

Optimal Securitization with Moral Hazard *

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Abstract

This paper considers the optimal design of mortgage backed securities (MBS) in dynamic setting with moral hazard. A mortgage underwriter with limited liability can engage in costly effort to screen for low risk borrowers and can sell loans to a secondary market. Secondary market investors cannot observe the effort of the mortgage underwriter, but they can make their payments to the underwriter conditional on the mortgage defaults. We find the optimal contract between that the underwriter and the investors involves a single payment to the underwriter after a waiting period. The dynamic setting of our model admits three new findings. First, the timing of payments to the underwriter is the key mechanism to provide incentives to the underwriter. Second, the duration of the optimal contract for mortgage underwriters can be short even though the duration of the mortgages is long. Third, mortgage pooling arises from an information *enhancement effect*.

Keywords: Security design, mortgage backed securities, moral hazard.

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1 Introduction

Mortgage underwriters face a dilemma: either to implement high underwriting standards and underwrite only high quality mortgages or relax underwriting standards in order to save on expenses. For example, an underwriter can collect as much information as possible about each mortgage applicant and fund only the most creditworthy borrowers. Alternatively, an underwriter could collect no information at all and simply make loans to every mortgage applicant. Clearly, the second approach, while less costly in terms of underwriting expenses, will result in higher default risks for the underwritten mortgages. Moreover, mortgage underwriters typically wish to sell their loans in a secondary market rather than hold loans in their portfolio. Investors do not observe the underwriter's effort and consequently do not observe the quality of the mortgages they are buying.¹ This paper examines how security design of mortgage backed securities (MBS) can address this agency conflict.

To address this issue we consider an optimal contracting problem between a mortgage underwriter and secondary-market investors.² At the origination date, the underwriter can choose to undertake costly effort that results in low expected default rate of the underwritten mortgages. If the underwriter chooses to shirk, the mortgages will have a high expected default rate. Moreover, by selling loans, rather than holding them in their portfolio, the mortgage underwriter can exploit new investment opportunities, i.e., make more loans. We model this feature by assuming that the underwriter is impatient, as in DeMarzo and Duffie (1999), so that the underwriter has a higher discount rate than the investors.³ Investors do not observe the actions of the underwriter, however the timing of mortgage defaults is publicly observable and contractable.

We derive the optimal contract between the underwriter and the investors that imple-

¹Recent empirical work in Keys, Mukherjee, Seru, and Vig (2008) finds that the securitized subprime loans have a higher default rate than loans held on portfolio. For an good description of the market for securitized subprime loans, see Ashcraft and Schuermann (2009).

²Although we refer to the agent in our model as the underwriter we can think of this agent as any issuer of asset backed securities who must pay a cost to screen assets. For example, we could think of the issuer as an aggregator or arranger who purchases single loans from originators but can pay a cost to undertake due diligence on the credit quality of the loans.

³An alternative motivation for the relative impatience of underwriters are regulatory capital requirements which induce a preference for cash over risky assets.

ments costly effort and maximizes the expected payoff for the underwriter, provided the investors are making non-negative profits in expectation. We do not make restrictive assumptions on the form of the contract. Instead, we include all possible payment schedules between the investors and the underwriter in the space of admissible contracts, so long as they depend only on the realization of mortgage defaults and provide limited liability to the underwriter.

Although the contracting problem is complicated, the optimal contract takes a simple form: The investors receive the entire pool of mortgages at time zero and make a single lump sum transfer to the underwriter after a waiting period provided no default occurs. If a single default occurs during the waiting period, the investors keep the mortgages, but no payments are made to the underwriter.

The timing and structure of the optimal contract is a tradeoff between incentive provision through delaying payment and efficiency through accelerating payment. By making the payment for mortgages contingent upon an initial period of no default, the investors can provide incentives for underwriters to underwrite low risk mortgages since high risk mortgages will be more likely to default during the initial period. However, delaying payments past the initial waiting period is suboptimal since the underwriter is impatient.

The optimal contract can be implemented using a credit default swap (CDS) and a risk free bond. Under this implementation, the investors pay the underwriter for the pool of mortgages at time zero. In addition, a CDS on the pool of mortgages is issued by the underwriter and given to the investors. The proceeds from the sale of the pool of mortgages are invested in the risk-free bond, which is held as a capital requirement for the CDS until it expires. If default occurs before the expiration of the CDS, then the investors receive the bond and the underwriter receives nothing.

Interestingly, the optimal contract calls for the underwriter to *pool* mortgages and include a bundled derivative security in the sale rather than sell each mortgage individually. By observing the timing of a single default, the investors learn about the quality of the remaining mortgages. As a result, the investors can infer the quality of the mortgages sooner by observing the entire pool rather than a single mortgage at a time, which we call the

information enhancement effect of pooling as is a consequence of our moral hazard setting. By making payment contingent upon the performance of the entire pool rather than each individual mortgage, it is possible to speed up payment to the underwriter while maintaining incentive compatibility. This result is not driven by any benefits from diversification.

Our findings are in stark contrast with the previous literature on security design with asymmetric information that primarily focuses on a static setting, e.g., Leland and Pyle (1977), DeMarzo and Duffie (1999) and DeMarzo (2005). We show that the timing of payments plays an extremely important role when the information about the underlying assets is revealed over time. In the dynamic setting of this paper, the optimal contract is about *when* the underwriter is paid, rather than what kind of piece of the underlying assets it retains.

Moreover, the timing of the optimal incentive contract has a much shorter duration than the underlying mortgages, which is very appealing from a practical standpoint. For reasonable parameter values the optimal contract has a duration of about two and a quarter years while the mortgages have a duration of about twenty years.

The literature has taken various approaches to explain the structure of asset backed securities. In one approach, the security design problem faced by an issuer of ABS is similar to the capital structure problem faced by a firm in the presence of asymmetric information over a fixed investment opportunity as characterized by Myers and Majluf (1984). In such a setting firms try to minimize the mispricing of their offered securities. The classic “folklore” hypothesis states that securities can be ranked by their informational sensitivity, and hence lemons related discount, in a “pecking order.” Nachman and Noe (1994) present a rigorous frame work for when a given a security design minimizes mispricing due to asymmetric information, showing standard debt to optimal over a very broad class of security design problems. Building on the pecking order intuition, Riddiough (1997) discusses the optimal design of asset backed securities when sellers have private information about asset value and thus face a lemons marketing problem. The issuer can increase her proceeds from securitization by creating multiple securities, or tranches, with differing levels of exposure to the issuers private information. The issuer then sells the least informationally sensitive

securities. Moreover, pooling assets that are not perfectly correlated can provide some diversification benefits and thus reduce the lemons discount.

Another approach considers the role of costly signaling in the basic security design problem considered by Leland and Pyle (1977). They model an informed and risk adverse entrepreneur who seeks external financing for a risky project. Investors are risk neutral and hence can better bear the risk of the project but demand a lemons discount. By retaining a portion of the project, or equity stake, the entrepreneur can signal the quality of the project. This signal is credible because retention is costly due to the difference in risk preferences of the entrepreneur and investors. Expanding on the role of costly signaling in security design, DeMarzo and Duffie (1999) presents a model of ex ante security design with ex post signaling. In the security issuance phase, an issuer can signal her private information by retaining a portion of the security she offers to investors. The issuer chooses the design of offered securities *before* receiving information about underlying cash flows to balance the cost of retaining residual cash flows and illiquidity caused by informational sensitivity.

Applying the costly signaling framework of DeMarzo and Duffie (1999), DeMarzo (2005) explains the pooling and tranching structure of ABS. In his model, an issuer of ABS can signal her private information about a pool of assets by retaining a portion of a security which is highly sensitive to that information. This signaling mechanism explains the multiclass, or tranching structure, of ABS. The bundling, or pooling, of multiple assets arises due to the balance of two forces. The first force is the information destruction effect of pooling, whereby selling bundled assets reduces value of information by limiting the issuers ability to use that information differently for different assets. The second force is the risk diversification effect whereby an issuer can create a low risk security from a large pool. As the number of assets grows large the latter effect dominates the former and informed intermediaries benefit from tranching and pooling.

