Financial Fragility

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Introduction

Phenomenon

• Massive withdrawals / capital outflows in financial systems.
• Spread beyond the financial system and affect the real economy.

Types of Crises

• Bank runs.
• Currency attacks.
• Equity market crashes.

Leading Explanations

• Panic / Self-fulfilling expectations.
• Information/ Fundamentals.
Examples

- Bank runs in the USA in late 19\textsuperscript{th} century and early 20\textsuperscript{th} century.
  - Leading source of current regulation.
- East Asian crisis in late 90’s.
- Open-end real-estate funds in Germany.

Related Phenomena

- Contagion.
- Twin Crises.
Panic-Based Bank Runs (Diamond and Dybvig, JPE 1983)

- Diamond and Dybvig provide a seminal model of bank runs that is inherently tied to the basic role of banks.

- Banks Create liquid claims on illiquid assets using demand-deposit contracts.
  - Enable investors with early liquidity needs to participate in long-term investments.
  - Provide risk sharing.

- Drawback: These contracts expose banks to panic-based bank runs.
Types and Preferences

- Three periods (0,1,2), one good, continuum [0,1] of agents.
- Each agent is born at period 0 with an endowment of 1.
- Consumption occurs only at periods 1 or 2.
- Agents can be of two types:
  - Impatient (probability $\lambda$) – enjoys utility $u(c_1)$,
  - Patient (probability 1-$\lambda$) – enjoys utility $u(c_1 + c_2)$.
- Types are i.i.d., privately revealed to agents at the beginning of period 1.
- Risk aversion coefficient: $-cu''(c)/u'(c) > 1$ for any $c \geq 1$. ($u(0) = 0$).
Technology

- 1 unit of input at period 0 $\Rightarrow$ 1 unit of output at period 1

  or $R$ units at period 2 with probability $p(\theta)$.

- $\theta$ is distributed uniformly over $[0,1]$, revealed at period 2.

- $p(\theta)$ is increasing in $\theta$.

- The technology yields (on average) higher returns in the long run:

  \[ E_\theta [p(\theta)]u(R) > u(1). \]

- In autarky, impatient agents consume in period 1, while patient agents wait till period 2.
First-Best Allocation (if types were verifiable)

Period-1 consumption of impatient agents: \( c_1 \).

Period-2 consumption of patient agents: \( c_2 = \frac{(1-\lambda c_1)}{1-\lambda} R \) (Probability \( p(\theta) \)).

Planner maximizes expected utility: \( \lambda u(c_1) + (1-\lambda)u\left(\frac{1-\lambda c_1}{1-\lambda}\right) E_\theta [p(\theta)] \).

First order condition: \( u'(c_1^{FB}) = Ru'\left(\frac{1-\lambda c_1^{FB}}{1-\lambda}\right) E_\theta [p(\theta)] \).

Result: \( c_1^{FB} > 1 \).

(Note that at \( c_1^{FB} = 1 \): \( 1 \cdot u'(1) > R \cdot u'(R) E_\theta [p(\theta)] \).)
The Role of Banks

Overcome the fact that types are not verifiable using demand deposit contracts. Payments are given as follows:

<table>
<thead>
<tr>
<th>Period</th>
<th>$n &lt; 1/r_1$</th>
<th>$n \geq 1/r_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$r_i$</td>
<td>$r_i \cdot \text{prob} \frac{1}{nr_i}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0 \cdot \text{prob} \left(1 - \frac{1}{nr_i}\right)$</td>
</tr>
<tr>
<td>2</td>
<td>${ \frac{(1-nr_i)}{1-n} \text{prob} p(\theta) }$</td>
<td>0</td>
</tr>
</tbody>
</table>
Free entry to banking sector $\Rightarrow$ banks maximize welfare of agents.

Suppose the bank sets $r_i = c_i^{FB}$.

(Assume incentive compatibility holds: $E_\theta [p(\theta)]u\left(\frac{1 - \lambda r_i}{1 - \lambda} R\right) > u(r_i)$.

**‘Good’ equilibrium**: Agents act according to their types;

First best allocation achieved.

**‘Bad’ equilibrium**: Bank run. All agents demand early withdrawal.

$1/r_1$ of agents consume $r_1$, the rest consume 0.

*Worse than autarky!*

The optimal contract under the assumption that runs *never* occur: $r_i = c_i^{FB}$. 
Complementarities and Fragility

- The source of fragility is **strategic complementarities** among depositors:
  - When $n<1/r_1$, the incentive to withdraw early increases in the number of other depositors who withdraw early.
    - The run depletes the bank’s resources and hurts those who stay.
    - Note that this reverses when $n\geq 1/r_1$.

- As a result, runs are **panic-based**: They occur as a result of the self-fulfilling beliefs that other depositors are going to run.

- Moreover, they are **unrelated to fundamentals**.
  - Some tend to attribute them to sunspots.
Problem in the Model

The model provides no tools to determine which equilibrium is more likely to occur.

- If the probability of runs is non-negligible (Cooper and Ross, JME1998):
  ⇒ ‘Optimal’ contract may be not optimal!
  ⇒ Demand deposit contract may be not even desirable.

- If the probability depends on contract (Goldstein and Pauzner, JF 2005):
  ⇒ It would affect the optimal contract.
  ⇒ Demand deposit contract may be even more undesirable.
Solutions to Fragility – Suspension of Convertibility:

• Suppose that the bank announces that after $\lambda$ depositors withdraw in period 1, no one else gets money in this period.

• The good equilibrium becomes the unique equilibrium.
  
  o Patient agents know that no matter what others do, they are guaranteed to get $E_{\theta}[p(\theta)u\left(\frac{1-\lambda r_i}{1-\lambda} R\right)] > u(r_i)$.

