Short Term Debt and Incentives in Banks
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Short-term debt in financial intermediation

- One of the most distinct features of banks is their reliance on short-term debt
  - Deposits represent over three-quarters of funding of US commercial banks (Hanson, Shleifer, Stein, and Vishny, 2015)
  - Not limited to deposits: banks and shadow banks rely on creditors in wholesale funding markets (Adrian and Shin, 2010)
- Reliance on short-term debt makes banks and other financial institutions prone to fragility and runs
- Two lines of theories highlight different bank functions and roles of short-term debt:
  - Banks’ core function is to provide liquidity to their depositors, which is amplified by government guarantees
  - Banks’ short-term debt provides market discipline against risk shifting, increasing the efficiency of banks’ investments
- Both lines of theories exhibit key role for incentives in shaping banks’ capital structure choices raising questions about optimality of short-term debt and implications for fragility and welfare
Providing guidance using theory

- These issues involve complex equilibrium interactions
- Developing a model to evaluate the full scope of the problem requires understanding of:
  - (a) How runs and fragility respond to banks’ choices of short-term debt
  - (b) Given (a), how banks determine short-term debt
    - For a given original motivation, such as liquidity provision, discipline, guarantees
  - (c) Given (a) and (b), how other conditions are determined
    - For example, government guarantees, general equilibrium behavior in banking sector, etc.
- The two models I will cover in detail provide recent analyses of this kind for the two leading approaches:
  - Allen, Carletti, Goldstein, Leonello (2018): Short-term debt is driven by liquidity provision and government guarantees
  - Eisenbach (2017): Short-term debt is driven by market discipline
Government Guarantees and Financial Stability

Allen, Carletti, Goldstein, Leonello

Journal of Economic Theory, 2018
Liquidity creation, fragility, and guarantees

- Liquidity creation, fragility, and guarantees (Diamond and Dybvig, 1983):
  - Banks provide risk sharing against early liquidity needs to depositors, by offering demandable debt, thus improving their welfare
  - But, the deposit contracts expose banks to the risk of a run as depositors may withdraw early (coordination failure)
  - Government guarantees, such as deposit insurance, have been proposed as a way to address the problem and eliminate panic

- The problem with guarantees:
  - They are costly when runs do occur
  - They encourage banks to increase short-term debt (Calomiris, 1990), fragility (Demirgüç-Kunt and Detragiache, 1998), and/or risk (Gropp, Grundl, and Guttler, 2014)

- Goal: understand equilibrium interactions, fragility, Banks’ choices, and desirability of guarantees
Modelling framework

- Follow Goldstein and Pauzner (2005), where:
  - Depositors’ withdrawal decisions and probability of runs are determined by the banking contract using global-games methodology
  - Banks set deposit contract to provide risk sharing against early liquidity need, taking into account the effect on fragility

- Two inefficiencies:
  - Inefficient runs destroy good investments
  - Banks scale down liquidity creation (e.g., reducing deposit rates) in the attempt of limiting runs

- Introduce different schemes of guarantees to analyze interaction between fragility, banks’ choices, and guarantees
  - Previous theoretical literature (e.g., Keeley, 1990; Cooper and Ross, 2002; Keister, 2016) does not endogenize run probability, banks’ choices, and guarantees at the same time
Results in a nutshell

- Guarantees against panic runs (similar to Diamond and Dybvig, 1983):
  - Can eliminate panics altogether, but induce banks to increase demandable debt
  - This increases the probability of fundamental-based runs and may increase the probability of runs overall
  - But, this is not indication of moral hazard, as guarantees are never paid in equilibrium
  - Guarantees allow banks to provide more risk sharing and liquidity, increasing welfare despite greater fragility

- Guarantees against panic runs and fundamental failures
  - More realistic and potentially more desirable
  - They are costly and so limited; reduce probability of runs but do not eliminate them
  - They distort banks’ choices, since banks do not internalize the effect on cost to government
  - Usually, banks choose too little demandable debt, as they do not internalize that runs can reduce fundamental failures and reduce cost to government
Three date \( t = 0, 1, 2 \) economy with a continuum \([0, 1]\) of banks and a continuum \([0, 1]\) of consumers in every bank.

At date 0, banks raise one unit of funds from consumers in exchange for a demandable deposit contract and invest in a risky project.