Other studies of asset backed securities have focused on different types of asymmetric information. For example, Axelson (2007) considers a setting in which *investors* have superior information about the distribution asset cash flows. The author gives conditions for which pooling may be an optimal response to investor private information and for which

single asset sale is preferred.

The literature on security design and ABS presented above utilizes mostly one period models of securitization. In contrast to the previous literature, we take a different approach by modeling mortgages which can default after some time has elapsed. This additional aspect allows us to show that the timing of payments from mortgage securitization can be a key incentive mechanism and that the duration of the optimal contract can be short while the duration of the mortgages is long.

Another important difference is that there is little previous work on costly hidden actions in underwriting practices. Such a moral hazard problem is likely to be important in private securitization markets where both the quality of assets and the operations of issuers are extremely difficult to verify. Indeed, some empirical studies, such as Mian and Sufi (2009), suggest that “mispriced agency conflicts” may have played a crucial role in the current mortgage crisis. In addition, evidence presented in Keys, Mukherjee, Seru, and Vig (2008) suggests that securitization of subprime loans led to lax lender standards, especially when there is “soft” information about borrowers which determines default risk but is not easily verifiable by investors.

2 The Model

2.1 Preferences, Technology, and Information

Time is infinite, continuous, and indexed by t . A risk-neutral agent (the underwriter⁴) originates N mortgages that she wants sell to a risk-neutral principal (the investors) immediately after origination. The underwriter has the constant discount rate of γ and the investors have the constant discount rate of r . We assume $\gamma > r$. This assumption could proxy for a preference for cash or additional investment opportunities of the underwriter (DeMarzo and Duffie 1999).

The underwriter may undertake an action $e \in \{0, 1\}$ at cost ce at the origination of

⁴Although we refer to the agent in our model as the underwriter, our setting applies equally well to other actors than mortgage underwriters. The defining characteristic of the agent in our model is that she can undertake costly hidden action to screen out high risk mortgages or assets

the pool of mortgages ($t = 0$). This action is hidden from the investors and hence not contractable. Each asset generates constant cash flow u until default which occurs according to an exponential random variable with parameter $\lambda \in \{\lambda_H, \lambda_L\}$ such that $\lambda = \lambda_L$ if $e = 1$ and $\lambda = \lambda_H$ if $e = 0$ and $\lambda_H > \lambda_L$. Upon default, all assets pay a lump sum recovery of R . All defaults are mutually independent conditional on effort. Later, when discuss the incentive compatibility constraint, we will show that the assumption that low effort leads to all mortgages defaulting with exponential distribution with parameter λ_H rather than a mixture of exponential distributions is not important.

A contract consists of transfers from the investors to the underwriter depending on mortgage defaults. Specifically, let D_t denote the total number of defaults that have occurred by time t and \mathcal{F}_t the filtration generated by D_t . Formally, contract is an \mathcal{F}_t -measurable process X_t giving the cumulative transfer to the underwriter by time t . We restrict our attention to contracts satisfying the limited liability constraint $dX_t \geq 0$. The underwriter thus has the following utility for a given contract X_t and effort e

$$E \left[\int_0^\infty e^{-\gamma t} dX_t | e \right] - ce.$$

All integrals will be Stieljes integrals. It will also be convenient to define the following sequence of stopping times

$$\tau_n = \inf\{t : D_t \geq n\}.$$

The underwriter must receive at least some promised utility a_0 from the contract.⁵ To guarantee the underwriter receives her promised utility, we must have the following participation constraint constraint

$$a_0 \leq E \left[\int_0^\infty e^{-\gamma t} dX_t | e \right] - ce \tag{1}$$

which states that the expected payment conditional on a given effort e less the cost of effort

⁵Note that a_0 is not necessarily equal to the value the underwriter places on holding the mortgages. a_0 can be greater than the value the underwriter places on holding the assets on portfolio. If the market for underwriters is perfectly competitive, then the underwriter does not receive any of the surplus created by selling the assets, and a_0 is equal to the value the underwriter places on holding the assets on portfolio.

must be greater than or equal to the promised utility of the underwriter.

Given a contract X_t , the agent will choose an effort level which maximizes the expected present value of the the contract net of her effort costs. Formally we have the following incentive compatibility constraint

$$e \in \arg \max_{e \in \{0,1\}} \left\{ E \left[\int_0^\infty e^{-\gamma t} dX_t | e \right] - ce \right\} \quad (2)$$

We say that e is incentive compatible given X_t if e solves (2) given X_t . In order to generate a non-trivial contracting problem, we assume that it is optimal for the investors to provide incentives to the underwriter to choose $e = 1$.

Assumption 1 *There exists a contract X_t such that $e = 1$ is incentive compatible given X_t and*

$$N \frac{u + \lambda_L R}{r + \lambda_L} - E \left[\int_0^\infty e^{-rt} dX_t | 1 \right] \geq N \frac{u + \lambda_H R}{r + \lambda_H} - a_0. \quad (3)$$

The left hand side of inequality (3) is the profit the investors receive if the contract implements high effort and is the difference of two terms. The first term is the present value of the mortgages under high effort with respect to the investors' discount rate. The second term is the cost of a contract that provides incentives to exert effort. Notice that all surplus in the model arises from exploiting the difference in discount rates between the investors and the underwriter. Hence the least costly contract that *does not* provide incentives for high effort simply delivers the underwriter's promised value in a lump sum transfer at time $t = 0$. So that the right hand side of (3) is the difference between the present value of the mortgages under low effort with respect to the investors' discount rate and the promised value of the underwriter.

2.2 Optimal contracts

The investors' problem is to maximize profits subject to providing incentives to expend effort and delivering a certain promised level of utility to the underwriter. Once we restrict our attention to incentive compatible contracts, the value the investors place on holding the

mortgages is fixed since the contract cannot affect the distribution of mortgage cash flows other than to guarantee that the underwriter only originates low risk mortgages. Hence, the investors maximize profits by choosing the incentive compatible contract with the lowest expected cost under their discount rate. In other words, the investor chooses the least costly incentive compatible contract. We state this formally in the following definition.

Definition 1 *Given a promised utility a_0 to the underwriter, a contract X_t is optimal if it solves the following problem*

$$b(a_0) = \min_{dX_t \geq 0} E \left[\int_0^\infty e^{-rt} dX_t | e = 1 \right] \quad (\text{I})$$

such that

$$E \left[\int_0^\infty e^{-\gamma t} dX_t | e = 0 \right] \leq E \left[\int_0^\infty e^{-\gamma t} dX_t | e = 1 \right] - c. \quad (\text{IC})$$

and

$$a_0 \leq E \left[\int_0^\infty e^{-\gamma t} dX_t | e = 1 \right] - c \quad (\text{PC})$$

It is important to note that Definition 1 is equivalent to a definition where we hold the cost to the investor fixed and maximize the utility of the underwriter. As an alternative to Definition 1, we could fix the cost paid by the investor $b(N)$ and find the contract which maximizes the underwriters initial utility a_0 . In the analysis that follows, we will find a one-to-one relationship between the cost paid by the investors and the value delivered to the underwriter; for any level of initial promised utility of the underwriter, we know the cost paid by the investors under the optimal contract and vice versa.

