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Credit Rating Inflation and Firms' Investments

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ABSTRACT

We analyze credit rating effects on firm investments in a rational bond financing game that features a feedback loop. The credit rating agency (CRA) inflates the rating, providing a biased but informative signal to creditors. Creditors' response to the rating affects the firm's investment decision and thus its credit quality, which is reflected in the rating. The CRA might reduce ex ante economic efficiency, which results solely from its strategic effect: the CRA assigns more firms high ratings and allows them to gamble for resurrection. We derive empirical predictions on the determinants of rating standards and inflation and discuss policy implications.

CREDIT RATING AGENCIES (CRAS) HAVE BEEN criticized for playing a central role in financial failures, such as the collapse of Enron and WorldCom in 2002 and the 2007–2009 financial crisis that led the Financial Crisis Inquiry Report to conclude that "the failures of CRAs were essential cogs in the wheel of financial destruction." Critics claim that CRAs assign overgenerous ratings, and several empirical studies find support for this view.[1] These studies argue that the documented credit rating inflation may be due to conflicts of interest arising from the use of an "issuer-pays" business model, whereby CRAs are

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[1] See, for example, Jiang, Stanford, and Xie (2012), Strobl and Xia (2012), and Cornaggia and Cornaggia (2013).

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paid by the issuers they are assessing. The concern is that by misleading creditors, inflated credit ratings help risky investments get funded and as a result have negative real effects.

Thinking about these arguments through the lens of models with rational creditors, it is not clear why inflated credit ratings would have negative real effects. Credit ratings must provide creditors with some valuable information, as otherwise the ratings would be ignored and CRAs would have no effect. But if CRAs provide informative (though potentially biased) signals, they should be able to increase, rather than decrease, economic efficiency, even if they do not lead to the first-best outcome. The question then is whether CRAs with a motive to inflate ratings can have negative effects on economic efficiency in a world with rational creditors.

In this paper, we develop a model to analyze this question. We show that in equilibrium, a CRA with an incentive to inflate ratings pools firms with economic fundamentals above a threshold and assigns them a rating indicative of high credit quality, despite the fact that some of them will choose risky projects and thus have low credit quality. Accordingly, the high credit rating is inflated but informative. High-rating firms enjoy lower financial costs. However, given lower financial costs, some firms that would have defaulted efficiently without the CRA will gamble for resurrection, leading to lower economic efficiency.[2]

Our model is parsimonious but rich enough to capture the essential factors of the interactions between a CRA, creditors, and an issuing firm. First, we consider a CRA that, by assigning a higher rating, earns higher revenue but incurs a higher cost if the firm fails. In particular, if the CRA assigns a high rating to a firm that has extremely bad economic fundamentals and that will default immediately despite the high rating, the CRA will incur an extremely high cost. This high cost in the event of firm failure prevents the CRA from assigning high ratings to firms with extremely bad economic fundamentals and thereby imposes a restriction on the CRA's rating strategy. We refer to this restriction as the *partial verifiability constraint*. Given this constraint, the CRA's rating, while perhaps biased, contains valuable information. Second, the credit rating is used by rational creditors who have dispersed beliefs about a firm's economic fundamentals and decide whether to buy bonds issued by the firm based on their private information and the CRA's credit rating. Third, the creditors' decisions together affect the firm's cost of capital and in turn affect the firm's investment decision, which determines the firm's credit quality. Fourth, when setting the rating, the CRA accounts for the effects of the rating on the decisions of creditors and the firm and the effects of these decisions on the firm's credit quality. As a result, there is a feedback loop whereby the rating affects creditors' behavior, which affects the behavior of the issuer and its credit quality, which is reflected in turn in the rating.

[2] For some model parameters, the CRA may have an incentive to "deflate" ratings. In such a case, it labels firms whose economic fundamentals are above the threshold with a rating that suggests low credit quality, but the effects on economic efficiency are the same as in the case with an incentive to inflate ratings. We focus on the case of inflation rather than deflation because inflation is the phenomenon of interest both empirically and in policy circles.

In our view, CRAs' strategic behavior in the feedback loop is central to understanding the effects of CRAs. Given their market power, CRAs are in a unique position to provide information that affects firm investment decisions and thus firm credit quality. Indeed, CRAs claim that their ratings are forward-looking, emphasizing that they are based on the potential impact of foreseeable future events, which include the effects of the ratings themselves. For example, in a document that explains its rating process, Moody's explicitly acknowledges "the effect of the rating action on the issuer, including the possible effect on issuer's market access or conditional obligations," and that "the level of rating that Moody's assigns to an issuer that might experience potential changes in market access or conditional obligations will reflect Moody's assessment of the issuer's creditworthiness, including such considerations." Another important component of the feedback loop is the endogenous firm investment, which determines firm credit quality. The firm's investment choice thus represents a moral hazard problem in the model. In the Internet Appendix we show that if firm credit quality is exogenous, the information provided by credit ratings always increases economic efficiency.[3]

In our model, a high rating, even though potentially inflated, provides positive information to creditors because it implies that the firm does not belong to the group of particularly low-quality firms, for which the partial verifiability constraint binds.[4] Hence, a high rating makes creditors more optimistic about the firm and more likely to invest in the firm's bonds, which reduces the firm's financial costs and impacts its investment decisions. This is how the ratings end up having real effects. For those firms for which financial costs are relatively high, the reduction in financial costs leads to inefficient risk-taking, as lower financial costs allow them to gamble for resurrection and pursue investments with low expected returns but high potential upside. For firms for which financial costs are relatively low, the reduction in financial costs provides more skin in the game, which encourages a shift from high-risk, low-expected-return investments to low-risk, high-expected-return investments. The implications for economic efficiency are negative in the first case and positive in the second. Hence, the overall effect of the CRA on economic efficiency depends on the relative strength of these opposing effects. This depends in turn on the model parameters.

Varying the parameters of the model, we show that the overall expected real effects of the CRA can be positive or negative. A key result is that the CRA's expected real effect is more likely to be negative when the upside of the risky inefficient investment is high, as a high upside makes gambling for resurrection more attractive and more likely to follow from a reduction in the cost of capital.

[3] The Internet Appendix may be found in the online version of this article.

[4] Hence, a high credit rating generated by a lax rating strategy is not cheap talk as in Crawford and Sobel (1982). Due to the partial verifiability constraint, the high rating provides creditors with a public signal about the firm. The public signal is endogenous and takes a different form from that in Morris and Shin (2002). In particular, it truncates the supports of creditors' interim beliefs from below.

Another important insight of our model emerges when we decompose the CRA's ex ante real effects into two components, namely, an *informational effect* and a *strategic effect*. The informational effect is the effect that obtains when the CRA does not incorporate the effect of its rating on the firm's investment and credit quality, that is, when the CRA provides the (biased) information that pertains to the equilibrium without a CRA. The strategic effect is the additional effect that arises from the CRA behaving strategically and taking into account the effect of its ratings on creditors' and the firm's decisions, in which case its rating incorporates these responses in maximizing its expected profit.

We show that the informational effect always increases economic efficiency. When the CRA acts in a reflecting way, simply providing (biased) information to creditors, it helps achieve a more efficient outcome. The negative implications for economic efficiency thus come purely from the strategic effect: When taking creditors' and firm's responses to the ratings into account, the CRA finds it beneficial to assign high ratings to more firms, allowing them to gamble for resurrection. When they gamble for resurrection, the CRA can assign them a high rating without violating the partial verifiability constraint, and thereby achieve higher values of its objective function at the expense of lower economic efficiency. The CRA essentially uses its market power in providing information to shape economic outcomes, and because of the inflation motive, economic efficiency might be sacrificed. This result highlights that the introduction of strategic effects into models of credit ratings is crucial for understanding the overall effects of CRAs.

