Commodity Financialization and Information Transmission

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Abstract
We provide a model to understand the effects that commodity futures financialization has on various market variables. We distinguish between financial speculators and financial hedgers and study their separate and combined effects on the informativeness of futures prices, the futures price bias, the comovement of futures prices with other markets, and the predictiveness of financial trading. We capture the interactions between commodity futures financialization and the real economy through spot prices and production decisions. A dynamic extension illustrates how key variables change over time in a period of acute financialization in a way that is consistent with observed empirical patterns.

Keywords: Commodity financialization, supply channel, price informativeness, feedback effect

JEL Classifications: D82, G14

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1 Introduction

The twenty-first century has seen many developments and changes in finance. A prominent one among them has been the financialization of commodity futures markets. Traditionally, these markets served mostly commodity producers and users looking to hedge their exposures and trade on their information. A new trend emerged around 2004, where financial investors—such as commodity index traders (CITs), commodity trading advisers, and hedge funds—entered these markets and became dominant players in them. The distinct feature of these financial traders is that, unlike the traditional players in these markets, they have no direct exposure to commodities in production/consumption activities. This trend led to a surge in academic studies, including work by Tang and Xiong (2012), Cheng and Xiong (2014), Basak and Pavlova (2016), and Bhardwaj, Gorton, and Rouwenhorst (2016).

The interest in the financialization of commodity futures markets stems to a large extent from concerns over what it might imply for price discovery in the futures markets, spillovers to the spot markets, and consequences for production and consumption of commodities. The so-called “Masters Hypothesis” provided by hedge fund manager Michael W. Masters in his testimonies before the U.S. Congress and U.S. Commodity Futures Trading Commission (CFTC) claims that the large inflow of financial capital into commodity futures markets is responsible for the 2007-2008 spike in commodity futures prices (see Irwin, 2012; Irwin and Sanders, 2012). An overview in the 2011 Report of the G20 Study Group on Commodities (p. 29) notes that “(t)he discussion centers around two related questions. First, does increased financial investment alter demand for and supply of commodity futures in a way that moves prices away from fundamentals and/or increase their volatility? And second, does financial investment in commodity futures affect spot prices?”. A burgeoning empirical literature tracks the effect of financialization on risk premia, market efficiency, correlations between commodity markets and equity markets, the return predictiveness of financial trading, operating profits of commodity producers and users, and other variables. Interestingly, the various papers in this literature, many of which we mention below, often come up with conflicting messages on the implications of financialization.

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1This kind of complaints prompted CFTC to add Commodity Index Trader (CIT) position supplement to the traditional weekly Commitments of Traders (COT) reports, starting in 2007.

Given the stage of development of the empirical literature and the debates within it, there is need for theoretical frameworks providing a unified approach to understand the various mechanisms and help guide and interpret the empirical work. In this paper, we attempt to provide such a unified framework. Our model is built in the tradition of the classic papers by Danthine (1978) and Grossman and Stiglitz (1980), but tailored to address the question of how an increase in financialization of commodity futures markets affects the various parameters of commodity markets and real outcomes.

At the outset, it is important to recognize that financial traders are not all made alike. In particular, there are two distinct motives for trading, which are characterizing different financial traders to different degrees: speculation and hedging (see, e.g., Cheng and Xiong, 2014). First, hedge funds and other financial traders have been investing a lot to acquire information on the fundamental developments of commodity demand and supply to guide their speculative trading in these markets. This is a main attraction for them in entering these markets, as they provide new opportunities for speculative gains. Second, for some financial traders, the main attraction of coming into these markets has been the ability to diversify and hedge exposures they have in other investments. Unlike commodity producers and users, they are not directly involved with commodity spot markets, but rather attempt to gain higher efficiency on their portfolios by adding commodity futures to their other investments. Hence, in our model, we are exploring the effects of introducing two groups of financial traders—one motivated by speculation and one motivated by hedging—for the various parameters of commodity markets and their real effects. Importantly, we also explore the implications of financialization overall by studying the effect of a wave of financialization, bringing financial speculators and hedgers in constant proportions into the market. This exercise is motivated by the premise that financialization was not geared towards any particular group, but rather made financial traders at large aware of these new trading opportunities, bringing both hedgers and speculators to the market.

To demonstrate the main effects, we start with a static framework, featuring one commodity good and two periods ($t = 0$ and $1$). The spot market of the commodity opens at date 1 and the spot price is determined based on the commodity supply and demand. The commodity demand is random, reflecting preference shocks to date-1 commodity consumers.

3We review existing theoretical work and explain our distinct angle below.
The commodity supply is determined endogenously based on commodity producers’ decisions, which are made at date 0 conditional on the equilibrium futures price. At date 0, the commodity futures market opens and the futures price is determined to clear the market. All commodity producers can trade futures contracts alongside financial traders and noise traders. Commodity producers have private information about the later commodity demand and thus they speculate on their information when trading futures. In addition, they trade to hedge the risk they are exposed to in their production. Financial traders belong to two types: financial speculators who trade on private information about the later commodity demand and financial hedgers who trade to hedge their positions in other assets such as stocks.

We analyze the effect of financialization on price informativeness, broadly thought of as market efficiency. As expected, financial speculators help improving price informativeness, whereas financial hedgers push it down. A priori, it is less clear what a wave of financialization, bringing the two types of traders into the market in constant proportions, would do to price informativeness. Our model offers an answer. We show that the effect of financial speculators dominates only as long as the total size of the financial traders population is relatively small. Initially, as financialization is small in magnitude, the additional noise brought by financial hedgers, wishing to hedge other positions, does not affect futures prices much. But, as financialization grows in magnitude, the additional noise becomes a more prominent determinant of futures prices. Hence, our comparative statics results suggest that a process of increased financialization first increases and then decreases price informativeness. We discuss how this result can help reconcile mixed empirical findings that commodity financialization improved market efficiency in the U.S. crude oil futures market (Raman, Robe, and Yadav, 2017) but harmed market efficiency in broader commodity index markets (Brogaard, Ringgenberg, and Sovich, 2019).

We then explore the implications for the futures price bias, which, in our model, is captured by the deviation of the futures price from the expected later spot price. Commodity financialization affects the magnitude of the bias through multiple channels. First, adding more financial traders facilitates risk sharing, which tends to reduce the bias. Second, as

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4 Consistent with our model, Ready and Ready (2018) indeed find that commodity index investors are moving the market through their hedging trades.
mentioned above, commodity financialization also affects price informativeness, which affects the magnitude of the bias through the amount of uncertainty that market participants have to hedge against. Interestingly, financialization can also change the sign of the bias in our model, as it affects the overall direction in which hedging pressure pushes futures prices. We characterize how the various effects interact and when futures prices drift further away from spot prices as a result of financialization (which is one of the key concerns expressed by the G20 in the quote above).

A variable of great interest in the empirical literature is the comovement between the commodity futures market and the equity market (see Tang and Xiong, 2012; Büyükşahin and Robe, 2014; Cheng and Xiong, 2014; and Bhardwaj, Gorton, and Rouwenhorst, 2016). Our model demonstrates how financial hedgers generate correlation between the two markets, whereas financial speculators have a more ambiguous effect. Another key variable is the extent to which financial traders’ positions predict futures prices (see Singleton, 2014; Cheng, Kirilenko, and Xiong, 2015; and Hamilton and Wu, 2015). In our model, consistent with the empirical evidence, the positions of financial speculators are much more reliable than those of financial hedgers as a source of such predictiveness.

Turning to the effect of futures markets on spot markets and the real economy, we highlight a supply channel, whereby a higher futures price induces commodity producers to supply more of the commodity, which in turn presses down the later spot price through the market-clearing mechanism in the spot market. This direct feedback effect differs from that in the vast literature reviewed by Bond, Edmans, and Goldstein (2012), where financial-market prices have a real effect through the information they provide, although this more indirect effect is also present in our model. Taking these effects together, our model features strong spillovers from the futures markets to spot markets and the real economy, underscoring the importance of understanding the impact of financialization, as in the second question from the G20’s quote above. In this light, we explore the implications of financialization for the profits and welfare of commodity producers. When financialization leads to an increase (decrease) in price informativeness in our model, commodity producers see higher (lower) operating profits. However, at the same time, their welfare decreases (increases), as a re-

\footnote{As we discuss, our mechanism is related to but distinct from that in Leland (1992), where an increase in the stock price causes the firm to issue more equity shares and make more real investments, but the asset payoff is exogenous.}
result of the decrease (increase) in trading and risk sharing opportunities. These results are important for interpreting empirical evidence and for policy. Brogaard, Ringgenberg, and Sovich (2019) show that the decrease in informativeness that followed financialization led to a decrease in operating profits of commodity producers. While consistent with our model, we show that the welfare implications for the producers, who participate in the futures markets, are opposite.

In the last part of the paper, we extend the model to a dynamic framework. While the basic forces behind commodity financialization and their effects on market outcomes can be understood through the lens of the static framework, the empirical literature explores time variation in the variables of interest, and so a dynamic framework can better map to this literature. We study an overlapping-generations (OLG) setting following the static framework, where an endogenous decision on commodity storage dynamically links the different periods. To capture the nature of financialization, we set the model up so that the population sizes of financial traders are small at the outset, and are growing gradually over the phase of the financialization, before reaching steady state. Other than the economic analysis, the computation of a non-stationary equilibrium, where the sizes of investors populations are growing over time, is a methodological contribution. Computing this equilibrium allows us to characterize how the key objects studied in the static framework change over time during the phase of the financialization.

Using variables from the empirical literature on commodity futures markets and from broader literature on trading in financial markets, we calibrate the dynamic model for the crude oil market, considering the phase between 2004 and 2009 as the phase of growing financialization. Our results demonstrate how our model generates an increase in price informativeness, a decrease in futures price bias, and an increase in the correlation with the equity market during this financialization phase, all consistent with the broad findings of the empirical literature mentioned above. They also replicate the usefulness of financial speculators’ positions in predicting futures prices and the lack of predictive power from the positions of financial hedgers. While we focus on one commodity and a phase of growing financialization that reaches steady state after six years, the methodology is flexible and can be adjusted to allow for multiple commodities or for cycles in the size of financial-traders populations in the commodity futures markets (so that the economy features both financial-
ization and de-financialization). Future research can explore such adjustments building on our framework.

**Related Literature**  Our paper is broadly related to three strands of literature. The first is the literature on commodity financialization, which is largely empirical and documents the trading behavior of financial traders in futures markets and their pricing impact. The theoretical research on the subject remains scarce. Basak and Pavlova (2016) construct dynamic equilibrium models to study how commodity financialization affects commodity futures prices, volatilities, and in particular, correlations among commodities and between equity and commodities. Fattouh and Mahadeva (2014) and Baker (2021) calibrate macrofinance models of commodities to quantify the effect of commodity financialization. Gorton, Hayashi, and Rouwenhorst (2012) and Ekeland, Lautier, and Villeneuve (2017) consider a combination of hedging pressure theory and storage theory to study commodity financialization. Knittel and Pindyck (2016) study a reduced-form setting of commodity financialization using a simple model of supply and demand in the cash and storage markets. Tang and Zhu (2016) model commodities as collateral for financing in a two-period economy with multiple countries and capital controls. Chari and Christiano (2017) develop a model to show that financial traders and traditional commodity traders insure each other. While these existing models offer important insights, they all feature symmetric information, and hence do not address the key channels of our model involving price informativeness and learning.

Three existing theoretical studies also analyze the effects of informational frictions in the context of commodity financialization. Sockin and Xiong (2015) focus on information asymmetry in the spot market. They show that a high spot price may further spur the commodity demand through an informational channel and that in the presence of complementarity, this informational effect can be so strong that commodity demand can increase with the price. Goldstein, Li, and Yang (2014) argue that financial traders and commodity producers may respond to the same fundamental information in opposite directions, such that commodity financialization may have a negative informational effect. Leclercq and Praz (2014) consider how the entry of new speculators affects the average and volatility of spot prices. We view our paper as complementary to these papers since it highlights different channels through which financialization affects prices and real outcomes. In particular, the feedback effect
from futures markets to the real economy in our model happens through the production decisions of commodity producers. Moreover, financial trading injects both information and noise into the futures market, through the behavior of different types of financial traders. These channels are empirically motivated and they generate very different implications, as our analysis demonstrates.

The second strand of related literature is the classic literature on futures markets (see Section 1.1 of Acharya, Lochstoer, and Ramadorai (2013) for a brief review of this literature). This literature has developed theories of “hedging pressure” (Keynes, 1930; Hicks, 1939; Hirshleifer, 1988, 1990) or “storage” (Kaldor, 1939; Working, 1949) to explain futures prices. Notably, the literature has also developed asymmetric information models on futures markets (e.g., Grossman, 1977; Danthine, 1978; Bray, 1981; Stein, 1987). However, because commodity financialization is just a recent phenomenon, these early models have focused on different research questions. The analysis in our model centers on the implications of financial trading for various parameters in commodity markets and real outcomes. This question is very relevant in today’s markets and has not been addressed by the older literature.

Finally, our dynamic analysis contributes to the recent literature that develops dynamic noisy-rational-expectations-equilibrium models to understand financial markets and the real economy. David, Hopenhayn, and Venkateswaran (2016) link imperfect information to resource misallocation and quantify the losses in productivity and output due to the informational friction. Begenau, Farboodi, and Veldkamp (2018) show that big data disproportionately benefits big firms, because a larger firm has produced more data, which attracts more financial analysis, reducing the firm’s cost of capital and enabling the larger firm to grow larger. Benhabib, Liu, and Wang (2019) show that the mutual learning between financial markets and the real economy creates a strategic complementarity in information production, generating self-fulfilling surges in economic uncertainties. Farboodi and Veldkamp (2020) explore how improvements in data processing shape investors’ information choices about future asset values or about others’ demands. They find that unbiased technological change can explain a market-wide shift in data collection, but in the long run, as data processing technology becomes increasingly advanced, both types of data continue to be processed. Brunnermeier, Sockin, and Xiong (2020) examine active government intervention in financial markets. In their model, the noise in government intervention becomes a factor driving asset prices, which
may divert investor attention away from studying fundamentals, leading to negative consequences such as worse informational efficiency. Our paper complements those studies by providing a dynamic framework to examine how financial traders affect commodity markets through an informational channel. We provide an approach to computing a non-stationary equilibrium, and use our analysis to understand the time variations in variables of interest.

2 The Baseline Model

In this section, we present a simple static model with multiple types of financial traders (speculators and hedgers) and multiple types of shocks (demand shocks, supply shocks, and financial market shocks). We interpret commodity financialization as an increase in the population sizes of different types of financial traders in the commodity futures market.

2.1 Setup

The baseline model lasts two periods: $t = 0$ and 1. The timeline of the economy is described by Figure 1. At date 0, the financial market opens, where financial speculators and financial hedgers trade futures contracts against commodity producers and noise traders. Commodity producers make their decisions on commodity production at date 0, which in turn determine the commodity supply at the spot market that operates later at date 1.

2.1.1 The Spot Market

There is one commodity good in our setting, such as oil or copper. The spot market opens at date 1. The supply of commodity will be determined by the production decisions of commodity producers, which we will discuss shortly. Following Hirshleifer (1988) and Goldstein, Li, and Yang (2014), we assume that the demand for the commodity is implicitly derived from the preference of some (unmodeled) consumers and it is represented by the following linear demand function:

$$ y = \tilde{\theta} + \tilde{\delta} - \tilde{v}. $$

(1)

Here, $\tilde{v}$ is the commodity spot price, which will be endogenously determined in equilibrium. Variables $\tilde{\theta}$ and $\tilde{\delta}$ represent exogenous shocks to consumers’ commodity demand.
Demand shocks $\bar{\theta}$ and $\bar{\delta}$ are normally distributed and mutually independent; that is, $\bar{\theta} \sim N(\bar{\theta}, \tau_\theta^{-1})$ and $\bar{\delta} \sim N(0, \tau_\delta^{-1})$, where $\bar{\theta} \in \mathbb{R}$, $\tau_\theta > 0$, and $\tau_\delta > 0$. We have normalized the mean of $\bar{\delta}$ to 0 since its mean can be absorbed by the mean of $\bar{\theta}$. We assume that traders can learn information about $\bar{\theta}$ but not about $\bar{\delta}$. The learnable component $\bar{\theta}$ represents factors on which there are many sources of information available that traders can purchase and analyze. In contrast, the unlearnable component $\bar{\delta}$ represents factors that are hard to predict given available data sources.

2.1.2 The Futures Market

At date 0, the financial market opens. There are two tradable assets: a futures contract on the commodity and a risk-free asset. We normalize the net risk-free rate as zero. The payoff on the futures contract is the date-1 spot price $\bar{v}$ of the commodity. Each unit of futures contract is traded at an endogenous price $\bar{p}$. Commodity producers, financial traders, and noise traders participate in the financial market. Noise traders represent random transient demands in the futures market and they as a group demand $\bar{\xi}$ units of the commodity futures, where $\bar{\xi} \sim N(0, \tau_\xi^{-1})$ with $\tau_\xi > 0$. We next describe in detail the behavior and information structure of commodity producers and financial traders.
Commodity Producers  There is a continuum of commodity producers, indexed by \( i \). We normalize the mass of commodity producers as 1. Commodity producers are risk averse so that they have hedging motives in the futures market. Specifically, commodity producer \( i \) derives expected utility from her final wealth \( \tilde{W}_{P,i} \) at the end of date 1; she has a constant-absolute-risk-aversion (CARA) utility over wealth: \(-e^{-\beta \tilde{W}_{P,i}}\), where \( \beta > 0 \) is the risk-aversion parameter. Commodity producers make two decisions at date 0. First, they decide on the quantity of commodities to produce, which will in turn determine the commodity supply at the date-1 spot market. Second, they decide on the investment in futures contracts in the date-0 futures market. This investment serves to hedge their commodity production and to speculate on their private information.

When commodity producer \( i \) decides to produce \( x_i \) units of commodities, she pays a production cost:

\[
C(x_i) = \tilde{c}x_i + \frac{1}{2h}x_i^2,
\]

where \( h \) is a positive constant, and \( \tilde{c} \sim N(\bar{c}, \tau_\tilde{c}^{-1}) \) with \( \bar{c} \in \mathbb{R} \) and \( \tau_\tilde{c} > 0 \). Random variable \( \tilde{c} \) represents a supply shock, and we assume that it is public information. Although this supply shock is not crucial in driving the results in our baseline model, it is useful for us to calibrate parameters in the dynamic model analyzed in Section 4, since in reality commodity price patterns are affected by supply shocks. Commodity producers are endowed with private information about the demand shock \( \tilde{\theta} \). Specifically, commodity producer \( i \) receives a private signal \( \tilde{s}_i \) which takes the following form:

\[
\tilde{s}_i = \tilde{\theta} + \tilde{\varepsilon}_i.
\]

Here, \( \tilde{\varepsilon}_i \sim N(0, \tau_{\tilde{\varepsilon}}^{-1}) \) (with \( \tau_{\tilde{\varepsilon}} > 0 \)) and \( \left\{ \tilde{\varepsilon}_i \right\} , \tilde{\theta}, \tilde{\delta}, \tilde{c} \) are mutually independent. The futures price \( \tilde{p} \) is observable to all market participants and thus, commodity producer \( i \)’s information set is \( \{ \tilde{s}_i, \tilde{c}, \tilde{p} \} \).

Commodity producer \( i \)’s problem is then to choose commodity production \( x_i \) and futures investment \( d_{P,i} \) (and investment in the risk-free asset) to maximize

\[
E\left(-e^{-\beta \tilde{W}_{P,i}} \mid \tilde{s}_i, \tilde{c}, \tilde{p}\right)
\]

subject to

\[
\tilde{W}_{P,i} = \tilde{v}x_i - C(x_i) + (\tilde{v} - \tilde{p})d_{P,i}.
\]

Here, \( \tilde{v}x_i - C(x_i) \) is the profit from producing and selling \( x_i \) units of commodities: selling \( x_i \) units of commodities at a later spot price \( \tilde{v} \) generates a revenue of \( \tilde{v}x_i \), which, net of the
production cost $C(x_i)$, gives rise to the operating profit $\tilde{v}x_i - C(x_i)$. The term $(\tilde{v} - \bar{p})d_{P,i}$ is the profit from trading $d_{P,i}$ units of futures contracts. Specifically, at date 0, buying a futures contract is equivalent to buying an asset that costs $\bar{p}$ and generates a payoff equal to the date-1 commodity spot price $\tilde{v}$. In equation (5), we have normalized commodity producer $i$’s initial endowment as 0, which is without loss of generality given the CARA preference.

**Financial Traders** There are two types of financial traders: financial speculators and financial hedgers. Both types of financial traders derive CARA utility from their final wealth at the end of date 1, with risk-aversion coefficients $\gamma_S > 0$ and $\gamma_H > 0$, respectively. We use $\Lambda_S > 0$ and $\Lambda_H > 0$ to respectively denote the masses of financial speculators and financial hedgers. In our setting, what matters is the ratios $\lambda_s \equiv \frac{\Lambda_S}{\gamma_S}$ and $\lambda_H \equiv \frac{\Lambda_H}{\gamma_H}$, and thus in the subsequent analysis, we focus on these two ratios.

Financial speculators trade futures to exploit their superior information. We assume that financial traders observe $\tilde{\theta}$ perfectly. The idea that they are more informed than other market participants is realistic to the extent that financial speculators, such as hedge funds, generally have more sophisticated information-processing capacities. Financial speculators also observe public information $\{\tilde{c}, \bar{p}\}$ and thus, their information set is $\{\tilde{\theta}, \tilde{c}, \bar{p}\}$. Their problem is to choose investment $d_S$ in futures to maximize

$$E \left[-e^{-\gamma_S(\tilde{v} - \bar{p})d_S}\mid \tilde{\theta}, \tilde{c}, \bar{p}\right].$$

Again, without loss of generality, we have normalized the initial endowment of financial speculators to be zero.