The project returns 1 if liquidated at date 1 and \( \tilde{R} \) at date 2 with

\[
\tilde{R} = \begin{cases} 
R > 1 & \text{w. p. } p(\theta) \\
0 & \text{w. p. } (1 - p(\theta))
\end{cases}
\]

Fundamental shock: \( \theta \sim U[0, 1] \) is the fundamental of the economy; realized at date 1 and become public at date 2.

Probability of success: assume \( p'(\theta) > 0 \) and \( E_\theta[p(\theta)]R > 1 \)

For simplicity, \( p(\theta) = \theta \)

Banking sector is competitive, so that deposit contracts maximize consumers' welfare; not taking into account externalities.
Preferences

- Consumers are risk-averse \((RRA > 1\) for any \(c \geq 1\)) and endowed with 1 unit each at date 0
- At date 0 they deposit at the bank in exchange for a deposit contract \((c_1, \tilde{c}_2)\)
- Consumers are ex ante identical but each has probability \(\lambda\) of suffering a liquidity shock and having to consume at date 1
  - Uncertainty is resolved privately at the beginning of date 1
- Consumers derive utility both from consuming at date 1 or 2 and from enjoying a public good \(g\)

\[
U(c, g) = u(c) + v(g)
\]

with \(u'(c) > 0\), \(v'(g) > 0\), \(u''(c) < 0\), \(v''(g) < 0\), \(u(0) = v(0) = 0\) and

\[
u'(1) < v'(g) < u'(0)
\]
Depositors’ information

- At the beginning of date 1, each depositor receives a private signal $x_i$ regarding the fundamental of the economy $\theta$ of the form
  
  $$x_i = \theta + \epsilon_i,$$

  with $\epsilon_i \sim U[-\epsilon, +\epsilon]$ being i.i.d. across agents. Most of the time, we focus on $\epsilon$ very close to 0.

- Based on the signal, depositors update their beliefs about the fundamental $\theta$ and the actions of the other depositors.
  - Early depositors always withdraw at date 1.
  - Late depositors withdraw at date 1 if they receive a low enough signal.

- The bank satisfies early withdrawal demands by liquidating its investments. If proceeds are not enough, depositors receive a pro-rata share.
Decentralized equilibrium

- Combination of
  - Bayesian Nash equilibrium among depositors at $t = 1$
  - Competitive equilibrium among banks at $t = 0$

- At date 1:
  - Fraction of depositors who withdraw: $n \geq \lambda$
  - Depositor payoffs (depending on bank liquidity):

<table>
<thead>
<tr>
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<th>liquid: $n \leq \frac{1}{c_1}$</th>
<th>illiquid: $n &gt; \frac{1}{c_1}$</th>
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<tbody>
<tr>
<td>wait</td>
<td>$\frac{1-nc_1}{1-n}R$ w. p. $\theta$</td>
<td>0</td>
</tr>
<tr>
<td>withdraw</td>
<td>$c_1$</td>
<td>$\frac{1}{n}$</td>
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- Unique equilibrium: $n = 1$ below $\theta^*$; $n = \lambda$ above $\theta^*$

- At date 0:
  - Banks set $c_1^D$ to maximize expected utility of depositors
The decentralized solution: Depositors’ withdrawals

\[ \theta(c_1) \] is the boundary for "fundamental runs"; determined as the indifference point assuming others don’t run:

\[
\theta(c_1) = \theta u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right)
\]

\[ \theta^*(c_1) \] is the cutoff for "panic runs"; determined as the indifference point assuming uniform distribution on depositors who withdraw:

\[
\int_{n=\lambda}^{\frac{1}{c_1}} \theta^* u \left( \frac{1 - nc_1}{1 - n} R \right) = \int_{n=\lambda}^{\frac{1}{c_1}} u(c_1) + \int_{n=\frac{1}{c_1}}^{1} u \left( \frac{1}{n} \right)
\]

Both thresholds \( \theta(c_1) \) and \( \theta^*(c_1) \) increase in \( c_1 \).
The decentralized solution: Types of crisis