The model leads to several empirical implications. First, a key insight that emerges from our analysis is that lax rating standards and rating inflation are two distinct endogenous terms that do not necessarily move in the same direction. A laxer rating standard corresponds to a rating strategy whereby the CRA is more likely to assign a high rating to firms with a given set of fundamentals. However, this does not necessarily imply higher rating inflation, which is defined as a larger measure of firms with low credit quality being assigned a high rating in equilibrium. The reason relates to the CRA's real effects: When the CRA changes its rating policy, it also affects the firm's credit quality, and so inflation, which is the difference between reported credit quality and actual credit quality, could change in either direction. Future empirical work should take this observation into account. Second, our comparative statics analyses, which support this view, deliver new empirical predictions about CRAs' credit rating standards and credit rating inflation. In particular, a decrease in firm transparency, an increase in the upside returns of risky projects, and an increase in market liquidity will lead to laxer rating standards (i.e., assigning high ratings to more firms). However, such changes in economic environment will not necessarily lead to higher rating inflation. Specifically, a decrease in firm transparency has an ambiguous effect on rating inflation, an increase in the upside returns of risky projects will lead to higher rating inflation, and an increase in market liquidity will lead to lower rating inflation.

Our analysis also has policy implications. A rating agency's equilibrium rating strategy depends on the ratio of its incremental revenue to incremental potential cost due to a rating upgrade. When this ratio is high, the rating agency will inflate ratings, which generates the inefficiency highlighted in our paper. However, it is not the case that an effective policy should just simply reduce this ratio (e.g., by making the incremental cost extremely large), because the rating agency would then have an incentive to deflate the rating, which can end up leading to the same efficiency implications as when the ratio is large and the rating agency inflates ratings. Therefore, to get to a truth-telling CRA in equilibrium, a policymaker should target a ratio of the incremental revenue to the incremental cost due to a rating upgrade such that it falls within a particular range. Unfortunately, this range might be difficult for policymakers to calibrate.

An additional question that arises concerning credit rating real effects is whether they should be expected to persist in a framework in which various signals besides credit ratings are available to creditors in the corporate bond market. We argue that even if other public signals exist, the CRA's real effects are still significant given creditors' heterogeneous private signals about a firm's economic fundamentals. Creditors' heterogeneous private signals imply that they hold dispersed beliefs. Hence, a high credit rating, by truncating creditor belief supports, will affect some creditors' beliefs about the firm's investment choice. As a result, the credit rating affects the firm's financial cost, investment decision, and credit quality, which in turn affects other creditors' behavior (even though credit ratings have little direct effect on their beliefs), leading to the significant real effects of the CRA. We show that dispersion in beliefs among creditors is crucial for the rating agency to play a role by demonstrating that the rating agency's effects become negligible when creditors have precise homogeneous information.

The real effects of CRAs have been documented empirically. For example, Sufi (2009) finds that introducing bank loan ratings increases firms' asset growth, cash acquisitions, and investment in working capital; Bannier, Hirsch, and Wiemann (2012) show that firms reduce (raise) their investment rates around negative (positive) rating events; and Almeida et al. (2017) find that firms reduce their investment due to an increased cost of debt capital following a sovereign rating downgrade. While such real effects are largely absent in theoretical models of credit ratings, several previous papers introduce different forms of feedback, in particular Boot, Milbourn, and Schmeits (2006), Manso (2013), Goel and Thakor (2015), and Holden, Natvik, and Vigier (2018). A key difference between our study and these previous papers is that in our model the real effect is a result of information transmission from the rating agency to creditors, whereas in these papers it is a result of changing the focal point for equilibrium selection, contractual features that affect the firm when the rating changes, or the CRA's incentives to balance the issuer's payoff and social welfare. While these are interesting dimensions to explore, the informational role of the rating is fundamental, going back to the basic motivation for introducing ratings to begin with. Another key difference is that in these other papers, there is no or limited rating inflation and the CRA seeks to provide accurate ratings. Our research question, in contrast, centers on the positive and negative real effects of a CRA with an inflation motive. As we show, these effects are driven by information transmission and would not arise under the frameworks considered in the other feedback papers.

Another paper that explores CRAs' real effects is Daley, Green, and Vanasco (2020), who find that the availability of credit ratings can reduce bank lending standards and lead to an oversupply of credit, but in most cases total welfare increases with rating accuracy. In their paper, credit ratings are modeled as exogenous public signals, and thus CRAs' incentives do not play a role in their real effects. Similarly, Donaldson and Piacentino (2018) and Parlour and Rajan (2020) view credit ratings as exogenous public signals and investigate credit rating real effects through contracting. Our model differs from these papers in that credit ratings are endogenously determined and the CRA has incentives to inflate ratings. Hence, rational creditors need to account for the CRA's incentives to understand the information content of credit ratings.

Several theoretical papers also study rating inflation. These papers typically attribute credit rating inflation to creditors' imperfect rationality (Skreta and Veldkamp (2009); Bolton, Freixas, and Shapiro (2012)) or to regulations tied to ratings (Opp, Opp, and Harris (2013)).[5] Hence, in these models, inflated credit ratings are not informative signals for creditors, as creditors are naïve or have regulatory motives. Again, while we think that bounded rationality and regulatory constraints are important, our aim is to analyze the role of the CRA in a rational environment.

We model the firm's credit market as a global game that features dispersed beliefs. This modeling choice allows for equilibrium uniqueness, which simplifies comparative statics analysis on the real effects of credit ratings. Although the global game is not essential in this paper, our paper contributes to the literature on global games. Our model differs from traditional global game models (Carlsson and van Damme (1993); Morris and Shin (2003)) in the endogenous information provided by the CRA to creditors. Dispersed beliefs give the information provided by the CRA relevance even when creditors are very well informed. Several papers endogenize the information structure in a global game setting in different ways. Angeletos, Hellwig, and Pavan (2006) and Angeletos and Pavan (2013) model the signaling effects of the government's preemptive defensive policies, which pools very strong governments and very weak governments together, and Edmond (2013) considers a dictator's costly private information manipulation, where all revolutionaries' interim beliefs have full support. The belief updating in these models therefore differs from that in our model. Indeed, the belief updating in our model is closer to that in Angeletos, Hellwig, and Pavan (2007) and Huang (2017). Nevertheless, our model has a unique equilibrium because the CRA's incentives to inflate credit ratings

[5] One exception is Frenkel (2015), who shows that credit rating inflation may be generated by CRAs' "double reputation." However, one necessary condition in Frenkel (2015) is that CRAs have different possible behavioral types (in his model, "honest" or "corrupt"). In contrast, in our paper the conflicts of interest caused by the issuer-pays business model are commonly known by all creditors. This may be a better description of the credit market, especially given what happened in the subprime crisis.

generate new dominant regions of not investing. Our model is also related to Bouvard, Chaigneau, and De Motta (2015), who examine how the government chooses public signals (without commitment) to shape bank depositors' posterior beliefs. Our model differs from theirs mainly in that the firm's investment choices and in turn credit quality are affected by credit ratings assigned by the CRA, and the CRA takes such effects into account to strategically choose the optimal rating strategy.

Finally, our paper is related to Goldstein and Huang (2016), who show how the government persuades investors not to attack a regime by committing to abandon the regime when it is below some cutoff level. Our current model differs in several ways. For instance, in our model the CRA cannot commit to a rating strategy, and the firm has moral hazard problems that interact with the rating policy. As a result, unlike the government in Goldstein and Huang (2016), the CRA in our paper may have negative ex ante real effects. These features also make our model different from those in the literature on Bayesian persuasion, such as Kamenica and Gentzkow (2011). On the one hand, the CRA in our model cannot commit to a rating rule, which is consistent with the empirical regularity but leads to different results. For example, when the CRA cannot commit, some rating is not assigned in equilibrium, and the equilibrium rating must be monotonic in the firm's fundamentals. On the other hand, senders in the literature on Bayesian persuasion usually disclose information about exogenous variables, while the CRA in our model discloses information about endogenous firm credit quality, which is affected in turn by its disclosure.

The rest of this paper is organized as follows. In Section I, we describe our model of corporate credit ratings. Section II establishes a benchmark without a CRA. In Section III, we analyze credit ratings' informativeness and the CRA's equilibrium rating strategy. Section IV compares the firm's equilibrium investments in the model with the CRA to those in the benchmark model without a CRA to derive the CRA's real effects. It then further studies the CRA's informational effects and strategic effects. Section V presents some empirical implications, and Section VI discusses the assumption of dispersed information. Section VII concludes. All proofs appear in the Appendix.

I. A Model of Corporate Credit Ratings

We study a model of a CRA that assigns a credit rating to a firm. There are three dates, $t = 0, 1, 2$. At the beginning of date 0, the firm needs to make a payment of \$1 for current liabilities such as unpaid wages. To finance the \$1, the firm can issue bonds at relatively low cost or borrow through an alternative financing channel at relatively high cost. One example of the alternative financing channel is a predetermined bank credit line, which we focus on in this paper for ease of exposition.

