Abstract

The asset-market evidence suggests that investors are concerned with large downward moves in equity prices, which occur about once every one or two years in the data. This evidence is puzzling as there are no concurrent jumps in macroeconomic fundamentals at such frequencies. I estimate a confidence-risk model where consumption shocks are Gaussian, and agents use a constant gain specification to learn about the unobserved expected growth from the cross-section of signals. Investors’ uncertainty (confidence measure) is time-varying and is subject to jumps, which endogenously leads to large negative moves in equity prices and expensive out-of-the-money puts. The model provides a good fit to macroeconomic, asset-price, and forecast data.

JEL Codes: E44, G13, C58

Key words: option prices, jumps, recursive utility, confidence, learning

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1 Introduction

One of the central findings in option markets is that the deep out-of-the-money index put options are too expensive relative to standard models (see e.g. Rubinstein, 1994). This suggests that investors are willing to pay a sizeable premium to hedge against large downward movements in the underlying asset prices. These large moves in the asset markets drive the equity prices down and concurrent market volatilities up at frequencies of once one to two years in the data. The jump evidence from option and equity markets is puzzling from the perspective of economic models. There is no persuasive support for large contemporaneous moves in the real economy at the considered frequencies in the data, which presents a challenge for an economic explanation of jump risks in financial markets.

In this paper, I show that there is a significant link in the data between option prices, equity jumps and the observed confidence risk of investors, measured from the cross-sectional variation in the forecasts about future macroeconomic growth. Drops in investors’ confidence (high cross-sectional uncertainty) are associated, on average, with negative moves in market returns and significant increases in current and future implied volatilities in the option markets. Motivated by this evidence, I set up and estimate a structural model for the option markets, which can explain the observed link between the confidence measure and the variation in option prices and jumps in returns. In the model, there are no jumps hardwired in the aggregate consumption and dividends processes, and the economic source of jump risks stems from sharp increases in investors’ uncertainty regarding future growth.

In the estimation, I find that the confidence risk model provides a good fit to the real

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2This paper focuses on relatively frequent, once every one or two year large moves in equity markets, which are different from very large rare disasters considered in Barro (2006) and Rietz (1988).
consumption, equity, risk-free rate, the option-implied volatilities at difference strikes and maturities, and the expected growth and the confidence measure from the forecast data. The model can quantitatively explain asset-price anomalies in derivative markets and account for the observed large moves in returns at economically plausible preference and model parameters.

The economy setup follows the confidence risks model of Bansal and Shaliastovich (2009) and Bansal and Shaliastovich (2010). As in the long-run risks model of Bansal and Yaron (2004), the dynamics of the true consumption growth is conditionally Gaussian, and features a persistent expected growth component and a time-varying volatility of consumption shocks. However, unlike in a standard model, expected growth is not directly observable, and investors form an estimate of future growth using a cross-section of signals. The time-varying precision of signals determines the uncertainty of investors about their estimate of expected growth. To help distinguish this uncertainty from other measures in the literature, I refer to it to as "the confidence measure." The confidence measure fluctuates over time, and is subject to positive Poisson jumps.

To model learning from the signals, I assume that the agents assign a constant weight to the recent information. Constant gain specification is considerably more tractable than the fully Bayesian specification, and has been used in a variety of economic studies. It can be justified in the environment with possible structural breaks at unknown dates. Alternatively, a constant gain specification is consistent with the recency bias evidence in the behavioral literature. This recency bias evidence in Hogarth and Einhorn (1992), De Bondt and Thaler (1990), Kahneman and Tversky (1973) suggests that the agents have a tendency to over-weight recent information and under-weight less salient data such as long-term averages. De Bondt and Thaler (1990) further find that investors overweigh recent information more in states of high uncertainty. The constant gain specification captures the intuition from these studies: the weight to the new information does not decrease with
signal quality, as in the Bayesian approach, and therefore the agents over-weight the impact of recent information on their forecasts.

I solve the equilibrium model which features confidence jump risks and the constant gain/recency bias learning specification. In the model, investors demand risk compensation for consumption, expected growth, and consumption volatility risks. The novel aspect of the model is that the confidence risks are also priced in the equilibrium, so that when agents have a preference for early resolution of uncertainty, states with higher uncertainty about expected growth are discounted more heavily. Notably, the confidence jump shocks receive risk compensation even though there are no jump risks in consumption. Fluctuating confidence and confidence jumps can explain the option pricing puzzles and jump in returns. In equilibrium, positive jumps in the confidence measure translate into negative jumps in returns and positive jumps in the market volatility. Out-of-the-money put options hedge confidence jump risks and thus appear expensive relative to at-the-money options. This can account for the cross-section of option prices in the data, where Black-Scholes implied volatilities from options are decreasing in the moneyness of the contract. Further, endogenous jumps in equilibrium asset prices can account for the evidence of large downward moves and heavy-tailed unconditional distribution of equity returns in the data.

To show a direct evidence for the link between the confidence measure and asset prices, I use a cross-section of forecasts of real consumption from the Survey of Professional Forecasters and construct the empirical proxies for the average signal and the confidence measure in the data. Consistent with the theoretical model, the average signal corresponds to the average forecast in the data, while the confidence measure is computed from the cross-sectional variance in the forecasts, adjusted by the number of forecasts. The confidence measure contains significant information about option implied volatilities in the data. Its contemporaneous correlations with option variances range from 50% to 60% across the contracts, and it significantly predicts future implied variances 1 and 3 quarters ahead.
even controlling for the current value of the option variance. These findings supplement Bansal and Shaliastovich (2009) who show the evidence for a jump-like component in the confidence measure which is related to large moves in returns and return volatility.

I formally assess the quantitative fit of the model to the asset price, macroeconomic, and forecast data. My asset-price data consist of monthly real market returns, the market price-dividend ratios, the real interest rates, and the option-implied volatilities with moneyness ranging from 0.90 to 1.10 and with 1- and 3-month maturities. I use monthly observations of real consumption to measure aggregate growth, and I include quarterly expected growth and the confidence measure from the forecast data. The estimation exercise is quite challenging due to mixed frequency of observations, non-Gaussian dynamics of the latent states, and a non-linear relation between the data measurements and the economic states. To deal with these issues, I use a Bayesian MCMC, mixed frequency, particle filter estimation approach, which is most closely related to Schorfheide, Song, and Yaron (2013) and Song (2014). Johannes and Polson (2009) provide a handbook treatment of the Bayesian methods with applications in finance.

The quantitative results from the estimation provide empirical support for the confidence risks model. I obtain plausible preference parameters, which indicate that investors have a preference for early resolution of uncertainty. The median posterior estimate of the relative risk aversion is 10.7 and the intertemporal elasticity of substitution is 2.99. The estimated model parameters suggest that the confidence measure significantly fluctuates over time; moreover, nearly all the variation in the series is driven by Poisson jumps. Large moves in uncertainty about future growth translate to large negative jumps in equity returns. The estimated frequency of jumps in asset prices, driven endogenously by jumps in the confidence measure, is once every 2 months, and the average jump in returns is \(-2.5\%\), monthly. Even though the estimated jumps are quite frequent, many of them are quite small to lead to large detectable moves in equity returns. Both in the model and in the
data, the probability of observing a large, two-standard deviation move in asset prices is about 4%, or once every two years.

In the model, the expected growth and the confidence risks contribute the most to the asset risk compensation. The compensation for the expected growth risks is about two-thirds of the market risk premium, while the confidence risks capture about a quarter of the equity risk premium, or 1.7%. As most of the confidence fluctuations are due to jumps, the confidence risk compensation thus captures the jump risk premium in the economy. These estimates of the jump risk premium are consistent with Pan (2002) and Broadie et al. (2007), who find that jump risks account for about one-third of the total equity risk premium.

The model with the confidence jump risks can quantitatively explain the cross-section of option prices and the variation in option-implied volatilities. Based on the median estimates, the unconditional at-the-money volatility is 18.1% in the model relative to 20.1% in the data (19.1% in the pre-crises sample). The difference between the out-of-the-money and at-the-money option volatilities is equal to 6.2% in the model relative to 6.7% in the data at a 1-month horizon, and it is 5.2% in the model and 5.6% in the data at a 3-month horizon. The in-sample root-mean squared errors range from 1.2% for at-the-money contracts to 2.8% for out-of-the-money contracts at 1 month to maturity. The confidence measure is the most significant driver of the variation in the out-of-the-money volatilities, while consumption volatility becomes more important for at- and in-the-money contracts.

Overall, confidence jumps play a significant role in explaining the cross-section of option prices. Without confidence jumps the model is unable to explain prices of the out-of-the-money option contracts, as the implied volatility curve is nearly flat across the strikes. I further consider an additional long-run volatility factor, as in Drechsler and Yaron (2011) and Bates (2012). While the addition of the factor improves the model fit to the option data, still it cannot substitute confidence jumps to account for the option price patterns in
Related Literature. Earlier structural models which aim to explain option prices and large moves in returns typically introduce jumps directly into consumption fundamentals. Eraker and Shaliastovich (2008) show that when investors have preference for early resolution of uncertainty, jumps in consumption fundamentals are priced in equilibrium and affect asset valuations and returns. In particular, positive jumps in consumption volatility endogenously translate into negative jumps in equilibrium prices, which can capture the shape of the implied volatility curve in option prices. Benzoni, Collin-Dufresne, and Goldstein (2005) consider jumps in expected consumption, which they show can also rationalize the volatility smirk observed in the data. Eraker (2007) and Drechsler and Yaron (2011) further argue that jumps in the conditional moments of consumption can account for certain features of the risk-neutral variance of returns implied by the cross-section of option prices in the data.

In a related literature, Liu, Pan, and Wang (2005) introduce rare jumps into the endowment dynamics and argue that concerns for model uncertainty can explain the over-pricing of out-of-the-money puts and the smirk pattern of option prices in the data. Drechsler (2013) shows that investor’s ambiguity about cash-flow jumps and volatility can account for the cross-section of option prices and variance premium. This implied volatility pattern can also be generated in a rare disaster model with a time-varying probability of a crash, as discussed by Gabaix (2012). In a similar vein, Bollerslev and Todorov (2011) argue that the compensation for rare events contributes to a large portion of the equity and variance risk premia, and Santa-Clara and Yan (2010) estimate risks of investors implied from the option markets and argue for substantial Peso issues in measuring jumps from the stock market data alone. Bates (2008) studies the equilibrium implications of the model which features jump news in dividends and crash-averse investors with heterogeneous attitudes towards crash risk. In an earlier study, Naik and Lee (1990) analyze general-equilibrium
option pricing when the underlying dividend follows a jump-diffusion process. Relative to
the above literature, I do not entertain the possibility of jumps in cash flows; rather, I
show that fluctuating confidence of investors can account for the empirical jump evidence
in option and equity data. This is consistent with the findings in Backus, Chernov, and
Martin (2011) that asset-market jumps arrive more frequently than the rare disasters in
the aggregate consumption data.

Other approaches which incorporate learning and option prices include Buraschi, Tro-
jan, and Vedolin (2013), who study the implications of learning, difference of opinion
and disagreement for the option prices. Campbell and Li (1999) considers learning about
volatility regimes, and Guidolin and Timmermann (2003) studies Bayesian learning im-
plications for option pricing in context of the equilibrium model. A number of papers
highlight the importance of information in option prices to learn about the risks in finan-
cial markets. The empirical evidence presented in Bollerslev, Tauchen, and Zhou (2009),
Todorov (2010), Buraschi and Jackwerth (2001), Bakshi and Kapadia (2003), as well as
from parametric models of asset prices, suggest that the risk premia in options cannot be
explained by compensation for diffusive stock market risk alone. A number of papers also
use option market data to study the characteristics of investor preferences; these works
include Bondarenko (2003), Garcia, Luger, and Renault (2003), Ait-Sahalia, Wang, and

The rest of the paper is organized as follows. In the next Section I set up the confidence
risk model. Solutions to the equilibrium asset and option prices are shown in Section 3.
Section 4 describes the data and the empirical evidence on the option pricing puzzles and
large moves in asset prices. I present the estimation results and model implications to asset
prices Section 5. Conclusion follows.
2 Economic Model

2.1 Preferences

I consider a discrete-time real endowment economy. Investor’s preferences over the uncertain consumption stream $C_t$ are described by the Kreps and Porteus (1978) recursive utility of Epstein and Zin (1989) and Weil (1989):

$$U_t = \left\{ (1 - \delta)C_t^{1-\gamma} + \delta(E_t[U_{t+1}^{1-\gamma}])^{1/\theta} \right\}^{\theta/\gamma},$$

(1)

where $\gamma$ is a measure of a local risk aversion of the agent, $\psi$ is the intertemporal elasticity of substitution, and $\delta \in (0, 1)$ is the subjective discount factor. The conditional expectations are taken with respect to the date-$t$ information set of the agent which I describe later in the paper. For notational simplicity, parameter $\theta$ is defined as

$$\theta = \frac{1 - \gamma}{1 - \frac{1}{\psi}}.$$  

(2)

When $\theta = 1$, that is, $\gamma = 1/\psi$, the recursive preferences collapse to a standard expected utility. As is pointed out in Epstein and Zin (1989), in this case the agent is indifferent to the timing of resolution of uncertainty in the consumption path. When risk aversion exceeds the reciprocal of the intertemporal elasticity of substitution, investors prefer early resolution of uncertainty; otherwise they prefer late resolution of uncertainty. Preference for the timing of the resolution of uncertainty has important equilibrium implications for prices of risk in the economy. In the long-run risk model agents prefer early resolution of uncertainty.

As shown in Epstein and Zin (1989), the logarithm of the intertemporal marginal rate
of substitution for these preferences is given by

\[ m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \]  

where \( \Delta c_{t+1} = \log(C_{t+1}/C_t) \) is the log growth rate of aggregate consumption, and \( r_{c,t+1} \) is the log of the return (i.e., continuous return) on an asset which delivers aggregate consumption as its dividends. This return is not observable in the data. It is different from the observed return on the market portfolio as the levels of market dividends and consumption are not equal: aggregate consumption is much larger than aggregate dividends. To solve the model, I assume an exogenous process for consumption growth and use a standard asset pricing restriction

\[ E_t[\exp(m_{t+1} + r_{t+1})] = 1, \]  

which holds for any log return \( r_{t+1} = \log(R_{t+1}) \) to calculate the equilibrium asset prices in the economy.

### 2.2 Economy Setup

Following Bansal and Yaron (2004), the true dynamics for log consumption growth \( \Delta c_{t+1} \) incorporates a time-varying mean \( x_t \) and stochastic volatility \( \sigma_t^2 \):

\[ \Delta c_{t+1} = \mu + x_t + \sigma_t \eta_{t+1}, \]  
\[ x_{t+1} = \rho x_t + \varphi_c \sigma_t \epsilon_{t+1}, \]  
\[ \sigma_{t+1}^2 = \sigma^2 + \nu_c (\sigma_t^2 - \sigma^2) + \varphi_w \sigma_t w_{c,t+1}, \]

where \( \eta_t, \epsilon_t \) and \( w_{c,t+1} \) are independent standard Normal shocks which capture short-run, long-run, and consumption volatility risks, respectively. The coefficient \( \sigma^2 \) determines the
long-run level of consumption volatility. It is a fixed parameter in the benchmark model. In Section 5.6 I allow the long-run volatility to vary over time. Parameters $\rho$ and $\nu_c$ determine the persistence of the conditional mean and variance of the consumption growth rate, while $\varphi_c$ and $\varphi_w$ govern their scale. The empirical motivation for the time-variation in the conditional moments of the consumption process comes from the long-run risks literature, see e.g. Bansal and Yaron (2004) and Hansen, Heaton, and Li (2008).

As in Bansal and Shaliastovich (2009), I assume that the agent knows the structure and parameters of the model and can observe consumption volatility $\sigma_t^2$, however, the true expected growth factor $x_t$ is not directly observable. Instead, that agents receive $n$ signals about the expected growth $x_{i,t}$, for $i = 1, 2, \ldots, n$. Each signal deviates from the true state $x_t$ by a random noise $\xi_{i,t}$,

$$x_{i,t} = x_t + \xi_{i,t}. \tag{8}$$

The errors $\xi_{i,t}$ are randomly drawn from a Normal distribution with zero mean and a time-varying variance, and are uncorrelated with all the other economic shocks. The time-variation in the variance of the signals captures the fluctuations in the quality of information about future growth.