• Hence, the run is prevented without even triggering suspension.

• **Problem:** What if the number of impatient agents is not known?
  
  o Suspension of convertibility may severely hurt impatient agents.
Solutions to Fragility – Deposit Insurance:

• Suppose that the government provides insurance to the bank in case of excess withdrawals.
  
  o To maintain the assumption of ‘closed’ economy, suppose that the government obtains this amount by taxing depositors.

• Again, the good equilibrium becomes the unique equilibrium.
  
  o Patient agents know that the withdrawal by others is not going to harm their long-term return.

• **Problem:** Deposit insurance might generate moral hazard: Banks make too risky investments or set deposit rate too high.
Information-Based Bank Runs (Chari and Jagannathan, JF 1988)

• Motivated by similar events, Chari and Jagannathan wish to explain runs on the basis of an informational story.
  
  o Agents run not because they fear others will run and will deplete the resources of the bank.

  o They run when they see others run, because they know others may have information about the fundamentals.

• The result is that runs are linked to fundamentals, but inefficiencies still occur.

  o Sometimes bad liquidity event prompts widespread run.
Types and Preferences

- Three periods (0,1,2), one good, continuum [0,1] of agents.
- Each agent is born at period 0 with an endowment of 1.
- Consumption occurs only at periods 1 or 2.
- Agents can be of two types:
  - Impatient (probability $\tilde{t}$) – enjoys utility $u(c_1)$,
  - Patient (probability $1 - \tilde{t}$) – enjoys utility $u(c_1 + c_2)$.
  - $\tilde{t} \in \{0, t_1, t_2\}$ with probabilities $r_0, r_1, r_2$.
- Types are i.i.d., privately revealed to agents at the beginning of period 1.
Technology and Information

• \(\{k_0, k\}\) represent investments in periods 0 and 1; \(K\) is aggregate investment.

• \(\{y_1, y_2\}\) represent outputs in periods 1 and 2.

• Liquidation in period 1 is costly; the cost depends on how many liquidate:

\[
y_1 = \begin{cases} 
  k_0 - k & \text{if } K \geq \bar{K} \\
  (1-a)(k_0-k) & \text{if } K < \bar{K}
\end{cases}
\]

• In period 2, the investment returns \(y_2 = \tilde{R}k\).
  
  \(\circ\) \(R=H\) with probability \(p\), and \(L=0\) otherwise; \(pH>1\).

• In period 1, \(\bar{\alpha}\) patient agents become informed about the return:
  
  \(\circ\) \(\bar{\alpha} \in \{0, \bar{\alpha}\}\) with probabilities \(1-q\) and \(q\).
Investment Choices

- In period 0, everyone invests.

- In period 1:
  - Impatient and negatively informed agents withdraw.
  - Positively informed agents invest.
  - Uninformed agents observe $K$, and make decision accordingly.

- Assume that $t_1 = \overline{\alpha}$ and $t_2 = t_1 + \overline{\alpha}(1 - t_1)$

  $\Rightarrow$ Uninformed agents cannot tell if withdrawals come from negative information or from liquidity shocks.

- The main feature of equilibrium outcomes is that runs are correlated with bad fundamentals, but there are unjustified runs in equilibrium (panic?)
Focusing on Unjustified Runs

Table I

<table>
<thead>
<tr>
<th>State of Nature</th>
<th>The aggregate investment demand function $K_D(\cdot)$ defined in equation (9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
<td>$\theta = {t, R, \alpha}$</td>
</tr>
<tr>
<td>1</td>
<td>$(0, R, 0)$</td>
</tr>
<tr>
<td>2</td>
<td>$(0, H, \bar{\alpha})$</td>
</tr>
<tr>
<td>3</td>
<td>$(0, L, \bar{\alpha})$</td>
</tr>
<tr>
<td>4</td>
<td>$(t_1, R, 0)$</td>
</tr>
<tr>
<td>5</td>
<td>$(t_1, H, \bar{\alpha})$</td>
</tr>
<tr>
<td>6</td>
<td>$(t_1, L, \bar{\alpha})$</td>
</tr>
<tr>
<td>7</td>
<td>$(t_2, R, 0)$</td>
</tr>
<tr>
<td>8</td>
<td>$(t_2, H, \bar{\alpha})$</td>
</tr>
<tr>
<td>9</td>
<td>$(t_2, L, \bar{\alpha})$</td>
</tr>
<tr>
<td></td>
<td>$k(K)$</td>
</tr>
<tr>
<td></td>
<td>$\bar{\alpha} + (1 - \bar{\alpha})k(K)$</td>
</tr>
<tr>
<td></td>
<td>$(1 - \bar{\alpha})k(K)$</td>
</tr>
<tr>
<td></td>
<td>$(1 - t_1)k(K)$</td>
</tr>
<tr>
<td></td>
<td>$(1 - t_1){\bar{\alpha} + (1 - \bar{\alpha})k(K)}$</td>
</tr>
<tr>
<td></td>
<td>$(1 - t_1)(1 - \bar{\alpha})k(K)$</td>
</tr>
<tr>
<td></td>
<td>$(1 - t_2)k(K)$</td>
</tr>
<tr>
<td></td>
<td>$(1 - t_2){\bar{\alpha} + (1 - \bar{\alpha})k(K)}$</td>
</tr>
<tr>
<td></td>
<td>$(1 - t_2)(1 - \bar{\alpha})k(K)$</td>
</tr>
</tbody>
</table>
• An equilibrium with unjustified runs implies that agents run in one of the states: 1, 4, or 7.

• Suppose that they do not run in state 7: $K(7) > 0$.