Financial hedgers trade futures to hedge positions they have in other assets whose payoffs are correlated with the commodity market (and hence the payoffs on commodity futures). We follow Wang (1994), Easley, O’Hara, and Yang (2014), and Han, Tang, and Yang (2016) in modelling this hedging behavior of financial hedgers. Formally, we assume that at date 0, in addition to the risk-free asset and the futures contract, financial hedgers can invest in another asset or private technology. This can represent a stock index in which financial hedgers typically invest. Another real-world example is commodity-linked notes (CLNs) that are traded over the counter and have payoffs linked to the price of commodity or commodity futures. As documented by Henderson, Pearson, and Wang (2015), the regular issuers of CLNs are big investment banks, who often invest in commodity futures to hedge
their issuance of CLNs. More broadly, introducing this additional asset is a modeling device that is meant to capture the important feature that financial hedgers trade futures partly for their own portfolio diversification and risk management goals, as emphasized by Cheng, Kirilenko, and Xiong (2015).

The net return on the private technology is \( \tilde{\alpha} + \tilde{\eta} \), where \( \tilde{\alpha} \sim N(\bar{\alpha}, \tau_{\alpha}^{-1}) \) and \( \tilde{\eta} \sim N(0, \tau_{\eta}^{-1}) \) with \( \bar{\alpha} \in \mathbb{R}, \tau_{\alpha} > 0, \) and \( \tau_{\eta} > 0 \). Similar to commodity demand shocks, the net return on the private technology also has two components: forecastable component \( \tilde{\alpha} \) and unforecastable component \( \tilde{\eta} \). We normalize the mean of \( \tilde{\eta} \) to 0 since its mean can be absorbed by the mean of \( \tilde{\alpha} \). Variable \( \tilde{\alpha} \) is independent of all other random variables and is privately observable to financial hedgers. Variable \( \tilde{\eta} \) is unforecastable, and importantly, it is correlated with the unforecastable commodity demand shock \( \tilde{\delta} \). We denote the correlation coefficient between \( \tilde{\eta} \) and \( \tilde{\delta} \) as \( \rho \in (-1, 1) \). This correlation is the modeling ingredient that generates the hedging motive of financial traders in the futures market. Financial hedgers’ problem is to choose investment \( d_H \) in futures and investment \( Z_H \) in the private technology (and investment in the risk-free asset) to maximize

\[
E \left[ -e^{-\gamma_H [(\tilde{\delta} - \tilde{\rho}) d_H + (\tilde{\alpha} + \tilde{\eta}) Z_H)]} \mid \tilde{\alpha}, \tilde{\eta}, \tilde{\delta} \right].
\]

Here, \((\tilde{\delta} - \tilde{\rho}) d_H \) captures the profit from trading futures and \((\tilde{\alpha} + \tilde{\eta}) Z_H \) captures the profit from investing in the private technology.

### 2.2 Equilibrium Characterization

In our setting, \((\tilde{\theta}, \tilde{\delta}, \tilde{\xi}, \{\tilde{\varepsilon}_i\}, \tilde{\alpha}, \tilde{\eta}, \tilde{c})\) are the underlying random variables that characterize the economy. They are mutually independent, except that \( \tilde{\delta} \) and \( \tilde{\eta} \) are correlated with each other with correlation coefficient \( \rho \in (-1, 1) \). The tuple \( \mathcal{E} \equiv (\lambda_s, \lambda_H, \beta, h, \tilde{\theta}, \tilde{c}, \tilde{\alpha}, \rho, \tau_\theta, \tau_\delta, \tau_\varepsilon, \tau_\xi, \tau_\alpha, \tau_\eta, \tau_c) \) defines an economy. Given an economy, an equilibrium consists of two subequilibria: the date-1 spot-market equilibrium and the date-0 futures-market equilibrium. A formal definition of an equilibrium is given as follows:

**Definition 1** An equilibrium consists of a spot price function, \( v(\tilde{\theta}, \tilde{\delta}, \tilde{c}, \tilde{\rho}) : \mathbb{R}^4 \rightarrow \mathbb{R} \); a futures price function, \( p(\tilde{\theta}, \tilde{\alpha}, \tilde{c}, \tilde{\xi}) : \mathbb{R}^4 \rightarrow \mathbb{R} \); a commodity production policy, \( x(\tilde{s}_i, \tilde{c}, \tilde{\rho}) : \mathbb{R}^3 \rightarrow \mathbb{R} \); a trading strategy of commodity producers, \( d_P(\tilde{s}_i, \tilde{c}, \tilde{\rho}) : \mathbb{R}^3 \rightarrow \mathbb{R} \); a trading strategy of financial speculators, \( d_S(\tilde{\theta}, \tilde{c}, \tilde{\rho}) : \mathbb{R}^3 \rightarrow \mathbb{R} \); a trading strategy of financial hedgers,
$d_H(\tilde{\alpha}, \tilde{c}, \tilde{p}) : \mathbb{R}^3 \rightarrow \mathbb{R}$; and a strategy of financial hedgers’ investment on the private technology, $Z_H(\tilde{\alpha}, \tilde{c}, \tilde{p}) : \mathbb{R}^3 \rightarrow \mathbb{R}$, such that:

(a) At date 1, the spot market clears, i.e.,
\[ \tilde{\theta} + \tilde{\delta} - v(\tilde{\theta}, \tilde{\delta}, \tilde{c}, \tilde{p}) = \int_0^1 x(\tilde{s}_i, \tilde{c}, \tilde{p}) \, di, \text{ almost surely;} \]  
(b) At date 0, given that $\tilde{v}$ is defined by $v(\tilde{\theta}, \tilde{\delta}, \tilde{c}, \tilde{p})$,
(i) $x(\tilde{s}_i, \tilde{c}, \tilde{p})$ and $d_P(\tilde{s}_i, \tilde{c}, \tilde{p})$ solve for commodity producers’ problem given by (4) and (5);
(ii) $d_S(\tilde{\theta}, \tilde{c}, \tilde{p})$ solves financial speculators’ problem (6);
(iii) $d_H(\tilde{\alpha}, \tilde{c}, \tilde{p})$ and $Z_H(\tilde{\alpha}, \tilde{c}, \tilde{p})$ solve financial hedgers’ problem (7); and
(iii) the futures market clears, i.e.,
\[ \int_0^1 d_P(\tilde{s}_i, \tilde{c}, \tilde{p}) \, di + \Lambda_Sd_S(\tilde{\theta}, \tilde{c}, \tilde{p}) + \Lambda_Hd_H(\tilde{\alpha}, \tilde{c}, \tilde{p}) + \tilde{\xi} = 0, \text{ almost surely.} \]  

We next construct an equilibrium in which the price functions $v(\tilde{\theta}, \tilde{\delta}, \tilde{c}, \tilde{p})$ and $p(\tilde{\theta}, \tilde{\alpha}, \tilde{c}, \tilde{\xi})$ are linear. As standard in the literature, we solve the equilibrium backward from date 1.

### 2.2.1 Spot Market Equilibrium

The commodity demand is given by equation (1). The commodity supply is determined by commodity producers’ date-0 investment decisions. The commodity producers’ problem, given by (4) and (5), can be decomposed as follows:

\[
\max_{x_i+d_{P,i}} \left[ E(\tilde{v} - \tilde{p}| \tilde{s}_i, \tilde{c}, \tilde{p}) (x_i + d_{P,i}) - \frac{\beta \text{Var}(\tilde{v}| \tilde{s}_i, \tilde{c}, \tilde{p}) (x_i + d_{P,i})^2}{2} \right] + \max_{x_i} [\tilde{p}x_i - C(x_i)].
\]  

Solving (10), we have:
\[ x_i + d_{P,i} = \frac{E(\tilde{v}| \tilde{s}_i, \tilde{c}, \tilde{p}) - \tilde{p}}{\beta \text{Var}(\tilde{v}| \tilde{s}_i, \tilde{c}, \tilde{p})}, \]  
\[ x_i = h(\tilde{p} - \tilde{c}). \]

The above expressions are similar to those in Danthine (1978). The intuition is as follows: since both real investment $x_i$ and financial investment $d_{P,i}$ expose a commodity producer to the same risk source $\tilde{v}$, her overall exposure to this risk is given by the standard demand function of a CARA investor, as expressed in (11). In it, the producer chooses a positive (negative) position when the expected spot price is above (below) the futures price, and the size of the position decreases in the risk it entails. Expression (12) says that after controlling the total exposure given by (11), commodity producers essentially treat the futures price $\tilde{p}$ as the commodity selling price when making real production decisions.
Aggregating (12) across all commodity producers delivers the aggregate commodity supply at the spot market:

\[ \int_0^1 x_i \, di = h (\tilde{p} - c). \]  

By the market-clearing condition (8) and equations (1) and (13), we can solve for the spot price \( \tilde{v} \), which is given by the following lemma:

**Lemma 1** (Spot prices) The date-1 spot price \( \tilde{v} \) is given by

\[ \tilde{v} = \tilde{\theta} + \tilde{\delta} + h\tilde{c} - h\tilde{p}. \]  

The second maximization problem in commodity producers’ problem (10) and its solution in (12) demonstrate the feedback effect of the futures market on commodity producers’ production activities. This effect says that an increase in the futures price \( \tilde{p} \) directly encourages commodity producers to supply more commodities. It is related to but distinct from the real effect of financial markets in Leland (1992). In Leland’s setting, a firm who issues shares to maximize profits faces a similar problem as the second maximization problem in (10). As a result, an increase in the stock price causes the firm to issue more equity shares (and implicitly make more real investments). However, in Leland’s setting, the asset payoff is exogenous; in contrast, in our setting, the payoff \( \tilde{v} \) on the futures contract is endogenously affected by the feedback effect, as formalized by Lemma 1. The 2011 G20 Report on Commodities raised the following key question: “(D)oes financial investment in commodity futures affect spot prices?” In our setting, such an effect indeed exists because financial traders’ investments in commodity futures will alter the futures price, which in turn changes the later spot prices. Chen and Linn (2017) find that changes in oil and natural gas field investment measured by drilling rig use respond positively to changes in the futures prices of oil and natural gas. This finding is consistent with the supply channel in (12) and the feedback effect in (14).

### 2.2.2 Futures Market Equilibrium

We conjecture the following linear futures price function:

\[ \tilde{p} = B_0 + B_c \tilde{c} + B_{\theta} \tilde{\theta} + B_{\alpha} \tilde{\alpha} + B_{\xi} \tilde{\xi}, \]  

where the \( B \)-coefficients are endogenous. We next compute the demand function of futures market participants and use the market-clearing condition to construct such a linear price function.
By (11) and (12), commodity producer \(i\)'s demand for the futures contract is

\[
\begin{align*}
    d_P (\tilde{s}_i, \tilde{c}, \tilde{p}) &= \frac{E (\tilde{v} | \tilde{s}_i, \tilde{c}, \tilde{p}) - \tilde{p}}{\beta \text{Var} (\tilde{v} | \tilde{s}_i, \tilde{c}, \tilde{p})} - h (\tilde{p} - \tilde{c}).
\end{align*}
\]

(16)

As mentioned before, a commodity producer trades futures for two reasons. First, she hedges her real commodity production of \(x_i = h (\tilde{p} - \tilde{c})\). Second, because she also has private information \(\tilde{s}_i\) on the later commodity demand and so the later spot price \(\tilde{v}\), she speculates on this private information. The expressions in (16) show how the total demand of the producer in the futures market can be decomposed into these two motives.

By (15), the information contained in the futures price is equivalent to the signal \(\tilde{s}_p\) in predicting demand shock \(\tilde{\theta}\):

\[
\tilde{s}_p = \tilde{p} - B_0 - B_\alpha \tilde{c} - B_\alpha \tilde{\alpha} = \tilde{\theta} + \frac{\tilde{\alpha} - \tilde{\alpha}}{m_{\alpha}} + \frac{\tilde{\xi}}{m_{\xi}},
\]

(17)

where

\[
    m_{\alpha} = \frac{B_{\theta}}{B_{\alpha}} \text{ and } m_{\xi} = \frac{B_{\theta}}{B_{\xi}}.
\]

(18)

Signal \(\tilde{s}_p\) is normally distributed with mean \(\tilde{\theta}\) and precision \(\tau_p\), where

\[
    \tau_p = \left( m_{\alpha}^{-2} \tau_{\alpha}^{-1} + m_{\xi}^{-2} \tau_{\xi}^{-1} \right)^{-1}.
\]

(19)

Precision \(\tau_p\) measures how informative the futures price \(\tilde{p}\) is about the later commodity demand “fundamental” \(\tilde{\theta}\), and so we refer to \(\tau_p\) as “price informativeness.” Using the expression of \(\tilde{v}\) in (14) and applying Bayes’ rule to compute the conditional moments in commodity producer \(i\)'s demand function (16), we can obtain

\[
\begin{align*}
    d_P (\tilde{s}_i, \tilde{c}, \tilde{p}) &= \frac{\tau_{\alpha} \tilde{\theta} + \tau_{\alpha} \tilde{s}_i + \tau_p \tilde{\eta}}{\tau_{\alpha} + \tau_{\alpha} + \tau_p} + h \tilde{c} - (h + 1) \tilde{p} - h (\tilde{p} - \tilde{c}).
\end{align*}
\]

(20)

Solving financial speculators’ problem in (6), we can compute their futures demand as follows:

\[
\begin{align*}
    d_S (\tilde{\theta}, \tilde{c}, \tilde{p}) &= \frac{E (\tilde{v} | \tilde{\theta}, \tilde{c}, \tilde{p}) - \tilde{p}}{\gamma_s \text{Var} (\tilde{v} | \tilde{\theta}, \tilde{c}, \tilde{p})} = \frac{\tau_{\delta}}{\gamma_s} \left[ \tilde{\theta} + h \tilde{c} - (h + 1) \tilde{p} \right],
\end{align*}
\]

(21)

where the second equation follows from the expression of \(\tilde{v}\) in (14) and applying Bayes’ rule.

Using the expression of \(\tilde{v}\) in (14), we can solve financial hedgers’ problem in (7) and find their futures demand as follows:

\[
\begin{align*}
    d_H (\tilde{\alpha}, \tilde{c}, \tilde{p}) &= \frac{E (\tilde{v} | \tilde{\alpha}, \tilde{c}, \tilde{p}) + h \tilde{c} - (h + 1) \tilde{p}}{\gamma_H \left[ 1 - \rho^2 \tau_{\delta}^2 \text{Var} (\tilde{\theta} | \tilde{\alpha}, \tilde{c}, \tilde{p}) \right]} - \frac{\rho \sqrt{\tau_{\eta}}}{\gamma_H \left[ 1 - \rho^2 \tau_{\delta}^2 \text{Var} (\tilde{\theta} | \tilde{\alpha}, \tilde{c}, \tilde{p}) \right] \sqrt{\tau_{\delta}}} \tilde{\alpha}.
\end{align*}
\]

(22)

Financial hedgers invest in futures contracts also for two reasons. First, they have made investment on their private technology, whose payoff is correlated with the commodity market
and thus, financial hedgers trade futures to hedge their investment in the private technology. Second, financial hedgers also speculate on their private information \( \tilde{\alpha} \), because they can use their knowledge about \( \tilde{\alpha} \) to draw inference about fundamental \( \tilde{\theta} \) from the futures price \( \tilde{p} \). Specifically, by (15), financial hedgers’ information set \( \{\tilde{\alpha}, \tilde{c}, \tilde{p}\} \) is equivalent to the signal \( \tilde{q} \) in predicting demand shock \( \tilde{\theta} \):

\[
\tilde{q} = \frac{\tilde{p} - B_0 - B_c \tilde{c} - B_\alpha \tilde{\alpha}}{B_\theta} = \tilde{\theta} + \frac{\tilde{\xi}}{m_\xi},
\]

with \( m_\xi \) being defined in equation (18). Signal \( \tilde{q} \) is normally distributed with mean \( \tilde{\theta} \) and precision \( \tau_q \), where

\[
\tau_q = m_\xi^2 \tau_\xi.
\]

Applying Bayes’ rule to compute \( E(\tilde{\theta}|\tilde{q}) \) and \( \text{Var}(\tilde{\theta}|\tilde{q}) \) and inserting these moments into (22) to replace \( E(\tilde{\theta}|\tilde{\alpha}, \tilde{c}, \tilde{p}) \) and \( \text{Var}(\tilde{\theta}|\tilde{\alpha}, \tilde{c}, \tilde{p}) \), we obtain

\[
d_H(\tilde{\alpha}, \tilde{c}, \tilde{p}) = \frac{\gamma_H \left( \frac{1-\rho^2}{\tau_\delta^2} + \frac{1}{\tau_\theta + \tau_q} \right)}{\gamma_H \left( \frac{1-\rho^2}{\tau_\delta^2} + \frac{1}{\tau_\theta + \tau_q} \right)} \sqrt{\tau_q} \tilde{\alpha}.
\]

We derive the equilibrium futures price function following the standard approach in the literature. That is, we insert demand functions (20), (21), and (25) into the market-clearing condition (9) to solve the price in terms of \( \tilde{c}, \tilde{\theta}, \tilde{\alpha}, \) and \( \tilde{\xi} \), and then compare with the conjectured price function in equation (15) to obtain a system defining the unknown \( B \)-coefficients. Solving this system yields the following proposition:

**Proposition 1 (Futures market equilibrium)** For any given masses \( (\lambda_S, \lambda_H) \in \mathbb{R}^2_{++} \) of financial traders, there exists a linear financial market equilibrium with price function (15), where the \( B \)-coefficients are given in the appendix. The equilibrium is characterized by variable \( m_\xi \in \left( \lambda_S \tau_\delta, \frac{1}{\beta \frac{1}{\tau_\theta + \tau_q} + \frac{1}{\tau_\delta} + \lambda_S \tau_\delta} \right) \), which is determined by

\[
m_\xi = \frac{\tau_\xi}{\beta \frac{1}{\tau_\theta + \tau_q} + \tau_\delta + \lambda_S \tau_\delta},
\]

which is equivalent to a 7th order polynomial of \( m_\xi \). In addition, if \( 8\tau_\theta (1 - \rho^2) > \tau_\delta \), then the equilibrium is the unique linear equilibrium.

The original system characterizing the equilibrium is defined in terms of two ratios, \( m_\xi \) and \( m_\alpha \). It turns out that we can express \( m_\alpha \) as a function of \( m_\xi \), so that the entire system is simplified into a single equation in terms of \( m_\xi \) in Proposition 1. We can further show that \( m_\xi \) is bounded, which will be useful for some analytical proofs in the subsequent section. Although we cannot demonstrate uniqueness for all parameter values, we have tried various
parameter configurations, in particular those that violate the condition $8\tau_\theta (1 - \rho^2) > \tau_\delta$, and found that the equilibrium is always unique.

3 Implications of Commodity Financialization

In this section, we conduct two comparative statics exercises to examine the implications of commodity financialization. First, we treat the mass $\lambda_S$ of financial speculators and the mass $\lambda_H$ of financial hedgers as free parameters and conduct comparative statics with respect to these two parameters, respectively. This exercise helps to gauge the separate effects of financial speculators and financial hedgers. Second, we fix the composition of financial traders and conduct comparative statics with respect to the total size of the financial traders population. Formally, let $\lambda_S + \lambda_H = \bar{\lambda}, \lambda_S = \phi_S \bar{\lambda}, \lambda_H = \phi_H \bar{\lambda}$, where $\bar{\lambda} \geq 0, \phi_S \in (0, 1), \phi_H \in (0, 1)$, and $\phi_S + \phi_H = 1$. We conduct comparative statics with respect to $\bar{\lambda}$. This exercise uses one parameter $\bar{\lambda}$ to proxy for commodity financialization, which therefore gives an idea of the overall effect of financialization.

3.1 Price Informativeness

As mentioned in Section 2.2.2, we use $\tau_p$ to measure price informativeness. It characterizes how much extra information the prevailing futures price $\bar{p}$ conveys about the futures contract’s “fundamental” (which is the commodity demand shock $\tilde{\theta}$ in our setting) to an outsider who observes only public information $\tilde{c}$. Our price informativeness measure is broadly consistent with the concept of “market efficiency,” which refers to the extent to which the prevailing market prices are informative about the future value of the traded assets.\footnote{For example, Brown, Harlow, and Tinic (1988, p. 355-356) write: “the efficient market hypothesis (EMH) claims that the price of a security at any point is a noisy estimate of the present value of the certainty equivalents of its risky future cash flows.” Another relevant quote is: “A market in which prices always ‘fully reflect’ available information is called ‘efficient.’” (Fama, 1970, p. 383). Due to its relation to information and prices, market efficiency is also termed as “informational efficiency” or “price efficiency.”} In this section, we examine the effect of commodity financialization on price informativeness. This is a question that received large attention in the empirical literature, e.g., Raman, Robe, and Yadav (2017), and Brogaard, Ringgenberg, and Sovich (2019).

As shown by equation (19), $\tau_p$ is positively related to two ratios: $m_\xi \equiv \frac{B_\theta}{B_\xi}$ and $m_\alpha \equiv \frac{B_\theta}{B_\alpha}$. 

6 For example, Brown, Harlow, and Tinic (1988, p. 355–356) write: “the efficient market hypothesis (EMH) claims that the price of a security at any point is a noisy estimate of the present value of the certainty equivalents of its risky future cash flows.” Another relevant quote is: “A market in which prices always ‘fully reflect’ available information is called ‘efficient.’” (Fama, 1970, p. 383). Due to its relation to information and prices, market efficiency is also termed as “informational efficiency” or “price efficiency.”
Because $B_\theta > 0$, $B_\xi > 0$, and $B_\alpha < 0$ (by Proposition 1), we have $m_\xi > 0$ and $m_\alpha < 0$. Thus, we will examine $m_\xi$ and $|m_\alpha|$. A higher ratio $m_\xi$ indicates that the price $\tilde{p}$ is more sensitive to the fundamental $\tilde{\theta}$ relative to the exogenous noise trading $\tilde{\xi}$. A higher ratio $|m_\alpha|$ says that the price is more sensitive to $\tilde{\theta}$ relative to the endogenous noise $\tilde{\alpha}$ injected by financial hedgers. Both exogenous noise trading $\tilde{\xi}$ and endogenous noise trading $\tilde{\alpha}$ are important for driving the behavior of price informativeness, as well as other results discussed in subsequent subsections. The following proposition characterizes the effect of commodity financialization on $m_\xi$, $|m_\alpha|$, and $\tau_p$.

**Proposition 2** (Price informativeness)

(a) Suppose $8 \tau_\theta (1 - \rho^2) > \tau_\delta$. When $\lambda_H$ is fixed, increasing $\lambda_S$ will increase $m_\xi$, $|m_\alpha|$, and $\tau_p$. When $\lambda_S$ is fixed, increasing $\lambda_H$ will increase $m_\xi$ but will decrease $\tau_p$ and $|m_\alpha|$.  

