Credit Rating Inflation and Firms’ Investments

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Abstract

We analyze credit ratings’ effects on firms’ investments in a rational debt-financing game that features a feedback loop. The credit rating agency (CRA) inflates the rating, providing a biased but informative signal to creditors. Creditors’ response to the rating affects the firm’s investment decision and credit quality, which is reflected in the rating. The CRA might reduce ex-ante economic efficiency, which results solely from the feedback effect of the rating: The CRA assigns more firms high ratings and allows them to gamble for resurrection. We derive empirical predictions on the determinants of rating standards and inflation and discuss policy implications.

Key Words: Credit rating agency, rating inflation, real effect, feedback effect

JEL Classification: D82, D83, G24, G32

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1 Introduction

Credit rating agencies (CRAs) have been criticized for playing a central role in financial failures. Prominent examples include the collapse of companies like Enron and WorldCom in 2002, and the crisis of 2007 – 2009 that led the Financial Crisis Inquiry Report to conclude that “the failures of credit rating agencies were essential cogs in the wheel of financial destruction.” CRAs are often blamed for assigning overgenerous ratings, and this has been documented by several empirical studies.\footnote{See, for example, Jiang, Stanford, and Xie (2012), Strobl and Xia (2012), and Cornaggia and Cornaggia (2013).} These studies argue that the credit rating inflation can be attributed to the conflicts of interest caused by the “issuer-pays” business model, according to which CRAs are paid by the issuers they are assessing. The concern is then that inflated credit ratings might mislead creditors, help bad investments get funded, and thus have negative real effects.

Thinking about these claims through the lens of models with rational creditors, it is not clear why inflated credit ratings would have negative real effects. To mislead creditors, credit ratings must provide some valuable information. Otherwise, the ratings would be ignored and CRAs would have no effect. But, if CRAs are providing creditors with informative (though potentially biased) signals, they should be able to promote, rather than hurt, economic efficiency, even if they do not reach the first-best outcome. The question then is whether CRAs with a motive to inflate ratings can have negative effects on economic efficiency in a world with rational creditors.

In this paper, we provide a model to analyze this question. Our model is parsimonious, but rich enough to capture the essential elements of the interaction between a CRA, creditors, and an issuing firm. First, we consider a CRA who, by assigning a higher rating, earns higher revenue but incurs higher cost when the firm fails. In particular, if the CRA is verified to knowingly assign inflated ratings, its cost will be so high that it will never assign a high rating to a firm who will surely fail. By contrast, if the rating manipulation cannot be verified, the CRA’s cost of assigning a high rating to the firm that fails is relatively low. The restriction that the CRA’s cost depends on the verifiability of its rating behavior is therefore called the \textit{partial verifiability constraint}. Thanks to this constraint, the CRA’s rating may be biased but contains valuable information. Second, the audience for the rating is a group of rational creditors who have dispersed beliefs about the firm’s economic fundamentals and have to make a decision whether to invest in the debt of the firm or not based on their own private information and the credit rating. Third, the aggregate action taken by the creditors affects the cost of capital of the firm, which affects the firm’s investment decision and, through that, its credit quality. Fourth,
when setting the rating, the CRA accounts for the effect of the rating on the actions of creditors and the firm and their effects on the credit quality. Hence, there is a feedback loop, whereby the rating affects creditors’ behavior, which affects the behavior of the issuer and its credit quality, which in turn is reflected in the rating.

In our view, this feedback loop is central to understanding the effects of CRAs. After all, CRAs, given their market power, are in a unique position to provide information that ends up affecting the credit quality of the firms that they rate. Indeed, credit rating agencies claim that their ratings are forward-looking, emphasizing that they will assess the potential impact of foreseeable future events that include the impacts of the ratings themselves. For example, Moody’s, in a document that explains its rating process, explicitly acknowledges “the effect of the rating action on the issuer, including the possible effect on issuer’s market access or conditional obligations.” It goes on to note that “the level of rating that Moody’s assigns to an issuer that might experience potential changes in market access or conditional obligations will reflect Moody’s assessment of the issuer’s creditworthiness, including such considerations.” As we argue below, this feedback loop is largely missing from the existing literature on credit rating agencies. We show that it plays a critical role in our results.

In our model, a high rating, even though potentially inflated, provides positive information to creditors because it implies that the firm does not belong to a group of particularly low quality, for which the partial verifiability constraint binds. Hence, a high rating makes creditors more optimistic and likely to invest in the debt, which reduces the firm’s financial costs and changes its investment decision. This is how the rating ends up having a real effect. For some firms, for which financial costs are relatively high, the reduction in financial costs leads to inefficient risk taking. Lower financial costs enable them to gamble for resurrection and take an investment with low expected return but high potential upside. For other firms, for which financial costs are relatively low, the reduction in financial costs provides more skin in the game, encouraging a shift from risky low-expected-return investments to less risky high-expected-return investments. The implications for economic efficiency are negative in the first case and positive in the second. Hence, the overall effect of the CRA on economic efficiency depends on how strong these effects are expected to be relative to each other. This depends on model parameters.

2Hence, a high credit rating generated by the lax rating strategy is not a cheap talk as in Crawford and Sobel (1982). Due to the partial verifiability constraint, the high rating provides the creditors with a public signal about the firm. Such a public signal is endogenous and takes a different form from that in Morris and Shin (2002): It truncates the supports of creditors’ interim beliefs from below.
Varying the parameters of the model, we can show that the overall expected real effects of the CRA can be positive or negative. A key result that we identify is that the CRA’s expected real effect is more likely to be negative when the upside of the risky inefficient investment is higher. A high upside makes gambling for resurrection more attractive and more likely to follow from a reduction in the cost of capital.

An important insight of our model emerges when we decompose the CRA’s ex-ante real effects into two components. The first one is the CRA’s pure informational effect and the second one is the CRA’s feedback effect. The informational effect is obtained when the CRA does not recognize the effect of its rating on the firm’s investment and credit quality, and just provides the (biased) information that would pertain to the equilibrium without a CRA. The feedback effect is the additional effect coming from the fact that the CRA is strategic and takes into account how the rating affects the creditors and the firm. It then assigns a rating that takes advantage of these responses in maximizing its expected profit.

We show that the informational effect always increases economic efficiency. When the CRA acts in a reflecting way, just providing the biased information to the creditors, it helps them achieve a more efficient outcome. The negative implications for economic efficiency thus come purely from the feedback effect: When taking into account the creditors’ and the firm’s responses to the ratings, the CRA finds it beneficial to assign high ratings to more firms, allowing them to gamble for resurrection. When they gamble for resurrection, the CRA can assign them a high rating without violating the partial verifiability constraint, and so achieve higher values of its objective function at the expense of lower economic efficiency. The CRA essentially uses its market power in information provision to shape economic outcomes, and because of the inflation motive, economic efficiency might be sacrificed. This proves that the introduction of feedback effects into models of credit ratings is indeed crucial for understanding the overall effects of CRAs.

We derive several empirical implications out of the model. First, a key insight that emerges out of our analysis is that lax rating standards and rating inflation are two different endogenous terms and they do not necessarily move in the same direction. Laxer rating standards correspond to a case where the CRA is more likely to give a high rating to a given firm with given fundamentals. However, this does not necessarily imply higher rating inflation. The reason is the feedback effect: When the CRA changes the rating policy, it also affects the credit quality of the firm, and so inflation, which is the difference between reported credit quality and actual credit quality, could change in either direction. This is an important point to consider in future empirical work. Second, we conduct several comparative static analyses, which demon-
strate this point and provide new empirical predictions about CRAs’ credit rating standards and credit rating inflation. In particular, a decrease in the firm’s transparency, an increase in upside returns of risky projects, and an increase in the market liquidity will all lead to laxer rating standards (assigning high ratings to more firms). However, these changes of economic environments do not necessarily cause higher rating inflation. Specifically, a decrease in the firm’s transparency has an ambiguous effect on rating inflation, an increase in upside returns of risky projects will cause higher rating inflation, and an increase in the market liquidity leads to lower rating inflation.

We also derive policy implications. The form of the rating agency’s equilibrium rating strategy depends upon the ratio of its incremental revenue to its incremental potential cost due to a rating upgrade. When such a ratio is too high, the rating agency will inflate ratings and cause the inefficiency highlighted in our paper. However, it is not the case that an effective policy should just reduce such a ratio (for example, a policy makes the incremental cost extremely large), because the rating agency would then have an incentive to deflate the rating, and this ends up having the same efficiency implications as when the ratio is sufficiently large and the rating agency inflates ratings. Therefore, in order to get to a truth telling CRA in equilibrium, the ratio of the incremental revenue to the incremental cost due to a rating upgrade has to be set in a particular range. Unfortunately, this might be difficult for policymakers to calibrate.

The fact that rating inflation and rating deflation end up having the same efficiency implications highlights that the key effect of the CRA is that of pooling different firms together. This is intuitive, given that in our model, creditors are all rational and update based on the equilibrium strategy of the CRA. Hence, the label that is put on firms that are pooled together should not matter; what matters is which firms are pooled together. This is different from other models, where the effect of inflation is assumed via creditors’ irrationality or regulatory arbitrage. We stick to the focus on inflation rather than deflation because inflation is the phenomenon that has been discussed empirically and in policy circles. But, it is important to highlight that in a model where the real effect is driven by rational updating, inflation should not have different efficiency implications than deflation.

Finally, a question that arises concerning the real effects of credit ratings is whether they should be expected to persist in a framework where, in addition to credit ratings, there are various other signals available to creditors in the corporate bond market. We argue that even if there are other public signals, the CRA’s real effects are still significant given creditors’ heterogeneous private signals about the firm’s economic fundamentals. Creditors’ heterogeneous private signals imply that they hold dispersed beliefs. Hence, a high credit rating, by truncating
creditors belief supports, will surely affect some creditors’ beliefs about the firm’s investment choice. As a result, the credit ratings will affect the firm’s financial cost, investment decision, and credit quality, which in turn will affect other creditors’ behavior (even though their beliefs are hardly affected by the credit ratings directly), leading to the significant real effects of the CRA. We show that the dispersed beliefs among creditors are crucial for the rating agency to play a role by demonstrating that the rating agency’s effects become negligible when creditors have precise homogeneous information.

The real effects of the CRAs have been documented empirically. For example, Sufi (2009), Bannier, Hirsch, and Wiemann (2012), and Almeida, Cunha, Ferreira, and Restrepo (2017) show credit ratings’ causal effects on firm investment decisions. While such feedback effects are largely absent in models of credit ratings, several previous papers introduced different forms of feedback, in particular Boot, Milbourn, and Schmeits (2006), Manso (2013), Goel and Thakor (2015), and Holden, Natvik, and Vigier (2018). A key difference between our paper and these previous papers is that in our paper the feedback effect is a result of information transmission from the rating agency to creditors, whereas in these papers it is a result of changing the focal point for equilibrium selection, or of contractual features that affect the firm when the rating changes, or of the CRA’s incentives to balance the issuer’s payoff and the social welfare. While we think these are interesting dimensions to explore, we believe that the informational role of the rating is fundamental, going back to the basic motivation of introducing ratings to begin with, and so we focus on it here. Another key difference is that in these other papers there is no or limited rating inflation and the CRA wants to provide accurate ratings. Our research question, on the other hand, centers on the positive and negative real effects of a CRA with an inflation motive. As we show, these effects are all driven by the information transmission, and would not arise in the frameworks of the other feedback papers.

Another strand of literature, including Donaldson and Piacentino (2018) and Parlour and Rajan (2019), investigates credit ratings’ real effects through contracting. In this literature, credit ratings are viewed as exogenous public information, and therefore agents with information asymmetry will have incentives to contract on them. In this paper, we endogenize credit ratings but abstract away from the contracting between the firm and the creditors.

There are also several theory papers that study rating inflation. Usually, they attribute credit rating inflation to creditors’ imperfect rationality (Bolton, Freixas, and Shapiro 2012; Skreta and Veldkamp 2009), or regulations tied to ratings (Opp, Opp, and Harris 2013).3 Hence, in these

3One exception is Frenkel (2015) who shows credit rating inflation may be generated by CRAs’ “double reputation.” However, one necessary condition in Frenkel (2015) is that the CRA has different possible behavioral types,
models, inflated credit ratings are not informative signals to the creditors who are naïve or have regulatory motives. Again, while we think that bounded rationality and regulatory constraints are important, our aim is to analyze the role of the CRA in a rational environment.

We model the firm’s credit market as a global game that features dispersed beliefs. This is mainly a modeling device for equilibrium uniqueness, which largely simplifies comparative static analysis about the CRA’s real effects. Although the global game is not essential in this paper, our paper still contributes to the literature on global games. Our model differs from traditional global game models (Carlsson and van Damme 1993; Morris and Shin 2003) in the endogenous information provided by the CRA to the creditors. The aspect of dispersed beliefs gives the information provided by the CRA prominence even when creditors are very well informed. Several papers endogenized the information structure in different ways in a global game setting. Angeletos, Hellwig, and Pavan (2006) and Angeletos and Pavan (2013) model the signaling effects of the government’s preemptive defending policies, which pools very strong governments and very weak ones together. Edmond (2013) discusses a dictator’s costly private information manipulation, where all revolutionaries’ interim beliefs have full supports. Hence, the belief updating in these models differs from that in our model. In fact, the belief updating in our model is closer to that in Angeletos, Hellwig, and Pavan (2007) and Huang (2017). Nevertheless, our model has a unique equilibrium, because the CRA’s incentives to inflate credit ratings generate new dominant regions of not investing. Our model is related to Bouvard, Chaigneau, and De Motta (2015) who study how the government chooses public signals (without commitment) to shape bank depositors’ posterior beliefs. Our model differs from theirs mainly in that the firm’s investments and thus its credit quality are affected by credit ratings assigned by the CRA, and the CRA will take such effects into account to strategically choose the optimal rating rule.

Finally, our paper is also related to Goldstein and Huang (2016) who show how the government persuades investors not to attack a regime by committing to abandon the regime when it is below some cutoff level. Our current model is different in several ways, such as that the CRA cannot commit to a rating strategy, and that the firm has moral hazard issues that interact with the rating policy. As a result, unlike the government in Goldstein and Huang (2016), the CRA in the current paper may have negative ex-ante real effects. These features also make our model different from those in the literature on Bayesian persuasion, such as Kamenica and Gentzkow

“honest” or “corrupt.” In contrast, in our paper, the conflicts of interest caused by the issuer-pays business model are commonly known by all creditors. This may be a better description of the credit market, especially given what happened in the subprime crisis.
On the one hand, the CRA in our model cannot commit to a rating rule, which is consistent with the empirical regularity but also leads to different results. For example, when the CRA cannot commit, some rating is not assigned in equilibrium, and the equilibrium rating must be monotonic in the firm’s fundamentals. On the other hand, senders in the literature on Bayesian persuasion usually disclose information about exogenous variables, while the CRA in our model is disclosing information about the endogenous firm credit quality that is affected by its disclosure.

2 A Model of Corporate Credit Ratings

We study a model of a CRA that is assigning credit ratings to a firm. There are three dates, \( t = 0, 1, 2 \). At the beginning of date 0, the firm needs to make a payment of $1 for current liabilities, such as unpaid wages. To finance $1, the firm can issue debt (with relatively low costs) or borrow through an alternative financing channel (with relatively high costs). One example of the alternative financing channel is a predetermined bank credit line, which we use in this paper for ease of exposition.

Once the firm pays back its current liabilities, the firm will make a new investment. Independent of the project it invests, the firm incurs an operation cost. Such an operation cost can be viewed as the wage payable to the employees for the new investment. The operation cost is determined by the firm’s fundamentals \( \theta \in \mathbb{R} \) and will be paid at date 2 if the firm does not default. We assume that all agents have a common improper uniform prior over \( \theta \), and \( \theta \) is not realized until the CRA works with the firm for the credit ratings.

At date 0, the CRA assigns credit ratings to the firm. Observing the rating, a continuum of creditors in the debt market simultaneously decide whether to buy the debt or not. At date 1, depending on the financial cost and its private knowledge about its economic fundamentals, the firm may choose to default or to continue investing. In the latter case, the cash flow is realized at date 2, and, if possible, the firm pays the new debt, as well as the operation cost, in full.

