

# Financing Risk and Innovation Waves\*

Ramana Nanda  
Harvard University  
Boston MA

Matthew Rhodes-Kropf  
Harvard University  
Boston MA

October 2009  
Preliminary Draft

## Abstract

The great waves of innovation are often associated with investment in startup firms. The inherent uncertainty in these firms leads investors to stage investments which causes investors to face a unstudied type of risk. We introduce the importance of *financing risk* and show how it can both cause and amplify waves of innovation in the real economy. Financing risk occurs when investors with limited resources must rely on future investors to fund a project at later stages. The project NPV then depends not only on the fundamentals of the project, but also on a given investor's belief about other investors' willingness to fund the project at later stages. When the risk that future investors will not fund the project becomes high, then like in a bank run, current investors flip to an equilibrium in which no one invests. Financing risk is particularly costly for innovative projects with substantial real option value, where the financing constraint is not easily overcome by a large investment *ex ante*. Therefore, the most innovative projects in the economy are particularly vulnerable to waves of investment activity. Our model provides a fundamental rationale for bubbles of economic activity in new sectors that does not rely on mispriced assets.

---

\*Nanda: Harvard University, Rock Center, Boston, Massachusetts 02163 Phone: 617-496-2783. Email: rnanda@hbs.edu. Rhodes-Kropf: Harvard University, Rock Center 313, Boston, Massachusetts 02163. Phone: (617) 496-3911. Email: mrhodeskropf@hbs.edu, respectively. All errors are our own.

*“The fundamental impulse that sets and keeps the capitalist engine in motion comes from the new consumers’ goods, the new methods of production or transportation, the new markets, the new forms of industrial organization that capitalist enterprise creates.”* Schumpeter (1942)

## Introduction

The technological revolutions that lead to Schumpeter’s (1942) waves of creative destruction are a fundamental driver of productivity growth in the economy (Aghion and Howitt 1992; King and Levine 1993). Startup firms are a central part of these technological revolutions. For example, in the last century the introduction of motor cars, semiconductors and computers, the internet, biotechnology, and clean technology have all been associated with ‘startup’ firms. Entrepreneurs and small firms are thus central to the process of creative destruction and a key aspect of the innovation cycle is the ability of new ideas to get financing in their infancy.

A natural consequence of the inherent uncertainty in innovative projects is the need for investors to stage their investments - i.e., provide limited capital to the firm in each round, and learn more about the project’s potential before providing more financing.<sup>1</sup> However, the act of staging investments introduces a risk faced by such investors that finance theory has not previously considered. Investors with limited capital must forecast the probability that their investment may not find follow-on funding from other investors at its next stage, even if the fundamentals of the project at the next stage are sound.

In this paper we introduce the notion of *financing risk* and explore its implications for the innovation cycle. We emphasize that financing risk is part of a rational equilibrium and all investors in our model will use an  $NPV \geq 0$  investing rule. Even with this standard rule we show that spikes in investing activity are more common in the innovative sectors of the economy. These spikes in investment by financiers can both cause or magnify waves of innovation in the real economy by impacting new firm formation, the resources for R&D and patenting, the timing of IPOs and so forth.

The intuition behind financing risk is relatively straight forward: when a project requires multiple periods of investment that are cumulatively more than any individual investor has (or

---

<sup>1</sup>see Gompers (1995), Bergemann and Hege (2005), Bergemann et al. (2008).

is willing to allocate), current investors need to rely on future investors to continue funding the project in order to realize the value of their investment. We document that there is a class of projects where the extent to which a given investment is positive or negative NPV depends not only on the fundamentals of the project (that is, a high enough probability of a realizing a good outcome), but also on the current investors' beliefs that future investors will choose to participate in follow on funding for the project. For these projects, financing risk is the risk that future investors will not fund the project at its next stage *even if the fundamentals of the project have not changed*.

To any one investor, financing risk is exogenous; however, in equilibrium it becomes endogenous. Each investor becomes less willing to make an investment because they are worried that others won't support the investment in the future. Thus, like in a bank run, if current investors believe that future investors will withdraw financing from such a project, they should also withdraw their investment, even though all investors would be better off in the equilibrium in which everyone invests. This is not an irrational decision and furthermore, does not depend on information asymmetries. There are simply two equilibria – one in which everyone invests in a sector and one in which no one does.

An important facet of this model is that each equilibrium is inherently unstable as it depends on the beliefs of others. Even when investors are in the 'good' financing equilibrium, investors realize that there is a potential to jump to the other equilibrium. In fact, financing risk is precisely the risk that the 'good equilibrium' switches after a given investor has funded a project but before returns can be realized. Investors thus estimate a transition probability that the state switches from the 'good' to the 'bad' financing equilibrium or vice versa.

The literature on venture capital has documented the extreme variation in venture capital investment (Gompers and Lerner (2004)) and fundraising (Gompers and Lerner (1998)), that are correlated with high market values, hot IPO markets or past returns (Kaplan and Schoar (2005)). And furthermore, technological revolutions seem to be associated with 'hot' financial markets (Perez (2002)). Prior work has suggested that these correlations could be overreaction by investors (Gompers and Lerner (1998)), rational reactions to fundamentals (Gompers et al. (2008), Pastor and Veronesi (2009)), herd behavior for reputational concerns (Scharfstein and Stein (1990)) or even reverse causality (Hobijn and Jovanovic (2001)).

Our model provides a natural mechanism for the extreme variation in venture capital markets and links ‘hot’ and ‘cold’ financial markets to waves of innovation in the real economy. It also provides an explanation for bubbles of investing activity that does not need to depend on mispriced assets. In ‘good times’, when financing risk is low, project NPVs and thus asset prices should be high, as investors impute low financing risk into higher prices. In bad times the opposite is true. In our model, this boom and bust activity does not imply an irrational asset pricing bubble. Rather, the boom bust cycle is the inevitable outcome of multiple potential equilibria. We suggest that what may look like (or be partially driven by) over or under reaction or even a reaction to changing fundamentals may instead be a jump from a high investing to a low investing equilibrium.

Accounting for financing risk in the innovation cycle provides several novel insights and implications that do not arise from previous models of innovation waves. The first implication of our model is that financing risk has the greatest impact on the most innovative projects in the economy, or ones that have the highest real option value for investors. Assuming that it is costly to withdraw money from a project once it is invested (e.g. Guedj and Scharfstein (2006)), the ideal financing strategy for very innovative projects is to provide frequent funding – thereby providing the projects with the required capital if things go well and retaining the real option to terminate the project if interim results are not promising (see Bergemann et al. (2008), Bergemann and Hege (2005) , Gompers (1995)). Thus, innovative projects are frequently raising money and hence are exposed far more to financing risk. Less innovative projects are not as impacted by financing risk as a larger ex ante investment can function as a commitment device to keep funding such projects through a low funding equilibrium. For less innovative projects, increasing funding increases the project NPV and can even eliminate financing risk completely. However, projects with high real option value benefit little from the large ex ante investment, as this destroys the option to terminate the project if interim results are bad. In fact, when financing risk is high, our model suggests that the mix of projects that get funded should shift towards less innovative ones and that conditional on being funded in a time of high financing risk, these less innovative projects should receive larger ex-ante investments (relative to their burn rate) as compared to times when financing risk is low.

The second implication of this model is that the mix of investors should change in periods

of high financing risk, relative to periods of low financing risk. Early round investors of very innovative projects are subject to a greater amount of financing risk as the chance that the state switches to a ‘bad’ equilibrium after they have invested but before a liquidity event is realized, is higher. Their investing activity should be particularly impacted by hot and cold financial markets. Our model also predicts that the mix of investors should shift towards smaller investors (with less capital to deploy) when financing risk is low, as the smaller and more frequent investments in periods of low financing risk are particularly well suited to smaller investors.

A third implication of accounting for financing risk is that any given investor should not rush to invest into *all* projects in a sector that is out of favor. Conventional wisdom suggests that when money leaves a sector it is a good time to invest, and when a lot of money enters it is just the time to leave. This intuition arises because the flood of money lowers the discipline of external finance and allows lower quality projects to get capital (Gompers and Lerner (2000); Nanda (2008)). However, accounting for financing risk makes it clear that investors cannot rush to invest into all projects in a sector that is out of favor. In particular, innovative projects have a low probability of receiving future funding and become NPV negative once financing risk is taken into account. We should note that it is still true in our model that on average, ‘better’ projects are funded during a ‘bad’ funding equilibrium. This occurs because only the very best projects can attract financing even in bad times and hence are positive NPV even in the low funding equilibrium. However, it also shows why fundamentally sound projects, particularly those with high real option value, can go unfunded in some periods but be funded in others.

The fourth implication of our model is that some extremely novel but NPV positive technologies or projects may in fact *need* ‘hot’ financial markets to get through the initial period of diffusion, because otherwise the financing risk for them is too extreme. This provides a more positive interpretation to waves of financial activity and may also explain the historical link between the initial diffusion of many very novel technologies (e.g. canals, railways, telephones, motor cars, internet, clean technology) being associated with heated financial market activity (Perez (2002)). Related to this, our model also provides a non-behavioral explanation for why asset prices can fall precipitously after rising steadily for long periods, even when the fundamentals of a firm have not changed (Pastor and Veronesi (2009)). If a sector stays in the ‘good’ equilibrium longer than expected or if the expected probability of remaining in the ‘good’ equi-

librium increases, then asset prices will rise and returns will be high, even if the fundamentals remain similar. When the ‘bad’ equilibrium eventually occurs, returns will be far lower than that predicted simply by looking at fundamentals since the low funding equilibrium implies a fall in NPV and hence asset prices.