Notice that the assumption that low effort leads to only high risk mortgages is not important for Definition 1. On the one hand, this assumption can easily be relaxed so that low effort leads to a mixture of high risk and low risk mortgages with a proportion ρ of the

mortgages being high risk. In this case constraint (2) would become

$$\rho E \left[\int_0^\infty e^{-\gamma t} dX_t | \lambda = \lambda_H \right] + (1 - \rho) E \left[\int_0^\infty e^{-\gamma t} dX_t | \lambda = \lambda_L \right] \leq E \left[\int_0^\infty e^{-\gamma t} dX_t | \lambda = \lambda_L \right] - c. \quad (4)$$

In light of (4), the optimal contract simply replaces c by $\tilde{c} = c/\rho$ in Definition 1 when we allow for a mixture of risk types if the underwriter applies low effort. On the other hand, it might not be too unrealistic to assume that low effort results in only high risk mortgages for the following reason. If the underwriter does not exert effort low risk borrowers will have to pay the same price for a mortgage as high risk borrowers. Under mild assumptions on competition in the market for underwriters, all low risk borrowers will go to an underwriter who exerts effort and can charge a lower price to low risk borrowers accordingly. Hence, if the underwriter does not exert effort, she will be left with only high risk type borrowers. Both parties are risk neutral and no new information is revealed after the last default, hence it can never be optimal to have transfers from the investors to the underwriter after the last default due to the inefficiency of delaying transfers. We state this result as the following lemma.

Lemma 1 $dX_t = 0$ for all $t \geq \tau_N$.

Proof Follows directly from the fact that no new information is revealed after τ_N and $\gamma > r$. ■

Lemma (1) allows us to restrict our attention to specifying transfers that occur before the last default.

We start by giving a heuristic derivation of the optimal contract when $N = 2$. This base case provides the basic intuition we will use throughout our solution. By Lemma 1, we can restrict our attention to contracts with $dX_t = 0$ for $t \geq \tau_2$. The problem is then to find the least costly (for the investors) payment rule dX_t before and after τ_1 . Intuitively, some payment must be contingent on τ_1 and τ_2 to provide incentives to exert effort. If all payment occurs regardless of the realization of τ_1 and τ_2 , then there is no incentive for the

underwriter to exert effort. Specifically, the contract should reward the underwriter when τ_1 and τ_2 is a relatively larger and punish the underwriter when τ_1 and τ_2 is relatively smaller since τ_1 and τ_2 are more likely to be large when the underwriter exerts effort. At the same time, the optimal contract should completely pay off the underwriter as quickly as possible to exploit the difference in discount rates of the investor and underwriter.

Given the intuition above, it is clear that the optimal contract will balance providing incentives with front loading payment. To that end, suppose we only consider contracts of the form $dX_t = 0$ for $t \neq t_0$ and $dX_{t_0} = \mathbb{I}((\tau_1, \tau_2) \in A)y_0$ for some $t_0 \geq 0$ and set of events A chosen such that X_t is \mathcal{F}_t -measurable.⁶ Further suppose that both the participation and incentive compatibility constraints bind. We then have the following two equations

$$e^{-\gamma t_0} y_0 P((\tau_1, \tau_2) \in A | e = 1) = a_0 + c \quad (5)$$

$$e^{-\gamma t_0} y_0 P((\tau_1, \tau_2) \in A | e = 0) = a_0 \quad (6)$$

where $P(\cdot | e)$ denotes probability conditional on effort. Such a contract will cost the investors

$$e^{-r t_0} y_0 P((\tau_1, \tau_2) \in A | e = 1) = e^{-(r-\gamma)t_0} (a_0 + c) \quad (7)$$

which is increasing in t_0 and independent of A . This implies that the optimal contract of this form will choose the set A to minimize t_0 subject to satisfying equations (5) and (6). To do so, we choose A such that $\mathbb{I}((\tau_1, \tau_2) \in A)$ depends *only* on τ_1 since any dependence on τ_2 necessarily increases t_0 . Moreover, we guess that A should take on as simple structure as possible. So let $A = \{\tau_1 \geq t_0\}$, then equations (5) and (6) reduce to the following

$$e^{-(\gamma+\lambda_L)t_0} y_0 = a_0 + c \quad (8)$$

$$e^{-(\gamma+\lambda_H)t_0} y_0 = a_0. \quad (9)$$

We can easily solve (8) and (9) for t_0 and y_0 . This heuristic argument has transformed, subject to verification, a contracting problem that requires optimization over a very general

⁶ $\mathbb{I}(\cdot)$ is the indicator function.

space of stochastic processes to a simple problem of solving two equations for two unknowns. We formally state the optimal contract in the following proposition.

Proposition 1 *An optimal contract X_t is given by*

$$dX_t = \begin{cases} 0 & \text{if } t \neq t_0 \\ y_0 \mathbb{I}(t_0 \leq \tau_1) & \text{if } t = t_0 \end{cases} \quad (10)$$

where

$$t_0 = \frac{1}{N(\lambda_H - \lambda_L)} \log \left(\frac{a_0 + c}{a_0} \right) \quad (11)$$

$$y_0 = \left(\frac{a_0 + c}{a_0} \right)^{\frac{\gamma + N\lambda_L}{N(\lambda_H - \lambda_L)}} (a_0 + c) \quad (12)$$

Moreover

$$b(a_0) = (a_0 + c) \left(\frac{a_0 + c}{a_0} \right)^{\frac{\gamma - r}{N(\lambda_H - \lambda_L)}} \quad (13)$$

Proof Sketch For complete proof see the Appendix. After verifying that the proposed contract is indeed incentive compatible, the essential part of this proof comes down to finding the following inequality that any incentive compatible contract X must satisfy

$$\frac{1}{a_0 + c} E \left[\int_0^\infty t e^{-\gamma t} dX_t | e = 1 \right] \geq t_0, \quad (14)$$

which puts a lower bound on the risk adjusted duration of any incentive compatible contract. Once we have this inequality in hand, we can use the convexity of the exponential function to put a lower bound on the cost of an incentive compatible contract. We can then show that the proposed contract achieves this lower bound and hence must be optimal.

■

The intuition behind the proof of Proposition 1 is the following. On the one hand, delaying payment is costly due to the difference in discount rates of the underwriter and investors. On the other hand, accelerating payment decreases the sensitivity of payments to the underwriter's choice of effort. These two forces imply that the optimal contract should

feature the most accelerated payment structure that preserves the minimum sensitivity to effort choice needed to provide incentives. Inequality (14) summarizes this point by showing that the minimum duration (with respect to the risk adjusted discount rate) of any incentive compatible contract is exactly t_0 , which turns out to be the duration of the optimal contract. Hence, the proof of Proposition 1 shows that optimal contracting problem we consider comes down to a duration minimization problem.

In order to find the minimum-duration contract we note that we can always improve on an arbitrary incentive compatible contract by delaying payment that occurs before t_0 and accelerating payment that occurs after t_0 until all payment occurs at t_0 . Doing so reduces the duration of the contract. Eventually we will have all payment occurring at time t_0 . The resulting contract is the optimal contract stated in Proposition 1.

The contract calls for no transfers from the investors to the underwriter to take place until the time t_0 given by equation (11). If the first default time $\tau_1 \geq t_0$ then the contract calls for a payment of y_0 given by equation (12) at time t_0 . Equation (11) is the product of two terms. The first term is the inverse of the difference between the arrival intensity of τ_1 given low effort and the arrival intensity of t_0 given high effort. The second term is the difference in the logs of the present value (gross of the cost of effort) of the contract from high effort and low effort respectively. Thus, t_0 is set so that the expected present value of a transfer of y_0 at t_0 under the underwriter's discount rate is exactly equal to $a_0 + c$ under high effort, and a_0 under low effort.

The cost of the contract to the investors is given by $b(a_0)$ in equation (13) and is the product of two terms. The first term is the expected present value of transfers under the discount rate of the underwriter. The second term adjusts this expected present value to the discount rate of the investors.

The classic principal-agent intuition applies equally well to our dynamic setting. The investors act as if they are performing inference on the action of the underwriter given the history of default events. Consider the following likelihood ratio

$$\mathcal{L}(t) = \frac{P(e = 0 | \tau_1 > t)}{P(e = 1 | \tau_1 > t)} = \frac{e^{-N\lambda_H t}}{e^{-N\lambda_L t}} = e^{-N(\lambda_H - \lambda_L)t}.$$

Transfer from the investors to the underwriter takes place when this likelihood ratio reaches a lower threshold, i.e., it occurs when the investor can be reasonably confident that the true default intensity is λ_L .

