Capital Structure

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Debt and Equity

- There are two main types of financing: **debt** and **equity**.
- Consider a two-period world with dates *0* and *1*. At date 1, the firm's assets are worth X. The firm has debt at face value of D.
- The value of debt at date *1* will be $min\{(1+r)D, X\}$, and the value of equity will be $max\{X-(1+r)D, 0\}$. Debt is the **senior claimant** to the firm's returns and equity is the **residual claimant**.
- Capital structure theory asks what is the optimal composition between debt and equity.

Modigliani and Miller (1958): Irrelevance Theorem

- A benchmark striking result is that under fairly general conditions, the value of the firm – defined as the sum of value of debt and equity – does not change as we change the capital structure.

- Under risk neutrality, this is very easy to see:

$$V = D + E = \frac{min\{(1+r)D, X\}}{1+r} + \frac{max\{X - (1+r)D, 0\}}{1+r} = \frac{X}{1+r}$$

- Under risk aversion, things are more challenging as the discount rates are different. The proof is by **no-arbitrage** arguments.

- Take two firms, *1* and *2*, where *1* is unlevered and *2* is levered: $V_1 = E_1$ and $V_2 = D_2 + E_2$. Both have the same asset generating X.
- Take an investor holding share s of firm *2*, for a total of sE_2. This gives him a total payoff of $s \cdot max\{X - (1+r)D_2, 0\}$ at date *1*.
- Suppose that he wants to buy $s(D_2 + E_2)$ in equity of firm *1*. He does that by selling his shares of firm *2* and borrowing sD_2. This gives him a payoff of $max\left\{\frac{s(D_2+E_2)}{E_1}X - s(1+r)D_2, 0\right\}$ at date *1*.
- If $V_2 = D_2 + E_2 > E_1 = V_1$, this is an arbitrage profit, which cannot exist in equilibrium.

- Now suppose that an investor holds share s of firm *1*, for a total of sE_1. This gives him a total payoff of $s \cdot X$ at date *1*.
- Suppose that he wants to sell his shares and buy $\frac{E_2}{D_2+E_2} sE_1$ equity of firm *2* and $\frac{D_2}{D_2+E_2} sE_1$ debt of firm *2*. This gives him a payoff of $s \frac{E_1}{D_2+E_2} \cdot max\{X-(1+r)D_2, 0\} + s \frac{E_1}{D_2+E_2} \cdot min\{(1+r)D_2, X\}$.
- If $V_2 = D_2 + E_2 < E_1 = V_1$, this is an arbitrage profit, which cannot exist in equilibrium.
- Hence, $V_1 = V_2$ must hold in equilibrium!

- While striking, the Modigliani-Miller result is based on some strong assumptions. Here are some of them:
  - Fixed investment policy;
  - No informational asymmetries or heterogeneous expectations;
  - Equal access to borrowing and lending;
  - No taxes, bankruptcy costs, agency costs.
- Further research on the theory of capital structure uses Modigliani-Miller as a benchmark, and then relaxes some of the assumptions.

Capital Structure and Taxes

- The first line of attack on the irrelevance result uses the argument that taxes provide incentives to firms to use debt. This was already acknowledged by Modigliani and Miller themselves.

- Denote the corporate tax rate as τ_C, the tax code in most countries deducts interest payments (but not dividend payments) from the tax liability, leading to an increase of $\tau_C rD$ in annual cash flow, or an increase of $\tau_C D$ in value (in case of perpetual debt).

- Based on this, firms want a *D/V* ratio of 100%.

- This is obviously not realistic. Hence, theories of capital structure often assume some cost of bankruptcy that increases in debt, and derive optimal capital structure as the balance between the tax benefit and the bankruptcy cost. This is often referred to as the **tradeoff theory**.
- Another way to generate balanced conclusions is to account for personal taxes.
    - Personal taxes create an incentive for equity, as the personal tax rate on equity income is often lower than that on debt income.

Miller (1977)

- Denoting the personal tax rate on income from stocks as τ_{PS} and that on income from bonds as τ_{PB}, Miller (1977) derives the following formula for the increase in firm value for perpetual debt D:

$$\left[1 - \frac{(1-\tau_C)(1-\tau_{PS})}{1-\tau_{PB}}\right] D$$

- Intuitively, the benefit of debt increases in corporate tax rate (higher tax shield), increases in personal tax rate on income from stocks (equity is more expensive), and decreases in personal tax rate on income from bonds (debt is more expensive).