(b) Fix $(\phi_S, \phi_H)$. For sufficiently small $\bar{\lambda}$, an increase in $\bar{\lambda}$ will increase $m_\xi$ and $\tau_p$, but will decrease $|m_\alpha|$. For sufficiently large $\bar{\lambda}$, an increase in $\bar{\lambda}$ will increase $m_\xi$, but will decrease $|m_\alpha|$ and $\tau_p$.

Part (a) of Proposition 2 examines the effects of separately changing $\lambda_S$ and $\lambda_H$. As demonstrated by the demand function (21) of financial speculators, their speculative trading injects information $\bar{\theta}$ into the price $\tilde{p}$. So, an increase in the mass $\lambda_S$ of financial speculators makes the price more sensitive to $\bar{\theta}$ relative to both $\tilde{\xi}$ and $\tilde{\alpha}$, thereby increasing $m_\xi$, $|m_\alpha|$, and $\tau_p$. By the demand function (22) of financial hedgers, their hedging-motivated trading injects noise $\tilde{\alpha}$ into the price $\tilde{p}$. Thus, an increase in the mass $\lambda_H$ of financial hedgers makes the price more sensitive to $\tilde{\alpha}$. This directly decreases $|m_\alpha|$ and price informativeness $\tau_p$. In response to the lower price informativeness, commodity producers rely less on the price and more on their own private information $\tilde{s}_i$ about $\bar{\theta}$ when trading futures, which therefore injects more information $\bar{\theta}$ into the price and increases $m_\xi$. Overall, the direct effect of a lower $|m_\alpha|$ dominates the indirect effect of a higher $m_\xi$, so that price informativeness $\tau_p$ decreases with $\lambda_H$.

Part (b) of Proposition 2 considers the effect of increasing the total population size $\bar{\lambda}$ of financial traders while fixing the proportions of financial speculators and financial hedgers. This exercise corresponds to a natural thought experiment in which both types of financial capital flow into the futures market roughly in the same speed. As Part (a) of Proposition 2
shows, financial speculators and financial hedgers have opposite effects on price informativeness $\tau_p$, and thus, analyzing the overall effect of $\tilde{\lambda}$ is non-trivial and new to the literature. Part (b) of Proposition 2 suggests that increasing the population size $\tilde{\lambda}$ of financial traders first improves price informativeness and then harms price informativeness.

To understand this result, we examine in detail the demand functions of commodity producers and financial traders, which are given by equations (20), (21), and (25), respectively. We use $\kappa_{P,\theta}$ to measure the sensitivity of commodity producers’ aggregate order flow to information $\tilde{\theta}$, i.e.,

$$\kappa_{P,\theta} \equiv \frac{\partial \int_0^1 \tilde{d}_P (\tilde{s}_i, \tilde{c}, \tilde{p}) \, d\tilde{i}}{\partial \tilde{\theta}} = \frac{\tau_{\theta}}{\tau_{\theta} + \tau_{\epsilon} + \tau_p} \beta \left( \frac{1}{\tau_{\theta} + \tau_{\epsilon} + \tau_p} + \frac{1}{\tau_{\theta}} \right),$$

where the last equality follows from equation (20). Similarly, we can compute the sensitivity of financial speculators’ total order flow to information $\tilde{\theta}$ and the sensitivity of financial hedgers’ total order flow to noise $\tilde{\alpha}$ as follows:

$$\frac{\partial \lambda_S d_S (\tilde{\theta}, \tilde{c}, \tilde{p})}{\partial \tilde{\theta}} = \lambda_S \tau_{\delta} = \tilde{\lambda} \phi_S \tau_{\delta},$$

$$- \frac{\partial \lambda_H d_H (\tilde{\alpha}, \tilde{c}, \tilde{p})}{\partial \tilde{\alpha}} = \lambda_H \kappa_{H,\alpha} = \tilde{\lambda} \phi_H K_{H,\alpha},$$

with $\kappa_{H,\alpha} \equiv \frac{\tau_{\theta}}{(\tau_{\theta} + \tau_{\epsilon}) m_{\alpha}} + \frac{\rho \sqrt{\tau_{\eta}}}{1 - \rho^2 + \frac{1}{\tau_{\theta} + \tau_{\epsilon}} + \frac{1}{\tau_{\theta} + \tau_{\epsilon}} \sqrt{\tau_{\delta}}}.

Equipped with these notations and inserting the demand functions (20), (21), and (25) into the market-clearing condition (9), we have

$$\kappa_{P,\theta} \tilde{\theta} + \tilde{\lambda} \phi_S \tau_{\delta} \tilde{\theta} - \tilde{\lambda} \phi_H \kappa_{H,\alpha} \tilde{\alpha} + \tilde{\xi} - L (\tilde{p}, \tilde{c}) = 0, \quad (27)$$

where $L (\tilde{p}, \tilde{c})$ is a known linear function that absorbs all the other terms unrelated to information or noise in the order flows of market participants. In the above market-clearing condition, the trading of commodity producers and of financial speculators injects information $\tilde{\theta}$ into the aggregate demand, the trading of financial hedgers injects endogenous noise $\tilde{\alpha}$ into the aggregate demand, and noise trading injects exogenous noise $\tilde{\xi}$ into the aggregate demand.

In (27), moving $L (\tilde{p}, \tilde{c})$ to the right-hand side and dividing both sides by $\kappa_{P,\theta} + \tilde{\lambda} \phi_S \tau_{\delta}$ lead to the following signal in predicting fundamental $\tilde{\theta}$:

$$\tilde{\theta} - \frac{\tilde{\lambda} \phi_H \kappa_{H,\alpha} \tilde{\alpha}}{\kappa_{P,\theta} + \tilde{\lambda} \phi_S \tau_{\delta}} + \frac{1}{\kappa_{P,\theta} + \tilde{\lambda} \phi_S \tau_{\delta}} \tilde{\xi} = \frac{L (\tilde{p}, \tilde{c})}{\kappa_{P,\theta} + \tilde{\lambda} \phi_S \tau_{\delta}} = \tilde{s}_p. \quad (28)$$
This signal gives the informational content in the aggregate order flow. In equilibrium, it must coincide with \( \tilde{s}_p \) given by equation (17).

In equation (28), it is clear that increasing \( \tilde{\lambda} \) has two offsetting effects on the informativeness of \( \tilde{s}_p \): first, it lowers the noise \( \frac{1}{\kappa_{P_\theta} + \lambda S \tau_d} \tilde{\xi} \) that is related to the exogenous noise trading; second, it increases the noise \( \frac{\lambda \phi_H \kappa_{H,\alpha}}{\kappa_{P_\theta} + \lambda S \tau_d} \tilde{\alpha} \) brought in endogenously by financial hedgers. When \( \tilde{\lambda} \) is small—for instance, when \( \tilde{\lambda} \approx 0 \)—the endogenous noise \( \frac{\lambda \phi_H \kappa_{H,\alpha}}{\kappa_{P_\theta} + \lambda S \tau_d} \tilde{\alpha} \) added by financial hedgers is relatively small and thus, the main effect of increasing \( \tilde{\lambda} \) is to lower \( \frac{1}{\kappa_{P_\theta} + \lambda S \tau_d} \tilde{\xi} \). As a result, the price signal \( \tilde{s}_p \) becomes more informative about \( \tilde{\theta} \) when \( \tilde{\lambda} \) increases from a very small value. In contrast, as \( \tilde{\lambda} \) becomes very large, the added noise \( \frac{\lambda \phi_H \kappa_{H,\alpha}}{\kappa_{P_\theta} + \lambda S \tau_d} \tilde{\alpha} \) eventually dominates the noise \( \frac{1}{\kappa_{P_\theta} + \lambda S \tau_d} \tilde{\xi} \), and the price signal \( \tilde{s}_p \) becomes less informative about the fundamental \( \tilde{\theta} \).

The hump-shaped relation between \( \tilde{\lambda} \) and \( \tau_p \) sheds light on recent empirical evidence finding mixed results on the question of how commodity financialization affects market efficiency. Raman, Robe, and Yadav (2017) document that the electronification of the U.S. crude oil futures trading in 2006 brought about a massive growth in intraday activity by “non-commercial” institutional financial traders. In their sample, this financialization of intraday trading activity had a positive impact on price efficiency. In contrast, Brogaard, Ringgenberg, and Sovich (2019) examine the financialization of commodity index markets and find that financialization distorts the informational content in the futures price. One possibility to reconcile the two based on our findings is that the U.S. crude oil futures market is the world’s largest commodity market, and so an influx of financial capital into this market corresponds to a relatively small value of \( \tilde{\lambda} \), and so the positive effect on price informativeness in Raman, Robe, and Yadav (2017) is expected in our model. In other markets, \( \tilde{\lambda} \) may be relatively large and thus increasing \( \tilde{\lambda} \) lowers \( \tau_p \), as documented in Brogaard, Ringgenberg, and Sovich (2019).

Figure 2 graphically illustrates Proposition 2 for the following parameter configuration: \( \tau_\theta = \tau_\alpha = 1, \tau_\epsilon = \tau_\delta = \tau_\xi = \tau_\eta = 10, \gamma_S = \gamma_H = \beta = 10, h = 1, \) and \( \rho = 0.5 \). Consistent with Proposition 2, price informativeness \( \tau_p \) increases with \( \lambda_S \), decreases with \( \lambda_H \), and is hump-shaped in \( \tilde{\lambda} \).

Although we focus on comparative static analysis with parameter \( \lambda \)'s, some other parameters deliver similar results. One such parameter is \( \rho \), which is the correlation coefficient.
This figure plots the effects of commodity financialization on price informativeness. In Panels a1-a3, we set $\lambda_H = 0.1$. In Panels b1-b3, we set $\lambda_S = 0.1$. In Panels c1-c3, we set $\phi_H = \phi_S = 0.5$. The other parameters are: $\tau_\theta = \tau_\alpha = 1, \tau_\epsilon = \tau_\delta = \tau_\xi = \tau_\eta = 10, \gamma_S = \gamma_H = \beta = 10, h = 1$, and $\rho = 0.5$.

between the unlearnable commodity demand shock $\tilde{\delta}$ and the unlearnable private technology shock $\tilde{\eta}$. By the demand function (22) of financial hedgers, parameter $\rho$ controls the strength of financial hedgers’ hedging motive. Thus, increasing the value of $|\rho|$ is similar to increasing the population size $\lambda_H$ of financial hedgers. Figure 3 confirms this statement using the same parameter values as those in Figure 2.

### 3.2 Futures Price Biases

The literature has long been interested in “futures price bias,” which is the deviation of the futures price from the expectation of the later spot price, $E(\tilde{\delta} - \tilde{p})$. A downward bias in the futures price is termed “normal backwardation,” while an upward bias in the futures price is
This figure plots the effects of the hedging motive parameter $\rho$ on price informativeness. The other parameters are: $\tau_\theta = \tau_\alpha = 1$, $\tau_\varepsilon = \tau_\delta = \tau_\xi = \tau_\eta = 10$, $\gamma_S = \gamma_H = \beta = 10$, $h = 1$, and $\lambda_S = \lambda_H = 0.1$.

termed “contango.” A major branch of literature on futures pricing has attributed bias to hedging pressures of commodity producers (e.g., Keynes, 1930; Hicks, 1939; Hirshleifer, 1988, 1990). Hamilton and Wu (2014) document that the futures price bias in crude oil futures on average decreased since 2005. Regulators are also very concerned about how commodity financialization affects the average futures price. As mentioned in the Introduction, the 2011 G20 Report on Commodities asked: “(D)oes increased financial investment alter demand for and supply of commodity futures in a way that moves prices away from fundamentals and/or increase their volatility?” We now explore how the futures price bias is affected by financialization in our model in light of the risk sharing and information effects that we highlight. The following proposition characterizes the futures price bias in our setting.

7In practice, the terms “normal backwardation” and “contango” are also used to refer to the bias between contemporaneous spot price and futures price. Capturing this definition exactly in our model would require us to extend the setting, and so, to keep it simple, we follow the literature such as Hirshleifer (1990) and define those terms as the difference between the current futures price and the expected value of the later spot price.

8The empirical variable examined by Hamilton and Wu (2014) is the futures risk premium, which is equivalent to the futures price bias in our analysis.
Proposition 3 (Futures price bias) The futures price bias is

\[ E(\tilde{v} - \tilde{p}) = \frac{h(\tilde{\theta} - \tilde{c})}{h+1} + \frac{\lambda H \rho \sqrt{\tau_\alpha \delta}}{(1 - \rho^2 + \frac{1}{\tau_\theta + \tau_q}) \sqrt{\tau_\delta}} \]  

\[ \text{risk sharing by speculators} \] 

\[ + \frac{\lambda_H}{\tau_\delta} + \frac{\beta}{\tau_\theta + \tau_q} \frac{(\tau_\theta + \tau_\epsilon + \tau_p + \tau_\delta)}{\tau_\theta + \tau_q} + \frac{h}{h+1} \]  

\[ \text{risk sharing by hedgers} \] 

\[ \text{learning by commodity producers} \]

Thus, \( E(\tilde{v} - \tilde{p}) > 0 \) if and only if

\[ \frac{h(\tilde{\theta} - \tilde{c})}{h+1} + \frac{\lambda H \rho \sqrt{\tau_\alpha \delta}}{(1 - \rho^2 + \frac{1}{\tau_\theta + \tau_q}) \sqrt{\tau_\delta}} > 0. \]  

In equation (29), commodity financialization affects the bias through three channels. The first channel is captured by the term \( \frac{\lambda_H \rho \sqrt{\tau_\alpha \delta}}{(1 - \rho^2 + \frac{1}{\tau_\theta + \tau_q}) \sqrt{\tau_\delta}} \) in the numerator of equation (29), which says that the hedging-motivated trading from financial hedgers either strengthens or offsets the hedging needs of commodity producers, depending on the sign of \( \tilde{\alpha} \). In particular, through this channel, commodity financialization not only affects the absolute magnitude of the bias but also its sign.

We can see that, depending on the sign of \( \frac{h(\tilde{\theta} - \tilde{c})}{h+1} + \frac{\lambda H \rho \sqrt{\tau_\alpha \delta}}{(1 - \rho^2 + \frac{1}{\tau_\theta + \tau_q}) \sqrt{\tau_\delta}} \), there can be either a downward bias or an upward bias in futures prices, that is, \( E(\tilde{v} - \tilde{p}) > 0 \) if and only if condition (30) is satisfied. Intuitively, when the average commodity demand shock \( \tilde{\theta} \) is high relative to the average production cost shock \( \tilde{c} \), commodity producers tend to produce more commodities and thus they will short more futures to hedge their commodity production. Similarly, when the average return \( \tilde{\alpha} \) on financial hedgers’ private investment technology is high, they tend to short more futures to hedge their investment in the private technology. Both shorting forces from commodity producers and financial hedgers push down the futures price, which leads to a downward bias in futures price (normal backwardation). By contrast, when \( \frac{\tilde{\theta} - \tilde{c}}{2} \) and \( \tilde{\alpha} \) are relatively small, the futures price is biased upward, leading to a contango.

Fama and French (1987) used 21 commodities to test the futures risk premium hypothesis, and indeed, they found that some markets feature “normal backwardation,” while others feature “contango.”

The other two channels through which commodity financialization affects the futures price bias are reflected in the denominator of equation (29). Through these two channels,
commodity financialization only affects the absolute magnitude of the bias but not its sign. First, the terms $\lambda_S \tau_\delta$ and $\frac{\lambda_H}{1 - \rho^2 \tau_\delta}$ in the denominator of equation (29) capture that the newly added financial speculators and financial hedgers directly share more risk that is loaded off from the hedging needs of commodity producers and financial hedgers. Second, the term $\frac{(\tau_{\theta} + \tau_{\Sigma} + \tau_{\delta}) \tau_\delta}{\beta (\tau_{\theta} + \tau_{\Sigma} + \tau_{\rho} + \tau_{\delta})}$ in the denominator of equation (29) means that the presence of financial traders affects price informativeness $\tau_p$, which in turn changes the risk perceived by commodity producers through their learning from the futures price. If we focus on these two denominator-related channels by setting $\bar{\alpha} = 0$, we can find the following sufficient conditions under which commodity financialization reduces the futures price bias.

**Corollary 1** Suppose $\bar{\alpha} = 0$ and $8\tau_\theta (1 - \rho^2) > \tau_\delta$. When the mass $\lambda_H$ of financial hedgers is fixed, increasing the mass $\lambda_S$ of financial speculators will decrease $|E(\hat{v} - \hat{p})|$. When the mass $\lambda_S$ of financial speculators is fixed, increasing the mass $\lambda_H$ of financial hedgers will decrease $|E(\hat{v} - \hat{p})|$ for sufficiently large $\lambda_H$.

Once we add back the numerator effect by setting $\bar{\alpha} \neq 0$, the patterns become more complicated. We use Figure 4 to illustrate the results by plotting the bias $E(\hat{v} - \hat{p})$ and its absolute value $|E(\hat{v} - \hat{p})|$ against the masses of financial traders using the same parameter values as in Figure 2. Here, we set $\bar{\alpha} = 1, \bar{\theta} = 0$, and $\bar{c} = 1$. Under this parameter configuration, condition (30) in Proposition 3 is violated for small values of $\lambda_H$, so that $E(\hat{v} - \hat{p}) < 0$. This immediately implies that the bias starts from a negative value in Panels a2 and a3 of Figure 4. In Panel a2, as $\lambda_H$ gradually increases, condition (30) is eventually satisfied, so that the bias becomes positive for large values of $\lambda_H$. In Panel a3, as $\lambda$ gradually increases, $\lambda_H$ increases accordingly, and condition (30) is again eventually satisfied; thus, the bias becomes positive for large values of $\lambda$ as well.

Interestingly, the bias also switches its sign in Panel a1 of Figure 4, where we vary $\lambda_S$ for a fixed $\lambda_H$. This result comes from the learning behavior of financial hedgers. Specifically, in Panel a1, we have set $\lambda_H = 0.1$, and under this $\lambda_H$ value, condition (30) is violated for small values of $\lambda_S$, so that $E(\hat{v} - \hat{p})$ starts as negative. As $\lambda_S$ gradually increases, the trading of financial speculators injects more information into the futures price, and so financial hedgers can read more information from the price and trade more aggressively, which strengthens their hedging motive. Formally, when $\lambda_S$ increases, $m_\xi$ increases (according to Proposition 2).
This figure plots the effects of commodity financialization on futures price biases. In Panels a1 and b1, we set $\lambda_H = 0.1$. In Panels a2 and b2, we set $\lambda_S = 0.1$. In Panels a3 and b3, we set $\phi_H = \phi_S = 0.5$. The other parameters are: $\tau_\theta = \tau_\alpha = 1$, $\tau_\epsilon = \tau_\delta = \tau_\xi = \tau_\eta = 10$, $\gamma_S = \gamma_H = \beta = 10$, $\tilde{\theta} = 0$, $\tilde{c} = \tilde{\alpha} = 1$, $h = 1$, and $\rho = 0.5$.

and thus, by equation (24), $\tau_q$ increases, leading to a higher $\lambda_H \rho \sqrt{\tau_\xi \tau_\eta}$. This result also highlights the importance of exogenous noise trading $\tilde{\xi}$, because the value of $\tau_q$ is determined by parameters governing $\tilde{\xi}$.

In Panels b1-b3 of Figure 4, $|E(\tilde{v} - \tilde{p})|$ exhibits non-monotone patterns due to the interactions among the three channels mentioned above. In particular, $|E(\tilde{v} - \tilde{p})|$ can increase with commodity financialization parameters $\lambda_S$, $\lambda_H$, and $\bar{\lambda}$, which provides some justification to the concerns voiced in policy circles.

### 3.3 Commodity-Equity Market Comovement

The empirical literature has been actively debating whether commodity financialization strengthens the comovement between the commodity futures market and the equity mar-
ket. Gorton and Rouwenhorst (2006) demonstrate that before 2004, commodity returns had negligible correlations with equity returns. Tang and Xiong (2012) document that the correlation between the Goldman Sachs Commodity Index (GSCI) and the S&P 500 stock returns rose after 2004, and was especially high in 2008, which is concurrent with the financialization of commodities. Cheng and Xiong (2014) suggest that commodity financialization has contributed to the sharp spike in the commodity-equity correlation during 2009–2011. Büyüksahin and Robe (2011, 2014) further link the increased correlation between commodities and stocks to the trading of hedge funds, especially those funds that are active in both equity and commodity futures markets. However, Bhardwaj, Gorton, and Rouwenhorst (2016) argue that the commodity-equity correlation falls back to its normal level after 2011, and they instead point to business cycles as the driving force of commodity-equity correlation patterns.

Our model can help shed light on the commodity-equity market comovement by interpreting financial hedgers’ additional investment opportunity as stocks. By construction, the return on stocks is simply $\tilde{\alpha} + \tilde{\eta}$ (investing one dollar at date 0 becomes $1 + \tilde{\alpha} + \tilde{\eta}$ dollars at date 1). We measure the return on futures by $\tilde{v} - \tilde{p}$: buying a futures contract at date 0 costs $\tilde{p}$; the contract matures at date 1, and its date-1 price changes to $\tilde{v}$ accordingly. Thus, this measure is effectively consistent with the empirical practice of constructing futures returns from the futures price data. We can capture the commodity-equity comovement by the covariance $\text{Cov}(\tilde{v} - \tilde{p}, \tilde{\alpha} + \tilde{\eta})$. The hedging-motivated trading of financial hedgers injects the forecastable component $\tilde{\alpha}$ in stock returns into the futures price $\tilde{p}$, which leads to extra comovement between futures returns $\tilde{v} - \tilde{p}$ and stock returns $\tilde{\alpha} + \tilde{\eta}$, as formalized by the following proposition.

**Proposition 4** (Commodity-equity market comovement)

(a) The covariance between stock returns $\tilde{\alpha} + \tilde{\eta}$ and futures returns $\tilde{v} - \tilde{p}$ is positive if and only if the correlation $\rho$ between the unforecastable component $\tilde{\eta}$ in stock returns and the unforecastable component $\tilde{\delta}$ in commodity demand is positive. That is, $\text{Cov}(\tilde{v} - \tilde{p}, \tilde{\alpha} + \tilde{\eta}) > 0$ if and only if $\text{Cov}(\tilde{\delta}, \tilde{\eta}) > 0$.

