for young and mature firms as a function of the shock X for an optimally chosen coupon. The beta for young firms is represented by the solid line, while the beta for mature firms is represented by the dashed line. Parameter values in the example are $r = 0.05, \mu = 0.0, \tau = 0.2, \sigma = 0.2, \alpha = 0.65, K_0 = 1, K_1 = 5$. The recovery rate on debt is set to $\xi = 0.0$ to generate realistic leverage ratios at issuance. These parameter choices generate an investment trigger $x_I = 0.2167$.

Figure 3: : Optimal Policies



This figure summarizes the optimal investment and financing policies as a function of existing debt (B) and firm size (K). The bottom pictures show the resulting value of the firm to equity holders. The dashed line refers to a realization of the aggregate shock, X , that is one standard deviation above its mean, the dotted line holds the aggregate shock fixed at its long-run mean, while the solid line refers to a realization of the aggregate shock that is one standard deviation below its mean.

Figure 4: : **Returns**



This figure shows the spread on corporate bonds, implied default probabilities and the equity betas implied by the corporate strategies of the firm for each possible level of current assets (K) and debt (B). The dashed line refers to a realization of the aggregate shock, X , that is one standard deviation above its mean, the dotted line holds the aggregate shock fixed at its long-run mean, while the solid line refers to a realization of the aggregate shock that is one standard deviation below its mean.

Table 2: : **Sample Moments**

Variable	Data	Model
Annual risk-free rate	0.018	0.025
Annual volatility of risk-free rate	0.030	0.019
Annual Equity Premium	6.00	7.81
Investment-to-asset ratio	0.15	0.17
Mean Market-to-Book	1.49	1.81
Book Leverage	0.67	0.74
Market Leverage	0.29	0.35
Frequency of Equity Issuance	0.09	0.15
Default Rate	0.02	0.02

This table reports unconditional sample moments generated from the simulated data of some key variables of the model. Data moments on asset returns come from Campbell, Lo, and McKinlay (1997). The data moments on the investment-to-asset ratio and the market-to-book ratio are taken from Gomes (2001). Leverage and aggregate default rate are taken from Covas and Den Haan (2006). All data are annualized

Table 3: : **Cross-Sectional Leverage Regressions**

Book Leverage Regressions		
Log sales	0.10 (1.94)	-
Log assets	-	0.06 (1.55)
Q	-0.33 (-2.13)	-0.41 (-2.66)
Profitability	-0.71 (-2.05)	-0.56 (-2.23)
Market Leverage Regressions		
Log sales	0.14 (2.09)	-
Log assets	-	0.09 (2.19)
Q	-0.49 (-2.45)	-0.57 (-2.28)
Profitability	-0.37 (-2.16)	-0.44 (-2.75)

This table reports results from Fama-Macbeth cross-sectional regressions of leverage measures on various measures of size, Tobin's Q and profitability. The leverage measure in the upper panel is book leverage, and the lower panel reports analogous results for market leverage. Our size measures are (log) sales and (log) assets, respectively. The results for our artificial dataset are generated by simulating the model with 2000 firms over 500 periods. The procedure is repeated 50 times

Table 4: : Univariate Portfolio Sorts

Variable	Mean Monthly Returns				
	Actual Data				
	Low	2	3	4	High
Book Leverage	0.48	0.49	0.52	0.52	0.50
Market Leverage	0.38	0.46	0.54	0.60	0.73
	Simulated Data				
	Low	2	3	4	High
Book Leverage	0.62	0.69	0.63	0.67	0.66
Market Leverage	0.57	0.68	0.72	0.77	0.82

This table reports average monthly realized returns of portfolios sorted first by either book leverage (top row) or by market leverage (bottom row). The top panel reports the empirical results for the CRSP/Compustat data set using the procedure and data definitions in Fama and French (1992). The bottom panel reports the results for our artificial dataset generated by simulating the model with 2000 firms over 500 periods. This procedure is repeated 50 times and the average results reported in the Table. Book leverage is defined as the ratio between book debt and the book value of equity plus book debt. Market leverage is the ratio between book debt and the market value of equity plus book debt.