After the firm makes its current liabilities payment, the firm invests in a new project. Independent of the investment choice, the firm incurs an operation cost. The operation cost can be thought of the wages payable to employees working on the new investment project. The operation cost is determined by the firm's fundamentals $\theta \in \mathbb{R}$ and is paid at date 2 if the firm does not default. We assume that all agents have a common improper uniform prior over θ, and that the firm does not learn θ until after the CRA has assigned its credit rating.

At date 0, the CRA assigns a credit rating to the firm. Upon observing the rating, a continuum of creditors in the bond market decide whether to buy the firm's bonds. At date 1, based on the financial cost and its private knowledge about its economic fundamentals, the firm chooses whether to default or to continue investing. In the latter case, the cash flow is realized at date 2, and, if possible, the firm repays the new bonds as well as its operation cost in full.

We view the model as a model of issuer credit ratings. In practice, CRAs assign credit ratings to both "issuers" and "issues." While we focus on firm-level credit ratings, in the model they are the same as bond-level credit ratings because the firm has only one bond issue in the model. These ratings differ, however, from credit ratings assigned to structured finance products such as mortgage-backed securities, as the real effects of corporate credit ratings are more salient.

A. Firm Investment

Following Boot, Milbourn, and Schmeits (2006), we assume that if the firm fully repays its current liabilities, it can invest in either a low-risk "viable" project (VP) or a high-risk project (HR) at date 1. VP generates a cash flow of $V > 0$ with probability $p \in (0, 1)$ but fails with probability $1 - p$. Similarly, HR generates a cash flow of $H > V$ with probability $q \in (0, p)$ but fails with probability $1 - q$. The firm receives zero cash flow if the project fails. Since both VP and HR fail with positive probabilities, the firm's investment choice between VP and HR is unobservable and unverifiable.[6]

At date 1, instead of investing in VP or HR, the firm may choose to default. In such a case, the firm will not withdraw from its credit line and its liquidation value is $L \in (0, 1)$. We assume that the liquidation value and the funds from the newly issued bonds are used to repay the current liabilities, since employees usually have higher repayment priority than unsecured creditors when a firm goes bankrupt. If the firm defaults at date 1, the game ends and thus its early default decision is publicly observable and verifiable.

We assume that the expected cash flow generated by VP is greater than one, but HR is unlikely to generate a positive cash flow (q is sufficiently small). Specifically, we assume that

$$pV > 1 > L > qH. \tag{1}$$

This assumption is important when we rank the firm's investments according to their efficiency implications.

[6] In practice, creditors may know the name of the project that the firm invests in, but they usually lack the professional knowledge to judge whether the project is VP or HR. Therefore, the choice between VP and HR is unverifiable even ex post.

B. Financing

There is a continuum of creditors with measure $1 - \gamma$ in the bond market, each having \$1. Here, γ captures the liquidity of the bond market, with a larger γ indicating a lower liquidity level. We assume that $\gamma \in (L, 1)$, so that even if all creditors buy the firm's bonds, without withdrawing from the bank credit line, the firm still cannot fully repay its current liabilities.[7]

The bonds are zero-coupon bonds with face value $F > 1$. They mature at date 2. So long as the firm does not default either endogenously at date 1 or exogenously at date 2, the creditors who buy the new bonds will be fully repaid. Here, to focus on the role of the CRA, we follow He and Xiong (2012) and assume that the bond face value, F, is exogenously given. This assumption does not change our insights about the real effects of credit ratings. Indeed, the key mechanism through which credit ratings affect the firm's investment decisions is through their effects on the firm's total cost of financing, rather than merely through the bond face value. In our model, credit ratings affect the firm's total financing cost (by affecting the measure of creditors who invest in the bonds), even if the bond face value is exogenous.

To examine the robustness of our results, we study models with endogenous bond face value. We find that a model with dispersed beliefs among multiple creditors is rather intractable, since creditors' learning about firm fundamentals from the endogenous bond face value cannot be explicitly analyzed. However, we obtain some tractability in a model with a competitive creditor. We present some equilibrium properties of such a model in the Internet Appendix. We find that if we focus on the equilibrium in which the firm borrows from the creditor at the lowest equilibrium bond face value, a high rating reduces the firm's financial cost by reducing the bond face value. As a consequence, the CRA's real effects are similar to those in our core model. This alternative model is still less tractable and elegant than our core model, so we proceed with our core model in which the face value of debt is fixed.

We assume that $pF > 1$, and thus if any creditor i knows that the firm will invest in VP, he will buy the firm's bonds. In contrast, the probability that HR is successful is so low ($qF < 1$) that creditor i will not buy the bonds if he knows that the firm will surely invest in HR. Creditor i will also not buy the bonds if he knows that the firm will default early. We denote by $a_i \in \{0, 1\}$ creditor i's bond-investment decision, where $a_i = 1$ indicates that creditor i buys the bonds and $a_i = 0$ indicates that creditor i does not buy.

The firm can withdraw up to \$1 from the credit line at the constant marginal cost $M > F$. The firm therefore wants to finance more from the bond market to lower its financial cost. However, we assume that even if no creditor buys the firm's bonds, the firm is still willing to withdraw \$1 from the bank credit line to pay its current liabilities and invest in VP, when it has a sufficiently

[7] This assumption is for simplicity. Under this assumption, when the firm defaults at date 1, the largest possible amount of funds available is $1 - \gamma + L$, which is less than \$1. Hence, any creditor who buys the bonds will get nothing, which implies global strategic complementarities among creditors.

low operation cost; formally, we assume that $M < \frac{pV-qH}{p-q}$. We denote by W the measure of creditors who buy the bonds. It follows that the firm needs to finance $1-W$ from the bank credit line. The firm's financial cost is then given by $WF + (1-W)M$.

The operation cost of a new investment is $f(\theta)$. We assume that the function $f(\cdot)$ is differentiable, strictly decreasing, and strictly convex. When the firm's economic fundamentals are extremely good, the operation cost is extremely low, that is, $\lim_{\theta\to+\infty} f(\theta) = 0$. However, when the firm's fundamentals are extremely bad, it will incur an unbounded operation cost for a new investment, so $\lim_{\theta\to-\infty} f(\theta) = +\infty$. Therefore, if the firm decides to invest in either VP or HR, its total cost at date 2 is

$$K(\theta) = f(\theta) + WF + (1-W)M. \tag{2}$$

Importantly, the measure of creditors who buy the bonds (W) is endogenously determined and is a function of θ and the credit rating in equilibrium. In addition, the firm's economic fundamentals, θ, do not directly affect creditor payoffs. Creditors care about θ only because it indirectly affects the firm's project choice and in turn the firm's default probability.

C. Firm's Payoff

The firm has limited liability. If it defaults, whether endogenously at date 1 or exogenously at date 2 (when the project fails), its payoff is zero. If the firm generates a positive cash flow at date 2, the firm needs to repay the bonds. The firm's payoff, U, therefore depends on its own investment choice, its operation cost, and its financial cost:

$$U = \begin{cases} 0, & \text{if the firm defaults at date 1,} \\ p\left[V - \left(f(\theta) + WF + (1-W)M\right)\right], & \text{if the firm invests in VP,} \\ q\left[H - \left(f(\theta) + WF + (1-W)M\right)\right], & \text{if the firm invests in HR.} \end{cases} \tag{3}$$

D. Information Structure

The firm's economic fundamentals, θ, are its own private information that remains unknown to creditors. We assume that the firm learns θ after it issues bonds, so the fact that the firm issues bonds is not informative about θ for creditors. Before deciding whether to buy a bond, each creditor i observes a private signal $x_i = \theta + \xi_i$, where $\xi_i \sim \mathcal{N}(0, \beta^{-1})$ is independent of θ and independent across all creditors. Since we aim to explore credit rating effects on rational, well-informed creditors, we focus on the case in which β is sufficiently large. In addition to their private signals, creditors observe a public credit rating by a CRA.