Recall that we must impose Assumption 1 to guarantee that is optimal for the investor to provide incentives to exert effort. We can now alter that assumption to contain specific parameter restrictions as follows.

Assumption 2 *The parameters of the model satisfy*

$$N \frac{u + \lambda_L R}{r + \lambda_L} - \left(\frac{a_0(N) + c}{a_0(N)} \right)^{\frac{\gamma-r}{N(\lambda_H - \lambda_L)}} (a_0(N) + c) \geq N \frac{u + \lambda_H R}{r + \lambda_H} - a_0. \quad (15)$$

The left hand side of inequality (15) is the profit the investors receive if the contract implements high effort and is optimal and is the difference of two terms. The first term is the present value of the mortgages under the investors discount rate. The second term is the cost of the optimal contract. The right hand side of (15) is the same as (3). If the parameters of the model violate Assumption (2), then it is not optimal for the investors to provide an incentive compatible contract. When no asymmetric information exists, high effort is efficient from the standpoint of the investors if

$$N \frac{u + \lambda_L R}{r + \lambda_L} - c \geq N \frac{u + \lambda_H R}{r + \lambda_H}$$

therefore when

$$\left(\frac{a_0(N) + c}{a_0(N)} \right)^{\frac{\gamma-r}{N(\lambda_H - \lambda_L)}} (a_0(N) + c) - a_0 > N \frac{u + \lambda_L R}{r + \lambda_L} - N \frac{u + \lambda_H R}{r + \lambda_H} > c$$

effort will be undersupplied relative to first best.

2.3 Implementation

In this section we show how to implement the optimal contract using a credit default swap and a bond.

Proposition 2 *Let $A_0 = (a_0 + c)e^{(\gamma+n\lambda_L)t_0}$. The optimal contract can be implemented as follows:*

- *At time $t = 0$ the underwriter transfers the assets to the investors along with a credit default swap (CDS) on the time of first default of the pool of assets with a notional amount of $\frac{A_0(r+\lambda_L)}{u}$ and an expiration date of t_0 .*
- *At time $t = 0$ the investors pays A_0 to the underwriter in the form of a risk free bond that pays a continuous coupon rA_0 which flows to the investors. The bond is used a capital requirement for the CDS contract and cannot be liquidated before expiration.*

Proof The proposed implementation yields the same net transfers as the optimal contract, so they are equivalent.

■

The implementation of the optimal contract given in Proposition 2 yields the following interpretation. The investors would like to pay the underwriter for the pool at time zero to exploit the difference in discount rates, but doing so would fail to provide incentives to the underwriter to expend effort. In order to commit to high effort the underwriter sells insurance to the investors on the pool value at a fair price given high effort. Due to the limited liability constraint, the underwriter cannot commit to having enough capital to cover the insurance if a default occurs before expiration. To overcome this problem the proceeds from the sale of the bundled assets and insurance are placed in the risk free bond, the balance of which is transferred to the underwriter at the expiration of the CDS.

2.4 The benefits of pooling

One important feature of MBS is the process of “pooling” and “tranching.” In this process, an issuer of an MBS first pools together many mortgages to form a collateral pool and then issues two derivative securities (tranches) on the collateral. This security design contrasts with individual loan sale in which an issuer simply sells each loan separately. Individual loan sale means that the transfers corresponding to the sale of one loan cannot effect the transfers corresponding to the sale of another. Hence, in the context of our model, individual loan

sales correspond to a contract which is the sum of N individual contracts, each of which depends on only one mortgage. We can denote such a contract X_t as

$$dX_t = \sum_{n=1}^N dX_t(n),$$

where $X_t(n)$ is measurable with respect to the filtration generated by the n th mortgage cash flows. Since each mortgage is independent and identically distributed after the underwriter chooses effort, it is natural to only consider individual loan sale contracts of the form $dX(n)_t = dX(m)_t$ for all n, m which implies that X_t is measurable with respect to the filtration generated by D_t . Thus, individual loan sale contracts are contained in the contract space we consider in the derivation of the optimal contract. This fact leads us to state the following important corollary to Proposition 2.

Corollary 1 *Pooling mortgages and bundling with a CDS contract dominates individual loan sale.*

Notice that Corollary 1 does not depend on the number of mortgages, N , in the pool. Moreover, the benefits of pooling do not arise from risk diversification benefits. On the contrary, the choice of effort induces cross-sectional correlation among the defaults. This correlation means that the optimal contract increases efficiency over any alternatives by making all transfers contingent on the earliest signal that provides information about the overall quality of the pool. Other results in the literature, for example DeMarzo (2005), attribute pooling and tranching to minimizing mispricing through reducing the sensitivity of offered securities to private information. The so called *risk diversification* effect of pooling dominates the *information destructive* effect of pooling. In addition, DeMarzo (2005) can only generate pooling and tranching when the number of assets is large enough and diversification benefits exist. In our model, pooling is a consequence of providing incentives in the least costly manner. By pooling the mortgages, the optimal contract maximizes the correlation of mortgage risk types, which in turn allows the investors to speed up payment to underwriter while maintaining incentive compatibility. Corollary 1 may seem to be an

artifact of the all-or-nothing effort technology of the current setting. In Section 3.2 we show that Propositions 1 and 2, and hence Corollary 1, are robust to a more flexible effort technology.

2.5 The promised value of the agent and the number of mortgages in the pool

We now shift our focus to the promised value for the agent and suppose that capital markets are perfectly competitive so that investors make zero profits as follows

$$\underbrace{(a_0 + c) \left(\frac{a_0 + c}{a_0} \right)^{\frac{\gamma-r}{N(\lambda_H - \lambda_L)}}}_{\text{Cost of Contract}} = \underbrace{N \frac{u + \lambda_L R}{r + \lambda_L}}_{\text{Present Value of Assets}} . \quad (16)$$

Equation (16) pins down the contract in terms of the parameters $N, c, u, \gamma, r, \lambda_L$, and λ_H , however it does not admit a closed form solution for a_0 in terms of c .⁷ We can use Equation (16) to explore how the promised value of the underwriter a_0 depends on the number of mortgages N and the cost of effort c . Although we cannot solve explicitly for a_0 , we numerically solve Equation (16) for a particular set of parameters. We can see that when effort costs are convex, as in Figure 1, the promised value may be a *concave* function of the number of assets in the pool. In fact, in this particular example, there exists a finite maximum for a_0 which means there exists a finite optimal pool size when effort costs are convex, although the choice of N is outside the model. Additionally, a_0 is maximized for a larger N than when the underwriter holds the assets in portfolio which suggests that when effort costs are convex, the underwriter will form larger assets pools when preparing assets for securitization than when planning to hold assets on portfolio. This effect is driven by the fact that the investors are relatively more patient and hence value the mortgages more than the underwriter so that the cost of effort makes up a relatively smaller portion of the value of the mortgages to the investor than the underwriter. In contrast, as is shown in Figure 2, a_0 is strictly increasing when effort costs exhibit increasing returns to scale.

⁷There is a unique solution to (16) once we require a_0 to be above some positive minimum value.