(b) For fixed $\lambda_S$, we have $\frac{\partial}{\partial \lambda} \text{Cov}(\tilde{v} - \tilde{p}, \tilde{\alpha} + \tilde{\eta}) > 0$ for sufficiently small $\lambda_H$. For fixed $(\phi_S, \phi_H)$, we have $\frac{\partial}{\partial \lambda} \text{Cov}(\tilde{v} - \tilde{p}, \tilde{\alpha} + \tilde{\eta}) > 0$ for sufficiently small $\bar{\lambda}$.
Figure 5: Commodity-Equity Market Comovement

This figure plots the effects of commodity financialization on commodity-equity market comovement. In Panel a, we set $\lambda_H = 0.1$. In Panel b, we set $\lambda_S = 0.1$. In Panel c, we set $\phi_H = \phi_S = 0.5$. The other parameters are: $\tau_\theta = \tau_\alpha = 1$, $\tau_e = \tau_\delta = \tau_\xi = \tau_\eta = 10$, $\gamma_S = \gamma_H = \beta = 10$, $h = 1$, and $\rho = 0.5$.

Figure 5 provides a graphical illustration for the effect on commodity-equity market comovement. Here, we plot a normalized measure, the correlation coefficient between stock returns and futures returns conditional on public information $\tilde{c}$, $\text{Corr}(\tilde{\alpha} + \tilde{\beta} \tilde{\epsilon}, \tilde{\nu}, \tilde{\nu}_{\tilde{p}})$, against the masses of financial traders. The parameter values are the same as those in Figure 2. We highlight two observations in Figure 5. First, consistent with Proposition 4, as we fix $\lambda_S$ and only increase $\lambda_H$ in Panel b, or increase both $\lambda_H$ and $\lambda_S$ in a fixed proportion in Panel c, the hedging-motivated trading from more financial hedgers strengthens the commodity-equity market comovement. Second, the effect of $\lambda_S$ differs from the effect of $\lambda_H$. Specifically, in Panel a, increasing the mass $\lambda_S$ of financial speculators can either strengthen or weaken the commodity-equity market comovement. This is due to the interaction between two forces. First, as $\lambda_S$ increases, the futures market is more driven by its own fundamental $\tilde{\theta}$ rather than noise $\tilde{\alpha}$, which weakens the commodity-equity market comovement. Second, as $\lambda_S$ increases, the futures price is more informative about $\tilde{\theta}$, which encourages financial hedgers to trade more aggressively, injecting more noise $\tilde{\alpha}$ into the futures market and thereby strengthening the commodity-equity market comovement.

In our theory, the hedging-motivated trades of financial traders injects the forecastable component $\tilde{\alpha}$ in stock returns into the futures price $\tilde{p}$, which leads to extra comovement between futures returns $\tilde{\nu} - \tilde{p}$ and stock returns $\tilde{\alpha} + \tilde{\eta}$. Our theory therefore predicts that fi-
Financialization can indeed increase the commodity-equity correlation, as illustrated by Panels b and c of Figure 5. Also note that in our settings, it is financial hedgers, active in both equity and commodity futures markets, who connect further the commodity and equity markets. This is consistent with the empirical channel documented by Büyükşahin and Robe (2011, 2014).\footnote{This view also complements Basak and Pavlova (2016) who obtain the increase in equity-commodity co-movement through benchmarking institutional investors to a commodity index that serves as a new common factor on which all assets load positively.} Under our theory, the cyclicality of financialization can potentially drive the cyclicality of commodity-equity correlation. For instance, if the market first became financialized in 2009–2011 and then de-financialized afterwards, the commodity-equity correlation would exhibit the pattern documented by Bhardwaj, Gorton, and Rouwenhorst (2016), which we have verified using our dynamic model as a laboratory. This provides a testable view complementary to the business-cycle based explanation suggested by Bhardwaj, Gorton, and Rouwenhorst (2016).

### 3.4 Return Predictiveness of Financial Trading

Another key part of the debate on commodity financialization is about whether there is a positive correlation between the trading of financial traders and commodity futures prices. For instance, Singleton (2014) provides evidence of positive price impact of CITs on futures prices of crude oil, while Büyükşahin and Harris (2011) and Hamilton and Wu (2015) find little evidence of CIT positions being predictive of futures prices.

To capture whether and how financial trading predicts futures returns, we compute the correlation coefficients between futures returns and financial speculators’ trading positions and financial hedgers’ trading positions, conditional on public information $\tilde{c}$ as follows:

\[
\text{Predictiveness of speculators’ trading} \quad : \quad Corr(\Lambda_S d_S, \tilde{v} - \tilde{p}|\tilde{c});
\]

\[
\text{Predictiveness of hedgers’ trading} \quad : \quad Corr(\Lambda_H d_H, \tilde{v} - \tilde{p}|\tilde{c}).
\]

In Figure 6, we plot the above two correlation coefficients against the total mass $\bar{\lambda}$ of financial traders for the same parameter values as those in Figure 2. Financial speculators’ trading and financial hedgers’ trading predict futures returns in different ways. Specifically, $Corr(\Lambda_S d_S, \tilde{v} - \tilde{p}|\tilde{c})$ is always positive while $Corr(\Lambda_H d_H, \tilde{v} - \tilde{p}|\tilde{c})$ can be either positive or negative. Intuitively, according to demand function (21), the trading $\Lambda_S d_S$ of financial
This figure plots the effects of commodity financialization on the correlation between financial positions and commodity returns. The parameter values are: \( \tau_\theta = \tau_\alpha = 1, \tau_\epsilon = \tau_\delta = \tau_\xi = \tau_\eta = 10, \gamma_S = \gamma_H = \beta = 10, \bar{\theta} = 0, \bar{\epsilon} = 1, h = 1, \rho = 0.5, \) and \( \phi_H = \phi_S = 0.5. \)

speculators is driven by information \( \bar{\theta} \), which will be reflected in the later spot price \( \bar{v} \), and thus, \( \Lambda_S d_S \) is positively related to futures returns \( \bar{v} - \bar{p} \). By the demand function (22), the trading \( \Lambda_H d_H \) of financial hedgers has two components, a hedging-motivated component and a speculative component. The hedging-motivated component generates the endogenous noise \( \alpha \) in the futures market and negatively predicts futures returns. The speculative component is driven by information and positively predicts futures returns.

Figure 6 suggests two messages. First, it is easier to use financial speculators to identify return predictiveness than financial hedgers, since the former has an unambiguous sign. To the extent that CITs and managed money correspond respectively to financial hedgers and financial speculators in our model, the return predictiveness of managed money is easier to be identified than that of CITs. This is consistent (i) with Büyükşahin and Robe (2014) and Cheng, Kirilenko, and Xiong (2015), who find strong evidence linking hedge fund trading to commodity future returns, and (ii) with Büyükşahin and Harris (2011) and Hamilton and Wu (2015), who find little evidence of CIT positions being predictive of futures prices.
Second, if one could differentiate the speculative trading and hedging-motivated trading of financial hedgers (CITs in practice), then one could in principle find a positive correlation between the speculative trading and futures returns and a negative correlation between the hedging-motivated trading and futures returns. This idea is broadly consistent with Cheng, Kirilenko, and Xiong (2015) who emphasize that the trading motives of financial traders are important for identifying their predictiveness for futures returns. Cheng, Kirilenko, and Xiong (2015) use CBOE Volatility Index (VIX) as an exogenous shock to identify whether financial traders initiate the trades or trade to accommodate other traders and find strong relation between financial trading and futures price changes. Our analysis suggests that speculative trading, either from financial speculators or from financial hedgers, positively predicts futures returns, and that hedging-motivated trading from financial hedgers negatively predicts futures returns.

3.5 Operating Profits and Producer Welfare

We now turn to analyze the effect of commodity financialization on the profits and welfare of commodity producers. Such questions have been discussed in the empirical literature (e.g., Brogaard, Ringgenberg, and Sovich, 2019). We measure the welfare of commodity producers by the ex ante certainty equivalent, $CE_P \equiv -\frac{1}{\beta} \ln \left[ E \left( e^{-\beta [\bar{v} x(\bar{s}, \bar{c}, \bar{p}) - C(x(\bar{s}, \bar{c}, \bar{p})) + (\bar{v} - \bar{p}) d_P(\bar{s}, \bar{c}, \bar{p})])} \right) \right]$, where $d_P(\bar{s}, \bar{c}, \bar{p})$ and $x(\bar{s}, \bar{c}, \bar{p})$ are the equilibrium trading strategy and production policy. The expected operating profit of commodity producers is $E[\pi_i] \equiv E[\bar{v} x_i - C(x_i)]$, which is an easier object to analyze in empirical research.

In Figure 7, we plot these two variables against the masses of financial traders for the same parameter values as those in Figure 2. Financial speculators and financial hedgers affect producer welfare in different ways. Specifically, as more financial speculators enter the futures market, commodity producers are worse off in Panel a2. In contrast, as more financial hedgers enter the futures market, commodity producers are better off in Panel b2.

We next focus on the bottom panels, where both financial speculators and financial hedgers increase and are thus closer to reality. Panel c1 shows that operating profits of commodity producers are hump-shaped in $\bar{\lambda}$, which has a similar pattern as price informativeness (see Part (b) of Proposition 2 and Panel c3 of Figure 2). This is consistent with Brogaard, Ringgenberg, and Sovich (2019). They find that after the spike in commodity

30
This figure plots the effects of commodity financialization on operation profits and producer welfare. In Panels a1 and a2, we set $\lambda_H = 0.1$. In Panels b1 and b2, we set $\lambda_S = 0.1$. In Panels c1 and c2, we set $\phi_H = \phi_S = 0.5$. The other parameters are: $\tau_\delta = \tau_\alpha = 1$, $\tau_\epsilon = \tau_\xi = \tau_\eta = 10$, $\gamma_S = \gamma_H = \beta = 10$, $\tilde{\lambda} = 0$, $\tilde{c} = 1$, $h = 1$, and $\rho = 0.5$.

Financialization in 2004, the informational efficiency of futures index prices decreased and those firms using index commodities saw a decrease in their profits. The intuition is that lower price informativeness causes commodity producers to make less efficient production decisions, which leads to a decrease in their operating profits.

However, a lower (higher) profit does not necessarily translate into a lower (higher) welfare for producers. In fact, Panel c2 of Figure 7 shows that the pattern of producer welfare $CE_P$ is opposite to the pattern of operating profits $E[\pi_i]$. The welfare pattern is a result of the effect of price informativeness on producers’ trading opportunities. As futures prices become more informative, commodity producers have fewer opportunities to exploit their information.
advantage and so their trading gains will deteriorate. In addition, their hedging and risk sharing opportunities are diminished when prices are more informative. This is related to the well-known Hirshleifer effect (1971).\textsuperscript{10} These effects end up dominating the benefit from more information in prices. To further examine this welfare result, we have considered an extension in which some commodity producers trade futures while others do not. In this extended setting, we find that for those commodity producers who do not trade futures, welfare and operating profits exhibit the same pattern as price informativeness, consistent with the intuition above that more informative futures prices allow more efficient production decisions.

Taken together, our analysis suggests that researchers should carefully differentiate among price efficiency, operating profits, and welfare when making normative statements. For instance, in Brogaard, Ringgenberg, and Sovich (2019), both price efficiency and operating profits deteriorate after 2004. This may suggest that in practice, commodity financialization harms those commodity producers who do not trade futures. However, for those commodity producers who do trade futures, they may actually benefit from commodity financialization. To make a welfare statement, a formal model such as ours is needed.

We can also analyze the implications of commodity financialization for the welfare of financial traders, the welfare of noise traders, and aggregate welfare. We use the ex ante certainty equivalents $CE_S$ and $CE_H$ to measure the welfare of financial speculators and financial hedgers. We do not have a utility function for noise traders and so we cannot compute their welfare. Instead, we follow the microstructure literature (e.g., Chowdhry and Nanda, 1991; Subrahmanyam, 1991; Leland, 1992; Easley, O’Hara, and Yang, 2016) and use expected revenue $ER_N$ as a proxy for their welfare: $ER_N \equiv E[(\tilde{v} - \tilde{p}) \tilde{\xi}]$. The aggregate welfare is thus defined as the summation of all agents’ payoff, that is, $WEL \equiv CE_P + \Lambda_S CE_S + \Lambda_H CE_H + ER_N$. We find that the results for $CE_S$, $CE_H$, $ER_N$, and $WEL$ are sensitive to parameter values, so that no general conclusions can be drawn.

\textsuperscript{10}See Kurlat and Veldkamp (2015) and Goldstein and Yang (2017) for more discussions on the negative welfare effect of reduction in trading opportunities.
4 The Dynamic Model

The static model provides a simple framework that allows us to see how different forces of commodity financialization interact to determine market outcomes. Because the issues examined in the empirical literature deeply involve time variation in variables of interest, we expand our model into a dynamic OLG setting to better map to the empirical settings and show how the interaction between different forces drives time variations in the key variables of interest. Methodology wise, our analysis also provides an approach to computing a non-stationary equilibrium in which the sizes of investor populations increase over time.

4.1 The OLG Setting

Time is discrete, \( t = 0, 1, 2, \ldots \). The timeline of the economy is described by Figure 8. As in the baseline model, there are one commodity good and a one-period futures contract on the commodity. In each period \( t \), three groups of agents—a continuum of date-\( t \) commodity producers, a mass \( \Lambda_{S,t} \) of date-\( t \) financial speculators, and a mass \( \Lambda_{H,t} \) of date-\( t \) financial hedgers—enter the economy and are active in periods \( t \) and \( t + 1 \). Financial speculators and financial hedgers are active in the date-\( t \) futures market, while commodity producers are active in the date-\( t \) futures market and in the date-\( t \) and date-\( t + 1 \) spot markets. All types of agents derive CARA utilities from their date-\( t + 1 \) wealth respectively with risk aversion coefficients \( \beta \) (commodity producers), \( \gamma_S \) (financial speculators), and \( \gamma_H \) (financial hedgers).

The information structure is similar to the baseline model. In the date-\( t + 1 \) spot market, the demand from commodity consumers is still represented by a linear demand function,

\[
y_{t+1} = \bar{\theta}_{t+1} + \bar{\delta}_{t+1} - \bar{v}_{t+1},
\]

where \( \bar{v}_{t+1} \) is the endogenous commodity spot price, \( \bar{\theta}_{t+1} \sim N(\bar{\theta}, \tau_{\theta}^{-1}) \) (with \( \bar{\theta} \in \mathbb{R} \) and \( \tau_{\theta} > 0 \)) is the learnable demand shock, \( \bar{\delta}_{t+1} \sim N(0, \tau_{\delta}^{-1}) \) is the unlearnable demand shock. We assume that \( \bar{\theta}_{t+1} \) and \( \bar{\delta}_{t+1} \) are mutually independent and that \( (\bar{\theta}_{t+1}, \bar{\delta}_{t+1}) \) is independent and identically distributed (i.i.d.) over time.

The date-\( t \) financial speculators observe \( \bar{\theta}_{t+1} \) and trade futures to exploit this private information. The date-\( t \) financial hedgers trade futures and invest in a private technology with net return \( \bar{\alpha}_{t+1} + \bar{\eta}_{t+1} \), where \( \bar{\alpha}_{t+1} \sim N(0, \tau_{\alpha}^{-1}) \) and \( \bar{\eta}_{t+1} \sim N(0, \tau_{\eta}^{-1}) \) with \( \tau_{\alpha} > 0 \) and \( \tau_{\eta} > 0 \). Variables \( \bar{\alpha}_{t+1}, \bar{\theta}_{t+1}, \) and \( \bar{\eta}_{t+1} \) are mutually independent, but \( \bar{\delta}_{t+1} \) and \( \bar{\eta}_{t+1} \) are
correlated with coefficient $\rho \in (-1, 1)$. To simplify analysis, we have assumed the mean of $\tilde{\alpha}_{t+1}$ to be zero. The date-$t$ financial hedgers still observe private information $\tilde{\alpha}_{t+1}$. Date-$t$ commodity producer $i$ observes a private signal $\tilde{s}_{t,i}$ about the next period demand shock $\tilde{\theta}_{t+1}$ as follows:

$$\tilde{s}_{t,i} = \tilde{\theta}_{t+1} + \tilde{\varepsilon}_{t,i},$$

where $\tilde{\varepsilon}_{t,i} \sim N(0, \tau_\varepsilon^{-1})$ (with $\tau_\varepsilon > 0$) and $\tilde{\theta}_{t+1}$ and $\tilde{\varepsilon}_{t,i}$ are mutually independent. She exploits this information in the futures market.

A date-$t$ commodity producer makes three decisions at date $t$. The first two decisions are the same as those in the baseline model, a production decision $x_{t,i}$ and a futures investment decision $d_{P,t,i}$. Again, when date-$t$ commodity producer $i$ produce $x_{t,i}$ units of commodities, she pays a production cost, $C'(x_{t,i}) = \tilde{c}_c x_{t,i} + \frac{1}{2h} x_{t,i}^2$, where $h > 0$ and $\tilde{c} \sim N(\bar{c}, \tau_c^{-1})$ with $\bar{c} \in \mathbb{R}$ and $\tau_c > 0$. Supply shock $\tilde{c}_t$ is public information and observable to commodity producers and financial traders. In addition to production and futures investment, we also allow commodity producers to store commodities through inventory. Specifically, date-$t$ commodity producer $i$ can go to the spot market to purchase $z_{t,i}$ units of commodities, and
carry them to the next period. Storage incurs a cost according to cost function,
\[ K (z_{t,i}) = \frac{1}{2k} z_{t,i}^2, \quad \text{with } k > 0. \]  
(33)
Parameter \( k \) controls the easiness of storage. Our baseline model corresponds to the limiting case of \( k \to 0 \), where storage is impossible. In effect, when \( k \to 0 \), the OLG setting corresponds to the repeated static baseline setting. For tractability, we follow Basak and Pavlova (2016) and abstract away from inventory stockouts; thus, we allow \( z_{t,i} \) to take negative values.

In the date-\( t \) futures market, date-\( t \) financial traders and date-\( t \) commodity producers trade against noise trading \( \tilde{\xi}_t \), where \( \tilde{\xi}_t \sim N \left( 0, \tau_{\xi}^{-1} \right) \) with \( \tau_{\xi} > 0 \). The random vector \( (\tilde{\theta}_{t+1}, \tilde{\delta}_{t+1}, \tilde{\alpha}_{t+1}, \tilde{\eta}_{t+1}, \{ \tilde{\varepsilon}_{t,i} \}, \tilde{c}_t, \tilde{\xi}_t) \) is i.i.d. over time.

4.2 Equilibrium Characterization

The public information at date \( t \) is \( \mathcal{I}_t \equiv (\{\tilde{\nu}_{t-s}, \tilde{p}_{t-s}\}_{s=0}^{\infty}, \{\tilde{\theta}_{t-s}, \tilde{\delta}_{t-s}\}_{s=0}^{\infty}, \{\tilde{c}_{t-s}\}_{s=0}^{\infty}) \). Consider date-\( t \) commodity producer \( i \). Her information set is \( \mathcal{I}_{P,t,i} = \{\mathcal{I}_t, \tilde{s}_{t,i}\} \). She chooses commodity production \( x_{t,i} \), futures investment \( d_{P,t,i} \), and inventory \( z_{t,i} \) to maximize \( E (-e^{-\beta \tilde{\nu}_{P,t+1,i} | \mathcal{I}_{P,t,i}}) \), subject to \( \tilde{W}_{P,t+1,i} = [\tilde{\nu}_{t+1} x_{t,i} - C (x_{t,i})] + (\tilde{\nu}_{t+1} - \tilde{p}_t) d_{P,t,i} + [(\tilde{\nu}_{t+1} - \tilde{v}_t) z_{t,i} - K (z_{t,i})] \), where the first two terms represent her profits from real production and futures investment, and the third term represents her profits from storing and selling commodities. Similar to the baseline model, the commodity producer’s problem can be decomposed into three maximization problems as follows:

\[
\max_{x_{t,i} + d_{P,t,i} + z_{t,i}} \left[ E(\tilde{\nu}_{t+1} - \tilde{p}_t | \mathcal{I}_{P,t,i}) (x_{t,i} + d_{P,t,i} + z_{t,i}) - \frac{\beta}{2} (x_{t,i} + d_{P,t,i} + z_{t,i})^2 \text{Var}(\tilde{\nu}_{t+1} | \mathcal{I}_{P,t,i}) \right] \\
+ \max_{x_{t,i}} [\tilde{p}_t x_{t,i} - C (x_{t,i})] + \max_{z_{t,i}} [(\tilde{p}_t - \tilde{v}_t) z_{t,i} - K (z_{t,i})].
\] 
(34)

Solving (34), we have:

\[
x_{t,i} + d_{P,t,i} + z_{t,i} = \frac{E(\tilde{\nu}_{t+1} | \mathcal{I}_{P,t,i}) - \tilde{p}_t}{\beta \text{Var}(\tilde{\nu}_{t+1} | \mathcal{I}_{P,t,i})},
\] 
(35)
\[
x_{t,i} = h (\tilde{p}_t - \tilde{c}_t),
\] 
(36)
\[
z_{t,i} = k (\tilde{p}_t - \tilde{v}_t).
\] 
(37)

The aggregate quantities that date-\( t \) commodity producers produce and store are, \( x_t = \int_0^1 x_{t,i} di = h (\tilde{p}_t - \tilde{c}_t) \) and \( z_t = \int_0^1 z_{t,i} di = k (\tilde{p}_t - \tilde{v}_t) \), respectively. They carry \( x_t + z_t \) to the date-\( t + 1 \) spot market, to meet the consumption demand \( y_{t+1} \) from date-\( t + 1 \) consumers and the storage demand \( z_{t+1} \) from date-\( t + 1 \) commodity producers. That is, the date-\( t + 1 \)
The spot market clearing condition is
\[ x_t + z_t = y_{t+1} + z_{t+1}. \]  

(38)

Financial speculators have information set \( \mathcal{I}_{S,t} = \{ \mathcal{I}_t, \tilde{\theta}_{t+1} \} \). They choose investment in futures \( d_{S,t} \) to maximize \( E \left[ -e^{-\gamma_S(\tilde{v}_{t+1} - \tilde{p}_t) d_{S,t} | \mathcal{I}_{S,t}} \right] \), which delivers their demand function as follows:
\[ d_{S,t} = \frac{E(\tilde{v}_{t+1} | \mathcal{I}_{S,t}) - \tilde{p}_t}{\gamma_S Var(\tilde{v}_{t+1} | \mathcal{I}_{S,t})}. \] 

(39)

Financial hedgers have information set \( \mathcal{I}_{S,t} = \{ \mathcal{I}_t, \tilde{\alpha}_{t+1} \} \). They choose investment \( d_{H,t} \) in futures contracts and investment \( Z_{H,t} \) in the private technology to maximize their expected utility \( E \left[ -e^{-\gamma_H((\tilde{v}_{t+1} - \tilde{p}) d_{H,t} + (\tilde{\alpha}_{t+1} + \tilde{q}_{t+1}) Z_{H,t}) | \mathcal{I}_{S,t}} \right] \). Solving this problem delivers the financial hedgers’ demand for futures, \( d_H(\mathcal{I}_t, \tilde{\alpha}_{t+1}) \), given by equation (A41) in the appendix. The date-\( t \) futures market clearing condition is
\[ \int_0^1 d_{P,t,i} di + \Lambda_{S,t} d_{S,t} + \Lambda_{H,t} d_{H,t} + \tilde{\xi}_t = 0. \] 

(40)

The equilibrium is jointly determined by the optimal demand functions and the market-clearing conditions. We still consider linear equilibria in which the price functions are linear in signals. We can show that the public information in the futures price \( \tilde{p}_t \) can be summarized by a single state variable and that the futures price \( \tilde{p}_t \) and the spot price \( \tilde{v}_t \) are linearly related. The following proposition characterizes the linear equilibria.

