Financial Contracting

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The Design of Securities

- Traditional capital-structure literature studies the tradeoff between debt and equity.
- A much deeper question is about why these securities emerge in the first place.
- The financial contracting literature addresses this question by developing the optimality of these securities from primitive assumptions.

Costly State Verification

- One approach to this question was pioneered by Townsend (1979) and followed by Gale and Hellwig (1985).
- Entrepreneur and investor write a contract by which the investor provides financing to a project, and the entrepreneur pays back an amount that depends on the state of the world.
- The state of the world, however, is observed only by the entrepreneur, and the investor can observe it only by incurring a monitoring cost. Hence, there is **costly state verification**.

- A standard debt contract is shown to be optimal (maximize entrepreneur’s wealth subject to the participation constraint of the investor) in this environment.
- Here,
    - The investor receives a fixed amount and does not verify the state when the return is above a certain threshold.
    - The investor receives the full return of the project (lower than the fixed amount above) and verifies the state when the return is below the threshold. This can be thought of as bankruptcy.

Formal Analysis: Tirole (Ch. 3.7)

Framework

- Entrepreneur's own wealth is given by A, and so to make an investment I he needs to raise I-A from an investor.
- The investment yields a random return R drawn from density $p(R)$ on $[0, \infty)$. This return is ex-post known to entrepreneur without cost, and the investor can verify it at cost of K.
- According to an arbitrary financing contract:

- The investor provides *I*-*A*,
- The entrepreneur invests yielding return *R* and reporting $\hat{R}$.
- Conditional on $\hat{R}$, the investor doesn't audit (to verify the return) with probability $y(\hat{R})$.
- The investor receives payment $R_1(\hat{R})$ in case of no auditing and $R_1(\hat{R}, R)$ in case of auditing.

- The entrepreneur's return in case of no auditing is $w_0(\hat{R}, R) = R - R_1(\hat{R})$, and in case of auditing is $w_1(\hat{R}, R) = R - R_1(\hat{R}, R)$.

Truthful Reporting

- According to the **revelation principle** (Myerson (1979)), we can restrict attention to contracts that provide incentive for the entrepreneur to report the return truthfully.
    - This principle states that under certain conditions (including the ability to commit to an action based on the report) any outcome can be replicated by a contract that induces truth telling.
- Then, we write the expected payoff of the entrepreneur as:

$$w(R) = y(R)w_0(R,R) + \big(1 - y(R)\big)w_1(R,R)$$

Optimal Contract

- The optimal contract maximizes the entrepreneur's expected income subject to the incentive constraint that the entrepreneur reports truthfully and to the participation constraint of the investor:

$$\max_{y(\cdot),w_0(\cdot,\cdot),w_1(\cdot,\cdot)} \left\{ \int_0^\infty w(R)p(R)dR \right\}$$

$$\text{s.t. } w(R) = \max_{\hat{R}} \left\{ y(\hat{R})w_0(\hat{R}, R) + \left(1 - y(\hat{R})\right) w_1(\hat{R}, R) \right\} \quad \text{(IC)}$$

$$\int_0^\infty [R - w(R) - [1 - y(R)]K]p(R)dR \geq I - A \qquad \text{(IR)}$$

- Since IR is binding, we can rewrite the maximization problem as:

$$\max_{y(\cdot),w_0(\cdot,\cdot),w_1(\cdot,\cdot)} \left\{\int_0^{\infty} w(R)p(R)dR\right\} =$$

$$\max_{y(\cdot),w_0(\cdot,\cdot),w_1(\cdot,\cdot)} \left\{\int_0^{\infty} [R - I + A - [1 - y(R)]K]p(R)dR\right\} =$$

$$\min_{y(\cdot),w_0(\cdot,\cdot),w_1(\cdot,\cdot)} \left\{\int_0^{\infty} [1 - y(R)]p(R)dR\right\}$$

  - That is, we wish to minimize the expected audit costs subject to the constraints.

Standard Debt Contract

- A standard debt contract looks as follows:
  - Debt payment D.
  - $y(R) = 1$ if $R \geq D$.
  - $y(R) = 0$ if $R < D$.
  - $w(R) = max(R - D, 0)$; $R_1(R) = min(R, D)$.
- We go on to show that under **deterministic audit**, that is, $y(R) = 0 \; or \; 1 \; for \; all \; R$, the optimal contract is a standard debt contract.

Proof

- Under the deterministic audit assumption, there are two regions of returns: the no-audit region $\mathcal{R}_0$ and the audit region $\mathcal{R}_1$.
  - $\mathcal{R}_0 \cap \mathcal{R}_1 = \emptyset$ and $\mathcal{R}_0 \cup \mathcal{R}_1 = [0, \infty)$.
- **First Property:** The payoff to the investor $R_1(R)$ is constant in the no-audit region $\mathcal{R}_0$. We denote this payoff D, implying that $\mathcal{R}_0 \subseteq [D, \infty)$
  - Suppose to the contrary that $R', R \in \mathcal{R}_0$ and $R_1(R') > R_1(R)$.

- Then, the entrepreneur would report R when observing R'. Because there is no audit he gets to pay less.
- This violates the incentive compatibility condition required for truthful disclosure.

- **By the same argument:** The payoff to the investor $R_1(R)$ in the audit region $\mathcal{R}_1$ cannot exceed D.
  - Otherwise, observing $R \in \mathcal{R}_1$, the report would be $R' \in \mathcal{R}_0$.
- **Economically,** no-audit regions are needed to save costs. They can only be sustained if they induce maximum payoff to investor.

- **Second Property:** the no-audit region $\mathcal{R}_0 = [D, \infty)$ and the audit region $\mathcal{R}_1 = [0, D)$.