2.6 Comparing the optimal contract to a standard equity contract

It is interesting to ask is how closely we can approximate the optimal contract using an alternative, and perhaps more standard, contract. To answer this question, we compare the optimal contract to a contract in which the underwriter retains an portion of the pool of mortgages and receives a lump sum transfer at time $t = 0$. For the remainder of this subsection, we assume the lump sum recovery upon default $R = 0$ for simplicity. This assumption is easily relaxed and plays no role in the analysis. Note that the total cash flow from the pool of mortgages at time t is $u(N - D_t)$. Hence a contract which calls for the underwriter to receive a time zero cash payment of K and retain a portion α of the pool of mortgages must take the following form

$$dX_t = \begin{cases} \alpha u(N - D_t)dt + K & t = 0 \\ \alpha u(N - D_t)dt & t \geq 0 \end{cases}. \quad (17)$$

It will also be useful to compute the expected present value of the contract under the underwriter's discount rate given high effort and low effort.

$$\begin{aligned} E \left[\int_0^\infty e^{-\gamma t} dX_t | e = 1 \right] &= K + N\alpha \frac{u}{\gamma + \lambda_L} \\ E \left[\int_0^\infty e^{-\gamma t} dX_t | e = 0 \right] &= K + N\alpha \frac{u}{\gamma + \lambda_H} \end{aligned}$$

Intuitively, an optimal contract of the form given in (17) must make both the participation constraint and the incentive compatibility constraint bind. Hence we have the following system of equations

$$\begin{aligned} a_0 + c &= K + \alpha N \frac{u}{\gamma + \lambda_L} \\ a_0 &= K + \alpha N \frac{u}{\gamma + \lambda_H}. \end{aligned}$$

Which we can solve to get

$$\alpha = \frac{c(\gamma + \lambda_H)(\gamma + \lambda_L)}{Nu(\lambda_H - \lambda_L)}$$

$$K = a_0 - \frac{c(\gamma + \lambda_L)}{(\lambda_H - \lambda_L)}.$$

The cost of providing such a contract, which we denote by $b^E(a_0)$, will then be

$$b^E(a_0) = a_0 - \frac{c(\gamma + \lambda_L)}{(\lambda_H - \lambda_L)} + \frac{c(\gamma + \lambda_H)(\gamma + \lambda_L)}{(r + \lambda_L)(\lambda_H - \lambda_L)}.$$

We can compare the cost of this contract to the cost of the optimal contract to evaluate how closely it approximates the optimal contract. For simplicity we assume that the market for underwriters is perfectly competitive so that all surplus from securitization flows to the investors. We define $\phi(N)$ to be the loss of efficiency from using an equity contract versus the optimal contract in terms of percentage of costs paid by the investors as follows

$$\phi(N) = \frac{b^E(a_0) - b(a_0)}{b(a_0)}.$$

Figure 3 plots $\phi(N)$ and the extra cost paid by the investors $b^E(a_0) - b(a_0)$ by using an equity contract. As the number of mortgages in the pool increases, using an equity like contract becomes less efficient, ranging for 1.5% for a one mortgage pool to almost 3% for a 100 mortgage pool. In percentage terms this may seem like a small loss in efficiency. However, in dollar terms this can amount to a large dollar amount of loss in inefficiency.

Another important consequence of restricting the contract space to equity-like contracts is that in doing so we must place an additional restriction on the parameters of the model to guarantee that providing incentives for high effort is optimal. There is a larger range of parameters for which providing incentives to exert effort is optimal under the optimal contract of Proposition 1 than under an equity-like contract.

3 Extensions

3.1 An Initial Capital Constraint

In this section we consider the optimal security design problem when the underwriter faces an initial capital constraint. This case arises when the underwriter does not have sufficient initial wealth to originate the mortgages. For certain parameter restrictions, the structure of the contract remains largely unchanged, except for the addition of a transfer at $t = 0$.

Suppose the underwriter requires K in initial capital at $t = 0$ to originate the mortgages. Specifically we add the following constraint

$$dX_0 \geq K \tag{18}$$

to Definition 1. The following proposition states the solution to the optimal contracting problem.

Proposition 3 *Suppose Assumption ?? holds. The optimal contract X_t that satisfies constraint (18) is given by*

$$dX_t = \begin{cases} K & \text{for } t = 0 \\ 0 & \text{for } 0 < t < t_0 \text{ and } t_0 < t \\ \mathbb{I}(\tau_1 \geq t_0)e^{(\gamma+N\lambda_L)t_0}(a_0 + c - K) & \text{for } t = t_0 \end{cases}$$

where

$$t_0 = \frac{1}{N(\lambda_H - \lambda_L)} \log \left(\frac{a_0 + c - K}{a_0 - K} \right).$$

Proof See appendix ■

The intuition behind Proposition 3 is essentially the same as for Proposition 1. After providing the initial required capital K , the contract makes all subsequent transfers dependent on the realization of the first default time to provide incentives to the underwriter to exert effort. For this incentive scheme to work, the underwriter must have some promised utility left after receiving the initial capital K , which we guarantee by imposing Assumption ?. Once a long enough period has passed before the first default time, it is optimal

to completely pay off the underwriter due to the difference in discount rate between the investors and the underwriter.

3.2 Partial Effort

The underwriter is endowed with an all-or-nothing effort technology in our model. In other words, the agent must choose a single effort level for each mortgage. Alternatively, we can consider a specification in which the underwriter can apply effort to some and not all of the mortgages. The optimal contract remains unchanged when we allow for such a deviation given a reasonable restriction on the cost of effort c detailed below. Specifically, suppose the underwriter can choose to apply effort to $n \leq N$ of the mortgages resulting in a pool of N mortgages in which n mortgages default according to an exponential distribution with parameter λ_L , and $N - n$ mortgages default according to an exponential distribution with parameter λ_H . We will refer to such a strategy as applying partial effort. We alter notation to let a partial effort strategy be denoted $e = n$ so that $e = 0$ corresponds to zero effort and $e = N$ corresponds to full effort. The cost of applying a partial effort strategy is given by c . We modify Definition 1 to include incentive compatibility constraints for each possible partial effort strategy as follows

Definition 2 *Given a promised utility a_0 to the underwriter, a contract $\{x_n\}_{n=0}^N$ that implements $e = n$ is optimal if it solves the following problem*

$$b(a_0) = \min_{dX_t \geq 0} E \left[\int_0^\infty e^{-\gamma t} dX_t | e = n \right] \quad (19)$$

such that

$$E \left[\int_0^\infty e^{-\gamma t} dX_t | e = m \right] - c(m) \leq E \left[\int_0^\infty e^{-\gamma t} dX_t | e = n \right] - c(n) \quad \text{for all } m \leq N \quad (20)$$

and

$$a_0 \leq E \left[\int_0^\infty e^{-\gamma t} dX_t | e = n \right] - c(n). \quad (21)$$

Definition 2 characterizes the least costly contract that implements each level of effort. In other words, for each possible level of effort $e = n$, Definition 2 calls a contract optimal over all contracts which implement $e = n$ if it is the least costly contract for the investor for which $e = n$ is an incentive compatible response.

Before stating the solution to problem (19)-(21) we impose the following assumption.

Assumption 3 *The cost function $c(\cdot)$ satisfies $nc(m) \leq mc(n)$ for all m, n such that $n \leq m$.*

Note that Assumption 3 is not overly restrictive as it includes all concave cost functions. The solution is given in the following proposition.

Proposition 4 *Suppose Assumption 3 holds. The optimal contract that implements full effort is that of Proposition 1.*

Proof See appendix ■

Proposition 4 relies on the fact that under the contract detailed in Proposition 1 and the flexible effort technology, the underwriter does not gain more by deviating to a strategy in which she applies effort to some and not all of the mortgages than a strategy in which she applies zero effort. This implies that the original optimal contract satisfies the additional incentive compatibility constraints ruling out partial effort deviations since it satisfies the original incentive compatibility constraint ruling out the zero effort strategy. Moreover, the space of contracts that satisfy (20) is contained in the set of contracts which satisfy (IC) since constraint (IC) is contained in (20). Since the contract of Proposition 1 satisfies the additional constraints induced by partial effort strategies and is optimal over the larger space of contracts satisfying constraint (20), it is optimal over the smaller space of contracts satisfying the additional constraints.

Note that in Proposition 4 we are careful to limit ourselves to characterizing the optimal contract that implements full effort. In doing so, we stopped short of stating that the *investors* cannot be made better off by choosing a contract which only provides incentives for partial effort. To make such a statement, we need to characterize the optimal contract

that implements effort $e = n$, and compare the profit from that contract to the profit from the contract providing incentives to exert full effort. We have the following proposition.