Our final implication relates to direct measures of innovation such as patenting that occur in great waves of activity (see Griliches (1990)). There are many explanations for why innovative output might cluster in certain periods of time even though we expect ideas to occur at random.<sup>2</sup> While these traditional explanations clearly have merit, combining our model of financing risk with the direct evidence from Kortum and Lerner (2000) on the link between financial market activity and patent suggest that financial markets may play an much larger and under-studied role in the creation and magnification of innovation waves.

While this paper is primarily focused on outlining a theory of financing risk and the set of testable implications arising from this concept, we do provide some descriptive evidence that is consistent with our model. There is much future work to be done to comprehensively study the link between financing risk, investment activity and innovation in the real economy.

The remainder of the paper is organized as follows. Section I. outlines a simple model of investing and illuminates the existence of the two potential investing equilibria. Section II. expands the model to a general equilibrium and shows how accounting for the transition probabilities from one equilibrium to the other affects the funding strategy of investors. Section III. allows complete, state contingent contracts and commitment among investors in an attempt to overcome financing risk, and shows why in a world of incomplete contracts, it is the innovative projects in the economy that are most impacted by financing risk. Section IV. summarizes the key implications and extensions of our model and Section V. concludes.

## I. A Model of Investment

The central goal of our model is to delineate the impact of financing risk on investment decisions. Financing risk is the risk that future investors will not fund a firm at its next stage *even if the*

---

<sup>2</sup>The classical explanations focus on sudden breakthroughs that lead to a cascade of follow-on inventions (e.g. Schumpeter (1939); Kuznets (1940); Kleinknecht (1987); Stein (1997)) or on changes in sales and profitability (or potential profitability) that stimulate investment in R&D and then drive concentrated periods of innovation (e.g. Schmookler (1966)). See Stoneman (1979) for a discussion of the supply versus demand considerations.

*fundamentals of the project have not changed*, leading a viable firm with good fundamentals to go bankrupt. We emphasize that financing risk is part of a rational equilibrium and show why innovative projects are particularly susceptible to financing risk.

## A. Setup

We model a single early stage project inside a broader economy. For simplicity, we equate this project with a firm. By early stage we aim to capture the fact that the firm does not have the cash flows to be self sufficient and hence requires outside investment to survive. The burn rate for the firm is assumed to be  $\$x$ , so that it requires an investment of at least  $\$x$  in each period in order not to go bankrupt and continue on to the next period. A second aspect of early stage firms is that they are not guaranteed to viable in the future. Intermediate results might show that its technology, processes or target market is not viable. Thus, in each period the firm continues to be viable with probability  $\gamma < 1$ .

We model two idealized types of investors in this economy, which we call venture capitalists (VCs) and public investors. Consistent with the view that VCs are thought to have a number of skills relating to the finding and nurturing new companies (Hsu (2004); Kaplan et al. (2009)), we assume that VCs focus on investing in early stage firms. VCs can determine if an early stage firm is viable or not. If the project is not viable, the VCs do not invest and the firm goes bankrupt. Conditional on it being viable, VCs choose whether or not to invest the  $\$x$  to take it through to the next period.<sup>3</sup> If they choose to make an investment, the cash infusion and nurturing by the VCs increases the firm's value by  $V_t$  in period  $t$ . Since a firm of infinite value makes little economic sense, we assume  $\sum_{k=1}^{t \rightarrow \infty} V_k < \bar{V}$ , where  $\bar{V}$  is some constant less than infinity and can be thought of as the potential of the project.<sup>4</sup>

The second class of investors are public investors, who are willing to purchase the firm (and hence generate liquidity for the VCs), but only if the project is still viable and is also sufficiently well-developed so as not to require the presence of VCs.<sup>5</sup> Since the early stage firm is not

---

<sup>3</sup>Initially, we assume that each VC is constrained and has only  $\$x$  to invest, but there are enough VCs such that each VC earns only the required rate of return,  $r$ , which for simplicity is set to zero.

<sup>4</sup>On average, the marginal value of each new investment is decreasing. But in any particular period value may increase significantly more than in the following period.

<sup>5</sup>Since the value created by the VCs is steadily decreasing, selling the project to public investors and spending time on a new project is a natural transition and should occur before  $V_t \leq x$  although we do not model the

*guaranteed* to reach a liquidity event in any given period, we model this uncertainty with the notion that public investors only arrive each period with probability  $(1 - \alpha)$ , where  $\alpha$  is the probability that the firm will require further VC involvement before public investors are willing to purchase it.

If public investors arrive, they pay  $\sum_{k=1}^t V_k$  for the firm in period  $t$ . However, we allow for the price offered by public investors to vary according to their bargaining power in period  $n$ . Specifically, when financial markets are ‘cold’, so that the firm has no alternative funding source (because it cannot rely on the VCs for continued investment in order to remain viable), public investors only pay  $\lambda \sum_{k=1}^t V_k$ .  $\lambda < 1$  represents the reduced bargaining power firms have when they are out of money and have no alternative source of financing.

The timing of the model is as follows. In each period, the project is viable with probability  $\gamma$  and receives an offer from public investors with probability  $(1 - \alpha)$ . If the project is not viable, it goes bankrupt. If the project is viable, VCs decide whether or not to invest  $\$x$ . We hypothesize (and later confirm) that there are two equilibria - one in which VCs choose to fund a viable project and one when they do not. Thus a project that is viable in any given period faces 4 possible outcomes. If the firm receives a purchase offer from public investors *and* an offer to invest from a VC, then they sell for the full value that has been created up to that point,  $\sum_{k=1}^t V_k$ . If the firm only receives an offer from public investors, but no investment from the VCs then it sells for the fraction  $\lambda \sum_{k=1}^t V_k$  of its full value. If the firm does not receive an offer from public investors, but VCs choose to invest, it accepts the investment of  $\$x$  and again is viable with probability  $\gamma$  and receives an offer from public investors with probability  $(1 - \alpha)$  in the following period. If the firm is viable but receives no investment from either the public investors or the VCs, it goes bankrupt and for simplicity, is assumed to have a liquidation value of 0.

To summarize, a project is therefore defined by (i) its vector of value increments in each period subject to an investment,  $\{V\}$ , (ii) its burn rate of  $\$x$ , (iii) the probability it is a viable project in the next period,  $\gamma$ , (iv) the probability that conditional on an investment, it is sufficiently well developed in the next period to attract public investors,  $(1 - \alpha)$ , and (v) the reduction in value,  $\lambda$ , if public investors arrive in a ‘cold’ investing market. We will show that as each of

---

optimal time to sell and instead assume VCs sell whenever the opportunity presents.

these facets of a project change, the impact on financing risk will differ.<sup>6</sup>

## B. Multiple Equilibria

In this section, we prove our hypothesis that there are two equilibria - one in which VCs choose to fund a viable project and one when they do not invest, even when the project is viable. We will refer to these as the ‘Invest’ and the ‘No-Invest’ equilibria. Specifically, we hypothesize – and later confirm – that VCs will choose to invest if and only if they are in the ‘Invest’ equilibrium and will not invest if and only if they are in the ‘No-Invest’ equilibrium.

To see this is the case, first note from the section above that the offer made by the public investors to buy the firm depends on whether the VCs would be willing to invest (support the project) in that period or not. The offer function of the public investors,  $O_t(I)$ , is thus:

$$O_t(I) = I * \sum_{k=1}^t V_k + (1 - I) * \lambda \sum_{k=1}^t V_k \quad (1)$$

where  $I$  is an indicator that equals 1 if the VCs are in the Invest equilibrium, and 0 if they are in the No-Invest equilibrium.

Since VCs invest in a given period on the expectation that they can liquidate their investment in a future period without making a loss, the expected profit for a VC from investing in period  $t$ ,  $\Pi_t$  is also a function of the offer function from the public investors. The expected profit for a VC from investing in period  $t$ ,  $\Pi_t$  is:

$$\Pi_t = \gamma [(1 - \alpha)O_t(I) + \alpha I \Pi_{t+1}] - x \quad (2)$$

That is, there is a  $\gamma$  chance that the project will still be viable in the next period and hence generate a return. Conditional on being viable, there is a  $(1 - \alpha)$  chance that public investors will make an offer,  $O_t(I)$ . If the project is viable, there is also an  $\alpha$  probability that the public investors will not make an offer to buy the firm. In this scenario, a ‘No-Invest’ equilibrium will imply no investment from the VCs and will lead to a 0 value as the firm will go bankrupt. If the investors are in the Invest equilibrium,  $I = 1$ , then the VCs will invest \$x in the firm and it will

---

<sup>6</sup>For simplicity x,  $\gamma$ ,  $\lambda$  and  $\alpha$  are assumed to be the same in each period for a given project.

be worth  $\Pi_{t+1}$  next period.<sup>7</sup> Hence in equation (2),  $\Pi_t$  is the net present value of the project. A VC therefore invests in the firm if this NPV is positive.

We emphasize that the decision rule of every investor is a simple NPV rule. What we will see is that the project may become NPV negative if future investors will not invest.