- He also derives a notion of equilibrium, later known as a **Miller equilibrium**, where the above benefit from debt is *0* for the marginal investor
  - Different investors have different tax rates.
  - Firms will issue debt as long as benefit is positive, until they reach marginal investor.
- This restores some of the spirit of the old irrelevance theorem.

Theories Based on Asymmetric Information

- Suppose that managers know more about the prospects of their firms than outside investors.
- They may use debt to signal their strength, as, for example, raising debt shows they are not concerned about bankruptcy.
- They may raise debt since raising equity appears as selling ‘lemons’.
- Theories are based on the early literature on asymmetric information and signaling: Akerlof (1970), Spence (1973).

Ross (1977): The Signaling Role of Debt

Basic Setup

- There are two types of firms, *A* and *B*.
- There are two dates, *0* and *1*. At date *1*, firm *A* will have a total value of *a*, and firm *B* will have a total value of *b*, where $a>b$.
- There is a probability *q* (*1*-*q*) that a firm is of type *A* (*B*).
- Market prices are determined by risk-neutral investors with a discount rate of *r*.

Managers

- A firm's manager has better information than any outsider, and is assumed to be the only one knowing the type of the firm.

- The manager receives the following compensation:

$$M = (1+r)\gamma_0 V_0 + \gamma_1 \begin{Bmatrix} V_1 & if \;\; V_1 \geq F \\ V_1 - L & if \;\; V_1 < F \end{Bmatrix}$$

  - The manager's compensation is based on date-*0* and date-*1* values, where γ_0 and γ_1 denote weights.

- V_0 is determined in the financial market based on the perceived value, and V_1 is the realized value (which can be a or b).
- F denotes the value of debt issued by a firm at date *0*; L is the penalty assessed on the manager if the firm is bankrupt in date *1*.
- Contract can be endogenized to address various incentive and signaling issues.

- The manager sets F to maximize his compensation above. The market observes F and thus this will affect V_0 in addition to the direct effect on the possibility of being penalized.

Signaling Equilibrium

- A signaling equilibrium is a **separating equilibrium**, where firm A issues more debt than firm B. The market is then able to identify the type of the firm and price its security correctly at date 0.

- Firm A issues more debt to separate itself from firm B. A separating equilibrium is sustained if firm B does not find it worthwhile to mimic firm A. This may be the case here because increasing the level of debt is more costly for firm B than for firm A.

- For some parameters a separating equilibrium may not exist, and we may find ourselves in a **pooling equilibrium** where both firms do the same and the market cannot separate them.

- Equilibrium concept is the **Perfect Bayesian equilibrium**.

    - “Perfect” implies that in a sequential game, players’ actions constitute a **best response** to others’ strategies in every stage.

    - “Bayesian” implies that players don’t know the types of others. Their actions constitute a **best response** to other types’ strategies given their beliefs about other players’ types.

- Consider an equilibrium where the market believes that there is a critical level of debt F^*, $b \leq F^* < a$, such that A-type firms issue $F > F^*$ and B-type firms issue $F \leq F^*$.

- Observing $F^A > F^*$ and $F^B \leq F^*$, the market correctly prices:

$$V_0(F^A) = \frac{a}{1+r};\ V_0(F^B) = \frac{b}{1+r}$$

- It remains to check that the market's belief is consistent with the firms' optimal choice of debt.

- The compensation of a manager of firm *A* is:

$$M(F^A) = \begin{cases} (\gamma_0 + \gamma_1)a & if \quad F^* < F^A \le a \\ \gamma_0 b + \gamma_1 a & if \quad F^A \le F^* \end{cases},$$

So he clearly wants to choose $F^A > F^*$.

- The compensation of a manager of firm *B* is:

$$M(F^B) = \begin{cases} \gamma_0 a + \gamma_1 (b - L) & if \quad F^B > F^* \\ (\gamma_0 + \gamma_1) b & if \quad F^B \le b \le F^* \end{cases},$$

So for him choosing $F^B \leq b \leq F^*$ is optimal as long as $\gamma_0 a + \gamma_1(b - L) < (\gamma_0 + \gamma_1)b$, that is:

$$\gamma_0(a - b) < \gamma_1 L.$$

- This is the condition needed for the separating equilibrium to hold. Otherwise there can be a pooling equilibrium, or even no equilibrium.