Table 5: : Market Leverage Sorts

Mean Monthly Returns															
Actual Data															
		Market leverage								Market leverage					
		Low	2	3	4	High	All			Low	2	3	4	High	All
Size	Small	0.39	0.88	0.93	1.03	1.12	0.86	Book to Market	Low	0.32	0.26	0.31	0.38	0.29	0.31
	2	0.54	0.67	0.95	0.83	0.98	0.80		2	0.66	0.50	0.43	0.59	0.51	0.52
	3	0.54	0.51	0.70	0.74	0.83	0.67		3	0.57	0.68	0.64	0.56	0.75	0.64
	4	0.62	0.58	0.59	0.62	0.76	0.64		4	0.80	0.63	0.73	0.78	0.76	0.72
	Large	0.41	0.30	0.46	0.51	0.51	0.41		High	1.05	0.81	0.86	1.14	1.19	0.89
	All	0.41	0.35	0.52	0.57	0.63			All	0.46	0.47	0.45	0.52	0.51	
Simulated Data															
		Market leverage								Market leverage					
		Low	2	3	4	High	All			Low	2	3	4	High	All
Size	Small	0.71	0.85	0.87	0.91	0.96	0.81	Book to Market	Low	0.48	0.49	0.51	0.55	0.57	0.53
	2	0.63	0.77	0.82	0.84	0.92	0.74		2	0.56	0.57	0.62	0.69	0.70	0.60
	3	0.56	0.68	0.72	0.79	0.81	0.63		3	0.62	0.66	0.71	0.73	0.75	0.77
	4	0.52	0.60	0.64	0.74	0.72	0.61		4	0.71	0.73	0.77	0.82	0.85	0.82
	Large	0.47	0.57	0.63	0.69	0.70	0.57		High	0.81	0.83	0.82	0.89	0.91	0.84
	All	0.55	0.62	0.67	0.72	0.74			All	0.65	0.64	0.67	0.72	0.71	

This table reports average monthly realized returns of portfolios sorted first by size and then market leverage (left panels) or first by book-to-market and then market leverage (right panels). The top panels report the empirical results for the CRSP/Compustat data set using the procedure and data definitions in Fama and French (1992). The bottom panels report the results for our artificial dataset generated by simulating the model with 2000 firms over 500 periods. The procedure is repeated 50 times and the average results reported in the Table. Market leverage is defined as the ratio between book debt and the market value of equity plus book debt.

Table 6: : Book Leverage Sorts

Mean Monthly Returns															
Actual Data															
		Book leverage								Book leverage					
		Low	2	3	4	High	All			Low	2	3	4	High	All
Size	Small	0.68	0.96	0.90	0.82	0.89	0.86	Book to Market	Low	0.45	0.31	0.37	0.34	0.21	0.31
	2	0.78	0.78	0.89	0.73	0.80	0.80		2	0.72	0.54	0.35	0.56	0.56	0.52
	3	0.67	0.61	0.78	0.65	0.64	0.67		3	0.59	0.68	0.67	0.54	0.71	0.64
	4	0.60	0.67	0.73	0.56	0.62	0.64		4	0.90	0.59	0.70	0.78	0.80	0.72
	Large	0.39	0.42	0.48	0.41	0.39	0.41		High	1.16	0.82	0.85	0.91	1.28	0.89
	All	0.42	0.46	0.56	0.48	0.47			All	0.52	0.49	0.45	0.47	0.52	
Simulated Data															
		Book leverage								Book leverage					
		Low	2	3	4	High	All			Low	2	3	4	High	All
Size	Small	0.76	0.78	0.75	0.74	0.81	0.78	Book to Market	Low	0.52	0.53	0.50	0.51	0.50	0.51
	2	0.64	0.68	0.73	0.72	0.72	0.70		2	0.64	0.66	0.66	0.67	0.65	0.66
	3	0.63	0.67	0.70	0.71	0.70	0.68		3	0.71	0.73	0.76	0.74	0.73	0.73
	4	0.58	0.61	0.60	0.64	0.65	0.62		4	0.79	0.81	0.80	0.81	0.77	0.79
	Large	0.57	0.56	0.59	0.60	0.58	0.58		High	0.88	0.90	0.92	0.87	0.86	0.89
	All	0.62	0.64	0.67	0.65	0.67			All	0.68	0.69	0.71	0.72	0.70	

This table reports average monthly realized returns of portfolios sorted first by size and then book leverage (left panels) or first by book-to-market and then book leverage (right panels). The top panels report the empirical results for the CRSP/Compustat data set using the procedure and data definitions in Fama and French (1992). The bottom panels report the results for our artificial dataset generated by simulating the model with 2000 firms over 500 periods. The procedure is repeated 50 times and the average results reported in the Table. Book leverage is defined as the ratio between book debt and the book value of equity plus book debt.