Solving iteratively (see appendix A.i.) we can write the expected profit from investing in period  $t$  as:

$$\Pi_t = \frac{(1-\alpha)\gamma(I+(1-I)\lambda)}{(1-\alpha\gamma I)} \left( \sum_{k=1}^t V_k + \sum_{k=1}^{\infty} (\alpha\gamma I)^k V_{k+t} \right) - \frac{x}{(1-\alpha\gamma I)}. \quad (3)$$

This leads directly to our understanding that there are two equilibria<sup>8</sup>

**Proposition 1** *There are some projects  $\{V, x, \gamma, \lambda, \alpha\}$  for which there are two possible equilibria - one in which the VCs invest (and they believe other VCs will invest) and another in which VCs do not invest (and they believe other VCs will not invest).*

**Proof.** The VCs invest if

$$\Pi_t = \frac{(1-\alpha)\gamma(I+(1-I)\lambda)}{(1-\alpha\gamma I)} \left( \sum_{k=1}^t V_k + \sum_{k=1}^{\infty} (\alpha\gamma I)^k V_{k+t} \right) - \frac{x}{(1-\alpha\gamma I)} > 0. \quad (4)$$

If the VC believes that other VCs will invest then the project NPV in period  $t$  becomes

$$\Pi_t|_{I=1} = \frac{(1-\alpha)\gamma}{(1-\alpha\gamma)} \left( \sum_{k=1}^t V_k + \sum_{k=1}^{\infty} (\alpha\gamma)^k V_{k+t} \right) - \frac{x}{(1-\alpha\gamma)}. \quad (5)$$

And if the VC believes that other VCs will not invest then the project NPV in period  $t$  becomes

$$\Pi_t|_{I=0} = (1-\alpha)\gamma\lambda \sum_{k=1}^t V_k - x. \quad (6)$$

---

<sup>7</sup>If investors do not invest,  $I = 0$ , then even though the project may be worth something in period  $t + 1$  the investor in period  $t$  gets no value because the project is bankrupt. We must confirm in equilibrium that no investor invests and the firm does indeed go bankrupt.

<sup>8</sup>When we say equilibria we mean pure strategy equilibria as mix strategy equilibria have no economic meaning here since the investor must decide to invest or not.

So, VCs who believe other VCs will not invest will not invest themselves in period  $t$  if

$$(1 - \alpha)\gamma\lambda \sum_{k=1}^t V_k < x. \quad (7)$$

While a VC who believes that other VCs will invest will invest themselves in period  $t$  if

$$(1 - \alpha)\gamma \left( \sum_{k=1}^t V_k + \sum_{k=1}^{\infty} (\alpha\gamma)^k V_{k+t} \right) \geq x. \quad (8)$$

However, it is only rational for each VC to believe that future VCs either will or will not invest if equations (7) and (8) hold for the next period. And in the future period it is again only rational if equations (7) and (8) hold for the period after that. Thus, for both equilibria to be rational equations (7) and (8) must hold for all future periods.

Since  $\alpha\gamma \leq 1$ , then  $\sum_{k=1}^t V_k + \sum_{k=1}^{\infty} (\alpha\gamma)^k V_{k+t}$  is an increasing function of  $t$ , and therefore as long as equation (8) holds in one period it holds in all future periods. Furthermore, since  $\sum_{k=1}^{t \rightarrow \infty} V_k < \bar{V}$  then for  $x > (1 - \alpha)\gamma\lambda\bar{V}$  equation (7) also holds for all periods. And it is possible for both to simultaneously hold for all future periods as long as

$$\lambda\bar{V} < \left( \sum_{k=1}^t V_k + \sum_{k=1}^{\infty} (\alpha\gamma)^k V_{k+t} \right), \quad (9)$$

which can be true for small enough  $\lambda$  and, if it is true one period is true for all future periods since the RHS is increasing with  $t$ . Q.E.D. ■

The two equilibria are similar to the dual equilibria in the banking literature where depositors can ‘run’ on a bank as in Diamond and Dybvig (1983). Depositors leave money in the bank unless they believe others will withdraw. Once a depositor believes others will withdraw, the only rational response is to attempt to withdraw first. Depositors are better off in the ‘deposit’ equilibrium, but this equilibrium is inherently unstable, as anything that makes depositors think others will withdraw makes everyone withdraw and makes everyone worse off. Our argument is that when investors must rely on other investors to fund projects, a similar phenomena can occur. That is, if investors believe that other investors will not invest in the firm, then they

themselves will not invest, leading to a self fulfilling equilibria in which everyone is worse off.

The central difference between our model and a bank or currency run model is in the time delay between investor actions. In a bank or currency run model each player is concerned about the current actions of other players and furthermore, simultaneous actions are strategic complements. In our model investors in the future know the actions of investors in the past but are concerned about investors further into the future. Thus, it is uncertainly about when the project will sell or become cash flow positive that cuases each investor to be concerned about the investor in the future.

### C. Can a wealthier investor overcome the ‘No Invest’ equilibrium?

If it is the reliance on other investors that leads to the problem, the question arises as to whether a VC with more money or a syndicate of VCs can overcome the No-Invest equilibrium. In this section we show that is not the case. To see this, consider a VC who has  $\$2x$  instead of just  $\$x$ . One might imagine that this VC faces less financing risk in the first period they invest because they can be sure to invest in the next period. In this case one might think that even in the No-Invest equilibrium the expected value of the project in the first period is

$$\Pi_1 = \gamma [(1 - \alpha)O_1(1) + \alpha\Pi_2] - x, \tag{10}$$

which is the NPV from equation (2) with I set to 1 in period 1, in spite of the No-Invest equilibrium (because of the second  $\$x$  held by the VC). If this were the case, then the NPV in period 1 would be greater for a VC with  $\$2x$  since the purchase offer would be larger if it occurs and the funding is sure to come if no offer arrives.

However, the proof to Proposition 1 demonstrated that the conditions for the No-Investing equilibrium must hold for *all* future periods. Therefore, in the next period when the VC goes to invest their second  $\$x$  they will find that if other VCs are in the No-Invest equilibrium then it is not NPV positive to invest the second  $\$x$ . Given this, the rational response for the VC will be to not invest the second  $\$x$ . Using backward induction, the VC will realize that they will not invest the second  $\$x$  and therefore, will reevaluate their decision to invest the first  $\$x$ . Since the second  $\$x$  will only be invested in the Invest equilibrium, the decision to invest the first  $\$x$

is the same for VCs with either \$x or \$2x. This same backward induction tells us that (in the absence of commitment) no finite amount of money held by one VC or syndicate of VCs can break the No-Invest equilibrium.

We believe that showing this existence of multiple equilibria – that are not overcome purely by syndicates or wealthy investors – is a key contribution of our paper. Our theory suggests that waves of investment activity are self-fulfilling and that hence innovation waves are the inevitable outcome for projects that rely on future investors to continue funding projects in order for them to be NPV positive for initial investors.

In the next section, we examine two important scenarios to see how they impact the financing strategy of VCs. First, what happens if everyone expects the equilibrium to flip from the No-Invest to the Invest equilibrium at some point in the future (or vice versa)? Second, what happens if investors can write complete contracts that commit themselves to continue investing in a firm even if they are in a No-Invest equilibrium in the future? We will turn first to the idea of transition probabilities from state-to-state and then to the idea of commitment. We will see how these additional ideas allow us to establish that financing risk is more important for innovative projects.

## II. Transitions from State to State

An important facet of this model is that each equilibrium is inherently unstable as it depends on the beliefs of others. Given this fact, VCs will also need to forecast the possibility of a jump to the other equilibrium and a jump back when calculating the NPV of their investment. VCs that forecast a possibility of the No-Invest equilibrium will prepare for it. And if a project does not need to survive an infinite No-Invest period, then more money may help prevent the No-Invest equilibrium from affecting the project.

We assume that either cheap talk among VCs or an exogenous signal causes investors to believe that the world is in one equilibrium or the other. Since this common belief becomes self-fulfilling, the equilibrium will jump whenever this exogenous signal occurs. Examples of such signals might relate to a key invention in a sector, future industry growth expectations or alternatively a signal that some other sector is hot and thus money will head there. We think

of these signals as relating to an industry or area of investing such as bio-tech, green-tech, or high-tech but they could also occur at a more or less granular level. For example, we would argue that part of the dramatic decline in venture investing that began in late 2008 is due to an equilibrium that is economy wide in which investors cannot invest because they do not believe others will be there to support the projects. On the other hand, several VCs continued to invest in clean energy projects because of their belief that this sector would continue to be ‘hot’ despite the overall market downturn.<sup>9</sup>

In our model, individual investors see the world as having an exogenous transition probability  $(1 - \theta)$  that an industry or sector shifts from the Invest to the No-Invest equilibrium and a probability  $\phi$  that an industry transitions back to the Invest equilibrium.<sup>10</sup> Using similar techniques as above, we can solve for the NPV of an investment, accounting for the probability that the state may jump and possibly jump back at some future date (see appendix A.ii.).

$$\Pi_t|_{I=1} = \frac{(1 - \alpha)\gamma(\theta + (1 - \theta)\lambda)}{(1 - \alpha\gamma\theta)} \left( \sum_{k=1}^t V_k + \sum_{k=1}^{\infty} (\alpha\gamma\theta)^k V_{k+t} \right) - \frac{x}{(1 - \alpha\gamma\theta)}. \quad (11)$$

$$\Pi_t|_{I=0} = (1 - \alpha)\gamma(\phi + (1 - \phi)\lambda) \sum_{k=1}^t V_k - x + \alpha\gamma\phi\Pi_{t+1}|_{I=1} \quad (12)$$

The potential for the No-Invest state to end improves the value of an investment in the No-Invest state. Thus the following proposition is similar to the first proposition, but must account for the probability that the No-Invest equilibrium might not last forever.