We view the model as a model about issuer credit ratings. In practice, CRAs assign credit ratings to both “issuers” and specific “issues.” While we focus on credit ratings to the firm, in the model they are consistent with credit ratings to the corporate bond, because the firm has only one bond issue in the model. We think that these ratings, however, are different from credit ratings to structured finance products, such as MBSs, whose real effects are not as salient as corporate credit ratings.
2.1 Firm Investment

Following Boot, Milbourn, and Schmeits (2006), we assume that if the firm fully repays the current liabilities, it can continue investments either in a viable (i.e., low-risk) project VP or a high-risk alternative HR at date 1. VP generates a cash flow $V > 0$ with probability $p \in (0, 1)$; however, it fails with probability $1 - p$. Similarly, HR generates a cash flow $H > V$ with probability $q \in (0, p)$ but fails with probability $1 - q$. The firm will receive a zero cash flow if the project fails. Since both VP and HR fail with positive probabilities, the firm’s investment choice between VP and HR is unobservable and unverifiable.\footnote{In practice, creditors may know the name of the project the firm invests in, but they usually lack the professional knowledge to judge whether the project is VP or HR. Therefore, the choice between VP and HR is unverifiable even ex post.}

At date 1, instead of investing in VP or HR, the firm may choose to default. In such a case, the firm will not withdraw from the credit line, and its liquidation value is $B \in (0, 1)$. We assume that the liquidation value and the funds from the newly issued debt are used to repay the current liabilities, since the employees usually have higher priorities than unsecured creditors to get repayments when the firm goes bankrupt. If the firm defaults at date 1, the game ends, and thus its early default decision is publicly observable and verifiable.

We assume that the expected cash flow generated by VP is greater than one, but HR is unlikely to generate a positive cash flow ($q$ is sufficiently small). Specifically, we assume that

$$pV > 1 > B > qH.$$ \hspace{1cm} (1)

This assumption is important when we rank the firm’s investments according to their efficiency implications.

2.2 Financing

There is a continuum of creditors with measure $1 - \gamma$ in the debt market, each having $1$. Here, $\gamma$ measures the liquidity of the debt market, with a larger $\gamma$ meaning a lower liquidity level of the debt market. We assume that $\gamma \in (B, 1)$, so that even if all creditors buy the debt, without withdrawing from the bank credit line, the firm still cannot fully pay the current liabilities.\footnote{This assumption is for simplicity. By this assumption, when the firm defaults at date 1, the largest possible amount of funds available is $1 - \gamma + B$, which is less than $1$. Hence, any creditor who buys the debt will get nothing, which implies global strategic complementarities among creditors.}

The debt is a zero-coupon bond with the face value $F > 1$. It matures at date 2. So long as the firm does not default either endogenously at date 1 or exogenously at date 2, the creditors
who buy the new debt will get full repayment. Here, in order to focus on the role of the credit rating agency, we follow He and Xiong (2012) to assume that $F$, the face value of the debt contract, is exogenously given. This assumption does not change the insights about credit ratings’ real effects. Indeed, the key mechanism by which credit ratings affect the firm’s investment decisions is through their effects on the firm’s total cost of financing, rather than merely through the face value of the debt. In our model, credit ratings do affect the firm’s total financing cost (by affecting the measure of creditors who invest in the debt), even if the face value of the debt contract is exogenous.

We further demonstrate that endogenizing debt face value will not add new insights by studying a model with one big competitive creditor and endogenous debt face value in an online appendix.\(^6\) We show that in that model, if we focus on the equilibrium with the firm borrowing from the creditor at the lowest debt face value (if it exists), the CRA will lower the firm’s financial cost, and its real effects are similar to that in our core model. However, such a model has several clear weaknesses: It is much less tractable; it may not have a nontrivial equilibrium in which the firm borrows from the creditor; and even if a nontrivial equilibrium exits, there may be multiple such equilibria, making the comparative statics not as clean as in our core model.

We assume $pF > 1$, and so if any creditor $i$ knows that the firm will invest in $VP$, he will buy the debt. On the other hand, the probability that $HR$ is successful is so low ($qF < 1$) that creditor $i$ will not buy the debt, if he knows that the firm will surely invest in $HR$. Obviously, if creditor $i$ knows that the firm will default early, he does not buy the debt either. We denote by $a_i \in \{0, 1\}$ creditor $i$’s debt-investment decision, where $a_i = 1$ means creditor $i$ buys the debt, and $a_i = 0$ means creditor $i$ does not buy.

The firm can withdraw up to $1$ from the credit line with the constant marginal cost $M > F$. Hence, the firm wants to finance more from the debt market to lower its financial cost. However, we assume that even if no creditor buys the debt, the firm is still willing to withdraw $1$ from the bank credit line to pay the current liabilities and to invest in $VP$, when it has sufficiently low operation cost; formally, we assume that $M < \frac{pV-qH}{p-q}$. We denote by $W$ the measure of creditors who buy the debt, and so the firm needs to finance $1 - W$ from the bank credit line. Then, the firm’s financial cost is $WF + (1 - W)M$.

The operation cost of a new investment is $f(\theta)$. We assume that the function $f(\cdot)$ is differ-

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\(^6\)In a model with dispersed beliefs among creditors and endogenous debt face value, creditors learn from the debt face value. As it will become clear, the effects of credit ratings on creditors’ belief updates make such a model rather intractable.
entiable, strictly decreasing, and strictly convex. When the firm’s economic fundamentals are extremely good, the operation cost is extremely low; that is, \( \lim_{\theta \to +\infty} f(\theta) = 0 \). However, when the firm’s fundamentals are extremely bad, it will incur an unbounded operation cost for a new investment, so \( \lim_{\theta \to -\infty} f(\theta) = +\infty \). Therefore, if the firm decides to invest in either VP or HR, its total cost at date 2 is

\[
K(\theta) = f(\theta) + WF + (1 - W)M. \tag{2}
\]

Important, the measure of creditors who buy the debt (\( W \)) is endogenously determined and will be a function of \( \theta \) and the credit rating in equilibrium.

### 2.3 Firm’s Payoff

The firm has limited liability. If it defaults, whether endogenously at date 1 or exogenously at date 2 (when the project fails), its payoff is zero. If the firm generates a positive cash flow at date 2, the firm needs to repay the creditors according to the debt contract. Therefore, the firm’s payoff \( U \) depends on its own investment choice, its operation cost, and its financial cost:

\[
U = \begin{cases} 
0, & \text{if the firm defaults at date 1;} \\
p \left[ V - (f(\theta) + WF + (1 - W)M) \right], & \text{if the firm invests in VP;} \\
q \left[ H - (f(\theta) + WF + (1 - W)M) \right], & \text{if the firm invests in HR.} 
\end{cases} \tag{3}
\]

### 2.4 Information Structure

The firm’s economic fundamentals, \( \theta \), are its private information, which remain unknown to creditors. We assume that the firm gets to know \( \theta \) after it issues the debt, so that the fact that the firm is issuing the debt is not informative about \( \theta \) to the creditors. Before deciding whether to buy the debt, each creditor \( i \) observes a private signal \( x_i = \theta + \xi_i \), where \( \xi_i \sim \mathcal{N}(0, \beta^{-1}) \) is independent of \( \theta \) and independent across all creditors. Since we aim to analyze credit ratings’ effects on rational, well-informed creditors, in this paper, we focus on the case when \( \beta \) is sufficiently large. Besides their private signals, creditors also observe a public credit rating by a CRA.

### 2.5 Credit Rating Agency

The CRA assigns the firm a credit rating \( R \). Following Boot, Milbourn, and Schmeits (2006), we restrict the space of ratings to \( \{0, q, p\} \), because these are the only possible credit qualities of
the firm: Early default at date 1 means the firm will certainly default, and thus the firm’s credit quality is 0; similarly, the firm investing in HR has a credit quality \( q \), and the firm investing in VP has a credit quality \( p \). This assumption is without loss of generality. In our model, the CRA cannot commit to a rating rule, and it will take into account rating effects on firm credit quality when assigning ratings. So, even if the CRA is allowed to assign ratings in a flexible space, the number of effective ratings cannot be strictly more than three in equilibrium. For example, if the CRA is announcing a \( \theta' \) directly, it will announce the highest \( \theta' \) that will lead to the same credit quality of the firm.

We assume that the CRA knows \( \theta \) perfectly.\(^7\) In addition, we consider pure strategies. Hence, the CRA can perfectly predict the firm’s choice and its corresponding default probability at date 0. Our model captures an important feature of credit ratings — forward-looking. The ratings take into account the effect they have on the firm’s action and success.

We denote by \( V^R \) the CRA’s rating revenue and by \( C^R \) its potential rating cost when it assigns a rating \( R \). Then, the CRA’s expected payoff by assigning the rating \( R \) is

\[
V^R - \mathbb{E}(C^R).
\]

Importantly, both the rating revenue and the potential rating cost are rating-varying. We shall specify the detailed assumptions about \( V^R \) and \( C^R \) in Section 4.1.

### 2.6 Economic Effects

We aim to analyze the effects of the CRA on the economic efficiency, which are measured by the difference between the sums of all agents’ ex-ante payoffs (except the CRA) with and without the CRA. Ultimately, the effective economic efficiency is ranked by the firm’s expected revenue, which is solely determined by the firm’s investment decision. This follows from the fact that the ex-post repayments are all transfers from one group of agents to another, provided that the firm does not default. For example, if the firm invests in HR, and there are \( W \) measure of creditors who buy the debt, the ex-ante payoffs to the firm, the creditors, the bank, and the employees are \( q[H - f(\theta) + WF + (1 - W)M], (1 - W) + WqF, (1 - W)qM - (1 - W), \) and \( qf(\theta) \), respectively. Then, the sum of ex-ante payoffs in this case is \( qH \), which is the firm’s expected revenue from investing in HR.

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\(^7\) The assumption that the CRA knows \( \theta \) perfectly is mainly for tractability. If the CRA observes a noisy signal about \( \theta \), when the noise is sufficiently small, the CRA’s incentives rarely change. However, creditors’ belief updating becomes rather complicated, making the model intractable. In addition, by assuming that the CRA knows \( \theta \) perfectly, the inflated rating will only arise from the CRA’s incentives, rather than the bias of the CRA’s signal.
Hence, when the firm invests in VP, the economic efficiency is $pV$; when the firm invests in HR, the economic efficiency is $qH$; and if the firm defaults at date 1, the economic efficiency is $B$. It then follows from equation (1) that VP leads to the highest economic efficiency, and an investment in HR leads to an even lower economic efficiency than early default does.

2.7 Timeline and Equilibrium

We summarize the model’s timeline in Figure 1 below. The CRA’s rating strategy, denoted by $R$, maps the firm’s fundamentals to the rating space $\{0, q, p\}$; the firm’s strategy maps its fundamentals, the CRA’s rating, and the measure of creditors investing in the new debt to project choices; and creditors’ strategies map their own private signals and the CRA’s rating to their debt-investment decisions.

The solution concept of the model is a monotone Perfect Bayesian equilibrium.

**Definition 1** The CRA’s rating strategy, the firm’s investment strategy, and the creditors’ debt-investment strategies constitute an equilibrium, if

1. given the firm’s investment strategy and the creditors’ debt-investment strategies, the CRA chooses the rating $R$ to maximize its rating profits $V^R - \mathbb{E}(C^R)$ for all $\theta \in \mathbb{R}$;

2. given the total repayment at date 2 in equation (2), the firm’s investment strategy maximizes the firm’s expected profits;

3. given the CRA’s rating strategy, the firm’s investment strategy, and other creditors’ strategies, any creditor $i$’s strategy is monotonic in his private signal $x_i$ and maximizes his expected payoff;

4. and, creditors use Bayes’ rule to update their beliefs.
3 The Benchmark: No CRA

We first set up a benchmark that excludes the CRA. In such a benchmark, when deciding whether to buy the debt, all creditors make choices solely based on their own private information. After observing the measure of creditors who invest in the debt, the firm makes its investment choice. Such a benchmark is similar to the debt-run model by Morris and Shin (2004), with the key difference being that the firm’s choice is not binary.

We first analyze the firm’s behavior in this benchmark model. Because of the law of large numbers, given the creditors’ strategies, the measure of the creditors who buy the debt is a deterministic function $W(\theta)$. Hence, any $\theta$-firm’s total repayments at date 2 is deterministic:

$$K(\theta) = f(\theta) + W(\theta)F + (1 - W(\theta))M. \quad (5)$$

Since $H > V$, the $\theta$-firm will default early, if and only if,\(^8\)

$$K(\theta) > H. \quad (6)$$

Conditional on that the $\theta$-firm decides to continue investing, it invests in VP rather than HR, if and only if,

$$p[V - K(\theta)] \geq q[H - K(\theta)] \Rightarrow K(\theta) \leq \frac{pV - qH}{p - q}. \quad (7)$$

As a result, given the creditors’ strategies, the $\theta$-firm’s optimal investment strategy is

$$\begin{cases} 
\text{early default,} & \text{if } K(\theta) > H; \\
HR, & \text{if } K(\theta) \in \left(\frac{pV - qH}{p - q}, H\right]; \\
VP, & \text{if } K(\theta) \leq \frac{pV - qH}{p - q}.
\end{cases} \quad (8)$$

Recall that we assume $M < \frac{pV - qH}{p - q}$. Hence, when the firm’s economic fundamentals are extremely good ($\theta \to +\infty$), its operation cost is almost zero, and so it will choose VP, even if no creditor buys the debt. This establishes a dominant region of investing for all creditors: When a creditor receives a very positive private signal, he will believe that the firm is going to invest in VP and hence will buy the debt even if all other creditors refrain from doing so. On the other hand, if $M > \frac{pV - qH}{p - q}$, the firm will not default at date 1 even if no creditor buys the debt.

\(^8\)We assume that the firm will default at date 1, if its total repayment at date 2 is larger than the highest possible cash flow the firm can generate. This reflects some very small cost incurred by the manager in case he continues.
hand, when the firm has extremely bad economic fundamentals and thus unlimited operation cost \((\lim_{\theta \to -\infty} f(\theta) = +\infty)\), it will choose to default at date 1, even if all creditors buy the debt. This establishes a dominant region of not investing: When a creditor receives a very negative private signal, he will believe that the firm will default at date 1, and hence will not buy the debt even if all other creditors choose to buy. Therefore, as in other global game models, in a monotone equilibrium, any creditor employs a cutoff strategy with the threshold \(\hat{x}\), such that he invests in the debt, if and only if \(x_i \geq \hat{x}\).

Given \(\theta\) and the creditors’ cutoff strategy, the measure of creditors who invest is

\[
W(\theta) = (1 - \gamma) \Pr(x \geq \hat{x}|\theta) = (1 - \gamma) \left\{1 - \Phi\left[\sqrt{\beta}(\hat{x} - \theta)\right]\right\},
\]

where \(\Phi(\cdot)\) is the CDF of the standard normal distribution. Then, the \(\theta\)-firm’s total repayment at date 2 is

\[
K(\theta) = f(\theta) + (1 - \gamma) \left\{1 - \Phi\left[\sqrt{\beta}(\hat{x} - \theta)\right]\right\} F + \left[\gamma + (1 - \gamma)\Phi[\sqrt{\beta}(\hat{x} - \theta)]\right] M
\]

\[
= f(\theta) + [(1 - \gamma)F + \gamma M] + (1 - \gamma)\Phi\left[\sqrt{\beta}(\hat{x} - \theta)\right] (M - F). \tag{9}
\]

In equation (9), the first term is the operation cost of the \(\theta\)-firm, the second term is the financial cost resulting from insufficient liquidity in the debt market, whereas the third term is the endogenous financial cost resulting from the creditors’ strategic uncertainties.

It directly follows from equation (9) that the firm’s total repayment at date 2 is strictly decreasing in its fundamentals. Specifically, as the firm’s fundamentals improve (that is, as \(\theta\) increases), its operation cost decreases (since \(f(\theta)\) is strictly decreasing); also, more creditors receive private signals landing above the threshold \(\hat{x}\) and thus choose to buy the debt, leading to a lower financial cost. The monotonicity of \(K(\theta)\) turns out to be critical for the equilibrium characterization.

First, given the creditors’ strategies, the firm will choose to default early if and only if \(\theta < \tilde{\theta}_1\). This implies that

\[
K(\tilde{\theta}_1) = f(\tilde{\theta}_1) + [(1 - \gamma)F + \gamma M] + (1 - \gamma)\Phi\left[\sqrt{\beta}(\tilde{x} - \tilde{\theta}_1)\right] (M - F) = H. \tag{10}
\]

Because \(K(\theta)\) is strictly decreasing, for any \(\theta < \tilde{\theta}_1\), the firm’s total repayment at date 2 will be greater than \(H\), the upside cash flow of HR; as a result, the firm would default at date 1. But if \(\theta \geq \tilde{\theta}_1\), the firm can at least choose HR in order to receive a non-negative expected payoff due to its limited liability, and thus the firm will not default early.
When $\theta \geq \theta_1$, the firm needs to choose between VP and HR. From equation (8) and the fact that $K(\theta)$ is strictly decreasing in $\theta$, there must be a $\tilde{\theta}_2 > \tilde{\theta}_1$, such that the firm will choose VP if and only if $\theta \geq \tilde{\theta}_2$. Hence,

$$K(\tilde{\theta}_2) = f(\tilde{\theta}_2) + [(1 - \gamma)F + \gamma M] + (1 - \gamma)\Phi \left[ \sqrt{\beta}(\tilde{x} - \tilde{\theta}_2) \right] (M - F) = \frac{pV - qH}{p - q}. \quad (11)$$

Following the above arguments, in a monotone equilibrium, the firm will default early if $\theta < \tilde{\theta}_1$, invest in HR if $\theta \in [\tilde{\theta}_1, \tilde{\theta}_2)$, and invest in VP if $\theta \geq \tilde{\theta}_2$.

