Durable Goods, Inflation Risk and the Equilibrium Term Structure

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Abstract

High inflation predicts a decline in future consumption, and the inflation non-neutrality is stronger for durable than for non-durable goods. This suggests that durables is an important channel for inflation premium in the economy. We derive and estimate an equilibrium two good nominal economy with non-separable utility over durable and nondurable consumption, persistent variations in real expected growth and inflation, inflation non-neutrality, and recursive utility of investors. Our model can explain unconditional moments and conditional movements in the nominal term structure. A large part of the positive risk premium on nominal bonds is due to the durable channel. In the model, as in the data, yields are negatively related to expected durable consumption, which cannot be explained in one good or expected utility restrictions of the model. We further show that model-implied equilibrium real yields are upward sloping for a range of numéraire choices.

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1 Introduction

Empirically, the consumption of durable goods is more sensitive to economic fluctuations than non-durable goods. It is intuitive that consumers would hold off on the purchase of a durable good, such as a car, in response to an adverse income shock, rather than non-durable goods such as food. This has motivated the development of two good consumption-based asset pricing models, such as Yogo (2006) and Yang (2010), whose models explain several key features of equity markets.

In this paper we analyze the relation between the term structure of interest rates, inflation, and growth rates of durable and non-durable consumption. We argue that durable consumption is important in capturing movements and risk premia in the nominal and real yields. There are three empirical observations that lead us to believe that durable consumption is important in explaining interest rates:

- Durable consumption is more sensitive to shocks in inflation than non-durable. In particular, we show that shocks to *expected inflation* have a larger impact on future consumption of durable than on non-durable consumption goods.
- Shocks to durable consumption growth are more persistent that shocks non-durable consumption growth.
- Movements in the nominal yield curve predict future real consumption growth of durable goods. The predictability is stronger for durables than for non-durables.

The first point suggests that consumption of durable goods is potentially more important than non-durables in pricing long maturity bonds, which are inflation sensitive. The second point is important for risk premia in long run risk models since the premia magnify in the persistence of state-variables such as non-durable and durable expected growth rates. The third is an equilibrium outcome. If investors worry about how expected inflation shocks impact future real consumption of either good, these risks will be embedded in the prices of bonds.

We develop a model aimed at explaining these empirical regularities in the data. Our model is a two good nominal version of a long-run risks economy of Bansal and Yaron (2004). Preferences are described by the general recursive preferences of Kreps and Porteus (1978), Epstein and Zin (1989) and Weil (1989) over future consumption of the two goods. The key ingredients of our model are non-separability between durable and nondurable goods in the preferences, persistent fluctuations in expected growth rates, and non-neutrality of inflation for future consumption. With these features, we show that investors are concerned with the risks in expected non-durable consumption, expected durable consumption, and expected inflation. As in standard one good models, yields load positively on expected non-durable consumption. However, we show that when the intratemporal elasticity of substitution between the goods is below the intertemporal elasticity of substitution between the periods, bond loadings on expected durables are negative. Hence, when the two goods are relatively hard to substitute because of composition risk, bond yields have different responses to the expected durable consumption relative to the expected non-durable consumption growth. The durable risk channel generates a positive risk premium on real bonds, and because of the inflation non-neutrality of durable growth, it amplifies the amount of inflation risk premium in nominal bonds.

Our model of the macro-economy can be seen as a VAR(1) model of the three expected growth components, or alternatively as a VARMA(0,1) on the observed growth rates. We estimate this model using Bayesian methods for sampling on the posterior of the parameter space. Our benchmark estimation is a two-stage estimation of the model. In the first stage, we estimate the model parameters and extract the latent states that govern the dynamics of macro variables using only the time series of observable macro variables. In the second step, we estimate the preference parameters using nominal bond yield data. Thus, the estimation of macro dynamics is independent of the equilibrium model specification and based only on the observed macro data. Hence, the implications for the term structure can be viewed as effectively "out-of-sample." As a robustness check, we also conduct a joint estimation of the model by estimating both macro and preference parameters in one stage using macro and yield data.

We find that our macroeconomic model captures the observed macroeconomic data very well. Expected growth rates are estimated to be persistent with an autocorrelation of 0.41 for expected non-durable growth, 0.92 for durable growth, and 0.94 for inflation. Inflation shocks have a negative impact on non-durable consumption, and a significantly larger and more permanent impact on durable consumption. The preference parameters are estimated in the second step from the term structure data; the estimated risk aversion coefficient is about 24 on quarterly frequency, and the intertemporal elasticity of substitution is about 2.6. At these preference parameters, we find that the model can explain the unconditional and conditional features of the nominal yields in the data quite well.

Our model features an upward sloping nominal term structure. The positive slope is due to an inflation risk premium, and a large portion of this premium comes through the durable channel. Our general model matches exactly the average term spread of 60 basis point we observe in the data. Restricting the specification to a one good economy decreases the spread from 60 basis points to 11 basis points. Further, we show that model-implied equilibrium bond yield loadings on expected non-durable, expected durable consumption and expected inflation match their estimates in the data very well. In particular, the bond yield loading on expected durable consumption is negative both in the model and in the data; this, we show, cannot be obtained in the restrictions of the models to one non-durable consumption good, or in the case of expected CRRA utility. The implied real term structure is U-shaped when the real bond is defined to pay one unit of non-durable consumption. Changing numéraire to be a durable good, or an optimal combination of the two goods, produces an upward sloping real term structure. The real term structure in a one good case, or with a CRRA utility, is downward sloping.

Our paper is related to recent literature that explores the asset-pricing implications of recursive utility with several goods. In the context of general equilibrium long-run risks type models, Yang (2010) specifies a model where non-durable consumption is a random walk and durable consumption has a persistent, long run risk component. Yang (2010) calibrates his model to the unconditional moments of equity markets and notes that his model produces an upward sloping average real yield curve. Pakos (2007) highlights the implications of a high intratemporal complementarity between non-durables and durables for the asset prices and risk premia, and shows that with a preference for early resolution of uncertainty, the durable good channel goes a long way to explain the equity premium, the risk-free rate puzzle and size and value puzzles. Colacito and Croce (2011) study the implications of a two good economy in the international context. Yogo (2006) uses the stochastic discount factor implied by the recursive preference structure to study the cross-section of asset returns, while Lustig and Verdelhan (2007) explores it in the cross-section of currency returns. Gomes, Kogan, and Yogo (2009) addresses the implications of durability of goods for the cross-section of asset returns in a production setting. These papers do not focus on the implications of durable risks for the term structure of interest rates.

In a CRRA expected utility framework, Piazzesi, Schneider, and Tuzel (2007) build a two good model to analyze the implications of the composition risk in housing consumption for the asset prices in the economy. Some of the earlier prominent literature on multiple consumption goods includes Eichenbaum and Hansen (1990), Dunn and Singleton (1986), Ogaki and Reinhart (1998). In particular, Dunn and Singleton (1986), based on term-structure data, find evidence against a specification of expected non-separable utility over durables and non-durables. Our specification features a long-run risks economy with recursive utility and fluctuations in expected growth components, which gives rise to additional risk premia components due to long-run risks and enables to better capture the risk and return in financial markets. Notably, the restrictions of expected utility in our setting makes the market prices of expected growth risks to equal zero so that the resulting real and nominal term structures are flat.

There is a large amount of literature on structural, consumption-based term structure models. In the long-run risks one good economy, papers which analyze the implications of long-run consumption risks include Bansal and Yaron (2004), Piazzesi and Schneider (2006), Bansal and Shaliastovich (2011), Eraker (2006), Doh (2010), and Hasseltoft (2010). Ulrich (2011) shows that real term structure can become upward sloping in ambiguity model. Wachter (2006) addresses the term structure implications in the habits model. To the best of our knowledge, the term-structure link to durable risks has not been entertained in a fully-specified general equilibrium context.

The paper is organized as follows: the next section presents the preference model setup, and the equilibrium solution to the model. In Section 3, we present and solve a benchmark model of the economy to highlight the qualitative role of the durable risk channel. Section 4 focuses on the empirical estimation results and model implications for the term structure. Section 5 concludes the paper. Model derivations are given in the Appendix.

2 Model Setup

2.1 Preferences

We specify an infinite-horizon, discrete-time, endowment economy where investors consume durable and non-durable goods. The investors' preferences over future consumption are described by the Kreps-Porteus, Epstein-Zin recursive utility function (see Epstein and Zin, 1989; Kreps and Porteus, 1978):

$$U_t = \left[(1-\beta)u_t^{1-\frac{1}{\psi}} + \beta\left(E_t U_{t+1}^{1-\gamma}\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}}, \tag{1}$$

where U_t is the life-time utility function, u_t is the intra-period consumption aggregator, β is the subjective discount factor, ψ is the elasticity of intertemporal substitution (IES), and γ is the relative risk aversion coefficient. For ease of notations, we define $\theta = (1-\gamma)/(1-1/\psi)$. Note that when $\theta = 1$, that is, when $\gamma = 1/\psi$, the recursive preferences collapse to a standard CRRA expected utility.

In our economy, the agent derives utility from non-durable consumption C_t and a service flow from durable goods, which we assume is proportional to the stock of durables S_t (for a similar assumption, see e.g. Ogaki and Reinhart (1998); Yogo (2006); Yang (2010)). The intra-period consumption aggregator takes a constant elasticity of substitution (CES) form, and thus can be expressed in the following way:

$$u(C, S) = \left[(1-\alpha)C^{1-\frac{1}{\epsilon}} + \alpha S^{1-\frac{1}{\epsilon}}\right]^{\frac{1}{1-\frac{1}{\epsilon}}}. \tag{2}$$

Parameter $\alpha \in [0, 1]$ determines the relative importance of durable consumption: with $\alpha = 0$ the economy collapses to a standard specification with a single perishable good. Parameter ϵ captures the intratemporal elasticity of substitution between the two goods. High values of ϵ indicate that the two goods can be easily substituted by the agent, while small values for ϵ capture the complementarity between the two goods.