As the signals in the cross-section are ex-ante identical, investors need to rely only on the average signal $\bar{x}_t$:

$$\bar{x}_t \equiv \frac{1}{n} \sum x_{i,t} = x_t + \xi_t. \tag{9}$$

The average signal noise $\xi_t = \frac{1}{n} \sum \xi_{i,t}$ has a Normal distribution whose time-varying conditional variance is denoted by $V_t$:

$$V_t = Var_t(\xi_t) = \frac{1}{n} Var(\xi_{it}). \tag{10}$$
The uncertainty $V_t$ determines the confidence of investors about their estimate of expected growth, and is referred to as the "confidence measure." In the model, the confidence measure is assumed to be observable to investors. An unbiased proxy for this measure in the data can be constructed from the cross-section of individual signals about future growth. Indeed, the signal equation (8) implies that

$$V_t = \frac{1}{n} E \left( \frac{1}{n-1} \sum_{i=1}^{n} (x_{i,t} - \bar{x}_t)^2 \right),$$

so that the cross-sectional variance of the signals adjusted by the number of signals $n$ can provide an estimate of the confidence measure $V_t$ in the data. In this case, the subsequent measurements of the confidence in the data from the cross-section of growth forecast are directly compatible with the theoretical notion of the uncertainty in the paper.

The confidence measure in the model captures the uncertainty that the agents have about their estimate of future growth. The variation in the confidence measure across time reflects the fluctuations in the quality of information in the economy, so that at times when information is poor, signals are less precise and the uncertainty is high ($V_t$ increases). The time-variation in the confidence measure and ensuing confidence risks are the novel contribution of the confidence risk model. Standard learning models feature a constant level of imprecision in observed signals; see for example David (1997), Veronesi (2000), Brennan and Xia (2001), Ai (2010) and David and Veronesi (2013).

### 2.3 Confidence Measure Dynamics

The specification of the confidence measure is a key ingredient of the model. The main features of the confidence measure dynamics are the persistent fluctuations and occasional large positive moves. These features are motivated by the theoretical literature on this issue and the empirical evidence in the data. In terms of theoretical work, Veldkamp (2006) and
Van Nieuwerburgh and Veldkamp (2006) present a model with endogenous learning, which features large discrete moves in the information about future economy. The large, discrete moves in investors’ uncertainty about future economy can also be obtained in the costly learning models due to lumpy information acquisition, as shown in Bansal and Shaliastovich (2011). Finally, the empirical evidence discussed in Section 4.2 provides a direct support for the fluctuations and large moves in the confidence measure.

Based on these considerations, I follow Bansal and Shaliastovich (2009) and set up a discrete-time jump-diffusion model for the confidence measure, which features persistence and jump-like innovations:

\[
V_{t+1} = \sigma_v^2 + \nu(V_t - \sigma_v^2) + \sigma_w \sqrt{V_t} w_{t+1} + Q_{t+1}. \tag{12}
\]

The parameters \(\sigma_v^2\) is the mean value of \(V_t\), \(\nu\) captures its persistence, and \(\sigma_w\) determines the volatility of a smooth Gaussian shock \(w_{t+1}\). The non-Gaussian innovation in the confidence process is denoted by \(Q_{t+1}\). I model it as a compensated Poisson jump,

\[
Q_{t+1} = \sum_{i=1}^{N_{t+1}} J_{i,t+1} - \mu_j \lambda_t, \tag{13}
\]

where \(N_{t+1}\) is the Poisson process, whose intensity \(\lambda_t \equiv E_t N_{t+1}\) corresponds to the probability of having one jump in the continuous-time model, while \(J_{i,t+1}\) determines the distribution of the size of the jump. Parameter \(\mu_j\) is the unconditional mean of jump size, so subtracting \(\mu_j \lambda_t\) on the right-hand side of the above equation ensures that the jump innovation \(Q_{t+1}\) is conditionally mean zero. In the estimation of the model, I consider an exponential distribution for jumps, which is convenient as it is fully described by a single

\[
E_t(Q_{t+1}) = E_t(E_t(Q_{t+1} | N_{t+1})) = E_t(\mu_j N_{t+1}) - \mu_j \lambda_t = 0.
\]

\(^3\text{Indeed,}

\[
E_t(Q_{t+1}) = E_t(E_t(Q_{t+1} | N_{t+1})) = E_t(\mu_j N_{t+1}) - \mu_j \lambda_t = 0.
\]
parameter $\mu_j$.

To capture the dependence of jump probability on the level of the confidence measure, I assume that the arrival intensity $\lambda_t$ is linear in $V_t$,

$$\lambda_t = \lambda_0 + \lambda_1 V_t. \quad (14)$$

When $\lambda_1 > 0$, the probability of jumps increases in the level of the confidence measure, so jumps are more frequent when the uncertainty about expected growth is high, which leads to clustering of jumps. This specification of the uncertainty dynamics is similar to the self-exciting variance jumps models considered in Ait-Sahalia, Cacho-Diaz, and Laeven (2014), Andersen, Fusari, and Todorov (2014), and Carr and Wu (2011).

### 2.4 Beliefs Specification

In the model, investors do not observe the true expected growth and thus have to learn about it using the past history of the data and the information in the current signals. To model a belief-updating process of the agents, consider a general specification, discussed in Hogarth and Einhorn (1992), where the prior estimate of investors is linearly adjusted by the impact of the recent news:

$$\hat{x}_t = \rho \hat{x}_{t-1} + K_t (\bar{x}_t - \rho \hat{x}_{t-1}). \quad (15)$$

In this updating equation, $\hat{x}_t$ is the investors’ estimate of the true expected growth $x_t$, $\rho \hat{x}_{t-1}$ corresponds to the agents’ prior belief about the expected state as of the last period, and $K_t$ denotes the adjustment weight which measures the sensitivity of agents’ expectations to the recent information in the average signal. Special cases of this belief-updating setup,

\footnote{In principle, agents can also use the history of consumption data in their update. However, at economically reasonable parameter values signals are much more informative about expected growth than...}
which correspond to different choices of $K_t$, include the Bayesian Kalman Filter as well as a constant-gain learning where $K_t$ is set to a constant value.

Constant gain specification is more tractable than the Bayesian learning specification, and is used in a variety of studies. Economically, it can be justified in the environment where investors are concerned about possible structural breaks at unknown dates. A constant gain specification gives more weight to recent observations, it can also be motivated behaviorally, based on the recency-biased expectation formation discussed in Kahneman and Tversky (1973) and De Bondt and Thaler (1990). With Bayesian learning, a Kalman Filter weight $K_t$ given to news today declines as the uncertainty $V$ rises: Bayesian investors optimally choose to put less weight to the recent information when its quality is low (cross-sectional variance is high). While in high confidence risk states Bayesian investors continue to put positive weight on the signals, however, the Bayesian learning would quantitatively suppress the effects of the recent information and of confidence risks on asset prices. In contrast, in a constant gain specification expectations overweight recently received information and does not lead to a decrease in the importance of signals at times of high confidence risks.

Based on these considerations, I adopt a constant gain, recency-bias specification for the beliefs of the agents. I set the weight that investors give to recent news to a constant $K$ equal to the steady-state value from a standard Kalman Filter. Under the recency bias consumption, so allowing the agent to learn from consumption growth does not have any material impact.\footnote{Constant gain learning specifications were used in Williams (2003), Primiceri (2006), Orphanides and Williams (2004). Malmendier and Nagel (2013) show recent evidence for the overweighting of the recent observations in individual expectations. For a broader survey of the behavioral literature see Barberis and Thaler (2003).}
specification, investor’s expectation formation can be expressed in the following way:

\[ \Delta c_{t+1} = \mu + \hat{x}_t + a_{c,t+1}, \]  
\[ \bar{x}_{t+1} = \rho \hat{x}_t + a_{x,t+1}, \]  
\[ \hat{x}_{t+1} = \rho \hat{x}_t + Ka_{x,t+1}. \]  
\[ (16) \]
\[ (17) \]
\[ (18) \]

The innovations \( a_c \) and \( a_x \) in equations (16)-(18) correspond to short-run and long-run shocks, and satisfy the following equations:

\[ a_{c,t+1} = x_t - \hat{x}_t + \sigma_t \eta_{t+1}, \]  
\[ a_{x,t+1} = \rho (x_t - \hat{x}_t) + \varphi \sigma_t \xi_{t+1} + (\bar{x}_{t+1} - x_{t+1}). \]  
\[ (19) \]

Given investors’ information, these innovations are conditionally Gaussian with zero mean and a time-varying variance driven by the consumption volatility \( \sigma_t^2 \) and the confidence measure \( V_t \). Notably, these short and long-run shocks depend on the prediction error of the investors \( (x_t - \hat{x}_t) \). As shown in the Appendix, the variance of the prediction error, denoted \( \omega_t^2 \), is directly related to the confidence measure:

\[ \omega_t^2 = KV_t. \]  
\[ (20) \]

The recency bias expectation formation ensures that the variance of the prediction error increases linearly with \( V_t \). In contrast, under Bayesian learning, periods of high confidence risks are associated with lower \( K_t \), which would dampen the role of confidence risks.


3 Model Solution

3.1 Discount Factor

To solve the model, I first use the dynamics of the economy given the information set of the agent and Euler equation (4) to calculate the price of the consumption claim. Using a standard Campbell-Shiller log-linearization of returns, the equilibrium price-consumption ratio is linear in the expected growth state, aggregate consumption volatility, and the confidence level of the investors:

\[ pc_t = B_0 + B_x \hat{x}_t + B_v V_t + B_\sigma \sigma_t^2, \] (21)

where the expressions for the loadings are provided in Appendix A.

The loading \( B_x \) measures the sensitivity of the price-consumption ratio to expected growth. It is positive for \( \psi > 1 \), so that when the substitution effect dominates the income effect, prices rise following a positive news about the expected consumption, similar to a standard long-run risks model. The loadings \( B_v \) and \( B_\sigma \) capture the effects of the confidence measure and consumption volatility on asset valuations. When the agent has a preference for early resolution of uncertainty (\( \gamma > 1/\psi \)), these loadings are negative. In this case, lack of confidence about the expected growth state and high aggregate uncertainty decrease equilibrium asset valuations.

The relative magnitudes of the loadings of the price-consumption ratio on the aggregate volatility and confidence measure depend on the quality of signal information about expected growth. In the complete information case, the true expected state is known and the consumption volatility factor \( \sigma_t^2 \) alone determines the conditional variation of short-run and long-run consumption shocks. On the other hand, with learning, the volatilities of these shocks are now driven by two factors, \( \sigma_t^2 \) and \( V_t \) (see equation (19)), so that the volatility
channel is now represented by consumption volatility and confidence measure states. This reduces the price of consumption volatility risks and the risk compensation for consumption volatility shocks relative to the complete information case.

Using the equilibrium solution to the consumption asset, I can express the discount factor in (3) in terms of the underlying states and shocks in the economy. In equilibrium, the log discount factor is equal to,

$$m_{t+1} = m_0 + m_x \hat{x}_t + m_v V_t + m_\sigma \sigma_t$$

$$- \lambda_c a_{c,t+1} - \lambda_x K a_{x,t+1} - \lambda_v \left( \sigma_w \sqrt{V_t w_{t+1}} + Q_{t+1} \right) - \lambda_\sigma \phi_w \sigma_t w_{c,t+1},$$

where the expressions for the discount factor loadings and prices of risks are pinned down by the model and preference parameters of the investors. Their expressions are provided in Appendix A.

Innovations in the discount factor determine the risks that investors face in the economy. As in standard long-run risks model with complete information, short-run, long-run and consumption volatility risks are priced. With preference for early resolution of uncertainty, the market price of the short-run and long-run risks are positive, while the market price of consumption volatility risk is negative. The novel dimension of the model is that the confidence shocks also receive risk compensation; in particular, the confidence jump risks $Q_{t+1}$ are priced even though there are no jumps in fundamental consumption. The market price of confidence risks is negative: the agent dislikes uncertainty, so an increase in $V_t$ raises marginal utility of the investors.

Using the solution for the discount factor, I can derive the expressions for the equilibrium risk-free rates in the economy. Real interest rates with $n$ periods to maturity are linear in

\[^6\text{As investors cannot observe the true long-run risks shocks, the price of long-run risk decreases, while the price of short-run consumption risk increases relative to complete information; this is consistent with Croce, Lettau, and Ludvigson (2006).}\]
the expected growth state, investors’ confidence and consumption variance:

\[ rf_{t,n} = -F_{0,n} - F_{x,n}\hat{x}_t - F_{v,n}V_t - F_{\sigma,n}\sigma_t^2, \]

(23)

where the bond coefficients are given in the Appendix A. In particular, real yields increase in the expected growth state, and decrease with positive shocks to the confidence measure.

### 3.2 Equity Prices

To obtain implications for equity prices, I consider a dividend process of the form

\[ \Delta d_{t+1} = \mu_d + \phi(\Delta c_{t+1} - \mu) + \phi_d\sigma_t\eta_{d,t+1}, \]

(24)

where \( \eta_{d,t+1} \) is a dividend shock independent from all the other innovations in the economy.

The equilibrium price-dividend ratio is linear in the expected growth state, consumption volatility and the level of the confidence measure of the investors:

\[ pd_t = H_0 + H_x\hat{x}_t + H_vV_t + H_\sigma\sigma_t^2, \]

(25)

where the solutions for the loadings are provided in Appendix A. Similar to the valuation of the consumption asset, equity prices increase in expected growth factor and decrease when the confidence measure or the aggregate volatility are high. In particular, large positive moves in \( V_t \) endogenously translate into large jumps in asset returns. Indeed, the equilibrium log market return satisfies

\[ r_{d,t+1} = \mu_r + b_x\hat{x}_t + b_vV_t + b_\sigma\sigma_t^2 + \phi a_{c,t+1} + \kappa_{d,1}H_xK\alpha_{x,t+1} \]
\[ + \kappa_{d,1}H_v\left( \sigma_w\sqrt{V_{t+1}w_{t+1} + Q_{t+1}} \right) + \kappa_{d,1}H_\sigma\phi_w\sigma_{t+1}^2 + \phi_d\sigma_t\eta_{d,t+1}, \]

(26)
for certain loadings $b_x, b_v, b_\sigma$, and a log-linearization parameter $\kappa_{1,d} \approx 1$. As the return beta to the confidence measure is negative ($H_v < 0$), when investors lose confidence about their estimate of expected growth, changes in the confidence measure are substantially magnified due to investors’ concerns about the long-run growth, and have a large negative impact on the equilibrium asset prices. Hence, positive uncertainty jumps in the confidence measure translate into large negative jumps in equity prices. Notably, the assumption of a constant gain $K$ plays an important role to quantitatively magnify the confidence risks and the impact of jumps in the economy.

3.3 Option Prices

The equilibrium asset-pricing framework can be used to compute prices of options written on the dividend claim. Let $S$ denote the strike of the option, and $P_t$ the price level of the dividend claim. In Appendix A.4 I show that the option price $C_t(S/P_t, n)$ of a put option contract with moneyness $S/P_t$ and maturity $n$ depends on the underlying expected growth, confidence measure and the aggregate volatility states:

$$
C_t(S/P_t, n) = \frac{1}{2\pi P_t} \int_{iz_i-\infty}^{iz_i+\infty} \frac{e^{G_{0,n}+G_{x,n}\hat{z}_t+G_{v,n}V_t+G_{\sigma,n}\sigma_t^2+i\hat{z}\log(S/P_t)} - iz^2}{iz - z^2} dz,
$$

where $z_i \equiv Im(z) < 0$, and complex-valued loadings $G$ depend on the model and preference parameters. The option price can be easily computed numerically for given states and parameters of the economy.