Any creditor $i$, receiving a private signal $x_i$ about $\theta$, first updates his belief about $\theta$ according to Bayes’ rule:

$$\theta|x_i \sim \mathcal{N}\left(x_i, \frac{1}{\beta}\right).$$

Then, given the firm’s strategy described above, creditor $i$ calculates his return from investing in the debt:

$$\left\{ \Phi \left[ \sqrt{\beta}(\tilde{x} - x_i) \right] - \Phi \left[ \sqrt{\beta}(\tilde{x}_1 - x_i) \right] \right\} qF + \left\{ 1 - \Phi \left[ \sqrt{\beta}(\tilde{x} - x_i) \right] \right\} pF.$$

Given the dominant regions of investing and not investing, there must be a marginal creditor who is indifferent between investing and not investing in equilibrium. Because any creditor will receive the payoff 1 if he does not invest, and his expected payoff from investing is strictly increasing in his private signal, the marginal creditor must have the private signal $\tilde{x}$ that makes his indifference condition hold:

$$\left\{ \Phi \left[ \sqrt{\beta}(\tilde{\theta}_2 - \tilde{x}) \right] - \Phi \left[ \sqrt{\beta}(\tilde{\theta}_1 - \tilde{x}) \right] \right\} qF + \left\{ 1 - \Phi \left[ \sqrt{\beta}(\tilde{\theta}_2 - \tilde{x}) \right] \right\} pF = 1. \quad (12)$$

Proposition 1 below characterizes the equilibrium of the benchmark model.

**Proposition 1 (The Unique Equilibrium in the Benchmark Model)** There exists a $\bar{\beta} > 0$, such that for all $\beta > \bar{\beta}$, the benchmark model without a CRA has a unique equilibrium described by $(\tilde{\theta}_1, \tilde{\theta}_2, \tilde{x})$, where $\tilde{\theta}_1 < \tilde{\theta}_2$. In particular,

1. the firm’s investment strategy is

$$\begin{cases} 
VP, & \text{if } \theta \geq \tilde{\theta}_2; \\
HR, & \text{if } \theta \in [\tilde{\theta}_1, \tilde{\theta}_2); \\
\text{early default}, & \text{if } \theta < \tilde{\theta}_1;
\end{cases}$$

2. and, any creditor $i$ buys the debt if and only if $x_i \geq \tilde{x}$. 

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4 Credit Ratings

We now consider our core model where the CRA strategically chooses its optimal rating strategy. We first specify details about the rating revenue and the potential rating cost. Then, as a first step to solve for an equilibrium, we discuss possible equilibrium rating strategies. In particular, we identify the condition under which the CRA will assign overgenerous ratings.

We then follow the literature on credit ratings and the empirical observations to focus on the case where the CRA may inflate credit ratings. We show that rating inflation must emerge in equilibrium, but credit ratings are still informative about firm fundamentals.

We finally solve for the unique equilibrium under rating inflation. In equilibrium, when assigning credit ratings, the CRA will take into account the effects of the ratings on the creditors’ debt investment decisions and thus the firm’s investment choices. As we show later in Section 5, this feature is the key to understand the credit ratings’ real effects.

4.1 Equilibrium Rating Strategies

Because of the prevailing “issuer-pays” business model in the credit rating industry, we assume that the CRA gains higher revenue by assigning the firm a higher credit rating. Hence, $V^p > V^q > V^0 = 0$, where we normalize the revenue from assigning the rating $R = 0$ to be zero.

On the other hand, the CRA’s potential rating cost (denoted by $C^R$), which may be viewed as a legal or reputation cost, occurs only when the firm defaults. Importantly, such a cost depends not only on the credit rating $R$, but also on the verifiability of the firm’s investment choice. First, if the firm with a rating $R > 0$ defaults endogenously at date 1, the CRA incurs a cost $C^D$. Because the firm’s early default is observable and verifiable, the CRA that assigns a rating $R > 0$ can be verified as wrong and will be heavily punished (White 2013). Hence, we assume

$$C^D > V^p$$

and name such an assumption the partial verifiability constraint imposed on credit ratings.

Second, if the firm with a rating $R > 0$ defaults at date 2 after the investment, the CRA incurs a cost $C^R$ if it assigns the firm the rating $R (R = p, q)$. We assume $V^q > C^p > C^q > 0$. Here, we assume that the CRA’s reputation costs are exogenous. They may arise from the government regulation or the repeated interactions between the creditors and the CRA.\(^\text{10}\)

\(^\text{9}\)Credit ratings are viewed as CRAs’ “free speech.” So, protected by the First Amendment, CRAs are not liable for any losses incurred by the inaccuracy of their ratings, unless it is proven that they know the ratings were false.\(^\text{10}\)Mathis, McAndrews, and Rochet (2009) show that reputation cannot fully address the rating inflation issue.
The partial verifiability constraint (equation (13)) implies that the CRA will assign the rating \( R = 0 \) if it foresees that the firm will default at date 1 even with a rating \( R' > 0 \). It then follows from the assumption \( V^p > V^q > C^p > C^q > 0 \) that if the firm does not default at date 1, the CRA will assign the rating \( p \) or the rating \( q \). Hence, in such a case, the CRA will issue the rating \( p \) if and only if \( V^p - \mathbb{E}(C^p) \geq V^q - \mathbb{E}(C^q) \).

Lemma 1 below shows that, the CRA’s equilibrium rating strategy depends on the ratio of the incremental revenue to the incremental cost due to the rating upgrading from \( q \) to \( p \).

**Lemma 1** The CRA’s equilibrium rating strategy depends on the ratio \( (V^p - V^q) / (C^p - C^q) \). There are three cases.

1. If \( \frac{V^p - V^q}{C^p - C^q} \geq 1 - q \), the equilibrium rating strategy is in the form
   \[
   \mathcal{R}(\theta) = \begin{cases} 
   p, & \text{if } \theta \geq \theta^I; \\
   0, & \text{if } \theta < \theta^I.
   \end{cases}
   \]
   \[ (14) \]

2. If \( \frac{V^p - V^q}{C^p - C^q} \leq 1 - p \), the equilibrium rating strategy is in the form
   \[
   \mathcal{R}(\theta) = \begin{cases} 
   q, & \text{if } \theta \geq \theta^D; \\
   0, & \text{if } \theta < \theta^D.
   \end{cases}
   \]
   \[ (15) \]

3. If \( \frac{V^p - V^q}{C^p - C^q} \in (1 - p, 1 - q) \), the equilibrium rating strategy is in the form
   \[
   \mathcal{R}(\theta) = \begin{cases} 
   p, & \text{if } \theta \geq \theta^p; \\
   q, & \text{if } \theta \in [\theta^q, \theta^p); \\
   0, & \text{if } \theta < \theta^q,
   \end{cases}
   \]
   \[ (16) \]
   where \( \theta^q \leq \theta^p \).

Part 1 of Lemma 1 shows that if the benefit of upgrading the rating from \( q \) to \( p \) is high enough (relative to the increase in the reputation cost), the CRA “inflates” the ratings assigned to the firms that will invest in HR. Therefore, in this case, the rating \( q \) will not be assigned in equilibrium. Intuitively, if a \( \theta \)-firm receives the rating \( q \), it will not default at date 1 in equilibrium. Then, when the CRA upgrades its rating to \( R = p \), either the \( \theta \)-firm will have

They also show that reputation cycles emerge in an infinite-horizon setup: Initially, the CRA’s reputation cost is high; but when its reputation is sufficiently high, its reputation cost becomes lower.
an even lower financial cost (if firms with the rating \( p \) are believed to have better economic fundamentals) or its financial cost is lower than some other firms that receive the rating \( p \). Therefore, after such an upgrading, the \( \theta \)-firm will not default at date 1. Then, the condition 
\[
\frac{V_p - V_q}{C_p - C_q} \geq 1 - q
\]
implies that upgrading the \( \theta \)-firm to the \( p \)-rating group will be a profitable deviation. Hence, the CRA will not assign the rating \( R = q \) in equilibrium.

Another important feature of the equilibrium rating strategies when 
\[
\frac{V_p - V_q}{C_p - C_q} \geq 1 - q
\]
is that the rating is monotonic in the firm’s economic fundamentals. This is also intuitive. For a \( \theta \)-firm with the rating 0, if there is another firm that has worse economic fundamentals but is assigned the rating \( p \) (and so will not default at date 1), it will be profitable for the CRA to also assign the rating \( p \) to the \( \theta \)-firm, since the \( \theta \)-firm, when assigned the rating \( p \), will have lower total repayments at date 2 and will not default either.

Part 2 of Lemma 1 shows that when the revenue of upgrading the rating from \( q \) to \( p \) is sufficiently small (relative to the increase in the reputation cost), the CRA will “deflate” the ratings assigned to the firms. In such a case, only ratings 0 and \( q \) will be assigned in equilibrium.

Importantly, in both the rating inflation case and the rating deflation case, the CRA pools all the firms that do not default at date 1 together and separate them with the firms that default at date 1. Therefore, the CRA’s effects on the economic efficiency would be identical in these two cases, because in our model, creditors are all rational and update their beliefs based on the equilibrium strategy of the CRA. Hence, the label that is put on firms that are pooled together should not matter; what really matters is which firms are pooled together.

Part 3 of Lemma 1, however, presents a very different case. When the ratio of the revenue increment of upgrading to the cost increment is in a medium range, the CRA will assign all three possible ratings in equilibrium. In particular, in such a case, the CRA will convey the accurate information about the firm’s investment choice and its credit quality. We then call the CRA in such a case a self-disciplined CRA. Obviously, a self-disciplined CRA can completely eliminate information asymmetry between the creditors and the firm and thus achieve the first-best economic efficiency.

The fact that whether the CRA will inflate ratings, deflate ratings, or honestly assign ratings depends on the ratio 
\[
\frac{V_p - V_q}{C_p - C_q}
\]
has important policy implications. Suppose that the government wants to regulate the credit rating industry by designing the cost scheme (conditional on the investment failure of the firm) such that the CRA is self-disciplined. In order to implement such a policy, the government has to accurately calibrate \( V_p - V_q \). While this may be difficult, Lemma 1 shows that punishing the CRA that assigns the highest rating too much (i.e., setting a very large \( C_p - C_q \)) is not an effective policy, since the CRA will deflate the ratings, and the
resulting effects of the CRA will be the same as the case in which the CRA inflates ratings.

4.2 Rating Inflation and Rating Informativeness

Since the effects of the CRA are straightforward when it is self-disciplined, and the effects of the “deflating” CRA are identical to those of the “inflating CRA,” we will focus on the “inflating” CRA case in the rest of the paper. We stick to the focus on rating inflation also because rating inflation is the phenomenon that has been discussed empirically and in policy circles. Formally, we will maintain the following assumption:

\[
\frac{V^P - V^q}{C^P - C^q} \geq 1 - q. \tag{17}
\]

With such an assumption, Lemma 1 implies that the CRA’s equilibrium rating strategy is

\[
\mathcal{R}(\theta) = \begin{cases} 
  p, & \text{if } \theta \geq \theta_1^*; \\
  0, & \text{if } \theta < \theta_1^*.
\end{cases} \tag{18}
\]

Therefore, the CRA’s equilibrium rating strategy can be characterized by a \(\theta_1^* \in \mathbb{R}\), with \(\mathcal{R}(\theta) = p\) when \(\theta \geq \theta_1^*\) and \(\mathcal{R}(\theta) = 0\) when \(\theta < \theta_1^*\). When \(\theta_1^*\) decreases, the CRA assigns more firms with the high rating \(p\). So for two rating strategies \(\mathcal{R}_1\) with the threshold \(\theta_1^*\) and \(\mathcal{R}_2\) with the threshold \(\theta_2^*\), we say that the rating strategy \(\mathcal{R}_2\) is laxer than the rating strategy \(\mathcal{R}_1\) if and only if \(\theta_2^* < \theta_1^*\). However, the laxer rating strategy \(\mathcal{R}_2\) may not lead to higher credit rating inflation, which refers to the fact that the nominal rating is strictly higher than the real credit quality. Formally:

**Definition 2** A credit rating assigned to a \(\theta\)-firm is inflated, if in equilibrium, the \(\theta\)-firm chooses HR and thus has the credit quality \(q\), but the CRA assigns the rating \(p\). In addition, a rating strategy is inflated, if credit ratings assigned according to the rating strategy are inflated for a non-negligible subset of fundamentals; and a credit rating strategy is more inflated, if for a larger measure of fundamentals, credit ratings assigned according to the rating strategy are inflated.

In equilibrium, the firm that is assigned the rating \(p\) does not default at date 1. However, the rating \(p\) cannot guarantee that the firm will invest in VP. Indeed, if all creditors believe that the firm with the rating \(p\) will surely invest in VP, they will all buy the debt, leading to the lowest possible financial cost. Then, the assumption that the \(\theta_1^*\)-firm will invest in VP implies that \(\theta_1^*\)-firm’s total repayments at date 2 (the sum of the financial cost and the operation cost) are
less than \( \frac{pV-qH}{p-q} \) and thus strictly less than \( H \). In consequence, the firms with the fundamentals slightly lower than \( \theta^*_1 \) will not default at date 1, if they are assigned the rating \( p \). So it is profitable for the CRA to deviate to assign the rating \( p \) to such firms. Therefore, in equilibrium, some firms with the rating \( p \) will invest in HR, implying credit rating inflation in equilibrium. Formally:

**Lemma 2 (Rating Inflation)** There is no monotone equilibrium in which all \( \theta \)-firms receiving a rating \( R = p \) invest in VP.

While rating inflation inevitably appears in equilibrium, credit ratings are still informative to creditors. The CRA’s equilibrium rating strategy (equation (18)) implies that if \( R = p \), all creditors know that \( \theta \geq \theta^*_1 \). So, the rating \( p \) guarantees creditors that the firm’s fundamentals are not extremely bad.

**Corollary 1 (Creditors’ belief supports following \( R = p \))** Following the credit rating \( R = p \), regardless of his private signal \( x_i \), the support of any creditor \( i \)’s interim belief about \( \theta \) is truncated from below by \( \theta^*_1 \).

### 4.3 Equilibrium under Rating Inflation

In this subsection, we characterize the unique equilibrium under rating inflation. As shown in Part 1 of Lemma 1, with equation (17), only the rating 0 and the rating \( p \) are assigned in equilibrium. Due to the partial verifiability constraint, the CRA assigns the rating 0 if and only if it knows that the firm will default early even with the rating \( p \). Therefore, when creditors observe the rating 0, they all believe that the firm will default early, and so they refrain from buying the debt. Hence, following \( R = 0 \), there is a unique equilibrium in which no creditor invests in the debt, and the firm defaults at date 1. Since the rating strategy assigns the rating 0 to the firm if and only if \( \theta < \theta^*_1 \), we must have \( K(\theta) = f(\theta) + M > H, \forall \theta < \theta^*_1 \). Then, by the continuity of \( f(\cdot) \), we have the first equilibrium condition:

\[
  f(\theta^*_1) \geq H - M. \tag{19}
\]

We now focus on the case following the rating \( p \). Since given creditors’ strategies, the firm’s total repayment at date 2 is strictly decreasing with its fundamentals, there must be a threshold \( \theta^*_2 > \theta^*_1 \) so that the \( \theta \)-firm invests in VP if \( \theta \geq \theta^*_2 \) but invests in HR if \( \theta \in [\theta^*_1, \theta^*_2) \). Note that Lemma 2 implies that \( \theta^*_2 \) must be strictly greater than \( \theta^*_1 \), because some firms with the rating \( R = p \) will invest in HR.
Creditors, on the other hand, will employ a cutoff strategy. Corollary 1 implies that given the CRA’s rating strategy, after observing the rating \( p \), all creditors believe that the firm’s true fundamentals are above \( \theta_1^* \). Hence, any creditor \( i \) will buy the debt if and only if his private signal lands above a threshold \( x^* \in \mathbb{R} \). We then call the creditor with the private signal \( x^* \) the marginal creditor. Importantly, the marginal creditor is indifferent between buying the debt or not.

Finally, given the firm’s and the creditors’ strategies, the CRA needs to choose \( \theta_1^* \) to maximize its expected rating profit. Since it will assign the rating \( R = p \) to the firm if and only if the firm will not default at date 1 with such a rating, \( \theta_1^* \) must be chosen so that the firm is indifferent between early default and HR.