2.2 Model Solution

In each period the agent consumes C_t of non-durable goods, makes purchases E_t of durable goods at a relative price P_t^d, and buys h_{it} shares of asset i whose prices is P_{it} and dividend payment is D_{it}, for $i = 1, 2, \dots N$. The budget constraint of the agent can be written in the usual way which stipulates that the beginning of period financial wealth of the agent is used to finance durable and non-durable purchases and acquisition of the new assets:

$$\sum_{i=1}^{N} (P_{it} + D_{it}) h_{i,t-1} = C_t + P_t^d E_t + \sum_{i=1}^{N} P_{it} h_{it}. \tag{3}$$

Notably, we choose non-durable consumption as the numéraire, so all the prices above are in units of non-durable consumption.

The total stock of durables is equal to the total amount last period net of depreciation at the rate δ, plus the new purchases E_t :

$$S_t = (1 - \delta) S_{t-1} + E_t. \tag{4}$$

Let W_t denote the beginning-of-period total wealth of the agent, which includes financial wealth, and the value of undepreciated durable good stock:

$$W_t = \sum_{i=1}^{N} (P_{it} + D_{it}) h_{i,t-1} + (1 - \delta) P_t^d S_{t-1}. \tag{5}$$

In the above equations, P_t^d indicates a purchase price of durable goods. As durable goods provide utility benefits beyond the current period, let us define a user cost of durable goods Q_t which fully captures the utility value of a durable good to the agent. The user cost is given by the ratio of the marginal utilities of durable to non-durable consumption:

$$Q_t = \frac{u_{st}}{u_{ct}} = \frac{\alpha}{1 - \alpha} \left(\frac{S_t}{C_t} \right)^{-\frac{1}{\epsilon}}. \tag{6}$$

In equilibrium, the user cost of durable goods is equal to the rental price of durables, and thus can be determined through the intertemporal optimality condition:

$$Q_t = P_t^d - (1-\delta) E_t M_{t+1} P_{t+1}^d, \tag{7}$$

where M_{t+1} denotes the equilibrium stochastic discount factor.

The stochastic discount factor in a two good economy can be written in the following way:

$$M_{t+1} = \beta^\theta \left(\frac{Z_{t+1}}{Z_t}\right)^{\frac{\theta}{1-\frac{1}{\epsilon}}\left(\frac{1}{\psi}-\frac{1}{\epsilon}\right)} \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R_{c,t+1}^{\theta-1}, \tag{8}$$

where Z_t denotes the relative share of non-durable consumption of the agent:

$$Z_t = \frac{C_t}{C_t + Q_t S_t}, \tag{9}$$

and $R_{c,t+1}$ is the return on the total wealth portfolio:

$$R_{c,t+1} = \frac{W_{t+1}}{W_t - (C_t + Q_t S_t)}. \tag{10}$$

The equilibrium discount factor can be used to price any asset in the economy, including the return on the wealth portfolio, using the standard Euler equation:

$$E_t M_{t+1} R_{i,t+1} = 1. \tag{11}$$

In a one good economy with only non-durable consumption ($\alpha = 0$), we obtain a standard expression for the discount factor featuring non-durable consumption risks and risks in the wealth portfolio:

$$M_{t+1}^{Non-Dur} = \beta^\theta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R_{c,t+1}^{\theta-1}. \tag{12}$$

As can be seen from the stochastic discount factor expressions in (8) and (12), compared to a single good economy, in a two good economy, the agent is further concerned about the composition risk due to the relative movements in the two goods, captured by the fluctuations in their relative share Z_t.

In the next section, we specify the exogenous dynamics of the macroeconomic inputs which drive the non-durable and durable consumption growth rates and solve for the equilibrium asset prices using the Euler condition in Equation (11).

3 Model Specification

3.1 Economy Specification

Denote g_t, the vector of non-durable consumption growth, non-durable inflation rate, and growth rate of stock of durables: $g_t = \begin{bmatrix} \Delta c_t & \Delta \pi_t & \Delta s_t \end{bmatrix}'$. Following the long-run risks model (Bansal and Yaron (2004)), we model their dynamics by incorporating a time-varying expected growth component x_t :

$$g_{t+1} = \mu_g + x_t + \Sigma_g \eta_{t+1}, \tag{13}$$

where η_{t+1} is a three-dimensional vector of independent Gaussian shocks, μ_g is the vector of unconditional means of the variables, and Σ_g is the volatility matrix. The three-dimensional state vector x_t captures the persistent variations in expected growth of non-durable consumption, expected inflation and expected growth of durable stock. We model x_t as a flexible VAR(1) process:

$$x_{t+1} = \Pi x_t + \Sigma_x u_{t+1}, \tag{14}$$

where Σ_x is the volatility matrix and u_{t+1} is a three-dimensional vector of independent innovations in expected growth which are assumed to be uncorrelated with short-run news η_{t+1}. Π is the persistence matrix which captures the persistence and feedback effects between the expected growth rates of non-durable and durable consumption and expected inflation. In particular, an important feature of the data is a non-neutrality of inflation, so that an increase in expected inflation forecasts a decline in future expected consumption of durables and non-durables. Such an inflation non-neutrality operating both through durable and non-durable channels can be captured by our expected growth specification above.

Our two good economy dynamics extend typical specifications of the model in the long-run risk literature. The original specification in Bansal and Yaron (2004) features a real economy with a single non-durable good. Bansal and Shaliastovich (2011), Eraker (2006), and Piazzesi and Schneider (2006) consider a nominal economy with a single consumption good and specify a bi-variate model for the dynamics of expected consumption and expected inflation. In the two good real economy of Yang (2010), the dynamics of durable and non-durable consumption are driven by a single expected growth component. In our specification of a nominal two good economy, we allow for separate processes in expected non-durable consumption growth, the expected durable consumption growth, and the expected inflation rate. We filter these expected growth rates out from the observed data using Kalman filtering.

For parsimony, in our model, the volatilities of all of the shocks in the economy are constant so that the asset price volatilities and the asset risk premia are constant as well. It is important to note that that the key focus of our paper is on the unconditional levels of the inflation premia, their source as a durable versus non-durable consumption, and their unconditional implication for the levels of the nominal term structure. To highlight these effects, we present a flexible model specification for the expected growth state x_t, and choose to shut down the time-variation in the stochastic volatility as it has second-order implications for the unconditional levels of risk premia and yields. The extension of our model to include stochastic volatility, along the lines of Bansal and Shaliastovich (2011) and Hasseltoft (2010), is straightforward and left for future research.

3.2 Discount Factor

To obtain closed-form analytical solutions to the asset prices, we rely on a standard log-linearization of the return on the wealth portfolio (see Appendix A for the details), and we further log-linearize the relative share process[1]:

$$\begin{aligned}\Delta z_{t+1} &= \log \frac{Z_{t+1}}{Z_t} \approx \chi(\Delta q_{t+1} + \Delta s_{t+1} - \Delta c_{t+1}) \\ &= \chi \left(1 - \frac{1}{\epsilon}\right) (i_s - i_c)' g_{t+1},\end{aligned} \tag{15}$$

where i_c and i_s pick out non-durable and durable consumption growth from g_t, and the parameter $\chi \in (0, 1)$ is an approximating constant equal to the average expenditure on durables in the economy, $\chi = \frac{\bar{Q}\bar{S}}{\bar{Q}\bar{S}+\bar{C}}$. This parameter captures the importance of durable goods in the economy. In particular, for $\chi = 0$, the specification reduces to a one good economy.

The equilibrium price-consumption ratio is a linear function of the economic states x_t :

$$pc_t = A_0 + A_x' x_t. \tag{16}$$

Using the Euler equation for the consumption asset, we obtain that the price-consumption loadings satisfy:

$$A_x = \left(1 - \frac{1}{\psi}\right) (I - \kappa_1 \Pi')^{-1} \left((1 - \chi) i_c + \chi i_s\right), \tag{17}$$

where $\kappa_1 \in (0, 1)$ is the log-linearization coefficient whose solution is provided in Appendix A. When the intertemporal elasticity of substitution ψ is above one, the substitution effect dominates the wealth effect. Hence, price of the consumption claim increases with a positive shock to expected non-durable consumption or long-run expected durable consumption. This intuition naturally extends a standard single-

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[1] The linearization of the relative share shuts off the variation in the asset volatilities and risk premia due to the relative share movements (see Cochrane, Longstaff, and Santa-Clara (2008)); these fluctuations are not likely to be important for the unconditional levels of prices which is the key focus of our paper. In Section 5.4, we further document that the approximation is quite accurate since the relative share of durables is quite stable in the data.

good long-run risks model. Furthermore, because positive expected inflation shocks forecast negative future real growth, the loading on the expected inflation is negative.

The real stochastic discount factor, expressed in units of non-durable numéraire, can be written in terms of the fundamental states and shocks in the economy in the following way:

$$m_{t+1} = m_0 + m_x' x_t - \lambda_g' \Sigma_g \eta_{t+1} - \lambda_x' \Sigma_x u_{t+1}, \tag{18}$$

where m_x captures the loadings of the discount factor on the expected growth components, and λ_g and λ_x are the market prices of immediate and expected growth risks. To gain further intuition on the sources and compensation for the aggregate risks in the economy, we can decompose discount factor loadings and the market prices of risks into the components related to non-durable and durable consumption state variables. Specifically, the discount factor loading on the expected growth satisfies

$$m_x = -\left(\frac{1}{\psi}(1-\chi) + \frac{1}{\epsilon}\chi\right) i_c + \chi\left(\frac{1}{\epsilon} - \frac{1}{\psi}\right) i_s. \tag{19}$$

The two components in brackets capture the loadings of the discount factor on the expected non-durable consumption and expected durable consumption, respectively. When $\chi = 0$ the specification reduces to a one good non-durable model, and the discount factor loading is equal to the negative of the reciprocal of the IES. With durable goods, both the intertemporal and intratemporal elasticities of substitution determine the response of the discount factor to the underlying economic states. In a two good economy, similar to a one good economy, the loading on expected non-durable consumption is negative. On the other hand, when $\epsilon < \psi$ the loading on the expected durable consumption is positive: when two goods are relatively hard to substitute, an expected increase in durable consumption for a given expected consumption of non-durables actually results in an increase in the expected marginal utility of the agent. Thus, because of the complementarity between the two goods, the shocks in expected durable and expected non-durable consumption can have opposite effects on the discount factor.