- Note that the parameters γ_0, γ_1, and L may be chosen endogenously here by the type-A manager, who creates the conditions that will enable him to separate himself.

A Richer Model

- The model becomes more interesting when managers have uncertainty about their firm's value and there are many types.
- Suppose that the firm's value at date *1* is *X*, which is uniformly distributed on $[0, t]$, the level of *t* is known only to the manager, and there is a continuum of types: $t \in [c, d]$.
- The incentive structure is similar to before:

$$M = (1 + r)\gamma_0 V_0 + \gamma_1 E \begin{Bmatrix} X & if \ \ X \geq F \\ X - L & if \ \ X < F \end{Bmatrix}$$

- If a firm's type is known for sure, then its current value is:

$$V_0 = \frac{t}{2(1+r)}$$

- We are looking for a separating equilibrium where the level of debt F perfectly reveals the firm's type $a(F)$. Then, a manager's compensation is given by:

$$\frac{1}{2}\gamma_0 a(F) + \gamma_1 \left[\frac{1}{2}t - L\frac{F}{t}\right]$$

- For the manager to choose F, we need:

$$\frac{1}{2}\gamma_0 a'(F) = \gamma_1 \frac{L}{t}$$

- In equilibrium, the signal leads to identification, and thus:

$$a(F) = t.$$

- Hence, we can write:

$$\frac{1}{2}\gamma_0 t' = \gamma_1 \frac{L}{t}, \text{ or}$$

$$F' = \frac{1}{2}\frac{\gamma_0}{\gamma_1}\frac{t}{L}$$

- The solution to this differential equation is:

$$F_t = \frac{1}{4}\frac{\gamma_0}{\gamma_1}\frac{t^2}{L} + b$$

- We know that firms of the lowest type will have *0* leverage, $\mathrm{F_c} = 0$, and thus:

$$F_t = \frac{\gamma_0}{4\gamma_1 L}[t^2 - c^2]$$

- This gives us the choice of leverage of every type of firm in the model.

Some Implications

- Firms of higher type have more debt ($F_t = \frac{\gamma_0}{4\gamma_1 L}[t^2 - c^2]$) and higher value ($V_0 = \frac{t}{2(1+r)}$).

- The debt to value ratio, $\frac{\frac{\gamma_0}{4\gamma_1 L}[t^2 - c^2]}{1+r} \Big/ \frac{t}{2(1+r)} = \frac{\gamma_0}{2\gamma_1 L}\left[t - \frac{c^2}{t}\right]$ is increasing in type.

- The probability of bankruptcy $\frac{F}{t} = \frac{\gamma_0}{4\gamma_1 L}\left[t - \frac{c^2}{t}\right]$ is increasing in type.

- High value is linked to high leverage and probability of bankruptcy!

Leland and Pyle (1977): A Different Version of Signaling

- Risk averse entrepreneur needs to finance a project with a mix of debt and equity.
- The quality of the project is known only to the entrepreneur.
- Raising more debt and less equity is costly, because the entrepreneur has to maintain a bigger share of equity and take more risk.
- This cost is lower to entrepreneurs who know the quality is high.
- Hence, taking debt can serve as a signal of high quality.

Myers and Majluf (1984): The Pecking Order Theory

- Raising equity to finance investments might be perceived by outside markets as an attempt to sell a 'lemon': **adverse selection**.
- This might increase the cost of equity so much that the firm will have to forego positive NPV projects.
- Hence, a pecking-order emerges, where the firm will use internal capital first, then raise debt (which is less exposed to the problem of asymmetric information) and then raise equity.

Basic Model

- The firm has an existing asset and an investment opportunity requiring an amount of I. The firm has financial slack S. Making the investment requires the firm to issue equity at the amount of $E = I - S$ (for now, only equity is considered).
- The expected value of the asset in place is $\overline{A} = E(\tilde{A})$. The realization is $a>0$ which is known at $t=0$ only to managers.
- The expected NPV of the investment is $\overline{B} = E(\tilde{B})$. The realization is $b>0$ which is known at $t=0$ only to managers.