• Then, $K(6) = K(7) > 0$.

• In fact, $K(6)$ can only be equal to $K(7)$ and $K(8)$.

• Then, when observing this level, the expected value from continuing is:

\[
\frac{r_2pH}{r_1(1-p)q + r_2(1-q) + r_2pq}
\]

• Thus, when this is less than $1-a$, it is never an equilibrium to invest when the state is 6 or 7.

• So, there must be unjustified runs.
The Role of Suspension of Convertibility

• Runs are bad in this model because the value of the investment in period 1 decreases when many agents run in period 1.
  ○ This is an externality.

• Suppose that there is a device that does not allow liquidation of more than $t_1$, i.e., Suspension of Convertibility.
  ○ Advantage: reducing the damage from runs.
  ○ Disadvantages: preventing runs when they are efficient and when agents really need to consume.

$\Rightarrow$ Suspension of convertibility is desirable when $a$ is sufficiently large.
Empirical Evidence

• For a long time, people thought that the empirical distinction between the two types of bank runs is that information-based bank runs are correlated with the fundamentals, whereas panic-based bank runs are not.

• Most empirical studies found a strong link between runs and various types of fundamentals.

• Kaminsky and Reinhart (AER, 99)
  - Study episodes of banking and currency crises in developing and developed countries between 1970 and 1995.
  - Find that banking crises and currency crises are interrelated and aggravate each other.
Both are driven by deteriorating fundamentals, as captured by variables like output, terms of trade, and stock prices.

- **Schumacher (JME, 00)**
  - Studies runs on Argentine banks after the 1994 Mexican crisis.
  - Finds that failing banks suffered more withdrawals than surviving banks.
  - These banks were ex-ante ‘bad’, as measured by variables like capital adequacy, asset quality, liquidity, performance, and size.
• **Martinez-Peria and Schmukler (JF, 01)**
  
  o Study the behavior of bank deposits and interest rates in Argentina, Chile, and Mexico in the 90’s.
  
  o Find that depositors discipline banks, in that they withdraw deposit and/or demand high interest rate when fundamentals deteriorate, as captured by variables like capital adequacy, non-performing loans, and profitability.

• **Calomiris and Mason (AER, 03)**
  
  o Study bank failures in the US between 1929 and 1931.
  
  o Show that the duration of survival can be explained by size, asset quality, leverage, and other fundamentals.
Are Panic-Based Crises Irrelevant?

- The empirical evidence described above point towards the relevance of information-based crises. That is, crises are driven by bad fundamentals, and are not random events.

- But, panic-based bank runs are not ‘treated fairly’.

- The view was that the theory of panic-based runs has no empirical predictions, since it imposes no restrictions on the data.

- Recent theoretical developments enable tighter analysis of panic-based models and generate testable empirical implications.

- These developments are due to the global-game literature that started with Carlsson and van Damme (Econometrica, 93).
The Global-Games Approach (Morris and Shin, AER 98)

A Model of Currency Attacks

- There is a continuum of speculators $[0,1]$ and a government.
- The exchange rate without intervention is $f(\theta); f'(\theta) > 0; \theta \sim U[0,1]$.
- The government maintains the exchange rate at $e^* > f(\theta), \forall \theta$.
- Speculators may choose to attack the currency.
  - The cost of attack is $t$.
  - The benefit in case the government abandons the regime is $e^* - f(\theta)$.
- The government’s payoff from maintaining the regime is: $v - c(\alpha, \theta)$.
  - $c(\alpha, \theta)$ is increasing in $\alpha$ (proportion of attackers) and decreasing in $\theta$. 
Equilibria under Perfect Information

- Define $1 > \bar{\theta} > \theta > 0$:
  - $c(0, \theta) = v$.
  - $e^* - f(\bar{\theta}) = t$.

- Three ranges of the fundamentals:
  - When $\theta < \bar{\theta}$, unique equilibrium: all speculators attack.
  - When $\theta > \bar{\theta}$, unique equilibrium: no speculator attacks.
  - When $\bar{\theta} > \theta > \bar{\theta}$, multiple equilibria: Either all speculators attack or no speculator attacks (for this, assume $c(1,1) > v$).

- The problem of multiplicity is similar to that in Diamond and Dybvig.
  - Originates from strategic complementarities.
Introducing Imperfect Information

- Suppose that speculators observe $\theta_i = \theta + \varepsilon_i$, where $\varepsilon_i \sim U[-\varepsilon, \varepsilon]$.

- Speculators choose whether to attack or not based on their signals.

- The signals serve to inform them about the fundamentals, but also about the signals (and thus the actions) of others.

- The key aspect is that because they only observe imperfect signals, they must take into account what others will do at other signals.

- This will ‘connect’ the different fundamentals and determine optimal action at each.

- As a result, we will show that there is a unique equilibrium where speculators attack if and only if they observe a signal below the cutoff $\theta^*$. 
Definitions

- Payoff from attacking as a function of fundamentals and aggregate attack:

\[
h(\theta, \alpha(\theta)) = \begin{cases} 
    e^* - f(\theta) - t & \text{if } \alpha(\theta) > a(\theta) \\
    -t & \text{if } \alpha(\theta) \leq a(\theta)
\end{cases}
\]

where \( c(a(\theta), \theta) = v \).

