Financial Markets

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Trading and Price Formation

- This line of the literature analyzes the formation of prices in financial markets in a setting where traders possess pieces of information and trade on it.
- Depending on the **market microstructure**, the price gets to reflect the information of traders with certain degree of precision.
- This line of the literature is distinct from traditional asset pricing, where prices are assumed efficient (reflecting all available information) and are set by risk-return considerations.

Rational-Expectations Equilibrium: Grossman and Stiglitz (1980)

- Grossman and Stiglitz (1980) develop a canonical **rational-expectations equilibrium** model of financial markets.
- The key ingredients of their model are:
  - There are two types of traders – informed and uninformed – trading an asset with uncertain value.
  - Each trader submits a demand curve for the asset – quantity for every price.

- Agents are price takers: they don’t consider their effect on the price.
- But, they learn from the price: the quantity they demand for a given price depends on the information the price reveals about the value of the asset.
- There is an exogenous noisy supply for the asset.
- In equilibrium, prices are set such that the market clears: the supply equals the demand.
- Agents choose whether to pay the cost of becoming informed.

- The fundamental insight is that prices cannot be fully efficient; they cannot reflect all the information available to informed agents.
  - If prices reflected all the information available to informed traders, then no trader would pay the cost to become informed, as he could just learn the information from the price.
  - But, of course, if no one produced information, then prices would reflect no information, and it would be profitable to produce information.
- Hence, prices reveal information with some noise, making traders willing to acquire the information.

The Model

- There is a safe asset yielding a return R and a risky asset yielding a random return u:

$$u = \theta + \varepsilon$$

  - θ can be thought of as the fundamental of the asset. It can be observed by an individual at a cost c. ε is independent noise.

- In equilibrium, a proportion λ of individuals choose to become informed. Their demand for the risky asset is a function of the price P and of the fundamental θ.

- A proportion $(1-\lambda)$ of agents choose to remain uninformed. Their demand for the risky asset is a function of the price P, which serves two roles:
    - Directly affects their payoff by determining how much they pay.
    - Indirectly affects their expected payoff by revealing information about the fundamental θ.
- There is an exogenous supply of the asset x.
- The equilibrium price is set such that demand equals supply. For each level of informed trading, we then get a price as a function of supply and fundamentals: $P_\lambda(\theta, x)$.

Endowments and Preferences

- Trader *i* has initial wealth W_{0i}:

$$W_{0i} \equiv \overline{M}_i + P\overline{X}_i$$

  Where $\overline{M}_i$ is his initial endowment of the safe asset, $\overline{X}_i$ is his initial endowment of the risky asset, and P is the equilibrium price of the risky asset (the price of the safe asset is normalized to *1*).

- The trader chooses new quantities of the assets M_i and X_i to maximize his expected utility from final wealth W_{1i}:

$$W_{1i} = RM_i + uX_i$$

Subject to the budget constraint:

$$M_i + PX_i = W_{0i}$$

- The utility function is assumed to exhibit constant absolute risk aversion (CARA):

$$V(W_{1i}) = -e^{-aW_{1i}}$$

Where a is the coefficient of risk aversion.

Demand Functions and Trading Equilibrium

- Assuming that all variables are normally distributed (with variances σ_θ^2, σ_ε^2, and σ_x^2 and means $\overline{\theta}$, *0*, and *0*), and using the properties of the CARA function and normal distribution, an agent's demand for the risky asset can be generally developed as follows:
- Conditional on his information, the agent maximizes:

$$E\big(V(W_{1i})\big) = -exp\left(-a\left[E(W_{1i}) - \frac{a}{2}Var(W_{1i})\right]\right)$$

$$= -exp\left(-a\left[RW_{0i} + X_i[E(u) - RP] - \frac{a}{2}{X_i}^2Var(u)\right]\right)$$

- This yields the following general demand function:

$$X_i = \frac{E(u) - RP}{aVar(u)}$$

- The informed agent knows θ and doesn't need to learn anything from the price. Hence, his demand function is:

$$X_I = \frac{E(u|\theta) - RP}{aVar(u|\theta)} = \frac{\theta - RP}{a\sigma_\varepsilon^2}$$