**Proposition 2** *As long as  $\phi$  is small enough and  $\theta$  is large enough, there are some projects  $\{V, x, \gamma, \lambda, \alpha\}$  for which there are two possible equilibria - one in which the VCs invest (and they believe other VCs will invest) and another in which VCs do not invest (and they believe other*

---

<sup>9</sup>One might think that the global games refinement proposed in Carlsson and van Damme (1993), and used in interesting papers such as Morris and Shin (1998), Goldstein and Pauzner (2005) and Goldstein and Pauzner (2004) would be useful here. The refinement results in a unique equilibria given the fundamentals rather than a unique equilibria given a signal. This refinement will not work in a model of investment across time because future investors know the actions of past investors and so there is no sense in which they are concerned about what action they may take or what signal they got. In the global games refinement investors today are concerned about the actions of other investors today because coordinated action can prevent or create the currency crises or bank run. With investment across time investors are concerned about the actions of future investors, so the assumptions required for the global games refinement do not hold.

<sup>10</sup>Economic logic dictates that  $\theta > \phi$  since either equilibrium is more likely to occur in a subsequent period if investors are currently in that equilibrium.

*VCs will not invest*).

**Proof.** See Appendix A.iii. ■

The intuition of the proof is straight forward. If the transition probability,  $\phi$ , is zero, then the No-Invest equilibrium will last forever, and therefore the conditions for the No-Invest equilibrium to be an equilibrium are the same as in Proposition 1. Thus, for  $\phi$  that is  $\epsilon$  greater than zero the No-Invest state is still an equilibrium. Likewise, if the transition probability to the No-Invest state,  $(1 - \theta)$  is zero, then the Invest equilibrium will last forever. Therefore, for a  $(1 - \theta)$  that is  $\epsilon$  greater than zero, the Invest state is still an equilibrium. The equilibrium actions eventually break down as  $\phi$  and  $(1 - \theta)$  become large enough, because a high enough probability of a transition to a state effectively causes participants to behave as if the state occurs today.

It would seem that with a probability that the No-Invest period ends, a wealthier VC or a syndicate of VCs could now overcome the No-Invest equilibrium for some projects. However, we show again that this is not the case. To see this, note that the very last \$x that the VC has will only be spent if the equilibrium has jumped back to the Invest equilibrium. The VC knows this in the period before the last period and also knows that if the industry is still in the No-Invest equilibrium in the period just before this last period, this means that  $\phi$  is not large enough to make investing the second to last \$x a good idea (as if it were large enough, it would cause VCs to behave as if the state occurs today and hence have caused the equilibrium to flip). Therefore, in the period just before this last period, the VC understands that the last \$x will only be spent if the equilibrium changes. So the second to last \$x is not invested either. Continuing this backward induction eventually brings us back to the first \$x. Thus (in the absence of commitment), neither an investor with more money nor a syndicate can break the No-Invest equilibrium. However, we will see the importance of commitment in the next section.

### III. The Benefit and Cost of Commitment

Increasing the dollars held by one investor or forming a syndicate does not help the company get over the No-Invest equilibrium because in each period the investment decision is made rationally and so a syndicate or even one investor with more money makes no decision differently than the market. After all, sunk costs are sunk. Therefore, if the market is rationally in the No-Invest

equilibrium, then any investor would make the same decision as the market.

However, we show that commitment to invest through a No-Invest equilibrium can change this result. We now allow an investor to commit to invest in the next period regardless of the equilibrium established by other investors. This increases the offer the firm will get in a sale during the No-Invest equilibrium from  $\lambda \sum_{k=1}^t V_k$  to  $\hat{\lambda} \sum_{k=1}^t V_k$  where  $\hat{\lambda} > \lambda$  due to the increase in bargaining power provided by the funding cushion. For simplicity we will assume  $\hat{\lambda} = 1$ .<sup>11</sup>

Initially we will also assume that contracts are complete and that there are no information asymmetries – so that the investor who has committed to invest in the second period does not invest if the project turns unviable (probability  $(1 - \gamma)$ ), but will invest if the project is viable and the equilibrium has jumped to the No-Invest equilibrium. Alternatively, an equivalent contract is a state contingent contract where investors give a project \$2x or more in a period and the project commits to return any unused funds if the project becomes unviable but not if the state transitions to the No-Invest equilibrium.

Commitment trades off the potential increase in sale price with the potential loss from having to invest during the bad equilibrium. If an investor only invests a single \$x then we know from above that the expected project NPV is

$$\Pi_t|_{I=1} = (1 - \alpha)\gamma(\theta + (1 - \theta)\lambda) \sum_{k=1}^t V_k + \alpha\gamma\theta\Pi_{t+1}|_{I=1} - x \quad (13)$$

If instead an investor or syndicate commits to invest in both the first and the second period the expected project NPV is

$$\Pi_t|_{I=1}^{n=2} = (1 - \alpha)\gamma \sum_{k=1}^t V_k + \alpha\gamma (\theta\Pi_{t+1}|_{I=1} + (1 - \theta)\Pi_{t+1}|_{I=0}) - x, \quad (14)$$

where the  $n = 2$  signifies the investment commitment for two periods. This equation differs from equation (13) in two ways. First, there is no loss in bargaining power if the state transitions because funding is certain.<sup>12</sup> Second, the investor has agreed to provide financing in the bad

---

<sup>11</sup> $\hat{\lambda}$  might be less than one if offers made during the No-Invest equilibrium are still lower than offers in the Invest equilibrium even to companies that are well funded.

<sup>12</sup>No loss of bargaining power is assumed for convince and all results hold if bargaining power is lower if the state changes but not as low as with no funding.

state. Therefore, if the project doesn't sell and the bad state occurs, the investor makes an expected loss since  $\Pi_{t+1}|_{I=0} < 0$ .

Thus, the question of whether it is better to commit to a second \$x is a question of whether profits with commitment are bigger than profits without. Subtracting the two profit equations, the question is reduced to whether

$$(1 - \alpha)(1 - \lambda) \sum_{k=1}^t V_k + \alpha \Pi_{t+1}|_{I=0} > 0 ? \quad (15)$$

The gain from committing a second \$x is the higher purchase price if a buyer arrives but the state has transitioned. The loss from committing a second \$x is the NPV from investing if no buyer arrives but the state has transitioned,  $\Pi_{t+1}|_{I=0}$ . If the state has not transitioned there is no gain or loss because the commitment did not matter.

The following proposition shows the impact of this trade-off.

**Proposition 3** *If investors or syndicates can commit to invest in future periods and contracts are complete then there are some projects  $\{V, x, \gamma, \lambda, \alpha\}$  for which -*

1. *Commitment increases the project NPV.*
2. *If the commitment of a single extra \$x increases the NPV of a project in every period, then committing more dollars increases the project NPV by more- i.e., the change in project NPV is always an increasing function of the dollars committed to the project.*
3. *If the commitment of an extra \$x increase the NPV of a project in every period, then there are some projects for which only the Invest equilibrium is an equilibrium, i.e. they no longer suffer from financing risk.*

**Proof.** See Appendix A.iv. ■

For many projects, commitment increases the NPV of the project. This is because providing more funding gives a project more bargaining power during a sale. The down side is that a committed investor must invest even if the state jumps to the No-Invest equilibrium. Thus, for many projects, providing the company with an extra \$x of funding increases the NPV.

Interestingly, if the investor or syndicate commits even more (i.e., to a third  $\$x$ ) then the investment of the second  $\$x$  is not as bad as it would have been without the third  $\$x$  because the third  $\$x$  increases the sale price in the second period even if the state has changed. So, if it always makes sense in each period to commit an extra  $\$x$ , then it makes sense to commit all the money up front.

Therefore, large investors and syndicates can actually increase the NPV of the projects they fund by giving them more dollars or implicitly or explicitly committing to fund them for longer.

Furthermore, for some projects, enough committed dollars make the project NPV positive even in the No-Invest state. That is, if enough investors join together, then a large enough investment in the bad state becomes NPV positive. For these projects, the only equilibrium is the Invest equilibrium and commitment eliminates financing risk.

The logic above would seem to suggest that all projects should get significant up front funding. However, as noted above, we have so far assumed that an investor or syndicate that commits to fund a project can withdraw funding if the project becomes unviable, i.e. the commitment only relates to the state of the world and not to the project.

The analogous venture capital contract is a tranching investment, in which the investors have committed to fund a project if certain milestones are reached. These type of contracts provide the investor with a real option, but we believe they are also an attempt to overcome financing risk as they commit the investor to invest if the company has done well even if the world has done poorly. However, they rarely cover more than one future financing, and for many projects (particularly innovative ones), it very difficult to articulate and delineate a clear milestone. Thus, it is unrealistic to assume that complete state-contingent contracts can be written for all future funding dates at the start of a project. The next section explores the trade-offs under the more realistic scenario of incomplete contracts.

## **A. Incomplete Contracts and the Lost Real Option**

Complete contracts are unrealistic as investors cannot contract on every future funding need at the start of a project. In this section, we assume that contracts are incomplete (a la Grossman and Hart (1986); Hart and Moore (1990)). We assume that it is not possible to either write down or verify all future states in which funding should or should not occur. For example, it

might be the case that states of nature are observable by the investors but not verifiable by a court. Specifically we define an incomplete contract as follows.