- Banks fail when they are not able to repay the promised repayment
  - It can occur either at date 1 or 2
- At date 1, banks fail because of runs
  - Low fundamentals below $\theta(c_1)$ — anticipation of low returns at date 2
  - Panic between $\theta(c_1)$ and $\theta^*(c_1)$ — coordination failure among depositors
- At date 2, banks fail when their asset returns 0
  - Project fails with probability $(1 - \theta)|\theta > \theta^*$
The decentralized solution: The bank’s choice

- Given depositors’ withdrawal decisions, at date 0 each bank chooses $c_1$ to maximize:
  \[
  \int_0^{\theta^*(c_1)} u(1) \, d\theta + \int_{\theta^*(c_1)}^1 \left[ \lambda u(c_1) + (1 - \lambda) \theta u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] \, d\theta \\
  + \nu(g)
  \]

- The optimal $c_1^D > 1$ trades off:
  - Better risk sharing; transferring consumption from patient to impatient agents
  - Against higher probability of runs $\left( \frac{\partial \theta^*(c_1)}{\partial c_1} > 0 \right)$

- Two inefficiencies related to panics:
  - Banks offer too little risk sharing (liquidity creation) in anticipation of the run: $c_1^D$ is lower than first best
  - Runs lead to inefficient liquidation of bank investment for $\theta \in (\theta(1), \theta^*(c_1^D))$

- Another inefficiency comes due to the fact that depositors are not protected against fundamental failure
Government guarantees against panics

- A natural starting point to demonstrate the effect of government guarantees is a scheme that guarantees against panic
  - This is closest to Diamond-Dybvig, except that banking contract will react to the scheme

- Specifically, depositors are guaranteed to receive $\bar{c}_s = \frac{1-\lambda c_1}{1-\lambda} R$ when the bank's project is successful at date 2, irrespective of how many depositors have withdrawn at date 1

- Panic runs are eliminated but fundamental runs remain for $\theta \in [0, \theta(c_1)]$

- Bank chooses $c_1^P$ to maximize

$$\int_0^{\theta(c_1)} u(1) d\theta + \int_{\theta(c_1)}^1 \left[ \lambda u(c_1) + (1 - \lambda) \theta u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] d\theta$$

$$+ \int_0^1 v(g) d\theta$$
Deposit contract under guarantees against panics

- Under guarantees against panic, \( c_1^P \) solves:
  \[
  \lambda \int_{\theta(c_1)}^{1} \left[ u'(c_1) - \theta u' \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] d\theta + \\
  - \frac{\partial \theta(c_1)}{\partial c_1} \left[ \lambda u(c_1) + (1 - \lambda)\theta u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(1) \right] = 0
  \]

- In decentralized solution, \( c_1^D \) solves:
  \[
  \lambda \int_{\theta^*(c_1)}^{1} \left[ u'(c_1) - \theta u' \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] d\theta + \\
  - \frac{\partial \theta^*(c_1)}{\partial c_1} \left[ \lambda u(c_1) + (1 - \lambda)\theta^* u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(1) \right] = 0
  \]

- Result: \( c_1^P > c_1^D \). Thus, \( \theta(c_1^P) > \theta(c_1^D) \) and possibly \( \overline{\theta}(c_1^P) > \theta^*(c_1^D) \)

- Note: No distortion in the choice of \( c_1^P \) as the guarantee entails no disbursement for the government
Runs and welfare under the guarantees against panics

- As $c_1^P > c_1^D$, guarantees
  - Increase the probability of fundamental runs and possibly runs overall

- Two scenarios depicted below:

- But, guarantees increase depositors’ expected utility from the private good and increase overall welfare
  - Increased short-term debt is not evidence of moral hazard
  - It reflects better ability of banks to provide liquidity and risk sharing
Adding guarantees against bank failure at date 2

- Still keep $\bar{c}_s = \frac{1-\lambda c_1}{1-\lambda} R$ at $t = 2$ iff the project succeeds
- Introduce guarantee $\bar{c}_f \neq \bar{c}_s$ at date 2 if the bank project fails
  - $\bar{c}_f > 0$ insures agents against fundamental risk and reduces probability of fundamental runs
  - But, it is costly as bank failures can occur and the government has to reduce $g$ to pay for the guarantee

Questions:
- Does the government want to set $\bar{c}_f > 0$?
- How do banks respond?
Runs and deposit contract under additional guarantee