E. Credit Rating Agency

The CRA assigns the firm a credit rating, R. Following Boot, Milbourn, and Schmeits (2006), we restrict the space of ratings to $\{0, q, p\}$, because these are the only possible credit qualities of the firm: early default at date 1 means that the firm will certainly default and thus the firm's credit quality is zero, the firm investing in HR has credit quality q, and the firm investing in VP has credit quality p. This assumption is without loss of generality. In our model, the CRA cannot commit to a rating rule, and it will take into account rating effects on the firm's credit quality when assigning ratings. Thus, even if the CRA is allowed to assign ratings in a flexible space, the number of effective ratings cannot be strictly more than three in equilibrium. For example, if the CRA is announcing θ' directly, it will announce the highest θ' that will lead to the same credit quality of the firm.

We assume that the CRA knows θ perfectly.[8] In addition, we consider pure strategies. Hence, the CRA can perfectly predict the firm's choice and its corresponding default probability at date 0. Our model captures an important feature of credit ratings—they are forward-looking, that is, the CRA takes into account the effects of the credit rating it assigns on the firm's choice and success.

We denote by V^R the CRA's rating revenue and by C^R its potential rating cost when it assigns a rating, R. The CRA's expected payoff by assigning the rating R is thus

$$V^R - \mathbb{E}(C^R). \tag{4}$$

Importantly, both the rating revenue and the potential rating cost vary with the rating. We specify detailed assumptions about V^R and C^R in Section IIIA.

F. Economic Effects

We seek to analyze the effects of the CRA on economic efficiency, effects that are captured by the difference between the sums of all agents' ex ante payoffs (except the CRA) with and without the CRA. Ultimately, the effective economic efficiency is ranked by the firm's expected revenue, which is determined solely by the firm's investment decision. This follows from the fact that the ex post repayments are all transfers from one group of agents to another, provided that the firm does not default. For example, if the firm invests in HR, and there are measure W of creditors who buy the bonds, then the ex ante payoffs to the firm, creditors, the bank, and employees are $q[H - (f(\theta) + WF + (1 - W)M)]$,

[8] We assume that the CRA knows θ perfectly mainly for tractability. If the CRA observes a noisy signal about θ, then when the noise is sufficiently small, the CRA's incentives rarely change. However, creditors' belief updating becomes rather complicated, making the model intractable. In addition, by assuming that the CRA knows θ perfectly, the inflated rating will arise only from the CRA's incentives, and not from the bias of the CRA's signal.

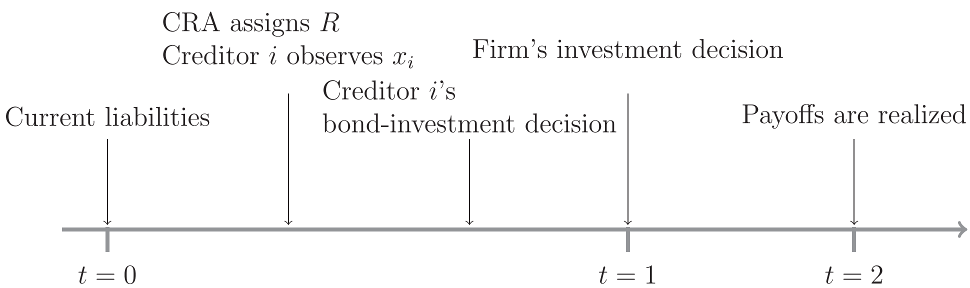

Figure 1. Timing.

$(1 - W) + WqF$, $(1 - W)qM - (1 - W)$, and $qf(\theta)$, respectively. In this case, the sum of ex ante payoffs is qH, which is the firm's expected revenue from investing in HR.

Hence, if the firm invests in VP, the economic efficiency is pV; if the firm invests in HR, the economic efficiency is qH; and if the firm defaults at date 1, the economic efficiency is L. It follows from equation (1) that VP leads to the highest economic efficiency, and an investment in HR leads to even lower economic efficiency than does early default.

G. Timeline and Equilibrium

We summarize the model's timeline in Figure 1. The CRA's rating strategy, denoted by $\mathcal{R}$, maps the firm's fundamentals to the rating space $\{0, q, p\}$; the firm's strategy maps its fundamentals, the CRA's rating, and the measure of creditors investing in the bonds to project choices; and creditors' strategies map their own private signals and the CRA's rating to their bond-investment decisions.

The solution concept of the model is a *monotone perfect Bayesian equilibrium*.

DEFINITION 1: The CRA's rating strategy, the firm's investment strategy, and creditors' bond-investment strategies constitute an equilibrium if:

(1) Given the firm's investment strategy and creditors' bond-investment strategies, the CRA chooses the rating R to maximize its rating profits $V^R - \mathbb{E}(C^R)$ for all $\theta \in \mathbb{R}$,
(2) Given the total repayment at date 2 in equation (2), the firm's investment strategy maximizes the firm's expected profits,
(3) Given the CRA's rating strategy, the firm's investment strategy, and other creditors' strategies, any creditor i's strategy is monotonic in his private signal x_i and maximizes his expected payoff,
(4) Creditors use Bayes' rule to update their beliefs.

II. The Benchmark: No CRA

We first set up a benchmark that excludes the CRA. In such a benchmark, when deciding whether to buy the firm's bonds, creditors' decisions are based solely on their own private information. After observing the measure of creditors who invest in its bonds, the firm makes its investment choice. Such a benchmark is similar to the debt run model of Morris and Shin (2004), with the key difference being that here the firm's choice is not binary.

We first analyze the firm's behavior in this benchmark model. Because of the law of large numbers, given creditors' strategies, the measure of creditors who buy the firm's bonds is a deterministic function $W(\theta)$. Hence, any θ-firm's total repayment at date 2 is deterministic:

$$K(\theta) = f(\theta) + W(\theta)F + \big(1 - W(\theta)\big)M. \tag{5}$$

Since $H > V$, the θ-firm will default early if and only if[9]

$$K(\theta) > H. \tag{6}$$

Conditional on the θ-firm deciding to continue investing, it invests in VP rather than HR if and only if

$$\begin{aligned} p\left[V - K(\theta)\right] &\geq q\left[H - K(\theta)\right] \\ \Rightarrow K(\theta) &\leq \frac{pV - qH}{p - q}. \end{aligned} \tag{7}$$

As a result, given creditors' strategies, the θ-firm's optimal investment strategy is

$$\begin{cases} \text{early default}, & \text{if} \quad K(\theta) > H, \\ \text{HR}, & \text{if} \quad K(\theta) \in \left(\frac{pV-qH}{p-q}, H\right], \\ \text{VP}, & \text{if} \quad K(\theta) \leq \frac{pV-qH}{p-q}. \end{cases} \tag{8}$$

Recall that we assume $M < \frac{pV-qH}{p-q}$. Hence, when the firm's economic fundamentals are extremely good ($\theta \to +\infty$), its operation cost is almost zero and so it will choose VP, even if no creditor buys the bonds. This establishes a *dominant region of investing* for all creditors: When a creditor receives a very positive private signal, he will believe that the firm is going to invest in VP and hence will buy the firm's bonds even if all other creditors refrain from doing so. In contrast, when the firm has extremely bad economic fundamentals and thus unlimited operation cost ($\lim_{\theta \to -\infty} f(\theta) = +\infty$), it will choose to default at date 1, even if all creditors buy the bonds. This establishes a *dominant region of not*

[9] We assume that the firm will default at date 1 if its total repayment at date 2 is larger than the highest possible cash flow the firm can generate. This reflects some very small cost incurred by the manager if he continues.

investing: When a creditor receives a very negative private signal, he will believe that the firm will default at date 1 and hence will not buy the bonds even if all other creditors choose to buy. Therefore, as in other global game models, in a monotone equilibrium, any creditor employs a cutoff strategy with the threshold $\tilde{x}$, such that he invests in the bonds if and only if $x_i \geq \tilde{x}$.

Given θ and creditors' cutoff strategy, the measure of creditors who invest is

$$W(\theta) = (1-\gamma)\Pr(x \geq \tilde{x}|\theta) = (1-\gamma)\left\{1 - \Phi\left[\sqrt{\beta}(\tilde{x}-\theta)\right]\right\},$$

where $\Phi(\cdot)$ is the CDF of the standard normal distribution. The θ-firm's total repayment at date 2 is thus

$$\begin{aligned} K(\theta) &= f(\theta) + (1-\gamma)\left\{1 - \Phi\left[\sqrt{\beta}(\tilde{x}-\theta)\right]\right\}F \\ &\quad + \left[\gamma + (1-\gamma)\Phi[\sqrt{\beta}(\tilde{x}-\theta)]\right]M \\ &= f(\theta) + \left[(1-\gamma)F + \gamma M\right] + (1-\gamma)\Phi\left[\sqrt{\beta}(\tilde{x}-\theta)\right](M-F). \end{aligned} \tag{9}$$

In equation (9), the first term is the operation cost of the θ-firm, the second term is the financial cost resulting from insufficient liquidity in the bond market, and the third term is the endogenous financial cost resulting from creditors' strategic uncertainty.