- The uninformed agent only knows the price, and so:

$$X_U = \frac{E(u|P) - RP}{aVar(u|P)}$$

- In equilibrium, the demand has to equal the supply:

$$\lambda X_I(P_\lambda(\theta, x), \theta) + (1-\lambda) X_U(P_\lambda(\theta, x), P_\lambda) = x$$

- To solve for an equilibrium, we conjecture a linear price function:

$$P_\lambda(\theta, x) = \alpha_{1\lambda} + \alpha_{2\lambda}\theta + \alpha_{3\lambda} x$$

- Given this price function, an uninformed trader observes a signal about the fundamental θ:

$$\frac{P - \alpha_{1\lambda}}{\alpha_{2\lambda}} = \theta + \frac{\alpha_{3\lambda}}{\alpha_{2\lambda}} x$$

- Then,

$$E(u|P) = \frac{\frac{1}{\sigma_\theta^2}\bar{\theta} + \frac{1}{\sigma_x^2}\left(\frac{\alpha_{2\lambda}}{\alpha_{3\lambda}}\right)^2 \frac{P - \alpha_{1\lambda}}{\alpha_{2\lambda}}}{\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_x^2}\left(\frac{\alpha_{2\lambda}}{\alpha_{3\lambda}}\right)^2}$$

$$Var(u|P) = \frac{1}{\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_x^2}\left(\frac{\alpha_{2\lambda}}{\alpha_{3\lambda}}\right)^2} + \sigma_\varepsilon^2$$

- We can rewrite the demand of an uninformed trader:

$$X_U = \frac{\frac{1}{\sigma_\theta^2}\overline{\theta}+\frac{1}{\sigma_x^2}\left(\frac{\alpha_{2\lambda}}{\alpha_{3\lambda}}\right)^2\left(\frac{P-\alpha_{1\lambda}}{\alpha_{2\lambda}}\right)-RP\left(\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_x^2}\left(\frac{\alpha_{2\lambda}}{\alpha_{3\lambda}}\right)^2\right)}{a+a\sigma_\varepsilon^2\left(\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_x^2}\left(\frac{\alpha_{2\lambda}}{\alpha_{3\lambda}}\right)^2\right)}.$$

- And then the equilibrium condition:

$$(1-\lambda)\frac{\frac{1}{\sigma_\theta^2}\overline{\theta}+\frac{1}{\sigma_x^2}\left(\frac{\alpha_{2\lambda}}{\alpha_{3\lambda}}\right)^2\left(\frac{P-\alpha_{1\lambda}}{\alpha_{2\lambda}}\right)-RP\left(\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_x^2}\left(\frac{\alpha_{2\lambda}}{\alpha_{3\lambda}}\right)^2\right)}{a+a\sigma_\varepsilon^2\left(\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_x^2}\left(\frac{\alpha_{2\lambda}}{\alpha_{3\lambda}}\right)^2\right)}+\lambda\frac{\theta-RP}{a\sigma_\varepsilon^2}=x.$$

- From this, we see that: $\alpha_{2\lambda} = -\frac{\lambda}{a\sigma_\varepsilon^2}\alpha_{3\lambda}$.

- Hence, the price is given by:

$$P_\lambda(\theta, x) = \alpha_{1\lambda} + \alpha_{2\lambda}\left(\theta - \frac{a\sigma_\varepsilon^2}{\lambda}x\right).$$

And we can solve for $\alpha_{1\lambda}$ and $\alpha_{2\lambda}$ from the equilibrium condition.

- This expression is quite powerful, as it tells us when the price is more informative about the fundamental θ. This is the case when there are more informed traders (high λ), when traders are less risk averse (low a), and when there is less noise in the payoff of the risky asset (low σ_ε^2).

Equilibrium in Information Market

- A trader decides to become informed if the difference between the expected utility of informed and uninformed traders is at least as high as the cost of information production.
- In an internal equilibrium (λ strictly between *0* and *1*), the traders are indifferent, implying that the benefit from informed trading is exactly offset by the cost of information production.
- It is shown in the paper that there is a unique such equilibrium, since the benefit from informed trading is decreasing in the proportion of informed traders:

- A higher proportion of informed traders means that the price is more informative; this information is seen by the uninformed.
- A higher proportion of informed traders implies that the gains are split among more traders.