**Definition 1** *In an incomplete contract, investors cannot contract on actions that differ between the No-Invest equilibrium,  $I = 0$  and project becoming unviable, (Prob  $(1 - \gamma)$ ).*

We still assume that it is costly for investors to renege on a commitment. Since one way to ‘commit’ to future funding is to provide extra funding today, the assumption that it is costly to renege on a commitment is the same as assuming that it is costly to shut down a project and return any unspent capital to investors. For simplicity we assume it is never optimal for the investor to fail to fund a contract. Effectively, this is the same as assuming commitment is enforceable.

Commitment was enforceable in the last section as well, but now, without complete contracts, project CEOs and investors are not able to write contracts that release the investor or return capital when bad project specific information arrives. We assume the CEO gets private benefits of control and is therefore biased toward continuation. Therefore, incomplete contracts create a world in which all dollars given or committed to a project are spent no matter what information arrives.<sup>13</sup> If all the money given to a project will be spent, then giving more money to a project destroys some of the value of the project’s real option to shut down in the event that intermediate information is not positive. On the other hand, more money better-protects a firm from the No-Invest equilibrium. Thus, it is those projects with more valuable real options for which protection from the No-Invest equilibrium is more costly.

In our model the real option value in a project depends on the probability that a project loses viability before it is sold. If  $\gamma = 1$  the project is always viable and there is no real option value. However, for lower values of  $\gamma$  it becomes valuable to give the project less up front funding (smaller commitment) and wait to learn that it is still a viable project in the next period.

We can see the effect of incomplete contracts and real options on the profitability of committing extra dollars to a project. In section III., when we assumed complete contracts, the profit

---

<sup>13</sup>The main tradeoff is the same if only a fraction of the committed dollars would be spent after the arrival of bad news.

from committing to invest an extra \$x was

$$\Pi_t|_{I=1}^{n=2} = (1 - \alpha)\gamma \sum_{k=1}^t V_k + \alpha\gamma (\theta\Pi_{t+1}|_{I=1} + (1 - \theta)\Pi_{t+1}|_{I=0}) - x. \quad (16)$$

With complete contracts if the project lost viability the investor would not lose the second committed \$x. Now, however, committing \$2x requires the investor to lose the second \$x if the project fails (i.e., it will be spent by the CEO). Thus, the expected profit from committing \$2x becomes

$$\Pi_t|_{I=1}^{n=2} = (1 - \alpha)\gamma \sum_{k=1}^t V_k + \alpha\gamma (\theta\Pi_{t+1}|_{I=1} + (1 - \theta)\Pi_{t+1}|_{I=0}) + \gamma x - 2x. \quad (17)$$

Compared to a profit function without the second committed \$x we see that there is an additional cost to commitment. This leads directly to our final proposition

**Proposition 4** *Incomplete contracts reduce the value of committing more money and the reduction in value is larger for more innovative projects (projects with more real option value).*

**Proof.** See Appendix A.v. ■

The central insight comes from comparing the profit equations with and without complete contracts (equations (16) and (17)). Note that if  $\gamma = 1$  there is no real option value and no difference between the two profit functions. With  $\gamma = 1$  there is no chance the project will fail so commitment only effects the No-Invest state of the world. Thus, just like in the last section, with  $\gamma = 1$  commitment trades off the cost of investing during the No-Invest equilibrium with the potential increase in sale price from doing so. However, the smaller  $\gamma$  becomes the more valuable it becomes to give the project only one period of funding to see if it fails. Thus, commitment becomes more and more costly.

Investors who give a project enough funding to get over the No-Invest equilibria lose the option to give the project a little funding and wait to see how it performs to give it more. Therefore, it is more costly to overcome the No-Invest equilibrium for innovative projects with high real option value. The less innovative projects can be given a larger amount of up-front financing in order to avoid the No-Invest equilibrium. But the innovative projects cannot be given significant funding up front or the loss of the real options may change it to an NPV negative

project. Therefore, more innovative projects should receive less funding up front and are more exposed to financing risk.

Thus, in a world with incomplete contracts, less innovative projects are not hurt as much by the prospect of financing risk. Instead it is the innovative end of the economy that is most impacted by waves of investor interest and disinterest in the sector. This does not require any behavioral explanation, although the effect could certainly be magnified by behavioral considerations. Rational investors know they face financing risk. They rationally try to mitigate that risk by forming syndicates and providing larger sums of money up-front. But for more innovative projects providing more money reduces the option value of the investment. Thus, innovative projects must be left exposed to the whims of the financial market.

## IV. Implications

### A. Innovation Waves and Project Mix

A key implication of our model is that we should see waves of innovative activity that are driven in part by fluctuations in the capital markets. Furthermore, with any increase in financing risk, some projects will become NPV negative while other projects will find it value enhancing to raise more money and thereby reduce the value of some of their real options but defend better against the potential No-Invest equilibrium. Therefore, projects that can do so will raise more money while projects which cannot raise enough money to defend against the No-Invest equilibria will become NPV negative. Thus among the projects that do get financed, we should see a difference in the amount of capital raised at each round of financing.

For any project, the value of committing an extra \$ $x$  is the difference between equation (17) and equation (13).

$$(1 - \alpha)\gamma(1 - \lambda)(1 - \theta) \sum_{k=1}^t V_k + \alpha\gamma(1 - \theta)\Pi_{t+1}|_{I=0} - (1 - \gamma)x \quad (18)$$

Since  $\Pi_{t+1}|_{I=0} < x$  (i.e., the worst an investor can do is lose \$ $x$ ), the derivative w.r.t  $\gamma$  must be positive. Since a larger  $\gamma$  is equivalent to a less innovative project more innovative projects gain less or lose more from an extra \$ $x$ .

Furthermore, the derivative of the value of committing an extra \$x with respect to  $\theta$  (the probability that the state will stay in the Invest equilibrium) is

$$-(1 - \alpha)\gamma(1 - \lambda) \sum_{k=1}^t V_k - \alpha\gamma\Pi_{t+1}|_{I=0} + \alpha\gamma(1 - \theta) \frac{\partial}{\partial\theta} \Pi_{t+1}|_{I=0}. \quad (19)$$

Thus, the value of committing an extra \$x may go up or down as the likelihood of jumping to the No-Invest equilibrium goes down. It depends to a large extent on whether or not the benefit of an expected increased sale price  $((1 - \alpha)\gamma(1 - \lambda) \sum_{k=1}^t V_k)$  outweighs the cost of investing the extra dollar in the No-Invest equilibrium  $(\alpha\gamma\Pi_{t+1}|_{I=0})$ . Plus, since  $\frac{\partial}{\partial\theta} \Pi_{t+1}|_{I=0} > 0$  the cost of investing in the No-Invest state goes down.

This tells us that if committing an extra \$x did not add value before the change in  $\theta$  then no decrease in  $\theta$  (prob of No-Invest state gets higher) will ever make an extra \$x add value. However, if it made since to commit the extra \$x before the change in  $\theta$  then any decrease in  $\theta$  will make an extra \$x more valuable because the need to survive the No-Invest state is more important.<sup>14</sup>

Thus, the most innovative projects will be the projects that cannot raise significant up-front financing because too much value is destroyed in the loss of their real options. This inability to raise an extra \$x will have a more negative impact when financing risk is larger. Therefore, as financing risk rises not only should fewer projects be financed but the mix of financed projects should become less innovative.

Furthermore, for projects that are funded, a variation in the financing risk may vary the amount of funding they receive. As financing risk rises, projects for which equation (19) is negative will find more value in raising an extra \$x. As financing risk falls eventually no project will find it valuable to raise an extra \$x. Thus, the additional firms that are financed when financing risk is low should receive *less* funding than the average firm financed when financing risk was higher.

If the world does jump to the No-Invest equilibrium when the No-Invest equilibrium seems

---

<sup>14</sup>Since  $-(1 - \gamma)x < 0$  an extra \$x will only add value if  $(1 - \alpha)\gamma(1 - \lambda) \sum_{k=1}^t V_k > \alpha\gamma\Pi_{t+1}|_{I=0}$  if this is not true, no change in  $\theta$  will cause an extra \$x to add value.

remote, firms will not have much of a cash cushion. With less of a cushion to survive the No-Invest equilibrium these firms will go out of business, and this will always be more true for more innovative firms.

## **B. Investor Mix**

Lower financing risk lowers the amount of capital firms need and should therefore also allow smaller investors with more limited capital to invest. Our model therefore suggests that the mix of investors should shift towards smaller and more early stage investors in times when financing risk is low. When financing risk is high, a large investor might be able to give a project more support and break them out of the No-Invest equilibrium but small investors don't even have this option and must therefore stop investing.

Thus, in the low investing times small angel investors should virtually disappear from the market as the coordination costs to bring together enough of them is too high. Further, the only projects we should see getting funded should be funded by larger investors and actually given a *larger* fraction of total money invested. So while less total money will enter the sector and fewer projects will get funded, the few projects that get funded will be well funded.

## **C. 'Herd Behavior' in Innovative Investments**

Conventional wisdom suggests that contrarian strategies might be good because following the crowd leads to a flood of capital in a sector and lowers returns. Our model implies that this is not true in every case. In our model, fully rational investors who only make NPV positive investments are optimally entering the market when prices are high (because the financing risk is low) and everyone else is also in the market. When financing risk is low, giving a project less money and seeing how it does makes sense. Smaller investors who face greater hurdles to forming large pools of money can find valuable investments in high real option companies that need only a little money *but only during good times*. Making this same investment when financing risk is high or during the No-Invest equilibrium is NPV negative. Thus for innovative projects with high real option value, it may actually make sense to invest with the crowd.