- Managers act to maximize the value for existing shareholders.
- If they issue equity, the price they receive is P':
    - The price reflects the value of the firm as perceived by the market. Crucial: it is affected by the inference drawn by market participants from the fact that the firm decided to issue equity.
- After the issuance, the value for old shareholders is:

$$V^{old} = \frac{P\prime}{P'+E}(E + S + a + b).$$

- Absent issuance, the value is: $V^{old} = S + a$.

- Shareholders are better off by issuance when:

$$\frac{E}{P'+E}(S+a) \leq \frac{P'}{P'+E}(E+b).$$

  - That is, when the share of the value of old assets that they give up is smaller than their share of the value of new assets.

- A simpler way to write this condition:

$$\frac{E}{P'}(S+a) \leq E+b.$$

- If the management follows this rule, P' will adjust accordingly:

$$P' = S + E\left(\tilde{A}\,\middle|\,\frac{E}{P'}(S+a) \leq E+b\right) + E\left(\tilde{B}\,\middle|\,\frac{E}{P'}(S+a) \leq E+b\right).$$

Example

- There are two equally likely states.
    - In state 1: $a=150$ and $b=20$, while in state 2: $a=50$ and $b=10$.
    - $I=100$, $S=0$, so the required equity to raise is $E=100$.
- Suppose that the firm issues equity and invests regardless of the state:
    - $P' = 115$.
    - The value of the firm in state 1 is 270 and in state 2 is 160.

- This implies that:
  - In state 1: $V^{old} = \frac{P'}{P'+E} V = \frac{115}{215} 270 = 144.42$; $V^{new} = \frac{E}{P'+E} V = \frac{100}{215} 270 = 125.58$.
  - In state 2: $V^{old} = \frac{P'}{P'+E} V = \frac{115}{215} 160 = 85.58$; $V^{new} = \frac{E}{P'+E} V = \frac{100}{215} 160 = 74.42$.
- This is not an equilibrium. In state 1, old shareholders can have a value of 150 without investment, and thus the firm will not invest.

- Hence, in equilibrium, the firm will only issue equity and invest in state 2, but then the market will realize this and set $P' = 60$.

- Overall, the firm gives up a positive NPV investment in state 1, and this leads to an expected loss of 10 (expected value is 105).

- Management wants to increase existing shareholders' wealth by not giving up equity in the good state, but this reduces what existing shareholders get in the bad state, and leads to an overall loss.

- The firm could avoid this waste if it financed the investment with internal resources, i.e., if $S=100$ (expected value would be 115).

Another Example

- The problem doesn't arise if the investment is sufficiently good.
- Suppose that in state 1: $a=150$ and $b=100$, while in state 2: $a=50$ and $b=10$ (the required equity to raise is still $E=100$).
- If the firm issues equity and invests regardless of the state:
  - $P' = 155$.
  - The value of the firm in state 1 is 350 and in state 2 is 160.
- This implies that:

- In state 1: $V^{old} = \frac{P'}{P'+E}V = \frac{155}{255}350 = 212.75$; $V^{new} = \frac{E}{P'+E}V = \frac{100}{255}350 = 137.25$.
- In state 2: $V^{old} = \frac{P'}{P'+E}V = \frac{155}{255}160 = 97.25$; $V^{new} = \frac{E}{P'+E}V = \frac{100}{255}160 = 62.75$.

- This is an equilibrium. In state 1, old shareholders can only have a value of 150 without investment, and thus even though they give up a too high share in this state, the investment is still too good to forego.

Some General Properties

- If there is no asymmetric information about a, then equity is always issued and there is no loss.
  - In this case, $P' = S + a + E\left(\tilde{B} \middle| \frac{E}{P'}(S + a) \leq E + b\right) \geq S + a$.
    - Recall that b is always positive.
  - Hence, $\frac{E}{P'}(S + a) \leq E + b$ always holds.
  - This suggests that firms should separate the claims on assets in place and investment opportunities.

- If there is no investment opportunity, b=0, then the market breaks down and there is never equity issuance, unless a is at a known minimum level.
    - For issuance to occur, we need $(S + a) \leq P'$.
    - Suppose that issuance occurs below some a'. Then, P' will account for all values of a below a', and will thus be below $(S + a')$. Hence, the firm would not issue equity at a', unless it is the lowest possible a.
- This is the Akerlof (1970) result.