**Proposition 5 (Dynamic equilibrium)** A linear equilibrium can be expressed as
\[ \tilde{p}_t = A_{0,t} + A_{1,t} \tilde{\Omega}_t + B_{\theta,t} \tilde{\theta}_{t+1} + B_{\alpha,t} \tilde{\alpha}_{t+1} + B_{\xi,t} \tilde{\xi}_t, \] 

(41)

\[ \tilde{v}_t = \frac{k}{1 + k} \tilde{p}_t + \tilde{G}_t, \] 

(42)

where
\[ \tilde{G}_t = \frac{\tilde{\theta}_t + \tilde{\delta}_t + h \tilde{v}_{t-1} + k \tilde{v}_{t-1} - (h + k) \tilde{p}_{t-1}}{1 + k}, \] 

(43)

\[ \tilde{\Omega}_t = (1 + k) \left( \tilde{G}_t + \frac{h}{k} \tilde{e}_t \right), \] 

(44)
where the price coefficients are determined by
\[ A_{0,t} = -\frac{\Phi^{P}_{0,t} + \Phi^{S}_{0,t} + \Phi^{H}_{0,t}}{(\Phi^{P}_{p,t} + \Phi^{S}_{p,t} + \Phi^{H}_{p,t}) + (\Phi^{P}_{v,t} + \Phi^{S}_{v,t} + \Phi^{H}_{v,t}) \frac{k}{1+k}}, \tag{45} \]
\[ A_{1,t} = -\frac{\Phi^{P}_{1,t} + \Phi^{S}_{1,t} + \Phi^{H}_{1,t}}{(\Phi^{P}_{p,t} + \Phi^{S}_{p,t} + \Phi^{H}_{p,t}) + (\Phi^{P}_{v,t} + \Phi^{S}_{v,t} + \Phi^{H}_{v,t}) \frac{k}{1+k}}, \tag{46} \]
\[ B_{\theta,t} = -\frac{\Phi^{P}_{\theta,t} + \Phi^{S}_{\theta,t} + \Phi^{H}_{\theta,t}}{(\Phi^{P}_{p,t} + \Phi^{S}_{p,t} + \Phi^{H}_{p,t}) + (\Phi^{P}_{v,t} + \Phi^{S}_{v,t} + \Phi^{H}_{v,t}) \frac{k}{1+k}}, \tag{47} \]
\[ B_{\alpha,t} = -\frac{\Phi^{P}_{\alpha,t} + \Phi^{H}_{\alpha,t}}{(\Phi^{P}_{p,t} + \Phi^{S}_{p,t} + \Phi^{H}_{p,t}) + (\Phi^{P}_{v,t} + \Phi^{S}_{v,t} + \Phi^{H}_{v,t}) \frac{k}{1+k}}, \tag{48} \]
\[ B_{\xi,t} = -\frac{\Phi^{P}_{\xi,t} + \Phi^{H}_{\xi,t} + 1}{(\Phi^{P}_{p,t} + \Phi^{S}_{p,t} + \Phi^{H}_{p,t}) + (\Phi^{P}_{v,t} + \Phi^{S}_{v,t} + \Phi^{H}_{v,t}) \frac{k}{1+k}}, \tag{49} \]
where the expressions of $\Phi$’s are given in the Internet Appendix.

### 4.3 Computation Methodology

Commodity financialization is reflected as an increase in the masses of financial traders. As in the baseline model, we consider the normalized masses, $\lambda_{S,t} = \frac{\lambda_{S,t}}{\gamma_S}$ and $\lambda_{H,t} = \frac{\lambda_{H,t}}{\gamma_H}$. We assume that $(\lambda_{S,t}, \lambda_{H,t})$ gradually increase from date 0 to date $T$, and then stay constant at $(\lambda_{S,T}, \lambda_{H,T})$ for the remaining dates. Thus, the economy is non-stationary from date 0 to date $T$, and then becomes stationary from date $T$ onward.

Since the futures price $\tilde{p}_t$ and the spot price $\tilde{v}_t$ are linearly related in Proposition 5, we only need to compute the futures price function. The date-$t$ price function is determined by the date-$t$ futures market-clearing condition and the date-$t$ futures demand functions. Note that the date-$t$ futures demand functions are involved with forecasts about the date-$t+1$ spot price $\tilde{v}_{t+1}$, which, by equation (42) (with one period forward), is a linear transformation of the date-$t+1$ futures price $\tilde{p}_{t+1}$. Hence, in order to compute $\tilde{p}_t$, we need to know $\tilde{p}_{t+1}$ and thus, we compute the futures price function backward. That is, we first compute the stationary equilibrium from date $T$ onward. Then, knowing the date-$T$ price coefficients allows us to compute the date-$T-1$ equilibrium. Similarly, once we figure out the date-$T-1$ price coefficients, we can compute the date-$T-2$ equilibrium. We continue this process till date 0.

The idea of computing the stationary equilibrium from $T$ onwards is as follows. In the stationary equilibrium, we have $(A_{0,t}, A_{1,t}, B_{\theta,t}, B_{\alpha,t}, B_{\xi,t}) = (A_{0,t+1}, A_{1,t+1}, B_{\theta,t+1}, B_{\alpha,t+1}, B_{\xi,t+1})$.
Table 1: Parameter Values of the Dynamic Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financialization</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_{S_{-1}}$</td>
<td>0.0757</td>
<td>Mass of financial speculators at date -1 (year 2003)</td>
</tr>
<tr>
<td>$\lambda_{H_{-1}}$</td>
<td>0.1586</td>
<td>Mass of financial hedgers at date -1 (year 2003)</td>
</tr>
<tr>
<td>$g_{S}$</td>
<td>0.6900</td>
<td>Growth rate of financial speculators</td>
</tr>
<tr>
<td>$g_{H}$</td>
<td>0.5539</td>
<td>Growth rate of financial hedgers</td>
</tr>
<tr>
<td>Preference</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta, \gamma_{S}, \gamma_{H}$</td>
<td>10</td>
<td>Risk aversion of commodity producers and financial traders</td>
</tr>
<tr>
<td>Information and technology</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h$</td>
<td>1</td>
<td>Production cost</td>
</tr>
<tr>
<td>$k$</td>
<td>1</td>
<td>Storage cost</td>
</tr>
<tr>
<td>$\tau_{\theta}$</td>
<td>4.2159</td>
<td>Precision of learnable commodity demand shock</td>
</tr>
<tr>
<td>$\tau_{\delta}$</td>
<td>2.1079</td>
<td>Precision of unlearnable commodity demand shock</td>
</tr>
<tr>
<td>$\tau_{e}$</td>
<td>4.2159</td>
<td>Precision of commodity producers’ private information</td>
</tr>
<tr>
<td>$\tau_{\xi}$</td>
<td>1.4053</td>
<td>Precision of noise trading in the futures market</td>
</tr>
<tr>
<td>$\tau_{\alpha}$</td>
<td>75</td>
<td>Precision of learnable component in stock returns</td>
</tr>
<tr>
<td>$\tau_{\eta}$</td>
<td>37.5</td>
<td>Precision of unlearnable component in stock returns</td>
</tr>
<tr>
<td>$\tau_{c}$</td>
<td>52.0833</td>
<td>Precision of commodity supply shock</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.5</td>
<td>Correlation between unlearnable commodity shock and unlearnable stock return shock</td>
</tr>
<tr>
<td>$\bar{\theta}$</td>
<td>0.8603</td>
<td>Average commodity demand shock</td>
</tr>
<tr>
<td>$\overline{\xi}$</td>
<td>0.5490</td>
<td>Average commodity supply shock</td>
</tr>
</tbody>
</table>

and we label these coefficients as $(A_0, A_1, B_\theta, B_\alpha, B_\xi)$. Equations (46)–(49) form the fixed point problem for $(A_1, B_\theta, B_\alpha, B_\xi)$. Once we figure out $(A_1, B_\theta, B_\alpha, B_\xi)$, we can use equation (45) to compute $A_0$.

When computing the date-$t$ non-stationary equilibrium, we take the computed date-$t+1$ price coefficients $(A_{0,t+1}, A_{1,t+1}, B_{\theta,t+1}, B_{\alpha,t+1}, B_{\xi,t+1})$ as inputs and proceed in the following two steps. First, we take $(A_{1,t+1}, B_{\theta,t+1}, B_{\alpha,t+1}, B_{\xi,t+1})$ as inputs and use equations (47)–(49) to form a fixed point problem of $(B_{\theta,t}, B_{\alpha,t}, B_{\xi,t})$ and compute these three coefficients. Second, we take $(A_{0,t+1}, A_{1,t+1}, B_{\theta,t+1}, B_{\alpha,t+1}, B_{\xi,t+1})$ and the computed $(B_{\theta,t}, B_{\alpha,t}, B_{\xi,t})$ as inputs and use equations (45) and (46) to compute $(A_{0,t}, A_{1,t})$.

### 4.4 Parameter Calibration

In order to connect the model to reality, we consider calibrated economies. We calibrate our model at an annual frequency. The calibrated parameter values are reported in Table
1. In calibration, we assume that economy experiences three phases as follows. The first phase is the pre-financialization phase, which corresponds to dates $-\infty$ to $-1$. In this phase, the economy stays at a stationary equilibrium in which the masses of financial traders are fixed at $(\lambda_{S,-1}, \lambda_{H,-1})$. The second phase is the financialization phase, lasting from date 0 to date $T$. In this phase, the masses of financial traders evolve according to the following deterministic process:

$$
\lambda_{S,t} = \lambda_{S,-1} (1 + g_{S})^{t+1} \quad \text{and} \quad \lambda_{H,t} = \lambda_{H,-1} (1 + g_{H})^{t+1}, \quad \text{for } t = 0, 1, ..., T, \tag{50}
$$

where $g_{S} > 0$ and $g_{H} > 0$. The third phase is the post-financialization phase, starting from date $T + 1$ onward, and the economy stays at a stationary equilibrium in which the masses of financial traders are fixed at $(\lambda_{S,T}, \lambda_{H,T})$.

The literature suggests that financialization started in year 2004 (e.g., Cheng and Xiong, 2014; Baker, 2021), and thus, we set year 2003 as date $-1$. We assume that the financialization process is complete in 2009 (i.e., $T = 5$). Starting from 2010, the economy becomes stationary.

We estimate parameters $\lambda_{S,-1}, \lambda_{H,-1}, g_{S}$, and $g_{H}$ using the Disaggregated Commitments of Traders Dataset (DCoT) provided by CFTC. This dataset provides information on long and short positions of “producer/merchant/processor/user,” “swap dealers,” and “managed money,” dating back to June 13, 2006, on a weekly basis. We interpret financial speculators as managed money and financial hedgers as swap dealers (who hedge their index exposures with clients). We measure $\lambda_{S,t}$ and $\lambda_{H,t}$ respectively as the ratios of the total positions of managed money and of swap dealers to the total positions of commodity producer/merchant/processor/user. Equipped with the weekly series of $\lambda_{S,t}$ and $\lambda_{H,t}$, we then run regressions to estimate (50) using the crude oil market data from 2006 to 2009. The estimation results generate $\lambda_{S,-1} = 0.0757, \lambda_{H,-1} = 0.1586, g_{S} = 0.69$, and $g_{H} = 0.5539$.

For risk aversion parameters $\beta, \gamma_{S}$, and $\gamma_{H}$ of CARA preference, the literature has chosen various values. We follow Benhabib, Liu, and Wang (2019) and note that absolute risk aversion can be calibrated as the relative risk aversion divided by wealth. The wealth level of a typical investor in our model is in the order of GDP, which has been normalized as 1.

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12 For instance, in an ascending order, the literature has chosen the following values: $3 \times 10^{-5}$ (Gârleanu and Pedersen, 2018), 0.05 (Farboodi and Veldkamp, 2020), 2 (Leland, 1992), 3 (Easley, O’Hara, and Yang, 2016), 8 (Benhabib, Liu, and Wang, 2019), 40 and 60 (Baker, 2021).
13 When we subsequently calibrate the average demand shock $\bar{\theta}$, we will consider average energy expenditure.
Thus, we can interpret $\beta$, $\gamma_S$, and $\gamma_H$ as relative risk aversion and set them to 10.

Yang (2013) has considered a production economy, and he calibrates the monthly volatility of demand shock and the monthly volatility of supply shock as 24.35% and 4%, respectively. We convert these values into annual frequency and thus set $\tau_{\theta}^{-1} + \tau_{\delta}^{-1} = 12 \times (24.35\%)^2$ and $\tau_c^{-1} = 12 \times (4\%)^2$. This immediately implies that $\tau_c = 52.0833$. In order to calibrate the values of $\tau_{\theta}$ and $\tau_{\delta}$, we rely on the literature on informed trading. Recall that financial speculators know private information $\tilde{\theta}$ and that commodity producers receive coarser information in the form of (3). If we define $\tilde{\theta} + \tilde{\delta}$ as the fundamental and write financial speculators’ information and commodity producers’ information in the form of fundamental plus noise, then their signal-to-noise ratios are $\frac{\tau_{\delta}}{\tau_{\theta}}$ and $\frac{\tau_{\epsilon}}{\tau_{\theta} + \tau_{\delta} + \tau_{\epsilon}}$, respectively. In Gennotte and Leland (1990), the investors’ signal-to-noise ratio is 0.2. In Gârleanu and Pedersen (2018), this ratio is 0.44444. Farboodi and Veldkamp (2020) estimate that this ratio is roughly 0.5 in 2015. We assign the highest value, 0.5, to financial speculators, and the lowest value, 0.2, to commodity producers. That is, $\frac{\tau_{\delta}}{\tau_{\theta}} = 0.5$ and $\frac{\tau_{\epsilon}}{\tau_{\theta} + \tau_{\delta} + \tau_{\epsilon}} = 0.2$. Together with $\tau_{\theta}^{-1} + \tau_{\delta}^{-1} = 12 \times (24.35\%)^2$, we can compute $\tau_{\theta} = 4.2159$, $\tau_{\delta} = 2.1079$, and $\tau_{\epsilon} = 4.2159$.

The estimate of noise trading has a wide range in the literature. Considering the ratio of noise trading precision to fundamental precision (i.e., $\frac{\tau_{\epsilon}^2}{(\tau_{\theta}^{-1} + \tau_{\delta}^{-1})^{-1}} = \frac{\tau_{\epsilon}^2}{\tau_{\theta}^2\tau_{\delta}^2}$), the literature has chosen the following values: 0.24663 (Farboodi and Veldkamp, 2020), 1 (Gârleanu and Pedersen, 2018), 4 (Leland, 1992), 235.29 (Gennotte and Leland, 1990), among many others. We simply set $\frac{\tau_{\epsilon}^2}{\tau_{\theta}^2\tau_{\delta}^2} = 1$, which implies that $\tau_{\epsilon} = 1.4053$.

We interpret the private investment technology of financial hedgers as the stock market. Therefore, we set the volatility of $\tilde{\alpha} + \tilde{\eta}$ as 20%, i.e., $\sqrt{\tau_{\alpha}^{-1} + \tau_{\eta}^{-1}} = 20\%$. Similar to financial speculators, we specify that financial hedgers’ information $\tilde{\alpha}$ has a signal-to-noise ratio of 0.5, which implies that $\frac{\tau_{\alpha}}{\tau_{\theta}} = 0.5$. Taken together, we can compute that $\tau_{\alpha} = 75$ and $\tau_{\eta} = 37.5$. We have little guidance about the correlation coefficient $\rho$ between the unlearnable commodity demand shock $\tilde{\delta}$ and the unlearnable stock return $\tilde{\eta}$. Nonetheless, we understand that this parameter drives the comovement between commodity and equity markets, and we set this parameter as 0.5, under which the equilibrium commodity-equity market correlation falls between 0.3 and 0.5.

Parameter $k$ governs commodity producers’ storage cost function (33). The storage as percentage of GDP, which implies that GDP in our model is normalized as 1.
cost can be understood as an adjustment cost of changing commodity producers’ inventory, and so we borrow the adjustment cost concept in the macro-finance literature to calibrate parameter $k$. Whited (1992) has estimated a quadratic adjustment cost function at annual frequency, and has reported a value of $\frac{1}{k}$ in the range of 0.5 and 2. We therefore choose $k = 1$.\footnote{Gardner and López (1996) have considered quadratic storage cost function to simulate price stabilization effects of interest-rate subsidies. They have set $\frac{1}{k} = 0.25$ in their simulations.} Similarly, parameter $h$ controls the quadratic component in commodity producers’ production function (2) and we set $h = 1$.

Finally, we calibrate the two level parameters, $\tilde{\theta}$ and $\bar{c}$, the average commodity demand shock and the average commodity supply shock, by matching the average consumption and the futures price bias. Using the U.S. data, Baker (2021) reports that the mean quarterly oil expenditure is 3.25% of GDP and the quarterly futures risk premium is 1.89% for the period of 1990–2003. We convert these numbers to annual frequency and match the counterparts in our model. The expected commodity consumption is $E(y_t)$ in our model. The gross return on futures is $\frac{\tilde{v}_{t+1}}{\tilde{p}_t}$, but its expectation is not well defined since both the numerator and the denominator are normally distributed. We therefore replace $\tilde{p}_t$ with $E(\tilde{p}_t)$ in computing the futures return, and obtain the risk premium in the model as $\frac{E(\tilde{v}_{t+1})}{E(\tilde{p}_t)} - 1$. Taken together, our calibration exercise implies that $\tilde{\theta} = 0.8603$ and $\bar{c} = 0.5490$.

### 4.5 Results

As mentioned above, we assume that the financialization phase lasts for 6 periods, which corresponds to year 2004 to 2009. Panels a1 and a2 of Figure 9 report the calibrated masses of financial traders. The mass $\lambda_{S,t}$ of financial speculators increases from $\lambda_{S,-1} = 0.0757$ to $\lambda_{S,5} = 1.7642$, while the mass $\lambda_{H,t}$ of financial hedgers increases from $\lambda_{H,-1} = 0.1586$ to $\lambda_{H,5} = 2.2327$. Since increasing $\lambda_{S,t}$ and increasing $\lambda_{H,t}$ often have countervailing effects on market outcomes, we now examine how the interaction between these two forces drives the time variations in the key variables of interest—price informativeness, the futures price bias, the commodity-equity market comovement, and the return predictiveness of financial positions. We here focus more on the patterns than on the magnitudes, since there is not always one-to-one matching between model variables and empirical variables in the literature and empirical papers do not always employ the same measure (e.g., Raman, Robe,
This figure plots the implications of simultaneously increasing the masses of financial speculators and financial hedgers in the dynamic model. The parameter values are given by Table 1.

and Yadav (2017) and Brogaard, Ringgenberg, and Sovich (2019) have adopted different empirical proxies for price efficiency).

In Panel b of Figure 9, as commodity financialization increases over time, price informativeness $\tau_{p,t}$ goes up, because in the calibrated economy, the positive price-informativeness effect of speculative financial trading dominates the negative price-informativeness effect of hedging-motivated financial trading. This result is consistent with Raman, Robe, and Yadav (2017) who document a positive impact of financialization on price efficiency in the U.S. crude oil futures market.

In Panel c of Figure 9, as commodity financialization increases over time, the futures price bias $\frac{E(\hat{v}_{t+1})}{E(\hat{p}_t)} - 1$ goes down. This is primarily driven by the risk sharing effect of adding
more players into the futures market. This result aligns with Hamilton and Wu (2014) who document that the futures price bias in crude oil futures on average decreased since 2005. In Panel d of Figure 9, as commodity financialization increases over time, the commodity-equity market comovement gradually becomes stronger. This result supports the argument that commodity financialization has contributed to the rise in the commodity-equity correlations (e.g., Tang and Xiong, 2012; Cheng and Xiong, 2014; Büyüksahin and Robe 2011, 2014).