The corollary to this view also provides a more positive interpretation to the bubbles of

activity that are associated with the initial diffusion of very radical new technologies, such as railways, motor cars, internet or clean technologies. Our model implies that such technologies may in fact need ‘hot’ financial markets, where financing risk is extremely low and lots of investors are in the market, to help with the initial diffusion of such technologies. Related to this, our model shows why asset prices in such times can steadily rise and then precipitously fall, even when the fundamentals of the projects have changed little. Since expectations of a low probability of a No-Invest equilibrium lead to high NPVs and hence high asset prices, a sudden change in the equilibrium will lead many projects to become negative NPV and lead asset prices to fall commensurately.

## V. Conclusion

Startups have been associated with the initial diffusion of several technological revolutions (semiconductors and computers, internet, motor cars, clean technology) and there is increasing evidence of the important role of startup firms in driving aggregate productivity growth in the economy (Foster et al. (2008)). This paper builds on the emerging research examining the role of the capital markets in driving innovation in startups (Kortum and Lerner (2000)) and provides a mechanism for why innovation in new firms might occur in waves of activity.

Innovators in early stage firms face two types of risk. First, they are involved in risky projects that have a high likelihood of failure. Second, they don’t know how much investment will be required to get to the finish line. Intermediate results may not be promising, or additional investments may be required to get to cash flow positive. Any investor in such startups with limited resources must therefore also rely on other investors to bring innovative projects to fruition. Because of this, such startups face two risks - fundamental risk (that the project gets an investment but turns out not to be viable) and financing risk (that the project needs more money to proceed but cannot get the financing even if it is fundamentally sound).

Financing risk is typically ignored in the finance literature because all projects with positive fundamental NPV are assumed to get funded. This ignores the fact that investing requires coordination across time between investors with limited resources. Investors must, therefore, forecast the probability that other investors will be there to fund the project in the future.

The impact of financing risk on a project can be reduced by giving the project more funding. However, this comes at a cost. A project with more funding will spend the money even in the event of intermediate information that the project is no longer viable. This cost is much more important for highly innovative projects where outcomes are uncertain and the real option to shut down the project is valuable. The more valuable the real option to shut down a project, the less funding the project should receive at a given time. Firms that receive less funding are more affected by a jump to the No-Invest equilibrium. Thus early round investors investing in innovative firms face an important trade-off between lowering financing risk and increasing real option value. The most innovative firms are thus most susceptible to financing risk as they are least able to acquire a ‘war chest’ to survive a down turn.

Since innovative firms are less protected from financing risk, innovative projects or sectors receive less financing during ‘cold’ financing environments. Innovative projects may also require ‘hot’ financing environments to help with their initial diffusion. Thus, financing risk both magnifies fundamental drivers of innovation such as new breakthroughs or business cycles and can potentially drive investment waves in innovation.

## References

- Bergemann, Dirk, and Ulrich Hege. “The Financing of Innovation: Learning and Stopping.” *RAND Journal of Economics* 36 (2005), pp. 719–752.
- Bergemann, Dirk, Ulrich Hege, and Liang Peng. “Venture Capital and Sequential Investments.” *Working Paper* (2008).
- Carlsson, Hans, and Eric van Damme. “Global Games and Equilibrium Selection.” *Econometrica* 61 (1993), pp. 989–1018.
- Diamond, Douglas, and Philip Dybvig. “Bank Runs, Deposit Insurance and Liquidity.” *Journal of Political Economy* 51 (1983), pp. 401–419.
- Foster, Lucia, John Haltiwanger, and Chad Syverson. “Reallocation, Firm Turnover and Efficiency: Selection on Productivity or Profitability.” *The American Economic Review* 98 (2008), pp. 394–425.
- Goldstein, Itay, and Ady Pauzner. “Contagion of self-fulfilling financial crises due to diversification of investment portfolios.” *Journal of Economic Theory* 119 (2004), pp. 151–183.
- Goldstein, Itay, and Ady Pauzner. “Demand-Deposit Contracts and the Probability of Bank Runs.” *Journal of Finance* LX (2005).
- Gompers, Paul. “Optimal Investment, Monitoring, and the Staging of Venture Capital.” *Journal of Finance* 50 (1995), pp. 1461–1489.
- Gompers, Paul, Anna Kovner, Josh Lerner, and David Scharfstein. “Venture capital investment cycles: The impact of public markets.” *Journal of Financial Economics* 87 (2008), pp. 1–23.
- Gompers, Paul, and Josh Lerner. “Venture capital distributions: short-run and long-run reactions.” *Journal of Finance* 53 (1998), pp. 2161–2183.
- Gompers, Paul, and Josh Lerner. “Money chasing deals? The impact of fund inflows on private equity valuations.” *Journal of Financial Economics* 55 (2000), pp. 281–325.

- Gompers, Paul, and Josh Lerner. *The Venture Capital Cycle*. Cambridge, MA and London: MIT Press (2004).
- Griliches, Zvi. “Patent Statistics as Economic Indicators: A Survey.” *Journal of Economic Literature* 28 (1990), pp. 1661–1707.
- Grossman, Sanford J., and Oliver D. Hart. “The costs and benefits of ownership: A theory of vertical and lateral integration.” *Journal of Political Economy* 94 (1986), pp. 691–719.
- Guedj, Ilan, and David Scharfstein. “Organizational Scope and Investment: Evidence from the Drug Development Strategies and Performance of Biopharmaceutical Firms.” *NBER working paper 10933* (2006).
- Hart, Oliver D., and John Moore. “Property rights and the nature of the firm.” *Journal of Political Economy* 98 (1990), pp. 1119–1158.
- Hobijn, Bart, and Boyan Jovanovic. “The Information-Technology Revolution and the Stock Market: Evidence.” *The American Economic Review* 91 (2001), pp. 1203–1220.
- Hsu, David. “What Do Entrepreneurs Pay for Venture Capital Affiliation?” *Journal of Finance* 59 (2004), pp. 1805–1844.
- Kaplan, Steven, and Antoinette Schoar. “Private Equity Performance: Returns, Persistence and Capital Flows.” *Journal of Finance* 60 (2005), pp. 1791–1823.
- Kaplan, Steven, Berk Sensoy, and Per Stromberg. “Should Investors Bet on the Jockey or the Horse? Evidence from the Evolution of Firms from Early Business Plans to Public Companies.” *Journal of Finance* 64 (2009).
- Kleinknecht, Arthur. *Innovation Patterns in Crisis and Prosperity*. McMillan (1987).
- Kortum, Samuel, and Josh Lerner. “Assessing the impact of venture capital on innovation.” *Rand Journal of Economics* 31 (2000), pp. 674–92.
- Kuznets, Simon. “Schumpeter’s Business Cycles.” *American Economic Review* 30 (1940), pp. 257–271.

- Morris, Stephen, and Hyun Song Shin. "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks." *American Economic Review* 88 (1998), pp. 587–597.
- Nanda, Ramana. "Entrepreneurship and the Discipline of External Finance." *HBS Working Paper* (2008).
- Pastor, Lubos, and Pietro Veronesi. "Technological Revolutions and Stock Prices." *American Economic Review* 99 (2009), pp. 1451–1483.
- Perez, Carlota. *Technological Revolutions and Financial Capital*. Edward Elgar (2002).
- Scharfstein, David, and Jeremy Stein. "Herd behavior and investment." *American Economic Review* 80 (1990), pp. 465–479.
- Schmookler, Jacob. *Invention and Economic Growth*. Harvard University Press (1966).
- Schumpeter, Joseph. *Business Cycles: A Theoretical, Historical and Statistical Analysis of the Capitalist Process*. McGraw Hill (1939).
- Schumpeter, Joseph. *Capitalism, Socialism and Democracy*. New York: Harper (1942).
- Stein, Jeremy. "Waves of Creative Destruction: Firm Specific Learning-by-Doing and the Dynamics of Innovation." *The Review of Economic Studies* 64 (1997), pp. 265–288.
- Stoneman, P. "Patenting Activity: A Re-evaluation of the Influence of Demand Pressures." *Journal of Industrial Economics* 27 (1979), pp. 385–401.