- **Third Property:** In the audit region $\mathcal{R}_1$, the payoff to the investor $R_1(R) = R$.

  - Take an arbitrary contract (characterized by regions $\mathcal{R}_0$ and $\mathcal{R}_1$), which satisfies the two constraints IC and IR, and in which these properties do not hold.

  - Compare it with a debt contract (characterized by $\mathcal{R}_0^*$ and $\mathcal{R}_1^*$) paying the same *D* in the no-audit region.

- The debt contract yields lower auditing costs since $\mathcal{R}_0 \subseteq \mathcal{R}_0^*$.
- The debt contract offers a higher payoff to the investor:
  - It pays him more at $R \in \mathcal{R}_1 \cap \mathcal{R}_0^*$ since the debt contract pays D and the arbitrary contract pays at most D.
  - It pays him more at $R \in \mathcal{R}_1 \cap \mathcal{R}_1^*$ since the debt contract pays the maximum available amount R.
  - It pays him the same (D) at $R \in \mathcal{R}_0$.
- The investor gets a strictly positive value with this debt contract:

$$[1 - P(D)]D + \int_0^D Rp(R)dR - P(D)K - (I - A) > 0$$

- Inspecting this expression, we can see that it is negative when $D=0$ and is continuous in D.

- Hence, there exists a debt contract with $D' < D$, such that the investor participation constraint is binding:

$$[1 - P(D')]D' + \int_0^{D'} Rp(R)dR - P(D')K - (I - A) = 0$$

- This contract is better than the original arbitrary contract since:

    - It has lower monitoring costs than debt contract with *D* which are lower than in the original arbitrary contract.
    - It has the investor participation constraint binding.
    - It satisfies the incentive compatibility constraint.
  - Hence, the arbitrary contract is dominated by a debt contract.
- **Economically**, it is optimal to minimize states where auditing cost is incurred. Paying full amount to investor when auditing happens helps reduce the amount *D* paid when auditing does not happen and thus reduce the probability of auditing.

Random Audits

- The optimality of the standard debt contract was established under the assumption that auditing happens with probability *0* or *1*.
- It turns out that when one allows for randomization in auditing decisions, the standard debt contract is no longer optimal, and randomization is expected to happen.
- To illustrate this, consider $R \in \mathcal{R}_1$ and $R' \in \mathcal{R}_0$ in the above problem ($R' > D > R$).

- Auditing in state R is important, since otherwise the entrepreneur would be better off misrepresenting in state R' and claiming it is R:

$$R' - D < R' - R$$

- But, when randomization is allowed, a probability of auditing between *0* and *1* at state R can be sufficient to deter the entrepreneur from misrepresentation while saving on auditing costs.

- In fact, a probability $y(R)$ in state R is going to be sufficient:

$$R' - D = y(R)(R' - R)$$

Other Limitations

- The model explains the optimality of outside debt and inside equity. It does not explain why outside equity may be issued.

- The model relies strongly on the notion of **commitment**:

    - Investors commit to audit if the firm reports earnings below D.
    - But suppose that **renegotiation** is allowed. Then, if ex post the entrepreneur reports a state lower than D and offers to pay D-K, the investor will not audit.

- This undermines incentive to report truthfully and changes the equilibrium.

- The interpretation of stealing and auditing has to be thought through.
  - An implicit assumption is that the entrepreneur can steal nothing from the firm before an audit takes place, but can take the residual income after repayment if an audit did not take place.
  - What is the underlying setting? Entrepreneur steals, but cannot consume for some time? Deriving utility from stolen income requires the firm not to shut down?

Incomplete Contracts: Aghion and Bolton (1992)

- Aghion and Bolton use the incomplete-contracts approach (Grossman and Hart (1986)) to explain financial contracting.
- They enrich the original framework by assuming that the entrepreneur has limited wealth and needs capital from an investor.
- Their focus is not on the payoffs that the contract provides, but on the control structure. Debt is a mechanism for **contingent control**:
    - Depending on the state of the world, control is shifted to the party whose interests are aligned with efficiency.

The Model (Simplified)

- A penniless entrepreneur seeks funding K from an investor at $t=0$.
- There are many investors, so all the bargaining power is at the hands of the entrepreneur:
  - The investor should get back at least K (participation constraint).
- At $t=1$, there is a realization of the state of the world $\theta \in \{\theta_g, \theta_b\}$, where θ_g happens with probability q. This is followed by an action taken by the control holder (cannot be contracted): $a \in \{a_g, a_b\}$.

- At $t=2$, there is realization of the return of the project $r \in \{0,1\}$.
    - Denote the expected return in state θ_i and given action a_j as: $y_j^i = Prob\left(r = 1 \middle| \theta = \theta_i, a = a_j\right)$.
- The (risk neutral) investor gets utility only from his share of the return: $U_I(r) = r$.
- The (risk neutral) entrepreneur also gets a private benefit: $U_E(r,a) = r + l(a,\theta)$.
    - Denote the private benefit in state θ_i and given action a_j as l_j^i.

Control Allocation and Payments

- Focus on three possibilities: Entrepreneur control, Investor control, Contingent control (depends on θ).
- Entrepreneur gets private benefits l and investor gets monetary benefits y. Fixed payment t can be made to meet participation constraint (only from investor to entrepreneur).
- This is a simplification of the model which assumes that θ is not verifiable and control depends on a signal s which is correlated with θ, and also allows a payment $t(s, r)$ which helps align incentives.