In Panels e1 and e2 of Figure 9, we plot the return predictiveness of financial positions. In the empirical literature (e.g., Singleton, 2014; Hamilton and Wu, 2015), researchers run regressions from the futures returns on financial positions and other control variables, and use the regression coefficient on financial positions to indicate return predictiveness. In our model, the return on futures is \( v_{t+1} \), and as mentioned above, we can approximate it with \( E(p_{t+1}) \) to facilitate moment computations. The total positions of financial speculators are \( S_{t,d} \). Since \( S_{t,d} \) increases over time, we normalize \( S_{t,d} \) with its mean. We use public information \((\tilde{G}_t, \tilde{c}_t)\) to work as controls. Thus, in our model, we measure the return predictiveness of speculative financial trading and of hedging financial trading as the following regression coefficients, respectively:

\[
\begin{align*}
    b_{S,t} &= \frac{Cov \left( \frac{\tilde{v}_{t+1}}{E(\tilde{p}_t)}, \frac{\Lambda_{S,t,d_{S,t}}}{E(S_{t,d_{S,t}})} \right) \left( \tilde{G}_t, \tilde{c}_t \right)}{Var \left( \frac{\Lambda_{S,t,d_{S,t}}}{E(S_{t,d_{S,t}})} \right) \left( \tilde{G}_t, \tilde{c}_t \right)} \\
    b_{H,t} &= \frac{Cov \left( \frac{\tilde{v}_{t+1}}{E(\tilde{p}_t)}, \frac{\Lambda_{H,t,d_{H,t}}}{E(H_{t,d_{H,t}})} \right) \left( \tilde{G}_t, \tilde{c}_t \right)}{Var \left( \frac{\Lambda_{H,t,d_{H,t}}}{E(H_{t,d_{H,t}})} \right) \left( \tilde{G}_t, \tilde{c}_t \right)}.
\end{align*}
\]

In Panel e1, \( b_{S,t} \) is always positive and gradually decreases over time. In Panel e2, \( b_{H,t} \) is positive at the early stage of financialization, but switches to be negative at the later stage of financialization. This is because, as we discussed in the baseline model, financial hedgers trade for two reasons, speculation and hedging; their speculative trading positively predicts futures returns, while their hedging-motivated trading negatively predicts futures returns. These observations echo our messages delivered in the static model. That is, it is easier to identify a positive return predictiveness for financial speculators (e.g., hedge funds) than for financial hedgers (e.g., CITs), as documented by Büyüksahin and Robe (2014) and Cheng, Kirilenko, and Xiong (2015). If, in the same spirit of Cheng, Kirilenko, and Xiong (2015), one could identify the trading motives of financial hedgers—speculation versus hedging—then one would be able to identify more significant return predictiveness for each component.

The analysis in this section illustrates how one can incorporate the different forces of
commodity financialization, as identified in our baseline model, into a dynamic setting. The presented results are broadly consistent with the findings documented in the existing empirical literature. The exercise in this section has considered only one commodity, crude oil, and assumed that financialization stabilizes after six years. Nonetheless, the framework and methodology are flexible. For instance, we can extend the model to a multi-commodity setting and calibrate parameters based on a cross-section of commodity markets; we can also specify that the process of $\{\lambda_{S,t}, \lambda_{H,t}\}_{t=0}^{\infty}$ exhibits some cycle features, so that the economy experiences both financialization and de-financialization. We leave the exploration of those interesting extensions for future research.

5 Conclusion

Commodity futures markets have seen a clear trend of financialization in the years 2004-2009, marked by the increased participation of financial traders, who are not otherwise exposed to commodity spot markets. Within the group of financial traders, both financial speculators (trading on information) and financial hedgers (trading to improve the efficiency of their broader financial portfolios) have been prevalent. In this paper we develop a model aimed at understanding the effects that these two types of traders and that financialization as a whole have on various market variables that capture a lot of empirical research. The model offers novel insights; for example, we show that financialization in its early stages is likely to improve price efficiency, but later-stage financialization is likely to decrease it. Our analysis highlights a supply channel through which the commodity futures market affects the spot market, and we show that the implications for the real economy are quite intricate: while commodity producers see higher operating profits when financialization improves market efficiency, they are overall worse-off by it due to reduced opportunities in the futures-market trading. A dynamic extension of the model provides a new methodology to characterize time-varying changes in the market in a period when acute financialization implies that market compositions are not stationary. This methodology can be used and extended in future work to understand cycles in financialization or interconnection across different commodities.
Appendix: Proofs

Proof of Proposition 1

We prove Proposition 1 as follows. We first characterize the equilibrium in a system of two unknowns \((m_\xi, m_\alpha)\), and then express \(m_\alpha\) as a function of \(m_\xi\), so that the equilibrium can be characterized in a single unknown \(m_\xi\). We then establish the existence of a linear equilibrium, and provide a sufficient condition under which the equilibrium is unique among the class of linear equilibria. Finally, we deliver the expressions of the \(B\)-coefficients.

The system characterizing the equilibrium. We plug demand functions (20), (21), and (25) into the market-clearing condition (9) to write the equilibrium price \(\tilde{p}\) as a function of \((\tilde{c}, \tilde{\theta}, \tilde{\alpha}, \tilde{\xi})\) as follows:

\[
B_0 = \frac{1}{D_p} \left[ \frac{\tau_\theta + \tau_p - B_0 - B_0 \alpha}{\tau_\theta + \tau_p + \tau_q} + \lambda H \frac{\tau_\theta + \tau_q - B_0}{\tau_\theta + \tau_q} \right], \quad (A1)
\]

\[
B_c = \frac{1}{D_p} \left[ \frac{\tau_p - B_0}{\tau_\theta + \tau_p + \tau_q} + h \beta \left( \frac{1}{\tau_\theta + \tau_p + \tau_q} + \frac{1}{\tau_\delta} \right) + h + \lambda H \frac{\tau_q - B_0}{\tau_\theta + \tau_q} + h \right], \quad (A2)
\]

\[
B_{\theta} = \frac{1}{D_p} \left[ \frac{\tau_\theta}{\tau_\theta + \tau_p + \tau_q} + \lambda S \tau_\delta \right], \quad (A3)
\]

\[
B_\alpha = \frac{\lambda H}{D_p} \left[ \frac{\tau_q}{\tau_\theta + \tau_q} - \frac{\rho \sqrt{\tau_q}}{\left( \frac{1}{\tau_\theta} + \frac{1}{\tau_\theta + \tau_q} \right) \sqrt{\tau_0}} \right], \quad (A4)
\]

\[
B_\xi = \frac{1}{D_p}, \quad (A5)
\]

where

\[
D_p = \frac{\tau_\theta + \tau_p}{\beta \left( \frac{1}{\tau_\theta + \tau_p + \tau_q} + \frac{1}{\tau_\delta} \right)} + h + \lambda H \frac{\tau_q}{\tau_\theta + \tau_q} + \lambda S \tau_\delta (h + 1). \quad (A6)
\]
By the above computed coefficients $B_\theta, B_\xi$, and $B_\alpha$, we have

$$m_\xi \equiv \frac{B_\theta}{B_\xi} = \frac{\tau_c}{\tau_\theta + \tau_c + \tau_p} + \lambda_S \tau_\delta,$$  \hspace{1cm} (A7)

$$m_\alpha \equiv \frac{B_\theta}{B_\alpha} = \frac{\tau_c}{\frac{1}{\tau_\theta + \tau_c + \tau_p} + \frac{1}{\tau_\delta}} + \lambda_S \tau_\delta,$$  \hspace{1cm} (A8)

Note that equation (A7) is the same as equation (26) in Proposition 1.

Using (A7) to replace the numerator of (A8) with $m_\xi$, and combing with the expression of $\tau_q$ in (24), we can express $m_\alpha$ as a function of $m_\xi$:

$$m_\alpha = -\left(\frac{m_\xi}{\lambda_H} + \frac{m_\xi^2 \tau_c}{\tau_\theta + m_\xi \tau_c} \right) \left(\frac{1 - \rho^2}{\tau_\delta} + \frac{1}{\tau_\theta + m_\xi \tau_c} \right) \sqrt{\tau_\delta}.$$  \hspace{1cm} (A9)

Inserting the above expression into equation (19), we can express $\tau_p$ as a function of $m_\xi$. Then, inserting the obtained expression of $\tau_p$ into equation (A7), we obtain the following 7th order polynomial of $m_\xi$:

$$J_7 m_\xi^7 + J_6 m_\xi^6 + J_5 m_\xi^5 + J_4 m_\xi^4 + J_3 m_\xi^3 + J_2 m_\xi^2 + J_1 m_\xi + J_0 = 0,$$  \hspace{1cm} (A10)

where the $J$-coefficients are given in the Internet Appendix.

**Existence of the equilibrium.** At $m_\xi = 0$, the left-hand-side (LHS) of equation (A7) is 0, while the right-hand-side (RHS) of equation (A7) exceeds $\lambda_S \tau_\delta$. As $m_\xi \to \infty$, the LHS of equation (A7) goes to $\infty$, the RHS of equation (A7) goes to $\lambda_S \tau_\delta$. Thus, by the intermediate value theorem, existence is established.

In addition, note that

$$0 < \frac{\tau_c}{\beta \left(\frac{1}{\tau_\theta + \tau_c + \tau_p} + \frac{1}{\tau_\delta}\right)} < \frac{\tau_c}{\beta \frac{1}{\tau_\theta + \tau_c} + \frac{1}{\tau_\delta}}.$$

So, by equation (A7), we have

$$\lambda_S \tau_\delta < m_\xi < \frac{\tau_c}{\beta \frac{1}{\tau_\theta + \tau_c} + \frac{1}{\tau_\delta}} + \lambda_S \tau_\delta,$$

which establishes the boundaries of $m_\xi$ in Proposition 1.
A sufficient condition for the uniqueness. When the RHS of equation (A7) is downward sloping, the equilibrium is unique among the class of linear equilibria. In particular, given the expression of $\tau_p$ in (19), if $m_\alpha^2$ determined by (A9) is increasing in $m_\xi$, then $\tau_p$ is increasing in $m_\xi$, so that the RHS of equation (A7) is decreasing in $m_\xi$. Using (A9), we can show that $\frac{\partial m_\alpha^2}{\partial m_\xi} > 0$ if and only if

$$(1 - \rho^2) \tau_\xi^2 m_\xi^4 + \tau_\xi \left(2 \left(1 - \rho^2\right) \tau_\theta - \tau_\delta\right) m_\xi^2 + 2 \lambda_H \tau_\theta \tau_\delta \tau_\xi m_\xi + \tau_\theta \left((1 - \rho^2) \tau_\theta + \tau_\delta\right) > 0. \quad (A11)$$

Treating the $4^{th}$, $2^{nd}$, and constant orders of $m_\xi$ of the LHS of the above condition as a quadratic polynomial of $m_\xi$, we can show that when $8 \tau_\theta (1 - \rho^2) > \tau_\delta$, the determinant of this quadratic polynomial is negative so that the value of this quadratic polynomial is always positive. As a result, when $8 \tau_\theta (1 - \rho^2) > \tau_\delta$, condition (A11) holds.

Expressions of the $B$-coefficients. Using the expressions of $B_\theta$ and $D_p$ in (A3) and (A6), we can compute

$$B_\theta = \frac{\tau_\xi \left(\frac{1}{\tau_\theta + \tau_\xi + \frac{1}{\tau_\delta}}\right)}{\beta \left(\frac{1}{\tau_\theta + \tau_\xi + \frac{1}{\tau_\delta}}\right)} + \lambda_H \frac{\tau_\xi}{\frac{1}{\tau_\theta + \tau_\xi + \frac{1}{\tau_\delta}}} + \lambda_S \frac{\tau_\xi}{\frac{1}{\tau_\delta}}.$$  

Note that the RHS of the above expression is known, since $\tau_p$ and $\tau_q$ are known given the values of $m_\xi$ and $m_\alpha$. Once we figure out $B_\theta$, we have:

$$m_\alpha = \frac{B_\theta}{B_\xi}, m_\xi = \frac{B_\theta}{B_\xi} \Rightarrow B_\alpha = \frac{B_\theta}{m_\alpha}, B_\xi = \frac{B_\theta}{m_\xi}.$$  

Also, given $B_\theta$, the value of $D_p$ is known by (A6). Finally, using equations (A1) and (A2), we can compute

$$B_0 = \frac{\tau_\xi \left(\frac{1}{\tau_\theta + \tau_\xi + \frac{1}{\tau_\delta}}\right)}{\beta \left(\frac{1}{\tau_\theta + \tau_\xi + \frac{1}{\tau_\delta}}\right)} + \lambda_H \frac{\tau_\xi}{\frac{1}{\tau_\theta + \tau_\xi + \frac{1}{\tau_\delta}}} + \lambda_S \frac{\tau_\xi}{\frac{1}{\tau_\delta}},$$

$$D_p + \frac{\tau_\xi}{\beta \left(\frac{1}{\tau_\theta + \tau_\xi + \frac{1}{\tau_\delta}}\right)} + \lambda_H \frac{\tau_\xi}{\frac{1}{\tau_\theta + \tau_\xi + \frac{1}{\tau_\delta}}} + \lambda_S \frac{\tau_\xi}{\frac{1}{\tau_\delta}} + \lambda_S \frac{\tau_\xi}{\frac{1}{\tau_\delta}},$$

$$B_c = \frac{\tau_\xi \left(\frac{1}{\tau_\theta + \tau_\xi + \frac{1}{\tau_\delta}}\right)}{\beta \left(\frac{1}{\tau_\theta + \tau_\xi + \frac{1}{\tau_\delta}}\right)} + \lambda_H \frac{\tau_\xi}{\frac{1}{\tau_\theta + \tau_\xi + \frac{1}{\tau_\delta}}} + \lambda_S \frac{\tau_\xi}{\frac{1}{\tau_\delta}} + \lambda_S \frac{\tau_\xi}{\frac{1}{\tau_\delta}}.$$
Proof of Proposition 2

**Part (a): The effect of \(\lambda_S\).**

From the proof of Proposition 1, we know that when \(8\tau_\theta (1 - \rho^2) > \tau_\delta\), the RHS of equation (26) in Proposition 1 is downward sloping in \(m_\xi\). Note that \(\frac{\tau_\xi}{\beta \tau_\theta + \tau_\epsilon + \tau_\delta}\) is not affected by \(\lambda_S\). So, an increase in \(\lambda_S\) will shift upward the RHS of equation (26) in Proposition 1. As a result, the equilibrium value of \(m_\xi\) will increase. In addition, from the proof of Proposition 1, the condition \(8\tau_\theta (1 - \rho^2) > \tau_\delta\) ensures that \(m_\alpha^2\) increase with \(m_\xi\). In consequence, \(|m_\alpha|\) increases with \(\lambda_S\). Since both \(m_\xi\) and \(|m_\alpha|\) increase with \(\lambda_S\), price informativeness \(\tau_p\) increases with \(\lambda_S\) too.

**The effect of \(\lambda_H\).**

Given \(m_\xi\), an increase in \(\lambda_H\) will decrease \(m_\alpha^2\) by equation (A9) and \(\tau_p\) by equation (19). This implies that the RHS of equation (26) shifts upward. Since under the condition \(8\tau_\theta (1 - \rho^2) > \tau_\delta\), the RHS of equation (26) is downward sloping, the equilibrium value of \(m_\xi\) increases.

By equation (26), \(m_\xi\) and \(\tau_p\) move in opposite directions in response to an increase in \(\lambda_H\). Hence, \(\partial m_\xi / \partial \lambda_H > 0\) implies that \(\partial \tau_p / \partial \lambda_H < 0\). By the expression of \(\tau_p\) in equation (19), \(\partial m_\xi / \partial \lambda_H > 0\) and \(\partial \tau_p / \partial \lambda_H < 0\) together imply that \(\partial |m_\alpha| / \partial \lambda_H < 0\).

**Part (b): When \(\bar{\lambda}\) is small.**

Fix \((\phi_S, \phi_H)\) and let \(\bar{\lambda} \to 0\). We first establish the limiting values of \(m_\xi, |m_\alpha|,\) and \(\tau_p\). Using equations (19), (26), and (A9), we can show that \(m_\xi\) converges to a finite value, which is the unique positive root of a cubic equation. Let us denote this limiting value as \(\hat{m}_{\xi,0}\). By equation (A9), we can compute

\[
|m_\alpha| \propto \frac{1}{\lambda} \left( \frac{1 - \rho^2}{\tau_\delta} + \frac{1}{\tau_\theta + \hat{m}_{\xi,0}^2 \tau_\xi} \right) \frac{\hat{m}_{\xi,0}}{\phi_H} \sqrt{\frac{\tau_\delta}{\rho^2 \tau_\eta}}, \tag{A12}
\]

where \("X \propto Y"\) means that \(\lim_{\lambda \to 0} \frac{X}{Y} = 1\). Similarly, by equation (19), we can compute

\[
\tau_p \propto \hat{m}_{\xi,0}^2 \tau_\xi. \tag{A13}
\]

We now compute the limiting values of derivatives \(\frac{\partial m_\xi}{\partial \lambda}, \frac{\partial \tau_p}{\partial \lambda},\) and \(\frac{\partial |m_\alpha|}{\partial \lambda}\). By the expression of \(\tau_p\) in (19), we have

\[
\frac{\partial \tau_p}{\partial \lambda} = \frac{1}{(m_\xi^2)^2 \tau_\alpha} \frac{\partial m_\alpha^2}{\partial \lambda} + \frac{1}{(m_\xi^2)^2 \tau_\xi} \frac{\partial m_\xi^2}{\partial \lambda} \left( \frac{1}{m_\xi^2 \tau_\alpha} + \frac{1}{m_\xi^2 \tau_\xi} \right)^2. \tag{A14}
\]

By (A9), we can compute \(\frac{\partial m_\alpha^2}{\partial \lambda} = -\frac{2|m_\alpha|m_\xi}{\phi_H} \sqrt{\frac{\tau_\alpha}{\rho^2 \tau_\eta}} O \left( \frac{1}{\lambda^2} \right),\) where \(O \left( \frac{1}{\lambda^2} \right)\) means that this term
has the same order as $\frac{1}{\lambda^2}$. Hence, by (A12), we have $\frac{1}{(m_\alpha^2)^2} \frac{\partial m_\alpha^2}{\partial \lambda} \to 0$ in (A14). As a result,

$$\frac{\partial \tau_p}{\partial \lambda} \propto \frac{1}{(m_{\epsilon,0}^2)^2 \tau_\epsilon} \frac{\partial m_\epsilon^2}{\partial \lambda} = 2\tau_\epsilon \dot{m}_{\epsilon,0} \frac{\partial m_\epsilon}{\partial \lambda}. \quad (A15)$$

Applying the implicit function theorem to equation (26), we have

$$\frac{\partial m_\xi}{\partial \lambda} = -\frac{\tau_\epsilon}{\beta} \frac{\tau_\delta}{(\tau_\theta + \tau_\epsilon + \tau_p + \tau_\delta)^2} \frac{\partial \tau_p}{\partial \lambda} + \phi_S \tau_\delta. \quad (A16)$$

Combined with equations (A13) and (A15), we can compute

$$\frac{\partial m_\xi}{\partial \lambda} \propto 1 + \frac{\tau_\epsilon}{\beta} \frac{\phi_S \tau_\delta}{(\tau_\theta + \tau_\epsilon + m_{\epsilon,0}^2 \tau_\epsilon + \tau_\delta)^2} 2\tau_\epsilon \dot{m}_{\epsilon,0} > 0, \quad (A17)$$

and

$$\frac{\partial \tau_p}{\partial \lambda} \propto 2\tau_\epsilon \dot{m}_{\epsilon,0} \frac{\partial m_\epsilon}{\partial \lambda} > 0. \quad (A18)$$

From (A9), we can compute

$$\frac{\partial m_\alpha^2}{\partial \lambda} \propto -\frac{2}{\lambda^3} \left( \frac{1 - \rho^2}{\tau_\delta} + \frac{1}{\rho^2 \tau_\eta} \right) \frac{\tau_\delta}{\phi_S^2 \phi_H^2} < 0. \quad (A19)$$

**When $\bar{\lambda}$ is large.**

Fix $(\phi_S, \phi_H)$ and let $\bar{\lambda} \to \infty$. First, by equation (26) and noting that $\tau_p$ is bounded, we have

$$m_\epsilon \propto \bar{\lambda} \phi_S \tau_\delta. \quad (A20)$$

Using (A9) and (19), we have

$$m_\alpha^2 \propto \left( \frac{1 - \rho^2}{\tau_\delta} \frac{\phi_S \tau_\delta}{\phi_H} + 1 \right) \frac{\tau_\delta}{\rho^2 \tau_\eta}, \quad (A21)$$

and

$$\tau_p \propto \left( \frac{1 - \rho^2}{\tau_\delta} \frac{\phi_S \tau_\delta}{\phi_H} + 1 \right) \frac{\tau_\delta}{\rho^2 \tau_\eta}, \quad (A22)$$

both of which are finite.

We next show that all of the three derivatives $\frac{\partial m_\alpha}{\partial \lambda}$, $\frac{\partial \tau_p}{\partial \lambda}$, and $\frac{\partial m_\alpha}{\partial \lambda}$ are finite, and at the same time, we sign them. Using (A20) and (A9), we can show

$$\frac{\partial m_\alpha^2}{\partial \lambda} \propto -2 \left( \frac{1 - \rho^2}{\tau_\delta} \frac{\phi_S \tau_\delta}{\phi_H} + 1 \right) \frac{(1 - \rho^2) \tau_\delta \phi_S}{\rho^2 \tau_\eta \phi_H} < 0. \quad (A23)$$
By (A14), (A20), (A21), and (A23), we can compute
\[
\frac{\partial \tau_p}{\partial \lambda} \propto \frac{-2 \tau_s \tau_\delta (1 - \phi_H) (1 - \rho^2) (\rho^2 \phi_H + 1 - \rho^2)}{\rho^2 \phi_H^2 \tau_\eta} < 0. \tag{A24}
\]

Finally, using (A16), (A22), and (A24), we have
\[
\frac{\partial m_\xi}{\partial \lambda} \propto \frac{\tau_\delta}{\beta} \left( \tau_\theta + \tau_\varepsilon + \left( \frac{1 - \rho^2}{\tau_\delta} \frac{\phi_0 \tau_\delta}{\phi_H} + 1 \right)^2 + \frac{\phi_S \tau_\delta}{\rho^2 \tau_\eta} \tau_\alpha + \tau_\delta \right)^2 + \phi_S \tau_\delta > 0. \tag{A25}
\]

**Proof of Proposition 3**

By demand functions (16), (21), and (22) as well as the market-clearing condition (9), we can show
\[
\begin{align*}
E(\tilde{v} - \tilde{p}) &= \int \left[ \frac{1}{\beta \text{Var}(\tilde{v} | \tilde{s}, \tilde{c}, \tilde{\bar{p}})} + \frac{\lambda_S}{\text{Var}(\tilde{v} | \tilde{\bar{\theta}}, \tilde{\bar{p}})} + \frac{\lambda_H - 1}{\beta \text{Var}(\tilde{\bar{\theta}} | \tilde{\bar{q}})} \right] E(\tilde{v} - \tilde{p}) \\
&= hE(\tilde{p} - \tilde{c}) + \lambda_H \rho \sqrt{\tau_\eta \frac{\tilde{\bar{\theta}} - \tilde{\bar{c}}}{\tilde{\bar{p}} + \tau_\varepsilon}} + \frac{1 - \rho^2}{\beta \text{Var}(\tilde{\bar{\theta}} | \tilde{\bar{q}})} \sqrt{\tau_\delta}.
\end{align*}
\]  

We then use the expression of \( \tilde{v} \) in (14) to obtain
\[
E(\tilde{p} - \tilde{c}) = \frac{\tilde{\bar{\theta}} - \tilde{\bar{c}}}{h + 1} - \frac{E(\tilde{v} - \tilde{p})}{h + 1}. \tag{A26}
\]

From equations (A26) and (A27), we can compute equation (29).