In a similar way, we can decompose the market prices of immediate and expected growth risks in the economy:

$$\begin{aligned}\lambda_z &= \left(\gamma(1-\chi) + \frac{1}{\epsilon}\chi\right) i_c + \left(\gamma - \frac{1}{\epsilon}\right) \chi i_s, \\ \lambda_x &= (1-\theta)\kappa_1 A_x.\end{aligned} \tag{20}$$

In a one good non-durable economy, $\chi = 0$, and the price of short-run consumption news is γ, while the price of expected growth news is given by $(1-\theta)\kappa_1 A_x$. Including durables, the price of non-durable consumption risks changes to a weighted average of the risk aversion and the inverse of intratemporal elasticity of substitution, where the weight is determined by the importance of durables in the agent utility. The price of a short-run durable risk depends on the relative magnitude of the risk-aversion coefficient and the inverse of intratemporal elasticity of substitution, and is expected to be positive when intratemporal elasticity ϵ is not too small. Due to a non-neutrality of inflation, the price of the expected inflation risks is non-zero. In particular, as we expect high inflation to be bad news for expected growth, the price of the expected inflation risks is negative.

It is important to note that with an expected utility ($\gamma = 1/\psi$), the market prices of expected durable and non-durable consumption risks are all equal to zero: $\lambda_x = 0$. In this case, only the short-run innovations in consumption are priced.

3.3 Equilibrium Bond Yields

Unlike a single good economy, in a multi-good economy there are arbitrarily many ways to define a risk-free asset, which depends on the choice of a basket of goods to be delivered in the future. Define Π_t^* the price of this basket in units of the numéraire, which in our case is a non-durable consumption. Then, the price of an $n-$period bond which delivers this basket satisfies the Euler equation:

$$P_{t,n} = E_t M_{t+1} \frac{\Pi_t^*}{\Pi_{t+1}^*} P_{t+1,n-1}. \tag{21}$$

Setting $\Pi_t^* = 1$ gives us a pricing equation for a risk-free bond which delivers one unit of nondurables. $\Pi_t^* = Q_t$ (or $\Pi_t^* = P_t^d$) enables us to derive a price of a claim to a unit of durable consumption in the future, where the price deflator uses rental cost (purchase price of durables). Following Piazzesi et al. (2007), we can further define Π_t^* to be the ideal price index which defines a basket based on the investors' preferences. Using the rental values, the ideal price index is given by,

$$\Pi_t^* = \left((1-\alpha)^{\epsilon} + \alpha^{\epsilon} \left(Q_t\right)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}. \tag{22}$$

The bond pricing equation in (21) also gives the prices of nominal bonds. In this case, Π_t^* corresponds to the nominal dollar price of a unit of non-durable good, i.e. $\Pi_t^* = \Pi_t$ where Π_t refers to a price index of non-durable consumption, consistent with our model specification in Section 3.1.

For all these numéraire choices we can derive analytical solutions for the bond prices (we use log-linearization in case of an ideal price index). The log bond prices are linear in the vector of economic states,

$$p_{t,n} = -B_{0,n} - B_{x,n}' x_t, \tag{23}$$

where the bond loadings depend on model and preference parameters and the choice of the numéraire. For example, using the non-durable consumption as the numéraire, ($\Pi_t^* = 1$), we obtain that a one-period bond yield satisfies,

$$r_t = const - m_x' x_t. \tag{24}$$

Following our discussion in the previous section, the yield on a one-period claim to a unit of nondurables positively responds to news about long-run expected non-durable consumption, and negatively to news of long-run expected durable consumption if $\epsilon < \psi$. It is well-known that in a single-good economy real bonds hedge news in expected consumption: the price of a real bond goes up when expected consumption is low (e.g., Bansal and Yaron, 2004). In a two good economy, a risk-free claim to non-durables still hedges the risks in expected non-durable consumption, but when the two goods are relatively complementary it is risky with respect to the fluctuations in expected durable consumption. This composition effect can play a significant role in determining the signs and levels of the risk premia on bonds in a two good relative to a single good case.

Denote $rx_{t+m,n}$ the excess log return on buying an n month bond at time t and selling it at time $t+m$ as an $n-m$ period bond:

$$rx_{t+m,n} = -p_{t,n} + p_{t+m,n-m} + p_{t,m}. \tag{25}$$

The expected excess return on n-period bonds is given by the covariance of the discount factor with the excess bond return up to Jensen's inequality term,

$$\begin{aligned} E_t rx_{n,t+1} + \frac{1}{2} Var_t rx_{n,t+1} &= -Cov_t(m_{t+1}, rx_{t+1,n-1}) \\ &= -B'_{x,n-1}\Sigma_x \Sigma'_x \lambda_x. \end{aligned} \tag{26}$$

The expected excess return on bonds depends on bond sensitivity to expected growth risks B_x, and the market compensation for these risks, $\tilde{\lambda}_x = \Sigma_x \Sigma'_x \lambda_x$. The bond risk premia capture the contribution of expected non-durable consumption risk, expected durable consumption risk, and risks in expected inflation. In the case of a risk-free bond with a unit payoff of non-durables, its yield loadings on expected non-durable risks and expected inflation are positive, and negative to expected durable growth. Hence, the expected non-durable risks lead to a negative risk compensation for this bond, similar to a standard one good case. The durable risks, on the other hand, contribute positively to the bond risk premia which can lead to a positive total risk premia and an upward slope of the term structure.

The risk premia equations for bonds delivering other consumption bundles are completely analogous. In particular, for nominal bonds, the expected inflation shocks constitute a prominent source of risk due to the non-neutrality of inflation and interaction of expected inflation with real growth. Indeed, consider a Fisher-type equation for nominal bonds:

$$y^{\$}_{t,n} = y_{t,n} + E_t \pi_{t \to t+n} - \frac{1}{2}\frac{1}{n} Var_t \pi_{t \to t+n} + \frac{1}{n} Cov_t(m_{t \to t+n}, \pi_{t \to t+n}), \tag{27}$$

where $\pi_{t \to t+n}$ and $m_{t \to t+n}$ denote the t to $t+n$ multi-period inflation rate and stochastic discount factor, respectively. Because expected inflation is non-neutral and high expected inflation predicts a persistent decline in expected real growth in the economy, the last term in the above equation which captures the inflation premium is positive and increasing at long maturities. In a single good economy the inflation premium arises only due to the long-run inflation's interaction with non-durable consumption. With multiple goods, however, the inflation non-neutrality, through its long-run negative covariation with durable consumption, gives rise to a second and even more significant component in the total inflation premium.

Notably, with expected CRRA utility, market prices of expected durable and non-durable risks are equal to zero. Therefore, bond premia are equal to zero, which leads to a flat term structure of interest rate, ignoring the Jensen's variance term.

4 Empirical Results

4.1 Data

We collect quarterly data on nominal non-durable goods and services expenditures, nominal durable goods expenditures, and non-durable and durable good price levels from the Bureau of Economic Analysis (BEA) from 1963Q1 to 2009Q4. Following the literature convention, we assume that consumers derive utility from the real service flow of consumption goods. The non-durable goods and services are consumed at the time of purchase, so the consumption of non-durable goods and services is equal to their expenditure. Durable goods, on the other hand, once purchased, are consumed over multiple periods. Since the BEA only reports the year-end durable good stock levels, we back out the quarterly durable good stock level using the depreciation and expenditure data as in Yogo (2006). Aggregate nominal service flows are deflated by the appropriate price levels and divided by the the total population to yield the real service flow data per capita. The service flows of consumption goods and the inflation rate exhibits strong seasonality at quarterly frequencies, which we remove from our data.

Table 1 presents basic descriptive statistics for our data. The durable good growth rate is about as volatile as the non-durable consumption growth in our sample, but it is significantly more persistent: the first-order autocorrelations of durable and non-durable growth rates are 0.76 and 0.42, respectively. This is similar to the evidence reported in Yang (2010). Moreover, the persistence in durable growth decays at a much slower rate than that of non-durables, as shown in Figure 1. For durable consumption growth, an autocorrelation coefficient remains positive and significant up to ten quarter lag, while the autocorrelation coefficient of non-durable consumption becomes insignificant at about a one-year horizon. As shown in Table 1, the inflation rate is quite volatile in the data: its standard deviation is 1.3% relative to the 1% for both non-durable and durable consumption growth. The inflation rate is quite persistent; its first-order auto-correlation coefficient is 0.78 and it decays very slowly with the horizon.

There is substantial evidence in the data for the long-run interaction between non-durable consumption growth rates, durable consumption growth rates, and the inflation rate. To measure these interactions, we first regress the cumulative average consumption growth on the current inflation rate:

$$\frac{1}{h}\sum_{j=1}^{h} g^i_{t+j} = const + b^i_h factor_t + error_{t\to t+h}, \tag{28}$$

where g^i_t stands either for a non-durable consumption growth Δc_t or for a growth rate of durable stock Δs_t, and $factor_t$ is non-durable inflation π_t. We report the slope coefficients and the R^2's in the regressions in Table 2. As shown in the Table, the slope coefficients are all negative and significant, so that high current inflation is bad news for future consumption. These findings suggest that inflation is non-neutral which is consistent with evidence reported in Piazzesi and Schneider (2006) and Bansal and Shaliastovich (2011) for the non-durable consumption. The novel evidence that motivates this paper is that inflation is also non-neutral for future durable consumption growth, and the inflation effect on durables seems much stronger than for non-durables. Indeed, the slope coefficient in non-durable consumption regressions is -0.80 for a one-year horizon, and it uniformly decreases, in absolute value, with the regression horizon to -0.14 at 5 years. The R^2's in these regressions decrease from 17% at one year to 2% at five years. For durable growth, the inflation slope coefficient is -0.73 at a one-year horizon; it increases to -1.14 at three years at which point it is almost three times as large as the corresponding coefficient in non-durable consumption regressions, and it finally decreases to about -0.93 at a five-year horizon.

The R^2's in the durable consumption regressions reach 25% at 3 to 5 year horizons. That is, while inflation is bad news for both non-durable and durable consumption, it affects future consumption of durables much more than that of non-durables. Intuitively, because durable purchases are long-lasting, they respond more significantly to price fluctuations relative to non-durables which are consumed in the same period.