For a given moneyness $S/P_t$ and maturity $n$, I convert option prices $C_t(S/P_t, n)$ into the Black-Scholes implied volatility units $\sigma_{BS,t}^2$ using model-implied interest rate $r_{f,t,n}$ and log price-dividend ratio $pd_t$ (see expressions (23) and (25), respectively). This transformation is convenient, as the implied volatilities are easier to interpret than the original option prices. Notably, the option-implied volatilities in the model are driven by the confidence
measure and consumption volatility, so positive jumps in the confidence measure endoge-
ously translate into positive jumps in the option-implied variance. The timing of these
moves corresponds to negative jumps in returns.

4 Empirical Evidence

4.1 Data

My asset-price data consist of option-implied volatilities, real market returns, the market
price-dividend ratios, and the real interest rates observed monthly from January 1996 to
June 2013.

I collect monthly data on European S&P 500 index option prices from the OptionMetrics
database. As is standard in the literature, to mitigate microstructure problems I exclude
contracts with option prices less than one eights of a dollar, with no trading volume,
with open interest less than 100 contracts, as well as those which violate basic arbitrage
restrictions. In my analysis I focus on put options as they are more actively traded than
call options, and the latter are redundant given the put-call parity relationship.

I convert option prices into the Black-Scholes implied volatility units. That is, using
the Black-Scholes formula, I solve for the implied volatility in the put option contract
given its observed price, moneyness, time to maturity, current index level, the interest
rate, and the dividend yield in the data. The implied volatilities represent a convenient
transformation of the dollar option price and are easier to interpret than the option prices
themselves. Following Dumas, Fleming, and Whaley (1998), I interpolate the implied
volatilities between strikes and times to maturity to back out the implied volatilities for 1-
and 3-month options with moneyness of 0.9, 0.95, 1.00, 1.05, and 1.10.

The data on market returns and price-dividend ratios come from the CRSP database.
The nominal market returns are deflated by inflation to obtain real returns. To proxy for the real risk-free rate, I adjust a short-term nominal rate by an estimate of the expected inflation, as in Bansal, Kiku, and Yaron (2011).

I obtain monthly data on real consumption growth rate from the BEA Tables. Additionally, I construct empirical measures of the expected growth and confidence of investors using the cross-section of individual forecasts from the Survey of Professional Forecasts at the Philadelphia Fed. Notably, unlike the asset-price and consumption data, the forecast data are available at a lower quarterly frequency. My measure of the expected growth corresponds to an average of four-quarters-ahead expectations of the real personal consumption expenditure growth. The corresponding confidence measure is computed from the cross-sectional variance of four-quarters-ahead forecasts, adjusted by the number of forecasts. These growth and dispersion measurements directly correspond to the notions of average signal and confidence developed in the economic model. Indeed, in the model the representative agent is confronted with the cross-section of signals about the expected growth – these correspond to the forecasts in the data. The average signal in the model hence maps directly into the average forecast in the data. The model confidence measure captures the volatility of the average signal, which can be computed from the cross-sectional variance of the forecasts divided by the number of forecasts. The construction of these measures follows Bansal and Shaliastovich (2009) and Bansal and Shaliastovich (2013), and the details of the computations are provided Appendix B.

The key features of the data are summarized in Table 1. The moments of the asset-price and macroeconomic data are quite standard. In my sample, the average log return is 5.2% and the real interest rate is 0.3%, so an estimate of the log equity risk premium is 4.8%. Market returns are quite volatile: the standard deviation of equity returns is 16.7% relative

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7 Related cross-sectional dispersion and uncertainty measures are also entertained in Drechsler (2013), David and Veronesi (2013), and Buraschi and Jiltsov (2006).
to 0.50% for the real interest rates. The interest rate and the market price-dividend ratio are both very persistent, with autocorrelation coefficients of 0.99 and 0.98, respectively. Consumption growth averages 1.4% and has a standard deviation of just below 1%. In my sample, the autocorrelation of consumption growth is −0.12. A negative autocorrelation in a higher-frequency consumption growth is a well-known feature of the data which stems from the measurement errors in consumption at monthly frequency (see Wilcox, 1992).

The Table further documents the summary statistics for the expected growth and confidence measure, and Figure 2 plots their time series in the data. The expected growth and confidence measure are quite persistent and volatile, with the autocorrelation coefficients of 0.8 and 0.7 at quarterly frequency, respectively. As shown in Figure, the expected growth tracks quite well the low-frequency movements in the realized consumption growth. It predicts one-year-ahead consumption growth with an $R^2$ of 30%, hence, it contains a meaningful information about future real growth in the economy.

### 4.2 Option Pricing Evidence

One of the key puzzles in option markets is that the out-of-the-money put options appear overpriced, so that the insurance for large downward movements in asset prices is too expensive relative to standard models (see e.g. Rubinstein, 1994). According to the Black-Scholes model, the option-implied volatilities across all strikes and maturities should be equal to the volatility of the underlying asset. Table 1 reports that in the data, the out-of-the-money option volatility of 26.8% exceeds at-the-money volatility of 20.1% by an average of 6.7% for 1-month contracts. For 3-months contracts, this difference is 5.6%. The evidence is robust to excluding the recent Financial Crisis and Great Recession observations. As shown in the bottom panel of Table 1, the magnitudes of the volatility smirk were similar in the pre-June 2008 sample, and equal to 6.3% and 5.2% for 1- and 3-month contracts,
respectively. Indeed, the volatility smirk is always positive in the sample, as can be seen from the time-series plot of the option volatilities in Figure 1.

Option implied volatilities also fluctuate significantly over time. The standard deviations of option volatilities range from 5.0% to 7.6%, and a sizeable variation and occasional large positive spikes in the series are apparent in Figure 1. Interestingly, the movements in option volatilities in the data are significantly related to the fluctuations in the confidence measure from the forecast data, as documented in Table 2. Specifically, I consider both the contemporaneous projections of option variances on the confidence measure in the data, and the predictive regressions of future option variances on their own lag and the lag of the confidence measure. I further standardize all the variables in the regressions, so the slope coefficients in the contemporaneous projections capture the correlations between the option variances and the confidence measure. As shown in the Table, these correlations are highly significant and range from 50% to 60%. In terms of the economic magnitudes, the at-the-money option volatility increases on average from 20% to 24.7% in periods when the confidence measure is above its 75% percentile (e.g. in 2001 and 2008-2009), while it decreases to 15.6% when the confidence measure falls below its 25% percentile (e.g. mid-2000s).

As shown in Table 2, the confidence measure has a similar pronounced impact on future option variances, even controlling for the current level of the implied variance. In 1 and 3 quarter ahead projections, the slope coefficient on the confidence measure is large and significant at all strikes and maturities, while the slope coefficient on the current value of option variance is small and is often insignificant or even negative. These findings are consistent with Buraschi and Jiltsov (2006) who show that the cross-sectional dispersion of option variances is significantly related to the confidence measure.

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8The regressions use the option-implied and confidence measure data expressed in the variance units, consistent with my model where the physical, risk-neutral, and up to a good approximation, Black-Scholes implied variances are linear in the variance states. The empirical evidence is robust to using standard deviations or logarithms of the variances.
of forecasts has information about the level and slope of the option smile and the volatility of returns.

The evidence for a large common variation in the confidence measure and the option variances in the data is robust to sub-sample analysis and the measurements of confidence in the data. As shown in Table D.1 in the Appendix, excluding the recent Financial Crisis does not materially affect the link between the confidence and option volatility. Indeed, the correlations between the confidence measure and option variances are around 45% in a pre-June 2008 period, comparable to a full sample. The evidence is further robust to using alternative macroeconomic variables to construct measures of confidence. For example, using real GDP forecasts or industrial production forecasts instead of real consumption leads to an average correlation of 52% and 59%, respectively. While the benchmark results use the option and confidence measures expressed in the variance units, the results are robust to expressing the variables as standard deviations or taking logs. For example, for 1-month to maturity ATM options, the contemporaneous correlation of option variance with confidence measure 0.60 in standard deviation units and 0.61 in log volatility units. Finally, I also verify that the confidence measure continues to contain important information about future option volatilities controlling for additional predictors beyond the current level of option volatility. For example, including the slope of the option volatility term structure (variance of the 3-month minus 1-month ATM options) has virtually no impact on the slope coefficients on the confidence measure at all strikes and horizons.

4.3 Large Move Evidence

The empirical findings of a positive and significant difference between the out-of-the-money and at-the-money option implied volatilities, and large spikes in the option volatilities in the data suggest the importance of jump risk factors to explain the cross-section of op-
tion prices in the data. The evidence for non-Gaussian moves in asset markets can be directly documented in the time series of option and equity prices and confidence measure themselves. As shown in Table 1, the historical distribution of returns is characterized by a negative skewness of $-0.85$ and a high kurtosis of 4.4 — for Normal distribution, these statistics are 0 and 3, respectively. Excess kurtosis and negative skewness are indicative of large downward moves in returns. Volatility measures, such as option volatility and confidence measure, exhibit a positive skewness and an excess kurtosis: confidence measures skewness is 2.2 and confidence measure kurtosis is 8.2, while the skewness of option volatilities range from 1.3 to 1.5, and the kurtosis from 6.0 to 6.6. This suggests that volatility measures contain large upward jumps. Sizeable variation across time and occasional large positive spikes are further apparent in the time series plots of the option-implied volatility in Figure 1 and the confidence measure in Figure 2.

In the data, large negative movements in equity returns are strongly related to large positive movements in the volatility measures. To proxy for large moves in the data, I identify two-standard-deviation or above innovations in the standardized series based on the AR(1)-GARCH(1,1) filter. In the data, the frequency of identified large moves in returns and option variances is once about every 2 years. More than three-quarters of identified large moves in equity returns are negative, while all of the large moves in implied variance are positive. The timing of large moves in implied variance and returns is highly related: all the negative large moves in returns are accompanied by a large positive increase in the option variance. These findings are consistent with jump evidence from the parametric models of asset prices discussed in Singleton (2006) and with empirical results in Tauchen and Todorov (2011) who present strong evidence for common jumps in stock price and option volatility from the high-frequency data. In the context of confidence measure

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9Statistical evidence on the importance of jumps for option prices is discussed in Bakshi et al. (1997), Bates (2000), Pan (2002), Broadie et al. (2007), Santa-Clara and Yan (2010)
jumps, Bansal and Shaliastovich (2009) analyze the confidence measure implications for the asset markets in a long sample starting in 1968. They find significant evidence for a jump-like component in the confidence measure and document that large moves in the confidence measure in the data are related to large moves in returns and return volatility.

Notably, while there is an evidence for common large negative moves in returns, positive moves in option volatilities and in the confidence measure, there is no strong empirical support for large moves in the real economy that can economically account for the jump features of financial data at the considered frequencies. In my sample, only one large negative move in equity returns coincides with a two standard deviation drop in real consumption, which occurred in June 2008. Half of all the other negative moves in equity returns actually occur at times of a positive shock to real consumption growth. Similar evidence is presented in Bansal and Shaliastovich (2009) and Bansal and Shaliastovich (2011), who argue that equity jump measures, at frequencies of one to two years, do not appear to be related to the fluctuations in the realized economic growth.

Hence, based on the empirical evidence, a measure of investors' uncertainty about future growth provides a plausible channel to explain option prices puzzles and the jump features of financial market data, without relying on jumps in consumption. In the next section, I formally assess the ability of the model with time-varying confidence jump risks to jointly explain the option, equity, macroeconomic and forecast data.

5 Model Estimation

The observed data consists of monthly observations of real consumption growth, real equity return, the market price-dividend ratio, the real risk-free rate, and the cross-section of option-implied volatilities at maturities of 1 and 3 months and moneyness of 0.90, 0.95, 1.00 and 1.05. I further use observations of the expected growth and the confidence mea-
sure from the forecast data, which are available at a quarterly frequency. The goal of the estimation is to identify the model parameters and the latent states which drive the dynamics of the expected growth, consumption volatility, and the confidence measure of the investors. The estimation exercise is quite challenging, due to 1) mixed frequency of the data observations (quarterly for forecasts versus monthly for asset-price and macro data); 2) non-Gaussian dynamics of the underlying factors (Poisson confidence jumps); 3) the latent nature of the factors, and 4) a non-linear relation between the observed data and the factors (implied volatilities). To deal with these issues, I adopt a Bayesian MCMC, mixed-frequency, particle-filter approach to estimate the model in the data. This approach is most closely related to Schorfheide et al. (2013) and Song (2014), who apply similar methods to estimate the long-run risks model using the equity and bond data, respectively.

5.1 State-Space Representation

I first consider a state-space representation of the model. A state-representation consists of a transition equation which describes the dynamics of the factors, and the measurement equation which links the observed data to the state variables.

Collect all the latent state variables into the vector $f_t$, which includes current and lagged observations of the true expected growth $x_t$, average signal $\bar{x}_t$, investors’ expectation $\hat{x}_t$, measurement error in consumption level $\xi_{c,t}$, confidence measure $V_t$, and the consumption variance $\sigma_t^2$. Denote the vector of model parameters by $\Theta$. Given the specification of the model dynamics in Section 2, the state vector $f_t$ follows a multidimensional autoregressive process with time-varying volatility and jumps:

$$f_{t+1} = f_0 + \Phi f_t + \zeta_{f,t+1}(f_t),$$

(28)

where the parameters of the process depend on the model parameters $\Theta$, and are specified
The innovation term $\zeta_{f,t+1}(f_t)$ emphasizes the dependence of the conditional variance and jump intensity of the state variables on the consumption volatility and the confidence measure in the vector $f_t$ itself.

To specify the measurement equation, let $Y_t$ denote the vector of observed macroeconomic and asset-price data, the cross-section of $m$ implied volatilities, and the forecast data:

$$Y_t^d = \begin{bmatrix} \Delta c_t^d & p t_d^d & r f_t^d & r_{d,t}^d & \{\sigma_{BS,j,t}^d\}_{j=1}^m & \tilde{z}_t^d & \sqrt{V_t^d} \end{bmatrix}'. \tag{29}$$

The superscript $d$ indicates that these are the data observations which are potentially contaminated by the measurement errors. Using the model solution, I can solve for theoretical counterparts to these measurements, $Y_t$, which depend on the model parameters and the economic states $f_t : Y_t = Y(\Theta, f_t)$. The measurement equation specifies that the data and theoretical measurements are equal up to the zero-mean shocks $\zeta_{Y,t}(f_{t-1})$:

$$A_t [Y_t^d] = A_t [Y(\Theta, f_t) + \zeta_{Y,t}]. \tag{30}$$

The zero-mean shocks $\zeta_{Y,t}$ capture the measurement errors in the data, or other economic innovations which are conditionally independent from the factor innovations $\zeta_{f,t}$. The deterministic matrix $A_t$ is designed to take into account a mixed frequency of the available data. For the end of the quarter observations (that is, every 3rd month) I observe both the forecast and asset-price data, so the matrix $A_t$ is identity. For the first and second month of the quarter, the forecast data is unavailable and the last two rows of the matrix $A_t$ are equal to zero.

Let me provide further intuition on the relation between the observed and theoretical variables in the measurement equation. Given the evidence for large measurement errors
in the monthly consumption data, I introduce a measurement error in the log consumption level:

$$c^d_t = c_t + \xi_{c,t},$$  \hspace{1cm} (31)

so that

$$\Delta c^d_t = \mu + x_{t-1} + \sigma_{t-1} \eta_t + \xi_{c,t} - \xi_{c,t-1}.$$  \hspace{1cm} (32)

The measurement error $\xi_{c,t}$ is i.i.d. Normal with mean zero and standard deviation $\sigma_{e,c}$. As the state vector $f_t$ includes the current and lagged values of $x_t$ and $\xi_{c,t}$, the consumption growth in the first measurement equation is thus linear in the states $f_t$, and the consumption-related innovation $\zeta_{Y,t}$ is equal to short-run consumption shock $\sigma_{t-1} \eta_t$. The equity return, whose dynamics is provided in (26), can similarly be expressed as a linear function of the current and lagged state variables $f_t$, so the return-related innovation $\zeta_{Y,t}$ just captures the dividend-specific shock $\varphi_d \sigma_{t-1} \eta_{d,t}$. The market-price dividend ratio and the real risk-free rate are linear functions of the expected growth $\hat{x}_t$, consumption volatility $\sigma^2_t$ and the confidence measure $V_t$; see equations (23) and (25). To avoid singularity, I assume Gaussian measurement errors for these observations, independent from all the other shocks in the economy. Let $\sigma_{e,rf}$ and $\sigma_{e,pd}$ denote the standard deviations of these measurement errors.