First-Best Actions

- The model assumes that the first-best action in θ_g is a_g, and the first-best action in θ_b is a_b:

$$y_g^g + l_g^g > y_b^g + l_b^g$$

$$y_b^b + l_b^b > y_g^b + l_g^b$$

- Also, the first best actions are feasible:

$$qy_g^g + (1-q)y_b^b > K$$

Comonotonic Benefits

- Suppose that $l_g^g > l_b^g$, $l_b^b > l_g^b$ (**private benefits are comonotonic**).
- Then, giving control to the entrepreneur can achieve first best:
  - The entrepreneur chooses action a_g in θ_g and action a_b in θ_b.
  - The payment t is set to meet the participation constraint of the investor: $qy_g^g + (1-q)y_b^b - t = K$.
- Similarly, when $y_g^g > y_b^g$ and $y_b^b > y_g^b$ (**monetary benefits are comonotonic**), giving control to the investor can achieve first best.

No Comonotonicity

- The interesting case arises when neither the entrepreneur nor the investor have incentives that are perfectly aligned with efficiency.

- Suppose that $l_g^g > l_b^g$, $l_b^b < l_g^b$, and $y_g^g < y_b^g$ and $y_b^b > y_g^b$:

  - Each one wants to do the efficient thing only in one state.

  - Real world interpretation: a_g represents expansion; θ_g represents good state. Expansion is efficient only in good state, whereas entrepreneur always want to expand and investor never wants to.

Entrepreneur Control

- Entrepreneur wants to choose a_g in θ_b.
- With ex-post **renegotiation** first-best can be restored. Yet, this might deprive the investor of adequate return, so he doesn't get *K*.
- In detail:
  - When θ_b is realized, entrepreneur has incentive to renegotiate.
  - Assuming entrepreneur has all the bargaining power, he makes a take-it-or-leave-it offer that keeps the investor indifferent.

- That is, he offers to provide the investor the monetary benefit y_b^b, in exchange for a payment of $y_b^b - y_g^b$.
- This is beneficial for the entrepreneur because $l_b^b + y_b^b - y_g^b > l_g^b$. He captures the total surplus from the efficient action.

- Hence, renegotiation guarantees that the efficient action is taken, but it doesn't guarantee the investor to get K back, as the investor is only getting y_g^b in θ_b. Hence, this doesn't work if:

$$qy_g^g + (1-q)y_g^b < K < qy_g^g + (1-q)y_b^b.$$

- The paper goes on to analyze the possibility of writing a **renegotiation-proof** contract.
  - Here, the payment to the entrepreneur is made contingent on the signal (correlated with the state of the world) and on the final monetary benefit.
  - This provides incentive for the entrepreneur to pick the ‘right’ action without leading to renegotiation.
  - This, however, is also costly, as the incentives reduce the return to the investor, leading to a problem as with renegotiation.

- For illustration, suppose that the contract specifies a payment $t(\theta, r) = t_\theta + t'_\theta r$ to the entrepreneur, where $t_\theta \geq 0$, and the investor gets the residual return.

- Designing a contract that makes the entrepreneur choose a_b in θ_b amounts to setting t'_{θ_b} high enough (t'_{θ_g} should be 0).

- To cause minimum damage to the investor's participation constraint, we want:

$$l_b^b + t'_{\theta_b}\, y_b^b = l_g^b + t'_{\theta_b}\, y_g^b$$

- This implies that:

$$t'_{\theta_b} = \frac{l_g^b - l_b^b}{y_b^b - y_g^b}$$

- But then the investor's return is capped at:

$$qy_g^g + (1-q)\left(1 - \frac{l_g^b - l_b^b}{y_b^b - y_g^b}\right)y_b^b$$

- So this arrangement is not feasible when:

$$qy_g^g + (1-q)\left(1 - \frac{l_g^b - l_b^b}{y_b^b - y_g^b}\right)y_b^b < K < qy_g^g + (1-q)y_b^b.$$

Investor Control

- With investor control, there will not even be any renegotiation. The investor chooses a_b in θ_g and first-best is not achieved.

- This leaves unexploited surplus of $y_g^g + l_g^g - y_b^g - l_b^g$

- In principle, the entrepreneur could pay $y_b^g - y_g^g$ to the investor and make him take action a_g.

- Yet, the entrepreneur cannot do it because he has no wealth and l isn't a monetary benefit.

Contingent Control

- The main point of the paper is that conditioning the control on the state of the world can achieve first-best when both unilateral control allocations fail to do so.

- Clearly, here, the first-best is achieved if control is given to the investor in θ_b and to the entrepreneur in θ_g.

- When the state of the world is not verifiable, the allocation can get very close to first-best if there is a verifiable signal highly correlated with the state of the world.

- This allocation of control resembles the allocation under a debt contract, where creditors get control when the firm is in bad shape.
- The underlying intuition is that by giving the investor control in the bad state, the entrepreneur is able to guarantee him a higher return, and this relaxes the financing constraint.
- One limitation of the model is that it doesn't have strong predictions about cash flow rights.
- Another limitation is that the event that triggers change in control is just a bad state, not the default by the firm.

The Number of Creditors: Bolton and Scharfstein (1996)

- After explaining the emergence of debt contracts, the structure of debt remains an important question, one aspect of which is the number of creditors.
- Bolton and Scharfstein (1996) develop a theory about the optimal number of creditors within the incomplete-contracts framework.
- The idea is that the number of creditors will affect the bargaining processes in the event of default. This will affect the liquidation value of the firm and the likelihood of default, leading to a tradeoff.

The Model

- A manager with no wealth needs to finance an investment project at $t=0$ at a cost of K. This will be spent on two assets A and B.

- At $t=1$, the project generates x with probability θ or 0 with probability $1-\theta$.

- At $t=2$, if the manager keeps running the project, it yields a cash flow of y. Alternatively, if the project is liquidated and run by the investors, it generates 0. If it is liquidated and run by other managers, it generates αy; $\alpha \leq 1$.