We further find that future durable consumption is significantly related to the levels of nominal yields in the data. We run the cumulative consumption regressions in Equation (28) on a nominal short rate, so $factor_t$ is one-quarter nominal yield. As shown in Table 2, for non-durable consumption, an increase in yield predicts a fall in non-durable consumption growth up to a three-year horizon, and the effect becomes insignificant afterwards. The R^2's in the regressions is 9% at a one-year horizon, and it decays to zero after three years. On the other hand, interest rates significantly and negatively forecast future durable goods growth. The regression slope coefficient is -0.24 and -0.13 at one-year and five-year horizons, respectively, and the R^2's increase from 15% at one-year to 20% at two and three years. Interestingly, the predictability of future durables by interest rate does not come entirely because of the inflation component in nominal yields. Indeed, as shown in the lower panel of Table 2, in multivariate regressions of future durables on both the short rate and the inflation rate, the slope on the yield remains negative and significant up to three-year horizon. These findings suggest that in the term structure data contain information about future durable growth rates above and beyond the inflation, which empirically motivates our model specification for separate expected durable and expected inflation factors which affect equilibrium asset prices. Our findings on the relative predictability of future durable consumption relative to non-durable consumption complement earlier evidence by Yogo (2006) and Yang (2010) in the case of price-dividend ratios.

4.2 Empirical Estimation

For our benchmark estimation we proceed in two stages. In the first stage, we estimate the model parameters and extract the latent states that govern the dynamics of macro variables in equations (13)-(14) using only the time series of observable macro variables. As using macro data alone does not allow us to identify preference parameters, given the first-stage estimates of the macro dynamics, we estimate preference parameters in the second step using the observations on yields. Thus, in the benchmark case the estimation of macro dynamics is independent of the structural model specification and based only on the observed macro data. This is economically appealing because 1) the implications for the term structure are effectively "out-of-sample" and subject only to the choice of the preference parameters; 2) the estimation of the macroeconomic model and the extraction of economic states using only the macro data is not affected by a possible mis-specification of the economic model; and 3) such an approach allows us for an easier comparison of alternative models keeping the fundamental macroeconomic dynamics unchanged. Naturally, ignoring yield information in the estimation of macro parameters is costly since it is hard to estimate precisely the small but persistent components in expected growth dynamics using macro data alone. As a robustness check, we therefore also conduct a joint estimation of the model by estimating both macro and preference parameters in one stage using macro and yield data.

Specifically, we carry out our first stage of estimation using Bayesian Markov-Chain-Monte-Carlo under uninformative (uniform) priors. The likelihood function is standard and is computed using Kalman filtering techniques; all the estimation details are provided in the Appendix. The advantage of Kalman filtering is that we recover estimates of the unobserved latent state-variables, x_i for $i = \{c, s, \pi\}$. Bayesian MCMC algorithm also provides a posterior distribution of estimated parameters and latent variables which can be used to construct confidence intervals for the estimators. Importantly, we construct the estimates of the filtered state variables and the model parameters without using the financial markets' data.

Table 3 reports the parameter estimates of our model. Consistent with our earlier findings, expected durable consumption and expected inflation are more persistent than expected non-durable growth. The implied autocorrelation for expected non-durable growth is 0.41, relative to 0.92 for durable and 0.94 for inflation. Overall, the estimated model can very well match the magnitude of the persistence and the decay in the autocorrelations in the macroeconomic variables with the horizon, as shown in Figure 1. Inflation shocks have a negative impact on non-durable consumption and a significantly larger and more permanent impact on durable consumption: the VAR loading of expected non-durable consumption on the lag of expected inflation is -0.06, relative to -0.10 for expected durables. Because of a high persistence of expected durables and expected inflation, inflation non-neutrality is magnified even further at longer frequencies. To highlight dynamic multi-horizon interactions between the expected growth rates, we plot impulse response functions for the three expected growth shocks in Figure 2. A one-standard deviation shock to non-durable expected growth increases future non-durable consumption growth, but the impact disappears very quickly after a year. Shocks to nondurable expected growth do not have significant impacts on future inflation and durable growth. On the other hand, shocks to expected inflation are very persistent and significantly affect the economy for up to five years. Specifically, positive shocks to expected inflation lower both future nondurable and durable growth. The negative impact of inflation, however, is much stronger at all the horizons and much more long lasting for expected durable growth than for expected growth of non-durables, which confirms our earlier findings in Table 2.

Overall, our macroeconomic model captures very well the dynamics of macroeconomic variables in the data. As shown in Figure 3, the filtered expected states track closely data realizations. The expected non-durable growth predicts the next-quarter realized non-durable consumption with an R^2 of 10%. The R^2's in the regressions of the next-period durable consumption and inflation on their corresponding expectations are 59% and 54%, respectively.

In the first stage of estimation, we obtain the macroeconomic model parameters and the time series of latent expected growth variables. This step does not allow us to identify the preference parameters δ, ψ, γ and ϵ and the relative importance of durables χ. We follow Ogaki and Reinhart (1998), Piazzesi et al. (2007) and Yogo (2006) and estimate the elasticity of intertemporal substitution ϵ from the regression of the user cost of durables on durable and non-durable consumption levels. Indeed, from the intratemporal condition for the user cost of durables in Equation (6) we obtain that

$$q_t = \frac{1}{\epsilon}(s_t - c_t), \tag{29}$$

which allows us to estimate the elasticity of substitution ϵ using the data measurements of user cost of capital and levels of goods. Our estimate of $\epsilon = 0.81$ agrees very well with the above studies. The durable parameter χ captures the relative expenditure of durable goods. We set χ to 20% which is equal to the average relative expenditure on durables in our sample. This parameter plays an important role in the model as it determines the strength of the durable goods' effect on the equilibrium asset prices, and we discuss the model implications at alternative values of χ in Section 5.2.

For identification purposes, we fix the subjective discount factor at 0.996.[2] In our benchmark estimation, we choose the remaining preference parameters ψ and γ in the second stage of estimation by minimizing the mean-squared error (MSE) calculated from the 1 and 5 year model implied and historical yield data. That is, let Y_t^{data} denote the vector of 1 and 5 year yield observed in the data, and let Y_t be the equilibrium yields in the model. In the second step, we estimate preference parameters to minimize the squared pricing errors:

$$\min_{\gamma,\psi} \sum_{t=1}^{T} (Y_t - Y_t^{data})'(Y_t - Y_t^{data}). \tag{30}$$

We compute the standard errors on the preference parameters accounting for the estimation error from the first stage. In particular, we perform a parametric bootstrap where we simulate data of the same size as our sample. We then apply the same two-stage estimation approach as in the data and obtain a new set of estimated preference parameters. The standard errors are computed by the standard deviation of the preference parameters across a number of model simulations. The estimates of preference parameters are reported in Table 3. At quarterly frequency, the estimated intertemporal elasticity of substitution is 2.56, and the risk aversion coefficient is 24.12. These estimates are similar to ones used elsewhere in the long-run risks literature (e.g. Bansal and Yaron (2004)). Notably, the risk aversion coefficient is estimated very imprecisely. This reflects the fact that it is hard to estimate accurately persistent risks from the macroeconomic data alone: a small decrease in the estimated persistence of durables and/or inflation would require a substantial increase in the risk aversion to match the slope of the term structure. Indeed, we show in Section 5 that risk aversion estimates become smaller and much more precise once we increase the share of durables in the economy. The precision of the risk aversion parameter also significantly improves in a one-stage estimation of the model.

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[2] This is related to the discussion of identification issues in Kocherlakota (1990) and is similar to the approach in Marshall (1992) and Bansal and Shaliastovich (2011).

4.3 Equilibrium Nominal Yields

The model implications for nominal yields are reported in Tables 4-6, while Figures 4 and 5 plot time-series of the short rate and term spread in the data and in the benchmark model. Unconditionally, our model matches perfectly the levels of 1 year and 5 year nominal yields, and the fit to 3-year yield is nearly exact as well. The volatilities of nominal yields are somewhat lower in the model (e.g., 1.9% in the model relative to 2.9% in the data for a 1-year yield); we show in Section 5.3 that the volatilities of yields in the model become much closer to the data in the joint macro-yield model estimation. Generally, the model-implied yields track the empirical yields in the data quite well, as shown in Figures 4 and 5. Some of the noticeable deviations of the model predictions to the data include the mid-eighties, where interest rates peaked significantly above what is predicted by our model, as well as the recent episodes in early and late 2000s, where the yields in the data were below the model predictions. These episodes have to do with particularities of the interest rate policies and the movements in the aggregate uncertainty which are outside our model.

Table 6 shows additional conditional implications of the model which we obtain by regressing a short nominal rate on the three filtered expected growth states. In the data, short term interest rates load positively on the expected non-durable growth and the expected inflation, and negatively on the expected durable growth. In our benchmark model, the signs and magnitudes of the bond loadings match the data quite well. Indeed, the slope coefficients on expected non-durable growth is 2.67 in the data compared to 2.25 in the model. The slope is 4.0 on expected inflation both in the model and in the data, and it is -1.31 on expected durables in the data relative to -0.69 in the model. These findings in the data are consistent with our earlier evidence for the predictability of future durable consumption growth by the yields (see Table 2): an increase in yields predicts a decrease in future durable consumption, even controlling for the current expected inflation in the economy. In the model, a negative response of yields to expected durable consumption is an important equilibrium implication which depends, among other things, on the magnitudes of elasticity of substitution parameters and the underlying preference structure. Indeed, as we discuss below, the negative response of yields to expected durable growth shocks cannot be obtained in a one good economy, or under the restriction of the preferences to power utility.