- Payoff as a function of the signal and aggregate attack:

\[
V(\theta, \alpha(\theta)) = \frac{1}{2\varepsilon} \int_{\theta_1-\varepsilon}^{\theta_1+\varepsilon} h(\theta, \alpha(\theta)) d\theta.
\]

- Aggregate attack when speculators follow threshold \( \theta' \):

\[
\alpha(\theta, \theta') = \begin{cases} 
    0 & \text{if } \theta > \theta' + \varepsilon \\
    \frac{\theta' + \varepsilon - \theta}{2\varepsilon} & \text{if } \theta' - \varepsilon \leq \theta \leq \theta' + \varepsilon \\
    1 & \text{if } \theta < \theta' - \varepsilon
\end{cases}
\]
Existence and Uniqueness of Threshold Equilibrium

- Function $V(\theta', \alpha(\theta, \theta'))$ is monotonically decreasing in $\theta'$; positive for low $\theta'$ and negative for high $\theta'$.

  $\Rightarrow$ There is a unique $\theta^*$ that satisfies $V(\theta^*, \alpha(\theta, \theta^*)) = 0$, and is a candidate for a threshold equilibrium.

- $V(\theta_i, \alpha(\theta, \theta^*)) < 0$, $\forall \theta_i > \theta^*$; because of strategic complementarities and the effect of fundamentals.

- Similarly, $V(\theta_i, \alpha(\theta, \theta^*)) > 0$, $\forall \theta_i < \theta^*$.

$\Rightarrow$ Threshold $\theta^*$ constitutes an equilibrium.
Ruling out Non-Threshold Equilibria

• Suppose speculators attack at signals above \( \theta^* \); denote the highest such signal as \( \theta'^* \) (we know it is below 1 because of upper dominance region).

• Denote the equilibrium attack as \( \alpha'(\theta) \), then: \( V(\theta'^*, \alpha'(\theta)) = 0 \).

• We know that \( \alpha'(\theta) \leq \alpha(\theta, \theta'^*) \).

• Then, due to strategic complementarities: \( V(\theta'^*, \alpha(\theta, \theta'^*)) \geq 0 \).

• But, this is in contradiction with \( V(\theta^*, \alpha(\theta, \theta^*)) = 0 \).

    \[ \Rightarrow \text{Hence, speculators do not attack at signals above } \theta^*. \]

• Similarly, one can show that they always attack at signals below \( \theta^* \).

\[ \Rightarrow \text{No non-threshold equilibria.} \]
Some Intuition

- These are the bounds on the proportion of attack imposed by the dominance regions:

  - Lower Dominance Region
  - Intermediate Region
  - Upper Dominance Region

- These bounds can be shifted closer together by iterative elimination of dominated strategies.
- The result is the equilibrium that we found:

\[
\alpha = 0 \quad \alpha = 1
\]

\[
\theta^* - \varepsilon \quad \theta^* \quad \theta^* + \varepsilon
\]

Total Attack Partial Attack No Attack
Important:

Although $\theta$ uniquely determines $\alpha$, attacks are still driven by bad expectations, i.e., still panic-based:

- In the intermediate region speculators attack because they believe others do so.
- $\theta$ acts like a coordination device for agents' beliefs.

A crucial point: $\theta$ is not just a sunspot, but rather a payoff-relevant variable.

$\Rightarrow$ Agents are *obliged* to act according to $\theta$. 
Why Is This Equilibrium Interesting?

• **First**, reconciles panic-based approach with empirical evidence that fundamentals are linked to crises.
  
  o Still very different from information-based approach.

• **Second**, panic-based approach can generate unique empirical implications.
  
  o Here, the probability of a crisis is pinned down by the value of $\theta^*$, which depends on variables like $t$, $v$, etc.

• **Third**, once the probability of crises is known, one can use the model for policy implications.

• **Fourth**, captures the notion of strategic risk, which is missing from the perfect-information version.
Back to Bank Runs (Goldstein and Pauzner, JF 05)

Goal
Use global-games approach to address the fundamental issues in the Diamond-Dybvig model.

Main Contributions

• **First**, the Diamond-Dybvig model violates the basic assumptions in the global-games approach. In particular, it does not satisfy global strategic complementarities.
  
  o Derive new proof technique that overcomes this problem.

• **Second**, once a unique equilibrium is obtained, study how the probability of a bank run is affected by the banking contract, and what is the optimal demand-deposit contract once the effect is taken into account.
Reminder, Payoff Structure

<table>
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<td>1</td>
<td>( r_i )</td>
<td>( \begin{cases} r_i &amp; \text{prob} \frac{1}{nr_i} \ 0 &amp; \text{prob} \ 1 - \frac{1}{nr_i} \end{cases} )</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{(1-nr_i)}{1-n} R ) ( \text{prob} \ p(\theta) ) ( \begin{cases} 0 &amp; \text{prob} \ 1 - p(\theta) \end{cases} )</td>
<td>0</td>
</tr>
</tbody>
</table>

- Global strategic complementarities do not hold: An agent’s incentive to run is highest when \( n = 1/r_1 \) rather than when \( n = 1 \).
Mathematically and graphically:

\[ v(\theta, n) = \begin{cases} 
  p(\theta)u\left(\frac{1-nr_1}{1-n} R\right) - u(r_1) & \text{if } \frac{1}{r_1} \geq n \geq \lambda \\
  0 - \frac{1}{nr_1} u(r_1) & \text{if } 1 \geq n \geq \frac{1}{r_1}
\end{cases} \]
• Our proof builds on *one-sided strategic complementarities*:
  \[ \Rightarrow \nu \text{ is monotonically decreasing whenever it is positive} \]

• which implies *Single crossing*:
  \[ \Rightarrow \nu \text{ crosses zero only once.} \]

• We show uniqueness by:
  
  o Showing that there exists a unique threshold equilibrium.
  
  o Showing that every equilibrium must be a threshold equilibrium.