## A. Appendix

### i. Solving Iteratively for Profits.

Profits in period  $t$  are

$$\Pi_t = (1 - \alpha)\gamma O_t(I) + \alpha\gamma I\Pi_{t+1} - x \quad (\text{A-1})$$

This can be rewritten as

$$\Pi_t = \Pi_{t-1} + (1 - \alpha)\gamma(O_t(I) - O_{t-1}(I)) + \alpha\gamma I(\Pi_{t+1} - \Pi_t). \quad (\text{A-2})$$

Remembering that

$$O_t(I) = I \sum_{k=1}^t V_k + (1 - I)\lambda \sum_{k=1}^t V_k \quad (\text{A-3})$$

profits in period  $t$  equals

$$\Pi_t = \Pi_{t-1} + (1 - \alpha)\gamma(I + (1 - I)\lambda)V_t + \alpha\gamma I * (\Pi_{t+1} - \Pi_t). \quad (\text{A-4})$$

Expanding  $\Pi_{t+1} - \Pi_t$  yields

$$\Pi_t = \Pi_{t-1} + (1 - \alpha)\gamma(I + (1 - I)\lambda)(V_t + \alpha\gamma I V_{t+1}) + \alpha\gamma I(\Pi_{t+2} - \Pi_{t+1}). \quad (\text{A-5})$$

Iteratively expanding yields

$$\Pi_t = \Pi_{t-1} + (1 - \alpha)\gamma(I + (1 - I)\lambda) \sum_{k=0}^{\infty} (\alpha\gamma I)^k V_{k+t}. \quad (\text{A-6})$$

Using equation A-1 we know that

$$\Pi_{t-1} = (1 - \alpha)\gamma O_{t-1}(I) + \alpha\gamma I\Pi_t - x. \quad (\text{A-7})$$

Substituting for  $\Pi_{t-1}$  we find

$$\Pi_t = (1 - \alpha)\gamma O_{t-1}(I) + \alpha\gamma I\Pi_t - x + (1 - \alpha)\gamma(I + (1 - I)\lambda) \sum_{k=0}^{\infty} (\alpha\gamma I)^k V_{k+t}. \quad (\text{A-8})$$

Substituting for  $O_{t-1}$  and subtracting  $\alpha\gamma I\Pi_t$  from both sides yields

$$\Pi_t - \alpha\gamma I\Pi_t = (1 - \alpha)\gamma(I + (1 - I)\lambda) \left( \sum_{k=1}^{t-1} V_k + \sum_{k=0}^{\infty} (\alpha\gamma I)^k V_{k+t} \right) - x. \quad (\text{A-9})$$

Or just

$$\Pi_t(1 - \alpha\gamma I) = (1 - \alpha)\gamma(I + (1 - I)\lambda) \left( \sum_{k=1}^t V_k + \sum_{k=1}^{\infty} (\alpha\gamma I)^k V_{k+t} \right) - x. \quad (\text{A-10})$$

Thus,

$$\Pi_t = \frac{(1-\alpha)\gamma(I+(1-I)\lambda)}{(1-\alpha\gamma I)} \left( \sum_{k=1}^t V_k + \sum_{k=1}^{\infty} (\alpha\gamma I)^k V_{k+t} \right) - \frac{x}{(1-\alpha\gamma I)}. \quad (\text{A-11})$$

## ii. Profits with State Transitions:

Profits in period  $t$  given that investors are currently in an investing equilibrium,  $I = 1$ , are

$$\Pi_t|_{I=1} = (1-\alpha)\gamma(\theta O_t(1) + (1-\theta)O_t(0)) + \alpha\gamma\theta\Pi_{t+1}|_{I=1} - x \quad (\text{A-12})$$

Remember that there is no  $(1-\theta)$  chance of getting  $\Pi_{t+1}|_{I=0}$  because we assume that the firm receives no funding in this state and goes bankrupt (which, of course, we must confirm in equilibrium, i.e.  $\Pi_{t+1}|_{I=0} < 0$ ).<sup>15</sup>

Profits can be rewritten as

$$\begin{aligned} \Pi_t|_{I=1} = \Pi_{t-1}|_{I=1} + (1-\alpha)\gamma[\theta(O_t(1) - O_{t-1}(1)) + (1-\theta)(O_t(0) - O_{t-1}(0))] \\ + \alpha\gamma\theta(\Pi_{t+1}|_{I=1} - \Pi_t|_{I=1}). \end{aligned} \quad (\text{A-13})$$

Remembering that

$$O_t(I) = I * \sum_{k=1}^t V_k + (1-I) * \lambda \sum_{k=1}^t V_k \quad (\text{A-14})$$

profits in period  $t$  equal

$$\Pi_t|_{I=1} = \Pi_{t-1}|_{I=1} + (1-\alpha)\gamma(\theta + (1-\theta)\lambda)V_t + \alpha\gamma\theta(\Pi_{t+1}|_{I=1} - \Pi_t|_{I=1}). \quad (\text{A-15})$$

Expanding  $\Pi_{t+1}|_{I=1} - \Pi_t|_{I=1}$  yields

$$\Pi_t|_{I=1} = \Pi_{t-1}|_{I=1} + (1-\alpha)\gamma(\theta + (1-\theta)\lambda)(V_t + \alpha\gamma\theta V_{t+1}) + \alpha\gamma\theta(\Pi_{t+2}|_{I=1} - \Pi_{t+1}|_{I=1}). \quad (\text{A-16})$$

Iteratively expanding yields

$$\Pi_t|_{I=1} = \Pi_{t-1}|_{I=1} + (1-\alpha)\gamma(\theta + (1-\theta)\lambda) \sum_{k=0}^{\infty} (\alpha\gamma\theta)^k V_{k+t}. \quad (\text{A-17})$$

Using equation (A-12) and substituting into equation (A-17) for  $\Pi_{t-1}|_{I=1}$  we find

$$\begin{aligned} \Pi_t|_{I=1} = (1-\alpha)\gamma(\theta O_{t-1}(1) + (1-\theta)O_{t-1}(0)) + \alpha\gamma\theta\Pi_t|_{I=1} - x \\ + (1-\alpha)\gamma(\theta + (1-\theta)\lambda) \sum_{k=0}^{\infty} (\alpha\gamma\theta)^k V_{k+t}. \end{aligned} \quad (\text{A-18})$$

---

<sup>15</sup>If this were not the case then there would be no no-invest equilibrium and profits in period  $t$  would be the same but with  $\theta = 1$

Substituting for  $O_{t-1}$  and subtracting  $\alpha\gamma\theta\Pi_t|_{I=1}$  from both sides yields

$$\Pi_t|_{I=1} - \alpha\gamma\theta\Pi_t|_{I=1} = (1 - \alpha)\gamma(\theta + (1 - \theta)\lambda) \left( \sum_{k=1}^{t-1} V_k + \sum_{k=0}^{\infty} (\alpha\gamma\theta)^k V_{k+t} \right) - x. \quad (\text{A-19})$$

Or just

$$\Pi_t|_{I=1} = \frac{(1 - \alpha)\gamma(\theta + (1 - \theta)\lambda)}{(1 - \alpha\gamma\theta)} \left( \sum_{k=1}^t V_k + \sum_{k=1}^{\infty} (\alpha\gamma\theta)^k V_{k+t} \right) - \frac{x}{(1 - \alpha\gamma\theta)}. \quad (\text{A-20})$$

In order to determine if this is an equilibrium we will also need profits in the no-invest equilibrium,  $I = 0$ ,

$$\Pi_t|_{I=0} = (1 - \alpha)\gamma(\phi O_t(1) + (1 - \phi)O_t(0)) + \alpha\gamma\phi\Pi_{t+1}|_{I=1} - x. \quad (\text{A-21})$$

Substituting for the offer,  $O_t$ , yields

$$\Pi_t|_{I=0} = (1 - \alpha)\gamma(\phi + (1 - \phi)\lambda) \sum_{k=1}^t V_k + \alpha\gamma\phi\Pi_{t+1}|_{I=1} - x. \quad (\text{A-22})$$

And finally substituting for  $\Pi_{t+1}|_{I=1}$  yields,

$$\begin{aligned} \Pi_t|_{I=0} &= (1 - \alpha)\gamma(\phi + (1 - \phi)\lambda) \sum_{k=1}^t V_k \\ &+ \alpha\gamma\phi \left[ \frac{(1 - \alpha)\gamma(\theta + (1 - \theta)\lambda)}{(1 - \alpha\gamma\theta)} \left( \sum_{k=1}^{t+1} V_k + \sum_{k=1}^{\infty} (\alpha\gamma\theta)^k V_{k+t+1} \right) - \frac{x}{(1 - \alpha\gamma\theta)} \right] - x \end{aligned} \quad (\text{A-23})$$

### iii. Proof of Proposition 2:

The VCs will invest if they believe others will invest if the NPV from doing so is positive,  $\Pi_t|_{I=1} \geq 0$ , or

$$(1 - \alpha)\gamma(\theta + (1 - \theta)\lambda) \left( \sum_{k=1}^t V_k + \sum_{k=1}^{\infty} (\alpha\gamma\theta)^k V_{k+t} \right) \geq x. \quad (\text{A-24})$$

And if the VC believes that other VCs will not invest then VC will not invest if the NPV from investing is negative,  $\Pi_t|_{I=0} < 0$  or

$$(1 - \alpha)\gamma(\phi + (1 - \phi)\lambda) \sum_{k=1}^t V_k + \alpha\gamma\phi\Pi_{t+1}|_{I=1} < x. \quad (\text{A-25})$$

Both condition (A-24) and (A-25) can simultaneously hold as long as  $\theta$  is large enough and  $\phi$  and  $\lambda$  are small enough. However, it is only rational for each investor to believe that future investors either will or will not invest if conditions (A-24) and (A-25) hold for the next period. And in the future period it is again only rational if conditions (A-24) and (A-25) hold for the

period after that. Thus, for both equilibria to be rational conditions (A-24) and (A-25) must hold for all future periods.

Since  $\alpha\gamma\phi \leq 1$ ,  $\sum_{k=1}^t V_k + \sum_{k=1}^{\infty} (\alpha\gamma\theta)^k V_{k+t}$  is an increasing function of  $t$ . Therefore, as long as condition (A-24) holds in one period it holds in all future periods. Furthermore, since  $\sum_{k=1}^{t \rightarrow \infty} V_k < \bar{V}$  and  $\lim_{n \rightarrow \infty} \sum_{k=1}^t V_k + \sum_{k=1}^{\infty} (\alpha\gamma)^k V_{k+n} < \bar{V}$ . Therefore,  $\Pi_t|_{I=0}$  is always less than

$$(1 - \alpha)\gamma(\phi + (1 - \phi)\lambda)\bar{V} + \alpha\gamma\phi \left[ \frac{(1 - \alpha)\gamma(\theta + (1 - \theta)\lambda)}{(1 - \alpha\gamma\theta)} \bar{V} - \frac{x}{(1 - \alpha\gamma\theta)} \right] - x \quad (\text{A-26})$$

which is less than zero as long as  $\phi$  is small enough and  $\theta$  is large enough and  $\lambda$  is small enough. That is, as long as the probability of a transition back from the no-invest equilibria ( $\phi$ ) is small enough and the probability of staying in the invest equilibria ( $\theta$ ) is large enough and the penalty for exiting during a no invest equilibria ( $\lambda$ ) is harsh enough then the no-invest equilibrium is an equilibrium, i.e. both condition (A-24) and (A-25) hold for all periods.