To highlight the role of the durable channel and the recursive utility for the nominal term structure, we first remove the durables from the preferences of the agent by setting their relative weight χ to zero. We refer to this model specification as $ND - EZ$ in the Tables. It is important to note that the dynamics of the macroeconomic variables is fixed through all the model variations, so all the changes in equilibrium asset prices are driven only by the change in the preference structure. In the recursive utility model specification based on a single non-durable good, the model can still generate an upward sloping term structure, but the term spread (11 basis points) is much smaller than in the data and in the full benchmark model (60 basis points). This confirms that a significant component of a positive risk premia on nominal bonds is a result of the durable channel. Notably, the one good version of the model can still deliver a sizable nominal bond risk premium if we increase the risk aversion coefficient to above 100 to magnify the inflation risk premium channel from the negative interaction between expected inflation and expected non-durable growth, as in Piazzesi and Schneider (2006).[3] However, in a single good version of the economy, the loading of short-term nominal yield on expected durable growth is zero at any risk aversion parameter, as shown in Table 6. That is, the non-durable model cannot capture an interaction between yields and durable consumption in the data.

To highlight the importance of recursive preference structure, we report the equilibrium implications for nominal yields in the expected CRRA utility case in a two good economy ('Dur-CRRA') as well as in a one good economy ('Nd-CRRA'). With power utility, the market prices of expected growth risks are all zero, so the risk premia on bonds are zero, up to Jensen's variance term. As shown in Table 4, the implied nominal term structure is flat and somewhat downward sloping, and the differences in one- and two good CRRA economies are quite minor. Notably, the CRRA economy with durable goods leads to a positive equilibrium loading on the expected durable growth component: now the elasticity of intertemporal substitution $\psi = 1/\gamma$ is below the elasticity of intratemporal substitution ϵ, so bond yields respond positively to a shock to expected durable growth. This is contrary to the empirical evidence.

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[3] The preference parameter implications are subject to the first-stage estimation of the macro model, and in particular, of persistence of the expected growth states. Bansal and Shaliastovich (2011) use forecast data to better capture the fluctuations in expected growth and report reasonable values of preference parameters in one good economy.

4.4 Equilibrium Real Yield

It is well known that in one good economy, the real yield curve is downward sloping when investors prefer early resolution of risk and dislike persistent shocks (see Bansal and Yaron (2004)). In two good economy, however, the real yield may become upward sloping due to the complementarities between the two goods and if the durable good channel is strong enough. When there are multiple consumption goods in the economy, there are many ways to define a risk-free asset depending on the choice of a consumption bundle to be delivered in the future. We first focus on the real bond which delivers a unit payoff of a non-durable consumption asset, and compare the implications for the term structure of the interest rates across different model specifications. As shown in Table 7, in our benchmark model, the real yield curve is nearly flat and U-shaped: it is downward sloping from 1.7% to 1.67% from 1 to 3 year maturities, and becomes upward sloping and goes up to 1.75% at 10 year horizon. The non-durable channel contributes negatively to the risk premia and term spread on real bonds, and its effect dominates in the short run. Over the long run, the durable channel which contributes positively to the risk premium on real yields takes over and makes the real yield curve upward sloping. As shown in the Table, all model restrictions to a single-good non-durable economy and/or CRRA preferences predict a downward-sloping term structure of real interest rates.

The lower panel of Table 7 examines the equilibrium real yields under the alternative consumption bundles in the benchmark model with recursive preferences and durable goods. When the real bond delivers one unit of non-durable good, the real yield curve is relatively flat. When real bonds are defined to deliver one unit of durable good at maturity, the real yield curve is steepest among the three and goes up from 0.35% for 1-year bonds to 0.69% for 10 year bonds. In this case, durable goods affect the prices through two channels: the durable consumption growth affects the SDF, and the durable goods price inflation determines the final payoff on bonds. Using the ideal price index generates the real yield curve in between the two, and the implied real yield curve is upward sloping. The real yield at a one-year horizon is 1.28%, and it increases to 1.45% at five-year maturity.

5 Robustness and Other Model Implications

5.1 Long-term Yields

As a robustness check we examine our model's implications for the equilibrium yields at very long maturities. The implied nominal yield at thirty-year maturity is 8%, while the real yield (based on non-durable consumption as a numéraire) is 1.87%. For a more comprehensive assessment of the long-term properties of the economy, we use an approach in Alvarez and Jermann (2005) to decompose the stochastic discount factor into a martingale and permanent component. Alvarez and Jermann (2005) show that the relative contribution of the variance of the permanent component to the total variance of the discount factor captures one minus the risk premium on a long-term bond with infinite maturity to the maximum risk premium in the financial markets. Empirically, the authors argue that this ratio should be close to one. Koijen, Lustig, Nieuwerburgh, and Verdelhan (2010) examine this ratio in the context of a single-good long-run risks model specification and conclude that it can impose a tight restriction on asset-pricing models.

In our benchmark model this ratio for a nominal pricing kernel is around 0.76, implying that the infinite horizon nominal bond risk premium is about 24% of the maximum nominal risk premium in this economy. The model-implied ratio for real discount factor is around 0.98, implying that the infinite horizon real bond risk premium is about 2% of the maximum real risk premium. These ratios are quite close to 1.

5.2 The Role of Durable Expenditure Share

The durable expenditure share χ is an important quantity as it determines the role of the durable good channel in the model. In our benchmark calibration, we fix this ratio to 0.2 using the estimate of the relative expenditure on durables in the data. However, the literature considers a wide range of values for this parameter. For example, the calibration in Yogo (2006) implies a durable expenditure share above 80%. Yang (2010) uses Cobb-Douglas utility with $\alpha = 0.5$, which implies a constant durable expenditure share of 50%. To investigate our model implications under different values of χ, we re-estimate the second stage of the model using higher values of χ. We find that increasing χ enhances the role of durables in the economy, which allows for a lower estimates of the risk aversion coefficient. For example, when $\chi = 0.5$, we find that the second-stage estimate of risk aversion drops to 12 (S.E. of 24.8) and that of intertemporal elasticity of substitution to 1.3 (S.E. of 0.53), so we need a lower risk aversion and IES to explain the term structure. In addition, real term structures become significantly more upward sloping due to an increase in the amount of durable risk.

5.3 Macro-Yield Joint Estimation

To check the robustness of our findings, we consider a one-step estimation of the model where we use the macroeconomic variables and three yields at 1, 3 and 5-year maturity bond yields to jointly identify the macroeconomic model and preference parameters; see Appendix C for econometric details.

We report point estimates in Table B.1 in the Appendix. The estimate of preference parameters is very close to what we obtain in a two-stage estimation: the estimated risk aversion is 27.2, the intertemporal elasticity of substitution is 2.39. The preference parameters, and in particular, the risk aversion, is estimated much more precisely relative to a two-stage approach. Using yield information somewhat changes the point estimates of the macroeconomic dynamics parameters, however, their implications for the macroeconomic data are quite similar to a two-stage estimation. The two methods further produce quite similar filtered sequences of expected growth variables. The correlation between two extracted expected inflation states is about 70%, and the correlation between the two time series of expected durable growth is 95%. The expected non-durable growth state variable is somewhat more persistent in the macro-yield joint estimation, and the correlation across the two estimations is around 30%.

Overall, the model implications based on one-stage or two-stage estimations are close to each other. The model implications based on the macro-yield joint estimation are presented in Table 8. Both estimations match the level of nominal yields nearly perfectly. Furthermore, the macro-yield joint estimation is able to match the volatility of nominal yield in the data. The MSE reported for macro-yield joint estimation is much lower than that in macro-only case. In Panel B of Table 8, we find that the real yield curve produced by two estimators are very close to each other. As the joint estimation identifies higher persistence in expected durables, the real term structure becomes upward sloping in a joint estimation.

5.4 Log-linearization of Relative Share

To solve the model analytically, we log-linearized the expression for the the relative share (see equation (15)). Because of the log-linearization, the relative share fluctuations do not impact the variations in the fluctuations in the volatilities and premia in the economy (see Cochrane et al. (2008)). To assess the quality of the approximation, we compare the true and log-linearized growth in log share z_t. In the data, the correlation between the two is 0.997, and their two volatilities are identical to each other, 0.0012. Thus, as the share of durables is relatively stable in the data, we do not expect the share log-linearization to have a material impact on the solution.

Conclusion

In the data, durable goods growth is persistent and highly predictable because it is negatively correlated with past nominal yields and inflation. These effects are much more pronounced for durable consumption than for nondurable consumption. That is, inflation appears non-neutral for future real growth, and more so for durable goods rather than non-durable goods. Motivated by these findings, we set up a two good, long-run risks-type nominal economy which features nonseparable utility over consumption of durable and nondurable goods, persistence fluctuations in expected growth rates, inflation non-neutrality, and recursive utility with preference for early resolution of uncertainty. We show that the model can successfully, and effectively out-of-sample, explain unconditional moments and the conditional movements in the term structure. Model-implied equilibrium real yields are upward sloping, which cannot be obtained in a one good economy, for a range of numéraire choices. Empirically, we find that most of the inflation premium in the model comes from a durable risk channel. Overall, our findings suggests that long-run durable risks play an important role in explaining the bond prices in the data.

Appendix

A Model Solution

The log-linearization parameter for the consumption asset κ_1 satisfies the following recursive equation:

$$\begin{aligned}\log \kappa_1 = {} & \log \beta + \left(1 - \frac{1}{\psi}\right) \left((1-\chi)i_c + \chi(i_a + i_s)\right)' \mu \\ & + \frac{1}{2}\theta \left(1 - \frac{1}{\psi}\right)^2 \left((1-\chi)i_c + \chi(i_a + i_s)\right)' \Sigma_g \Sigma_g' \left((1-\chi)i_c + \chi(i_a + i_s)\right) \\ & + \frac{1}{2}\theta \kappa_1^2 A_x' \Sigma_x \Sigma_x' A_x.\end{aligned} \tag{A.1}$$

The discount factor parameters are given by

$$m_0 = \theta \log \delta + (1-\theta)\log \kappa_1 - \lambda_g' \mu. \tag{A.2}$$

The nominal discount factor parameters satisfy

$$m_0^{\$} = m_0 - i_\pi \mu_g, \quad m_x^{\$} = m_x - F' i_\pi, \quad \lambda_g^{\$} = \lambda_g + i_\pi, \tag{A.3}$$

where $i_\pi = \begin{bmatrix} 0 & 1 \end{bmatrix}'$.