It follows directly from equation (9) that the firm's total repayment at date 2 is strictly decreasing in its fundamentals. Specifically, as the firm's fundamentals improve (i.e., as θ increases), its operation cost decreases (since $f(\theta)$ is strictly decreasing); also, more creditors receive private signals above the threshold $\tilde{x}$ and thus choose to buy the bonds, leading to a lower financial cost. The monotonicity of $K(\theta)$ turns out to be critical for the equilibrium characterization.

First, given creditors' strategies, the firm will choose to default early if and only if $\theta < \tilde{\theta}_1$. This implies that

$$K(\tilde{\theta}_1) = f(\tilde{\theta}_1) + \left[(1-\gamma)F + \gamma M\right] + (1-\gamma)\Phi\left[\sqrt{\beta}(\tilde{x}-\tilde{\theta}_1)\right](M-F) = H. \tag{10}$$

Because $K(\theta)$ is strictly decreasing, for any $\theta < \tilde{\theta}_1$, the firm's total repayment at date 2 will be greater than H, the upside cash flow of HR. As a result, the firm would default at date 1. But if $\theta \geq \tilde{\theta}_1$, the firm can at least choose HR to receive a nonnegative expected payoff due to its limited liability, and thus the firm will not default early.

When $\theta \geq \tilde{\theta}_1$, the firm needs to choose between VP and HR. From equation (8) and the fact that $K(\theta)$ is strictly decreasing in θ, there must be a $\tilde{\theta}_2 > \tilde{\theta}_1$ such that the firm will choose VP if and only if $\theta \geq \tilde{\theta}_2$. Hence,

$$K(\tilde{\theta}_2) = f(\tilde{\theta}_2) + \left[(1-\gamma)F + \gamma M\right] + (1-\gamma)\Phi\left[\sqrt{\beta}(\tilde{x}-\tilde{\theta}_2)\right](M-F) = \frac{pV - qH}{p-q}. \tag{11}$$

Following the above arguments, in a monotone equilibrium, the firm will default early if $\theta < \tilde{\theta}_1$, invest in HR if $\theta \in [\tilde{\theta}_1, \tilde{\theta}_2)$, and invest in VP if $\theta \geq \tilde{\theta}_2$.

Any creditor i receiving a private signal x_i about θ first updates his belief about θ according to Bayes' rule:

$$\theta | x_i \sim \mathcal{N}\left(x_i, \frac{1}{\beta}\right).$$

Given the firm's strategy described above, creditor i then calculates his return from investing in the bonds:

$$\left\{\Phi\left[\sqrt{\beta}(\tilde{\theta}_2 - x_i)\right] - \Phi\left[\sqrt{\beta}(\tilde{\theta}_1 - x_i)\right]\right\} qF + \left\{1 - \Phi\left[\sqrt{\beta}(\tilde{\theta}_2 - x_i)\right]\right\} pF.$$

Given the dominant regions of investing and not investing, there must be a marginal creditor who is indifferent between investing and not investing. Because any creditor will receive the payoff 1 if he does not invest, and his expected payoff from investing is strictly increasing in his private signal, the marginal creditor must have the private signal $\tilde{x}$ that makes his indifference condition hold:

$$\left\{\Phi\left[\sqrt{\beta}(\tilde{\theta}_2 - \tilde{x})\right] - \Phi\left[\sqrt{\beta}(\tilde{\theta}_1 - \tilde{x})\right]\right\} qF + \left\{1 - \Phi\left[\sqrt{\beta}(\tilde{\theta}_2 - \tilde{x})\right]\right\} pF = 1. \quad (12)$$

Proposition 1 characterizes the equilibrium of the benchmark model.

PROPOSITION 1 (The Unique Equilibrium in the Benchmark Model): *There exists a $\tilde{\beta} > 0$ such that for all $\beta > \tilde{\beta}$, the benchmark model without a CRA has a unique equilibrium described by $(\tilde{\theta}_1, \tilde{\theta}_2, \tilde{x})$, where $\tilde{\theta}_1 < \tilde{\theta}_2$. In particular:*

(1) The firm's investment strategy is

$$\begin{cases} VP, & \text{if } \theta \geq \tilde{\theta}_2, \\ HR, & \text{if } \theta \in \left[\tilde{\theta}_1, \tilde{\theta}_2\right), \\ \text{early default}, & \text{if } \theta < \tilde{\theta}_1. \end{cases}$$

(2) Any creditor i buys the firm's bonds if and only if $x_i \geq \tilde{x}$.

III. Credit Ratings

We now consider our core model in which the CRA strategically chooses its optimal rating strategy. We first specify details about the rating revenue and the potential rating cost. Then, as a first step toward solving for an equilibrium, we discuss possible equilibrium rating strategies. In particular, we identify the conditions under which the CRA will assign overgenerous ratings.

We next follow the literature on credit ratings and the empirical observations to focus on the case in which the CRA may inflate credit ratings. We show that rating inflation must emerge in equilibrium, but credit ratings are still informative about firm fundamentals.

We finally solve for the unique equilibrium under rating inflation. In equilibrium, when assigning credit ratings, the CRA takes into account the rating effects on creditors' bond investment decisions and thus firm investment choices. As we show in Section IV, this feature is key to understanding the real effects of credit ratings.

A. Equilibrium Rating Strategies

Because of the prevailing "issuer-pays" business model in the credit rating industry, we assume that the CRA receives more revenue by assigning the firm a higher credit rating. Hence, $V^p > V^q > V^0 = 0$, where we normalize the revenue from assigning the rating $R = 0$ to zero.

The CRA incurs a rating cost, C^R, which may be viewed as a legal or reputation cost, when the firm defaults. Importantly, the rating cost depends not only on the credit rating R, but also on the verifiability of the firm's investment choice. First, if a firm with rating $R > 0$ defaults endogenously at date 1, the CRA incurs a cost of C^D. Because the firm's early default is observable and verifiable, the rating $R > 0$ can be verified as wrong and the CRA will be heavily punished (White, 2013).[10] We therefore assume that

$$C^D > V^p. \tag{13}$$

We refer to this assumption as the *partial verifiability constraint* imposed on credit ratings.

Second, if a firm with rating $R > 0$ defaults at date 2 after making an investment, the CRA incurs a cost of C^R $(R = p, q)$. Here, we assume that $V^p > V^q > C^p > C^q > 0$. We further assume that the CRA's reputation cost is exogenous. This cost may arise from government regulation or from the repeated interactions between creditors and the CRA.[11]

The partial verifiability constraint (equation (13)) implies that the CRA will assign the rating $R = 0$ if it foresees that the firm will default at date 1 even with a rating of $R' > 0$. It then follows from the assumption $V^p > V^q > C^p > C^q > 0$ that if the firm does not default at date 1, the CRA will assign the rating p or q. In such a case, the CRA will issue the rating p if and only if $V^p - \mathbb{E}(C^p) \geq V^q - \mathbb{E}(C^q)$.

Lemma 1 shows that the CRA's equilibrium rating strategy depends on the ratio of the incremental revenue to the incremental cost of upgrading the rating from q to p.

[10] Credit ratings are viewed as CRAs' "free speech" protected by the First Amendment. As a result, CRAs are not liable for any losses incurred due to the inaccuracy of their ratings, unless it is proven that they know the ratings are false.

[11] Mathis, McAndrews, and Rochet (2009) show that reputation cannot fully address the rating inflation issue. They also show that reputation cycles emerge in an infinite-horizon setup: Initially the CRA's reputation cost is high, but when its reputation is sufficiently high, its reputation cost is lower.