Proposition 5 *Suppose Assumption 3 holds. An optimal contract which implements $e = n$ is given by*

$$dX_t = \begin{cases} 0 & \text{if } t \neq t_0 \\ y_0 \mathbb{I}(t_0 \leq \tau_1) & \text{if } t = t_0 \end{cases}$$

where

$$t_0 = \frac{1}{(N-n)\lambda_H - n\lambda_L} \log \left(\frac{a_0 + c(n)}{a_0} \right)$$

$$y_0 = \left(\frac{a_0 - c(n)}{a_0} \right)^{\frac{\gamma + n\lambda_L + (N-n)\lambda_H}{(N-n)\lambda_H - n\lambda_L}} (a_0 + c(n)).$$

Moreover

$$b_0(a(n)) = \left(\frac{a_0 + c(n)}{a_0} \right)^{\frac{\gamma - r}{n\lambda_H - (N-n)\lambda_L}} (a_0 + c(n)).$$

Proof Essentially the same as the proof for Proposition 4. ■

The contract detailed in Proposition 5 is of the same form as the contract of Proposition 1. However, this contract implements partial effort. Hence, the contract is given by t_0 and y_0 such that constraints (20) and (21) bind. Importantly, the structure of the contract remains qualitatively unchanged so that we may implement the optimal contract using a CDS contract and a bond as in Proposition 2 and pooling and bundling with a CDS remains optimal.

Proposition 5 allows us to state the following corollary.

Corollary 2 *Suppose*

$$N \frac{u + \lambda_L R}{r + \lambda_L} - (a_0 + c(N)) \left(\frac{a_0 + c(N)}{a_0} \right)^{\frac{\gamma - r}{N(\lambda_H - \lambda_L)}}$$

$$\geq n \frac{u + \lambda_L R}{r + \lambda_L} + (N - n) \frac{u + \lambda_H R}{r + \lambda_H} - \left(\frac{a_0 + c(n)}{a_0} \right)^{\frac{\gamma - r}{n\lambda_H - (N-n)\lambda_L}} (a_0 + c(N))$$

for all $n < N$, then it is optimal to implement full effort.

Corollary 2 gives specific parameter restrictions under which it is optimal for the investor to implement full effort. Specifically, the expected profit from implementing full effort must be weakly greater than the profit from implementing partial effort.

3.3 Adverse selection

Throughout the above analysis, we have focused on a moral hazard setting in which the underwriter makes a hidden effort choice that affects the risk of the mortgages she sells to the investors. In this section, we show how our model can be altered to address an adverse selection problem in which the underwriter is endowed with mortgages with a given default risk and wishes to sell them to secondary market investors. This setting is similar to other papers in the literature, notably DeMarzo (2005). The main result is that the optimal contract for an underwriter with low risk mortgages remains qualitatively unchanged.

In a standard adverse selection model of asset backed security design, the issuer, in our case the underwriter, has private information about the assets she wishes to sell. We model this by assuming that the underwriter has N mortgages to sell, all of which are either low risk or high risk where mortgage cash flows are given in section 2.1. We look for a separating equilibrium in which the underwriter can signal the quality of her mortgages by choosing a contract. Specifically, we look for a pair of contracts (X^H, X^L) such that an underwriter with high risk mortgages chooses the contract X^H , and an underwriter with low risk mortgages chooses the contract X^L . As such, we can view the choice of contract as the signal by the agent. In a static framework, such as that of DeMarzo (2005), the signal space is limited to the portion of the equity tranche the underwriter chooses to retain. In our setting, the signal space is much richer since it includes any payoff profile through that is adapted to the information filtration generated by the cumulative default process of mortgages.

We look for separating pair that is least costly for the investors. In principal, the contracting costs associated with X^H could affect the contracting costs associated with X^L and vice versa, we either rule this out by definition for simplicity.

The first constraint we must place on contracts is the following individual rationality

constraint

$$N \frac{u + \lambda_i R}{\gamma + \lambda_i} \leq E \left[\int_0^\infty e^{-\gamma t} dX_t^i | \lambda_i \right] \quad \text{for } i \in \{H, L\},$$

which states that an underwriter with mortgages of type i must weakly prefer the contract X^i to holding the mortgages on portfolio.

The next constraint is an incentive compatibility constraint for an underwriter with high risk mortgages

$$E \left[\int_0^\infty e^{-\gamma t} dX_t^H | \lambda_H \right] \geq E \left[\int_0^\infty e^{-\gamma t} dX_t^L | \lambda_H \right],$$

which states that an underwriter with high risk mortgages must weakly prefer the contract X^H to X^L .

The final constraint is an incentive compatibility constraint for an underwriter with low risk mortgages

$$E \left[\int_0^\infty e^{-\gamma t} dX_t^L | \lambda_L \right] \geq E \left[\int_0^\infty e^{-\gamma t} dX_t^H | \lambda_L \right],$$

which states that an underwriter with high risk mortgages must weakly prefer the contract X^H to X^L .

We assume the structure of the market is such that the underwriter offers a security design and the investors choose to either purchase the mortgages or not. With these constraints in hand, we can state the definition of an optimal contract pair.

Definition 3 *A contract pair X^H, X^L is optimal if it solves the following problem*

$$b(N) = \min_{dX_t^H, dX_t^L \geq 0} \left\{ \mathbb{I}(X = X^H) E \left[\int_0^\infty e^{-rt} dX_t^H | \lambda_H \right] + \mathbb{I}(X = X^L) E \left[\int_0^\infty e^{-rt} dX_t^L | \lambda_H \right] \right\} \quad (22)$$

such that

$$N \frac{u + \lambda_H R}{\gamma + \lambda_H} \leq E \left[\int_0^\infty e^{-\gamma t} dX_t^H | \lambda_L \right] \quad (23)$$

$$N \frac{u + \lambda_L R}{\gamma + \lambda_L} \leq E \left[\int_0^\infty e^{-\gamma t} dX_t^L | \lambda_L \right] \quad (24)$$

$$E \left[\int_0^\infty e^{-\gamma t} dX_t^H | \lambda_H \right] \geq E \left[\int_0^\infty e^{-\gamma t} dX_t^L | \lambda_H \right] \quad (25)$$

$$E \left[\int_0^\infty e^{-\gamma t} dX_t^L | \lambda_L \right] \geq E \left[\int_0^\infty e^{-\gamma t} dX_t^H | \lambda_L \right]. \quad (26)$$

Clearly it is optimal to set $dX_0^H = N \frac{u + \lambda_H R}{\gamma + \lambda_H}$ and $dX_t^H = 0$ for $t > 0$, since by choosing to offer X^H as a security design the underwriter signal that she has high risk mortgages. Accordingly, Definition 3 is equivalent to Definition 1 and we have the following proposition.

Proposition 6 *An optimal contract pair is given by $dX_0^H = N \frac{u + \lambda_H R}{\gamma + \lambda_H}$ and $dX_t^H = 0$ for $t > 0$, and*

$$dX_t^L = \begin{cases} 0 & \text{if } t \neq t_0 \\ y_0 \mathbb{I}(t_0 \leq \tau_1) & \text{if } t = t_0 \end{cases}$$

where

$$\begin{aligned} a_0 &= N \frac{u + \lambda_L R}{\gamma + \lambda_L} \\ t_0 &= \frac{1}{N(\lambda_H - \lambda_L)} \log \left(\frac{a_0 + c(N)}{a_0} \right) \\ y_0 &= \left(\frac{a_0 + c(N)}{a_0} \right)^{\frac{\gamma + N\lambda_L}{N(\lambda_H - \lambda_L)}} (a_0 + c(N)). \end{aligned}$$

Proof Follows directly from Proposition 1 ■

4 Conclusion

This paper studies a model of mortgage securitization in a moral hazard setting with dynamic cash flows. We find that the optimal contract in our most simple setup is a lump sum payment from the investors to the underwriter conditional on a period of no defaults.

We give a simple implementation of the contract using a risk free bond and a CDS. By bundling a CDS with the mortgage pool, the underwriter can in effect signal high effort by selling insurance which is priced fairly conditional on high effort.