- Then, results are derived on the effect of different parameters on the informativeness of the price system (taking into account the endogeneity of λ).
  - The informativeness of the price decreases in the cost of information production c.
    - A higher cost reduces the proportion of informed traders.

- The informativeness of the price does not change in the noise in the supply σ_x^2.
  - For a given level of informed trading, more noisy supply reduces informativeness, but noisy supply increases informed trading.
- The informativeness of the price increases as the fundamental θ is more informative about the future payoff (σ_θ^2 increases or σ_ε^2 decreases) or as risk aversion a decreases.
  - These changes make traders produce more information and/or trade more aggressively.

Bayesian-Nash Equilibrium: Kyle (1985)

- The rational-expectations equilibrium cannot be easily interpreted in the context of a game or a real-world financial market.
- In it, traders take the price as given and do not consider their effect on the price, while at the same time they realize that other traders impact the price and thus that they can learn from it.
- Kyle (1985) developed a model that is closer to a game-theoretic approach, where the equilibrium concept is similar to the Bayesian-Nash Equilibrium.

The Model

- There is a risky asset, whose payoff $\tilde{v}$ is normally distributed with mean p_0 and variance Σ_0.
- A noise trader trades quantity $\tilde{u}$, which is normally distributed with mean *0* and variance σ_u^2.
- $\tilde{v}$ and $\tilde{u}$ are independent.
- An informed trader, who knows the value of $\tilde{v}$ is trading an endogenous quantity $\tilde{x} = X(\tilde{v})$. The price set in the market is denoted as $\tilde{p}$.

- The price is set by a market maker who clears the market. The market maker observes the total order flow $\tilde{x}+\tilde{u}$, but not its components. The price is then a function of the total order flow: $\tilde{p}=P(\tilde{x}+\tilde{u})\equiv P(\tilde{y})$.
- The profit of the informed trader is given by: $\tilde{\pi}=(\tilde{v}-\tilde{p})\tilde{x}$.
- We denote the price and profit as functions of the strategies: $\tilde{\pi}(X,P)$, $\tilde{p}(X,P)$.
- Note that the informed trader is submitting a **market order**. That is, his order to the market maker is not a function of the price. The alternative – a **limit order** – is studied in other papers.

Equilibrium

- An equilibrium is a set of X and P that satisfies two conditions:
  - **Profit maximization:** the informed trader's trading strategy maximizes his expected profit, given the pricing rule and given his signal:

$$E\{\tilde{\pi}(X,P)|\tilde{v}=v\} \geq E\{\tilde{\pi}(X',P)|\tilde{v}=v\}$$

  - **Market Efficiency:** given the trading strategy and the order flow, the market maker sets the price to be equal to the expected value of the security:

$$\tilde{p}(X, P) = E\{\tilde{v} | \tilde{x} + \tilde{u}\}$$

- This is not quite a game theoretic concept, as the market make does not maximize an objective function, but rather sets the price to be equal to expected value.
- It can be thought of as a result of competition in the market-making sector, which is driving the profits for market makers to zero.

- Essentially, the trader chooses an optimal strategy, taking into account the pricing rule and his effect on the price.

Solution

- We conjecture a linear equilibrium, where:

$$P(y) = \mu + \lambda y;$$

$$X(v) = \alpha + \beta v$$

- Now, we can write the profit of the informed trader:

$$E\left\{\left(\tilde{v} - P(x + \tilde{u})\right)x \middle| \tilde{v} = v\right\} = (v - \mu - \lambda x)x$$

The trader internalizes that his order flow is going to affect the price against him, and this serves to restrain his order size.

- Solving for optimal profit, we get:

$$x = \frac{v - \mu}{2\lambda}$$

- Hence, we can express α and β:

$$\beta = \frac{1}{2\lambda},$$

$$\alpha = -\frac{\mu}{2\lambda} = -\mu\beta$$

When the market maker puts a higher weight on the order flow in setting the price, the trader puts a lower weight on his information.