- Only fundamental runs occur. The threshold $\theta$ is the solution to
  \[ u(c_1) = \theta u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) + (1 - \theta) u(\bar{c}_f) , \]
- The threshold $\theta$ increases in $c_1$ and decreases in $\bar{c}_f$
- Each bank sets $c_1^F$ to maximize
  \[ \int_0^\theta u(1) \, d\theta + \int_\theta^1 \left[ (1 - \lambda) \left[ \theta u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) + \right] + \lambda u(c_1) \right] \, d\theta + E \left[ v(g, c_1^*, \bar{c}_f) \right] \]
- Results show that $\frac{dc_1^F}{d\bar{c}_f} > 0$. Thus, $c_1^F > c_1^P$ for any $\bar{c}_f > 0$
- The bank does not internalize the reduction in $g$ for the provision of the guarantee
The government choice for additional guarantee

- Government chooses $\bar{c}_f$ to maximize depositors’ overall expected utility
  - Cost of the disbursement is internalized
  - The effect on the bank’s choice of $c_1^F$ is also taken into account
- The government chooses $\bar{c}_f > 0$ when $u'(0) - v'(g) > 0$
  - The government with a sufficiently large endowment wants to provide some guarantees to reduce runs
- Interestingly, there is a reverse moral hazard: the government would choose higher short-term commitment for the bank: $c_1^G > c_1^F$
  - This is because of lower expected utility from public good if no runs occur:
    $$\bar{\theta}v(g) + (1 - \bar{\theta})v(g - (1 - \lambda)\bar{c}_f) < v(g)$$
  - This is the only thing that is not internalized by the bank in the model
Deposit insurance

- Depositors are guaranteed to receive a $\bar{c}_s = \bar{c}_f = \bar{c}$ whenever their bank is not able to repay the promised repayment
  - More realistic; similar to a standard deposit insurance scheme with $\bar{c}$ being the lowerbound on depositors’ payment
  - Less desirable, because amount guaranteed is not tailored to the cause and because guarantee might also imply payment at date 1, which is never optimal

- Probability of both types of runs is reduced but both runs still occur
  - It is too costly to fully guarantee against panic when amount of guarantee is the same in all cases

- Providing guarantees is costly and the market solution is inefficient
  - Again, banks internalize the effect of their choices on the run probability, but not on the cost of providing the guarantee
Depositors’ withdrawal decisions with deposit insurance

- Fundamental runs occur for $\theta < \theta(c_1, \bar{c})$ where $\theta(c_1, \bar{c})$ solves

$$u(c_1) = \theta u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) + (1 - \theta) u(\bar{c})$$

- Panic runs occur now for $\theta < \theta^*(c_1, \bar{c})$ where

$$\theta^*(c_1, \bar{c}) = \frac{\int_{n=\lambda}^{\hat{n}} u(c_1) + \int_{n=\hat{n}}^{1} u(\frac{1}{n}) - \int_{n=\lambda}^{1} u(\bar{c})}{\int_{n=\lambda}^{\hat{n}} \left[ u \left( \frac{1 - n c_1}{1 - n} R \right) - u(\bar{c}) \right]},$$

and $\bar{n} = \frac{R - \bar{c}}{R c_1 - \bar{c}}$ and $\hat{n} = \frac{1}{c_1}$

- The thresholds $\theta(c_1, \bar{c})$ and $\theta^*(c_1, \bar{c})$ increase with $c_1$ and decrease with $\bar{c}$
Bank’s choice of the deposit contract under deposit insurance

- When $\bar{c} < 1$, each bank sets $c_1$ now to maximize

$$
\int_0^{\theta^*} u(1) \, d\theta + \int_{\theta^*}^1 [\lambda u(c_1) + (1 - \lambda)(\theta u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) + \\
+ (1 - \theta) u(\bar{c})] \, d\theta + E [\nu(g, c_1^*, \bar{c})]
$$

where $\theta^* = \theta^*(c_1, \bar{c})$, and

$$
E[\nu(g, c_1^*, \bar{c})] = \int_0^{\theta^*} \nu(g) \, d\theta + \\
+ \int_{\theta^*}^1 [\theta \nu(g) + (1 - \theta) \nu(g - (1 - \lambda)\bar{c})] \, d\theta
$$