- In an equilibrium where issuance happens with probability strictly between *0* and *1*, the price following equity issuance will always be below the price absent equity issuance.
  - The decision not to issue implies that $a > P'(1 + b/E) - S$, and hence that $P' < a + S$.
  - Since the price absent issuance is $S + E(\tilde{A})$, it has to be greater than P'.
  - When the firm chooses to issue equity, it shows that the value of the assets is lower than expected.

The Role of Debt

- In the above model:
  - The value for old shareholders following equity issuance is $S + a + b - \Delta E$, where ΔE is the capital gain to new equity holders after information is revealed.
  - In equilibrium, the firm issues equity only when $b \geq \Delta E$. Also, it is required that the expected level of ΔE is *0*.
- Suppose that the value of debt D is less sensitive to private information: $|\Delta E| > |\Delta D|$.

- In equilibrium, the firm will never issue equity, only debt:
  - The firm issues equity only in cases where $\Delta E < \Delta D$.
  - But, this happen only when $\Delta E < 0$.
  - Yet, in equilibrium the expected value of ΔE has to be *0*.
  - Hence, there is a contradiction.
- Thus, under the assumption that debt is less sensitive to private information than equity, the theory says that the firm will always prefer debt to equity. Recall that internal slack is preferable to both.

Theories Based on Agency Problems

- Existing theories highlight two types of **agency problems**:
  - Agency problem between debt holders and equity holders.
    - Equity holders would like to take riskier projects; known as **asset substitution problem**.
  - Agency problem between managers and equity holders:
    - Managers want to promote their agendas, build empires, etc.
- The first type of problem calls for less debt:

    - In the presence of debt, equity holders cannot resist taking excessive risks. This reduces the ex-ante value of debt and the ex-ante value of the firm.

- The second type of problem calls for more debt:
    - Debt financing enables the manager to retain more equity.
    - Debt financing reduces the free cash flow available to managers.
- These ideas are developed by Jensen and Meckling (1976) and Jensen (1986).

Stulz (1990): Using Debt to Control Managers

- The paper presents a simple tradeoff associated with debt financing:
  - On the positive side, debt payments prevent managers from taking bad projects for their private benefits (e.g., **empire building**).
  - On the negative side, debt payments prevent firms from taking good projects when they come along.
- Optimal capital structure finds optimal balance between the forces.

Basic Model

- There are three dates: *0*, *1*, and *2*.
- The firm's assets in place yield a random cash flow R at date *1*, drawn from a cumulative distribution function $G(R)$ and a density function $g(R)$.
- The firm invests an amount I in date *1*. The date-*2* expected payoff of investment is Z per unit for the first I^* units invested and Y per unit for every unit invested in excess of I^*. The assumption is that $Z>1>Y$. Managers get private benefits from any investment.

Firm's Benchmark Value

- Without access to capital markets, managers will simply invest R at date 1, and the value of the firm will be:

$$V = I^*(Z-1) + E(R) - \int_{I^*}^{\infty} (R - I^*)(1 - Y)g(R)dR$$

$$- \int_0^{I^*} (I^* - R)(Z - 1)g(R)dR.$$

  - The first two terms capture the value of the firm if the management acts to maximize shareholder value.

- The third term captures **the overinvestment cost of managerial discretion**:
  - When they have available resources, managers invest in negative-NPV projects.
- The fourth term captures **the underinvestment cost of managerial discretion**:
  - When managers do not have access to financial markets, they have to forego positive-NPV projects.

- Financial policy can be used to get better balance between the costs.

Positive Net Financing

- Suppose that shareholders set financial policy to allow managers access to additional N at date 1.
  - Assuming some cost of default at date 2, the optimal way to achieve this is with date-1 equity issuance.
- Value of the firm becomes:

$$V(N) = \int_{I^*-N}^{\infty} [(R + N - I^*)Y + I^*Z]g(R)dR$$

$$+ \int_{0}^{I^*-N} (R + N)Zg(R)dR - N.$$

- To find whether $N>0$ is optimal, we calculate the derivative of this expression with respect to N:

$$V_N = \int_{I^*-N}^{\infty} Yg(R)dR + \int_0^{I^*-N} Zg(R)dR - 1$$

Note that the derivatives of the boundaries cancel out.