- We now look at the price-setting rule:

$$\mu + \lambda y = E\{\tilde{v} | \alpha + \beta \tilde{v} + \tilde{u} = y\}$$

- Essentially, the market maker observes a normally-distributed signal about $\tilde{v}$:

$$\tilde{v} + \frac{\tilde{u}}{\beta} = \frac{y - \alpha}{\beta}$$

- Hence, his expectation for $\tilde{v}$ is:

$$\frac{\frac{\beta^2}{\sigma_u^2}\left(\frac{y-\alpha}{\beta}\right)+\frac{1}{\Sigma_0}\mathrm{p}_0}{\frac{\beta^2}{\sigma_u^2}+\frac{1}{\Sigma_0}}$$

- This can be written as:

$$\frac{\beta\Sigma_0 y+\sigma_u^2\mathrm{p}_0-\alpha\beta\Sigma_0}{\beta^2\Sigma_0+\sigma_u^2}$$

- Hence,

$$\lambda=\frac{\beta\Sigma_0}{\beta^2\Sigma_0+\sigma_u^2},$$

$$\mu = \mathrm{p}_0 - \frac{\beta^2\Sigma_0\mathrm{p}_0 + \alpha\beta\Sigma_0}{\beta^2\Sigma_0 + \sigma_u^2} = \mathrm{p}_0 - \lambda(\alpha + \beta\mathrm{p}_0)$$

- Using the expressions for α and β above, we can solve the system:

$$\lambda = \frac{1}{2}\left(\frac{\sigma_u^2}{\Sigma_0}\right)^{-1/2},$$

$$\beta = \left(\frac{\sigma_u^2}{\Sigma_0}\right)^{1/2},$$

$$\mu = \mathrm{p}_0,$$

$$\alpha = -\beta\mathrm{p}_0.$$

Liquidity and Informativeness

- The parameter of most interest in the model is λ, which is usually referred to as **Kyle's Lambda**.
  - This is commonly used as a measure of the **illiquidity** of financial markets.
  - It captures **price impact**: by how much a price moves with the order flow.
  - This is how the market maker protects himself against losing money to an informed trader.

- We can see that illiquidity increases in the amount of uncertainty about the fundamental $\tilde{v}$ and decreases in the amount of noise.
- Of course, the informed trader internalizes the effect that he has on the price, and wants to trade less aggressively when this effect is large. Hence, we see that β is inversely related to λ. This affects the overall informativeness of the price.
- To analyze the informativeness of the price, we look at how much variance there is still about the fundamental $\tilde{v}$ given the realization of the price. Using the above solution of the model, we can see that:

$$var\{\tilde{v}|\tilde{p}\} = \frac{1}{2}\Sigma_0$$

- Hence, half of the information of the informed trader gets incorporated into the price. This is unaffected by the volatility of noise traders.
  - Given the trader's strategy, more noise trading reduces informativeness, but more noise trading encourages the trader to trade more aggressively.
- The expected profit of the informed trader is given by $\frac{1}{2}(\Sigma_0\sigma_u^2)^{1/2}$, and is thus increasing in both types of uncertainty.

Noise Trading

- The assumption that some trade in the market comes from noise traders is crucial.
  - Without it, the information of the informed trader would be fully reflected in the price, not enabling him to make a trading profit.
- There are different ways to think of uninformed trade:
  - Hedging/liquidity needs.
  - Behavioral assumptions.
  - Learning how to trade.

A Different Game-Theoretic Approach: Glosten and Milgrom (1985)

- Glosten and Milgrom developed an alternative microstructure model that is often used to analyze trading and price formation.
- In their model, a market maker posts **bid and ask** prices, at which he is willing to buy and sell shares from traders.
- At a point in time, the market maker receives one trade request, which can be coming from an uninformed or from an informed trader (the market maker doesn't know the identity of the trader).

- The market maker in the Glosten-Milgrom model protects himself against losing to an informed trader by asking a higher price when he sells a share than when he buys a share.
    - When he sells (buys), he is likely to be trading against a positively (negatively) informed trader.
- The difference between the two prices is known as the **bid-ask spread**, which is the measure of illiquidity in the model.
    - A high bid-ask spread is attributed to a high (low) probability of informed (uninformed) trading.
    - It implies a high transaction cost for traders.