• Not by iterative dominance.
The Demand-Deposit Contract and the Viability of Banks

At the limit, threshold signal $\theta^*(r_1)$ is defined by:

\[
\int_{n=\lambda}^{1/r_1} u(r_1) + \int_{n=1/r_1}^{1} \frac{1}{nr_1} u(r_1) = \int_{n=\lambda}^{1/r_1} p(\theta^*) \cdot u(\frac{1-r^n}{1-n} R)
\]

**THEOREM 2**: $\theta^*(r_1)$ is increasing in $r_1$.

$\Rightarrow$ The bank becomes more vulnerable when it offers more risk sharing!

**Intuition:**

- With a higher $r_1$ the incentive of agents to withdraw early is higher.
- Moreover, other agents are more likely to withdraw at period 1, so the agent assesses a higher probability for a bank run.
Finding the optimal $r_1$

The bank chooses $r_1$ to maximize:

$$\lim_{\theta \to 1} \mathbb{E} U(r_1) = \int_0^{\theta^*(r_1)} \frac{1}{r_1} u(r_1) d\theta$$

$$+ \int_{\theta^*(r_1)}^1 \lambda \cdot u(r_1) + (1 - \lambda) \cdot p(\theta) \cdot u\left(\frac{1 - \lambda r_1}{1 - \lambda} R\right) d\theta$$

Now, the bank has to consider the effect that an increase in $r_1$ has on the expected costs of bank runs.

**Main question:** Are demand deposit contracts still desirable?

**THEOREM 3:** If $\theta(1)$ is not too large, the optimal $r_1$ must be larger than 1.
**Intuition:**
Increasing $r_1$ slightly above 1 generates one benefit and two costs:

**Benefit:** Risk sharing among agents.

⇒ Benefit is of first-order significance: Gains from risk sharing are maximal at $r_1=1$.

**Cost I:** Increase in the probability of bank runs beyond $\theta(1)$.

⇒ Cost is of second order: Liquidation at $\theta(1)$ is almost harmless.

**Cost II:** Increase in the welfare loss resulting from bank runs in $[0, \theta(1)]$.

⇒ Cost is small when $\theta(1)$ is not too large.

**Implication:** The optimal $r_1$ generates panic-based bank runs.
THEOREM 4: The optimal $r_1$ is lower than $C_1^{FB}$.

First order condition for $r_1$:

$$\lambda \int_{\theta^{*}(\eta)}^{1} \left[ u'(r_1) - p(\theta) \cdot R \cdot u' \left( \frac{1 - \lambda r_1}{1 - \lambda} R \right) \right] d\theta =$$

$$\frac{\partial \theta^{*}(r_1)}{\partial r_1} \left[ \left( \lambda u(r_1) + (1 - \lambda) p(\theta^{*}(r_1)) u \left( \frac{1 - \lambda r_1}{1 - \lambda} R \right) \right) - \frac{1}{r_1} u(r_1) \right] + \int_{\theta^{*}(\eta)}^{\theta^{*}(\eta)} \left[ \frac{u(r_1) - r_1 u'(r_1)}{r_1^2} \right] d\theta.$$  

LHS: marginal benefit from risk sharing.

RHS: marginal cost of bank runs.

Since marginal cost of bank runs is positive, and since marginal benefit is decreasing in $r_1$: The optimal $r_1$ is lower than $C_1^{FB}$. 
The Effect of a Large Investor (Corsetti, Dasgupta, Morris, and Shin, REStud 04)

- So far we analyzed situations with many small investors.

- A very relevant question is how things are going to be affected if large investors are present.

- Corsetti, Dasgupta, Morris, and Shin analyze this question motivated by the case of Soros.

- The key intuition can be understood by looking at what happens when instead of a continuum of small investors, there is only one large investor that decides whether to attack/run.
• In the Morris and Shin (1998) model, a large investor would choose to attack if and only if $\theta < \bar{\theta}$.
  
  o He can force the government to abandon the regime and gain $e^* - f(\theta) - t$, which is positive when $\theta < \bar{\theta}$.

• In the Goldstein and Pauzner (2005) model, a large investor would choose to run if and only if $\theta < \theta(1)$.
  
  o He knows that the bank can only pay him 1 in case he demands early withdrawal, which is optimal only when $\theta < \theta(1)$.

$\Rightarrow$ In a currency attack model, large investor generates more fragility, while in a bank run model, he generates more stability.
• The unifying theme is that the large investor is able to achieve the best outcome from his point of view.

  o In currency attacks, this means attack, whereas in bank runs, this means no run.

• What happens when the large investor is present alongside the small investors?

  o The qualitative effect is similar, albeit weaker.

  o Interestingly, the presence of a large investor, affects the behavior of small investors in the same direction.

• Knowing that he is there, they tend to attack more or run less, depending on the context.
Strategic Complementarities and Financial Fragility:
Empirical Evidence (Chen, Goldstein, and Jiang, WP 07)

• With global-games tools, one can test directly for the effect of strategic complementarities on financial fragility.

• Chen, Goldstein, and Jiang do this in the context of mutual funds.

• Why mutual funds?
  
  o Rich and detailed data.
  
  o Literature shows that, like in banks, payoff structure exhibits complementarities:
    
    ▪ e.g., Edelen (JFE 1999) and Johnson (JF 2004).
Premises

- Complementarities arise when funds experience net outflows, i.e., when the performance is low.
- Conditional on net outflows, complementarities are stronger when funds hold more illiquid assets.