The solution for real bond price loadings are given by,

$$\begin{aligned} B_{0,n} &= B_{0,n-1} - m_0 - \frac{1}{2}\lambda_g' \Sigma_g \Sigma_g' \lambda_g - \frac{1}{2}(\lambda_x + B_{x,n-1})' \Sigma_x \Sigma_x' (\lambda_x + B_{x,n-1}), \\ B_{x,n} &= \Pi' B_{x,n-1} - m_x, \end{aligned} \tag{A.4}$$

and similar for nominal bonds using the parameters of the nominal discount factor in Equation (A.3).

B Joint Estimation Evidence

Table B.1: Parameter Estimates: One-Step Joint Macro-Yield Estimation

Macro Variable Model Parameters:

Π

	Δc_t	$\Delta \pi_t$	Δs_t
Δc_t	1.0232 (0.0224)	−0.0195 (0.0026)	−0.0296 (0.0137)
$\Delta \pi_t$	0.4759 (0.0506)	0.8865 (0.0196)	−0.1324 (0.0308)
Δs_t	0.1169 (0.0843)	−0.0581 (0.0123)	0.8918 (0.0551)

	$\Sigma_x \times 1000$			$\text{diag}(\Sigma_g) \times 1000$
	Δc_t	$\Delta \pi_t$	Δs_t	
Δc_t	0.6608 (0.1022)			4.4906 (0.2805)
$\Delta \pi_t$	0.8104 (0.4031)	2.3287 (0.1986)		5.8059 (0.3407)
Δs_t	1.4633 (0.3208)	−0.4763 (0.1768)	1.0942 (0.2143)	1.8057 (0.2223)

Preference Parameters:

γ	ψ	δ
27.2 (12.00)	2.39 (0.20)	0.9956

Parameter estimates for the full model specification: $g_{t+1} = \mu_g + F x_t + \Sigma_g \eta_{t+1}, \quad x_{t+1} = \Pi x_t + \Sigma_x u_{t+1}$. Macroeconomic and preference parameters are estimated in one stage by MLE using Kalman filtering based on macro and yield data. Quarterly observations of non-durable consumption, durable consumption, inflation rate from 1963Q1 to 2009Q4.

C MCMC Estimation

In order to perform inference for the parameters in our model we estimate the posterior distributions of the model parameters using Bayesian MCMC.

Bayes' theorem says that the posterior $\pi(\Theta \mid Y)$ is proportional to the likelihood multiplied by the prior, $\pi(\Theta \mid Y) \propto \mathcal{L}(Y; \Theta)p(\Theta)$. The likelihood function, $\mathcal{L}$, can be computed through Kalman filtering as described in the following.

Let $\eta_t^* = \Sigma_g \eta_t$ is defined as the exogenous random shock to the macro-variables g_t in equation (13),

$$\eta_t^* = y_t - \mu_y - x_t.$$

Let Y_t^{data} denote a vector of observed zero coupon yields of different maturities. Define

$$u_t = Y_t^{\text{data}} - Y_t^{\text{model}}$$

where Y_t^{model} is our model implied counterpart, so that an n maturity zero is $Y_t^{\text{model}} = -p_{t,n}/n$ where $p_{t,n}$ is the log bond price. We assume

$$u_t \sim N(0, \Sigma_u)$$

where Σ_u is diagonal so that we force the pricing errors to be uncorrelated across bonds.

A vector of time t errors are now given by

$$\epsilon_t = \begin{bmatrix} \eta_t \\ u_t \end{bmatrix}$$

where $\epsilon_t \sim N(0, \Sigma)$, and

$$\Sigma = \begin{bmatrix} \Sigma_g & 0 \\ 0 & \Sigma_u \end{bmatrix}.$$

The dynamics of $Y_t = (g_t, Y_t^{\text{data}})$ forms a linear state-space model,

$$Y_t = \mu + F x_t + \epsilon_t$$

where

$$F = \begin{bmatrix} I_3 & 0 \\ 0 & -B_{x,n}'/n \end{bmatrix}.$$

and $\mu = (\mu_g, -B_{0,n}/n)$ and where $B_{0,n}$ and $B_{x,n}$ are given by the equations (A.4). We can now apply standard Kalman filtering to compute the likelihood function. Specifically, we perform Bayesian posterior simulations using MCMC sampling under an un-informative prior. Let V denote an estimate of the covariance of the posterior distribution. Draw $\Theta_p \sim N(\Theta_J, V)$. Set $\Theta = \Theta_J^p$ with probability $\alpha = \min(1, \pi(\Theta_J^p)/\pi(\Theta))$. In practice, we update fewer than all n parameters in one iteration of the sampler. This avoids the curse of dimensionality associated with sampling in high dimensional parameter spaces.

For the estimation of macroeconomic dynamics without the yield data, we omit Y_t^{data} in the specification of Y_t.

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Tables and Figures

Table 1: Summary Statistics

	Non-Dur Consumption	Non-Dur Inflation	Durable Consumption
Mean	2.04	4.20	4.36
Stdev.	0.95	1.29	0.97
Autocorr	0.42	0.78	0.76

Mean, volatility and autocorrelation of real non-durable consumption growth rate, non-durable inflation rate and durable good growth rate. Mean and volatility are annualized, in percent. Quarterly observations from 1963Q1 to 2009Q4.

Table 2: Consumption Growth Predictability

	1 yr	2 yr	3 yr	4 yr	5 yr
Non-durable Goods:					
By Inflation:					
Slope	-0.804	-0.593	-0.393	-0.237	-0.135
	(0.139)	(0.109)	(0.091)	(0.079)	(0.075)
R^2	0.170	0.153	0.102	0.052	0.019
By Short Rate:					
Slope	-0.135	-0.096	-0.042	0.002	0.025
	(0.033)	(0.026)	(0.022)	(0.018)	(0.017)
R^2	0.092	0.077	0.023	0.000	0.013
By Short Rate and Inflation:					
Slope (yld)	-0.061	-0.054	0.002	0.042	0.070
	(0.046)	(0.036)	(0.031)	(0.026)	(0.024)
Slope (infl)	-0.141	-0.079	-0.083	-0.077	-0.085
	(0.064)	(0.050)	(0.042)	(0.036)	(0.033)
R^2	0.118	0.091	0.045	0.027	0.051
Durable Goods:					
By Inflation:					
Slope	-0.729	-1.061	-1.135	-1.067	-0.931
	(0.203)	(0.177)	(0.156)	(0.141)	(0.130)
R^2	0.073	0.179	0.243	0.258	0.238
By Short Rate:					
Slope	-0.240	-0.271	-0.243	-0.189	-0.133
	(0.044)	(0.039)	(0.036)	(0.034)	(0.032)
R^2	0.152	0.225	0.214	0.156	0.094
By Short Rate and Inflation:					
Slope (yld)	-0.138	-0.152	-0.103	-0.033	0.0334
	(0.062)	(0.055)	(0.049)	(0.046)	(0.043)
Slope (infl)	-0.197	-0.227	-0.266	-0.297	-0.319
	(0.086)	(0.075)	(0.068)	(0.063)	(0.059)
Adj. R^2	0.179	0.266	0.281	0.256	0.234

Projections of future multi-horizon average consumption growth on inflation rate and 3-month nominal interest rate. Top panel is based on non-durable consumption data, while bottom panel is based on growth rate in durable good stock. Quarterly data from 1963Q1 to 2009Q4. Newey-West standard errors with 10 lags.

Table 3: Parameter Estimates: Two-stage Estimation

Macro Variable Model Parameters:

	Π		
	Δc_t	$\Delta \pi_t$	Δs_t
Δc_t	0.385 (0.091)	-0.060 (0.040)	-0.025 (0.029)
$\Delta \pi_t$	0.170 (0.072)	0.951 (0.004)	-0.026 (0.049)
Δs_t	0.003 (0.040)	-0.100 (0.016)	0.877 (0.020)

	$\Sigma_x \times 1000$			$\text{diag}(\Sigma_g) \times 1000$
	Δc_t	$\Delta \pi_t$	Δs_t	
Δc_t	4.056 (0.510)			0.999 (0.844)
$\Delta \pi_t$	-0.781 (0.412)	1.841 (0.399)		3.075 (0.343)
Δs_t	0.861 (0.217)	0.843 (0.319)	1.159 (0.193)	2.097 (0.166)

Preference Parameters:

γ	ψ	δ
24.12 (39.6)	2.56 (0.55)	0.9958

Parameter estimates for the full model specification: $g_{t+1} = \mu_g + F x_t + \Sigma_g \eta_{t+1}, \quad x_{t+1} = \Pi x_t + \Sigma_x u_{t+1}$. Macroeconomic parameters are estimated in the first stage by MLE using Kalman filtering based only on macro data on non-durable consumption growth, non-durable inflation and durable goods growth. Preference parameters are estimated from the Non-linear Least Square fit to 1 and 5 year nominal yields. Quarterly observations of non-durable consumption, durable consumption, inflation rate from 1963Q1 to 2009Q4.

Table 4: Nominal Yields: Data and Models

	Preferences			Yield Levels			Yield Volatility		
	γ	ψ	δ	1y	3y	5y	1y	3y	5y
Data				5.98	6.35	6.58	2.89	2.75	2.63
$Dur - EZ$	24.12	2.56	0.996	5.98	6.33	6.58	1.90	1.64	1.44
$Nd - EZ(1)$	24.12	2.56	0.996	6.42	6.48	6.53	1.81	1.50	1.29
$Nd - EZ(2)$	111.81	2.48	0.997	5.96	6.27	6.57	1.81	1.50	1.28
$Dur - CRRA(1)$	24.12	2.56	0.996	62.78	61.20	60.67	9.64	6.34	5.56
$Dur - CRRA(2)$	10.00	0.10	1.056	5.99	5.73	5.65	3.58	1.87	1.52
$Nd - CRRA(1)$	24.12	2.56	0.996	52.23	50.81	50.58	7.33	2.71	1.73
$Nd - CRRA(2)$	10.00	0.10	1.046	5.97	5.75	5.71	2.97	1.04	0.69

Levels and volatilities of nominal yields in the data and in the models. $Dur - EZ$ is the benchmark model with Epstein-Zin utility and non-durable and durable consumption; $Nd - EZ$ uses EZ utility with only non-durable consumption; $Dur - CRRA$ uses both non-durable and durable consumption but restricts the preferences to expected CRRA utility; and $Nd - CRRA$ model uses only non-durable consumption and CRRA utility. Models indexed (1) use preference parameters as in the benchmark model, and (2) re-estimate preference parameters to match nominal yields (risk aversion parameter γ is set to 10 in CRRA case).