- The deposit contract $c_1^{Dl} > c_1^D$, with $\frac{dc_1^{Dl}}{dc} > 0$ solves

$$
\lambda \int_{\theta^*}^1 \left[ u'(c_1) - \theta Ru' \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] \, d\theta + \\
- \frac{\partial \theta^*}{\partial c_1} \left[ \lambda u(c_1) + (1 - \lambda) \left( \theta^* u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) + (1 - \theta^*) u(\bar{c}) \right) - u(1) \right] = 0
$$
Government choice under deposit insurance

- The government has the same objective as the bank but internalizes the costs of providing the guarantee while taking $c_1^{DI}$ as given.
- It can be shown that $0 < \bar{c} < 1$ if $g$ is not too high.
- In this case, government would like to choose a $c_1^G > c_1^{DI}$ as
  \[
  \theta^* v(g) + (1 - \theta^*) v(g - (1 - \lambda)\bar{c}) < v(g)
  \]
- Liquidating banks early (e.g., via prompt corrective actions) can be optimal as it allows to minimize the costs associated with public intervention.
- Despite the inefficiency of the market solution, this scheme may lead to higher welfare than the decentralized solution.
Conclusions

- Government guarantees present a complicated trade-off and understanding it requires endogenizing banks’ choices and depositors’ behavior in response to government intervention.
- Increased demandable debt and fragility may be desirable as they reflect greater liquidity provision by banks.
- While banks’ choices may be distorted, in many cases more demandable debt is desirable.
- Theoretical framework sheds new light on empirical results and policy discussions.
Rollover Risk as Market Discipline: A Two-Sided Inefficiency

Eisenbach

Short-term debt and market discipline

- Underlying theory (Calomiris and Kahn, 1991; Diamond and Rajan, 2001):
  - Leverage provides an incentive for bank equity holders and managers to conduct risk shifting and not liquidate bad projects
  - Demandable debt provides discipline and induces liquidation if creditors run upon receiving bad news

- Problems with market discipline:
  - Insufficient discipline in good times (e.g. Admati et al., 2010):
    - Increasing reliance on short-term funding and increasingly risky activities
  - Excessive discipline during crisis (e.g. Gorton and Metrick, 2012):
    - Large-scale withdrawal of short-term funding affecting issuers unrelated to housing
Modelling framework and key results

- Banks optimally choose debt maturity structure
  - Short term debt disciplines risk taking
- Rollover risk modeled as global game
  - Resolve multiplicity at interim stage
  - Probability of a run can be characterized for underlying parameters and banks’ choices
- Embed in General equilibrium framework for amplification effects across banks
  - Excessive risk taking in good times
  - Excessive liquidation in bad times
Model

- Three periods $t = 0, 1, 2$, agents risk neutral, discount rate 0
- A continuum $[0, 1]$ of banks $(i)$ and a continuum $[0, 1]$ of creditors $(j)$ in every bank
- Every bank has a project:
Incentive problem

- Efficiency requires:

\[ \text{Continue } \iff \theta_i X > \ell \]

- However, if bank is financed by a combination of debt and equity, risk shifting incentives emerge (Jensen and Meckling, 1976), since liquidation proceeds go mostly to creditors
  - Banker continues even if \( \theta_i X < \ell \)

- For simplicity, assume that bank is financed only with debt (focus on maturity choice)
Financing

- Investment at $t = 0$ funded by competitive creditors
- Each bank $i$ has a continuum of creditors $j \in [0, 1]$
- Long-term debt:
  - Face value $B_i$ matures at $t = 2$
- Short-term debt:
  - Face value $R_i$ if withdrawn at $t = 1$
  - Face value $R_i^2$ if rolled over to $t = 2$
- Bank chooses maturity structure of debt:
  - Fraction of short-term debt $\alpha_i$
  - Fraction of long-term debt $1 - \alpha_i$
- Face values $B_i$ and $R_i$ adjust so creditors break even
Uncertainty and information

- Idiosyncratic risk for bank $i$:
  \[ \theta_i \text{ drawn i.i.d. from } F_s \]

- Aggregate risk state:
  \[ s \in \{H, L\} \text{ with } \Pr[s = H] = p \]