- If it was possible to write a complete contract, it would specify payments for different cash flows of the firm and ensure that liquidation doesn’t occur at $t=1$.
- Yet, in the spirit of the incomplete-contracts literature, this is assumed impossible.
- The possibility of liquidation in $t=1$ is necessary to deter the manager from diverting cash to himself.
- Note that at t=2, there is no way to make the manager pay back to the investor, as there is no more threat of liquidation.

Contract

- The most general contract then specifies the probability of liquidation β at *t=1* for a payoff R made by the firm to the investors. (Partial liquidations are shown to be inefficient.)
- Hence, if with cash flow x, the manager makes payment R_x, investors have the right to liquidate with probability $\beta(R_x)$, or β_x. Similarly, with cash flow 0, we get R_0 and β_0.
- Of course, payments cannot exceed cash flows: $R_0 \leq 0$ and $R_x \leq x$.
- It is straightforward to show that $\beta_x = 0$ and $R_0 = 0$.

- The goal is to maximize the firm's expected payoffs:

$$\theta[x - R_x + y] + (1 - \theta)(1 - \beta_0)y.$$

- Denoting the liquidation values as L_x and L_0, the investors' participation constraint is:

$$\theta R_x + (1 - \theta)\beta_0 L_0 - K \geq 0.$$

- The incentive compatibility constraint is:

$$x - R_x + y \geq x + \beta_0 S + (1 - \beta_0)y$$

S denotes benefit that the manager has from paying 0 with *x* cash.

- We can rewrite the incentive constraint as:

$$R_x \leq \beta_0(y - S)$$

- Since both constraints are binding, we plug one in the other:

$$\beta_0[\theta(y - S) + (1 - \theta)L_0] - K = 0$$

- Now, we can rewrite the objective function:

$$\theta[x - R_x + y] + (1 - \theta)(1 - \beta_0)y$$

$$= \theta x + y - \theta\beta_0(y - S) - \beta_0(1 - \theta)y$$

$$= \theta x + y - K - \beta_0(1 - \theta)(y - L_0)$$

- Here, the first three terms capture the expected profit in the first-best case of no liquidation, and the fourth term captures the loss from liquidation due to contract incompleteness.

- Clearly, the objective function is decreasing in β_0.

- Hence, we are going to set β_0 at the lowest possible level:

$$\beta_0 = \frac{K}{\theta(y - S) + (1 - \theta)L_0}$$

- There is liquidation in equilibrium in the bad state in order to incentivize the manager not to divert cash in the good state.

- Note that the arrangement becomes not feasible if

$$K > \theta(y - S) + (1 - \theta)L_0.$$

  - Here, the amount that needs to be financed is too large relative to the maximum amount that can be promised to the investors.
  - Hence, large projects (high K) with low continuation value (low y), low liquidation value (low L_0), and high managerial benefit from liquidation (high S) are less likely to be financed.
  - If financed, such projects will exhibit a higher probability of liquidation.

One Creditor

- We compute L_0, S, and β_0 in case where the firm has one creditor.
- $\boldsymbol{L_0}$:
    - Upon **liquidity default**, the creditor tries to sell the assets to an outside manager, for whom the continuation value is αy.
    - Suppose that the buyer incurs a cost c to get into bargaining over the assets, where c is uniformly distributed over $[0, \bar{c}]$.
    - The buyer will get into the process if c is less than his payoff.

- Assuming Nash bargaining, the surplus will be split between the creditor and the buyer such that each one of them gets $\frac{1}{2}\alpha y$.
- Therefore, the assets are sold with probability $\frac{\frac{1}{2}\alpha y}{\bar{c}}$.
- Hence, L_0 in case of one creditor is:

$$L_0(1) = \frac{\alpha^2 y^2}{4\bar{c}}.$$

- ***S***:
  - Under **strategic default**, the manager has cash and so he buys the firm.

- His continuation value is y, so under Nash bargaining both he and the creditor get:

$$S(1) = \frac{y}{2}.$$

- $\boldsymbol{\beta_0}$:

$$\beta_0(1) = \frac{K}{\theta \frac{y}{2} + (1-\theta)\frac{\alpha^2 y^2}{4\bar{c}}}$$

- We can see that the probability of default is decreasing in the probability of success θ.

Two Creditors

- We compute L_0, S, and β_0 in case where the firm has two creditors.
- The assumption is that each creditor is secured by another asset.
- The critical assumption is that the two assets are worth more together than apart:

$$\Delta = y - y^A - y^B > 0.$$

- Essentially, there are either increasing returns to scale or complementarities between the two assets.

- $\boldsymbol{L_0}$:
    - Now, there are three parties to the bargaining: outside manager and the two creditors.
    - The bargaining outcome is based on the Shapley values.
    - Creditor a's Shapley value is: $\frac{1}{2}\alpha y^A + \frac{1}{3}\alpha\Delta$.
        - With probability $\frac{1}{3}$ he is in coalition with the other creditor and the buyer contributing $\alpha y^A + \alpha\Delta$, while with probability $\frac{1}{6}$ he is in coalition with the buyer contributing αy^A.

- Creditor b's Shapley value is: $\frac{1}{2}\alpha y^B + \frac{1}{3}\alpha\Delta$.
- Hence, the Shapley value of the outside manager is $\frac{1}{2}\alpha y - \frac{1}{6}\alpha\Delta$.
  - He is getting less because of the complementarities between the two creditors. After teaming with one, he needs to pay more to the other one.
- Based on the logic before, the assets are sold with probability $\frac{\frac{1}{2}\alpha y - \frac{1}{6}\alpha\Delta}{\bar{c}}$, leading to liquidation value:

$$L_0(2) = \frac{\left(\frac{1}{2}\alpha y - \frac{1}{6}\alpha\Delta\right)\left(\frac{1}{2}\alpha y + \frac{1}{6}\alpha\Delta\right)}{\bar{c}} = \frac{\alpha^2 y^2}{4\bar{c}} - \frac{\alpha^2\Delta^2}{36\bar{c}} < \frac{\alpha^2 y^2}{4\bar{c}}.$$

- The liquidation value in case of two creditors is lower than in case of one borrower.
  - The creditors squeeze more in case they sell the assets, but this leads to a lower probability of selling them.
  - The second effect dominates.