  - The second derivative is:

$$V_{NN} = (Y - Z)g(I^* - N) < 0$$

  - We also know that the derivative is negative when $N = I^*$.

- Hence, if $V_N(0) > 0$, it is optimal to set *N>0* and there is a unique level of *N* that maximizes firm value.

- The condition is:

$$\int_{I^*}^{\infty} Yg(R)dR + \int_0^{I^*} Zg(R)dR - 1 > 0, \text{ or}$$

$$Y[1 - G(I^*)] + ZG(I^*) > 1$$

  - The expected return on an extra dollar available to the manager is a weighted average between the return on a good project and the return on a bad project, and it should be greater than 1.

Optimal Capital Structure

- If the above condition does not hold, the firm's value increases by restricting the manager's access to funds.

- This can be done by issuing debt at date *0*, and committing the manager to a cash payment of F at date *1*.

- This commitment is credible because if the firm cannot pay back the debt, it goes bankrupt and managers lose their job.

- The assumption is that similar commitment cannot be obtained for dividend payments.

- Value of the firm is:

$$V(F) = \int_{F+I^*}^{\infty} [(R - F - I^*)Y + I^*Z]g(R)dR$$

$$+ \int_{F}^{F+I^*} (R - F)Zg(R)dR + \int_{0}^{F} Rg(R)dR + \int_{F}^{\infty} Fg(R)dR.$$

  - Note the asymmetry relative to the case of positive net financing.
  - Here, if R falls below F, the firm goes bankrupt and no investment takes place.

- First order condition with respect to F is:

$$V_F = \int_{F+I^*}^{\infty} (1-Y)g(R)dR - \int_{F}^{F+I^*} (Z-1)g(R)dR$$

  - The marginal benefit of increasing debt is the decrease in bad investment when R is above $F+I^*$. The marginal cost is the decrease in good investment when R is between F and $F+I^*$.

- If at F=0: $Y[1-G(I^*)] + ZG(I^*) < 1$ (opposite condition than the case of positive net financing), then there exists F>0 that maximizes firm value (note that the sign of the second derivative is not clear).

Some Implications

- Suppose an internal solution F^*, satisfying:

$$V_{F^*} = \int_{F^*+I^*}^{\infty} (1-Y)g(R)dR - \int_{F^*}^{F^*+I^*} (Z-1)g(R)dR = 0.$$

- We want to analyze the effect of parameters Y, Z, and I^* on the optimal amount of debt.

- By the **implicit function theorem**, taking a parameter X: $V_{F^*X} + \frac{dF^*}{dX}V_{F^*F^*} = 0$, and thus:

$$\frac{dF^*}{dX} = -\frac{V_{F^*X}}{V_{F^*F^*}}.$$

- At an internal maximum, $V_{F^*F^*} < 0$, and thus the sign of $\frac{dF^*}{dX}$ is the same as the sign of V_{F^*X}.

- Since V_{F^*Y}, V_{F^*Z}, and $V_{F^*I^*}$ are negative, we learn that the amount of debt decreases in the profitability of the firm's investment (as captured by Y, Z, and I^*).
    - When the profitability improves there is higher cost and lower benefit from restricting resources for investment.

Theories Based on Product Market Interactions

- This line of theories integrates the industrial-organization literature with the literature on financial policy.
- In industrial organization, we study how firms determine prices and outputs, and how these are affected by the structure of the industry.
- The idea is that the financial structure of the firm will affect its position and behavior in the product market by affecting the firm's incentives or the incentives of its competitors/customers.

Brander and Lewis (1986): Using Debt as Commitment for Aggressive Product-Market Behavior

- Firms in Cournot competition determine quantity as best response to the quantity set by other firms.
- When the firm has more debt, its incentive to take more risk and set a higher quantity increases.
- As best response, other firms will reduce their quantities, and this will benefit the firm.

The Model

- Firms *1* and *2* produce competing products q_1 and q_2 and play **Cournot quantity competition**.

- The operating profit for firm i is denoted by $R^i(q_i, q_j, z_i)$, where z_i reflects the fundamentals of the firm and is distributed according to density function $f(z_i)$ and is independent of z_j.

- Usual assumptions: $R^i_{ii} < 0$ (concavity); $R^i_j < 0$ and $R^i_{ij} < 0$ (effect of competition); $R^i_z > 0$ and $R^i_{iz} > 0$ (effect of fundamentals).