Predictions

- **H1**: Conditional on low performance, funds that hold illiquid assets will experience more outflows.
- **H2**: Pattern weakens when fund is held by large investors.
  - i.e., large investors internalize the externality.
Liquidity and Outflows: Semi-Parametric Estimation
Liquidity and Outflows: Regression

<table>
<thead>
<tr>
<th>Variable for Perf</th>
<th>Full Sample</th>
<th>Subsample of negative performance</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>(1) Alpha1</td>
<td>(2) Alpha4</td>
</tr>
<tr>
<td>Perf</td>
<td>0.70</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>22.03</td>
<td>16.35</td>
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<tr>
<td>Illiq*Perf</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>3.65</td>
<td>3.27</td>
</tr>
</tbody>
</table>

Control variables:

| Flow(-1)         | 0.14        | 0.15        | 0.24         | 0.07         | 0.10         | 0.18           |
| Size(Ln)         | 0.11        | 0.12        | 0.13         | 0.06         | 0.09         | 0.08           |
| Age(Ln)          | -2.01       | -36.33      | -1.99        | -2.58        | -1.79        | -2.64          |
| Expense          | -0.30       | -6.51       | -0.32        | -0.27        | -0.62        | -0.56          |
| Load             | -0.05       | -4.75       | -0.05        | -0.02        | 0.00         | 0.06           |
| Inst             | -0.74       | -11.19      | -0.74        | -0.84        | -13.02       | -5.32          |
| Illiq            | 0.13        | 0.22        | 0.28         | 0.25         | 0.43         | 0.29           |
| Size*Perf        | 0.06        | 7.37        | 0.04         | 0.09         | 8.21         | 1.11           |
| Age*Perf         | -0.32       | -12.43      | -0.19        | -0.46        | -1.12        | -0.41          |
| Expense*Perf     | 0.03        | 1.05        | 0.05         | 0.08         | 1.95         | -0.14          |
| Load*Perf        | 0.01        | 0.86        | 0.00         | 0.02         | 1.61         | 0.05           |
| Inst*Perf        | -0.16       | -3.79       | -0.10        | -0.16        | -2.57        | 0.09           |

Year fixed-effects: Yes

#obs & R-sqr: 639,596, 0.07

639,596, 0.06

676,198, 0.13

344,127, 0.03

374,697, 0.03

384,123, 0.08
Some Discussion

- Importantly, there is a difference from bank runs: the outflow events that we capture are not total runs.

- Our story applies mostly to the marginal investor making a redemption decision, not to the average investor.
  - Evidence by Johnson (WP 2006) and Agnew, Pierluigi, and Sunden (AER 2003) show that most investors do not trade much.

- Total runs on mutual funds are very uncommon:
  - One example is the open-end real-estate funds in Germany. See Bannier, Fecht, and tyrell (WP 2006).
The Effect of Large Investors

<table>
<thead>
<tr>
<th></th>
<th>Institutional-oriented funds with</th>
<th>Retail-oriented funds with</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>%Large Investor &gt;= 75%</td>
<td>%Large Investor &lt;= 25%</td>
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<tr>
<td>Inst=1</td>
<td>MinPur250k=1</td>
<td>Inst=1 MinPur250k=1</td>
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<table>
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<td>( Alpha_1 )</td>
<td>0.27*</td>
<td>1.66</td>
<td>0.43**</td>
<td>2.26</td>
<td>0.24**</td>
<td>3.36</td>
<td>0.25**</td>
<td>3.68</td>
</tr>
<tr>
<td>( Illiq^* Alpha_1 )</td>
<td>0.02</td>
<td>0.18</td>
<td>0.06</td>
<td>0.33</td>
<td>0.20**</td>
<td>2.91</td>
<td>0.16**</td>
<td>2.71</td>
</tr>
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</table>
Financial Market Runs (Bernardo and Welch, QJE 04)

- The ‘run’ literature has focused on financial institutions. Yet, events in financial markets suggest that similar events occur there as well. The model by Bernardo and Welch describes such a situation.

- There are three dates:

<table>
<thead>
<tr>
<th>Date</th>
<th>Event Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 0$</td>
<td>Investors Trade. Focus of our Paper.</td>
</tr>
<tr>
<td>$t = 1$</td>
<td>Possible Liquidity Shock. Shocked</td>
</tr>
<tr>
<td></td>
<td>Investors Forced to Trade.</td>
</tr>
<tr>
<td>$t = 2$</td>
<td>Unshocked investors return. Asset value</td>
</tr>
<tr>
<td></td>
<td>is revealed.</td>
</tr>
</tbody>
</table>

- The return of the asset is $\tilde{Z} \sim N(\mu, \sigma^2)$. 
Agents in the Model

- Market makers:
  - Hold no inventory, risk averse, face no risk of liquidity shock.

- Individual investors:
  - Hold the supply of the asset (1), risk neutral, get a liquidity shock in period 1 with probability $s$.

- Investors can sell their assets to the market makers. Hence, market makers provide insurance to investors.
  - This is costly because market makers are risk averse.
  - The market making sector cannot expand to absorb massive sale.
Trading

- Execution order is not perfectly sequential. E.g.,
  - After market closure.
  - During crash.
  - In over-the-counter markets.

- Prices are determined so that market makers earn zero expected utility in each trading period. (They are myopic.)