The implied volatilities in the model are non-linear functions of the underlying states. For each strike and maturity contract $j$, $\sigma_{BS,j,t} = \sigma_{BS}(\Theta, f_t)$. Similar to the bond and equity prices, I assume measurement errors on the implied volatilities in the data. The measurement errors are i.i.d. Gaussian, and for parsimony, the standard deviation of the measurement errors is common across all contracts and denoted by $\sigma_{e,iv}$.

Finally, the expected growth and confidence measure in the data are linearly related
to the theoretical counterparts in the model, up to Gaussian measurement errors. In the
data the forecast horizon is 1 year, while the model horizon of observation is 1 month.
To address the forecast horizon differences, note that the 12-month expected growth is a
scaled version of the next-month expected growth
\[ E_t (\hat{x}_t + \hat{x}_{t+1} + \ldots + \hat{x}_{t+11}) = \alpha \hat{x}_t, \] (33)
for \( \alpha = (1 - \rho^{12})/(1 - \rho) \). Hence, aside from the measurement error, I set the expected
forecast in the data to be \( \alpha \) times the growth forecast in the model, and the confidence
measure in the data is \( \alpha^2 \) times the confidence state in the model. This allows me to take
into account the difference in the expected growth forecasts horizons in the model and in the
data.\(^{10}\) For ease of comparison, I match the square roots of the confidence measure in the
data and in the model. I denote \( \sigma_{e,x} \) and \( \sigma_{e,v} \) the standard deviations of the measurement
errors in the expected growth and confidence data, respectively.

5.2 Estimation Approach

I use an MCMC Bayesian methodology to estimate the vector of model parameters \( \Theta \)
and the latent states \( f_t \). The equations (28) and (30) describe the state transition and
observations equations for my estimation. As the state-space system is non-Gaussian and
non-linear, I cannot use standard Kalman filter techniques to directly evaluate the likeli-
hood function in the data. Instead, I approximate the likelihood function using a particle
filter approach. As described in Andrieu, Doucet, and Holenstein (2010), the approxima-
tion errors average out in the MCMC chain, and thus allow to sample from the posterior
distribution of the parameters and states. In addition to computing the likelihood, par-

\(^{10}\) Schorfheide et al. (2013) and Bansal, Kiku, and Yaron (2012) provide an extensive discussion of the
time-aggregation issues in the data.
ticle filter also allows me to obtain the filtered estimates of the latent state variables, and the predictive values for the variables in the measurement equation. As in Andrieu et al. (2010) and Fernandez-Villaverde and Rubio-Ramirez (2007), I embed a particle filter algorithm into the Bayesian MCMC estimation methodology, using a standard random walk Metropolis sampling scheme. I provide further details on the implementation of the estimation approach in Appendix C.

Table 3 summarizes the prior distributions for the estimated model parameters $\Theta$. Whenever possible, the prior distributions are centered at the rounded values common in the literature. The prior variances are selected to encompass plausible values in the literature, and to be large enough to allow for a meaningful role of the data likelihood in the estimation.

Specifically, the prior mean for the intertemporal elasticity of substitution $\psi$ is set to 2, and for the risk aversion $\gamma$ to 10. These values are directly comparable to the calibration values of 1.5 and 10, respectively, in Bansal and Yaron (2004). The 95% prior confidence interval for $\psi$ ranges from 0.5 to 4.4, and for $\gamma$ from 5.7 to 15.5, which cover reasonable values in the literature. The prior for both parameters follows a Gamma distribution which has a positive support. Following Bansal and Shaliastovich (2013), the remaining preference parameter $\delta$ is fixed at 0.999 to facilitate the identification of the model parameters.

I further fix the mean of consumption and dividend growth rates $\mu$ and $\mu_d$ to their unconditional values in the sample. The prior mean of the persistence of the expected consumption $\rho$ is centered at 0.98. A high persistence of the expected growth is a common feature of the long-run risks models; see e.g. Bansal and Yaron (2004) who calibrate its value to 0.979. A fairly large prior standard deviation of $\rho$ of 0.2 leads to a wide prior confidence interval of $[0.59; 1.00]$. The parameter which governs the volatility of the expected growth $\varphi_e$ is centered at 0.045, which is nearly identical to the value of 0.044 used in Bansal and Yaron (2004).
I center the prior level of consumption volatility $\sigma$ and of the confidence measure $\sigma_v$ to their unconditional estimates in the sample. The prior distributions for the persistence of the consumption volatility and of the confidence measure are quite dispersed, and cover the range from 0.70 to 1.00 for their 95% confidence interval. The prior means for the diffusive volatility parameters in the consumption variance and confidence measure dynamics ($\varphi_w$ and $\sigma_w$) are set to the corresponding prior means for the levels of the two variances. I choose large standard deviations in the prior distributions of the volatility parameters and rely on the data to identify them in the estimation. The other key parameters in the confidence measure dynamics govern the frequency and size of jumps. To be agnostic about the importance of jumps, I center the prior frequency of jumps at zero, while the 95% upper bound for the prior probability of a jump is as high as 78% per month (once every 1.3 month). Notably, I do not restrict the intensity of jumps to increase at times of high confidence risks: the prior distribution for the jump intensity loading $\lambda_1$ is drawn from a symmetric Normal distribution centered at zero. The prior mean of jump size is equal to the unconditional level of the confidence measure, and its 95% prior confidence interval ranges from essentially 0 to 9.5 times the confidence measure level.

I center the dividend loading parameters $\varphi$ and $\varphi_d$ at their typical value of 3.5. The prior standard deviation is set at 2 to cover a large range of plausible estimates in the data. Finally, in my implementation, I do not fix the variances of the measurement errors and estimate them together with all the other model parameters. To be conservative I set the prior means of the standard deviations of the measurement errors to be equal to the half of the variances of the underlying series in the data. In this case, I do not affect the ex-ante importance of each variable in the estimation, and let the estimation approach choose the best fit to the data. The prior distributions for the standard deviations are quite disperse: the prior volatility of each standard deviation is equal to its prior mean.
5.3 Estimation Results

The posterior distribution of the parameters is summarized in Table 3, which documents the median values as well as the 95% confidence intervals for each parameter from the MCMC chain.

The estimated preference parameters are quite standard in the long-run risks literature. The posterior median for the estimated risk aversion coefficient is 10.72, and for the intertemporal elasticity of substitution is 2.99. My estimate of the intertemporal elasticity of substitution is somewhat higher than the value of 1.5 used in Bansal and Yaron (2004), which is driven by a lower mean and volatility of the risk-free rates in my sample relative to the long-sample values in the literature. The preference parameter configuration ensures that the agents prefer early resolution of uncertainty, so that they dislike negative shocks to expected consumption and positive shocks to consumption volatility and the confidence measure.

The estimation can capture well consumption growth dynamics in the data. The posterior median of the consumption volatility is 0.95%, annualized, which matches the data. As shown in Table 3, the expected consumption growth is very persistent: the median parameter estimate for $\rho$ is 0.984, so the half-life of the expected growth shocks is about 3.5 years. The value for this parameter is very close to typical estimates in the literature, and is also comparable to the persistence of the expected growth in the survey data. Indeed, the AR(1) coefficient for the quarterly expected growth, shown in Table 1, is 0.82 (S.E. 0.09), which translates into $0.82^{1/3} \approx 0.94$ at a monthly frequency. Interestingly, while there is a large uncertainty in the estimate of the persistence of the expected growth in the data, using the asset-price data reduces the standard error and results in a fairly tight 95% confidence interval of $[0.982; 0.987]$. While very persistent, the expected growth is a small component of the realized consumption growth rate. The scale of the volatility
of the expected growth $\varphi_e$ is estimated at 0.054, so that the expected growth explains less than 10% of the total variance of consumption. This ensures that the model-implied, measurement-error adjusted consumption growth is only weakly persistent, with an AR(1) coefficient of 0.08. A small but positive persistence of the realized consumption growth is a common feature of the macroeconomic data at lower frequencies, as discussed in Bansal and Yaron (2004).

The estimated level of the confidence measure, in volatility units, is almost 40 times lower than the unconditional volatility of consumption growth, which is consistent with the evidence in the data. The level of the confidence measure is directly related to the uncertainty that investors face about their estimate of expected growth. Indeed, as shown in equation (20), the variance of the variance of the filtering error $\omega_t^2$ is proportional to the confidence measure, and the proportionality coefficient $K$ is equal to about 0.8, based on the median estimates in the data. Hence, the model-implied two standard deviations band around the investors’ estimate of expected consumption growth is on average $\pm 0.05\%$, annualized. The average uncertainty about expected growth is quite small; for comparison, the standard deviation of the consumption growth is 1% on an annual basis.

The estimated model parameters indicate that the confidence measure significantly fluctuates over time. The median posterior persistence of the confidence shocks is 0.88, which translates into a half-life of 5 months. This matches well the persistence of the confidence risks in the data. Indeed, as shown in Table I, the quarterly estimate of an AR(1) coefficient of the confidence measure in the data is 0.70, or $0.70^{1/3} \approx 0.89$ at a monthly frequency. The confidence measure is driven by both Gaussian and Poisson jump shocks; however, the estimation results suggest that most of the fluctuation in the confidence measure is driven by non-Gaussian jump shocks which account for almost 90% of the conditional variance of the series. Interestingly, this parallels the findings in Tauchen and Todorov (2011) who argue that the market volatility (VIX index) is mainly driven by jumps.
in the data. The estimated median frequency of large moves in confidence is about once every 2 months, and the median posterior jump size is about 1.9 of the level of confidence. While the estimated jumps are quite frequent, many of them are quite small to be detected as large moves in the data. For example, 30% of jumps are less than one-standard deviation of the Gaussian component of the confidence distribution, and only about 4% of jumps are large enough to cause a 2-standard deviation movement in the confidence measure. Hence, the implied frequency of large detectable confidence moves is about once every 2 years, which agrees with the evidence in the data discussed in Section 4.3. Finally, the results indicate that the probability of confidence jump is increasing when the confidence measure is high (high uncertainty): the jump intensity parameter $\lambda_1$ is estimated to be positive.

Compared to the confidence measure, the consumption volatility is moderately persistent, with a median posterior autoregression coefficient of 0.77. The persistence of the consumption volatility is imprecisely estimated. Its 95% confidence interval is quite large and ranges from 0.74 to 0.85. The estimated persistence of consumption volatility is lower than the values used in the long-run risks literature (see Bansal and Yaron (2004)). Relative to the earlier literature, my model features two stochastic volatilities, and the estimation identifies confidence jumps as the key source for the uncertainty-related risks relative to smooth movements in fundamental consumption volatility. In this sense, the importance and persistence of the consumption volatility risks diminish.

The remaining model parameters include the dividend leverage coefficients, and the standard deviations of the measurement errors. The median dividend leverage parameter $\phi$ is estimated at 5.05, while the scale parameter $\varphi_d$ of the volatility of the idiosyncratic

\[11\] The estimation further relies on a short data sample which starts from 1996. The recent sample does not feature low-frequency movements in the volatility, such as a gradual decline from the 1930s and in the Great Moderation from mid-80s, which play an important role in identifying a the persistence aggregate volatility process, as shown in Bansal, Kiku, Shaliastovich, and Yaron (2013). While a higher persistence of consumption volatility can be imposed through a tight prior, I opt to keep a prior specification relatively uninformative, and let the data determine the values of the coefficients in the sample.
dividend shock is estimated at 4.65. The leverage parameter $\phi$ is somewhat higher than the typical values in the literature of around 3.5. The model estimates a higher dividend leverage parameter to compensate for the reduction in the overall consumption volatility in the recent sample relative to a long sample starting in 1930 used in other studies. Indeed, consumption volatility in my sample is 1%, which is twice as small as its estimate of 2% over a long data sample, while the volatility of returns in both recent and long samples is about the same. Finally, Table 3 reports that the posterior scale of the measurement errors decrease relative to the prior for the market price-dividend ratio, implied option volatilities, and the confidence measures, and increase for consumption growth, risk-free rate, and the expected growth. At the posterior median parameters, the measurement errors range from about 10% of the unconditional volatility in the data for the confidence measure, to about a quarter of the volatility for the price-dividend ratio and the option-implied volatilities, to about two standard deviations for the risk-free rate.

The filtered estimates of the expected growth, confidence measure, and consumption volatility, alongside with the confidence intervals, are depicted in Figure 3. As shown in the Figure, the median expected growth state can capture well the low-frequency fluctuations in the average forecast in the data, and the correlation between the two is 60%. The 95% confidence interval for the expected growth state in the model nearly always contains the average forecast. Some of the noticeable deviations include a larger drops in the expected growth in the data in 2001 and 2008 periods. The filtered expected growth predicts next-month consumption in the data with an $R^2$ of 6%, and next-year consumption with an $R^2$ of 13%, so it contain meaningful information about low-frequency growth fluctuations in the data.

The filtered confidence measure and consumption volatility are plotted in the middle and bottom panel of Figure 3. The confidence measure exhibits substantial variation over time, spiking up during Russian Default/LTCM and the Great Recession, and nearly hitting
a zero boundary in mid 2000s. The filtered confidence measure consistently tracks the persistent movements in the confidence measure in the data; in fact, the correlation between the quarterly observations of the confidence measure in the data and in the model is in excess of 90%. The filtered consumption volatility, depicted in the bottom panel of the Figure, also tends to increase in bad economic times, such as the Great Recession in 2008-2009. However, unconditionally, the filtered consumption volatility is only weakly related to the filtered confidence measure: the correlation between the median estimates of the two is 0.30. This highlights the importance of separating the movements in aggregate volatility from the learning-induced confidence measure. Finally, note that there is a substantially higher statistical uncertainty in identifying consumption volatility relative to the confidence measure, as the confidence interval is quite larger for the former.

5.4 Implications for Equity Prices

Figure 4 shows the sample model fit to the market price-dividend ratio and the risk-free rate, and Table 4 reports the analytical population moments of the asset prices in the model computed using the posterior parameter distribution from the MCMC chain. As can be seen from the Figure and the Table, the model can capture well both the in-sample conditional dynamics of the market price-dividend ratio, and its unconditional moments in the data. The top panel of the Figure shows that the model-implied price-dividend ratio is nearly always on top of the data observations, and the correlation between the data and the posterior median estimate in the model is 95%. Table 4 reports the population values from the model for the key moments of the market prices, such as its mean, standard deviation, and persistence. These values are very close to the sample estimates in the data reported in Table 1.

The bottom panel of Figure 4 depicts the time-series of the real risk-free rate in the
data and in the model. The two series are similar on average, and comove positively with each other with a correlation coefficient of 30%. However, there are some noticeable differences between the risk-free rate in the model and in the data. The risk-free rate in the data is smoother and less volatile than the filtered estimate in the model, and the latter exhibits several pronounced negative spikes in bad economic times. The sample evidence is consistent with the unconditional moments in the model reported in Table 4. The posterior median of the mean risk-free rate in the model is 0.37 which closely matches the data estimate of 0.32. However, the model persistence of 0.88 is smaller than 0.99 in the data, and the model volatility of 1.63 is larger than its estimate of 0.50 in the data. As a result, the implied measurement error of the risk-free rate is quite large, and equal to about 2 times the volatility of the series. To understand the difference between the model and the data, it is important to keep in mind that the real rate in the data is only a proxy for the true risk-free rate. Indeed, there are no reliable market data on real bond prices going back to 1996, and the risk-free rate in the data is constructed by removing the estimates of expected inflation from the nominal yield. Further, some of the largest deviations between the real rates in the model and in the data occur post 2008 at times when the short rates are close to a zero lower bound and the Fed engages in unconventional monetary policy. These policy issues are outside the scope of the model, and can further affect the measurements of the real rates in the data and the fit of the model to the data. Hence, it is not surprising to find a larger measurement noise and larger deviations between the model and data real bond prices, relative to the equity market prices.