The above arguments lead to the indifference conditions of the firm, the marginal creditor, and the CRA, which are characterized by equations (20), (21), and (22), respectively, below.

\[
f(\theta_2^*) + (1 - \gamma) \left[ 1 - \Phi(\sqrt{\beta}(x^* - \theta_2^*)) \right] F + \left[ \gamma + (1 - \gamma)\Phi(\sqrt{\beta}(x^* - \theta_2^*)) \right] M = \frac{pV - qH}{p - q} \tag{20}
\]

\[
\frac{\Phi[\sqrt{\beta}(\theta_2^* - x^*)] - \Phi[\sqrt{\beta}(\theta_1^* - x^*)]}{1 - \Phi[\sqrt{\beta}(\theta_1^* - x^*)]} qF + \frac{1 - \Phi[\sqrt{\beta}(\theta_2^* - x^*)]}{1 - \Phi[\sqrt{\beta}(\theta_1^* - x^*)]} pF = 1 \tag{21}
\]

\[
f(\theta_1^*) + (1 - \gamma) \left[ 1 - \Phi(\sqrt{\beta}(x^* - \theta_1^*)) \right] F + \left[ \gamma + (1 - \gamma)\Phi(\sqrt{\beta}(x^* - \theta_1^*)) \right] M = H. \tag{22}
\]

Proposition 2 then shows that under the condition of rating inflation (equation (17)), the model has a unique equilibrium in which the CRA’s rating, the firm’s investment decision, and the creditors’ debt-investment decisions interact with one another.

**Proposition 2 (Unique Equilibrium under Rating Inflation)** With equation (17), there is a \( \beta^* > 0 \), such that when \( \beta > \beta^* \), the model has a unique equilibrium. The equilibrium is characterized by \((\theta_1^*, \theta_2^*, x^*)\) with \( \theta_2^* > \theta_1^* \), such that

1. the CRA will assign the rating \( R = p \), if the firm’s fundamentals \( \theta \) belongs to \([\theta_1^*, +\infty)\); and it will assign the rating \( R = 0 \), if the firm’s fundamentals \( \theta \) is strictly less than \( \theta_1^* \);

2. if \( R = 0 \), no creditor buys the debt, and the firm defaults at date 1;

3. if \( R = p \), a creditor invests in the debt if and only if his private signal lands above \( x^* \), and the firm will choose HR if \( \theta \in [\theta_1^*, \theta_2^*) \) and VP if \( \theta \in [\theta_2^*, +\infty) \); and

4. \((\theta_1^*, \theta_2^*, x^*)\) solves equations (20), (21), and (22).
The equilibrium uniqueness arises from creditors’ new dominant region of not investing, generated by the credit rating $p$. With equation (17), Lemma 2 implies that the CRA will assign the rating $p$ to the firm that has the fundamentals just above $\theta_1^*$, and such firms will invest in HR. Consequently, when creditors receive very negative signals, they believe that the firm has fundamentals landing within the HR-investment region, and so they refrain from investing in the debt. This endogenously generates a new dominant region of not investing, and thus, the creditors have a unique best response to the rating $p$.

Proposition 2 provides us with a clear measure of equilibrium rating inflation. When $\theta < \theta_1^*$, the CRA will assign the rating 0 to the firm. Since the firm will default early, the credit rating truly reflects the firm’s credit quality. When $\theta \geq \theta_2^*$, the firm’s fundamentals are sufficiently good, so it will invest in VP. In this case, the credit rating $p$ also equals the firm’s credit quality. However, when $\theta \in [\theta_1^*, \theta_2^*)$, the firm invests in HR and thus has the credit quality $q$, but it receives the high rating $p$. So the credit ratings assigned to such firms are inflated. Hence, the rating inflation can be measured by $\theta_2^* - \theta_1^*$.

5 The CRA’s Real Effects under Rating Inflation

We are now in a position to analyze the CRA’s real effects. For a given $\theta$-firm, if the assigned credit rating changes its investment (comparing to its investment in the benchmark model without a CRA), the CRA has effects on the economic efficiency. In such a case, we say that the CRA has real effects on the $\theta$-firm. Such effects are positive, if the CRA leads to higher economic efficiency; conversely, if the CRA leads to lower economic efficiency, the CRA’s real effects on the $\theta$-firm are negative. The CRA’s ex-ante real effects are then measured by the average change of the economic efficiency. Hence, the ex-ante real effects of the CRA are positive (negative) if the economic efficiency increases (decreases) with the CRA.

Lemma 3 below shows that, with the CRA, both the firm’s early default threshold and VP-investment threshold are lower than those in the benchmark model without a CRA.

**Lemma 3** Comparing the equilibrium of the model with a CRA (described in Proposition 2) to that of the benchmark model without a CRA (described in Proposition 1), we have $\theta_1^* < \bar{\theta}_1$, $\theta_2^* < \bar{\theta}_2$, and $x^* < \bar{x}$. However, the sign of $\theta_2^* - \bar{\theta}_1$ is undetermined.

We illustrate the CRA’s real effects in the case with $\theta_2^* > \bar{\theta}_1$ in Figure 2 below. When $\theta_2^* > \bar{\theta}_1$, there are two cases of the CRA’s real effects. First, when $\theta \in [\theta_1^*, \bar{\theta}_1)$, without the CRA, the firm’s financial cost is so high that it will default early; but when the CRA is present, it will
assign the firm the inflated rating $p$, leading to lower financial costs to the firm. Such a decrease in the financial cost encourages the firm to gamble for resurrection, rather than default early, which shows the CRA’s negative real effects. Second, when $\theta \in [\theta^*_2, \tilde{\theta}_2)$, because the high rating $p$ helps the firm reduce financial costs, the firm switches from HR to VP, which implies positive real effects.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2}
\caption{CRA's real effects when $\theta^*_2 > \tilde{\theta}_1$}
\end{figure}

When $\theta^*_2 \leq \tilde{\theta}_1$, the CRA’s real effects are similar, except that the range for the negative real effect is different. Proposition 3 below formally summarizes the CRA’s real effects in the case of rating inflation. Note that because of the improper uniform prior, when calculating the CRA’s ex-ante real effects, we consider Lebesgue measure for the number of firms whose investment decisions are affected by credit ratings.

**Proposition 3** With the assumption of equation (17), there are two cases of the analysis of the CRA’s real effects.

1. If $\theta^*_2 > \tilde{\theta}_1$, the CRA has positive real effects when $\theta \in [\theta^*_2, \tilde{\theta}_2)$ and has negative real effects when $\theta \in [\theta^*_1, \tilde{\theta}_1)$; hence, the CRA’s ex-ante real effects are

\[(\tilde{\theta}_2 - \theta^*_2)(pV - qH) + (\tilde{\theta}_1 - \theta^*_1)(qH - B).\]

2. If $\theta^*_2 \leq \tilde{\theta}_1$, the CRA has positive real effects when $\theta \in [\theta^*_2, \tilde{\theta}_2)$ and has negative real effects when $\theta \in [\theta^*_1, \tilde{\theta}_2)$; hence, the CRA’s ex-ante real effects are

\[(\tilde{\theta}_2 - \tilde{\theta}_1)(pV - qH) + (\tilde{\theta}_1 - \theta^*_1)(qH - B) + (\theta^*_2 - \theta^*_1)(qH - B).\]
Importantly, Proposition 3 shows that the CRA who employs an inflated rating strategy may have positive or negative real effects, depending on the firm’s fundamentals. The CRA’s ex-ante real effects then depend on the model’s parameters. In Figure 3 below, we numerically show the CRA’s ex-ante real effects as a function of the upside return of the risky project, $H$.

Figure 3: The CRA’s Real Effects as a Function of $H$. The parameters used in this figure are $F = 1.2$, $M = 1.5$, $V = 3$, $p = 0.9$, $q = 0.005$, $\gamma = 0.7$, $B = 0.7$, $\beta = 0.8$, and $f(\theta) = e^{-\theta}$.

In particular, Figure 3 shows that when the upside return of the risky project is relatively high, the CRA’s ex-ante real effects are negative. This is because when $H$ is large, the firm has stronger incentives to take risks by investing in HR and thus is less likely to default at date 1 efficiently. The CRA then will assign more firms the high rating $R = p$, which allows those firms to gamble for resurrection, and so have negative ex-ante real effects. When $H$ is relatively small, the CRA encourages more firms to switch from HR to VP and thus has positive ex-ante real effects.

5.1 Informational Effects and Feedback Effects

Proposition 2 suggests that the CRA affects a firm’s investment decision through two interacting channels. On the one hand, by assigning the rating $R = p$, the CRA separates firms
with fundamentals above a threshold from those with the fundamentals below the threshold. Hence, the rating $R = p$ provides the creditors with new information about the firm’s fundamentals. Such new information affects the creditors’ debt-investment decisions, and thus the firm’s financial cost and investment. We call such effects the CRA’s informational effects.

On the other hand, the CRA strategically chooses $\theta_1^*$ to pool firms investing in HR with those investing in VP. Hence, the set of types of the firm that invest in either HR or VP may differ in cases with and without a CRA. This also affects firms’ investment decisions. We call such effects the CRA’s feedback effects, since the CRA, when choosing $\theta_1^*$, takes into account the creditors’ and the firm’s best responses to the ratings.

In this subsection, We analyze how these two effects interact to determine the CRA’s real effects. We first analyze the CRA’s informational effects. Let’s consider the case in which the CRA commits to the following rating strategy:

$$
R(\theta) = \begin{cases} 
0, & \text{if } \theta < \hat{\theta}_1 \equiv \tilde{\theta}_1; \\
\theta, & \text{if } \theta \geq \hat{\theta}_1.
\end{cases}
$$

(23)

Here, $\hat{\theta}_1$, which is characterized in Proposition 1, is the early-default threshold of the firm when there is no CRA.

The committed rating strategy characterized in equation (23) just reflects the firm’s investment decision in the benchmark model without a CRA. So, for ease of exposition, we call such a CRA a reflecting CRA and the CRA analyzed in Section 4 a strategic CRA. Importantly, a reflecting CRA does not have feedback effects, because it does not strategically take into account its effects on the firm’s investment decision when committing to its rating strategy, although such a rating strategy may still be inflated. Therefore, the real effects of the reflecting CRA is just the informational effects of the strategic CRA. Then, by comparing the strategic CRA’s real effects with the reflecting CRA’s real effects, we can find the strategic CRA’s feedback effects.

Proposition 4 shows the firm’s equilibrium investment decision in the case with the reflecting CRA.

**Proposition 4** Given the committed rating strategy in equation (23), the generated credit ratings lead to two continuation plays. In particular,

1. following $R = 0$, there is a unique equilibrium play in which the firm defaults at date 1; and,

2. following $R = \theta$, in any equilibrium, the $\theta$-firm invests in VP if $\theta \geq \hat{\theta}_2$ and invests in HR if $\theta \in [\hat{\theta}_1, \hat{\theta}_2)$. Furthermore, if $\theta_2^* > \hat{\theta}_1$, we have $\hat{\theta}_2 < \theta_2^*$. 

25
The reflecting CRA’s real effects can be derived from the comparison between the equilibrium characterized in Proposition 4 and that characterized in Proposition 1. With the rating \( p \) assigned by a reflecting CRA, if \( \theta > \hat{\theta}_2 \) (which is strictly greater than \( \hat{\theta}_2 \)), the firm invests in VP, in both the case with a reflecting CRA and the case without a CRA. Therefore, for any \( \theta > \hat{\theta}_2 \), the reflecting CRA does not have real effects. Similarly, the reflecting CRA does not have any real effects when \( \theta \in [\hat{\theta}_1, \hat{\theta}_2) \).

However, the reflecting CRA will change the firm’s investment decision when \( \theta \in [\hat{\theta}_2, \tilde{\theta}_2) \). In particular, without a CRA, the firm invests in HR, but with a reflecting CRA, the firm will invest in VP. Therefore, the reflecting CRA has positive real effects, which are measured by \((\tilde{\theta}_2 - \hat{\theta}_2)(pV - qH)\). That is, the strategic CRA’s informational effects are always positive, precisely because its rating \( R = p \), though potentially inflated, provides the creditors with an informative signal and thus correctly guides creditors’ debt-investment decisions and with that influencing the firm’s investment.

The CRA has informational effects because credit ratings are informative public signals to creditors. Generally, public signals may have negative effects, as analyzed by Morris and Shin (2002) and others. In our model, however, the CRA’s informational effects are always positive. Given the credit rating rule committed by the reflecting CRA, a firm can receive a high rating only if it would invest in either VP or HR in the benchmark model without a CRA. Because a high rating reduces the firm’s financial costs, in such a case, it is possible for a firm to switch from HR to VP, but it is impossible for a firm to switch from efficient default to HR. Consequently, given a credit rating assigned by the reflecting CRA, the resulting firm credit quality is at least as high as that in the benchmark model.

We finally investigate the strategic CRA’s feedback effects. Similarly to Proposition 3, there are two cases: \( \theta^*_2 \geq \hat{\theta}_1 \) and \( \theta^*_2 < \hat{\theta}_1 \). In both cases, the strategic CRA’s feedback effects have a negative component. Specifically, the strategic CRA knows that when it assigns the rating \( R = p \), more creditors will buy the debt and the firm’s financial cost will decrease, so it can issue the high rating \( R = p \) to more firms. That is, in equilibrium, the strategic CRA will employ the rating strategy with the threshold \( \theta^*_1 < \min\{\hat{\theta}_1, \theta^*_2\} \). Such a manipulation leads firms with \( \theta \in [\theta^*_1, \min\{\hat{\theta}_1, \theta^*_2\}) \) to gamble for resurrection, and thus leads to adverse real effects.

In the case with \( \theta^*_2 \geq \hat{\theta}_1 \), the strategic CRA’s real effects have another negative component. Because \( \theta^*_1 < \hat{\theta}_1 \), the rating \( R = p \) assigned by the strategic CRA is less informative than the

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11 In the context of ratings, Daley, Green, and Vanasco (2019) recently show that credit ratings, as a source of public information, could reduce lending standards and lead to an oversupply of credit.
rating \( R = p \) assigned by the reflecting CRA. So, with the strategic CRA, after the rating \( R = p \), fewer creditors buy the debt (comparing to the case with the reflecting CRA), and so the firm’s financial cost increases. Hence, fewer firms (measured by \( \theta_2^* - \hat{\theta}_2 \)) switch from HR to VP. That is, in the case with \( \theta_2^* > \hat{\theta}_1 \), the strategic CRA’s rating strategy will weaken its information effects. The CRA’s feedback effects in the case with \( \theta_2^* \geq \hat{\theta}_1 \) are then illustrated in Figure 4 (with the green indicating positive real effects and the red indicating negative real effects).

**Figure 4: CRA’s feedback effects when \( \theta_2^* > \hat{\theta}_1 \)**

In the other case with \( \theta_2^* < \hat{\theta}_1 \), the second component of the strategic CRA’s feedback effects is positive. This is because by assigning the rating \( R = p \) to the firm with \( \theta \in [\theta_1^*, \hat{\theta}_1) \), it is possible for the firm to invest in VP. Indeed, when \( \theta \in [\theta_2^*, \hat{\theta}_2) \), the firm invests in VP, implying that the strategic CRA has positive feedback effects.

The above arguments are summarized in Proposition 5.

**Proposition 5** The strategic CRA’s real effects can be decomposed into its informational effects and its feedback effects. The strategic CRA’s informational effects, which are measured by \( (\hat{\theta}_1 - \theta_1^*) (pV - qH) \), are always positive. When the set of parameters are such that \( \theta_2^* \geq \hat{\theta}_1 \), the strategic CRA’s feedback effects, measured by

\[
(\hat{\theta}_1 - \theta_1^*) (qH - B) + (\theta_2^* - \hat{\theta}_2) (qH - pV),
\]

are negative; but given the set of parameters such that \( \theta_2^* < \hat{\theta}_1 \), the strategic CRA’s feedback effects are measured by

\[
(\theta_2^* - \theta_1^*) (qH - B) + (\hat{\theta}_1 - \theta_2^*) (pV - B) + (\hat{\theta}_2 - \hat{\theta}_1) (pV - qH),
\]
whose sign is undetermined.

Proposition 5 implies that credit rating inflation itself does not necessarily lead to negative ex-ante real effects. Because inflated ratings are informative signals, they do increase the market’s efficiency and lead to positive real effects. The negative real effects, however, arise from the CRA’s feedback effects. Because the CRA knows that the rating will reduce the firm’s financial costs and default likelihood, it will issue the high rating to more firms, providing them with the opportunities to gamble for resurrection.

6 Empirical Predictions

The theory we develop in this paper provides several new empirical predictions about CRAs’ rating strategies and credit rating inflation. In this section, we analyze how a CRA’s rating strategy and the rating inflation vary when the economic environment changes. That is, we perform comparative static analysis to provide empirical predictions about CRAs’ rating strategies and credit rating inflation.