Table 5: Nominal Yield and Term Spread: Data and Models

	Preferences			1y Yield			5y Term Spread		
	γ	ψ	δ	Mean	Std	MSE	Mean	Std	MSE
Data				5.98	2.89	0.00	0.60	0.86	0.00
$Dur - EZ$	24.12	2.56	0.996	5.98	1.90	1.96	0.60	0.47	0.82
$Nd - EZ(1)$	24.12	2.56	0.996	6.42	1.81	2.05	0.11	0.54	0.95
$Nd - EZ(2)$	111.81	2.48	0.997	5.96	1.81	2.00	0.60	0.54	0.82
$Dur - CRRA(1)$	24.12	2.56	0.996	62.78	9.64	57.82	-2.10	6.18	6.64
$Dur - CRRA(2)$	10.00	0.10	1.056	5.99	3.58	4.76	-0.34	2.61	2.72
$Nd - CRRA(1)$	24.12	2.56	0.996	52.23	7.33	46.95	-1.66	5.82	6.30
$Nd - CRRA(2)$	10.00	0.10	1.046	5.97	2.97	3.69	-0.26	2.39	2.65

1 year nominal yield and 5-1 year term spread in the data and in the models. $Dur - EZ$ is the benchmark model with Epstein-Zin utility and non-durable and durable consumption; $Nd - EZ$ uses EZ utility with only non-durable consumption; $Dur - CRRA$ uses both non-durable and durable consumption but restricts the preferences to expected CRRA utility; and $Nd - CRRA$ model uses only non-durable consumption and CRRA utility. Models indexed (1) use preference parameters as in the benchmark model, and (2) re-estimate preference parameters to match nominal yields (risk aversion parameter γ is set to 10 in CRRA case).

Table 6: Nominal Yield Loadings on Expected Growth States

	Exp. Non-Dur.	Exp. Infl.	Exp. Dur.
Data	2.67 (0.79)	4.03 (0.29)	-1.31 (0.42)
$Dur - EZ$	2.25	4.00	-0.69
$Nd - EZ(1)$	1.56	4.00	0.00
$Nd - EZ(2)$	1.61	4.00	0.00
$Dur - CRRA(1)$	78.21	4.00	18.30
$Dur - CRRA(2)$	33.00	4.00	7.00
$Nd - CRRA(1)$	96.51	4.00	0.00
$Nd - CRRA(2)$	40.00	4.00	0.00

Slope coefficients in the regression of one-quarter nominal yield on the estimated expected growth state variables, in the data and across the models.

Table 7: Equilibrium Real Yields

	1y	3y	5y	10y
Paying Unit of Non-Durables:				
$Dur - EZ$	1.70	1.67	1.69	1.75
$Nd - EZ(1)$	2.26	2.22	2.21	2.21
$Nd - EZ(2)$	1.43	1.23	1.18	1.13
$Dur - CRRA(1)$	58.70	57.12	56.51	55.33
$Dur - CRRA(2)$	1.92	1.67	1.58	1.39
$Nd - CRRA(1)$	48.15	46.71	46.44	46.19
$Nd - CRRA(2)$	1.91	1.69	1.65	1.61
For Different Consumption Baskets:				
Non-durable	1.69	1.67	1.70	1.75
Ideal	1.28	1.26	1.32	1.45
Durable	0.35	0.32	0.42	0.69

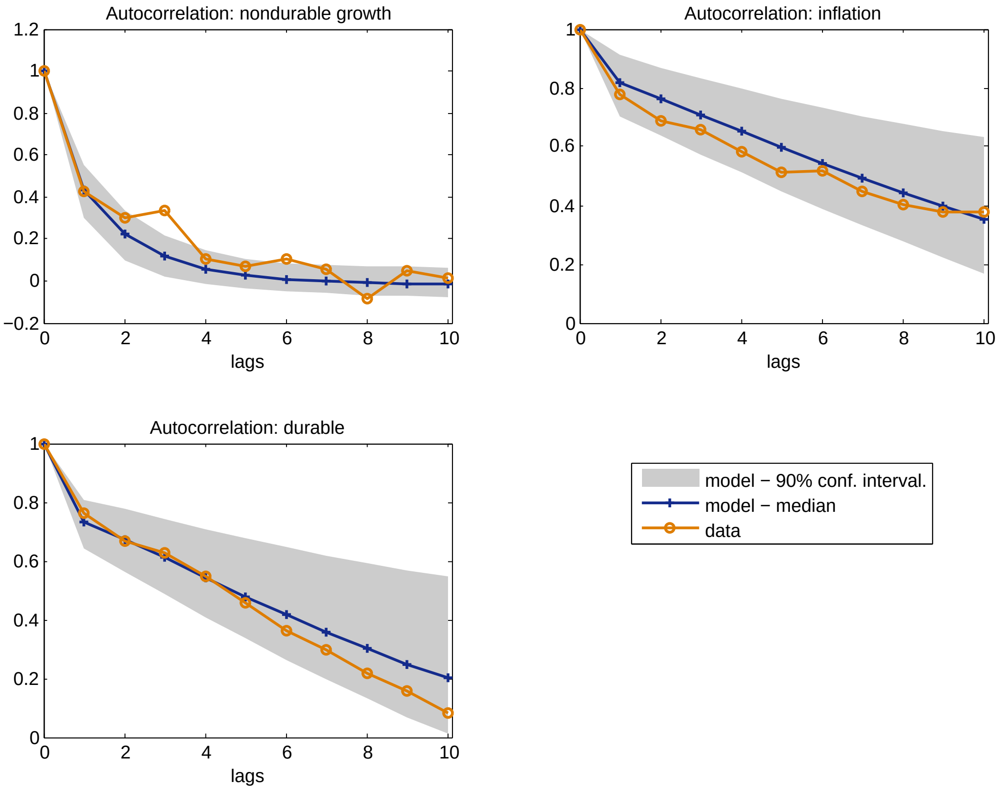
Model-implied term structure of interest rates on real bonds. A top panel compares the real yields on bonds which deliver one unit of non-durable consumption across different model specifications. Bottom panel reports real yields in the benchmark model with durable goods and recursive utility for different choices of consumption basket. "Non-durable" pays a unit of non-durable good, "Ideal" uses the ideal price index and "Durable" pays a unit of durable goods valued at a purchase price.

Table 8: Equilibrium Yields: 2-Stage v.s. Joint Estimation

	1y	3y	5y
Nominal Yields:			
Data	5.98	6.35	6.58
	(2.89)	(2.75)	(2.63)
2-Stage	5.98	6.33	6.58
	(1.90)	(1.64)	(1.44)
Joint	5.98	6.35	6.58
	(2.78)	(2.69)	(2.58)
Real Yields:			
2-Stage	1.70	1.67	1.69
	(0.23)	(0.16)	(0.14)
Joint	1.71	1.72	1.74
	(0.36)	(0.34)	(0.31)
	Mean	Stdev.	MSE
One Year Yield Fit:			
Data	5.98	2.89	0.00
2-Stage	5.98	1.90	1.96
Joint	5.98	2.78	0.99
Five Year Spread Fit:			
Data	0.60	0.82	0.00
2-Stage	0.60	0.47	0.82
Joint	0.60	0.71	0.39

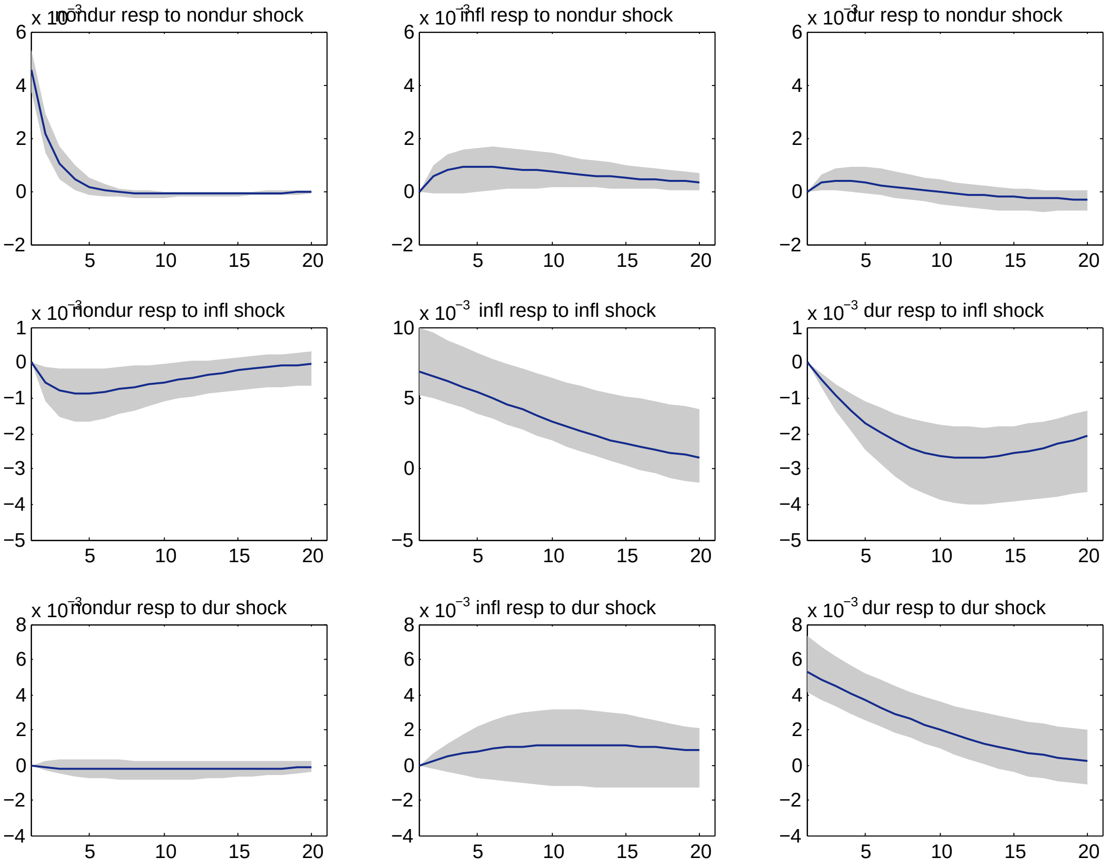
The level, volatilities and mean-squared errors for real and nominal yields in the data and in the model estimated in two stages, and in one step jointly using both macroeconomic and yield data.