- $\alpha$ investors sell in period 0; $q_1(\alpha)$ are forced to sell in period 1. Define the expected net benefit of selling at period 0:

\[
F(\alpha) = p_0(\alpha) - s \cdot p_1(q_1(\alpha); \alpha) - (1 - s) \cdot \mu.
\]
A CARA-Normal Example

- Market making sector has negative exponential utility:

\[ u(w) = -e^{-\gamma \cdot w} \]

- The share price at period 0 makes the market makers indifferent between buying \( \alpha \) shares at period 0 and maintaining 0 inventory of shares:

\[
E[-e^{-\gamma \cdot \bar{w}_2}] = E[e^{-\gamma \cdot \bar{w}_0}],
\]

\[
\Rightarrow E[W_0 + \alpha \cdot (Z - p_0)] - \gamma \cdot \text{var} [W_0 + \alpha \cdot (Z - p_0)]/2 = W_0
\]

\[
\Rightarrow p_0(\alpha) = \mu - \gamma \cdot \sigma^2 \cdot \alpha/2.
\]

- Suppose that liquidity shocks in period 1 are perfectly correlated, so that all remaining investors want to sell their shares with probability \( s \).
• The share price at period 1 makes the market makers indifferent between buying \((1 - \alpha)\) more shares and maintaining an inventory of \(\alpha\) shares:

\[
E[\hat{W}_2 + (1 - \alpha) \cdot (\hat{Z} - p_1)] - \gamma \\
\quad \cdot \text{var} [\hat{W}_2 + (1 - \alpha) \cdot (\hat{Z} - p_1)]/2 = E[\hat{W}_2] - \gamma \cdot \text{var} [\hat{W}_2]/2, \\
\Rightarrow p_1((1 - \alpha); \alpha) = \mu - [2 \cdot \alpha + (1 - \alpha)] \cdot \gamma \cdot \sigma^2/2.
\]

• Then, \(F(\alpha) = \frac{\sigma^2}{2} [s(1 + \alpha) - \alpha].\)

  \(\circ\) \(F(0) > 0.\)

  \(\circ\) \(F(1) > 0\) iff \(s > \frac{1}{2}.\)

  \(\circ\) \(F'(\alpha) < 0.\)

• Then, the unique symmetric Nash equilibrium is:

\[
\alpha^* = \begin{cases} 
(s/1 - s) & \text{for } s \leq \frac{1}{2} \\
1 & \text{for } s > \frac{1}{2}
\end{cases}
\]
Implications

- Even though it is efficient that market makers don’t hold any shares at period 0, the desire of investors to preempt other investors generates a run that forces market makers to hold shares at that time.

- The ‘total run’ equilibrium occurs when the probability of liquidity shocks in period 1 is high.
  - In this case, investors know that prices are going to be very low in period 1, and the incentive to preempt is stronger.

- This is not a model of strategic complementarities and multiple equilibria as in bank runs and currency attacks.
  - The incentive to run is highest when no one else does.
Are There Strategic Complementarities in Financial Markets?

• The basic structure of financial markets does not generate strategic complementarities.
  
  o When more traders sell, the price decreases, and the incentive of others to sell decreases as well.

• Strategic complementarities may arise in some settings, leading to bank-run type phenomena.

• **Loss Limits (Morris and Shin, RF 04)**
  
  o Traders in financial markets often don’t trade on their own money. As a result they are provided with incentive contracts to induce them to trade optimally.
In many cases, contracts imply that if the value of the stocks they hold falls below a certain threshold, they will be penalized.

When the price is close to the threshold, knowing that other traders are going to sell increases the incentive of each investor to sell as well, so that he doesn’t hold the asset when the price falls below the threshold.

• **Feedback Effect**

  o Stock prices may have an effect on real firm value:

    • They affect firms’ access to capital (Baker, Stein, and Wurgler, QJE 03).

    • They convey information that affects corporate investments (Luo, JF 05; Chen, Goldstein, and Jiang, RFS 07).
• They affect managerial incentives via stock-based compensation.

  o Incorporating this into models of financial markets changes basic results (Dow and Gorton, JF 97; Subrahmanyam and Titman, JF 99; Goldstein and Guembel, REStud forthcoming).

  o In particular, feedback effect can generate strategic complementarities just like in banks (Ozdenoren and Yuan, WP 07):

    • Sales by other traders reduce the price.

    • This will reduce the value of the firm via the feedback effect.

    • And increases the incentive of each trader to sell.

• Of course, in these settings complementarities are not the only force.
Contagion

- One of the most striking features of financial crises is that they spread across countries/institutions.
  - Banking crises in the US in 19th and early 20th century.
  - East Asian Crisis in late 90’s.

- Several leading explanations have been offered:
  - Information.
  - Interbank Connections.
  - Investors’ portfolios readjustments.
  - Behavioral explanations.
Contagion due to Interbank Connections (Allen and Gale, JPE 00)

- There are three dates: 0, 1, and 2; one good.

- Investment technology:
  - **Short term**: One unit invested in $t=0$ yields one unit in $t=1$.
  - **Long term**: One unit invested in $t=0$ yields $R$ in $t=2$, or $r$ in $t=1$; $0 < r < 1 < R$.

- There are four different regions: A, B, C, and D.
  - Each region has a continuum $[0,1]$ of agents, who might face liquidity shocks, as in Diamond and Dybvig.
• Utility is given by:

\[
U(c_1, c_2) = \begin{cases} 
    u(c_1) & \text{with probability } \omega \\
    u(c_2) & \text{with probability } 1 - \omega,
\end{cases}
\]

• The probability of a liquidity shock varies from region to region; there are two equally likely states:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>S_1</td>
<td>(\omega_H)</td>
<td>(\omega_L)</td>
<td>(\omega_H)</td>
<td>(\omega_L)</td>
</tr>
<tr>
<td>S_2</td>
<td>(\omega_L)</td>
<td>(\omega_H)</td>
<td>(\omega_L)</td>
<td>(\omega_H)</td>
</tr>
</tbody>
</table>

○ Regions are ex-ante identical.
Optimal Risk Sharing

• Social planner identifies types. Denote:

\[ \gamma = \frac{\omega_H + \omega_L}{2}. \]

Also, \( x \) and \( y \) are per capita amounts invested in long-term and short-term assets, respectively.