From these comparative static analysis, we show that laxer credit rating strategies are not necessarily accompanied by higher rating inflation. In our model, both the CRA’s rating strategy (measured by $\theta_1^*$) and the credit rating inflation (measured by $\theta_2^* - \theta_1^*$) are endogenously determined. Then, an exogenous economic environment change may lead to a laxer rating strategy and a lower financial cost to the firm at the same time. While the former effect may increase the rating inflation, the latter effect may encourage the firm to invest in VP, which reduces the rating inflation. Therefore, whether a laxer rating strategy is accompanied by higher rating inflation depends on which effect dominates. This can help interpreting recent empirical findings: Alp (2013) and Baghai, Servaes, and Tamayo (2014) find that CRAs become more conservative by using stricter rating standards, but Strobl and Xia (2012) show that the stricter rating standards do not reduce credit rating inflation.

Proposition 6 When $\beta$ is sufficiently large, a decrease in $\beta$, an increase in $H$, and a decrease in $\gamma$ will all lead to a decrease in $\theta_1^*$. However, a decrease in $\beta$ has ambiguous effects on $\theta_2^* - \theta_1^*$, an increase in $H$ increases $\theta_2^* - \theta_1^*$, and a decrease in $\gamma$ decreases $\theta_2^* - \theta_1^*$.

First, $\beta$ is the precision of creditors’ private signals, so it measures the firm’s transparency. Proposition 6 shows that for more opaque firms, the CRA employs laxer rating strategies. By

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12This is the condition for the equilibrium uniqueness, which is critical for comparative static analysis.
the properties of the truncated normal random variable’s mean, when creditors’ private signals become less precise, they believe that the firm is more likely to invest in VP. As a result, more creditors invest in the debt, and the firm’s financial cost decreases, which allows the CRA to employ a laxer rating strategy. This is consistent with a recent empirical finding in Fong, Hong, Kacperczyk, and Kubik (2014) that security analysts can discipline CRAs by providing creditors with more information.

While Proposition 6 implies that CRAs employ laxer rating strategies for more opaque firms, it does not imply that credit ratings assigned to more opaque firms are more inflated. Since creditors will decrease their debt-investment threshold when the firm is more opaque, the firm’s financial cost is lower, causing a smaller $\theta^*_2$, which is the firm’s VP investment threshold. Consequently, when creditors’ signals are less precise, the CRA is more likely to assign the rating $p$ to the firm, but the firm with a high rating is more likely to invest in VP. As a result, whether the rating inflation for a more opaque firm is higher or lower depends on which of these two effects dominates. This in turn depends on other parameters of the model.

Second, cross-sectionally, firms differ in the upside returns of their available projects. Equation (22) suggests that the highest upside return among all available projects may determine the credit rating assigned to the firm. Hence, it is interesting to consider how the firm’s upside return from HR affects the CRA’s rating strategy. An increase in $H$ does not directly affect the creditors’ behavior, because the creditors’ payoffs are solely determined by the debt contract, which does not involve the cash flow to the firm, conditional on the success of the investment. Yet, $H$ has direct effects on both the firm’s investment and the CRA’s rating strategy. On the one hand, an increase in $H$ increases the firm’s incentives to invest in HR, because the expected return from the HR is higher. On the other hand, an increase in $H$ decreases the firm’s incentives to default early, because the firm has limited liability. As a result, for fixed creditors’ strategies, when $H$ increases, the CRA’s rating strategy will be laxer, and the firm is more likely to invest in HR rather than VP, resulting in higher credit rating inflation.

Finally, consider an increase in the debt market liquidity (formally, a decrease in $\gamma$). Then, the measure of total potential creditors, $1 - \gamma$, increases. The direct effect is that the firm’s financial cost will surely decrease, because the firm needs to finance less money from the expensive non-debt sources, such as the bank credit line. In addition, a decrease in $\gamma$ will lead more creditors to buy the debt, due to the strategic complementarities among the creditors. This further reduces the firm’s financial cost. Then, the firm’s threshold of investing in VP will decrease, implying that fewer firms will invest in HR given the CRA’s credit rating strategy. In the meanwhile, the lower financial cost of the firm means that fewer firms may default early. As
a result, the CRA would want to employ a laxer rating strategy. Furthermore, as $\gamma$ decreases, the measure of firms that shift from HR to VP due to the reduced financial cost is greater than the measure of firms that gamble for resurrection because of the high credit rating, leading to lower credit rating inflation.

### 7 The Role of Dispersed Beliefs

In this section, we discuss how the belief dispersion among creditors plays a role in determining the CRA’s real effects.\(^\text{13}\) We analyze an environment in which all creditors, the firm, and the CRA share a common prior belief $\theta \in \mathcal{N}(\theta_s, \alpha^{-1})$. We will consider the case when $\alpha$ is sufficiently large and will focus on a symmetric pure-strategy equilibrium. Then, the creditors’ debt-investment decision will directly determine the firm’s financial cost. Specifically, the firm’s total repayments at date 2 are

$$K(\theta) = \begin{cases} 
  f(\theta) + (1 - \gamma)F + \gamma M, & \text{if all creditors invest in the debt;} \\
  f(\theta) + M, & \text{if all creditors refrain from investing.}
\end{cases}$$

Hence, when all creditors choose to invest in the debt, the firm’s optimal investment choice is

\[
\begin{align*}
\begin{cases}
\text{Default early,} & \text{if } f(\theta) + (1 - \gamma)F + \gamma M > H; \\
\text{HR,} & \text{if } f(\theta) + (1 - \gamma)F + \gamma M \in \left(\frac{pV - qH}{p-q}, H\right]; \\
\text{VP,} & \text{if } f(\theta) + (1 - \gamma)F + \gamma M \leq \frac{pV - qH}{p-q}.
\end{cases}
\end{align*}
\]

Denote by $y_1$ the solution to the equation $f(\theta) + (1 - \gamma)F + \gamma M = H$ and by $y_2$ the solution to the equation $f(\theta) + (1 - \gamma)F + \gamma M = (pV - qH)/(p - q)$. The firm’s optimal investment choice when all the creditors invest in the debt can be written as

\[
\begin{align*}
\begin{cases}
\text{Default early,} & \text{if } \theta < y_1; \\
\text{HR,} & \text{if } \theta \in [y_1, y_2); \\
\text{VP,} & \text{if } \theta \geq y_2.
\end{cases}
\end{align*}
\]

Similarly, we denote by $y'_1$ the solution to the equation $f(\theta) + M = H$ and by $y'_2$ the solution to the equation $f(\theta) + M = (pV - qH)/(p - q)$. The firm’s optimal investment choice when

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\(^{13}\)Since a continuum of creditors with a homogeneous belief is informationally equivalent to a big creditor, the analysis in this section may shed light on the CRA’s real effects when the firm is borrowing from a big creditor, such as a bank.
all the creditors choose not to invest in the debt can be written as

\[
\begin{align*}
\text{Default early,} & \quad \text{if } \theta < y'_1; \\
\text{HR,} & \quad \text{if } \theta \in [y'_1, y'_2); \\
\text{VP,} & \quad \text{if } \theta \geq y'_2.
\end{align*}
\]

Because \( M > (1 - \gamma)F + \gamma M \) for any \( \theta \), we have \( y_1 < y'_1 \) and \( y_2 < y'_2 \). When \( \alpha \) is sufficiently large, the creditors mainly rely on the public signal to make the debt-investment decision. Because the creditors’ behavior determines the firm’s investment choice, the public signal and the creditors’ behavior determine the CRA’s credit rating. Proposition 7 below shows the equilibrium credit rating strategy in this extension with homogeneous beliefs among creditors.

**Proposition 7 (Credit Ratings Affected by Public Signals)** There exists \( \bar{\alpha} > 0 \), such that for all \( \alpha > \bar{\alpha} \), the public signal determines the CRA’s equilibrium rating strategy. Specifically,

1. when \( \theta_s \geq y'_2 \), the CRA will employ the rating strategy \( \theta^*_1 = y_1 \);

2. when \( \theta_s < y_2 \), the CRA will employ the rating strategy \( \theta^*_1 = y'_1 \); and

3. when \( \theta_s \in [y_2, y'_2) \), the CRA will set \( \theta^*_1 = y_1 \) if all creditor invest in the debt after \( R = p \), while the CRA will set \( \theta^*_1 = y'_1 \) if all creditors refrain from investing in the debt after \( R = p \).

We observe in Proposition 7 that, when the public signal is very positive (\( \theta_s \geq y'_2 \)), the CRA employs a laxer rating strategy, meaning that the good rating is a less positive signal. When the public signal is very negative (\( \theta_s < y'_2 \)), the CRA employs a stricter rating strategy, meaning that the good rating is a more positive signal. Such a “substitution” results from the fact that the creditors rely more on the public signal when making the debt-investment decision. When the public signal is in the medium range, there will be multiple equilibria: If all creditors invest in the debt, the CRA will employ a more inflated credit rating strategy; and if all creditors refrain from investing in the debt, the CRA will employ a more conservative rating strategy.

It follows from Proposition 7 that, when all creditors share a precise common belief, the CRA hardly has any real effect: The creditors will ignore the information extracted from the credit ratings. By contrast, in the model presented in Section 2, even if we allow for a public signal or an informative prior, if creditors’ private signals are sufficiently precise (\( \beta \) is extremely large), the credit ratings will surely affect a positive measure of creditors’ decisions. This is because the continuum of creditors have dispersed beliefs caused by their conditionally independent private signals. Since credit ratings affect creditors’ beliefs by truncating their belief
supports, some creditors’ beliefs about the firm’s investment choice are surely affected. These creditors will change their debt-investment decision, which will affect the firm’s financial cost, investment choice, and thus credit quality. Then, the debt-investment decisions of some other creditors whose beliefs are hardly affected by credit ratings directly will also be affected, leading to the CRA’s significant real effects. Such a comparison shows the importance of creditors’ belief dispersion in our core model. This analysis also demonstrates that the feedback effect of credit rating agencies will stay large even if other sources of precise public information emerge.

8 Conclusion

We study credit rating agencies’ effects on firm investments. We show that high credit ratings, though commonly known to be potentially inflated, exclude extremely bad firms from creditor belief support. Therefore, high ratings make the creditors more optimistic, reduce the firm’s financial costs, and thus change its investments. That is, even in an environment with perfectly rational and well-informed creditors, inflated ratings still have significant real effects.

Such real effects, however, could be positive or negative. With the high ratings, some firms take risky projects instead of default efficiently, implying CRAs’ adverse real effects; but some other firms will switch from risky inefficient investments to safe efficient investments, implying CRAs’ positive real effects. CRAs’ overall ex-ante real effects then depend on the economic environment. Specifically, when the upside return of the risky project is high, CRAs’ overall ex-ante real effects are negative.

In order to better understand why the CRA may have negative ex-ante real effects, even though it provides informative signals to the corporate bond market, we further decompose its real effects into its informational effects and its feedback effects. We show that credit ratings that act as new informative signals do positively affect firms’ investment efficiency. Hence, the CRA’s negative real effects arise solely from its feedback effects. Indeed, the CRA takes advantage of the feedback between credit ratings and firm investments to assign high ratings to more firms, providing a chance for those firms to gamble for resurrection. Such a manipulation leads to negative real effects.

We emphasize that credit rating standards and credit rating inflation are two different concepts, and they are both endogenously determined. Furthermore, changes of economic environments that lead to laxer rating strategies do not necessarily cause higher rating inflation.

Our paper offers applied and theoretical contributions. From the applied perspective, we provide a rational framework, enabling us to analyze credit rating inflation and credit rating
agencies’ real effects in a model of feedback. While we focus on the credit ratings assigned to a firm in the paper, our model can also be applied to sovereign ratings. In fact, the assumption of the unverifiability of the firm’s investment (either VP or HR) may be very appropriate in the scenario of sovereign ratings: Because there are fewer data points of sovereign ratings, the inflated ratings are harder to be detected. Our model also generates several testable empirical predictions and some policy suggestions.

While we focus on credit ratings’ real effects in this paper, the intuition and key mechanism in this paper may be more broadly applicable. For example, in the bank stress tests (Goldstein and Leitner 2018; Inostroza and Pavan 2018), regulators may want to declare a larger number of banks solvent than truly are. This may be efficiency-enhancing for some banks (because of the reduced financial costs), but may lead some other banks to take risky projects. In addition, the economic environment we set up can be applied to many other scenarios, such as financial advising, firm disclosure, auditing, marketing, and academic grading and recommendation.

From the theoretical perspective, we analyze an expert information disclosure model with multiple audiences, who have dispersed beliefs. More importantly, the expert’s message will endogenously affect the fundamentals signaled by the message. This can motivate new research on general disclosure models.
A Proofs of lemmas and propositions

Proof of Proposition 1:

To show there is a unique equilibrium in this benchmark model, we only need to show that there is a unique solution $(\tilde{\theta}_1, \tilde{\theta}_2, \tilde{x})$ to equations (10), (11), and (12).

We first solve $\tilde{x}$ from equation (11). Define

$$\tilde{\Delta} = \frac{pV - qH}{p - q} - [(1 - \gamma)F + \gamma M] - f(\tilde{\theta}_2) \left( (1 - \gamma)(M - F) \right).$$

Because $f(\theta)$ is strictly decreasing, $\tilde{\Delta}$ is strictly increasing in $\tilde{\theta}_2$. Then we have

$$\tilde{x} = \tilde{\theta}_2 + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\tilde{\Delta}),$$

and $\tilde{x}$ is strictly increasing in $\tilde{\theta}_2$.

Plugging $\tilde{x}$ as a function of $\tilde{\theta}_2$ into equation (12), we have

$$\tilde{\Delta}(pF - qF) + \Phi \left[ \sqrt{\beta} \left( \tilde{\theta}_2 - \tilde{\theta}_1 + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\tilde{\Delta}) \right) \right] qF = 1.$$ 

The left-hand side of equation (25) strictly increases in $\tilde{\theta}_2$ and strictly decreases in $\tilde{\theta}_1$. So, we have $\partial \tilde{\theta}_2 / \partial \tilde{\theta}_1 > 0$ and $\partial \tilde{x} / \partial \tilde{\theta}_1 > 0$.

Let’s finally consider equation (10). The derivative of the left-hand side of equation (10) is

$$\frac{\partial K}{\partial \tilde{\theta}_1} + \frac{\partial K}{\partial \tilde{x}} \frac{\partial \tilde{x}}{\partial \tilde{\theta}_1},$$

where

$$\frac{\partial K}{\partial \tilde{\theta}_1} = f'(\tilde{\theta}_1) - (1 - \gamma)(M - F) \sqrt{\beta} \phi \left( \sqrt{\beta}(\tilde{x} - \tilde{\theta}_1) \right) < 0$$

$$\frac{\partial K}{\partial \tilde{x}} \frac{\partial \tilde{x}}{\partial \tilde{\theta}_1} = (1 - \gamma)(M - F) \sqrt{\beta} \phi \left( \sqrt{\beta}(\tilde{x} - \tilde{\theta}_1) \right) \frac{\partial \tilde{x}}{\partial \tilde{\theta}_1} > 0.$$

Note that $\tilde{\Lambda}$ is between 0 and 1. From equation (25), we have $\beta \to +\infty$, $\tilde{\Delta}$ is bounded away from both 0 and 1. To see this, suppose $\tilde{\Delta} \to 1$ first. Then, the left-hand side of equation (25) goes to $pF$, which is greater than 1, the right-hand side of equation (25). Similarly, if $\tilde{\Delta} \to 0$, the left-hand side of equation (25) is strictly less than 1.
Hence, from equation (24), $\tilde{x} \rightarrow \tilde{\theta}_2$. In addition, as $\beta \rightarrow +\infty$, $\tilde{\theta}_2$ cannot converge to $\tilde{\theta}_1$; otherwise, equation (10) and equation (11) cannot hold simultaneously. Therefore, as $\beta \rightarrow +\infty$, $\tilde{x} - \tilde{\theta}_1$ is bounded away from 0. This implies that

$$\lim_{\beta \rightarrow +\infty} \sqrt{\beta} \phi \left( \sqrt{\beta}(\tilde{x} - \tilde{\theta}_1) \right) = 0,$$

which further implies that $\frac{\partial \tilde{x}}{\partial \tilde{\theta}_1}$ and $\frac{\partial \tilde{x}}{\partial \tilde{\theta}_1}$ are converging to 0 as $\beta \rightarrow +\infty$. Therefore, though $\frac{\partial K}{\partial \tilde{x}} > 0$, when $\beta$ is large enough, $\frac{\partial K}{\partial \tilde{\theta}_1}$ is very close to 0. For the term of $\frac{\partial K}{\partial \tilde{\theta}_1}$, it will not go to 0 as $\beta$ goes to $\infty$, because $f'(\tilde{\theta}_1) < 0$. Therefore, there is a $\tilde{\beta} > 0$, such that for all $\beta > \tilde{\beta}$, the left-hand side of equation (10) strictly decreases in $\tilde{\theta}_1$. The left-hand side of equation (10) will be less than $M$ and thus less than $H$ as $\tilde{\theta}_1$ goes to $+\infty$ and will diverge to $+\infty$ when $\tilde{\theta}_1$ goes to $-\infty$. Then by the continuity of function $f(\cdot)$, there is a unique $\tilde{\theta}_1$. Then, there is a unique solution to equations (10), (11), and (12).