- First-order stochastic dominance:
  \[ F_H(\theta) < F_L(\theta) \text{ for all } \theta \in (0, 1) \]

- Information at $t = 1$:
  - Aggregate $s$: common knowledge
  - Idiosyncratic $\theta_i$: creditor $ji$ observes signal $x_{ji} = \theta_i + \sigma \epsilon_{ji}$
Liquidation value

- Aggregate asset sales $\phi \in [0, 1]$ used in secondary sector
- Liquidation value $= \text{marginal product:}$
  \[\ell(\phi) \quad \text{with} \quad \ell'(\phi) < 0\]
- In equilibrium:
  \[E_H[\theta X] > E_L[\theta X]\]
  \[\Rightarrow \quad \phi_H < \phi_L\]
  \[\Rightarrow \quad \ell_H > \ell_L\]
Equilibrium

Combination of

1. Bayesian Nash equilibrium among creditors at $t = 1$
2. Competitive equilibrium among banks at $t = 0$
Creditor Coordination

- Fraction of creditors who withdraw: $\lambda$
  - Bank illiquid if $\alpha \lambda R > \ell$
- Creditor payoffs

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<thead>
<tr>
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<th>liquid</th>
<th>illiquid</th>
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<tbody>
<tr>
<td>roll over</td>
<td>$\theta R^2$</td>
<td>0</td>
</tr>
<tr>
<td>withdraw</td>
<td>$R$</td>
<td>$\ell$</td>
</tr>
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Complication:

- Liquidation value $\ell$
  - enters payoff of all creditors at all banks
  - depends on coordination outcomes at all banks
→ All creditors at all banks are interacting
Creditor equilibrium

With symmetric banks, for \( \sigma \to 0 \), the unique Bayesian Nash equilibrium is in switching strategies around a threshold \( \hat{\theta} \) given by

\[
\hat{\theta} = \frac{(1 + \alpha) R - \ell}{R^2}
\]

- For realizations \( \theta_i > \hat{\theta} \):
  - All creditors \( ji \) roll over
  - Bank \( i \) is liquid and project continues

- For realizations \( \theta_i < \hat{\theta} \):
  - All creditors \( ji \) withdraw
  - Bank \( i \) is illiquid and project is liquidated
Creditor with signal $x = \hat{\theta}$ has to be indifferent:

$$\text{Pr} \left[ \text{liquid} \right] \times \hat{\theta} R^2 = \text{Pr} \left[ \text{liquid} \right] \times R + \text{Pr} \left[ \text{illiquid} \right] \times \ell$$

For $\sigma \rightarrow 0$, distribution of $\lambda \mid \hat{\theta}$ becomes uniform

$$\text{Pr} \left[ \text{liquid} \right] = \text{Pr} \left[ \lambda \leq \frac{\ell}{\alpha R} \right] = \frac{\ell}{\alpha R}$$

Resulting in:

$$\frac{\ell}{\alpha R} \times \hat{\theta} R^2 = \frac{\ell}{\alpha R} \times R + \left[ 1 - \frac{\ell}{\alpha R} \right] \times \ell$$

$$\Rightarrow \hat{\theta} = \frac{(1 + \alpha) R - \ell}{R^2}$$
Rollover risk

Ex ante rollover risk for bank $i$:

$$\Pr \left[ \theta_i \leq \frac{(1 + \alpha_i) R_i - \ell}{R_i^2} \right]$$

- Depends on maturity structure $\alpha_i$:
  - Directly $\rightarrow$ increasing
  - Indirectly through $R_i$

- Run is more likely for:
  1. Bad idiosyncratic news (low $\theta_i$)
  2. Bad aggregate news (low $\ell$)
No aggregate risk

- No aggregate risk, \( F_H = F_L =: F \)
  - liquidation value deterministic, \( \ell_H = \ell_L =: \ell \)
- Bank’s problem:

\[
\max_{\alpha} \int_{\hat{\theta}}^{1} \theta \left( X - \alpha R^2 - (1 - \alpha) B \right) \, dF(\theta)
\]

subject to \( F(\hat{\theta}) \ell + \int_{\hat{\theta}}^{1} \theta R^2 \, dF(\theta) = 1 \)

\( F(\hat{\theta}) \ell + \int_{\hat{\theta}}^{1} \theta B \, dF(\theta) = 1 \)