- ***S***:
  - Based on similar logic:.

$$S(2) = \frac{y}{2} - \frac{\Delta}{6} < \frac{y}{2}.$$

- The manager gets less out of strategic default with two creditors. The complementarities enable creditors to get a larger share.

- $\boldsymbol{\beta_0}$:

$$\beta_0(2) = \frac{K}{\theta\left(\frac{y}{2} + \frac{\Delta}{6}\right) + (1-\theta)\left(\frac{\alpha^2 y^2}{4\bar{c}} - \frac{\alpha^2 \Delta^2}{36\bar{c}}\right)}$$

- Relative to the probability of liquidation with one creditor, there are two effects: L_0 is lower leading to a higher probability of liquidation, but S is lower leading to a lower probability of liquidation.

Tradeoff

- The inefficiency in financing is given by: $\beta_0(1-\theta)(y-L_0)$.
- Increasing the number of borrowers has two effects:
  - **Positive:** Making strategic default less tempting, and thus relaxing the incentive constraint and enabling reduction in β_0.
  - **Negative:** Reducing the liquidation value in case of liquidity default which affects inefficiency both directly and indirectly via the increase in the probability of liquidation.

- Overall, having more creditors increases their bargaining power.
- This means they will get less in case of liquidity default (buyers are less likely to show up), but more in case of strategic default (manager is already there).
- Which effect dominates depends on the parameters.
- One can make cross-sectional comparisons:
    - Borrowing from one creditor is better when default risk is high (θ is low).

- In this case, the liquidation value upon liquidity default becomes more important.

o Borrowing from one creditor is better when complementarities are high (Δ is high).

- The effect on the liquidation value is convex.

o Borrowing from one creditor is better when outside managers are more efficient (α is high).

- In this case, the damage to liquidation value becomes more important.

The Diversity of Claims: Dewatripont and Tirole (1994)

- Several papers try to explain why a firm has multiple classes of external securities at the same time.
- Dewatripont and Tirole (1994) focus on **external equity** and debt.
- In their model, incentivizing the manager to exert effort requires commitment to actions that are ex-post not efficient. Since contracting on such actions is impossible, incentives are given to different security holders via **control and cash flow rights** to implement the optimal plan.

The Model

- There are two periods *t=1, 2.*

- At *t=1*, the manager chooses unobservable effort $e \in \{\underline{e}, \overline{e}\}$; $\underline{e} < \overline{e}$.

    - $\overline{e}$ is the efficient effort, but it costs the manager *K*; $\underline{e}$ costs *0.*

- The firm's verifiable profit in period *t* is π_t. The distribution of π_t is determined by *e*.

- π_1 is not a sufficient statistic for *e*. At *t=1*, investors can observe a non-contractible signal *u*, which is a sufficient statistic for π_2.

- After π_1 and u are realized the outsider in control of the firm can choose a non-contractible action $A \in \{S, C\}$, where S stands for "stopping" and C for "continuing".

- The effect of effort is modeled as follows:
    - The density of $\pi_1 \in \left[\pi_1^{min}, \pi_1^{max}\right]$ is denoted by $\overline{f}(\pi_1)$ for $\overline{e}$ and by $\underline{f}(\pi_1)$ for $\underline{e}$. Similarly, the density of $u \in [0,1]$ is denoted by $\overline{g}(u)$ or $\underline{g}(u)$.
    - MLRP holds: $\overline{f}(\pi_1)/\underline{f}(\pi_1)$ is increasing in π_1; $\overline{g}(u)/\underline{g}(u)$ is increasing in u.

- The effect of the outsider's action is modeled as follows:
  - For each u, the density of $\pi_2 \in \left[\pi_2^{min}, \pi_2^{max}\right]$ is $h_C(\pi_2|u)$ for action C and $h_S(\pi_2|u)$ for action S. The cumulative functions are H_C and H_S, respectively.
  - Action S is safer than action C: for each u, there exists $\hat{\pi}_2(u)$,

$$H_S(\pi_2|u) < H_C(\pi_2|u) \quad for\ \pi_2^{min} < \pi_2 < \hat{\pi}_2(u)$$

$$H_S(\pi_2|u) > H_C(\pi_2|u) \quad for\ \hat{\pi}_2(u) < \pi_2 < \pi_2^{max}$$

    - Hence, equity holders tilt to C and debt holders to S.

- Action C becomes more appealing for high signal u:

$$\frac{\partial[H_S(\pi_2|u)-H_C(\pi_2|u)]}{\partial u} > 0 \quad ,$$

$$\text{and } \exists\bar{u} \in [0,1],\ E_{H_S}(\pi_2|\bar{u}) = E_{H_C}(\pi_2|\bar{u})$$

- The manager is assumed to receive private benefit B as long as outsiders continue the project.
  - This is a simple assumption leading to the congruence of interests between the manager and shareholders.
  - The authors show that similar results are obtained with endogenous monetary benefits.

Managerial Incentive Scheme

- The program is set to minimize ex-post inefficiency in the choice of C vs. S subject to the constraint that the manager receives the incentive to choose $\bar{e}$.
  - The manager's outside utility is assumed to be *0*, and thus the participation constraint is not binding.
- Specifically, we find the optimal probability of continuation $x(\pi_1, u)$ for each π_1 and u.
- Since u is non-contractible, we implement it with capital structure.