Q.E.D.

**Proof of Lemma 1:**

We prove each part of this lemma.

1. The case $\frac{V_p - V_q}{C_p - C_q} \geq 1 - q$.

Because $p > q$, this implies

$$\frac{V_p - V_q}{C_p - C_q} \geq 1 - q > 1 - p. \quad (26)$$

We first show that the CRA will not assign the rating $q$ in equilibrium. Suppose that there is an equilibrium, in which the CRA assigns the rating $q$ to a $\theta$-firm when $\theta \in (\theta_1, \theta_2)$. There are two cases. In the first case, a firm with the rating $R = p$ is believed to have better economic fundamentals than a firm with the rating $R = q$. Consider that the CRA deviates to assign the rating $R = p$ to the $\theta$-firm. Then, observing the rating $p$, creditors are more optimistic about the firm’s fundamentals, and so more creditors will buy the debt. Consequently, the firm will have a lower financial cost. Therefore, after such a deviation by the CRA, the firm will not default at date 1. Then, the condition $\frac{V_p - V_q}{C_p - C_q} \geq 1 - q > 1 - p$ implies that if the firm invests in HR, we have $V_p - (1 - q)C_p \geq V_q - (1 - q)C_q$, and if the firm invests in VP, we have $V_p - (1 - p)C_p > V_q - (1 - p)C_q$. These imply that such a deviation is profitable to the CRA.
Now, consider the second case in which a firm with the rating $q$ is believed to have better economic fundamentals than a firm with the rating $p$. Then, there must exist intervals $[\theta_3, \theta_4)$ and $(\theta_1, \theta_2)$ with $\theta_3 < \theta_4 < \theta_1 < \theta_2$, such that the rating strategy specifies $R(\theta) = p$ when $\theta \in [\theta_3, \theta_4)$ and $R(\theta) = q$ when $\theta \in (\theta_1, \theta_2)$. Since the firms with the economic fundamentals in both $(\theta_1, \theta_2)$ and $[\theta_3, \theta_4)$ will not default at date 1, the $\theta_3$-firm does not default early. Then, if the CRA assigns the rating $p$ to any $\theta$-firm for $\theta \in (\theta_1, \theta_2)$, the $\theta$-firm’s financial cost is lower than the $\theta_3$-firm’s. Since such a $\theta$-firm will have a lower operation cost than the $\theta_3$-firm does, the $\theta$-firm does not default at date 1 after being assigned the rating $p$. Then, it follows from the condition $\frac{V^p - V^q}{C^p - C^q} \geq 1 - q$ that such a deviation is profitable to the CRA. These arguments show that the CRA will not assign the rating $q$ in equilibrium.

We then prove that the rating strategy must be monotonic. Suppose that there is an equilibrium, in which a $\theta$-firm does not default at date 1 if it is assigned the rating $p$. So the CRA should best respond by assigning the rating $R(\theta) = p$, because by assigning the rating $R = 0$, the CRA can get zero profit only. Let $W(p)$ be the measure of creditors who choose to buy the debt, after observing the credit rating $R(\theta)$ and their own private signals. Then the assumption that $\theta$-firm does not default early implies

$$K(\theta) = f(\theta) + W(p, \theta)F + (1 - W(p, \theta))M < H.$$  

Now, let’s consider any $\theta'$-firm with $\theta' > \theta$. Again, the CRA can only get zero payoff by assigning the rating $R = 0$, so it will assign the rating $R = p$ to the $\theta'$-firm if the $\theta'$-firm does not default at date 1. In a monotone equilibrium, any creditor $i$’s strategy is monotonic in his private signal $x_i$, and any creditor’s private signal conditional on $\theta'$ First-order Stochastic dominates that conditional on $\theta$. So $W(p, \theta') > W(p, \theta)$. Then we have

$$K(\theta') = f(\theta') + W(p, \theta')F + (1 - W(p, \theta'))M < f(\theta) + W(p, \theta)F + (1 - W(p, \theta))M < H.$$  

Therefore, the $\theta'$-firm does not default at date 1 either, implying that $R(\theta') = p$ in equilibrium.

Furthermore, independent of creditors’ decisions, when $\theta$ is very negative, the firm will default early, and when $\theta$ is very positive, the firm will not default early. As a result, in
any equilibrium (if exists), the CRA’s rating strategy must be in the form described by equation (14).

2. The case $\frac{V_p - V_q}{C_p - C_q} \leq 1 - p$.

This case is same as the previous one, except that we replace the rating $p$ by the rating $q$.

3. The case $\frac{V_p - V_q}{C_p - C_q} \in (1 - p, 1 - q)$.

In this case, we have

$$V_p - (1 - p)C_p > V_q - (1 - p)C_q > V_q - (1 - q)C_q > V_p - (1 - q)C_p.$$ 

Therefore, if a $\theta$-firm chooses VP (no matter whether it is assigned the rating $p$ or the rating $q$), the CRA will assign the rating $p$. Similarly, if the $\theta$-firm chooses HR, (no matter whether it is assigned the rating $p$ or the rating $q$), the CRA will assign the rating $q$. In addition, when the $\theta$-firm chooses VP following the rating $p$ and HR following the rating $q$, the CRA will assign the rating $p$. Furthermore, because for any particular $\theta$-firm, the financial cost following the rating $p$ is lower than the financial cost following the rating $q$, if the firm invests in HR following the rating $p$, it will invest in HR following the rating $q$.

Hence, the rating strategy must be in the form as in equation (16). Whether the rating $q$ can appear in the equilibrium depends on the model’s parameters, so $\theta^q$ could be equal to $\theta^p$, in which case the CRA will not assign the rating $q$ in equilibrium.

Q.E.D.

Proof of Lemma 2:

Suppose that there is an equilibrium in which the firm invests in VP for all $\theta$ such that $R(\theta) = p$. All creditors will invest in the debt, leading to the firm’s financial cost $(1 - \gamma)F + \gamma M$. For the firm to choose VP if and only if $\theta \geq \theta^*_1$, we must have

$$f(\theta^*_1) + (1 - \gamma)F + \gamma M \leq \frac{pV - qH}{p - q} < H.$$ 

But because $f(\cdot)$ is continuous and strictly decreasing, there exists $\hat{\theta}^*_1 < \theta^*_1$ such that,

$$\frac{pV - qH}{p - q} < f(\hat{\theta}^*_1) + (1 - \gamma)F + \gamma M < H.$$
That is, there is a subset of $\theta$’s with a positive measure that are greater than $\hat{\theta}^*_1$ but very close to $\hat{\theta}^*_1$, such that the firm will invest in HR. Since the firm’s investment choice HR is unverifiable, a deviation to the rating strategy with $\hat{\theta}^*_1$ is profitable to the CRA. Therefore, the rating strategy with $\theta^*_1$ such that $f(\theta^*_1) + (1 - \gamma)F + \gamma M \leq \frac{pV - qH}{p - q}$ cannot be part of an equilibrium. Therefore, in any monotone equilibrium (if it exists), the rating strategy must be inflated.

Q.E.D.

Proof of Proposition 2:

We first show that given the CRA’s rating strategy $\theta^*_1$, there is a unique equilibrium play of the firm and the creditors following the rating $R = p$. This is formally presented in Lemma 4 below.

**Lemma 4 (Debt Financing Following $R = p$)** There exists a $\beta^* > 0$, such that for any $\beta > \beta^*$, given the CRA’s rating strategy $\theta^*_1$, following the rating $p$, there is a unique solution $(\theta^*_2, x^*)$ with $\theta^*_2 > \theta^*_1$ to equation (20) and equation (21).

Proof of Lemma 4:

For a given $x^* \in \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$, the left-hand side of equation (20) is strictly decreasing in $\theta$. When $\theta \to +\infty$, the LHS of equation (20) goes to $(1 - \gamma)F + \gamma M$, which is strictly less than $\frac{pV - qH}{p - q}$, since $\frac{pV - qH}{p - q} > M > F$. However, if when $\theta = \theta^*_1$, the LHS is still less than $\frac{pV - qH}{p - q}$, the firm will always choose VP after the rating $R = p$. This contradicts Lemma 2. Therefore, for a given $x^* \in \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$, there is a unique $\theta^*_2 > \theta^*_1$, such that equation (20) holds. Then we can solve for $x^*$ from equation (20)

$$x^* = \theta^*_2 + \frac{1}{\sqrt{\beta}} \Phi^{-1} \left[ \frac{\frac{pV - qH}{p - q} - [f(\theta^*_2) + (1 - \gamma)F + \gamma M]}{(1 - \gamma) [M - F]} \right].$$

Denote

$$\Delta = \frac{\frac{pV - qH}{p - q} - [f(\theta^*_2) + (1 - \gamma)F + \gamma M]}{(1 - \gamma) [M - F]},$$

so $x^* = \theta^*_2 + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\Delta)$. Because $f(\cdot)$ is strictly decreasing, $\Delta$ is strictly increasing in $\theta^*_2$, and thus $x^*$ is strictly increasing in $\theta^*_2$.
Then, plugging $x^*$ as a function of $\theta_2^*$ into equation (21), we have

$$
\frac{\Delta}{\Phi \left[ \sqrt{\beta} \left( \theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}} \Phi^{-1} (\Delta) \right) \right]} (pF - qF) = 1 - qF.
$$

(27)

Differentiating the left-hand side of equation (27), the sign of this derivative would be the same as the sign of

$$
\frac{\partial \Delta}{\partial \theta_2^*} \frac{\Phi \left[ \sqrt{\beta} \left( \theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}} \Phi^{-1} (\Delta) \right) \right]}{\Delta} - \Phi \left[ \sqrt{\beta} \left( \theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}} \Phi^{-1} (\Delta) \right) \right] \left( \sqrt{\beta} + \frac{1}{\phi(\Delta)} \frac{\partial \Delta}{\partial \theta_2^*} \right) = \frac{\partial \Delta}{\partial \theta_2^*} \frac{pF - qF}{1 - qF} - \Phi \left[ \sqrt{\beta} \left( \theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}} \Phi^{-1} (\Delta) \right) \right] \left( \sqrt{\beta} + \frac{1}{\phi(\Delta)} \frac{\partial \Delta}{\partial \theta_2^*} \right).
$$

The first term is positive for any $\beta$, because $\Delta$ is not a function of $\beta$, and $\theta_2^*$ is bounded (and so $f'(\theta_2^*)$ is bounded away from 0).

The second, though is negative, will converge to 0 as $\beta \to +\infty$. This is because $\Phi \left[ \sqrt{\beta} \left( \theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}} \Phi^{-1} (\Delta) \right) \right]$ will converge to 0 higher order faster than $\sqrt{\beta}$. We need to consider three cases to prove this argument. First, as $\beta \to +\infty$, $\Delta$ must be bounded away from 1; otherwise, the left-hand side of equation (27) converges to $pF - qF$, which is strictly greater than $1 - qF$, the right-hand side of equation (27). Second, suppose that as $\beta \to +\infty$, $\Delta$ is also bounded away from 0. Then $x^* - \theta_2^* \to 0$. But it follows from equation (27) $\theta_2^* - \theta_1^*$ must be positive and bounded away from 0; otherwise, the left-hand of equation (27) converges to $pF - qF$, which is strictly greater than the right-hand side of equation (27). Hence, $\Phi \left[ \sqrt{\beta} \left( \theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}} \Phi^{-1} (\Delta) \right) \right]$ $\sqrt{\beta} = \Phi \left[ \sqrt{\beta} (x^* - \theta_1^*) \right]$ $\sqrt{\beta}$ must converge to 0. Finally, as $\beta \to +\infty$, $\Delta \to 0$. Then from equation (27), we must have $\Phi \left[ \sqrt{\beta} (x^* - \theta_1^*) \right] \to 0$ and thus $\sqrt{\beta} (x^* - \theta_1^*) \to -\infty$ as $\beta \to +\infty$. By L'Hôpital's rule, we have

$$
\lim_{\beta \to +\infty} \frac{1}{\sqrt{\beta} (x^* - \theta_1^*)} = \lim_{\beta \to +\infty} \frac{\beta^{-\frac{1}{2}}}{(x^* - \theta_1^*)} = \lim_{\beta \to +\infty} \frac{1}{2\beta^{\frac{3}{2}} \frac{\partial x^*}{\partial \beta}} = 0
$$

Therefore, $\lim_{\beta \to +\infty} 2\beta^{\frac{3}{2}} \frac{\partial x^*}{\partial \beta} = +\infty$. Then, simple algebra can lead to the result that $\Phi \left[ \sqrt{\beta} \left( \theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}} \Phi^{-1} (\Delta) \right) \right]$ $\sqrt{\beta}$ converges to 0, as $\beta \to +\infty$. Therefore, there is a $\beta^*$ such that when $\beta > \beta^*$, the left-hand side of equation (27) is strictly increasing in $\theta_2^*$. 39
Note by definition, \( \Delta \) must be a number in \([0, 1]\). Therefore, there are \( \theta \) and \( \bar{\theta} \) such that, \( \theta^*_1 < \theta < \bar{\theta} < +\infty \), \( \Delta(\bar{\theta}) = 1 \), and \( \Delta(\theta) = 0 \). Then when \( \theta^*_2 \to \bar{\theta} \), the left hand side of equation (27) is strictly greater than \( 1 - qF \); when \( \theta^*_2 \to \theta \), the left hand side of equation (27) is close to 0 and thus strictly smaller than \( 1 - qF \).

Therefore, there is a unique \( \theta^*_2 \), and thus there is a unique \( x^* \).

Q.E.D.

We then now argue that the creditor’s threshold \( x^* \) and the firm’s VP-investment threshold \( \theta^*_2 \) are both strictly increasing in \( \theta^*_1 \). Intuitively, when the CRA employs a laxer rating strategy (a smaller \( \theta^*_1 \)), the creditors will discount the positive information conveyed by the rating \( R = p \), and so are less likely to buy the debt (\( x^* \) increases). This increases the firm’s financial cost, and so the firm is less likely to invest in VP (\( \theta^*_2 \) increases). Such an argument is presented in Lemma 5 below.

**Lemma 5 (Laxer Rating Strategy)** For any \( \beta > \beta^* \), both \( x^* \) and \( \theta^*_2 \) are strictly decreasing in \( \theta^*_1 \).

**Proof of Lemma 5:**

The left-hand side of equation (27) is strictly increasing in \( \theta^*_1 \), fixing \( \theta^*_2 \). It also follows from the proof of Lemma 4 that the left-hand side of equation (27) is strictly increasing in \( \theta^*_2 \). Then, the Implicit Function Theorem implies that \( \theta^*_2 \) is strictly increasing in \( \theta^*_1 \). Since \( x^* \) is strictly increasing \( \theta^*_2 \), \( x^* \) is strictly decreasing in \( \theta^*_1 \) (given \( \theta^*_2 \), \( x^* \) is determined by equation \( x^* = \theta^*_2 + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\Delta) \)).

Q.E.D.