- The debt level of firm i is denoted as D_i. The firm has to pay this amount to creditors out of operating profits, or go bankrupt (in which case creditors receive the operating profits).

- The value that goes to shareholders after financing and production decisions is:

$$V^i(q_i, q_j) = \int_{\hat{z}_i}^{\bar{z}} \left(R^i(q_i, q_j, z_i) - D_i\right) f(z_i) dz_i,$$

Where $\hat{z}_i$ is defined by:

$$R^i(q_i, q_j, \hat{z}_i) - D_i = 0.$$

The Effect of Debt on Firm Output

- Each firm sets its quantity to maximize shareholder value given the predetermined debt levels and the belief about the other firm's quantity (Nash equilibrium):

$$V_i^i = \int_{\hat{z}_i}^{\bar{z}} R_i^i\big(q_i, q_j, z_i\big) f(z_i) dz_i = 0$$

- Taking the quantity of the other firm as given, we can see that a firm tends to set a higher quantity when it has more debt:

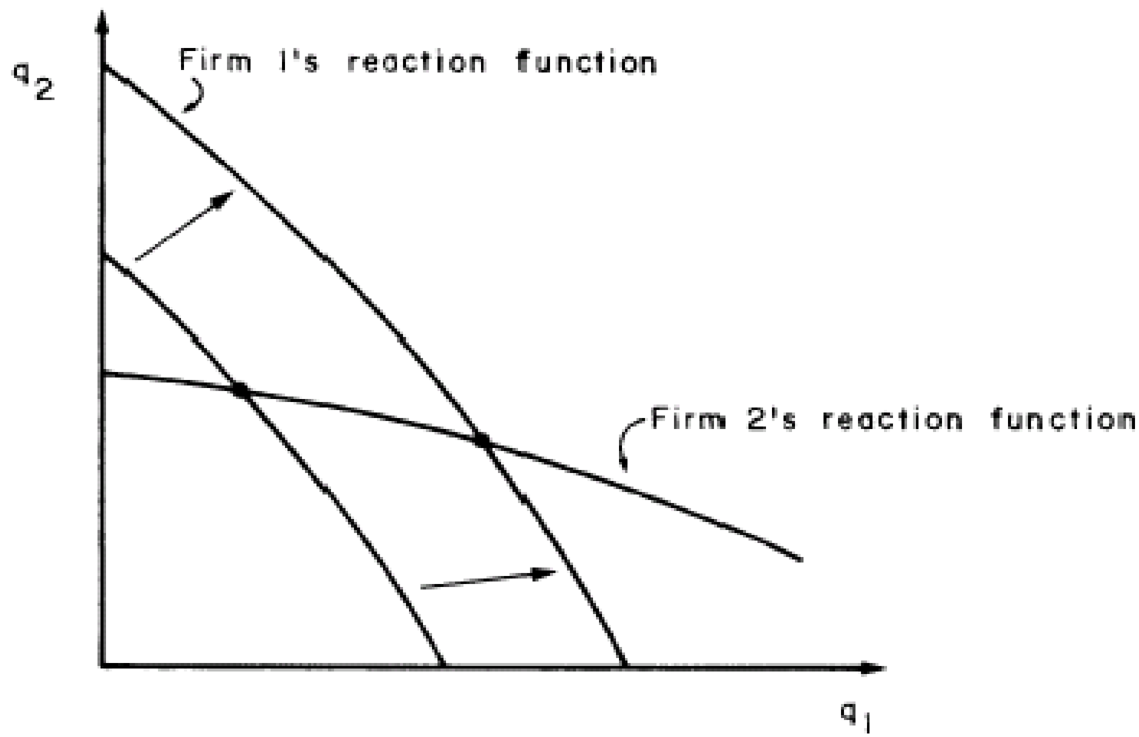
$$\frac{dq_i}{dD_i} = -\frac{V_{iD}^i}{V_{ii}^i} > 0.$$

- A sketch of the argument:
    - Since $V_{ii}^{i} < 0$ by concavity, we need $V_{iD}^{i} > 0$.
    - We know that $V_{iD}^{i} = -\frac{d\hat{z}_i}{dD_i} R_i^i(q_i, q_j, \hat{z}_i) f(\hat{z}_i)$, where $\frac{d\hat{z}_i}{dD_i} > 0$, and so we need $R_i^i(q_i, q_j, \hat{z}_i) < 0$.
    - This holds because $\int_{\hat{z}_i}^{\bar{z}} R_i^i(q_i, q_j, z_i) f(z_i) dz_i = 0$ and $R_{iz}^{i} > 0$.
- Intuitively, when a firm has more debt, it goes bankrupt in a larger range of z_i, and considers only the marginal profit at higher z_i's.