LEMMA 1: *The CRA's equilibrium rating strategy depends on the ratio* $(V^p - V^q)/(C^p - C^q)$. *There are three cases:*

(1) If $\frac{V^p - V^q}{C^p - C^q} \geq 1 - q$, *the equilibrium rating strategy takes the form*

$$\mathcal{R}(\theta) = \begin{cases} p, & \text{if } \theta \geq \theta^I, \\ 0, & \text{if } \theta < \theta^I. \end{cases} \tag{14}$$

(2) If $\frac{V^p - V^q}{C^p - C^q} \leq 1 - p$, *the equilibrium rating strategy takes the form*

$$\mathcal{R}(\theta) = \begin{cases} q, & \text{if } \theta \geq \theta^D, \\ 0, & \text{if } \theta < \theta^D. \end{cases} \tag{15}$$

(3) If $\frac{V^p - V^q}{C^p - C^q} \in (1 - p, 1 - q)$, *the equilibrium rating strategy takes the form*

$$\mathcal{R}(\theta) = \begin{cases} p, & \text{if } \theta \geq \theta^p, \\ q, & \text{if } \theta \in [\theta^q, \theta^p), \\ 0, & \text{if } \theta < \theta^q, \end{cases} \tag{16}$$

where $\theta^q \leq \theta^p$.

Part 1 of Lemma 1 shows that if the benefit of upgrading the rating from q to p is high enough (relative to the increase in reputation cost), the CRA will "inflate" the ratings assigned to firms that invest in HR. In this case, the rating $R = q$ will not be assigned in equilibrium. Intuitively, if a θ-firm receives the rating q, it will not default at date 1 in equilibrium. If the CRA upgrades its rating to $R = p$, either the θ-firm will have an even lower financial cost (if firms with rating p are believed to have better economic fundamentals) or its financial cost will be lower than that of other firms that receive the rating p. As a result, after such an upgrade, the θ-firm will not default at date 1. The condition $\frac{V^P - V^q}{C^p - C^q} \geq 1 - q$ therefore implies that upgrading the θ-firm to the p-rating group will be a profitable deviation, and hence the CRA will not assign the rating $R = q$ in equilibrium.

Another important feature of the equilibrium rating strategy when $\frac{V^P - V^q}{C^p - C^q} \geq 1 - q$ is that the rating is monotonic in the firm's economic fundamentals. This is also intuitive. For a θ-firm with the rating $R = 0$, if another firm that has worse economic fundamentals is assigned the rating p (and so it will not default at date 1), it will be profitable for the CRA to also assign the rating p to the θ-firm, since the θ-firm, when assigned the rating p, will have lower total repayments at date 2 and will not default either.

Part 2 of Lemma 1 shows that when the revenue of upgrading the rating from q to p is sufficiently small (relative to the increase in the reputation cost), the CRA will "deflate" the ratings assigned to firms. In this case, only ratings $R = 0$ and $R = q$ will be assigned in equilibrium.

Importantly, in both the rating inflation case and the rating deflation case, the CRA pools all firms that do not default at date 1 together, separating them from the firms that do default at date 1. The CRA's effects on economic efficiency are therefore identical in these two cases, because in our model, all creditors are rational and update their beliefs based on the equilibrium strategy of the CRA. As a result, the label that is put on the firms that are pooled together should not matter, what matters is which firms are pooled together.

Part 3 of Lemma 1 presents a very different case. When the ratio of the revenue increment of upgrading to the cost increment is in a medium range, the CRA may assign all three possible ratings in equilibrium. Importantly, in this case, in equilibrium the rating coincides with the firm's credit quality, that is, the CRA conveys accurate information about the firm's credit quality. We refer to such a CRA as a *self-disciplined* CRA.

By definition, a self-disciplined CRA will eliminate information asymmetry between the firm and creditors. It is then intuitive that the self-disciplined CRA will lead to a level of economic efficiency that is at least as high as in the case without a CRA, strictly promoting economic efficiency for some firm fundamentals. In particular, for firms that invest in VP if assigned the rating p and in HR if assigned the rating q, the condition $\frac{V^p - V^q}{C^p - C^q} \in (1-p, 1-q)$ implies that the CRA will assign them the rating p. Hence, a self-disciplined CRA will maximize the measure of firms that invest in VP, the socially optimal investment project.

The fact that whether the CRA inflates ratings, deflates ratings, or assigns accurate ratings depends on the ratio $\frac{V^p - V^q}{C^p - C^q}$ has important policy implications. Suppose that the government wants to regulate the credit rating industry by designing the cost scheme (conditional on the investment failure of the firm) such that the CRA is self-disciplined. To implement such a policy, the government has to accurately calibrate $V^p - V^q$. While this may be difficult, Lemma 1 shows that punishing a CRA that assigns the highest rating too much (i.e., setting a very large $C^p - C^q$) is not an effective policy, as the CRA will respond by deflating the ratings, with the resulting effects of the CRA the same as in the case in which the CRA inflates ratings.

B. Rating Inflation and Rating Informativeness

Since the effects of the CRA are straightforward when it is self-disciplined, and the effects of the "deflating" CRA are identical to those of the "inflating" CRA, we focus on the case of the "inflating" CRA in the rest of the paper. The case of rating inflation is also the case that is most often discussed empirically and in policy circles. Formally, we maintain the following assumption:

$$\frac{V^P - V^q}{C^p - C^q} \geq 1 - q. \tag{17}$$

Under such an assumption, Lemma 1 implies that the CRA's equilibrium rating strategy is

$$\mathcal{R}(\theta) = \begin{cases} p, & \text{if } \theta \geq \theta_1^*, \\ 0, & \text{if } \theta < \theta_1^*. \end{cases} \tag{18}$$

Therefore, the CRA's equilibrium rating strategy can be characterized by $\theta_1^* \in \mathbb{R}$, with $\mathcal{R}(\theta) = p$ when $\theta \geq \theta_1^*$ and $\mathcal{R}(\theta) = 0$ when $\theta < \theta_1^*$. When θ_1^* decreases, the CRA assigns more firms with the high rating p. So for rating strategies $\mathcal{R}_1$ with threshold θ_1^* and $\mathcal{R}_2$ with threshold θ_2^*, we say that the rating strategy $\mathcal{R}_2$ is *laxer* than the rating strategy $\mathcal{R}_1$ if and only if $\theta_2^* < \theta_1^*$. However, the laxer rating strategy $\mathcal{R}_2$ may not lead to higher *credit rating inflation*, which arises when the nominal rating is strictly higher than the real credit quality. Formally,

DEFINITION 2: *A credit rating assigned to a θ-firm is inflated if, in equilibrium, the θ-firm chooses HR and thus has credit quality q but the CRA assigns the rating p. In addition, a rating strategy is inflated if credit ratings assigned according to the rating strategy are inflated for a nonnegligible subset of fundamentals, and a credit rating strategy is more inflated if, for a larger measure of fundamentals, credit ratings assigned according to the rating strategy are inflated.*

In equilibrium, a firm that is assigned the rating p does not default at date 1. However, the rating p cannot guarantee that the firm will invest in VP. Indeed, if all creditors believe that the firm with rating p will surely invest in VP, they will all buy the firm's bonds, leading to the lowest possible financial cost. The assumption that the θ_1^*-firm will invest in VP then implies that the θ_1^*-firm's total repayments at date 2 (the sum of the financial cost and the operation cost) are less than $\frac{pV-qH}{p-q}$ and thus strictly less than H. As a result, firms with fundamentals slightly lower than θ_1^* will not default at date 1 if they are assigned the rating p. It is therefore profitable for the CRA to deviate and assign the rating p to such firms. Thus, in equilibrium, some firms with the rating p will invest in HR, implying credit rating inflation in equilibrium. Formally,

LEMMA 2: *There is no monotone equilibrium in which all θ-firms that receive a rating $R = p$ invest in VP.*

While rating inflation inevitably appears in equilibrium, credit ratings are still informative for creditors. The CRA's equilibrium rating strategy (equation (18)) implies that if $R = p$, all creditors know that $\theta \geq \theta_1^*$. So the rating p guarantees creditors that the firm's fundamentals are not extremely bad.