The model implications for the distribution of equity returns are summarized in the bottom panel of Table 4. The median estimate of the log market premium is 5.18%, and its 95% confidence interval includes [4.27%; 6.28%]. The unconditional log equity premium in the model matches well the estimate in the data of 4.84%. To assess the magnitude of economic risks driving the asset markets, I report the market prices of risks and the
contribution of each source of risk to the equity risk premium in Table 5. In the Table, the market risk prices are multiplied by the standard deviation of each risk shock, so they can be interpreted as a risk compensation per a unit exposure to risk. The confidence measure and consumption volatility have a negative market price of risk, and expected and realized growth shocks have a positive market price of risk. As agents have a preference for early resolution of uncertainty, they dislike increases in volatility and uncertainty, and reductions in expected growth. Quantitatively, the expected growth risk and the confidence risk receive the largest risk compensations. The expected growth and confidence measure further contribute the most to the equity premium, as shown in the bottom panel in the Table. Note that the equity premium computed in this Table is for the total level return. It is equal to the log market premium plus a Jensen’s adjustment which reflects the variability of market returns, that is why this estimate is above the log equity premium reported in Table 4. The risk compensation for expected growth risks comprise about two-thirds of the market equity premium. About a quarter of the equity premium, or 1.7%, is due to the confidence risks. Recall that most of the variation in the confidence measure is due to Poisson jumps, so the confidence risk compensation captures the compensation for jump risk in the economy. This magnitude is consistent with other studies. For example, Broadie et al. (2007) and Pan (2002) estimate the jump risk premium between 2% and 3.5%, or about one-third of the sample equity premium, comparable to my model estimate. Notably, the compensation for the consumption volatility risks is quite small. This is due to the fact that first, the confidence measure now drives a component of the uncertainty about the expected growth of the investors, which reduces the importance of consumption volatility. Second, the estimated persistence of consumption volatility is somewhat lower than in literature, so that the volatility shocks matter less to the investors.

As shown in Table 4, the median value for the volatility of market returns in the model is about 18.8%, which is similar, though a bit higher than the data estimate of 16.7%. To
assess the conditional movements in the market volatility implied by the model, Figure 6 depicts a model-implied time-series of the conditional volatility of returns alongside with a GARCH(1,1) estimate in the data. The two series co-move very closely, and both spike up in bad economic times, such as in 1998 and 2008. Relative to the GARCH estimate, the model estimate exhibits more pronounced positive spikes at these times.

I next assess the model fit to the higher-order moments of returns in the data. Based on the median estimate, the skewness of returns in the model is -0.93, which is very close to the value of -0.85 in the data. The kurtosis of market returns in the model is 3.52, with a 95% confidence interval of [2.94; 5.31], which includes the data estimate of 4.40. Figure 7 shows the return distribution implied by the model, and the estimate based on the historical data. The distribution of returns in the model and in the data match quite closely. In particular, the return distributions in the model and in the data exhibit heavy left tails due to large downward moves. In the model, the jumps in equilibrium market prices are driven by jumps in the confidence measure. The jumps arrive once about every 2 months, and the average jump in return is -2.5%, monthly. As discussed in Section 5.3, even though the estimated confidence jumps in the model are quite frequent, many of them are quite small to lead to large detectable moves in equity returns. Both in the model and in the data, the probability of observing a large, two-standard deviation move in asset prices is about 4%, or once every two years.

5.5 Implications for Option Prices

In the equilibrium model, out-of-the-money put options hedge large negative moves in returns which are driven by positive jumps in the confidence measure. This economic channel can quantitatively explain the cross-section of option prices in the data. Figure 5 shows the time series of the option-implied volatilities in the data and in the model,
and the top panel of Table 6 summarizes the in-sample and unconditional model fit to the option data. As shown in the Figure, the model can capture quite well the conditional movements in the implied volatilities in the data. The correlations between the model and data implied volatilities range from 93% for out-of-the-money options to 99% for in-the-money options. The model is able to generate both sharp positive spikes in the volatilities in the recessions, as well as low values for the volatilities in calm periods of mid-2000s. The timing and magnitudes of option volatility jumps correspond to the jumps in the confidence measure.

As shown in Table 6, the unconditional estimates of the level of implied volatility in the model range from 24.4% for out-of-the-money options to 18.1% for at-the-money options and 15.7% for in-the-money options at 1 month to maturity. For 3-month contracts, these estimates are 22.5%, 17.3% and 16.0%, respectively. The model captures quite well the average slope of the implied volatility curve, equal to the difference between the out-of-the money and at-the-money volatilities. Based on the median estimates, the slope is equal to 6.2% in the model relative to 6.7% in the data at a 1-month horizon, and it is 5.2% in the model and 5.6% in the data at a 3-month horizon. Unconditionally, the model underestimates the average values of the out- and at-the-money volatilities by about 2% at a 1-month horizon, and by about 3% at a 3-month horizon. It delivers a closer fit to the in-the-money implied volatilities, where the difference between unconditional volatility levels in the model and the data is about 0.25% at 1 month and 0.75% at 3 months, so that the data estimates are contained in the unconditional confidence interval of the model. Notably, there is a lot of fluctuations and positive spikes in the implied volatilities in the data, especially during the recent Great Recession, which produces a significant statistical uncertainty in the estimates of the average implied volatility in the data. Indeed, as shown in Table 6 restricting the data sample till June 2008 decreases the implied volatility curve by about 1.5% at a 1-month horizon, and by about 1% at a 3-month horizon. These pre-
crises values are comparable to the unconditional estimates in the model, and are almost always contained in the model confidence interval.

The top panel of Table 6 summarizes the in-sample model fit to the option price data. The in-sample median estimates of the implied volatilities are similar to the unconditional values, and are closer to the actual averages in the data. Consistent with a previous discussion, the model’s largest deviations are for out-of-the-money volatilities. The sample root mean-squared error (MSE) for these contracts is 2.8% at a 1-month and 2.4% at a 3-month horizon. The root mean-squared error is the lowest for at-the-money and intermediate in-the-money contracts with moneyness of 1.00 and 1.05, respectively. For these contracts, the MSE are 1.2% and 1.6% at 1 month, and 1.7% and 1.2% at 3 months. The average root MSE across the option contracts is consistent with the estimated scale of the measurement error of 1.82%. The model fit to option price is comparable to the literature. For comparison, Santa-Clara and Yan (2010) document root-square errors of 2%, while Pan (2002) reports absolute pricing errors in the range from 1% to 3%.

In the model, the implied volatilities are non-linear functions of the state variables. However, as the asset-pricing model is exponentially affine and the conditional volatility of market returns is linear in the confidence measure and consumption volatility, it turns out that the option-implied volatilities are also nearly linear in the confidence measure and the consumption volatility. Indeed, I numerically verify that these two states can linearly capture more than 95% of the fluctuation in the option volatility. To quantify the relative importance of each state for the option price, I project the option variance on the confidence measure and the consumption variance, and report the slope coefficients in the bottom panel of Table 6. I standardize all the left- and right-hand side variables in the regression to have mean zero and variance one, so that the slope coefficients are comparable across the factors and the regressions. As can be seen from the Table, the confidence measure is a predominant driver of the out-of-the-money implied volatilities: the median slope coefficient on the
confidence measure is 0.89 relative to 0.22 for consumption variance. As the moneyness of the option contract increases, the relative role of the confidence measure diminishes. For at-the-money options, the slope coefficients are 0.73 and 0.46 for the confidence measure and consumption variance, respectively, while for in-the-money contracts consumption volatility becomes more important, with a slope coefficient of 0.66 relative to 0.53 for the confidence measure. Similar results hold for the option contracts with a 3-month horizon.

The option-price evidence suggests for an important economic role of the learning-induced confidence measure to account for the average level and the fluctuations in the out-of-the-money option prices, relative to standard macroeconomic volatility measures.

5.6 Alternative Models

In this section I examine the importance of jumps in the confidence measures and alternative channels for volatility fluctuations to explain asset prices. All the alternative model specifications I entertain deliver comparable fit to the macroeconomic and equity price data, similar to the benchmark case. Indeed, a standard long-run risks model of Bansal and Yaron (2004) can already account for the key features of the macroeconomic and equity price evidence without jumps or extra volatility factors. So, it is not surprising that it is hard to discriminate the extended models based on their fit to the standard consumption, equity return, and the risk-free rate data. However, the alternative models produce quite different implications for the option prices. Given this evidence, in the interest of space I only focus on the model implications for the option prices, which is the main objective of this paper.

Specifically, I estimate three alternative model specifications. In the first model specification, I remove Poisson jumps from the confidence measure. I set the jump intensity parameters $\lambda_0$ and $\lambda_1$ in the confidence specification in (12) to zero, so the confidence
measures is only driven by Gaussian shocks. The parameter estimates for this model are reported in Table D.2 in the Appendix. They are generally similar to the benchmark case, and the most prominent change is an increase in the volatility of Gaussian shocks of the confidence measure to compensate for the exclusion of jumps in its dynamics. The implications for the option prices are summarized in the first panel of Table 7. Without jumps, the model is unable to explain the difference between the option volatilities in the cross section. The implied volatility curve is nearly flat across the strikes: the average out-of-the-money volatility is 17.8%, relative to 17.4% for at-the-money options, and 17.1% for in-the-money options. While there is a considerable confidence interval on these estimates, the slope of the volatility curve stays close to zero across all the parameter draws. Hence, the economic model cannot account for the option-price evidence in the data without confidence risk jumps.

Next, I enrich the model by allowing a long-run value of the consumption volatility to follow its own persistent process driven by Gaussian shocks. This volatility model specification is also entertained in Drechsler and Yaron (2011) in the context of the equilibrium model, and in Bates (2012) in the reduced-form setting. In this model specification, the consumption volatility dynamics in (7) is extended to,

\begin{align*}
\sigma_{t+1}^2 &= s_t^2 + \nu_c(s_t^2 - \sigma_t^2) + \varphi_w \sigma_t \epsilon_{c,t+1}, \\
s_{t+1}^2 &= s_0^2 + \nu_s(s_t^2 - s_0^2) + \varphi s \epsilon_{s,t+1}.
\end{align*}

(34)

(35)

I do not allow jumps in the long-run consumption volatility, so the only source of the jump risk is in the confidence measure. The solution to the equilibrium asset prices now extends to accommodate the third volatility factor, so the equity prices, risk-free rates, and the option volatilities depend on the long-run consumption volatility, in addition to the expected growth, confidence measure and the consumption volatility states. The solution
to this model is presented in the Appendix A. The parameter estimates are reported in Table D.2 and are similar to the common parameters in the benchmark model. Notably, the long-run volatility is estimated to be quite persistent: the posterior median value for $\nu_s$ is 0.93 relative to 0.70 for consumption volatility, and 0.87 for the confidence measure. As documented in Table 7, the addition of the persistent volatility component improves the model fit to the option prices. The implied volatilities increase by about 1%-2% across strikes, and are closer to the estimates in the data. The fit generally improves in sample, and the scale of the measurement error of the implied volatilities decreases relative to the benchmark model.

Next I evaluate whether the long-run volatility channel can actually account for option prices without confidence jumps. To this end, I maintain a three-volatility specification but remove Poisson jumps in the confidence measure. The option price implications are presented in the last panel of Table 7. With three volatilities, the model slope of the implied volatility curve is larger than in a 2-volatility model with no jumps. Indeed, now the average option volatilities range from 22.8% for out-of-the-money contracts to 21.8% for at-the-money contracts at a one-month horizon, and from 23.5% to 20.4% at a three-month horizon. However, these slopes are significantly smaller than in the benchmark model with confidence jumps and in the data. In a three-volatility model with no jumps, the slopes are 1.0% and 3.1% at 1- and 3-month horizon, relative to 6.2% and 5.2% in the full benchmark model. Hence, while an additional persistent volatility component helps match the option-price evidence, in my model, it cannot substitute confidence jumps to explain the cross-section of option prices.
6 Conclusion

I present an economic model which features fluctuating confidence of investors about their estimate of unobserved expected growth. Cross-sectional uncertainty about expected growth (confidence measure) is time-varying and subject to jump-like risks. With a constant gain learning specification, jumps in the confidence measure lead to concurrent equilibrium jumps in the market returns and market volatilities. The confidence jump risk channel can quantitatively account for the cross-section of option prices and large moves in asset prices, without hard-wiring jumps into consumption.

I provide empirical evidence that the confidence measure in the data contains significant information about current and future implied volatilities. I further adopt a Bayesian MCMC, mixed-frequency, particle filter approach to estimate the model using the asset-price, macroeconomic, and forecast data. The empirical results provide a support for the confidence risks model, and indicate an important role of confidence risks jumps to explain the dynamics of option and equity prices.
A Model Solution

I present a solution to the extended model which incorporates the fluctuations in the long-run consumption volatility, so that the consumption volatility dynamics in (7) is modified to (34). The solution to benchmark model obtains by setting \( s_t \) to a constant value of \( s_0^2 = \sigma^2 \), and setting \( \varphi_s = 0 \).

A.1 Kalman Filter

Given the dynamics of the underlying economy in (5)-(6) and the specification of signals in (8), the distribution of the states given the current information set and next-period confidence measure is conditionally Normal:

\[
\begin{bmatrix}
    x_{t+1} \\
    \Delta c_{t+1} \\
    \bar{x}_{t+1}
\end{bmatrix} \mid I_t, V_{t+1} \sim N \left( \begin{bmatrix}
    \mu + \hat{x}_t \\
    \rho \hat{x}_t \\
    \rho \hat{x}_t
\end{bmatrix}, \Sigma_{t+1} \right),
\]

where the variance-covariance matrix is given by,

\[
\Sigma_{t+1} = \begin{bmatrix}
    \rho^2 \omega_t^2 + \varphi_e^2 \sigma_t^2 & \rho \omega_t^2 & \rho^2 \omega_t^2 + \varphi_e^2 \sigma_t^2 \\
    \rho \omega_t^2 & \omega_t^2 + \sigma_t^2 & \rho \omega_t^2 \\
    \rho^2 \omega_t^2 + \varphi_e^2 \sigma_t^2 & \rho \omega_t^2 & \rho^2 \omega_t^2 + \varphi_e^2 \sigma_t^2 + V_{t+1}
\end{bmatrix}.
\]

The innovation representation of the system can then be written in the following way:

\[
\begin{align*}
    \Delta c_{t+1} &= \mu + \hat{x}_t + a_{c,t+1}, \\
    \bar{x}_{t+1} &= \rho \hat{x}_t + a_{x,t+1}, \\
    \hat{x}_{t+1} &= \rho \hat{x}_t + K_{1,t+1} a_{c,t+1} + K_{2,t+1} a_{x,t+1},
\end{align*}
\]

where the Kalman Filter weights and the update for the filtering variance \( \omega_t^2 \) satisfy standard equations

\[
\begin{align*}
    K_{t+1} &= \Sigma_{t+1}^{12} (\Sigma_{t+1}^{22})^{-1}, \\
    \omega_{t+1}^2 &= \Sigma_{t+1}^{11} - \Sigma_{t+1}^{12} (\Sigma_{t+1}^{22})^{-1} \Sigma_{t+1}^{21},
\end{align*}
\]

where the superscripts refer to the partitioning of \( \Sigma_{t+1} \) into four blocks, such that \( \Sigma_{t+1}^{11} \) is the (1, 1) element of the matrix, \( \Sigma_{t+1}^{12} \) contain the elements from the first row and second and third columns, etc. The explicit solutions for the Kalman Filter weights satisfy

\[
\begin{align*}
    K_{1,t+1} &= \frac{\rho \omega_t^2 V_{t+1}}{(\omega_t^2 + \sigma_t^2) V_{t+1} + (\varphi_e^2 \sigma_t^2 + (\varphi_e^2 + \rho^2) \omega_t^2) \sigma_t^2}, \\
    K_{2,t+1} &= \frac{(\varphi_e^2 \sigma_t^2 + (\varphi_e^2 + \rho^2) \omega_t^2) \sigma_t^2}{(\omega_t^2 + \sigma_t^2) V_{t+1} + (\varphi_e^2 \sigma_t^2 + (\varphi_e^2 + \rho^2) \omega_t^2) \sigma_t^2}.
\end{align*}
\]

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while the evolution of the variance of the filtering error is given by

$$\omega_{t+1}^2 = V_{t+1}K_{2,t+1}. \quad (44)$$

Notably, for the economically relevant parameter values, signals are much more informative about the expected growth than consumption data, the Kalman Filter weight on consumption news $K_{1,t+1}$ is much smaller than that on the average signal $K_{2,t+1}$. Hence, to simplify the exposition, one can assume that the agent learns only from the average signals. In the case when investors do not look at consumption data and only update based on the average signal, $K_{1,t+1} = 0$ and $K_{2,t+1} \equiv K_{t+1}$ simplifies to

$$K_{t+1} = \frac{\rho^2 \omega_t^2 + \varphi^2 \sigma_t^2}{\rho^2 \omega_t^2 + \varphi^2 \sigma_t^2 + V_{t+1}}. \quad (45)$$

Further, the evolution of the estimate of expected growth can be rewritten in the following way:

$$\hat{x}_{t+1} = (1 - K_{t+1})\rho \hat{x}_t + K_{t+1}\bar{x}_{t+1}. \quad (46)$$

In the recency-bias motivated solution of the model that is featured in the paper, the weight $K_{t+1}$ is constant and equal to steady-state value $K$. To solve for the steady state of the system, I plug the solution for filtering uncertainty in $\omega_t^2 = KV_t$ into the above equation and solve a quadratic equation for the constant value of $K$ when the volatility processes $V_t$ and $\sigma_t^2$ are set to their unconditional means.