- Define the net monetary gain from continuing given signal u:

$$\Delta(u) = \int \pi_2 [h_C(\pi_2|u) - h_S(\pi_2|u)]\, d\pi_2$$

$$= \int [H_S(\pi_2|u) - H_C(\pi_2|u)]\, d\pi_2,$$

where the second step is obtained after integration by parts.

    - $\Delta(\overline{u}) = 0$, $\Delta(u) < 0$ for $u < \overline{u}$, and $\Delta(u) > 0$ for $u > \overline{u}$.

- Defining $\Delta^+(u) = max(\Delta(u), 0)$ and $\Delta^-(u) = min(\Delta(u), 0)$, we can write the program as:

$$\min_{x(\cdot,\cdot)} \iint \left[\left(1 - x(\pi_1, u)\right)\Delta^+(u) - x(\pi_1, u)\Delta^-(u)\right]\overline{f}(\pi_1)\overline{g}(u)d\pi_1 du$$

Subject to

$$B \iint x(\pi_1, u)\left[\overline{f}(\pi_1)\overline{g}(u) - \underline{f}(\pi_1)\underline{g}(u)\right] d\pi_1 du \geq K$$

- Denote the Lagrange multiplier of this program as μ. Since $\Delta^+(u) + \Delta^-(u) = \Delta(u)$, we get the derivative of the Lagrangian:

$$\Delta(u)\overline{f}(\pi_1)\overline{g}(u) + \mu B\left[\overline{f}(\pi_1)\overline{g}(u) - \underline{f}(\pi_1)\underline{g}(u)\right]$$

- We can see that $x(\pi_1, u)$ is optimally either *0* or *1*.

- Then, since $\overline{f}(\pi_1)/\underline{f}(\pi_1)$ is increasing in π_1, $\overline{g}(u)/\underline{g}(u)$ is increasing in u, and $\Delta(u)$ is increasing in u, the solution is of the following form:

$$\begin{cases} x(\pi_1, u) = 0 \ \ if \ u < u^*(\pi_1) \\ x(\pi_1, u) = 1 \ \ if \ u \geq u^*(\pi_1) \end{cases},$$

where $u^*(\pi_1)$ is decreasing in π_1.

- Intuitively, low *u* and low π_1 lead to action *S* because they indicate lack of effort, and thus the manager should be penalized. In addition, low *u* makes action *S* more efficient.

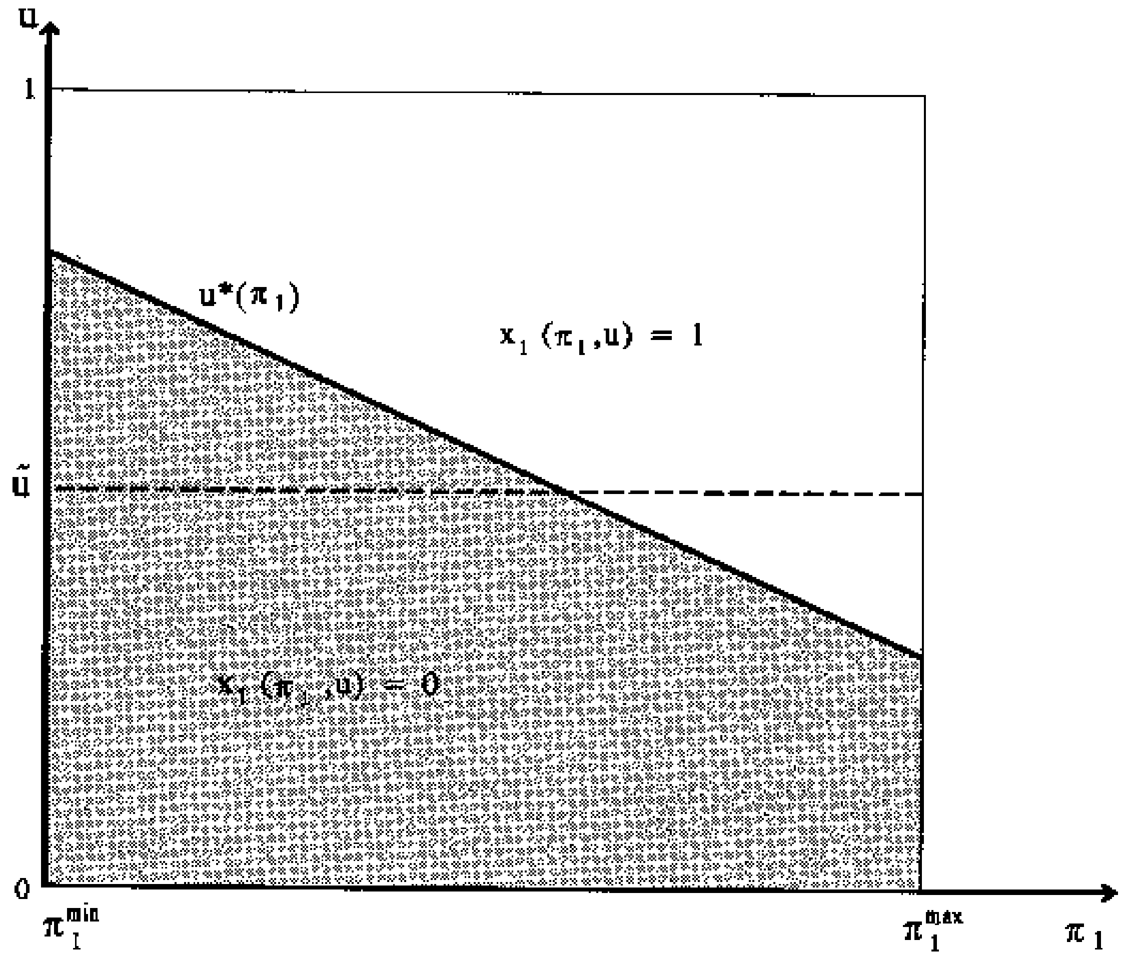


FIGURE II

Outsiders' Incentive Scheme

- Designing outsiders' incentives via the financial structure of the firm serves to implement the optimal managerial incentive schemes as characterized by $u^*(\pi_1)$.