COROLLARY 1: *Following the credit rating $R = p$, regardless of his private signal x_i, the support of any creditor i's interim belief about θ is truncated from below by θ_1^*.*

C. Equilibrium under Rating Inflation

In this subsection, we characterize the unique equilibrium under rating inflation. As shown in Part 1 of Lemma 1, given equation (17), only rating $R = 0$ and rating $R = p$ are assigned in equilibrium. Due to the partial verifiability constraint, the CRA assigns $R = 0$ if and only if it knows that the firm will default early, even with the rating p. Therefore, when creditors observe $R = 0$, they will infer that the firm will default early and hence will not buy the firm's bonds. As a result, following $R = 0$, there is a unique equilibrium in which no creditor invests in the bonds, and the firm defaults at date 1. Since the rating strategy assigns the rating $R = 0$ to the firm if and only if $\theta < \theta_1^*$, we must have that $K(\theta) = f(\theta) + M > H$, $\forall \theta < \theta_1^*$. Then, by the continuity of $f(\cdot)$, we have the first equilibrium condition:

$$f(\theta_1^*) \geq H - M. \tag{19}$$

We now focus on the case following the rating $R = p$. Since given creditors' strategies, the firm's total repayment at date 2 is strictly decreasing with its fundamentals, there must be a threshold $\theta_2^* > \theta_1^*$ such that the θ-firm invests in VP if $\theta \geq \theta_2^*$ but in HR if $\theta \in [\theta_1^*, \theta_2^*)$. Note that Lemma 2 implies that θ_2^* must be strictly greater than θ_1^*, because some firms with the rating $R = p$ will invest in HR.

Creditors, however, employ a cutoff strategy. Corollary 1 implies that given the CRA's rating strategy, after observing the rating p, all creditors believe that the firm's true fundamentals are above θ_1^*. Hence, any creditor i will buy the bonds if and only if his private signal lands above a threshold $x^* \in \mathbb{R}$. We refer to the creditor with private signal x^* the marginal creditor. Importantly, the marginal creditor is indifferent between buying the bonds or not buying the bonds.

Finally, given the firm's and creditors' strategies, the CRA chooses θ_1^* to maximize its expected rating profit. Since it will assign $R = p$ to the firm if and only if the firm will not default at date 1 with such a rating, θ_1^* must be chosen so that the firm is indifferent between early default and HR.

The above arguments lead to the indifference conditions of the firm, the marginal creditor, and the CRA, which are characterized by equations (20), (21), and (22), respectively:

$$\begin{aligned} f\left(\theta_2\right) + (1-\gamma)\left[1 - \Phi\left(\sqrt{\beta}\left(x - \theta_2\right)\right)\right] F \\ + \left[\gamma + (1-\gamma)\Phi\left(\sqrt{\beta}\left(x - \theta_2\right)\right)\right] M = \frac{pV - qH}{p - q}, \end{aligned} \tag{20}$$

$$\frac{\Phi[\sqrt{\beta}(\theta_2^* - x^*)] - \Phi[\sqrt{\beta}(\theta_1^* - x^*)]}{1 - \Phi[\sqrt{\beta}(\theta_1^* - x^*)]} qF + \frac{1 - \Phi[\sqrt{\beta}(\theta_2^* - x^*)]}{1 - \Phi[\sqrt{\beta}(\theta_1^* - x^*)]} pF = 1, \tag{21}$$

$$f(\theta_1^*) + (1-\gamma)\left[1 - \Phi(\sqrt{\beta}(x^* - \theta_1^*))\right]F + \left[\gamma + (1-\gamma)\Phi(\sqrt{\beta}(x^* - \theta_1^*))\right]M = H. \tag{22}$$

Proposition 2 shows that under the assumption of rating inflation (equation (17)), the model has a unique equilibrium in which the CRA's rating, the firm's investment decision, and creditors' bond-investment decisions interact with one another.

PROPOSITION 2: *Given equation (17), there is a $\beta^* > 0$ such that when $\beta > \beta^*$, the model has a unique equilibrium. The equilibrium is characterized by $(\theta_1^*, \theta_2^*, x^*)$, where $\theta_2^* > \theta_1^*$, such that:*

(1) The CRA will assign the rating $R = p$ if the firm's fundamentals θ belong to $[\theta_1^*, +\infty)$, and the rating $R = 0$ otherwise.
(2) If $R = 0$, no creditor buys the bonds, and the firm defaults at date 1.
(3) If $R = p$, a creditor invests in the bonds if and only if his private signal lands above x^*, and the firm will choose HR if $\theta \in [\theta_1^*, \theta_2^*)$ and VP if $\theta \in [\theta_2^*, +\infty)$.
(4) The triple $(\theta_1^*, \theta_2^*, x^*)$ solves equations (20), (21), and (22).

The equilibrium uniqueness arises from creditors' new dominant region of not investing, which is generated by the credit rating p. Given equation (17), Lemma 2 implies that the CRA will assign the rating p to the firm with fundamentals just above θ_1^*, and such firms will invest in HR. Consequently, when creditors receive very negative signals, they infer that the firm's fundamentals land within the HR investment region and hence they refrain from investing in the bonds. This endogenously generates a new dominant region of not investing, and thus creditors have a unique best response to the rating p.

Proposition 2 provides a clear measure of equilibrium rating inflation. When $\theta < \theta_1^*$, the CRA will assign the rating $R = 0$ to the firm. Since the firm will default early, the credit rating accurately reflects the firm's credit quality. When $\theta \geq \theta_2^*$, the firm's fundamentals are sufficiently good that it will invest in VP. In this case, the credit rating $R = p$ also indicates the firm's actual credit quality. However, when $\theta \in [\theta_1^*, \theta_2^*)$, the firm invests in HR and thus has credit quality q, but it receives the high rating p. The credit ratings assigned to such firms are inflated. Hence, rating inflation can be captured by $\theta_2^* - \theta_1^*$.

IV. The CRA's Real Effects under Rating Inflation

We are now able to analyze the CRA's real effects. For a given θ-firm, if the assigned credit rating changes its investment decision (compared to its investment in the benchmark model without a CRA), the CRA affects economic efficiency. In this case we say that the CRA has *real effects* on the θ-firm. Such effects are *positive* if the CRA leads to higher economic efficiency and *negative* if the CRA leads to lower economic efficiency. We capture the CRA's ex ante

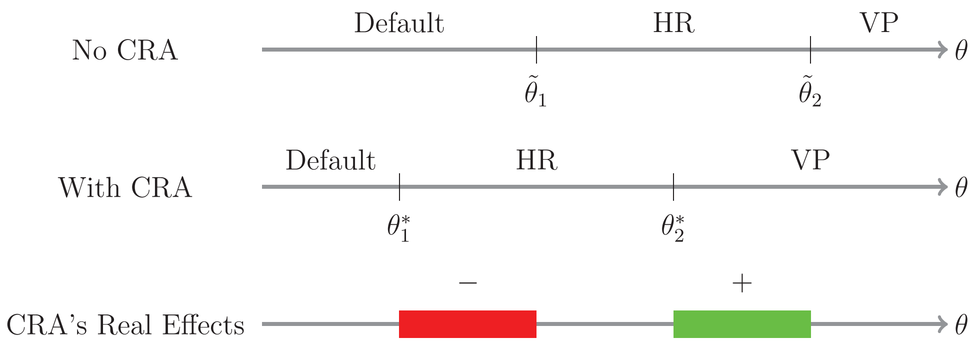

Figure 2. CRA's real effects when $\theta_2^* > \tilde{\theta}_1$. (Color figure can be viewed at wileyonlinelibrary.com)

real effects using the average change in economic efficiency. Hence, the ex ante real effects of the CRA are positive (negative) if economic efficiency is higher (lower) with the CRA.

Lemma 3 shows that, with the CRA, both the early default threshold and the VP investment threshold are lower than those in the benchmark model without a CRA.

LEMMA 3: *Comparing the equilibrium of the model with a CRA (described in Proposition 2) to that of the benchmark model without a CRA (described in Proposition 1), we have $\theta_1^* < \tilde{\theta}_1$, $\theta_2^* < \tilde{\theta}_2$, and $x^* < \tilde{x}$. However, the sign of $\theta_2^* - \tilde{\theta}_1$ is undetermined.*

Figure 2 illustrates the CRA's real effects in the case in which $\theta_2^* > \tilde{\theta}_1$. When $\theta_2^* > \tilde{\theta}_1$, there are two cases. First, when $\theta \in [\theta_1^*, \tilde{\theta}_1)$, without the CRA, the firm's financial costs are so high that it will default early, whereas when the CRA is present, it will assign the firm the inflated rating p, leading to lower financial costs to the firm. Such a decrease in financial costs encourages the firm to gamble for resurrection rather than default early, which implies negative real effects. Second, when $\theta \in [\theta_2^*, \tilde{\theta}_2)$, because the high rating p reduces the firm's financial costs, the firm switches from HR to VP, which implies positive real effects.