### A.2 Discount Factor

The aggregate consumption volatility $\sigma_t^2$ follows a square-root process specified in (34), while the dynamics of the confidence measure is given by a discrete-time jump-diffusion specification outlined in (12). The distribution of jump size $J_{i,t+1}$ is defined by its moment generating function,

$$l(y) \equiv Ee^{yJ_i}. \quad (47)$$

For example, when jump size follows exponential distribution with mean jump $\mu_j$,

$$l(y) = (1 - \mu_j y)^{-1}. \quad (48)$$

The conditional variance-covariance of consumption and expected growth shocks is given by,

$$\Sigma_{ex,t+1} = Var \left( \begin{bmatrix} a_{c,t+1} \\ a_{x,t+1} \end{bmatrix} \right) = \begin{bmatrix} KV_t + \sigma_t^2 & \rho KV_t \\ \rho KV_t & \rho^2 KV_t + \varphi^2 \sigma_t^2 + V_{t+1} \end{bmatrix}. \quad (49)$$

The log price-to-consumption ratio $pc_t$ is linear in the states of the economy:

$$pc_t = B_0 + B_x \hat{x}_t + B_{\sigma} V_t + B_{\sigma^2} \sigma_t^2 + B_{s^2} s_t^2. \quad (50)$$

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Using Euler equation (4), I can directly solve for the loading \( B_x \):

\[
B_x = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho},
\]

(51)

The loading on the confidence measure \( B_v \) satisfies non-linear equation

\[
\frac{1}{2} \theta \kappa_1^2 \sigma_w^2 z^2 - (1 - \kappa_1 (\nu - \lambda_1 \mu_j)) z + \frac{1}{2} \theta B_v^2 K ((1 - (1 - K) \kappa_1 \rho)^2 + \kappa_1 K) + \frac{\lambda_1}{\theta} (l(\theta \kappa_1 z) - 1) = 0,
\]

(52)

for \( z = B_v + \frac{1}{2} \theta \kappa_1 B_x^2 K^2 \). The loading \( B_\sigma \) solves a quadratic equation

\[
\frac{1}{2} \theta \kappa_1^2 \varphi_w^2 B_\sigma^2 - (1 - \nu_c \kappa_1) B_\sigma + \frac{1}{2} \theta \left( (1 - \frac{1}{\psi})^2 + \kappa_1^2 B_x^2 K^2 \varphi_w^2 \right) = 0.
\]

(53)

The loading \( B_s \) solves a quadratic equation

\[
\frac{1}{2} \theta \kappa_1^2 \varphi_s^2 B_s^2 - (1 - \nu_s \kappa_1) B_s + (1 - \nu_c) B_\sigma \kappa_1 = 0.
\]

(54)

Finally, the log-linearization parameter, which is pinned down by the equilibrium level of the price-consumption ratio, satisfies the following non-linear equation:

\[
\log \kappa_1 = \log \delta + (1 - \frac{1}{\psi}) \mu + (B_\sigma (1 - \kappa_1) + B_s (1 - \kappa_1 \nu_s)) s_0^2 \\
+ (B_v (1 - \kappa_1) + \kappa_1 (1 - \nu) z) \sigma_v^2 + \frac{\lambda_0}{\theta} (l(\theta \kappa_1 z) - \theta \kappa_1 z \mu_j - 1).
\]

(55)

As in Eraker and Shaliastovich (2008), because \( B_\sigma, B_s \) and \( B_v \) can have multiple roots, I choose the solution which is non-explosive as the variation in \( V_t, \sigma_v^2 \) and \( s_t \) is approaching zero.

Using the equilibrium solution to the price-consumption ratio, I can write down the expression for the discount factor in the following way:

\[
m_{t+1} = m_0 + m_x \tilde{x}_t + m_v V_t + m_\sigma \sigma_t^2 + m_s s_t^2 \\
- \lambda c a_{c,t+1} - \lambda_x K a_{x,t+1} - \lambda_v \left( \sigma_w \sqrt{V_t} \psi_{t+1} + Q_{t+1} \right) - \lambda_\sigma \varphi_w \sigma_t \psi_{c,t+1} - \lambda_s \varphi_s \sigma_t \psi_{s,t+1},
\]

(56)

where the discount factor loadings and the prices of risks are pinned down by the dynamics of factors and preference parameters of the investors. Their solutions are given by,

\[
m_x = \frac{1}{\psi}, \quad m_v = (1 - \theta) B_v (1 - \kappa_1 \nu), \quad m_\sigma = (1 - \theta) B_\sigma (1 - \kappa_1 \nu_s), \\
m_s = (1 - \theta) (B_s (1 - \kappa_1 \nu_s) - (1 - \nu_c \kappa_1 B_\sigma), \\
m_0 = \theta \log \delta + (1 - \theta) \log \kappa_1 - \gamma \mu - m_v \sigma_v^2 - m_\sigma \sigma_v^2 - m_s s_0^2,
\]

and

\[
\lambda_x = (1 - \theta) \kappa_1 B_x, \quad \lambda_\sigma = (1 - \theta) \kappa_1 B_\sigma, \quad \lambda_s = (1 - \theta) \kappa_1 B_s, \quad \lambda_v = (1 - \theta) \kappa_1 B_v.
\]

(58)
A.3 Asset Prices

Consider a log payoff tomorrow expressed as,

\[
p_{n-1,t+1} = F_{0,n-1} + F_{x,n-1} \hat{x}_{t+1} + F_{v,n-1} V_{t+1} + F_{\sigma,n-1} \sigma_{t+1}^2 + F_{s,n-1} s_{t+1}^2 \\
+ F_{g,n-1} \Delta \varepsilon_{t+1} + F_{d,n-1} \sigma \eta_{d,t+1}.
\]

(59)

Then, the solution for the coefficients in its log price today \( p_{n,t} \) satisfies

\[
F_{g,n} = F_{d,n} = 0, \\
F_{x,n} = m_x + F_{x,n-1} \rho + F_{g,n-1}, \\
F_{\sigma,n} = m_\sigma + F_{\sigma,n-1} \nu_c + \frac{1}{2} \left( (F_{g,n-1} - \lambda_c)^2 + \varphi_c^2 (F_{x,n-1} - \lambda_x)^2 K^2 + \varphi_w^2 (F_{\sigma,n-1} - \lambda_\sigma)^2 + F_{d,n-1}^2 \right), \\
F_{s,n} = m_s + F_{s,n-1} \nu_s + F_{\sigma,n-1} (1 - \nu_c) + \frac{1}{2} (F_{x,n-1} - \lambda_s)^2 \varphi_s^2, \\
F_{v,n} = m_v + \frac{1}{2} (F_{g,n-1} - \lambda_v + \rho (F_{x,n-1} - \lambda_x) K)^2 K + (q_{vex} + \lambda_v) \nu + \frac{1}{2} q_{vex}^2 \sigma_w^2 + \lambda (l(q_{vex}) - q_{vex} \mu_j - 1), \\
F_{0,n} = m_0 + F_{0,n-1} + F_{v,n-1} \mu + F_{s,n-1} s_{0}^2 (1 - \nu_s) + (q_{vex} + \lambda_v) \sigma_v^2 (1 - \nu) + \lambda_0 (l(q_{vex}) - q_{vex} \mu_j - 1)
\]

(60)

for \( q_{vex} = F_{x,n-1} - \lambda_v + \frac{1}{2} (F_{x,n-1} - \lambda_x)^2 K^2 \).

Setting \( F_{0,n-1} = F_{x,n-1} = F_{v,n-1} = F_{\sigma,n-1} = F_{s,n-1} = F_{g,n-1} = F_{d,n-1} = 0 \) in the above recursion, I can obtain the solution to \( n \)-period real risk-free rate.

On the other hand, the price-dividend ratio is given by,

\[
pd_t = H_0 + H_x \hat{x}_t + H_v V_t + H_\sigma \sigma_t^2 + H_s s_t^2,
\]

(61)

where the loadings satisfy the following equations:

\[
H_x = m_x + \kappa_{d,1} \rho H_x + \phi, \\
H_\sigma = m_\sigma + \kappa_{d,1} H_\sigma \nu_c + \frac{1}{2} \left( (\phi - \lambda_c)^2 + \varphi_c^2 (\kappa_{d,1} H_x - \lambda_x)^2 K^2 + \varphi_w^2 (\kappa_{d,1} H_\sigma - \lambda_\sigma)^2 + \varphi_d^2 \right), \\
H_s = m_s + \kappa_{d,1} H_s \nu_s + H_\sigma \kappa_{d,1} (1 - \nu_c) + \frac{1}{2} (\kappa_{d,1} H_s - \lambda_s)^2 \varphi_s^2, \\
H_v = m_v + \frac{1}{2} (\phi - \lambda_c + \rho (\kappa_{d,1} H_x - \lambda_x) K)^2 K + (q_{vex} + \lambda_v) \nu + \frac{1}{2} q_{vex}^2 \sigma_w^2 + \lambda (l(q_{vex}) - q_{vex} \mu_j - 1),
\]

(62)

for \( q_{vex} = \kappa_{d,1} H_v - \lambda_v + \frac{1}{2} (\kappa_{d,1} H_x - \lambda_x)^2 K^2 \), and the log-linearization parameter

\[
\log \kappa_{d,1} = m_0 + \mu_d + \left( H_v (1 - \kappa_{d,1} \nu) + \frac{1}{2} (\kappa_{d,1} H_x - \lambda_x)^2 K^2 (1 - \nu) \right) \sigma_v^2 \\
+ (H_x (1 - \kappa_{d,1} \nu) + H_\sigma (1 - \kappa_{d,1})) s_0^2 + \lambda_0 (l(q_{vex}) - q_{vex} \mu_j - 1).
\]

(63)

A.4 Option Prices

The option prices are computed using the approach in Lewis (2000). Unlike other methods in the literature, it relies on a single integration along the complex line, which reduces computational burden (see Eraker and Shaliastovich, 2008).
The option price with a strike \(S\) and maturity \(n\) is given by,

\[
C_t(S/P_t,n) = E_t \left[ M_{t,t+n} \max(e^{pt+n} - S, 0) \right] = \frac{1}{2\pi} \int_{iz\to\infty} E_t (M_{t,t+n}e^{-izp_t+n}) \hat{w}(z)dz ,
\]

(64)

where \(M_{t,t+n}\) is the discount factor which can be used to price \(n\)-period ahead payoffs, \(p_t\) is the log equity price and \(\hat{w}(z)\) is the generalized Fourier transform of the payoff function of the option equal to,

\[
\hat{w}(z) = \int_{-\infty}^{\infty} e^{izx} (e^x - S)^+ dx
= \frac{S^{iz+1}}{z^2 - iz} .
\]

(65)

The integration region is parallel to the real line in the complex plane, and \(z_i \equiv Im(z) > 1\) for call options and \(z_i < 0\) for put options.

Using the equilibrium solution to the discount factor and asset valuations, the expectation inside the integral in (64) is given by

\[
\log E_t e^{m_{t+n}-izp_t+n} = G_{0,n} + G_{x,n}\hat{x}_t + G_{v,n}V_t + G_{\sigma,n}\sigma_t^2 + G_{s,n}s_t^2 - izp_t ,
\]

(66)

where complex-valued loadings \(G_{0,n}, G_{x,n}, G_{v,n}\) and \(G_{\sigma,n}\) satisfy recursive equations similar to those computed in Appendix A.3.

Hence, the equilibrium put option price normalized by the equity price satisfies

\[
\frac{C_t(S/P_t,n)}{P_t} = \frac{1}{2\pi} \frac{S}{P_t} \left[ \int_{iz\to\infty} \frac{e^{G_{0,n} + G_{x,n}\hat{x}_t + G_{v,n}V_t + G_{\sigma,n}\sigma_t^2 + G_{s,n}s_t^2 + iz \log(S/P_t)}}{iz - z^2} dz \right] ,
\]

(67)

B Confidence Measure

I use a cross-section of individual forecasts from the Survey of Professional Forecasts to calculate the measures of expected growth and confidence for the period from 1996 to 2013. The survey started in the last quarter of 1968 as a joint project of the American Statistical Association and the National Bureau of Economic Research; in 1990 it was taken by the Federal Reserve Bank of Philadelphia. The data set contains quarterly forecasts on a variety of macroeconomic and financial variables made by the professional forecasters who largely come from the business world and Wall Street, see Croushore (1993) for details and Zarnowitz and Braun (1993) for a comprehensive study of the survey.

In my application, I use four-quarters-ahead forecasts of real personal consumption expenditure growth. This series is the closest, though not perfect, counterpart to the real consumption of nondurable goods in the model. I choose four-quarter forecast horizon to minimize potential data mis-alignment issues, as the forecasts are typically released within the first one or two months of the quarter, rather than by the beginning of the quarter. To make the inference robust to possible outliers and errors, I delete observations which are more than two standard deviations away from
the mean.

For each quarter \( t \) let \( RCONSUM_{i,t} \) denote the four-quarters-ahead forecasts of real personal consumption expenditure of a forecaster \( i \). If \( n_t \) is the number of forecasts, then my measure expected growth rate is

\[
\hat{x}_t = \frac{1}{n_t} \sum_{i=1}^{n_t} \log \frac{RCONSUM_{i,t}}{RCONSUM_t},
\]

(68)

where \( RCONSUM_t \) is the current value of the series.

The confidence measure is constructed from the cross-sectional variance of forecasts divided by the number of forecasts: (variance) in the average forecast:

\[
V_t = \frac{1}{n_t} \text{Var}_i \left( \log \frac{RCONSUM_{i,t}}{RCONSUM_t} \right)
= \frac{1}{n_t} \left( \frac{1}{n_t - 1} \sum_{i=1}^{n_t} \left( \log \frac{RCONSUM_{i,t}}{RCONSUM_t} - \hat{x}_t \right) \right)^2.
\]

(69)

The cross-sectional variance of the forecasts adjusted by the number of forecasts provides an unbiased estimate for the confidence measure in the model.

C  Model Estimation

As before, I provide the details for the extended model which allows for the time-variation in the long-run consumption volatility, as in \( \text{(34)} \). The benchmark model obtains by setting the variations in \( s_t \) to zero.