- Since the marginal profit increases in z_i, the firm finds production more profitable and hence increases the quantity produced.
- This is a variant of the asset substitution problem discussed above:
  - With more debt, equity holders want to take more risk.
  - They take actions that increase payoffs in good states of the world and reduce payoffs in bad states of the world.
  - This is because in bad states of the world, the firm goes bankrupt anyway, so they don't care what happens.

The Strategic Role of Debt

- The benefit from increasing the amount of debt is in the effect this has on the rival firm.
- Taking debt serves as a commitment device to increase output. This, in turn, leads the other firm to reduce its output.
- An algebraic proof is in the proof of Proposition 2 in the paper, but taking what we established above (that the quantity of firm i increases with its level of debt for a fixed quantity of firm j) and assuming decreasing response functions, we can see this graphically:



(a) $R^i_{iz} > 0$

Selection of Debt Levels

- Let us now analyze the choice of debt levels by firms.

- Firms choose their debt levels simultaneously in a Nash equilibrium. When choosing the debt levels, they take into account how these will affect the Nash equilibrium played later in the product market. Thus, the equilibrium is **subgame perfect**.

- Denote the dependence of output levels on debt levels as $q_i(D)$, where $D = (D_1, D_2)$.

- When setting debt levels, firms maximize total value – including the value of debt and equity. This is because the debt is sold at true value, and thus whatever cash flows debt holders expect to lose in the future they take out of the price they pay today.

- Hence, the firm sets D_i to maximize:

$$Y^i\big(q_i(D), q_J(D), D\big) = \int_{\underline{Z}}^{\hat{z}_i} R^i\big(q_i(D), q_j(D), z_i\big) f(z_i) dz_i$$

$$+ \int_{\hat{z}_i}^{\bar{z}} R^i\big(q_i(D), q_j(D), z_i\big) f(z_i) dz_i$$

- We can see that debt levels only affect value via the effect on future output levels. If output levels were fixed, debt policy would be irrelevant like in Modigliani-Miller.

- We can write the marginal effect of D_i on Y^i:

$$Y^i_{Di} = \left[\int_{\underline{Z}}^{\hat{z}_i} R^i_i(z_i)f(z_i)dz_i\right]\frac{dq_i}{dD_i} + \left[\int_{\hat{z}_i}^{\bar{z}} R^i_i(z_i)f(z_i)dz_i\right]\frac{dq_i}{dD_i}$$

$$\left[\int_{\underline{Z}}^{\hat{z}_i} R^i_j(z_i)f(z_i)dz_i + \int_{\hat{z}_i}^{\bar{z}} R^i_j(z_i)f(z_i)dz_i\right]\frac{dq_j}{dD_i}$$

- The second term is zero by the maximization problem in the product market.

- The first term is negative since $R_i^i(\hat{z}_i) < 0$ and $R_{iz}^i > 0$.

  - This is the effect of excessive risk taking on the value of debt. In the product market, shareholders take high risk and harm debt holders, but this reduces the price of debt and harms total value.

- The third effect is positive.

  - When firm i takes more debt, firm j reduces output and this is unambiguously good for firm i.

- Thus, there is a tradeoff associated with increasing debt levels. On the one hand, it exacerbates the conflict of interest between equity holders and debt holders, leading to excessive risk taking and inefficiency. On the other hand, it generates a commitment device for increased output that creates a strategic advantage in the product market.
- In general, we cannot say which effect dominates.
- However, we can say that at $D_i = 0$, the first effect is zero, and the second one is strictly positive.

- Hence, the conclusion is that in equilibrium firms will have strictly positive debt levels.
- This will push overall output up.
- Interestingly, firms would be better off by colluding not to issue debt.
    - In a non-cooperative equilibrium, firms issue debt to better compete against each other.
    - These effects sum up to zero, but the inefficiency from excessive risk taking remains a cost.