- We need at least two outside investors and two classes of securities, as absent these conditions, continuation will occur if and only if it is ex-post efficient, i.e., when $u \geq \overline{u}$.

  - This implies optimal policy only for profit level $\overline{\pi}_1$, which is defined by: $u^*(\overline{\pi}_1) = \overline{u}$.

- Note that ex-post optimal policy does not depend on π_1, but such dependence is crucial to implement ex-ante managerial incentives.

- We need to implement a policy that leads to ex-post excessive continuation when $\pi_1 > \overline{\pi}_1$ and to ex-post excessive liquidation when $\pi_1 < \overline{\pi}_1$.

- The authors show that the optimal plan can be implemented with the two standard financial instruments: debt and equity.

- Suppose that the firm has long-term debt due at *t=2* of D_2.

- Suppose that the investor in control of the decision holds a proportion α of the firm's debt and a proportion β of its equity. For a given u, a decision to continue yields:

$$\alpha\left[\int_{\pi_2^{min}}^{D_2} \pi_2 h_C(\pi_2|u)d\pi_2 + D_2\big(1 - H_C(D_2|u)\big)\right]$$

$$+\beta\left[\int_{D_2}^{\pi_2^{max}} (\pi_2 - D_2)h_C(\pi_2|u)d\pi_2\right]$$

- While a decision to stop yields:

$$\alpha\left[\int_{\pi_2^{min}}^{D_2} \pi_2 h_S(\pi_2|u)d\pi_2 + D_2\big(1 - H_S(D_2|u)\big)\right]$$

$$+\beta\left[\int_{D_2}^{\pi_2^{max}} (\pi_2 - D_2)h_S(\pi_2|u)d\pi_2\right]$$

- After integration by parts, we get the net payoff to continuing:

$$\alpha\int_{\pi_2^{min}}^{D_2} [H_S(\pi_2|u) - H_C(\pi_2|u)]d\pi_2$$

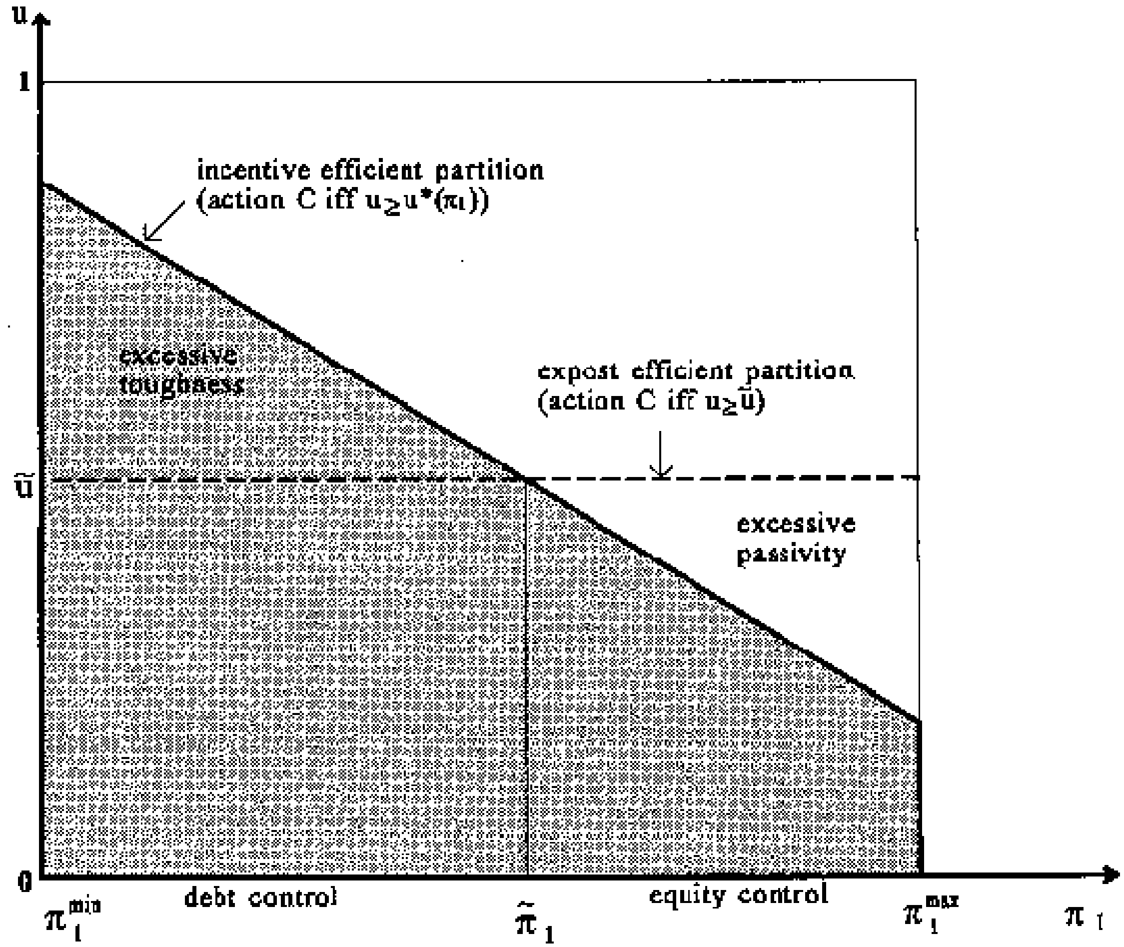
$$+\beta\left[\int_{D_2}^{\pi_2^{max}} [H_S(\pi_2|u) - H_C(\pi_2|u)]d\pi_2\right]$$

- Since continuation becomes more attractive for high u, the investor in control will continue if and only if u is above some threshold $u(\alpha, \beta, D_2)$.