C.1  State-Space Equations

For ease of exposition, decompose the vector of states \( f_t \) into the growth and volatility related states, \( f_{g,t} \) and \( f_{v,t} \) respectively:

\[
f_{g,t} = [x_t \quad \hat{x}_t \quad \xi_{c,t} \quad x_{t-1} \quad \hat{x}_{t-1} \quad \xi_{c,t-1}],
\]

\[
f_{v,t} = [V_t \quad \sigma^2_t \quad s_t^2 \quad V_{t-1} \quad \sigma^2_{t-1} \quad s_{t-1}^2].
\]

(70)

The transition dynamics of the state variables is as follows:

\[
f_{g,t+1} = \begin{bmatrix}
\rho & 0 & 0 & 0 & 0 & 0 \\
\rho & 0 & 0 & 0 & 0 & 0 \\
\rho K & 0 & (1 - \rho)K & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\varphi_{e} \sigma_{t-1} \xi_{t+1} \\
\varphi_{e} \sigma_{t} \xi_{t+1} + \xi_{t+1} \\
K(\varphi_{e} d_{t} \xi_{t+1} + \xi_{t+1}) \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix},
\]

(71)
The particle filter allows to approximate the latent state distribution. The posterior inference is based on the Bayesian MCMC particle filter to the state-space system.

C.2 Posterior Inference

The measurement equations for consumption, equity and risk-free rate data are given by:

\[
\begin{bmatrix}
\Delta c_{t+1}^d \\
pd_{t+1} \\
rf_{t+1} \\
r_{d,t+1}
\end{bmatrix} = \begin{bmatrix}
\mu \\
H_0 \\
-F_0 \\
\kappa_{1,d}H_v
\end{bmatrix} + \begin{bmatrix}
0 & 0 & 0 & 1 \\
0 & H_x & 0 & 0 \\
0 & 0 & F_x & 0 \\
0 & \kappa_{1,d}H_s & 0 & -H_x \\
\end{bmatrix} f_{g,t+1} + \begin{bmatrix}
\sigma_{t}\eta_{t+1} \\
\zeta_{pd,t+1} \\
\zeta_{rf,t+1} \\
\phi\sigma_{t}\eta_{t+1} + \varphi_d\sigma_{t}\eta_{d,t+1}
\end{bmatrix} f_{v,t+1}.
\]

(73)

The measurement equation for the forecast data are given by:

\[
\bar{x}_{t+1}^d = \begin{bmatrix}
0 \\
\alpha \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix} f_{g,t+1} + \zeta_{x,t+1}
\]

\[
\sqrt{V_{t+1}^d} = \sqrt{\begin{bmatrix}
\sigma^2 & 0 & 0 & 0 & 0 & 0 \\
0 & \sigma^2 & 0 & 0 & 0 & 0 \\
0 & 0 & \sigma^2 & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma^2 & 0 & 0 \\
0 & 0 & 0 & 0 & \sigma^2 & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma^2
\end{bmatrix}} f_{v,t+1} + \zeta_{v,t+1}.
\]

(74)

Finally, the measurement equations for the implied volatilities are given by:

\[
\sigma_{BS,j,t}^d = \sigma_{BS,j}(f_t, \Theta) + \zeta_{iv,t}.
\]

(75)

The measurement errors \(\xi_c, \zeta_{pd}, \zeta_{rf}, \zeta_x, \zeta_v\) and \(\zeta_{iv}\) are i.i.d. Normal.

The posterior inference is based on the Bayesian MCMC particle filter to the state-space system. The particle filter allows to approximate the latent state distribution \(\{p(f_t|Y_{1,t}^d)^T\}_{t=1}^T\), and get approximate likelihood functions \(\{p(Y_t^d|Y_{1,t}^d)^T\}_{t=1}^T\). I briefly summarize main steps below; for further details, refer to Schorfheide et al. (2013).

1. Initialize the Markov Chain at the parameter vector \(\Theta^0\).

2. Given the parameter vector \(\Theta^j\):

   a) I initialize a swarm of particles at time \(t = 0\) by drawing a large number of particles \(\{f_0^i\}_{i=1}^N\) from their unconditional distribution. The weight of each particle is set to \(\pi_0^i = 1/N\).
b) For a given time period \( t \), I propagate each particle \( f_{t-1}^i \) using the state transition in (28).

c) I correct the weight to each particle \( \tilde{\pi}_t^i \):

\[
\tilde{\pi}_t^i = \pi_{t-1}^i \times p(Y_t^d|f_t^i).
\]  

(76)

The time-\( t \) likelihood of the data given the current state vector \( f_t \) directly follows from the measurement equation, and it is conditionally Gaussian.

d) I normalize the weights, \( \pi_t^i = \tilde{\pi}_t^i / \sum \tilde{\pi}_t^j \), and resample the particles \( \{f_t^i\}_{i=1}^N \) under these new weights.

e) I repeat 2b) - d) from time \( t = 1 \) to \( T \). In the end, I can use the particles to compute (an approximate) likelihood function of the data:

\[
\log \hat{p}(Y_t^d|Y_{1:t-1}^d) = \log \hat{p}(Y_t^d|Y_{1:t-2}^d) + \log \sum \tilde{\pi}_t^i.
\]

Further, I can use particles to obtain filtered estimates of the latent states \( \hat{f}_t \), and the predictive values for the measurement equations \( \hat{Y}_t \).

3. I calculate the posterior \( p(\Theta|Y^d) \) by multiplying the computed likelihood \( p(Y|\Theta^j) \) by the prior \( p(\Theta^j) \).

4. I accept the parameter draw \( \Theta^j \) with probability \( \min(1, p(\Theta^j|Y^d)/p(\Theta^{j-1}|Y^d)) \).

5. I draw a new parameter draw \( \Theta^{j+1} \) using a random walk proposal for each parameter, and repeat steps 2-4.

In empirical implementation, I use 50,000 particles, and I checked that the results are not materially affected if I increase the number of particles.

**D Other Tables**
Table D.1: **Option Volatility Predictability: Pre-crisis period**

<table>
<thead>
<tr>
<th></th>
<th>Option Variance</th>
<th>Confidence Measure</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1m to maturity:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Out-of-the-Money</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contemp</td>
<td>0.44 (0.10)</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>1q ahead</td>
<td>0.48 (0.16)</td>
<td>-0.02 (0.15)</td>
<td>0.22</td>
</tr>
<tr>
<td>3q ahead</td>
<td>0.19 (0.16)</td>
<td>0.23 (0.12)</td>
<td>0.12</td>
</tr>
<tr>
<td><strong>At-the-Money</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contemp</td>
<td>0.45 (0.11)</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>1q ahead</td>
<td>0.42 (0.17)</td>
<td>0.04 (0.19)</td>
<td>0.20</td>
</tr>
<tr>
<td>3q ahead</td>
<td>0.18 (0.16)</td>
<td>0.29 (0.12)</td>
<td>0.15</td>
</tr>
<tr>
<td><strong>In-the-Money</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contemp</td>
<td>0.42 (0.12)</td>
<td>0.18</td>
<td></td>
</tr>
<tr>
<td>1q ahead</td>
<td>0.37 (0.14)</td>
<td>0.10 (0.20)</td>
<td>0.18</td>
</tr>
<tr>
<td>3q ahead</td>
<td>0.11 (0.13)</td>
<td>0.36 (0.11)</td>
<td>0.17</td>
</tr>
<tr>
<td><strong>3m to maturity:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Out-of-the-Money</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contemp</td>
<td>0.44 (0.14)</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>1q ahead</td>
<td>0.54 (0.12)</td>
<td>0.01 (0.13)</td>
<td>0.29</td>
</tr>
<tr>
<td>3q ahead</td>
<td>0.22 (0.15)</td>
<td>0.18 (0.10)</td>
<td>0.11</td>
</tr>
<tr>
<td><strong>At-the-Money</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contemp</td>
<td>0.45 (0.13)</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>1q ahead</td>
<td>0.52 (0.14)</td>
<td>0.03 (0.15)</td>
<td>0.28</td>
</tr>
<tr>
<td>3q ahead</td>
<td>0.18 (0.16)</td>
<td>0.23 (0.09)</td>
<td>0.12</td>
</tr>
<tr>
<td><strong>In-the-Money</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contemp</td>
<td>0.44 (0.13)</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td>1q ahead</td>
<td>0.50 (0.14)</td>
<td>0.06 (0.16)</td>
<td>0.28</td>
</tr>
<tr>
<td>3q ahead</td>
<td>0.11 (0.14)</td>
<td>0.30 (0.08)</td>
<td>0.14</td>
</tr>
</tbody>
</table>

The table reports slope coefficients, standard errors, and $R^2$’s in the projections of current and future option-implied variances on current option variance and the confidence measure. Option variances and confidence measures are standardized to have mean zero and variance 1. OTM refers to the out-of-the-money options with moneyness of 0.90; ATM denotes the at-the-money options, and ITM is the in-the-money options with moneyness of 1.10. Standard errors are Newey-West adjusted with 10 lags. Quarterly data from January 1996 to June 2008.
Table D.2: Parameter Estimates: Alternative Models

<table>
<thead>
<tr>
<th></th>
<th>2 vol, No J</th>
<th>3 vol</th>
<th>3 vol, No J</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5%</td>
<td>50%</td>
<td>95%</td>
</tr>
<tr>
<td><strong>Preferences:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\delta)</td>
<td>-0.999</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(\psi)</td>
<td>2.81</td>
<td>2.95</td>
<td>3.05</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>10.72</td>
<td>10.88</td>
<td>11.35</td>
</tr>
<tr>
<td><strong>Cash Flows:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\mu \times 10^3)</td>
<td>-</td>
<td>1.20</td>
<td>-</td>
</tr>
<tr>
<td>(\rho)</td>
<td>0.972</td>
<td>0.978</td>
<td>0.983</td>
</tr>
<tr>
<td>(\varphi_e \times 10^2)</td>
<td>5.16</td>
<td>5.48</td>
<td>5.75</td>
</tr>
<tr>
<td>(\sigma \times 10^3)</td>
<td>3.15</td>
<td>3.68</td>
<td>4.44</td>
</tr>
<tr>
<td>(\nu_c)</td>
<td>0.74</td>
<td>0.76</td>
<td>0.80</td>
</tr>
<tr>
<td>(\varphi_w \times 10^2)</td>
<td>0.20</td>
<td>0.21</td>
<td>0.22</td>
</tr>
<tr>
<td>(\nu_s)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(\varphi_s \times 10^2)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(\phi)</td>
<td>4.87</td>
<td>4.99</td>
<td>5.42</td>
</tr>
<tr>
<td>(\varphi_d)</td>
<td>4.62</td>
<td>4.84</td>
<td>4.95</td>
</tr>
<tr>
<td><strong>Confidence Measure:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma_v \times 10^4)</td>
<td>0.10</td>
<td>0.40</td>
<td>1.95</td>
</tr>
<tr>
<td>(\nu)</td>
<td>0.89</td>
<td>0.90</td>
<td>0.92</td>
</tr>
<tr>
<td>(\sigma_w \times 10^4)</td>
<td>1.39</td>
<td>1.49</td>
<td>1.52</td>
</tr>
<tr>
<td>(\lambda_0)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(\lambda_1 \times \sigma_v^2)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(\sqrt{\mu_j/\sigma_v})</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Measurement Errors:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma_{e,rf} \times 10^2)</td>
<td>0.25</td>
<td>0.26</td>
<td>0.27</td>
</tr>
<tr>
<td>(\sigma_{e,pd})</td>
<td>0.01</td>
<td>0.03</td>
<td>0.07</td>
</tr>
<tr>
<td>(\sigma_{e,c} \times 10^2)</td>
<td>0.28</td>
<td>0.30</td>
<td>0.31</td>
</tr>
<tr>
<td>(\sigma_{e,iv} \times 10^2)</td>
<td>2.43</td>
<td>2.56</td>
<td>2.89</td>
</tr>
<tr>
<td>(\sigma_{e,x} \times 10^2)</td>
<td>0.50</td>
<td>0.52</td>
<td>0.53</td>
</tr>
<tr>
<td>(\sigma_{e,v} \times 10^4)</td>
<td>0.29</td>
<td>0.31</td>
<td>0.32</td>
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</tbody>
</table>

The table summarizes the posterior distribution for the model parameter estimates across alternative model specifications. "No J" refers to the restriction of no Poisson jumps in the confidence measure. "3-vol" additionally introduces a long-run volatility component in consumption variance.
References


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Drechsler, Itamar, 2013, Uncertainty, time-varying fear, and asset prices, forthcoming in *Journal of Finance*.


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Song, Dongho, 2014, Bond market exposures to macroeconomic and monetary policy risks, working paper, University of Pennsylvania.


## Tables and Figures

### Table 1: Summary Statistics

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<tr>
<th></th>
<th>Mean</th>
<th>AR(1)</th>
<th>Std. Dev.</th>
<th>Skew</th>
<th>Kurt</th>
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<td></td>
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<td></td>
<td></td>
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<tr>
<td>Cons. growth</td>
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<td>-0.12</td>
<td>0.96</td>
<td>-0.61</td>
<td>8.02</td>
</tr>
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<td>Interest rate</td>
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<td>0.99</td>
<td>0.50</td>
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<td>Log price-div.</td>
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<td>0.98</td>
<td>0.23</td>
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<td>Log return</td>
<td>5.16</td>
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<td>16.67</td>
<td>-0.85</td>
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<td><strong>Forecast Data:</strong></td>
<td></td>
<td></td>
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<td>0.82</td>
<td>0.69</td>
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<td>0.05</td>
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<td><strong>Option Implied Volatility Data:</strong></td>
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<td></td>
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</tr>
<tr>
<td>OTM 1 month</td>
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<td>0.80</td>
<td>7.60</td>
<td>1.50</td>
<td>6.58</td>
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<td>0.83</td>
<td>7.33</td>
<td>1.49</td>
<td>6.54</td>
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<td>6.74</td>
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<td>6.12</td>
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<td><strong>Option Implied Volatility Data, Pre-Crisis:</strong></td>
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<td></td>
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<td>OTM 1 month</td>
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<td>0.78</td>
<td>6.27</td>
<td>0.81</td>
<td>3.87</td>
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<td>5.99</td>
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<td>3.59</td>
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<td>ITM 1 month</td>
<td>14.85</td>
<td>0.79</td>
<td>5.61</td>
<td>0.58</td>
<td>3.49</td>
</tr>
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<td>OTM 3 month</td>
<td>24.40</td>
<td>0.84</td>
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<td>0.72</td>
<td>3.72</td>
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<td>ATM 3 month</td>
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<td>0.86</td>
<td>5.41</td>
<td>0.48</td>
<td>3.14</td>
</tr>
<tr>
<td>ITM 3 month</td>
<td>16.05</td>
<td>0.85</td>
<td>4.95</td>
<td>0.42</td>
<td>3.03</td>
</tr>
</tbody>
</table>

The table shows summary statistics for macroeconomic, asset-price, forecast, and option volatility data. OTM refers to the out-of-the money options with moneyness of 0.90; ATM denotes the at-the-money options, and ITM is the in-the-money options with moneyness of 1.10. Mean and volatility of consumption growth, interest rate, and log market return are annualized, in per cent. Volatility of the expected growth is in per cent. Confidence measure is multiplied by $10^5$. Standard errors are Newey-West adjusted with 10 lags. Monthly observations from January 1996 to June 2013 (quarterly for forecast data). Pre-crisis period is from January 1996 to June 2008.
Table 2: Option Variance Predictability