For example, as  $\phi$  approaches zero and  $\theta$  approaches 1 both conditions hold as long as  $(1 - \alpha)\gamma\bar{V} \geq x$  and  $(1 - \alpha)\gamma\lambda\bar{V} \leq x$  which are both true for small enough  $\lambda$ . Q.E.D.

#### iv. Proof of Proposition 3:

The questions of whether it is better to commit to a second \$x is a questions of whether profits with commitment are bigger than profits without. Subtracting profits without commitment, equation (13), from profits with commitment, equation (14), the question is reduced to whether condition 15 holds, i.e.

$$(1 - \alpha)(1 - \lambda) \sum_{k=1}^t V_k + \alpha \Pi_{t+1}|_{I=0} > 0 ? \quad (\text{A-27})$$

The gain from committing a second \$x is the higher purchase price if a buyer arrives but the state has transitioned. The loss from committing a second \$x is the NPV from investing in the bad state,  $\Pi_{t+1}|_{I=0}$ . We know from above that

$$\begin{aligned} \Pi_{t+1}|_{I=0} &= (1 - \alpha)\gamma(\phi + (1 - \phi)\lambda) \sum_{k=1}^{t+1} V_k - x \\ &+ \frac{\alpha\gamma\phi}{(1 - \alpha\gamma\theta)} \left[ (1 - \alpha)\gamma(\theta + (1 - \theta)\lambda) \left( \sum_{k=1}^{t+2} V_k + \sum_{k=1}^{\infty} (\alpha\gamma\theta)^k V_{k+t+2} \right) - x \right] \end{aligned} \quad (\text{A-28})$$

Therefore, condition 15 holds when  $\alpha$  is small enough (for any values of  $\sum_{k=1}^t V_k$  and  $\Pi_{t+1}|_{I=0}$ ) because the investor gets a higher sale price from the reserved investment if the bad state occurs but is unlikely to have to make the bad investment because the project is likely to be sold.

Condition 15 is more likely to hold for smaller  $\lambda$  because the benefit of the second \$x in reserve is larger for smaller  $\lambda$ . The condition is also more likely to hold the larger  $\phi$ ,  $\theta$  and  $\gamma$  are because investing during the No-Invest equilibrium is not as negative NPV.

So if it is NPV positive to commit an extra \$x in every period then condition 15 must hold for all  $t$ .

Conditional on this being true we now ask if an investor or syndicate should commit to more than one extra \$x. We know from above that if an investor or syndicate commits to invest in both the first and the second period the expected project NPV is

$$\Pi_t|_{I=1}^{n=2} = (1 - \alpha)\gamma \sum_{k=1}^t V_k + \alpha\gamma (\theta\Pi_{t+1}|_{I=1} + (1 - \theta)\Pi_{t+1}|_{I=0}) - x. \quad (\text{A-29})$$

If instead the syndicate commits to invest in the first, second and third period then the expected project NPV is

$$\Pi_t|_{I=1}^{n=3} = (1 - \alpha)\gamma \sum_{k=1}^t V_k + \alpha\gamma (\theta\Pi_{t+1}|_{I=1}^{n=2} + (1 - \theta)\Pi_{t+1}|_{I=0}^{n=2}) - x. \quad (\text{A-30})$$

Subtracting,  $\Pi_t|_{I=1}^{n=3} - \Pi_t|_{I=1}^{n=2}$ , we find that conditional on the commitment of the second \$x, then committing to the third \$x is NPV positive if

$$\theta(\Pi_{t+1}|_{I=1}^{n=2} - \Pi_{t+1}|_{I=1}^{n=1}) + (1 - \theta)(\Pi_{t+1}|_{I=0}^{n=2} - \Pi_{t+1}|_{I=0}^{n=1}) > 0. \quad (\text{A-31})$$

We know

$$\Pi_{t+1}|_{I=1}^{n=2} - \Pi_{t+1}|_{I=1}^{n=1} = (1 - \alpha)(1 - \theta)(1 - \lambda)\gamma \sum_{k=1}^{t+1} V_k + \alpha\gamma(1 - \theta)\Pi_{t+2}|_{I=0} \quad (\text{A-32})$$

and

$$\Pi_{t+1}|_{I=0}^{n=2} - \Pi_{t+1}|_{I=0}^{n=1} = (1 - \alpha)(1 - \phi)(1 - \lambda)\gamma \sum_{k=1}^{t+1} V_k + \alpha\gamma(1 - \phi)\Pi_{t+2}|_{I=0}. \quad (\text{A-33})$$

Therefore if committing the second \$x is NPV positive at each point in time then

$$(1 - \alpha)(1 - \lambda) \sum_{k=1}^{t+1} V_k + \alpha\Pi_{t+2}|_{I=0} > 0 \quad (\text{A-34})$$

and both equation A-32 and A-33 are positive. Therefore, condition A-31 is true.

In general the difference in NPV from committing to fund  $m$  periods versus  $m+1$  periods is the probability that the equilibrium at time  $m$  is the Invest equilibrium  $I = 1$  times the value of the extra \$x commitment in that equilibrium plus the probability that they equilibrium at time  $m$  is in the No-Invest equilibrium  $I = 0$  times the value of the extra \$x commitment in that

equilibrium

$$Prob[I = 1|t = m + t](\Pi_{t+m+1}|_{I=1}^{n=2} - \Pi_{t+m+1}|_{I=1}^{n=1}) + \quad (A-35)$$

$$Prob[I = 0|t = m + t](\Pi_{t+m+1}|_{I=0}^{n=2} - \Pi_{t+m+1}|_{I=0}^{n=1}) > 0. \quad (A-36)$$

Thus, it is always NPV positive to commit more capital as long as it is NPV positive to commit one extra \$x in any period. The commitment is valuable thus committing at the project's start increases the NPV of the project.

The extra commitment also increases the NPV of investments that are made during the No-Invest equilibrium. Remember that dual equilibria required that the NPV from investing during the No-Invest state be negative,  $\Pi_t|_{I=0} < 0$  or

$$(1 - \alpha)\gamma(\phi + (1 - \phi)\lambda) \sum_{k=1}^t V_k + \alpha\gamma\phi\Pi_{t+1}|_{I=1} < x. \quad (A-37)$$

However with commitment this becomes  $\Pi_t|_{I=0}^{n=2} < 0$  or

$$(1 - \alpha)\gamma \sum_{k=1}^t V_k + \alpha\gamma(\phi\Pi_{t+1}|_{I=1} + (1 - \phi)\Pi_{t+1}|_{I=0}) < x. \quad (A-38)$$

But we know that if commitment is valuable then  $\Pi_t|_{I=0}^{n=2} > \Pi_t|_{I=0}$ . Furthermore, the NPV of an investment in the 'No-Invest' equilibrium increases with each dollar committed, i.e.  $\Pi_t|_{I=0}^{n=m} > \Pi_t|_{I=0}^{n=m-1}$ . Thus, for small enough  $\alpha$  a large enough commitment will make an investment during the 'No-Invest' equilibrium positive NPV. This would turn the No-Invest equilibrium into an Invest equilibrium and the project would no longer suffer from financing risk.

We have shown that for small  $\alpha$  commitment is valuable. If it is valuable then more commitment is more valuable. For a small enough  $\alpha$ , then enough commitment will mean that investing in the no-invest equilibrium is NPV positive and the project that can get a large syndicate will not suffer from financing risk. Q.E.D.

## v. Proof of Proposition 4:

With complete contracts we know from Proposition 3 that the value from commitment is

$$(1 - \alpha)\gamma(1 - \lambda)(1 - \theta) \sum_{k=1}^t V_k + \alpha\gamma(1 - \theta)\Pi_{t+1}|_{I=0}. \quad (A-39)$$

With incomplete contracts the profit function with commitment is

$$\Pi_t|_{I=1}^{n=2} = (1 - \alpha)\gamma \sum_{k=1}^t V_k + \alpha\gamma(\theta\Pi_{t+1}|_{I=1} + (1 - \theta)\Pi_{t+1}|_{I=0}) + \gamma x - 2x. \quad (A-40)$$

And the value from committing an extra \$x is

$$(1 - \alpha)\gamma(1 - \lambda)(1 - \theta) \sum_{k=1}^t V_k + \alpha\gamma(1 - \theta)\Pi_{t+1}|_{I=0} - (1 - \gamma)x \quad (\text{A-41})$$

If we subtract equation (A-39) from equation (A-41) we get

$$-(1 - \gamma)x. \quad (\text{A-42})$$

Thus, the value of committing and extra \$x with incomplete contracts is less than with complete contracts. Furthermore, the derivative of equation (A-42) with respect to  $\gamma$  is positive. Therefore, the reduction in value from incomplete contracts is larger for projects with smaller  $\gamma$ . Q.E.D.