**Proof of Corollary 1**

When \( \tilde{\bar{\alpha}} = 0 \), the effects of \( \lambda \)'s on \( |E(\tilde{v} - \tilde{p})| \) are determined by the denominator of equation (29), given by
\[
\text{DEN}_{|E(\tilde{v} - \tilde{p})|} = \frac{h}{h + 1} + \lambda_S \tau_\delta + \frac{1 - \rho^2}{\tau_\delta} + \frac{1}{\tau_\theta + \tau_\varepsilon} + \frac{(\tau_\theta + \tau_\varepsilon + \tau_\delta) \tau_\delta}{\beta (\tau_\theta + \tau_\varepsilon + \tau_\delta + \tau_\varepsilon)}. \tag{A28}
\]

So, in the following proof, we examine this denominator \( \text{DEN}_{|E(\tilde{v} - \tilde{p})|} \).

The effect of \( \lambda_S \). Parameter \( \lambda_S \) affects \( \text{DEN}_{|E(\tilde{v} - \tilde{p})|} \) through three terms: \( \lambda_S \tau_\delta \), \( \frac{\lambda_H}{\tau_\delta} \), and \( \frac{(\tau_\theta + \tau_\varepsilon + \tau_\delta) \tau_\delta}{\beta (\tau_\theta + \tau_\varepsilon + \tau_\delta + \tau_\varepsilon)} \). Clearly, an increase in \( \lambda_S \) directly increases the first term. Parameter \( \lambda_S \) affects the second term indirectly through \( \tau_\eta \) and the third term indirectly through \( \tau_\theta \). By
Part (a) of Proposition 2, we know that an increase in $\lambda_S$ will increase $m_\xi$ and $\tau_p$. By the expression of $\tau_q$ in (24), $\tau_q$ increases with $\lambda_S$. As a result, both the second term \( \frac{\lambda_H}{\tau_\delta + \frac{1}{\tau_\theta + \tau_q}} \) and the third term \( \frac{\lambda_H}{\beta(\tau_p + \tau_\theta + \tau_\xi + \tau_\delta)} \) increase with $\lambda_S$. Overall, $DEN_{|E(\tilde{v} - \tilde{p})|}$ increases in $\lambda_S$, and thus, $|E(\tilde{v} - \tilde{p})|$ decreases with $\lambda_S$.

The effect of $\lambda_H$. Parameter $\lambda_H$ affects $DEN_{|E(\tilde{v} - \tilde{p})|}$ through two terms, \( \frac{\lambda_H}{\tau_\delta + \frac{1}{\tau_\theta + \tau_q}} \) and \( \frac{\lambda_H}{\beta(\tau_p + \tau_\theta + \tau_\xi + \tau_\delta)} \), and it affects these two terms in opposite directions. (By Part (a) of Proposition 2, an increase in $\lambda_H$ will increase $m_\xi$ but decrease $\tau_p$. So, by the expression of $\tau_q$ in (24), $\tau_q$ increases with $\lambda_S$, and thus \( \frac{\lambda_H}{\tau_\delta + \frac{1}{\tau_\theta + \tau_q}} \) increases with $\lambda_H$. The term \( \frac{\lambda_H}{\beta(\tau_p + \tau_\theta + \tau_\xi + \tau_\delta)} \) decreases with $\lambda_H$ through $\tau_p$.) Using equation (26) in Proposition 1, we can rewrite $DEN_{|E(\tilde{v} - \tilde{p})|}$ as follows:

\[
DEN_{|E(\tilde{v} - \tilde{p})|} = \lambda_H \left( \frac{1 - \rho^2 + \frac{1}{\tau_\theta + \tau_q}}{\tau_\delta + \frac{1}{\tau_\theta + \tau_q}} \right) - \frac{\tau_\delta}{\tau_\xi} m_\xi + \left( \frac{\lambda_H}{\tau_\delta + \frac{1}{\tau_\theta + \tau_q}} \right) \left( \frac{2m_\xi \tau_\xi}{\tau_\theta + \frac{m_\xi \tau_\xi}{\tau_\delta}} \right) - \frac{\tau_\delta}{\tau_\xi} \frac{\partial m_\xi}{\partial \lambda_H}.
\]

Taking derivative, we get

\[
\frac{\partial DEN_{|E(\tilde{v} - \tilde{p})|}}{\partial \lambda_H} = \frac{1}{1 - \rho^2 + \frac{1}{\tau_\theta + \tau_q}} + \left[ \frac{\lambda_H}{(1 - \rho^2 + \frac{1}{\tau_\theta + \tau_q})^2} \left( \frac{2m_\xi \tau_\xi}{\tau_\theta + \frac{m_\xi \tau_\xi}{\tau_\delta}} \right) \right] \frac{\partial m_\xi}{\partial \lambda_H}.
\]

When $\lambda_H$ is sufficiently large, we must have \( \frac{\lambda_H}{(1 - \rho^2 + \frac{1}{\tau_\theta + \tau_q})^2} \left( \frac{2m_\xi \tau_\xi}{\tau_\theta + \frac{m_\xi \tau_\xi}{\tau_\delta}} \right) \frac{\partial m_\xi}{\partial \lambda_H} > 0 \) since $m_\xi$ is bounded. Given $\frac{\partial m_\xi}{\partial \lambda_H} > 0$, we know that $\frac{\partial DEN_{|E(\tilde{v} - \tilde{p})|}}{\partial \lambda_H} > 0$ for sufficiently large $\lambda_H$.

Proof of Proposition 4

Part (a): By equations (14) and (15), we have

\[
Cov(\tilde{\alpha} + \tilde{\eta}, \tilde{v} - \tilde{p}) = -(h + 1) Cov(\tilde{\alpha}, \tilde{p}) + Cov(\tilde{\eta}, \tilde{\delta}) = -\frac{(h + 1) B_\alpha}{\tau_\alpha} + \frac{\rho}{\sqrt{\tau_\delta \tau_\eta}}.
\]

From (A9), we have

\[
B_\alpha = -B_\theta \left( \frac{1 - \rho^2 + \frac{1}{\tau_\theta + \tau_q}}{\tau_\delta + \frac{1}{\tau_\theta + \tau_q}} \right)^{\frac{\rho \sqrt{\tau_\delta}}{\sqrt{\tau_\delta \tau_\eta}}},
\]

\[
\frac{m_\xi}{\lambda_H} + \frac{m_\xi}{1 - \rho^2 + \frac{1}{\tau_\theta + \tau_q}} \frac{\tau_\theta + \frac{m_\xi \tau_\delta}{\tau_\theta}}{\tau_\theta + \tau_\xi + \tau_\delta}.
\]
Thus,

\[
\text{Cov} (\tilde{\alpha} + \tilde{\eta}, \tilde{v} - \tilde{p}) = \left[ \frac{(h + 1) B_\delta}{\tau_\alpha} \frac{\tau_\eta}{\tau_\delta + \rho m_\xi + \rho m_\xi^2} \frac{1}{\tau_\theta + \rho m_\xi} + 1 \right] \frac{\rho}{\sqrt{\tau_\delta \tau_\eta}}.
\]

which implies that \(\text{Cov} (\tilde{\alpha} + \tilde{\eta}, \tilde{v} - \tilde{p}) > 0\) if and only if \(\rho > 0\).

**Part (b):** Without loss of generality, let us assume \(\rho > 0\). When \(\lambda_H = 0\), we have

\[
\text{Cov} (\tilde{\alpha} + \tilde{\eta}, \tilde{v} - \tilde{p}) = \frac{\rho}{\sqrt{\tau_\delta \tau_\eta}}.
\]

When \(\lambda_H > 0\), we have

\[
\text{Cov} (\tilde{\alpha} + \tilde{\eta}, \tilde{v} - \tilde{p}) > \frac{\rho}{\sqrt{\tau_\delta \tau_\eta}}.
\]

Thus, it must be the case that \(\text{Cov} (\tilde{\alpha} + \tilde{\eta}, \tilde{v} - \tilde{p})\) is increasing in \(\lambda_H\) at \(\lambda_H = 0\). This is true either when we treat \(\lambda_H\) as a free parameter or when we vary \(\lambda_H\) through varying \(\tilde{\lambda}\).

**Proof of Proposition 5**

Inserting the expressions of \(y_t\) in (31), of \(x_{t-1}\) in (36), and of \(z_t\) and \(z_{t-1}\) in (37) into the date-\(t\) spot market-clearing condition, \(y_t + z_t = x_{t-1} + z_{t-1}\), we obtain equation (42) in Proposition 5. Also, equation (42) implies that the spot price \(\tilde{v}_t\) does not contain additional information beyond the futures price \(\tilde{p}_t\).

The futures price \(\tilde{p}_t\) is determined by the system composed of the futures market-clearing condition and the three demand functions from commodity producers, financial speculators, and financial hedgers. In this system, the exogenous information is \(\{\tilde{\theta}_t, \tilde{\delta}_t, \tilde{c}_{t-1}, \tilde{v}_{t-1}, \tilde{\phi}_{t-1}, \tilde{\xi}_t\}\). We thus conjecture the following linear price function:

\[
\tilde{p}_t = A(I_t) + B(\tilde{\theta}_{t+1}, \tilde{\alpha}_{t+1}, \tilde{\xi}_t),
\]

where

\[
A(I_t) = A_{0,t} + A_{q,t} \tilde{\theta}_t + A_{\delta,t} \tilde{\delta}_t + A_{c,t} \tilde{c}_{t-1} + A_{v,t} \tilde{v}_{t-1} + A_{p,t} \tilde{p}_{t-1} + B_{c,t} \tilde{c}_t,
\]

\[
B(\tilde{\theta}_{t+1}, \tilde{\alpha}_{t+1}, \tilde{\xi}_t) = B_{\theta,t} \tilde{\theta}_{t+1} + B_{\delta,t} \tilde{\delta}_{t+1} + B_{\xi,t} \tilde{\xi}_t.
\]

Function \(A(I_t)\) corresponds to public information, and function \(B(\tilde{\theta}_{t+1}, \tilde{\alpha}_{t+1}, \tilde{\xi}_t)\) corresponds to private information. The \(A\)-coefficients and the \(B\)-coefficients are endogenous. In relation to equation (15) in the baseline model, coefficients \((B_{c,t}, B_{\theta,t}, B_{\delta,t}, B_{\xi,t})\) correspond to \((B_c, B_\theta, B_\delta, B_\xi)\) in (15), and the term \(A_{0,t} + A_{q,t} \tilde{\theta}_t + A_{\delta,t} \tilde{\delta}_t + A_{c,t} \tilde{c}_{t-1} + A_{v,t} \tilde{v}_{t-1} + A_{p,t} \tilde{p}_{t-1}\) correspond to \((A_0, A_q, A_\delta, A_c, A_v, A_p)\) in (15).
corresponds to $B_0$ in (15).

In order to compute the demand functions, we need to figure out the expression of $\tilde{v}_{t+1}$. Applying equation (42) one period forward and using the conjectured price function for $\tilde{p}_{t+1}$, we can compute

$$\tilde{v}_{t+1} = \frac{k}{1+k} \left( B_{\theta,t+1} \tilde{\theta}_{t+2} + B_{c,t+1} \tilde{c}_{t+1} + B_{\alpha,t+1} \tilde{\alpha}_{t+2} + B_{\xi,t+1} \tilde{\xi}_{t+1} \right)$$

$$+ \frac{1}{1+k} \left[ kA_{0,t+1} + (kA_{\theta,t+1} + 1) \tilde{\theta}_{t+1} + (kA_{\delta,t+1} + 1) \delta_{t+1} + (kA_{c,t+1} + h) \tilde{c}_t + k(A_v,t+1 + 1) \tilde{v}_t + [kA_{p,t+1} - (h + k)] \tilde{p}_t \right]. \quad (A29)$$

By equations (35)–(37), the futures demand of date-$t$ commodity producer $i$ is

$$d_{P,t,i} = \frac{E(\tilde{v}_{t+1} | \mathcal{I}_t, \tilde{s}_{t,i}) - \tilde{p}_t}{\beta \text{Var}(\tilde{v}_{t+1} | \mathcal{I}_t, \tilde{s}_{t,i})} - h (\tilde{p}_t - \tilde{c}_t) - k (\tilde{p}_t - \tilde{v}_t). \quad (A30)$$

Using (A29), we have

$$E(\tilde{v}_{t+1} | \mathcal{I}_t, \tilde{s}_{t,i}) = \frac{k}{1+k} \left( B_{\theta,t+1} \tilde{\theta} + B_{c,t+1} \tilde{c} \right)$$

$$+ \frac{1}{1+k} \left[ kA_{0,t+1} + (kA_{\theta,t+1} + 1) E(\tilde{\theta}_{t+1} | \mathcal{I}_t, \tilde{s}_{t,i}) + (kA_{c,t+1} + h) \tilde{c}_t \right] \quad (A31)$$

and

$$\text{Var}(\tilde{v}_{t+1} | \mathcal{I}_t, \tilde{s}_{t,i}) = \left( \frac{k}{1+k} \right)^2 \left( B_{\theta,t+1}^2 \frac{1}{\tau_{\theta}} + B_{c,t+1}^2 \frac{1}{\tau_c} + B_{\alpha,t+1}^2 \frac{1}{\tau_{\alpha}} + B_{\xi,t+1}^2 \frac{1}{\tau_{\xi}} \right)$$

$$+ \frac{1}{(1+k)^2} \left[ (kA_{\theta,t+1} + 1)^2 \text{Var}(\tilde{\theta}_{t+1} | \mathcal{I}_t, \tilde{s}_{t,i}) + (kA_{\delta,t+1} + 1)^2 \frac{1}{\tau_{\delta}} \right]. \quad (A32)$$

Date-$t$ commodity producer $i$ can read information from the prevailing futures price $\tilde{p}_t$, which is equivalent to the following signal to her:

$$\tilde{s}_{p,t} = \frac{\tilde{p}_t - A(\mathcal{I}_t)}{B_{\theta,t}} = \tilde{\theta}_{t+1} + \frac{B_{\alpha,t}}{B_{\theta,t}} \tilde{\alpha}_{t+1} + \frac{B_{\xi,t}}{B_{\theta,t}} \tilde{\xi}_t, \quad (A33)$$

which is normally distributed with mean $\tilde{\theta}_{t+1}$ and precision given by

$$\tau_{p,t} = \left[ \left( \frac{B_{\alpha,t}}{B_{\theta,t}} \right)^2 \frac{1}{\tau_{\alpha}} + \left( \frac{B_{\xi,t}}{B_{\theta,t}} \right)^2 \frac{1}{\tau_{\xi}} \right]^{-1}. \quad (A34)$$
Similar to the baseline model, parameter $\tau_{p,t}$ measure price informativeness of the date-$t$ financial market. Using Bayes’ rule, we can compute:

$$E(\tilde{\theta}_{t+1}|I_t, \tilde{s}_{t,i}) = E(\tilde{\theta}_{t+1}|s_{p,t}, \tilde{s}_{t,i}) = \frac{\tau_\theta \tilde{\theta} + \tau_{p,t} \tilde{s}_{p,t} + \tau_\varepsilon \tilde{s}_{t,i}}{\tau_\theta + \tau_{p,t} + \tau_\varepsilon},$$  \hspace{1cm} (A35)

$$Var(\tilde{\theta}_{t+1}|I_t, \tilde{s}_{t,i}) = Var(\tilde{\theta}_{t+1}|s_{p,t}, \tilde{s}_{t,i}) = \frac{1}{\tau_\theta + \tau_{p,t} + \tau_\varepsilon}.$$  \hspace{1cm} (A36)

Inserting equations (A31), (A32), (A33), (A35), and (A36) into equation (A30) and aggregating, we can express the aggregate demand function from commodity producers as follows:

$$\int_0^1 dP_{t,i}d = \Phi_{0,t}^P + \Phi_{\theta,t}^P \tilde{\theta}_{t+1} + \Phi_{c,t}^P \tilde{c}_t + \Phi_{\alpha,t}^P \tilde{\alpha}_{t+1} + \Phi_{\xi,t}^P \tilde{\xi}_t + \Phi_{p,t}^P \tilde{p}_t + \Phi_{v,t}^P \tilde{v}_t,$$  \hspace{1cm} (A37)

where the $\Phi$-coefficients are given in the Internet Appendix.

Date-$t$ financial speculators’ futures demand is given by equation (39). Using (A29), we can compute the conditional moments as follows:

$$E(\tilde{v}_{t+1}|I_t, \tilde{\theta}_{t+1}) = \frac{k}{1+k} (B_{\theta,t+1} \tilde{\theta} + B_{c,t+1} \tilde{c})$$

$$+ \frac{1}{1+k} \left[ kA_{0,t+1} + (kA_{\theta,t+1} + 1) \tilde{\theta}_{t+1} + (kA_{c,t+1} + h) \tilde{c}_t + k(A_{c,t+1} + 1) \tilde{v}_t + [kA_{p,t+1} - (h + k)] \tilde{p}_t \right],$$  \hspace{1cm} (A38)

$$Var(\tilde{v}_{t+1}|I_t, \tilde{\theta}_{t+1}) = \frac{(kA_{\delta,t+1} + 1)^2}{(1+k)^2} \frac{1}{\tau_\delta}$$

$$+ \left( \frac{k}{1+k} \right)^2 \left( \frac{B_{\theta,t+1}^2}{\tau_\theta} + \frac{B_{c,t+1}^2}{\tau_c} + \frac{B_{\alpha,t+1}^2}{\tau_\alpha} + \frac{B_{\xi,t+1}^2}{\tau_\xi} \right).$$  \hspace{1cm} (A39)

Inserting the above two moment expressions into equation (39), we can compute the total demand from financial speculators as follows:

$$\Lambda_{S,t}d_{S,t} = \Phi_{0,t}^S + \Phi_{\theta,t}^S \tilde{\theta}_{t+1} + \Phi_{c,t}^S \tilde{c}_t + \Phi_{p,t}^S \tilde{p}_t + \Phi_{v,t}^S \tilde{v}_t,$$  \hspace{1cm} (A40)

where the $\Phi$-coefficients are given in the Internet Appendix.

We can compute the demand function of date-$t$ financial hedgers as follows:

$$d_{H,t} = \frac{1}{\gamma_H} E(\tilde{v}_{t+1}|I_t, \tilde{\alpha}_{t+1}) - \tilde{p}_t - \frac{(kA_{\delta,t+1} + 1)\rho_\delta \sqrt{\tau_\delta}}{(1+k)\sqrt{\tau_\delta}} \tilde{\alpha}_{t+1}$$

$$- \frac{\left( \frac{(kA_{\delta,t+1} + 1)\rho_\delta^2}{(1+k)^2} \tau_\delta \right)}{\sqrt{\tau_\delta}} \tilde{v}_{t+1},$$  \hspace{1cm} (A41)
Using (A29), we have
\[
E(\tilde{\nu}_{t+1}|\mathcal{I}_t, \tilde{\alpha}_{t+1}) = \frac{k}{1+k} (B_{\theta,t+1} \tilde{\theta} + B_{c,t+1} \tilde{c}) \\
+ \frac{1}{1+k} \left[ (kA_{\theta,t+1} + 1) E(\tilde{\theta}_{t+1}|\tilde{q}_t) \\
+ kA_{b,t+1} + (kA_{c,t+1} + h) \tilde{c}_t \\
+ k(A_v,t+1 + 1) \tilde{v}_t + [kA_p,t+1 - (h + k)] \tilde{p}_t \right], \tag{A42}
\]
\[
Var(\tilde{\nu}_{t+1}|\mathcal{I}_t, \tilde{\alpha}_{t+1}) = \left( \frac{k}{1+k} \right)^2 \left( B_{\theta,t+1}^2 \frac{1}{\tau_\theta} + B_{c,t+1}^2 \frac{1}{\tau_c} + B_{a,t+1}^2 \frac{1}{\tau_a} + B_{\xi,t+1}^2 \frac{1}{\tau_\xi} \right) \\
+ \frac{(kA_{\delta,t+1} + 1)^2}{\tau_\delta} + (kA_{\theta,t+1} + 1)^2 Var(\tilde{\theta}_{t+1}|\tilde{q}_t) \\
\frac{(1+k)^2}{(1+k)^2}, \tag{A43}
\]
where \(\tilde{q}_t\) is the signal that financial speculators extract from the price \(\tilde{p}_t\):
\[
\tilde{q}_t = \frac{\tilde{p}_t - A(\mathcal{I}_t) - B_{\alpha,t} \tilde{\alpha}_{t+1}}{B_{\theta,t}} = \tilde{\theta}_{t+1} + \frac{B_{\xi,t}}{B_{\theta,t}} \tilde{\xi}_t, \tag{A44}
\]
which is normally distributed with mean \(\tilde{\theta}_{t+1}\) and precision given by
\[
\tau_{q,t} = \left( \frac{B_{\theta,t}}{B_{\xi,t}} \right)^2 \tau_\xi. \tag{A45}
\]
Inserting (A42)–(A44) into (A41), we can compute the aggregate demand from date-\(t\) financial hedgers as follows:
\[
\Lambda_{H,t} d_{H,t} = \Phi^H_{0,t} + \Phi^H_{\theta,t} \tilde{\theta}_{t+1} + \Phi^H_{c,t} \tilde{c}_t + \Phi^H_{a,t} \tilde{a}_{t+1} + \Phi^H_{\xi,t} \tilde{\xi}_t + \Phi^H_{p,t} \tilde{p}_t + \Phi^H_{v,t} \tilde{v}_t, \tag{A46}
\]
where the \(\Phi\)-coefficients are given in the Internet Appendix.