We also consider three important extensions to the basic model. The main result is the nature that our optimal contract is robust to altering the contracting problem in plausible ways.

The first extension introduces an initial capital constraint that arises due to the underwriter lacking sufficient initial capital to originate the mortgages. Up to an important restriction on the parameters of the model, the qualitative features of the contract remain unchanged after introducing this constraint. The only significant difference being a time zero transfer from the investors to the underwriter exactly equal to the capital required to originate the mortgages. After time zero, the optimal contract is identical to the optimal contract without the initial capital constraint except for the magnitude and timing of the one time lump sum transfer. For such a contract to be optimal, we require that the promised value of the underwriter be greater than the capital required to underwrite the mortgages. This restriction guarantees that the underwriter will have sufficient continuation value for the contract after date zero to provide incentives.

The next extension we consider is a flexible effort technology. It is possible that the underwriter could apply costly underwriting practices to some and not all of the mortgages. For concave costs, our contract is robust to such an effort technology and remains optimal.

Finally, we show how our result can be adapted to an adverse selection setting, with the main result that the optimal contract is unchanged.

A Appendix - Proofs

Proof of Proposition 1 Let X_t^* denote the proposed contract. First observe that X_t^* satisfies constraints (PC) since

$$\begin{aligned} E \left[\int_0^\infty e^{-\gamma t} dX_t^* | e = 1 \right] &= E[e^{-\gamma t_0} \mathbb{I}(t_0 \leq \tau_1) e^{(\gamma + N\lambda_L)t_0} (a_0 + c) | e = 1] \\ &= P(t_0 \leq \tau_1 | e = 1) e^{N\lambda_L t_0} (a_0 + c) \\ &= a_0 + c. \end{aligned}$$

Next observe that the proposed contract satisfies constraint (IC) since

$$\begin{aligned} E \left[\int_0^\infty e^{-\gamma t} dX_t^* | e = 0 \right] &= E[e^{-\gamma t_0} \mathbb{I}(t_0 \leq \tau_1) e^{(\gamma + N\lambda_L)t_0} (a_0 + c) | e = 0] \\ &= P(t_0 \leq \tau_1 | e = 0) e^{N\lambda_L t_0} (a_0 + c) \\ &= e^{-N(\lambda_H - \lambda_L)t_0} (a_0 + c) = a_0. \end{aligned}$$

Now suppose X_t is an arbitrary incentive compatible contract. Let $X_t^0 = X_t \mathbb{I}(t \leq \tau_1)$. Note that $X_t^0 : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is not a random variable, rather it a function of time which denotes the payment at time t given no defaults have yet occurred. We have

$$\begin{aligned} E \left[\int_0^{\tau_1} e^{-\gamma t} dX_t^0 | e = 0 \right] &= \int_0^\infty e^{-(\gamma + N\lambda_H)t} dX_t^0 \\ &= \int_0^\infty e^{N(\lambda_L - \lambda_H)t} e^{-(\gamma + N\lambda_L)t} dX_t^0 \\ &\geq e^{-N(\lambda_H - \lambda_L)t_0} \int_0^\infty (N(\lambda_H - \lambda_L)(t_0 - t) + 1) e^{-(\gamma + N\lambda_L)t} dX_t^0 \\ &= \frac{a_0}{a_0 + c} E \left[\int_0^{\tau_1} (N(\lambda_H - \lambda_L)(t_0 - t) + 1) e^{-\gamma t} dX_t^0 | e = 1 \right] \quad (27) \end{aligned}$$

where the second to last step follows from the fact that $e^{-N(\lambda_H - \lambda_L)t} \geq e^{-N(\lambda_H - \lambda_L)t_0} (N(\lambda_H - \lambda_L)(t_0 - t) + 1)$ since $e^{-N(\lambda_H - \lambda_L)t}$ is convex.

Now let $X_t^n(\mathbf{s}) = X_t \mathbb{I}(s_1 = \tau_1, \dots, s_n = \tau_n, t \leq \tau_{n+1})$ for $n = 1, \dots, N - 1$. Note again that $X_t^n(\mathbf{s}) : \mathbb{R}^{n+1} \times [s_n, \infty) \rightarrow \mathbb{R}^+$ is not a random variable, rather it is a function of time

and the vector \mathbf{s} which denotes the payment at time t given the first n defaults occurred at times given by the vector \mathbf{s} . Let $\mathcal{A} = \{\mathbf{s} \in \mathbb{R}^{n+1} | s_1 \leq s_2 \leq \dots \leq s_n\}$ and $d\mathcal{A} = ds_n \dots ds_1$.

We have

$$\begin{aligned}
& E \left[\int_{\tau_n}^{\tau_{n+1}} e^{-\gamma t} dX_t^1(\tau_1, \dots, \tau_n) | e = 0 \right] \\
&= \frac{\lambda_H^n N!}{(N-n)!} \int_{\mathcal{A}} \int_{s_n}^{\infty} \exp \left(-(\gamma + (N-n)\lambda_H)t - \sum_{k=1}^n \lambda_H s_k \right) dX_t^1(\mathbf{s}) d\mathcal{A} \\
&= \frac{\lambda_H^n N!}{(N-n)!} \int_{\mathcal{A}} \int_{s_n}^{\infty} \exp \left(-(\gamma + N\lambda_H)t + \sum_{k=1}^n \lambda_H(t - s_k) \right) dX_t^1(\mathbf{s}) d\mathcal{A} \\
&\geq \frac{\lambda_L^n N!}{(N-n)!} \int_{\mathcal{A}} \int_{s_n}^{\infty} \exp \left(-(\gamma + N\lambda_H)t + \sum_{k=1}^n \lambda_H(t - s_k) \right) dX_t^1(\mathbf{s}) d\mathcal{A} \\
&\geq \frac{\lambda_L^n N!}{(N-n)!} \int_{\mathcal{A}} \int_{s_n}^{\infty} \exp \left(-(\gamma + N\lambda_H)t + \sum_{k=1}^n \lambda_L(t - s_k) \right) dX_t^1(\mathbf{s}) d\mathcal{A} \\
&\geq e^{-N(\lambda_H - \lambda_L)t_0} \frac{\lambda_L^n N!}{(N-n)!} \times \\
&\int_{\mathcal{A}} \int_{s_n}^{\infty} (N(\lambda_H - \lambda_L)(t_0 - t) + 1) \exp \left(-(\gamma + N\lambda_L)t - \sum_{k=1}^n \lambda_L(s_k - t) \right) dX_t^1(\mathbf{s}) d\mathcal{A} \\
&= \frac{a_0}{a_0 + c} E \left[\int_{\tau_n}^{\tau_{n+1}} (N(\lambda_H - \lambda_L)(t_0 - t) + 1) e^{-\gamma t} dX_t^1(\tau_1, \dots, \tau_n) | e = 0 \right] \tag{28}
\end{aligned}$$

where the second to last step again follows from the fact that $e^{-N(\lambda_H - \lambda_L)t}$ is convex.

Note that by construction we have

$$X_t = \sum_{n=0}^{N-1} X_t^n \mathbb{I}(\tau_n < t \leq \tau_{n+1}),$$

hence inequalities (27) and (28) imply

$$E \left[\int_0^{\infty} e^{-\gamma t} dX_t | e = 0 \right] \geq \frac{a_0}{a_0 + c} E \left[\int_0^{\infty} (N(\lambda_H - \lambda_L)(t_0 - t) + 1) e^{-\gamma t} dX_t | e = 1 \right]. \tag{29}$$

Combining constraints (IC) and (PC) with inequality (29) we are left with the following inequality

$$\frac{1}{a_0 + c} E \left[\int_0^{\infty} t e^{-\gamma t} dX_t | e = 1 \right] \geq t_0 \tag{30}$$

Now consider the cost of this contract to the investors. Using a similar argument as above and the fact that $e^{-(\gamma-r)t}$ is convex we have

$$E \left[\int_0^\infty e^{-rt} dX_t | e = 1 \right] \geq e^{(\gamma-r)t_0} \left[E \left[\int_0^\infty (\gamma-r)(t-t_0)e^{-\gamma t} dX_t | e = 1 \right] + E \left[\int_0^\infty e^{-\gamma t} dX_t | e = 1 \right] \right]. \quad (31)$$

But inequality (30) implies that the first term on the right hand side of inequality (31) is greater than zero, which together with constraint (PC) implies

$$E \left[\int_0^\infty e^{-rt} dX_t | e = 1 \right] \geq e^{(\gamma-r)t_0} (a_0 + c).$$

But $e^{-(\gamma-r)t_0}(a_0 + c)$ is the cost to the investors of the proposed contract. So we have shown that the proposed contract costs less (or the same) to the investors than any alternative contract that satisfies (IC) and (PC), hence the proposed contract is optimal.