<table>
<thead>
<tr>
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<th>Confidence Measure</th>
<th>$R^2$</th>
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<tbody>
<tr>
<td></td>
<td>1m to maturity:</td>
<td></td>
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</tr>
<tr>
<td>OTM:</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Contemp</td>
<td>0.53 (0.11)</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>1q ahead</td>
<td>0.27 (0.14)</td>
<td>0.27 (0.17)</td>
<td>0.22</td>
</tr>
<tr>
<td>3q ahead</td>
<td>0.01 (0.14)</td>
<td>0.24 (0.08)</td>
<td>0.06</td>
</tr>
<tr>
<td>ATM:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contemp</td>
<td>0.58 (0.12)</td>
<td>0.34</td>
<td></td>
</tr>
<tr>
<td>1q ahead</td>
<td>0.20 (0.16)</td>
<td>0.37 (0.21)</td>
<td>0.26</td>
</tr>
<tr>
<td>3q ahead</td>
<td>-0.05 (0.15)</td>
<td>0.31 (0.08)</td>
<td>0.08</td>
</tr>
<tr>
<td>ITM:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contemp</td>
<td>0.60 (0.12)</td>
<td>0.37</td>
<td></td>
</tr>
<tr>
<td>1q ahead</td>
<td>0.13 (0.17)</td>
<td>0.46 (0.26)</td>
<td>0.31</td>
</tr>
<tr>
<td>3q ahead</td>
<td>-0.13 (0.15)</td>
<td>0.39 (0.09)</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
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<tr>
<td>3m to maturity:</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>OTM:</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Contemp</td>
<td>0.56 (0.13)</td>
<td>0.30</td>
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</tr>
<tr>
<td>1q ahead</td>
<td>0.37 (0.13)</td>
<td>0.31 (0.18)</td>
<td>0.35</td>
</tr>
<tr>
<td>3q ahead</td>
<td>0.05 (0.14)</td>
<td>0.25 (0.07)</td>
<td>0.08</td>
</tr>
<tr>
<td>ATM:</td>
<td></td>
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<tr>
<td>Contemp</td>
<td>0.60 (0.13)</td>
<td>0.36</td>
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<tr>
<td>1q ahead</td>
<td>0.32 (0.15)</td>
<td>0.37 (0.21)</td>
<td>0.38</td>
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<tr>
<td>3q ahead</td>
<td>-0.02 (0.15)</td>
<td>0.31 (0.07)</td>
<td>0.09</td>
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<td>ITM:</td>
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<tr>
<td>Contemp</td>
<td>0.62 (0.13)</td>
<td>0.40</td>
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<tr>
<td>1q ahead</td>
<td>0.26 (0.17)</td>
<td>0.44 (0.25)</td>
<td>0.41</td>
</tr>
<tr>
<td>3q ahead</td>
<td>-0.11 (0.15)</td>
<td>0.39 (0.08)</td>
<td>0.11</td>
</tr>
</tbody>
</table>

The table reports slope coefficients, standard errors, and $R^2$s in the projections of current and future option-implied variances on current option variance and the confidence measure. Option variances and confidence measures are standardized to have mean zero and variance 1, and are expressed in variance units. OTM refers to the out-of-the-money options with moneyness of 0.90; ATM denotes the at-the-money options, and ITM is the in-the-money options with moneyness of 1.10. Standard errors are Newey-West adjusted with 10 lags. Quarterly data from January 1996 to June 2013.
Table 3: Model Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>Cash Flows:</th>
<th>Confidence Measure:</th>
<th>Measurement Errors:</th>
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<td>Mean</td>
<td>Std. Dev.</td>
<td>5%</td>
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<tr>
<td>$\delta$</td>
<td>-</td>
<td>0.999</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\psi$ G</td>
<td>2</td>
<td>1.0</td>
<td>2.66</td>
<td>2.99</td>
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<tr>
<td>$\gamma$ G</td>
<td>10</td>
<td>2.5</td>
<td>10.41</td>
<td>10.72</td>
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<tr>
<td>$\mu \times 10^3$</td>
<td>-</td>
<td>1.20</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\rho N^T$</td>
<td>0.98</td>
<td>0.2</td>
<td>0.982</td>
<td>0.984</td>
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<tr>
<td>$\varphi_e \times 10^2$</td>
<td>G</td>
<td>4.5</td>
<td>2.0</td>
<td>4.11</td>
</tr>
<tr>
<td>$\sigma \times 10^3$</td>
<td>IG</td>
<td>3.0</td>
<td>1.0</td>
<td>2.33</td>
</tr>
<tr>
<td>$\nu_c N^T$</td>
<td>0.9</td>
<td>0.1</td>
<td>0.74</td>
<td>0.77</td>
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<tr>
<td>$\varphi_w \times 10^2$</td>
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<td>0.3</td>
<td>0.20</td>
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<tr>
<td>$\phi N$</td>
<td>3.5</td>
<td>2</td>
<td>4.82</td>
<td>5.05</td>
</tr>
<tr>
<td>$\varphi_d$</td>
<td>N</td>
<td>3.5</td>
<td>2</td>
<td>4.49</td>
</tr>
<tr>
<td>$\sigma_v \times 10^4$</td>
<td>IG</td>
<td>0.7</td>
<td>0.7</td>
<td>0.59</td>
</tr>
<tr>
<td>$\nu N^T$</td>
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<td>0.1</td>
<td>0.87</td>
<td>0.88</td>
</tr>
<tr>
<td>$\sigma_w \times 10^4$</td>
<td>IG</td>
<td>0.7</td>
<td>0.7</td>
<td>0.97</td>
</tr>
<tr>
<td>$\lambda_0 N$</td>
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<td>0.2</td>
<td>0.13</td>
<td>0.19</td>
</tr>
<tr>
<td>$\lambda_1 \times \sigma_v^2$</td>
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<td>0</td>
<td>0.2</td>
<td>0.23</td>
</tr>
<tr>
<td>$\sqrt{\mu_j / \sigma_v}$</td>
<td>G</td>
<td>1.0</td>
<td>3.0</td>
<td>1.68</td>
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<tr>
<td>$\sigma_{e,rj} \times 10^2$</td>
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<td>0.09</td>
<td>0.09</td>
<td>0.28</td>
</tr>
<tr>
<td>$\sigma_{e,pd}$</td>
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<td>0.1</td>
<td>0.04</td>
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<td>0.15</td>
<td>0.27</td>
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<tr>
<td>$\sigma_{e,iv} \times 10^2$</td>
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<td>3.0</td>
<td>1.78</td>
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<td>0.35</td>
<td>0.52</td>
</tr>
<tr>
<td>$\sigma_{e,v} \times 10^4$</td>
<td>IG</td>
<td>1.0</td>
<td>1.0</td>
<td>0.28</td>
</tr>
</tbody>
</table>

The table summarizes the prior and posterior distributions for the model parameter estimates. $G$ refers to Gamma distribution, $N$ to Normal distribution, $N^T$ is truncated (at one) Normal distribution, and $IG$ is Inverse-Gamma. Dashed line indicates that the parameter value is fixed.
Table 4: Model Implications for Asset Prices

<table>
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<tr>
<th></th>
<th>Model Pop</th>
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<th></th>
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<tbody>
<tr>
<td></td>
<td>5%</td>
<td>50%</td>
<td>95%</td>
<td>---</td>
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<tr>
<td><strong>Market price-dividend ratio:</strong></td>
<td></td>
<td></td>
<td></td>
<td>---</td>
</tr>
<tr>
<td>Mean</td>
<td>3.70</td>
<td>3.94</td>
<td>4.17</td>
<td>---</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.97</td>
<td>0.98</td>
<td>0.98</td>
<td>---</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.22</td>
<td>0.28</td>
<td>0.33</td>
<td>---</td>
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<td><strong>Risk-free rate:</strong></td>
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<td>0.37</td>
<td>0.92</td>
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</tr>
<tr>
<td>AR(1)</td>
<td>0.87</td>
<td>0.88</td>
<td>0.89</td>
<td>---</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>1.26</td>
<td>1.63</td>
<td>1.99</td>
<td>---</td>
</tr>
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<td><strong>Excess market return:</strong></td>
<td></td>
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<td></td>
<td>---</td>
</tr>
<tr>
<td>Mean</td>
<td>4.27</td>
<td>5.18</td>
<td>6.28</td>
<td>---</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>16.27</td>
<td>18.82</td>
<td>22.81</td>
<td>---</td>
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<tr>
<td>Skewness</td>
<td>-1.51</td>
<td>-0.93</td>
<td>-0.45</td>
<td>---</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.94</td>
<td>3.52</td>
<td>5.31</td>
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</tbody>
</table>

The table shows model implications for the market price-dividend ratio, the real risk-free rate, and real market return. The model statistics correspond to the population values from the model evaluated at the estimated model parameters.
Table 5: **Model Implications for Risk Compensation**

<table>
<thead>
<tr>
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<th>Model Pop</th>
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</tr>
</thead>
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<td></td>
<td></td>
<td>5%</td>
<td>50%</td>
</tr>
<tr>
<td><strong>Risk Compensation:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exp. growth risk:</td>
<td>9.83</td>
<td>10.65</td>
<td>12.69</td>
</tr>
<tr>
<td>Confidence risk:</td>
<td>-5.26</td>
<td>-4.25</td>
<td>-3.58</td>
</tr>
<tr>
<td>Cons. vol. risk:</td>
<td>-1.64</td>
<td>-1.28</td>
<td>-1.01</td>
</tr>
<tr>
<td>Cons. growth risk:</td>
<td>2.80</td>
<td>3.62</td>
<td>4.07</td>
</tr>
<tr>
<td><strong>Equity Risk Premium:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total:</td>
<td>6.21</td>
<td>6.91</td>
<td>8.09</td>
</tr>
<tr>
<td>Exp. growth risk:</td>
<td>3.70</td>
<td>4.48</td>
<td>5.39</td>
</tr>
<tr>
<td>Confidence risk:</td>
<td>1.18</td>
<td>1.66</td>
<td>2.52</td>
</tr>
<tr>
<td>Cons. vol. risk:</td>
<td>0.07</td>
<td>0.11</td>
<td>0.19</td>
</tr>
<tr>
<td>Cons. growth risk:</td>
<td>0.41</td>
<td>0.61</td>
<td>0.94</td>
</tr>
</tbody>
</table>

The table shows the magnitudes of risk compensation and the decomposition of the equity risk premium in the model into the components due to the expected consumption growth, confidence measure, consumption volatility, and the consumption growth risks. The risk compensation is measured as the market price of risk multiplied by the volatility of the risk source, in per cent. Equity premium is annualized, in per cent. The model statistics correspond to the population values from the model evaluated at the estimated model parameters.
Table 6: Model Implications for Option Prices

<table>
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<tr>
<th>Volatility Level:</th>
<th>Model Pop</th>
<th>Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5%</td>
<td>50%</td>
</tr>
<tr>
<td>0.90 mn, 1 m</td>
<td>23.00</td>
<td>24.36</td>
</tr>
<tr>
<td>0.95 mn, 1 m</td>
<td>19.52</td>
<td>20.82</td>
</tr>
<tr>
<td>1.00 mn, 1 m</td>
<td>16.89</td>
<td>18.13</td>
</tr>
<tr>
<td>1.05 mn, 1 m</td>
<td>15.22</td>
<td>16.62</td>
</tr>
<tr>
<td>1.10 mn, 1 m</td>
<td>14.05</td>
<td>15.72</td>
</tr>
<tr>
<td>0.90 mn, 3 m</td>
<td>21.07</td>
<td>22.47</td>
</tr>
<tr>
<td>0.95 mn, 3 m</td>
<td>18.41</td>
<td>19.52</td>
</tr>
<tr>
<td>1.00 mn, 3 m</td>
<td>16.20</td>
<td>17.29</td>
</tr>
<tr>
<td>1.05 mn, 3 m</td>
<td>14.87</td>
<td>16.13</td>
</tr>
<tr>
<td>1.10 mn, 3 m</td>
<td>14.80</td>
<td>16.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>5%</td>
<td>50%</td>
</tr>
<tr>
<td>0.90 mn, 1 m</td>
<td>0.88</td>
<td>0.89</td>
</tr>
<tr>
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<td>0.89</td>
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<td>0.65</td>
<td>0.68</td>
</tr>
<tr>
<td>1.10 mn, 3 m</td>
<td>0.51</td>
<td>0.55</td>
</tr>
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</table>

The table shows model implications for the unconditional level and variation in option volatilities. Top panel shows the model-implied level of the option volatility. Bottom panel shows the model-implied slope coefficients in the contemporaneous projections of option implied variances on the confidence measure and consumption variance. In these projections, the option variances, the confidence measure, and the consumption variance are standardized to have mean zero and variance 1, and are expressed in variance units. The model statistics reported in "Model Pop" correspond to the population values from the model evaluated at the estimated model parameters. The model statistics in "Sample" columns report the level of option volatility and root mean squared error evaluated in-sample based on the median estimated parameter and state values.
Table 7: Implications For Option Prices: Alternative Models

<table>
<thead>
<tr>
<th>Model Pop</th>
<th>5%</th>
<th>50%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark Model, No jump:</td>
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<tr>
<td>OTM, 1m</td>
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<td>18.15</td>
</tr>
<tr>
<td>OTM, 3m</td>
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<td>17.91</td>
<td>23.62</td>
</tr>
<tr>
<td>ATM, 3m</td>
<td>15.95</td>
<td>16.68</td>
<td>17.87</td>
</tr>
<tr>
<td>ITM, 3m</td>
<td>15.10</td>
<td>16.37</td>
<td>17.16</td>
</tr>
<tr>
<td>3-Vol Model:</td>
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<td></td>
</tr>
<tr>
<td>OTM, 1m</td>
<td>24.77</td>
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<td>19.39</td>
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<td>16.46</td>
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<td>3-Vol Model, No jump:</td>
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<td>ITM, 1m</td>
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<td>OTM, 3m</td>
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<tr>
<td>ATM, 3m</td>
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<td>20.42</td>
<td>25.61</td>
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<tr>
<td>ITM, 3m</td>
<td>15.16</td>
<td>17.87</td>
<td>23.00</td>
</tr>
</tbody>
</table>

The table shows the levels of option volatilities across alternative model specifications. "No jump" refers to the restriction of no Poisson jumps in the confidence measure. "3-vol" additionally introduces a long-run volatility component in consumption variance. OTM refers to the out-of-the-money options with moneyness of 0.90; ATM denotes the at-the-money options, and ITM is the in-the-money options with moneyness of 1.10. The model statistics correspond to the population values from the model evaluated at the estimated model parameters.
The figure shows the time series of the implied option volatilities in the data, which correspond to the out-of-the-money options with 0.90 moneyness (circles); at-the-money options (solid line), and in-the-money options with 1.10 option moneyness (dashed line). The maturity of options is 1 month (top panel) and 3 month (bottom panel). Monthly observations from January 1996 to June 2013.
Figure 2: Expected Growth and Confidence Data

Panel A: Real Growth:

The figure shows the realized (dashed line) and the expected consumption growth (solid line) from the forecast data in Panel A. Panel B shows the square root of the confidence measure in the data ($\sqrt{V}$). Monthly observations from January 1996 to June 2013 (quarterly for forecast data).
The figure shows the estimated expected real growth factor (top panel), square root of the confidence measure (middle panel), and consumption volatility (bottom panel). Solid line is the median estimate in the model, and dashed line denotes the estimate from the forecast data. Grey area indicates 5%-95% confidence interval band from the model. Monthly observations from January 1996 to June 2013 (quarterly for forecast data).
The figure shows the market price-dividend ratio (top panel) and the real risk-free rate (bottom panel) in the data and in the model. Solid line is the median estimate in the model, and dashed line denotes the data. Grey area indicates 5%-95% confidence interval band from the model. Monthly observations from January 1996 to June 2013.
Figure 5: Model Fit to Implied Volatilities

Panel A: 1-Month:

Panel B: 3-Month:

The figure shows the option implied volatility in the data and in the model. Solid line is the median estimate in the model, and dashed line denotes the data. Grey area indicates 5%-95% confidence interval band from the model. Monthly observations from January 1996 to June 2013.
Figure 6: Volatility of Equity Returns

The figure shows the volatility of equity returns in the data and in the model. Solid line is the median estimate in the model, and dashed line denotes the measurement in the data based on a GARCH(1,1) model. Grey area indicates 5%-95% confidence interval band from the model. Monthly observations from January 1996 to June 2013.

Figure 7: Equity Return Distribution

The figure shows the unconditional distribution of equity returns in the data and in the model. Solid line is the median estimate in the model, and dashed line denotes the measurement in the data. Grey area indicates 5%-95% confidence interval band from the model. Monthly observations from January 1996 to June 2013.