\( \hat{\theta} = \frac{(1+\alpha)R-\ell}{R^2} \)

Above conditions implicitly define \( \hat{\theta} \) as a function of \( \alpha \) with

\( \hat{\theta}'(\alpha) > 0 \)
Optimal maturity structure

Without aggregate risk

- Bank problem becomes:

$$\max_\alpha F(\hat{\theta}(\alpha)) \ell + \int_{\hat{\theta}(\alpha)}^{1} \theta X \, dF(\theta) - 1$$

- Bank chooses efficient liquidation:

$$\hat{\theta}(\alpha^*) = \frac{\ell}{X}$$
With aggregate risk

- With aggregate risk, $F_H(\theta) < F_L(\theta)$ for all $\theta \in (0, 1)$
  - liquidation value uncertain, $\ell_H > \ell_L$

- Two opposing effects:

  **Efficiency**: Want less liquidation in bad state
  \[
  \frac{\ell_H}{X} > \frac{\ell_L}{X}
  \]

  **Rollover risk**: Get more liquidation in bad state
  \[
  \frac{(1 + \alpha) R - \ell_H}{R^2} < \frac{(1 + \alpha) R - \ell_L}{R^2}
  \]
Optimal maturity structure
With aggregate risk

Bank trades off two inefficiencies:

\[ \hat{\theta}_H(\alpha^*) < \frac{\ell_H}{X} \quad \text{and} \quad \hat{\theta}_L(\alpha^*) > \frac{\ell_L}{X} \]
General equilibrium
Without aggregate risk

- Conditions implicitly defining $\hat{\theta}(\alpha)$ both depend on $\ell$
- Liquidation value $\ell$ depends on aggregate asset sales $\phi$
  → Explicitly $\hat{\theta}(\alpha, \phi)$
- Competitive banks take $\phi$ as given
  - choose $\alpha^*(\phi)$
  - yielding $\hat{\theta}(\alpha^*(\phi), \phi)$
- Symmetry:
  mass of assets sold = fraction of banks with $\theta_i < \hat{\theta}(\alpha^*(\phi), \phi)$
General equilibrium
Without aggregate risk

- Competitive equilibrium allocation:

\[ \phi^{CE} = F\left(\hat{\theta}\left(\alpha^*(\phi^{CE}), \phi^{CE}\right)\right) \quad \text{with} \quad \hat{\theta}\left(\alpha^*(\phi), \phi\right) = \frac{\ell(\phi)}{X} \]

- First-best allocation:

\[ \phi^{FB} = F\left(\frac{\ell(\phi^{FB})}{X}\right) \]

→ Without aggregate risk, CE achieves FB allocation
General equilibrium
With aggregate risk

- Competitive equilibrium allocation $\Phi = [\phi_H, \phi_L]$:
  $$\Phi^{CE} = \left[ F_H\left(\hat{\theta}_H \left( \alpha^*(\Phi^{CE}, \Phi^{CE}) \right) \right), F_L\left(\hat{\theta}_L \left( \alpha^*(\Phi^{CE}, \Phi^{CE}) \right) \right) \right]$$

- First-best allocation:
  $$\Phi^{FB} = \left[ F_H\left( \frac{\ell(\phi_H^{FB})}{X} \right), F_L\left( \frac{\ell(\phi_L^{FB})}{X} \right) \right]$$

  With $F_H(\theta) < F_L(\theta)$ and $F_s(\ell(\phi_s)/X)$ decreasing in $\phi_s$

- Amplification:
  $$\ell(\phi_H^{CE}) > \ell(\phi_H^{FB}) > \ell(\phi_L^{FB}) > \ell(\phi_L^{CE})$$
Feedback loops
With aggregate risk

State $H$
- Good news
  - Increased bank stability
  - Inflated liquidation values
  - Weaker market discipline
  - Fewer asset sales

Excessive risk taking

State $L$
- Bad news
  - Reduced bank stability
  - Depressed liquidation values
  - Stronger market discipline
  - More asset sales

Excessive liquidation
Conclusions

- Individual bank stability depends on
  1. News about idiosyncratic return
  2. News about aggregate conditions

- Efficiency and market discipline diverge
  → Two-sided inefficiency, in bad and good times