By Lemma 4 and Lemma 5, in order to prove Proposition 2, we only need to show that there is a unique \( \theta^*_1 \) such that equation (22) holds, given \( x^* \) as a function of \( \theta^*_1 \). When \( \beta \) is sufficiently large, by Lemma 5, \( x^* \) is strictly decreasing in \( \theta^*_1 \). Then, the derivative of the left hand side of equation (22) with respect to \( \theta \) is

\[
f'(\theta) - (1 - \gamma)\phi[\sqrt{\beta}(x^* - \theta)]\sqrt{\beta}(M - F) + (1 - \gamma)(M - F)\phi[\sqrt{\beta}(x^* - \theta)]\sqrt{\beta} \frac{\partial x^*}{\partial \theta} < 0.
\]

We know that when \( \theta \to +\infty \), \( f(\theta) \to 0 \), the left hand side of equation (22) converges to \( (1 - \gamma)F + \gamma M \), which is less than \( H \); when \( \theta \to -\infty \), \( f(\theta) \to +\infty \), the left hand side of equation (22) diverges to \( +\infty \), which is greater than \( H \). Therefore, the solution to equation (22) exists and is unique.
Because \( H > \frac{pV - qH}{p - q} \), equation (22) and equation (20) imply that \( \theta_2^* > \theta_1^* \). In addition, equation (22) also implies that \( f(\theta_1^*) + M > H \), because \( M > F \). These complete the proof of the uniqueness of the equilibrium of the model.

\[ Q.E.D. \]

**Proof of Lemma 3:**

Recall that the three equations determining the equilibrium of the model without the CRA are

\[
f(\theta_1) + [(1 - \gamma)F + \gamma M] + (1 - \gamma)\Phi \left[ \sqrt{\beta}(x - \theta_1) \right] (M - F) = H \quad (28)
\]

\[
f(\theta_2) + [(1 - \gamma)F + \gamma M] + (1 - \gamma)\Phi \left[ \sqrt{\beta}(x - \theta_2) \right] (M - F) = \frac{pV - qH}{p - q} \quad (29)
\]

\[
\{ \Phi \left[ \sqrt{\beta}(\theta_2 - x) \right] - \Phi \left[ \sqrt{\beta}(\theta_1 - x) \right] \} qF + \{ 1 - \Phi \left[ \sqrt{\beta}(\theta_2 - x) \right] \} pF = 1; \quad (30)
\]

and the three equations determining the equilibrium of the model with the CRA are

\[
f(\theta_1) + [(1 - \gamma)F + \gamma M] + (1 - \gamma)\Phi \left[ \sqrt{\beta}(x - \theta_1) \right] (M - F) = H \quad (31)
\]

\[
f(\theta_2) + [(1 - \gamma)F + \gamma M] + (1 - \gamma)\Phi \left[ \sqrt{\beta}(x - \theta_2) \right] (M - F) = \frac{pV - qH}{p - q} \quad (32)
\]

\[
\frac{\Phi[\sqrt{\beta}(\theta_2 - x)] - \Phi[\sqrt{\beta}(\theta_1 - x)]}{1 - \Phi[\sqrt{\beta}(\theta_1 - x)]}qF + \frac{1 - \Phi[\sqrt{\beta}(\theta_2 - x)]}{1 - \Phi[\sqrt{\beta}(\theta_1 - x)]}pF = 1 \quad (33)
\]

The difference between the equilibrium in the benchmark model and that in the model with the CRA stems from the difference between equation (30) and equation (33). That is, the creditors’ indifference conditions differ. If we change equation (30) by dividing both sides by the term \( 1 - \Phi[\sqrt{\beta}(\theta_1 - x)] \), we have

\[
\frac{\Phi[\sqrt{\beta}(\theta_2 - x)] - \Phi[\sqrt{\beta}(\theta_1 - x)]}{1 - \Phi[\sqrt{\beta}(\theta_1 - x)]}qF + \frac{1 - \Phi[\sqrt{\beta}(\theta_2 - x)]}{1 - \Phi[\sqrt{\beta}(\theta_1 - x)]}pF = 1
\]

\[
= \frac{1}{1 - \Phi[\sqrt{\beta}(\theta_1 - x)]} > 1.
\]

Solve \( x \) as a function of \( \theta_2 \) from equation (29) or equation (32), and plug it into equation (34) and equation (33). Then, for a same \( \theta_1 \), \( \theta_2 \) in equation (34) is greater than that in equation (33), because the left-hand sides of these two equations are strictly increasing in \( \theta_2 \). Hence, \( x \)
in equation (34) is greater than \( x \) in equation (33). Furthermore, because \( \theta_1 \) in the benchmark model is positively correlated to \( \theta_2 \), while \( \theta_1 \) in the model with the CRA is negatively correlated to \( \theta_2 \), we know \( \theta^*_1 < \tilde{\theta}_1 \). Moreover, we have \( \tilde{\theta}_2 > \theta^*_2 \) and \( \tilde{x} > x^* \).

However, the sign of \( \theta^*_2 - \tilde{\theta}_1 \) is undetermined. Consider equation (28) and equation (32). Both \( \tilde{\theta}_1 \) and \( \theta^*_2 \) are strictly increasing functions of \( x \). While we have shown that \( \tilde{x} > x^* \), the right-hand side of equation (28) is greater than that of equation (32). Therefore, without specifying parameters’ values, we cannot determine the sign of \( \theta^*_2 - \tilde{\theta}_1 \).

\[
\text{Q.E.D.}
\]

**Proof of Proposition 3:**

We denote by \( \Omega \) and \( \tilde{\Omega} \) the equilibrium economic efficiency in the model with the CRA and in the model without the CRA, respectively. We first consider the firm with \( \theta \geq \tilde{\theta}_2 \). It follows from Proposition 2 that the CRA assigns the rating \( p \) to the firm, and the firm will invest in VP. In the model without the CRA, the firm will also invest in VP, therefore, \( \Omega = \tilde{\Omega} \), and hence, the CRA has no effect on the expected NPV. Similarly, when \( \theta < \theta^*_1 \), the firm defaults at date 1 with or without the CRA. Therefore, \( \Omega = \tilde{\Omega} \). Hence, when \( \theta < \theta^*_1 \), the CRA has no effect on the economic efficiency either.

When \( \theta \in [\theta^*_1, \tilde{\theta}_2] \), the CRA’s effect depends on the parameters of the model. In the first case where \( \theta^*_2 > \tilde{\theta}_1 \), for \( \theta \in [\theta^*_1, \tilde{\theta}_1] \), \( \Omega - \tilde{\Omega} = qH - B < 0 \), because \( qH \) is assumed to be less than \( B \). For \( \theta \in [\tilde{\theta}_2, \theta^*_2] \), \( \Omega - \tilde{\Omega} = pV - qH > 0 \). For all \( \theta \in [\tilde{\theta}_1, \theta^*_2] \), the firm invests in HR with or without the CRA; thus, CRA has no effect on the economic efficiency. Therefore, in such a case, the CRA’s ex-ante real effects are

\[
(\tilde{\theta}_2 - \theta^*_2)(pV - qH) + (\tilde{\theta}_1 - \theta^*_1)(qH - B).
\]

In case 2 where \( \theta^*_2 \leq \tilde{\theta}_1 \), for \( \theta \in [\theta^*_1, \theta^*_2] \), \( \Omega - \tilde{\Omega} = qH - B \). For \( \theta \in [\theta^*_2, \tilde{\theta}_1] \), \( \Omega - \tilde{\Omega} = pV - qH \). Finally, for \( \theta \in [\tilde{\theta}_1, \tilde{\theta}_2] \), \( \Omega - \tilde{\Omega} = pV - qH \). Therefore, in this case, the CRA’s ex-ante real effects are

\[
(\tilde{\theta}_2 - \tilde{\theta}_1)(pV - qH) + (\tilde{\theta}_1 - \theta^*_2)(pV - B) + (\theta^*_2 - \theta^*_1)(qH - B).
\]

\[
\text{Q.E.D.}
\]

**Proof of Proposition 4:**
Part 1: We first consider the continuation play following the rating \( R = 0 \). It then follows from equation (23) that \( \theta < \hat{\theta}_1 = \tilde{\theta}_1 \). Suppose that all creditors refrain from investing. Then if the \( \theta \)-firm continues to make investments, its total repayments at date 2 are

\[
K(\theta) = f(\theta) + M \\
> f(\theta) + [(1 - \gamma)F + \gamma M] + (1 - \gamma) \Phi \left( \sqrt{\beta} (\bar{x} - \theta) \right) (M - F) \\
> f(\hat{\theta}_1) + [(1 - \gamma)F + \gamma M] + (1 - \gamma) \Phi \left( \sqrt{\beta} (\bar{x} - \hat{\theta}_1) \right) (M - F) \\
= H.
\]

Hence, if all creditors refrains from investing, the \( \theta \)-firm will default at date 1. On the other hand, given that any \( \theta \)-firm will default early, no creditor will invest in the debt, implying that there is an equilibrium in which the firm will default early when receiving the rating \( R = 0 \) assigned by the reflecting CRA.

We now show that in the continuation play following \( R = 0 \) the firm will not continue to invest in either HR or VP. Suppose that there is a (monotone) equilibrium in which a creditor with the private signal \( x_i \) invests in the debt if and only if \( x_i \geq x' \), when the rating is \( R = 0 \). Here, \( x' \in \mathbb{R} \). Since some creditors are willing to invest, they must believe that any \( \theta \)-firm will invest in VP if \( \theta \in [\theta'_1, \tilde{\theta}_1) \), and that by the continuity of the firm’s financial cost, any \( \theta \)-firm will invest in HR if \( \theta \in [\theta'_1, \theta'_2) \), where \( \theta'_1, \theta'_2 \in \mathbb{R} \) and \( \theta'_1 < \theta'_2 < \hat{\theta}_1 \). Therefore, such an equilibrium can be characterized by the following system of equations

\[
f(\theta'_1) + [(1 - \gamma)F + \gamma M] + (1 - \gamma) \Phi \left[ \sqrt{\beta} (\theta'_1 - x') \right] (M - F) = H \tag{35} \\
f(\theta'_2) + [(1 - \gamma)F + \gamma M] + (1 - \gamma) \Phi \left[ \sqrt{\beta} (\theta'_2 - x') \right] (M - F) = \frac{pV - qH}{p - q} \tag{36} \\
\frac{\Phi[\sqrt{\beta}(\theta'_2 - x')]}{\Phi[\sqrt{\beta}(\hat{\theta} - x')]} - \Phi[\sqrt{\beta}(\theta'_1 - x')] - \Phi[\sqrt{\beta}(\theta'_2 - x')] \left. \frac{qF + \Phi[\sqrt{\beta}(\hat{\theta} - x')] - \Phi[\sqrt{\beta}(\theta'_2 - x')]\right|_{\Phi[\sqrt{\beta}(\hat{\theta} - x')]} = 1 \tag{37}
\]

Comparing equation (36) with equation (11), we can see that since \( \theta'_2 < \hat{\theta}_1 = \tilde{\theta}_1 < \tilde{\theta}_2 \), \( x' \) must be strictly less than \( \bar{x} \).

Note that for any \( \hat{\theta}_1 \) (which may not be \( \tilde{\theta}_1 \)), given the committed rating rule (equation (23)), a monotone equilibrium with some \( \theta \)-firm investing in VP or HR must be characterized by the system of equations (35), (36), and (37). The following argument therefore relies on the comparative static analysis of the solution to such a system of equations.

Solving \( x' \) as a function of \( \theta'_2 \) from equation (36) and substituting it into equation (35) and
reflecting CRA, in the subgame following the rating $R$ if $\hat{\theta}$ in HR if
olating the equilibrium uniqueness result. when $\hat{\theta}$ not be an equilibrium in which a positive measure of types of the firm defaults early. Otherwise,
defaults early and all creditors run.
a reflecting CRA, the subgame following $R$ of Proposition 1 that the benchmark model has a unique equilibrium. Therefore, in the case with
in VP if $\theta$ in Proposition 2. Therefore, in an equilibrium of the subgame following $R$ benchmark model has another equilibrium with $x$.

Part 2: Similarly to the proof of Part 1, in the continuation play following $R = p$, there cannot be an equilibrium in which a positive measure of types of the firm defaults early. Otherwise, when $\hat{\theta}$ goes to $-\infty$, we show that the benchmark model will have two different equilibria, violating the equilibrium uniqueness result.

Now, suppose that $\theta_2^* > \hat{\theta}_1$. Also, suppose that there is $\hat{\theta}_2 \in [\hat{\theta}_1, +\infty)$, such that with the reflecting CRA, in the subgame following the rating $R = p$, the firm invests in VP if $\theta \leq \hat{\theta}_2$ and in HR if $\theta \in [\hat{\theta}_1, \hat{\theta}_2)$. It follows from Lemma 5 that $\hat{\theta}_2$ and $\hat{x}$ are both strictly decreasing in $\hat{\theta}_1$. If $\hat{\theta}_2 > \theta_2^*$ is part of an equilibrium, when the reflecting CRA has the rating strategy $\hat{\theta}_1 = \theta_1^*$, there is an equilibrium in which $\hat{\theta}_2 > \theta_2^*$, violating the equilibrium uniqueness conclusion in Proposition 2. Therefore, in an equilibrium of the subgame following $R = p$, the firm invests in VP if $\theta \leq \hat{\theta}_2$ and in HR if $\theta \in [\hat{\theta}_1, \hat{\theta}_2)$. Furthermore, if $\theta_2^* > \hat{\theta}_1$, we have $\hat{\theta}_2 < \theta_2^*$.

Q.E.D.
Proof of Proposition 6:

We first show the comparative static analysis with respect to $\beta$. Recall that

$$x^* = \theta_2^* + \frac{1}{\sqrt{\beta}} \Phi^{-1} [\Delta],$$

where

$$\Delta = \frac{pV - qH}{p-q} - [(1-\gamma)F + \gamma M] - f(\theta_2^*),$$

so $\partial \Delta / \partial \theta_2^* > 0$.

Substitute $x^*$ as a function of $\theta_2^*$ into equation (21) and equation (22), and denote $\sqrt{\beta}(\theta_2^* - \theta_1^*) + \Phi^{-1}(\Delta) = \Psi$ for simplicity, we have

$$\Delta(pF - qF) - \Phi[\Psi](1-qF) = 0 \quad (40)$$

$$f(\theta_1^*) + (1-\gamma)(M-F)\Phi[\Psi] = H - [(1-\gamma)F + \gamma M] \quad (41)$$

Total differentiation of the above two equations with respect to $\theta_2^*, \theta_1^*$, and $\beta$, we have

$$A \begin{bmatrix} \frac{\partial \Delta}{\partial \theta_2^*} \\ \frac{\partial \Delta}{\partial \theta_1^*} \\ \frac{\partial \Delta}{\partial \beta} \end{bmatrix} = \begin{bmatrix} \phi(\Psi)(1-qF) \frac{\theta_2^* - \theta_1^*}{2\sqrt{\beta}} \\ - (1-\gamma)(M-F)\Phi(\Psi) \frac{\theta_2^* - \theta_1^*}{2\sqrt{\beta}} \end{bmatrix}, \quad (42)$$

where

$$A = \begin{bmatrix} \frac{\partial}{\partial \theta_2^*}(pF - qF) - \phi(\Psi) \left( \sqrt{\beta} + \frac{1}{\Phi[\Psi]} \frac{\partial \Delta}{\partial \theta_2^*} \right) (1-qF) & \phi(\Psi) \sqrt{\beta}(1-qF) \\ (1-\gamma)(M-F)\Phi(\Psi) \left( \sqrt{\beta} + \frac{1}{\Phi[\Psi]} \frac{\partial \Delta}{\partial \theta_1^*} \right) & f'(\theta_1^*) - (1-\gamma)(M-F)\Phi(\Psi) \sqrt{\beta} \end{bmatrix} \quad (43)$$

As we have shown in the proofs of Lemma 4, when $\beta$ is large enough, $\phi(\Psi) \sqrt{\beta}$ is very close to 0. Therefore, when $\beta$ is sufficiently large, the determinant of the matrix $A$ is close to

$$\frac{\partial \Delta}{\partial \theta_2^*} (pF - qF) f'(\theta_1^*) < 0,$$

because $f'(\theta_1^*) < 0$.

Then further algebra shows that when $\beta$ is sufficiently large, the sign of $\partial \theta_1^* / \partial \beta$ is the same as that of

$$(1-\gamma)(M-F)(pF - qF)\frac{\partial \Delta}{\partial \theta_2^*},$$

which is positive. Therefore, $\theta_1^*$ is strictly increasing in $\beta$. 

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Now, let us consider the comparative static analysis with respect to $H$. Total differentiation of equation (40) and equation (41) with respect to $\theta_2^*, \theta_1^*$, and $H$, we have

$$A \begin{bmatrix} \frac{\partial \theta_2^*}{\partial H} \\ \frac{\partial \theta_1^*}{\partial H} \end{bmatrix} = \begin{bmatrix} -(pF - qF) \frac{\partial \Delta}{\partial H} + \phi(\Psi) \frac{1}{\phi(\Delta)} \frac{\partial \Delta}{\partial H} (1 - qF) \\ 1 - (1 - \gamma)(M - F) \phi(\Psi) \frac{1}{\phi(\Delta)} \frac{\partial \Delta}{\partial H} \end{bmatrix},$$

where $A$ is defined in equation (43).

Note that $\phi(\Psi) \sqrt{\beta}$ is very close to 0 when $\beta$ is sufficiently large, we have

$$\text{sign} \begin{bmatrix} \frac{\partial \theta_2^*}{\partial H} \\ \frac{\partial \theta_1^*}{\partial H} \end{bmatrix} = \text{sign} \begin{bmatrix} (pF - qF)f'(\theta_1^*) \frac{\partial \Delta}{\partial H} \\ -(pF - qF) \frac{\partial \Delta}{\partial \theta_2^*} \end{bmatrix}.$$  

Because $f'(\theta_1^*) < 0$, $\partial \Delta/\partial H < 0$, and $\partial \Delta/\partial \theta_2^* > 0$, we have $\partial \theta_1^*/\partial H < 0$ and $\partial \theta_2^*/\partial H > 0$. Therefore, $\partial (\theta_2^* - \theta_1^*)/\partial H > 0$.

We finally show the comparative static analysis with respect to $\gamma$. Similar to that about $H$, the total differentiation of of equation (40) and equation (41) with respect to $\theta_2^*, \theta_1^*$, and $\gamma$, we have

$$A \begin{bmatrix} \frac{\partial \theta_2^*}{\partial \gamma} \\ \frac{\partial \theta_1^*}{\partial \gamma} \end{bmatrix} = \begin{bmatrix} -(pF - qF) \frac{\partial \Delta}{\partial \gamma} + \phi(\Psi) \frac{1}{\phi(\Delta)} \frac{\partial \Delta}{\partial \gamma} (1 - qF) \\ (M - F) \left[ \Phi(\Psi) - 1 - (1 - \gamma) \phi(\Psi) \frac{1}{\phi(\Delta)} \frac{\partial \Delta}{\partial \gamma} \right] \end{bmatrix},$$

where $A$ is defined in equation (43).