Figure 1: Autocorrelation of Consumption Growth and Inflation



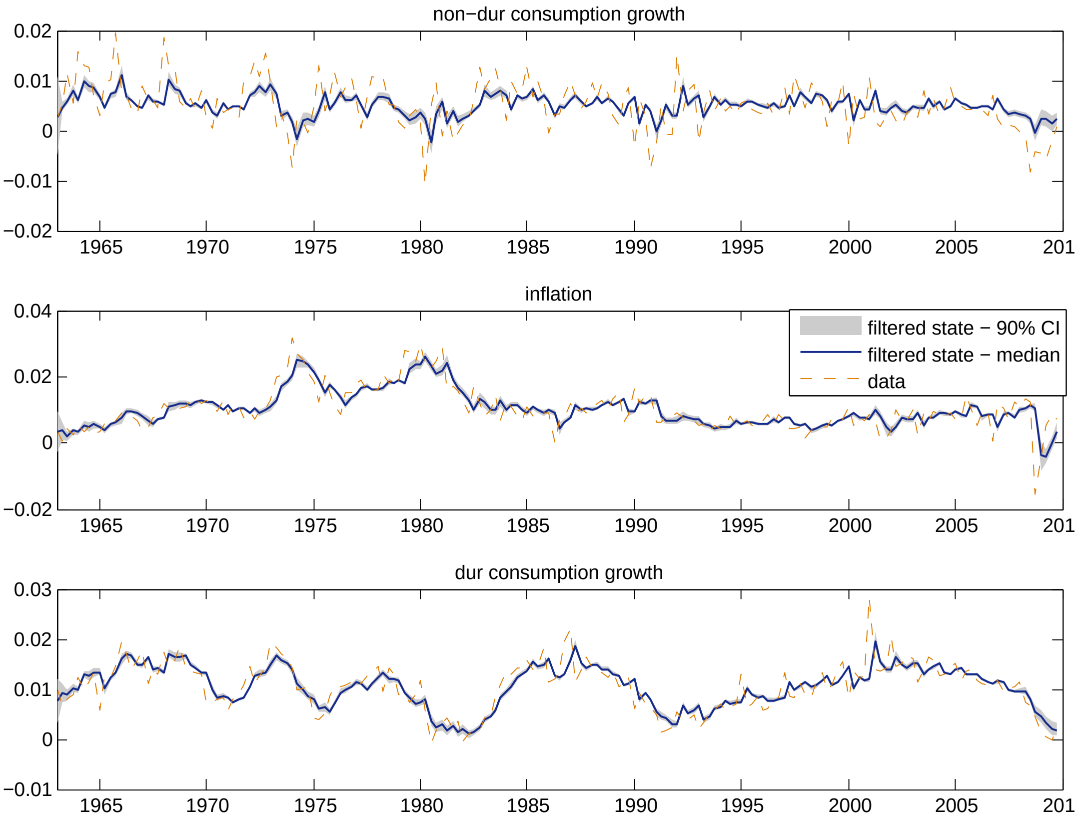
Autocorrelation functions of non-durable and durable consumption growth and non-durable inflation rate based on the estimates in the data and implied by the macroeconomic model. Quarterly observations from 1963Q1 to 2009Q4.

Figure 2: Impulse Response Functions of Expected Growth Shocks



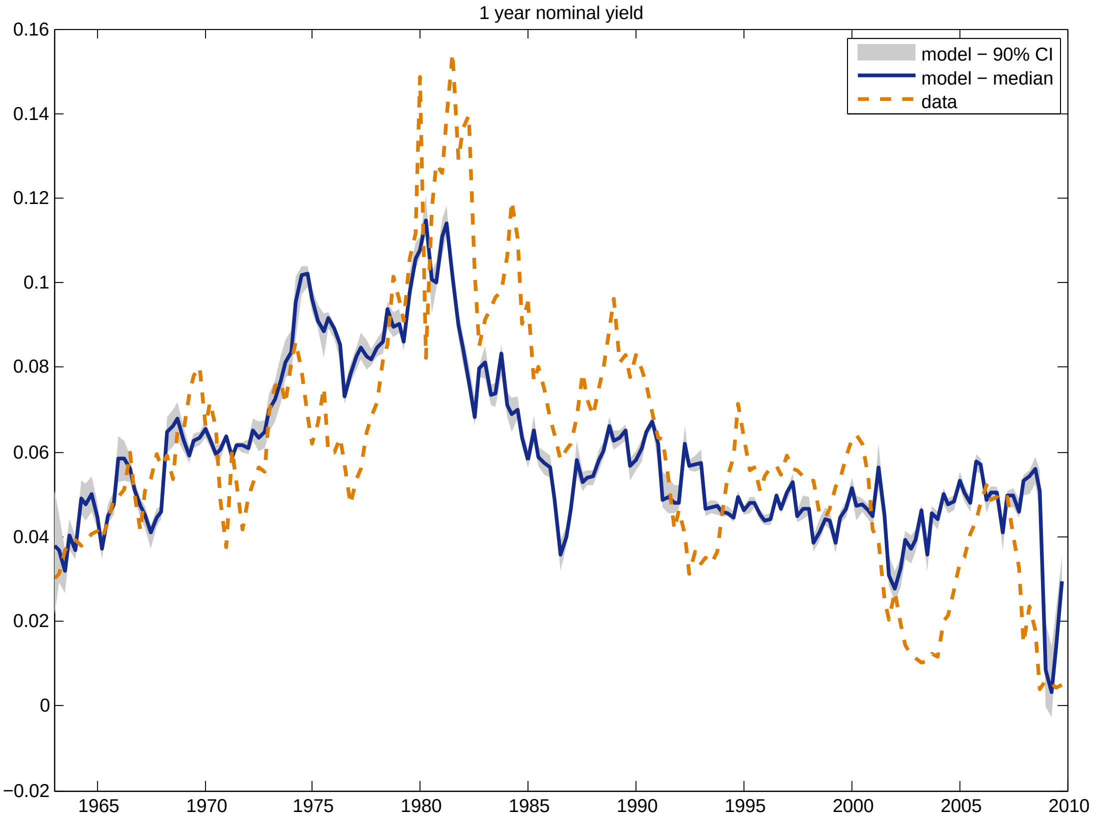
Impulse response functions for shocks to expected non-durable consumption, expected durable consumption and expected non-durable inflation, based on the estimated macroeconomic model. Grey regions correspond to 90% confidence interval.

Figure 3: Realized and Filtered Macroeconomic Variables



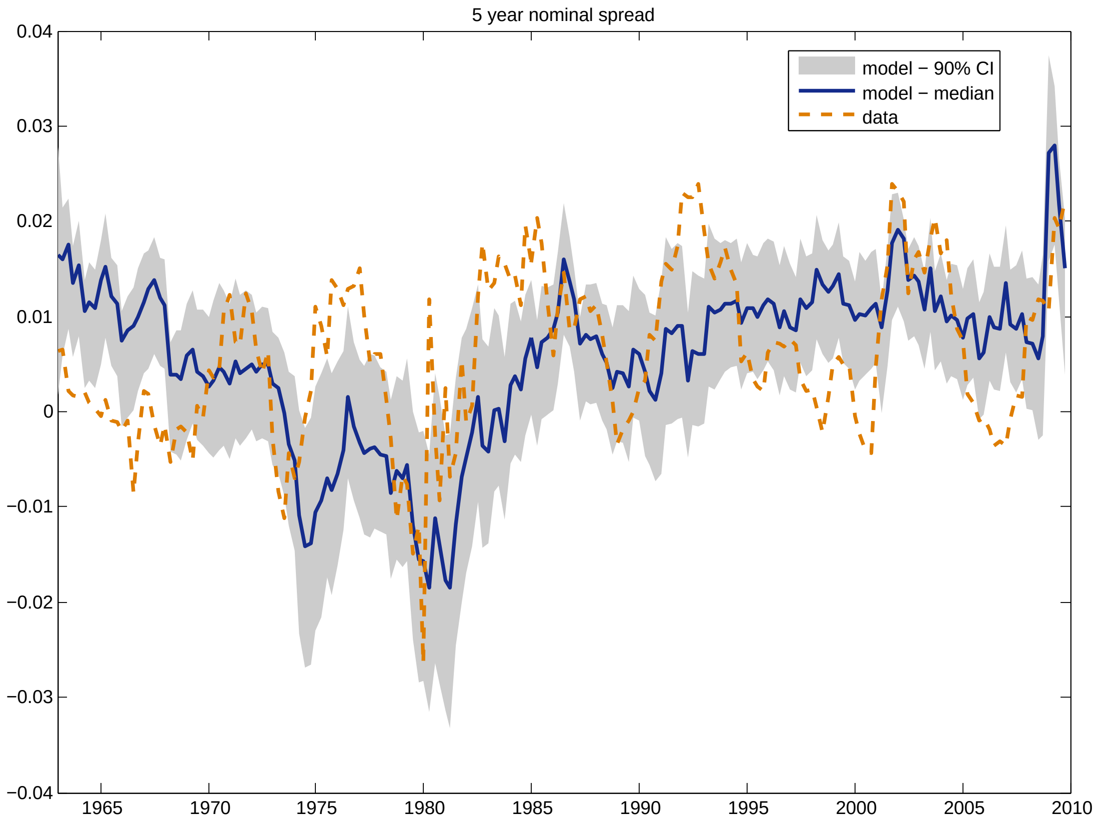
Realized and filtered non-durable consumption growth rate, non-durable inflation rate and durable goods growth rates.

Figure 4: One-Year Nominal Yield: Data and Model



Time series of one-year nominal yield in the data (dashed line) and in the model (solid line).

Figure 5: Nominal Yield Spread: Data and Model



Time series of five minus one year nominal spread in the data (dashed line) and in the model (solid line).