Inserting the demand functions (A37), (A40), and (A46) into the futures market-clearing condition (40), and using the expressions of \(\tilde{\nu}_t\) and \(\tilde{G}_t\) in equations (42) and (43), we can compute the implied price function. Then, comparing this implied price function with the conjectured price function, we obtain the following system of equations:
\[
A_{0,t} = -\frac{\Phi^P_{0,t} + \Phi^S_{0,t} + \Phi^H_{0,t}}{\left( \Phi^P_{p,t} + \Phi^S_{p,t} + \Phi^H_{p,t} \right) + \left( \Phi^P_{v,t} + \Phi^S_{v,t} + \Phi^H_{v,t} \right) \frac{k}{1+k}}, \tag{A47}
\]
\[
A_{\theta,t} = -\frac{\Phi^P_{\theta,t} + \Phi^S_{\theta,t} + \Phi^H_{\theta,t}}{\left( \Phi^P_{p,t} + \Phi^S_{p,t} + \Phi^H_{p,t} \right) + \left( \Phi^P_{v,t} + \Phi^S_{v,t} + \Phi^H_{v,t} \right) \frac{k}{1+k}}, \tag{A48}
\]
\[\text{55}\]
\[
A_{\delta,t} = \frac{\Phi^P_{v,t} + \Phi^S_{c,t} + \Phi^H_{v,t}}{(\Phi^P_{p,t} + \Phi^S_{p,t} + \Phi^H_{p,t}) + (\Phi^P_{v,t} + \Phi^S_{v,t} + \Phi^H_{v,t}) \frac{k}{1+k}},
\]
\[
A_{c,t} = \frac{\Phi^P_{v,t} + \Phi^S_{c,t} + \Phi^H_{c,t}}{(\Phi^P_{p,t} + \Phi^S_{p,t} + \Phi^H_{p,t}) + (\Phi^P_{v,t} + \Phi^S_{v,t} + \Phi^H_{v,t}) \frac{k}{1+k}},
\]
\[
A_{v,t} = \frac{\Phi^P_{v,t} + \Phi^S_{v,t} + \Phi^H_{v,t}}{(\Phi^P_{p,t} + \Phi^S_{p,t} + \Phi^H_{p,t}) + (\Phi^P_{v,t} + \Phi^S_{v,t} + \Phi^H_{v,t}) \frac{k}{1+k}},
\]
\[
A_{p,t} = \frac{\Phi^P_{v,t} + \Phi^S_{v,t} + \Phi^H_{v,t}}{(\Phi^P_{p,t} + \Phi^S_{p,t} + \Phi^H_{p,t}) + (\Phi^P_{v,t} + \Phi^S_{v,t} + \Phi^H_{v,t}) \frac{k}{1+k}},
\]
\[
B_{c,t} = \frac{\Phi^P_{c,t} + \Phi^S_{c,t} + \Phi^H_{c,t}}{(\Phi^P_{p,t} + \Phi^S_{p,t} + \Phi^H_{p,t}) + (\Phi^P_{v,t} + \Phi^S_{v,t} + \Phi^H_{v,t}) \frac{k}{1+k}},
\]
\[
B_{\theta,t} = \frac{\Phi^P_{\theta,t} + \Phi^S_{\theta,t} + \Phi^H_{\theta,t}}{(\Phi^P_{p,t} + \Phi^S_{p,t} + \Phi^H_{p,t}) + (\Phi^P_{v,t} + \Phi^S_{v,t} + \Phi^H_{v,t}) \frac{k}{1+k}},
\]
\[
B_{\alpha,t} = \frac{\Phi^P_{\alpha,t} + \Phi^H_{\alpha,t}}{(\Phi^P_{p,t} + \Phi^S_{p,t} + \Phi^H_{p,t}) + (\Phi^P_{v,t} + \Phi^S_{v,t} + \Phi^H_{v,t}) \frac{k}{1+k}},
\]
\[
B_{\xi,t} = \frac{\Phi^P_{\xi,t} + \Phi^H_{\xi,t} + 1}{(\Phi^P_{p,t} + \Phi^S_{p,t} + \Phi^H_{p,t}) + (\Phi^P_{v,t} + \Phi^S_{v,t} + \Phi^H_{v,t}) \frac{k}{1+k}}.
\]

Note that equation (A47) is equation (45) in Proposition 5 and that equations (A54)–(A56) are equations (47)–(49) in Proposition 5.

By equations (A48)–(A52), we can show

\[
A_{\delta,t} = A_{\theta,t}, A_{c,t} = hA_{\theta,t}, A_{v,t} = kA_{\theta,t}, \text{ and } A_{p,t} = -(h + k)A_{\theta,t}.
\]

By the expressions of \(A_{c,t}\) and \(B_{c,t}\) in (A50) and (A53) and the expressions of \(\Phi\)'s in the Internet Appendix, we can show that \(B_{c,t} = \frac{1+k}{k}A_{c,t}\). Define \(A_{1,t} = -A_{\theta,t}\). Equation (A48) then becomes equation (46) in Proposition 5.
References


Internet Appendix: Additional Materials

OA.1 The J-Coefficients in Equation (A10)

The 7th order polynomial of $m_\xi$ in equation (A10) characterizes the financial market equilibrium in the baseline model. The J-coefficients in this polynomial are as follows:

$$J_7 = -\beta \tau_\alpha \tau_\delta^2 (1 - \rho^2)^2,$$
$$J_6 = -\beta \tau_\alpha \tau_\delta \tau_\epsilon^2 (1 - \rho^2)(2\lambda_H - \lambda_S + \rho^2 \lambda_S),$$
$$J_5 = -\beta \tau_\delta^2 \left(\begin{array}{c}
3\tau_\theta \tau_\alpha + 3\tau_\alpha \tau_\delta + \tau_\alpha \tau_\epsilon \\
-6\rho^2 \tau_\theta \tau_\alpha + 3\rho^4 \tau_\theta \tau_\alpha - 4\rho^2 \tau_\alpha \tau_\delta \\
-2\rho^2 \tau_\alpha \tau_\epsilon + \rho^4 \tau_\alpha \tau_\delta + \rho^4 \tau_\alpha \tau_\delta + \lambda_H \tau_\alpha \tau_\delta^2 \tau_\epsilon \\
+\rho^2 \lambda_H \tau_\delta^2 \tau_\epsilon \tau_\eta + 2\lambda_H \lambda_S \tau_\alpha \tau_\delta^2 \tau_\epsilon + 2\rho^2 \lambda_H \lambda_S \tau_\alpha \tau_\delta^2 \tau_\epsilon
\end{array}\right),$$
$$J_4 = \tau_\delta^2 \left(\begin{array}{c}
3\beta \tau_\theta \tau_\alpha + 3\beta \tau_\alpha \tau_\delta + \beta \tau_\alpha \tau_\epsilon \\
-4\beta \lambda_H \tau_\theta \tau_\alpha + 4\beta \lambda_H \tau_\alpha \tau_\delta - 2\beta \lambda_H \tau_\alpha \tau_\epsilon \\
+3\beta \lambda_S \tau_\theta \tau_\alpha + 3\beta \lambda_S \tau_\alpha \tau_\delta + \beta \lambda_S \tau_\alpha \tau_\epsilon \\
+4\beta \rho^2 \lambda_H \tau_\theta \tau_\alpha + 2\beta \rho^2 \lambda_H \tau_\alpha \tau_\delta + 2\beta \rho^2 \lambda_H \tau_\alpha \tau_\epsilon \\
-6\beta \rho^2 \lambda_S \tau_\theta \tau_\alpha + 3\beta \rho^4 \lambda_S \tau_\theta \tau_\alpha - 4\beta \rho^2 \lambda_S \tau_\alpha \tau_\delta \\
-2\beta \rho^2 \lambda_S \tau_\alpha \tau_\epsilon + \beta \rho^4 \lambda_S \tau_\alpha \tau_\delta + \beta \rho^4 \lambda_S \tau_\alpha \tau_\epsilon \\
+\beta \lambda_H \lambda_S \tau_\alpha \tau_\delta^2 \tau_\epsilon + \rho^2 \lambda_H \tau_\delta \tau_\epsilon \tau_\xi + \beta \rho^2 \lambda_H \lambda_S \tau_\delta \tau_\epsilon \tau_\xi + \beta \rho^2 \lambda_H \lambda_S \tau_\delta \tau_\epsilon \tau_\xi
\end{array}\right),$$
$$J_3 = -\tau_\xi \left(\begin{array}{c}
3\beta \tau_\theta \tau_\alpha + 3\beta \tau_\alpha \tau_\delta + 6\beta \tau_\alpha \tau_\epsilon \\
+3\beta \rho^4 \tau_\theta \tau_\alpha - 2\beta \rho^2 \tau_\alpha \tau_\delta + 6\beta \tau_\theta \tau_\alpha \tau_\delta \\
+2\beta \tau_\theta \tau_\alpha \tau_\epsilon + 2\beta \tau_\alpha \tau_\delta \tau_\epsilon + \beta \lambda_H \tau_\alpha \tau_\delta^2 \tau_\epsilon \\
-8\beta \rho^2 \tau_\theta \tau_\alpha \tau_\delta - 4\beta \rho^2 \tau_\theta \tau_\alpha \tau_\epsilon + 2\beta \rho^4 \tau_\theta \tau_\alpha \tau_\epsilon \\
+2\beta \rho^4 \tau_\theta \tau_\alpha \tau_\delta - 2\beta \rho^2 \tau_\alpha \tau_\delta \tau_\epsilon + \beta \lambda_H \tau_\theta \tau_\alpha \tau_\delta^2 \tau_\epsilon \\
+\beta \lambda_H \lambda_S \tau_\alpha \tau_\delta^2 \tau_\epsilon + 4\beta \lambda_H \lambda_S \tau_\alpha \tau_\delta^2 \tau_\epsilon - 2\lambda_H \tau_\alpha \tau_\delta^2 \tau_\epsilon \\
+2\beta \rho^2 \lambda_H \lambda_S \tau_\alpha \tau_\delta^2 \tau_\epsilon + 2\rho^2 \lambda_H \tau_\alpha \tau_\delta^2 \tau_\epsilon \\
+2\beta \rho^2 \lambda_H \lambda_S \tau_\alpha \tau_\delta^2 \tau_\epsilon + 2\beta \rho^2 \lambda_H \tau_\alpha \tau_\delta^2 \tau_\epsilon
\end{array}\right).$$
$$J_2 = \tau_\delta \tau_\xi$$

\[
\begin{pmatrix}
2\tau_\theta \tau_\alpha \tau_\epsilon + 2\tau_\alpha \tau_\delta \tau_\epsilon - 2\beta \lambda H \tau_\theta^2 \tau_\alpha \\
-2\beta \lambda H \tau_\alpha \tau_\delta^2 + 3\beta \lambda S \tau_\theta^2 \tau_\alpha + 3\beta \lambda S \tau_\alpha \tau_\delta^2 \\
-4\rho^2 \tau_\theta \tau_\alpha \tau_\epsilon + 2\rho^2 \tau_\theta \tau_\alpha \tau_\epsilon - 2\rho^2 \tau_\alpha \tau_\delta \tau_\epsilon \\
+\beta \lambda H \lambda S \tau_\alpha \tau_\delta^2 \tau_\xi + \lambda^2 \lambda H \tau_\alpha \tau_\delta^2 \tau_\xi + 2\beta \rho^2 \lambda H \tau_\theta \tau_\delta \tau_\alpha \tau_\epsilon \\
-6\beta \rho^2 \lambda S \tau_\theta \tau_\delta \tau_\alpha \tau_\epsilon + 3\beta \rho^4 \lambda S \tau_\theta^2 \tau_\alpha \tau_\epsilon - 2\beta \rho^2 \lambda S \tau_\delta \tau_\alpha \tau_\delta \tau_\epsilon \\
-4\beta \lambda H \tau_\theta \tau_\alpha \tau_\delta - 2\beta \lambda H \tau_\theta \tau_\alpha \tau_\delta - 2\beta \lambda H \tau_\alpha \tau_\delta \tau_\epsilon \\
+6\lambda S \tau_\theta \tau_\alpha \tau_\delta \tau_\epsilon + 2\beta \lambda S \tau_\theta \tau_\delta \tau_\alpha \tau_\epsilon + 2\lambda S \tau_\alpha \tau_\delta \tau_\delta \\
+2\beta \rho^2 \lambda H \tau_\theta \tau_\alpha \tau_\delta + 2\beta \rho^2 \lambda H \tau_\theta \tau_\alpha \tau_\delta - 8\beta \rho^2 \lambda \tau_\alpha \tau_\delta \tau_\epsilon \\
-4\beta \rho^2 \lambda S \tau_\theta \tau_\alpha \tau_\delta + 2\beta \rho^4 \lambda S \tau_\theta \tau_\alpha \tau_\delta + 2\beta \rho^2 \lambda S \tau_\theta \tau_\alpha \tau_\delta \\
-2\beta \rho^2 \lambda S \tau_\theta \tau_\delta \tau_\alpha \tau_\epsilon + \beta \lambda^2 \lambda S \tau_\theta \tau_\delta \tau_\alpha \tau_\delta \tau_\epsilon + \beta \lambda^2 \lambda S \tau_\delta \tau_\alpha \tau_\delta \tau_\epsilon \\
+2\beta \rho^2 \lambda H \tau_\theta \tau_\alpha \tau_\delta \tau_\epsilon \tau_\eta + 2\beta \rho^2 \lambda H \tau_\theta \tau_\delta \tau_\delta \tau_\epsilon \tau_\eta \\
+2\beta \rho^2 \lambda^2 \lambda S \tau_\delta \tau_\delta \tau_\alpha \tau_\delta \tau_\epsilon \tau_\eta + 2\beta \rho^2 \lambda^2 \lambda S \tau_\theta \tau_\alpha \tau_\delta \tau_\epsilon \tau_\eta \\
\end{pmatrix}
\]

$$J_1 = -$$

\[
\begin{pmatrix}
\beta \rho^2 \tau_\theta \tau_\alpha \tau_\delta - 2\beta \rho^2 \tau_\delta^2 \tau_\alpha + \beta \rho^4 \tau_\delta \tau_\alpha \\
+3\beta \tau_\theta \tau_\alpha \tau_\delta^2 + 3\beta \tau_\theta \tau_\tau_\delta \tau_\delta - 2\beta \rho^2 \tau_\theta \tau_\delta \tau_\alpha \tau_\epsilon - 2\beta \rho^2 \tau_\delta \tau_\delta \tau_\alpha \tau_\epsilon \\
+\beta \rho^4 \tau_\theta \tau_\delta \tau_\alpha \tau_\epsilon + \beta \rho^4 \tau_\theta \tau_\delta \tau_\epsilon + 2\beta \theta \tau_\delta \tau_\alpha \tau_\delta \tau_\epsilon \\
-2\beta \lambda H \lambda S \tau_\theta \tau_\alpha \tau_\delta \tau_\delta \tau_\epsilon + 2\beta \lambda H \tau_\theta \tau_\alpha \tau_\delta \tau_\delta \tau_\epsilon + \beta \rho^2 \lambda^2 \tau_\theta \tau_\delta \tau_\delta \tau_\epsilon \tau_\eta \\
-4\beta \lambda H \lambda S \tau_\theta \tau_\alpha \tau_\delta \tau_\delta \tau_\epsilon + 2\beta \lambda H \tau_\theta \tau_\alpha \tau_\delta \tau_\delta \tau_\epsilon + 2\beta \lambda H \tau_\alpha \tau_\delta \tau_\delta \tau_\epsilon \tau_\eta \\
-2\beta \rho^2 \lambda H \tau_\theta \tau_\alpha \tau_\delta \tau_\delta \tau_\epsilon \tau_\eta + 2\beta \rho^2 \lambda H \tau_\theta \tau_\alpha \tau_\delta \tau_\delta \tau_\epsilon \tau_\eta + 2\rho^2 \lambda H \tau_\alpha \tau_\delta \tau_\delta \tau_\epsilon \tau_\eta \\
+2\beta \rho^2 \lambda H \tau_\alpha \tau_\delta \tau_\delta \tau_\epsilon \tau_\eta + 2\beta \rho^2 \lambda H \tau_\alpha \tau_\delta \tau_\delta \tau_\epsilon \tau_\eta + 2\beta \rho^2 \lambda H \tau_\alpha \tau_\delta \tau_\delta \tau_\epsilon \tau_\eta \\
\end{pmatrix}
\]

and

$$J_0 = \tau_\delta \left( \tau_\theta \tau_\alpha \tau_\delta - 2\rho^2 \tau_\delta^2 \tau_\alpha + \rho^2 \tau_\delta \tau_\alpha \\
+2\tau_\theta \tau_\alpha \tau_\delta - 2\rho^2 \tau_\tau_\alpha \tau_\delta + 3\beta \rho^2 \tau_\theta \tau_\delta \tau_\alpha \tau_\delta \tau_\epsilon \tau_\eta \right) \left( \tau_\epsilon + 2\beta \lambda S \tau_\theta + \beta \lambda S \tau_\delta + 2\lambda \xi \tau_\epsilon \right).$$

**OA.2 The Φ-Coefficients in Equations (A37), (A40), and (A46)**

The Φ-coefficients in the aggregate demand (A37) of date-t commodity producers are as follows:

1. **Constant**: $\Phi_{0,t} = \frac{1}{1 + \rho^2 \beta \vartheta} \left( B_{0,t+1} + B_{t+1,t+1} \right) + \frac{k_{A0,t+1}}{1 + \rho^2 \beta \vartheta} \left( \beta \vartheta \vartheta [t_{z+1} | \xi, \delta] \right) \left( \gamma_{p,t+1} \right)$

2. **Coefficient on $\hat{\theta}_{t+1}$**: $\Phi_{p,t} = \frac{1}{1 + \rho^2 \beta \vartheta} \left( \beta \vartheta \vartheta [t_{z+1} | \xi, \delta] \right) \left( \gamma_{p,t+1} \right)$

3. **Coefficient on $\tilde{c}_t$**: $\Phi_{c,t} = \frac{k_{A0,t+1}}{1 + \rho^2 \beta \vartheta} \left( \beta \vartheta \vartheta [t_{z+1} | \xi, \delta] \right) + h$

4. **Coefficient on $\tilde{a}_t$**: $\Phi_{a,t} = \frac{1}{1 + \rho^2 \beta \vartheta} \left( \beta \vartheta \vartheta [t_{z+1} | \xi, \delta] \right) \left( \gamma_{p,t+1} \right) \left( \beta \vartheta \vartheta [t_{z+1} | \xi, \delta] \right)$

5. **Coefficient on $\tilde{\xi}_t$**: $\Phi_{\xi,t} = \frac{k_{A0,t+1}}{1 + \rho^2 \beta \vartheta} \left( \beta \vartheta \vartheta [t_{z+1} | \xi, \delta] \right) \left( \gamma_{p,t+1} \right) \left( \beta \vartheta \vartheta [t_{z+1} | \xi, \delta] \right)$
(6) Coefficient on $\tilde{p}_t$: $\Phi_{p,t}^P \equiv \frac{kA_{p,t+1} - (h+k) - 1}{\beta Var(\tilde{v}_{t+1} | Z_t, \tilde{z}_t)} (h + k) ;$

(7) Coefficient on $\tilde{v}_t$: $\Phi_{v,t}^P \equiv \frac{k(A_{v,t+1} + 1)}{\beta Var(\tilde{v}_{t+1} | Z_t, \tilde{z}_t)} + k.$

The $\Phi$-coefficients in the aggregate demand (A40) of date-$t$ financial speculators are as follows:

(1) Constant: $\Phi_{0,t}^S \equiv \lambda_{S,t} \frac{kA_{0,t+1} + 1}{\sigma^2_{B_{0,t+1} + B_{e,t+1}}} + \frac{kA_{0,t+1}}{1 + k} ;$

(2) Coefficient on $\tilde{\theta}_{t+1}$: $\Phi_{\theta,t}^S \equiv \lambda_{S,t} \frac{kA_{\theta,t+1} + 1}{\sigma^2_{B_{\theta,t+1} + B_{e,t+1}}} ;$

(3) Coefficient on $\tilde{c}_t$: $\Phi_{c,t}^S \equiv \lambda_{S,t} \frac{kA_{c,t+1} + 1}{\sigma^2_{B_{c,t+1} + B_{e,t+1}}} ;$

(4) Coefficient on $\tilde{p}_t$: $\Phi_{p,t}^S \equiv \lambda_{S,t} \frac{kA_{p,t+1} + 1}{\sigma^2_{B_{p,t+1} + B_{e,t+1}}} ;$

(5) Coefficient on $\tilde{v}_t$: $\Phi_{v,t}^S \equiv \lambda_{S,t} \frac{kA_{v,t+1} + 1}{\sigma^2_{B_{v,t+1} + B_{e,t+1}}} .

The $\Phi$-coefficients in the aggregate demand (A46) of date-$t$ financial hedgers are as follows:

(1) Constant: $\Phi_{0,t}^H \equiv \lambda_{H,t} \frac{kA_{0,t+1} + 1}{\sigma^2_{B_{0,t+1} + B_{e,t+1}}} ;$

(2) Coefficient on $\tilde{\theta}_{t+1}$: $\Phi_{\theta,t}^H \equiv \lambda_{H,t} \frac{kA_{\theta,t+1} + 1}{\sigma^2_{B_{\theta,t+1} + B_{e,t+1}}} ;$

(3) Coefficient on $\tilde{c}_t$: $\Phi_{c,t}^H \equiv \lambda_{H,t} \frac{kA_{c,t+1} + 1}{\sigma^2_{B_{c,t+1} + B_{e,t+1}}} ;$

(4) Coefficient on $\tilde{p}_t$: $\Phi_{p,t}^H \equiv \lambda_{H,t} \frac{kA_{p,t+1} + 1}{\sigma^2_{B_{p,t+1} + B_{e,t+1}}} ;$

(5) Coefficient on $\tilde{v}_t$: $\Phi_{v,t}^H \equiv \lambda_{H,t} \frac{kA_{v,t+1} + 1}{\sigma^2_{B_{v,t+1} + B_{e,t+1}}} .

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