- Since continuation is riskier, the threshold increases in α/β:

    - For $\alpha/\beta = 1$, the threshold is $\bar{u}$.

- Specializing attention to pure debt or pure equity control, the optimal plan is to give control to the debt holders when $\pi_1 < \bar{\pi}_1$ and to the equity holders when $\pi_1 > \bar{\pi}_1$. This is done by setting a short-term debt level at $D_1 = \bar{\pi}_1$



u
1
incentive efficient partition
(action C iff u≥u*(π1))
excessive toughness
expost efficient partition
(action C iff u≥ũ)
ũ
excessive passivity
0
debt control
equity control

FIGURE III

- To finalize the implementation of the optimal incentive scheme, we need to pin down the long-term debt levels D_2.
    - These debt levels must vary with first-period profit.
- Under equity control ($\pi_1 > \overline{\pi}_1$), D_2 is determined by:

$$\int_{D_2}^{\pi_2^{max}} \left[H_S\left(\pi_2 \middle| u^*(\pi_1)\right) - H_C\left(\pi_2 \middle| u^*(\pi_1)\right) \right] d\pi_2 = 0.$$

- Under debt control ($\pi_1 < \overline{\pi}_1$), D_2 is determined by:

$$\int_{\pi_2^{min}}^{D_2} \left[H_S\left(\pi_2 \middle| u^*(\pi_1)\right) - H_C\left(\pi_2 \middle| u^*(\pi_1)\right) \right] d\pi_2 = 0.$$

- We can see that under both debt and equity control, long-term debt D_2 increases in first-period profit π_1.
  - In both cases, a higher long-term debt make the decision-maker consider higher levels of π_2 in his decision, which makes continuation more attractive.
  - Interestingly, while higher short-term debt tends to favor liquidation, higher long-term debt tends to favor continuation.
- Yet, there is a jump down in D_2 from π_2^{max} to π_2^{min} as we shift from the range of debt control to the range of equity control.

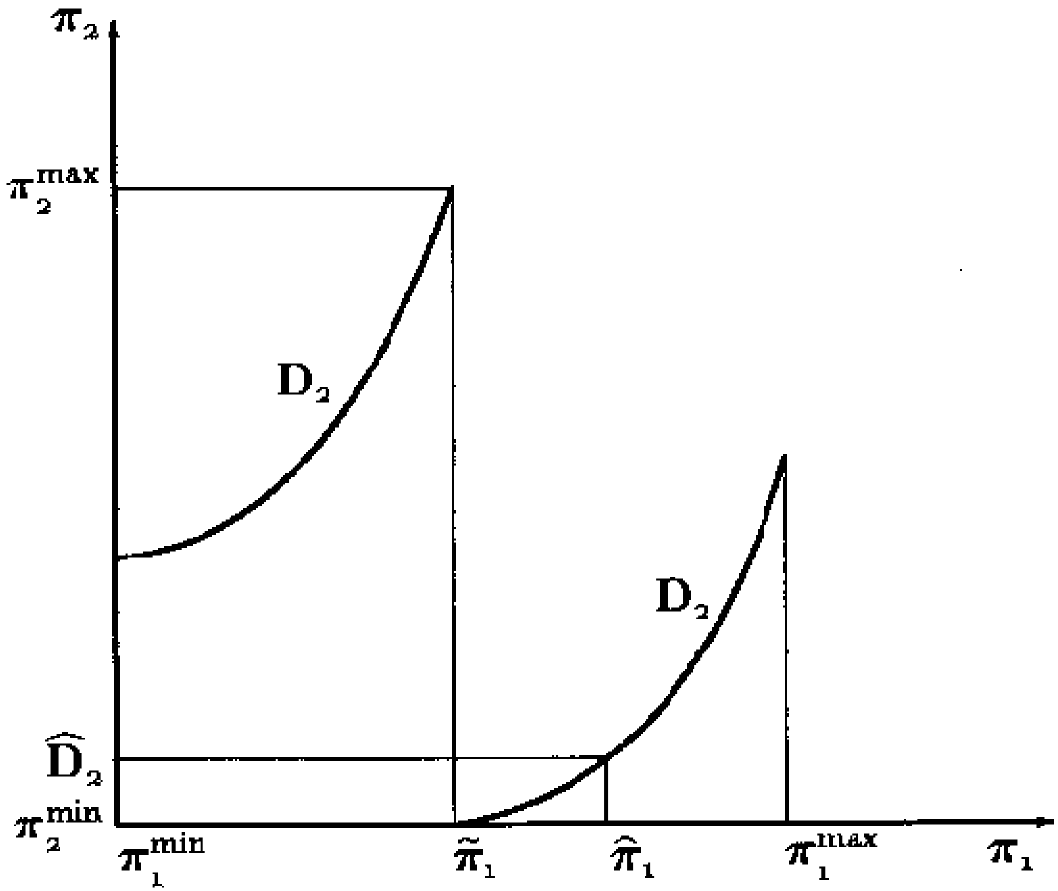

π_2
π_2^{max}
D_2
D_2
$\widehat{D}_2$
π_2^{min}
π_1^{min}
$\tilde{\pi}_1$
$\hat{\pi}_1$
π_1^{max}
π_1

FIGURE IV