Note that $\phi(\Psi) \sqrt{\beta}$ is very close to 0 when $\beta$ is sufficiently large. In addition, $\partial \Delta/\partial \gamma > 0$ and $(1 - \Phi(\Psi))/\phi(\Psi) \to 0$ when $\beta \to +\infty$. Then, simple algebra will show that $\partial \theta_1^*/\partial \gamma > 0$ and $\partial \theta_2^*/\partial \gamma > 0$. Further algebra will show that $\partial (\theta_2^* - \theta_1^*)/\partial \gamma > 0$.

Q.E.D.
References


B Online Appendix (not for Publication)

B.1 Endogenous Debt Face Value

In the model presented in Section 2, the debt face value $F$ is exogenously given. We have argued that such an assumption is not critical for our results but can largely simplify our analysis, as in He and Xiong (2012). To further demonstrate this point, in this appendix, we consider a model where the debt face value is endogenously determined. This model, however, may not have a nontrivial equilibrium in which the firm borrows in the credit market, and even if a nontrivial equilibrium exists, there may be multiple nontrivial equilibria. Despite such weaknesses of this model, if we focus on the equilibrium with the lowest debt face value (if it exists), the CRA’s real effects are similar to that derived in Section 5, implying that endogenizing debt face value will not change the key insight of our paper.

To keep the model simple, we assume that there is one big competitive creditor. Assuming one big competitive creditor is equivalent to assuming two big creditors who are in a Bertrand competition; it is also equivalent to assuming that there are infinitely many small creditors who can freely enter the debt market. Assume that the creditor, the firm, and the CRA observe the same public signal $\theta_s = \theta + u$, where $u \sim \mathcal{N}(0, \alpha^{-1})$. Hence, without the credit rating, all agents will have a common prior belief about $\theta$: $\theta \sim \mathcal{N}(\theta_s, \alpha^{-1})$.

Denote by $F$ the face value of the debt. Since the creditor is “competitive,” in equilibrium, if the firm finances through the debt market, the creditor must be indifferent given other players’ strategies. In addition, it is obvious that the firm will choose the financing channel with the lower cost; that is, the firm will finance all the $1 by debt, if $F \leq M$, and by the alternative financing channel, if $F > M$.

We first analyze the benchmark case without the CRA. Consider first that the firm does not finance in the debt market. Then, the financial cost will be $M$, if it wants to continue the

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1This modeling choice rules out the direct strategic information transmission from the firm to the creditor, which does not play a role in the mechanism in our core model. For example, if we allow the firm to set the debt face value, there will be signaling effects, since the face value may signal the firm’s knowledge about $\theta$. Second, if we let the creditor to name the face value of the debt, there will be a lemon problem: The fact that the firm accepts such a proposed face value suggests some information about $\theta$ to the creditor, which the creditor should take into account when proposing the face value. Therefore, we consider the model in this appendix the simplest natural information model with endogenous debt face value.
investment. Given this restriction, the firm’s optimal investment choice will be

\[
\begin{cases}
  \text{default at date 1,} & \text{if } \theta < \theta_1; \\
  \text{HR,} & \text{if } \theta \in [\theta_1, \theta_2); \\
  \text{VP,} & \text{if } \theta \in [\theta_2, \infty),
\end{cases}
\]  

where \( \theta_1 = f^{-1}(H - M) \) and \( \theta_2 = f^{-1}\left(\frac{pV - qH}{p - q} - M\right) \).

We also define

\[
\bar{M} = \frac{1}{p [1 - \Phi(\sqrt{\alpha(\theta_2 - \theta_s)})] + q \left[ \Phi(\sqrt{\alpha(\theta_2 - \theta_s)}) - \Phi(\sqrt{\alpha(\theta_1 - \theta_s)}) \right]}.
\]

Note that the denominator of the right-hand side of equation (45) is just the probability that the firm will not default.

We then analyze the firm’s financial cost if it finances in the debt market. There are two cases. The first case is when \( \bar{M} > M \). This may occur when the prior is sufficiently pessimistic (\( \theta_s \) is sufficiently small). In such a case, there is a trivial equilibrium, in which the creditor does not buy the debt, the firm finances only through the alternative financing channel (if continuing to invest), and the firm’s investment strategy is as described in equation (44).

It is then interesting to analyze whether there is a nontrivial equilibrium in which the firm finances through the debt market. We first establish the lower bound of an equilibrium debt face value, \( F \). Consider the firm’s investment cutoffs when the debt face value is \( F = 0 \). Then, the firm will default early if \( \theta < \theta_1 \), will invest in HR if \( \theta \in [\theta_1, \theta_2) \), and will invest in VP is \( \theta \geq \theta_2 \). Here, \( \theta_1 \) and \( \theta_2 \) are determined by

\[
\begin{align*}
  f(\theta_1) &= H, \\  f(\theta_2) &= \frac{pV - qH}{p - q}.
\end{align*}
\]

Then the smallest possible debt face value, \( F \), will be the one when creditors believe that the firm’s investment cutoffs are \( \theta_1 \) and \( \theta_2 \):

\[
F = \frac{1}{p [1 - \Phi(\sqrt{\alpha(\theta_2 - \theta_s)})] + q \left[ \Phi(\sqrt{\alpha(\theta_2 - \theta_s)}) - \Phi(\sqrt{\alpha(\theta_1 - \theta_s)}) \right]}.
\]

For any given debt face value \( F \in (\bar{F}, M) \), the firm’s investment cutoffs \( \theta_1 \) and \( \theta_2 \) are determined by

\[
\begin{align*}
  f(\theta_1) + F &= H, \\  f(\theta_2) + F &= \frac{pV - qH}{p - q}.
\end{align*}
\]
Substituting the solutions \((\theta_1, \theta_2)\) to the equation system \((49)\) and \((50)\) into the creditors’ indifference condition, we can define a function \(\tilde{T}(F)\):

\[
\tilde{T}(F) = \frac{1}{p \left[1 - \Phi\left(\sqrt{\alpha}(\theta_2 - \theta_s)\right)\right] + q \left[\Phi\left(\sqrt{\alpha}(\theta_2 - \theta_s)\right) - \Phi\left(\sqrt{\alpha}(\theta_1 - \theta_s)\right)\right]}.
\]

(51)

Therefore, there exists an equilibrium in which the firm finances through the debt market if and only if there exists a fixed-point \(\tilde{F} \in (E, M)\) such that \(\tilde{T}(\tilde{F}) = \tilde{F}\).

Obviously, \(\tilde{T}(F)\) is continuous over \((E, M)\); simple algebra will show that \(\tilde{T}(F)\) is strictly increasing; finally, \(\tilde{T}(E) > E\) and \(\tilde{T}(M) = \bar{M} > M\). Therefore, generically, either there is no \(\tilde{F} \in (E, M)\) such that \(T(\tilde{F}) = \tilde{F}\), or there are at least two \(\tilde{F}\)'s such that \(T(\tilde{F}) = \tilde{F}\). That is, either the model has a unique equilibrium in which the firm finances through the alternative financing channel, or there are multiple equilibria in which the firm finances through the debt market.

Consider now the case where the CRA is assigning the rating to the firm. Similarly to Lemma 1, with equation (17), the CRA will assign the rating \(p\) to the firm if \(\theta \geq \theta_1\); otherwise, it will assign the rating 0 to the firm. Therefore, following the rating \(p\), the creditors will correctly form the posterior belief that \(\theta\) is normally distributed with the mean \(\theta_s\) and the precision \(\alpha\), but truncated from below by \(\theta_1\).

Given a financial cost \(F\) that is smaller than \(M\) (otherwise, the firm will finance through the alternative channel), the CRA will set the threshold of its rating strategy \((\theta_1)\) such that

\[
f(\theta_1) + F = H.
\]

(52)

Also, given \(F\), the firm will invest in \(VP\), if and only if \(\theta \geq \theta_2\), and so the firm’s indifference condition is

\[
f(\theta_2) + F = \frac{pV - qH}{p - q}.
\]

(53)

Hence, the investor’s indifference condition is

\[
\Phi\left[\sqrt{\alpha}(\theta_2 - \theta_s)\right] - \Phi\left[\sqrt{\alpha}(\theta_1 - \theta_s)\right] qF + \frac{1 - \Phi\left[\sqrt{\alpha}(\theta_2 - \theta_s)\right]}{1 - \Phi\left[\sqrt{\alpha}(\theta_1 - \theta_s)\right]} \cdot pF = 1,
\]

which implies a function of \(F\) defined as

\[
T(F) = \frac{1 - \Phi\left[\sqrt{\alpha}(\theta_1 - \theta_s)\right]}{q \left\{\Phi\left[\sqrt{\alpha}(\theta_2 - \theta_s)\right] - \Phi\left[\sqrt{\alpha}(\theta_1 - \theta_s)\right]\right\} + p \left\{1 - \Phi\left[\sqrt{\alpha}(\theta_2 - \theta_s)\right]\right\}}
\]

\[
= \frac{1 - \Phi\left[\sqrt{\alpha}(\theta_1 - \theta_s)\right]}{p - (p - q) \Phi\left[\sqrt{\alpha}(\theta_2 - \theta_s)\right] - q \Phi\left[\sqrt{\alpha}(\theta_1 - \theta_s)\right]}.
\]

(54)
Then, with the CRA, there exists an equilibrium in which the firm finances through the debt market if and only if there is \(F^* \in (E, M)\) such that \(T(F^*) = F^*\).

We then differentiate \(T(F)\), and the derivative is

\[
\frac{dT}{dF} = D \left\{ \left[ p \left( 1 - \Phi\left( \sqrt{\alpha} (\theta_2 - \theta_3) \right) \right) + q \left( \Phi\left( \sqrt{\alpha} (\theta_2 - \theta_3) \right) - \Phi\left( \sqrt{\alpha} (\theta_1 - \theta_3) \right) \right) \right] \frac{1}{f'(\theta_1^*)} \\
- \left( p - q \right) \left( 1 - \Phi\left( \sqrt{\alpha} (\theta_2 - \theta_3) \right) \right) \frac{1}{f'(\theta_2^*)} - q \left( 1 - \Phi\left( \sqrt{\alpha} (\theta_1 - \theta_3) \right) \right) \frac{1}{f'(\theta_1^*)} \right\} \\
= -\left( p - q \right) D \frac{\sqrt{\alpha} \Phi\left( \sqrt{\alpha} (\theta_s - \theta_1^*) \right)}{f'(\theta_1^*)} \\
\left\{ \frac{\Phi\left( \sqrt{\alpha} (\theta_2^* - \theta_3) \right) f'(\theta_1^*)}{\Phi\left( \sqrt{\alpha} (\theta_1^* - \theta_3) \right) f'(\theta_2^*)} \Phi\left( \sqrt{\alpha} (\theta_s - \theta_1^*) \right) - \Phi\left( \sqrt{\alpha} (\theta_s - \theta_2^*) \right) \right\},
\]

(55)

where

\[
D = \frac{1}{\left\{ q \left\{ \Phi\left( \sqrt{\alpha} (\theta_2^* - \theta_3) \right) - \Phi\left( \sqrt{\alpha} (\theta_1^* - \theta_3) \right) \right\} + p \left\{ 1 - \Phi\left( \sqrt{\alpha} (\theta_2^* - \theta_3) \right) \right\} \right\}^2} > 0.
\]

Hence, when \(\alpha\) is sufficiently small, \(dT/dF > 0\). When \(\alpha\) is sufficiently large, the sign of \(dT/dF\) will depend on the functional form of the function \(f(\cdot)\). In particular, we show that if \(f(\theta) = e^{-\theta}\), \(dT/dF\) is also strictly positive.

Hence, similarly to that in the model without the CRA, if \(T(M) > M\) (which occurs when \(\theta_s\) is sufficiently small), there is always an equilibrium in which the creditor does not buy the debt (so the firm will finance through the alternative channel if it continues to invest), the CRA’s rating strategy will be characterized by \(\bar{\theta}_1\) (assigning \(R = 0\) if \(\theta < \bar{\theta}_1\) and assigning \(R = p\) if \(\theta \geq \bar{\theta}_1\)), and the firm’s investment strategy is characterized by equation (44). In addition, the monotonicity of \(T(\cdot)\) implies that \(T(E) > E\) because \(T(0) = E\). Hence, the equilibrium characterization would be similar to the case without the CRA.

From the two sets of equilibrium conditions (equations (49), (50), and (51) for an equilibrium without the CRA and equations (52), (53), and (54) for an equilibrium with the CRA), we find that the model is rather intractable. Therefore, we analyze the model numerically below.

Figure 5 below shows the case that with or without the CRA, there is a unique equilibrium in which the firm does not finance through the debt market. In Figure 5, the yellow straight line is the “45° line.” The blue curve and the red curve are the self-mappings \(\hat{T}(F)\) (without the CRA) and \(T(F)\) (with the CRA), respectively. As we argued, if and only if the self-mappings have intersections with the “45° line,” the model has equilibria in which the firm finances through the debt market. Obviously, Figure 5 shows the case that with or without the CRA, there is
no equilibrium in which the firm finances through the debt market. Hence, the model has a unique equilibrium in which the firm finances through the alternative financing channel and has the financial cost $M$.

**Figure 5: Unique Equilibrium**
Figure 6: Unique Equilibrium without the CRA but Multiple Equilibria with the CRA

Figure 6 shows the case that without the CRA, the model has a unique equilibrium in which the firm does not finance through the debt market. By contrast, with the CRA, there are two equilibria in which the CRA finances through the debt market (the red curve now has two intersections with the 45° line).

Finally, Figure 7 presents the case where with or without the CRA, the model has multiple equilibria. In particular, for the equilibrium with a higher financial cost, the firm’s financial cost is even higher with the CRA than that without the CRA. This seems inconsistent with the empirical regularity.
The equilibrium multiplicity arises from the self-fulfilling property. The intuition is similar to that in the literature on debt market and endogenous credit quality, (see, for example, Cole and Kehoe (2000)). Indeed, the CRA cannot commit to a rating policy, and the firm cannot commit to an investment strategy. Hence, the creditor best responds to her belief about the CRA’s cutoff and the firm’s cutoff. When these two cutoffs increase, the creditor’s belief that the firm will default is higher, and hence the debt face value must increase so that the creditor can be indifferent. The higher debt face value will in turn increase the firm’s financial cost, which will lead to higher cutoffs. Therefore, the model may have multiple equilibria: When the creditor is more pessimistic in the interim, the firm has a higher financial cost in equilibrium, and when the creditor is more optimistic in the interim, the firm has a lower financial cost in equilibrium.

The second case is when $\bar{M} < M$. This occurs when $\theta_s$ is sufficiently large. In such a case, with or without the CRA, the firm finances through the emergency financing channel is not part of an equilibrium. This is because given the financial cost $M$, the creditors would like to invest in the debt with an even lower financial cost $\bar{M}$, which causes the firm to finance through the debt market. Figure 8 illustrates this case. Based on the graph, it is likely that there is a unique equilibrium in this case, and the firm borrows in the debt market in equilibrium.
Figure 8: Equilibrium with an Optimistic Prior

Importantly, Figure 8 also shows $\tilde{F} > F^*$, implying that with the CRA, the firm will have a lower debt face value and thus a lower financial cost.

Proposition 8 In the model with one big competitive creditor who observes a public signal $\theta_s = \theta + u$, there is a $\tilde{\theta}_s$, which is the solution to $\bar{M} = M$, such that

1. when $\theta_s < \tilde{\theta}_s$, the model has an equilibrium in which the creditor does not buy the debt; there is either no other equilibrium, or multiple equilibria in which the creditor buys the debt; and

2. when $\theta_s > \tilde{\theta}_s$, the model has a unique equilibrium, in which the creditor buys the debt.

Despite the fact that a nontrivial equilibrium in which the firm borrows from the debt market may not exist in the model (illustrated in Figure 5 and Figure 6), and the fact that even if a nontrivial equilibrium exists, the model may have multiple nontrivial equilibria (illustrated in Figure 7), we still analyze the CRA’s real effects in this model. In particular, we focus on the equilibrium in which the firm borrows from the creditor at the lowest equilibrium debt face value. Both Figure 7 and Figure 8 show that the CRA helps lower the firm’s financial cost ($\tilde{F} > F^*$). It then follows from equations (52) and (53) that $\tilde{\theta}_1 < \theta^*_1$ and $\tilde{\theta}_2 < \theta^*_2$. That is, in the equilibrium with the firm borrowing from the creditor (if it exists), the CRA’s real effects are similar to that derived in Section 5.