• Planner maximizes:

\[ \gamma u(c_1) + (1 - \gamma) u(c_2) \]

• Subject to

\[ \gamma c_1 \leq y \]
\[ (1 - \gamma) c_2 \leq Rx. \]
• First-best allocation satisfies incentive-compatibility constraint:

\[ c_1 \leq c_2. \]

⇒ Thus, first-best can be achieved even if types are not observable.

• The planner will shift resources across regions.
  
  o In state 1,

\[ (\omega_H - \gamma) c_1 \]

moves from B and D to A and C in \( t=1 \), and

\[ (\omega_H - \gamma) c_2 \]

moves from A and C to B and D in \( t=2 \).
Decentralization

- In each region, consumers deposit their endowments in banks, who offer demand deposit contracts.

- Banks hold deposits in banks of other regions. Suppose the market is incomplete:

Fig. 2.—Incomplete market structure
• How can banks achieve the first best?
  
  o They make the investments and promise the returns as the planner.
  
  o They hold deposits at banks at the adjacent region at the amount:

\[ w_H - \gamma \]

  o In \( t=1 \) banks in regions with high liquidity needs liquidate the deposits at banks in regions with low liquidity needs.
  
  o In \( t=2 \) banks in regions with low liquidity needs liquidate the deposits at banks in regions with high liquidity needs.

• The fact that banks with low liquidity needs hold deposits in banks with high liquidity needs and vice versa guarantees efficient allocation.
Fragility

- Assume the same allocation as before, but a new state becomes possible:

| \( S_1 \) | \( \omega_H \) | \( \omega_L \) | \( \omega_H \) | \( \omega_L \) |
| \( S_2 \) | \( \omega_L \) | \( \omega_H \) | \( \omega_L \) | \( \omega_H \) |
| \( \hat{S} \) | \( \gamma + \epsilon \) | \( \gamma \) | \( \gamma \) | \( \gamma \) |

- The new state is assigned probability zero; in it, aggregate demand for liquidity requires liquidation of some long-term assets.

- Assume that deposits are liquidated before long-term assets:

\[ 1 < \frac{c_2}{c_1} < \frac{R}{r} \]
• If aggregate liquidity shock is large enough, banks in region A must go bankrupt:
  o They liquidate long-term assets to pay early withdrawals, and thus cannot pay enough to patient investors, who then decide to run.

• If liquidation value is sufficiently low, banks in region D will also go bankrupt.
  o The value of their deposits in region A is low, so they liquidate long-term assets and trigger a run.

• By induction, banks in regions B and C will also go bankrupt.

⇒ The failure of region A, triggers a failure of region D, which triggers a failure of Region C, which triggers a failure of region B.
Interbank Structures that Reduce Fragility

- No bank depends strongly on banks in region A.

⇒ Under similar parameters, the shocks that other regions suffer as a result of the failure of region A can be sufficiently small to be absorbed without generating failure.
• Failures are limited to regions A and B.

⇒ The link between market completeness and fragility of the system is non-monotone

  o Fragility occurs with intermediate level of completeness.

  o All banks are linked, but not directly.
Contagion due to Portfolio Diversification (Goldstein and Pauzner, JET 04)

• There is a continuum [0, 1] of identical agents.

• Their utility from consumption, \( u(c) \), is increasing, and satisfies decreasing absolute risk aversion: \(-u''(c)/u'(c)\) is decreasing.

• Each agent holds an investment of 1 in each country (1 and 2).

• An agent can choose when to withdraw each of her two investments:

  \[ \Rightarrow \quad \text{The (gross) return on investment in country } i \text{ is } 1 \text{ if withdrawn prematurely or } R \left( \frac{\theta_i}{n_i} \right) \text{ if withdrawn at maturity.} \]
• The fundamentals $\theta_1$ and $\theta_2$ are independent and drawn from a uniform distribution on $[0,1]$.

• Agent $j$ obtains a noisy signal $\theta_i^j$ on the fundamentals of country $i$:  
  $\theta_i^j = \theta_i + \varepsilon_i^j$

• $\varepsilon_i^j$ are error terms which are uniformly distributed over the interval $[-\varepsilon, \varepsilon]$ and independent across agents and countries.

• We will focus on the case in which signals are very precise, i.e. $\varepsilon$ is close to 0.

• There exist $0 < \underline{\theta} < \bar{\theta} < 1$ such that $R(\theta,0) = 1$ and $R(\bar{\theta},1) = 1$. 

Timeline

Agents hold investments in countries 1 and 2

\( \theta_1 \) is realized

\( \theta_1^j \) are observed

Agents decide whether to withdraw early in country 1

The aggregate outcomes in country 1 are realized and known to all agents

\( \theta_2 \) is realized

\( \theta_2^j \) are observed

Agents decide whether to withdraw early in country 2

The aggregate outcomes in country 2 are realized
The Behavior of Agents in Country 2: Contagion

- Using global-game techniques, there is a unique threshold in country 2, below which agents run.

- The threshold is higher if a crisis has happened in country 1.

- A crisis in country 1 reduces agents’ wealth, and makes them more averse to the strategic risk associated with investing in country 2.
The Behavior of Agents in Country 1: Correlation

- Focus on a threshold equilibrium. Due to contagion there is a correlation between the returns in country 1 and the returns in country 2.

- This is despite the fact that the fundamentals in the two countries are independent of each other.
Optimal Diversification

- There is externality associated with diversification. When agents diversify, they increase the correlation between the assets and reduce the benefits from diversification.

- Optimal level of diversification may be different from equilibrium level, and may be intermediate.

Fig. 8. Average welfare as a function of $\beta$. 