■

Proof of Proposition 3 We can rewrite the optimal contracting problem as follows

$$b(a_0) = \min_{dX_t \geq 0} \left\{ K + E \left[\int_0^\infty e^{-rt} d\hat{X}_t | e = 1 \right] \right\}$$

such that

$$\begin{aligned} K + E \left[\int_0^\infty e^{-\gamma t} d\hat{X}_t | e = 0 \right] &\leq K + E \left[\int_0^\infty e^{-\gamma t} d\hat{X}_t | e = 1 \right] - c, \\ a_0 &\leq K + E \left[\int_0^\infty e^{-\gamma t} d\hat{X}_t | e = 1 \right] - c, \\ d\hat{X}_t &= dX_t - \mathbb{I}(t=0)K \\ dX_0 &\geq K \end{aligned}$$

Since $a_0 > K$, the solution follows directly from Proposition 1.

■

Proof of Proposition 4 First we show that most profitable deviation for the underwriter under the contract of Proposition 1 is to exert no effort. Consider the utility the

underwriter receives from deviating to a strategy in which she applies effort to $n < N$ of the mortgages

$$\begin{aligned}
& P(\tau_1 \geq t_0 | e = n) e^{-\gamma t_0} e^{(\gamma + N\lambda_L)t_0} (a_0 + c) - c \\
&= e^{-(n\lambda_L + (N-n)\lambda_H)t_0} e^{(\gamma + N\lambda_L)t_0} (a_0 + c) - c \\
&= e^{-(N-n)(\lambda_H - \lambda_L)t_0} (a_0 + c) - c \\
&= \left(\frac{a_0 + c}{a_0} \right)^{-\frac{N-n}{N}} (a_0 + c) - c \\
&= a_0 \left(\left(\frac{a_0 + c}{a_0} \right)^{\frac{n}{N}} - \frac{c}{a_0} \right) \\
&\leq a_0 \left(\frac{n}{N} \left(\frac{a_0 + c}{a_0} - 1 \right) + 1 - \frac{c}{a_0} \right) \\
&= a_0 + \frac{1}{N} (nc - Nc) \\
&\leq a_0
\end{aligned}$$

This implies that the contract of Proposition 1 satisfies the constraints contained in (20). Now observe that constraint (IC) is contained in (20) and note that the objective functions of equation (I) and (19) are the same. Hence the contract of Proposition 1 minimizes the objective function of equation (19) over a set of weaker constraints and satisfies the set of stronger constraints, hence it minimize (19) over the set of stronger constraints.

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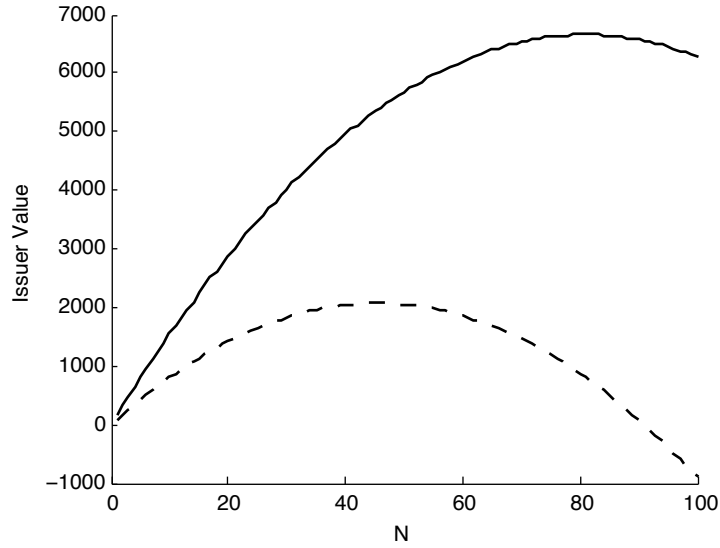


Figure 1: $a(N)$ versus the underwriter's value of holding the assets when effort exhibits decreasing returns to scale. The dashed curve is the underwriter's value for holding the assets in portfolio and the solid curve is her promised value from selling the assets. Parameter values: $u = 10, L = .01, H = .03, r = .05, \gamma = .10$ and $c = N^2$

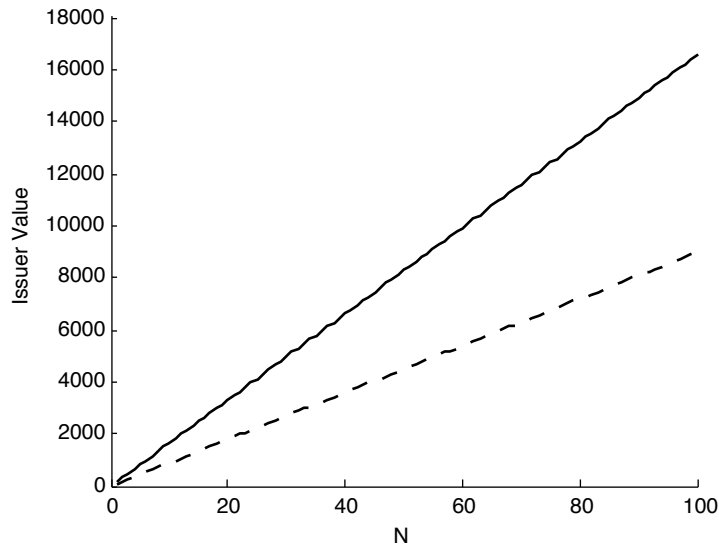


Figure 2: $a(N)$ versus the underwriter's value of holding the assets when effort exhibits increasing returns to scale. The dashed line is the underwriter's value for holding the assets in portfolio and the solid line is her promised value from selling the assets. Parameter values: $u = 10, L = .01, H = .03, r = .05, \gamma = .10$ and $c = 10 + N$

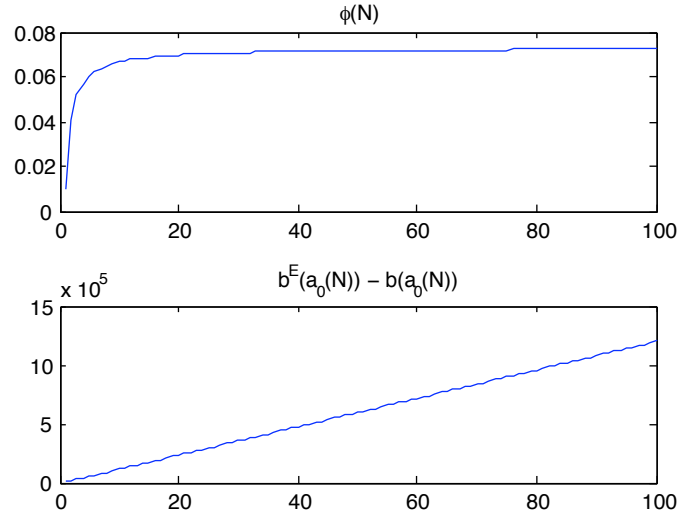


Figure 3: $\phi(N)$ and $b^E(a_0(N)) - b(a_0(N))$ versus N . Parameter values: $u = 10,000$, $L = .1\%$, $H = .05\%$, $r = .05$, $\gamma = .06$ and $c(N) = 500 * N$