When $\theta_2^* \leq \tilde{\theta}_1$, the CRA's real effects are similar, except that the range for negative real effects is different. Proposition 3 formally summarizes the CRA's real effects in the case of rating inflation. Note that because of the improper uniform prior, when calculating the CRA's ex ante real effects, we consider the Lebesgue measure for the number of firms whose investment decisions are affected by credit ratings.

PROPOSITION 3: *Under the assumption of equation (17), the CRA's real effects are summarized by two cases:*

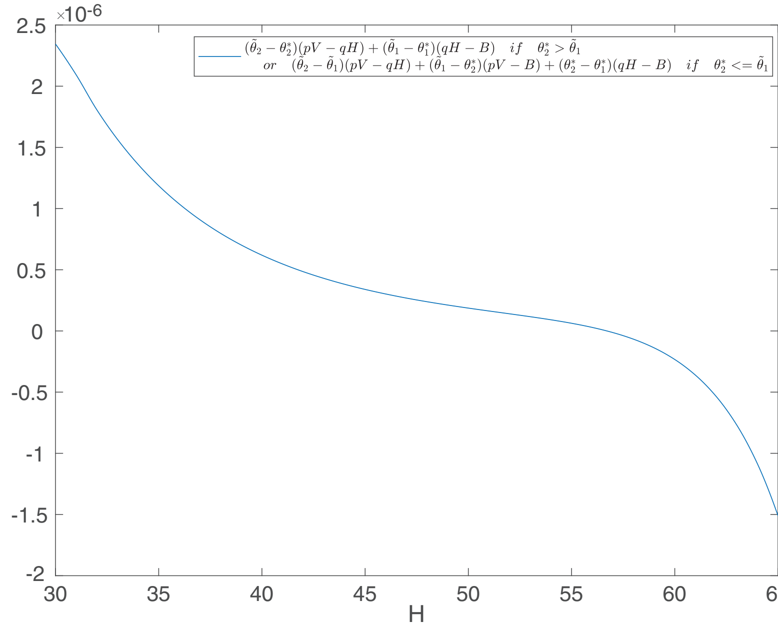

Figure 3. The CRA's real effects as a function of *H*. The parameters used in this figure are $F = 1.2, M = 1.5, V = 3, p = 0.9, q = 0.005, \gamma = 0.7, L = 0.7, \beta = 0.8$, and $f(\theta) = e^{-\theta}$. (Color figure can be viewed at wileyonlinelibrary.com)

(1) *If $\theta_2^* > \tilde{\theta}_1$, the CRA has positive real effects when $\theta \in [\theta_2^*, \tilde{\theta}_2)$ and negative real effects when $\theta \in [\theta_1^*, \tilde{\theta}_1)$, and hence the CRA's ex ante real effects are*

$$(\tilde{\theta}_2 - \theta_2^*)(pV - qH) + (\tilde{\theta}_1 - \theta_1^*)(qH - L).$$

(2) *If $\theta_2^* \leq \tilde{\theta}_1$, the CRA has positive real effects when $\theta \in [\theta_2^*, \tilde{\theta}_2)$ and negative real effects when $\theta \in [\theta_1^*, \theta_2^*)$, and hence the CRA's ex ante real effects are*

$$(\tilde{\theta}_2 - \tilde{\theta}_1)(pV - qH) + (\tilde{\theta}_1 - \theta_2^*)(pV - L) + (\theta_2^* - \theta_1^*)(qH - L).$$

Importantly, Proposition 3 shows that the CRA that employs an inflated rating strategy may have positive or negative real effects, depending on the firm's fundamentals. The CRA's ex ante real effects then depend on model parameters.

In Figure 3, we depict the CRA's ex ante real effects as a function of the upside return of the risky project, H. The figure shows that when the upside return of the risky project is relatively high, the CRA's ex ante real effects are negative. This is because when H is large, the firm has stronger incentives to take risks by investing in HR and thus is less likely to default at date 1 efficiently. The CRA will therefore assign more firms the high rating $R = p$, which allows those firms to gamble for resurrection and in turn have negative ex ante real effects. When H is relatively small, the CRA encourages more firms to switch from HR to VP and thus has positive ex ante real effects.

A. Informational Effects and Strategic Effects

Proposition 2 suggests that the CRA affects a firm's investment decision through two interacting channels. On the one hand, by assigning the rating $R = p$, the CRA separates firms with fundamentals above a threshold from those with fundamentals below the threshold. Hence, the rating $R = p$ provides creditors with new information about the firm's fundamentals. This new information affects creditors' bond-investment decisions, and thus the firm's financial costs and investment choice. We refer to such effects as the CRA's *informational effects*.

On the other hand, the CRA strategically chooses θ_1^* to pool the firms that invest in HR with those that invest in VP. Hence, the set of firm types that invest in HR or VP may differ in cases with and without a CRA. This also affects firm investment decisions. We refer to such effects as the CRA's *strategic effects*, since the CRA, when choosing θ_1^*, takes into account creditors' and the firm's best responses to the ratings.

In this subsection, we examine how these two effects interact to determine the CRA's real effects. We first analyze the CRA's informational effects. Consider the case in which the CRA commits to the following rating strategy:

$$\mathcal{R}(\theta) = \begin{cases} 0, \text{ if } \theta < \widehat{\theta}_1 \equiv \tilde{\theta}_1; \\ p, \text{ if } \theta \geq \widehat{\theta}_1. \end{cases} \tag{23}$$

Here, $\tilde{\theta}_1$, which is characterized in Proposition 1, is the early-default threshold of the firm when there is no CRA.

The committed rating strategy characterized in equation (23) simply reflects the firm's investment decision in the benchmark model without a CRA. For ease of exposition, we refer to such a CRA as a *reflecting* CRA and the CRA analyzed in Section III as a *strategic* CRA. Importantly, a reflecting CRA does not have strategic effects, because it does not strategically account for its effects on the firm's investment decision when committing to its rating strategy, although such a rating strategy may still be inflated. Therefore, the real effects of the reflecting CRA are just the informational effects of the strategic CRA. By comparing the strategic CRA's real effects with the reflecting CRA's real effects, we can identify the strategic CRA's strategic effects.

Proposition 4 characterizes the firm's equilibrium investment decision in the case with the reflecting CRA.

PROPOSITION 4: *Given the committed rating strategy in equation (23), the resulting credit ratings lead to two continuation plays:*

(1) Following $R = 0$, there is a unique equilibrium play in which the firm defaults at date 1.

(2) Following $R = p$, in any equilibrium, the θ-firm invests in VP if $\theta \geq \widehat{\theta}_2$ and in HR if $\theta \in [\widehat{\theta}_1, \widehat{\theta}_2)$. Furthermore, if $\theta_2^* > \tilde{\theta}_1$, we have $\widehat{\theta}_2 < \theta_2^*$.

The reflecting CRA's real effects can be derived from the comparison between the equilibrium characterized in Proposition 4 and that characterized in Proposition 1. With the rating p assigned by a reflecting CRA, if $\theta > \tilde{\theta}_2$ (which is strictly greater than $\widehat{\theta}_2$), then the firm invests in VP, both with a reflecting CRA and without a CRA. Therefore, for any $\theta > \tilde{\theta}_2$, the reflecting CRA does not have real effects. Similarly, the reflecting CRA does not have real effects when $\theta \in [\widehat{\theta}_1, \widehat{\theta}_2)$.

However, the reflecting CRA will change the firm's investment decision when $\theta \in [\widehat{\theta}_2, \tilde{\theta}_2)$. In particular, without a CRA, the firm invests in HR, but with a reflecting CRA, the firm will invest in VP. Therefore, the reflecting CRA has positive real effects, which are captured by $(\tilde{\theta}_2 - \widehat{\theta}_2)(pV - qH)$. The strategic CRA's informational effects are always positive precisely because its rating $R = p$, though potentially inflated, provides creditors with an informative signal and thus correctly guides creditors' bond-investment decisions and in turn the firm's investment decision.

The CRA has informational effects because credit ratings are informative public signals to creditors. Generally, prior literature shows that public signals may have negative effects, as demonstrated by Morris and Shin (2002) and others. In our model, however, the CRA's informational effects are always positive. Given the credit rating rule of the reflecting CRA, a firm can receive a high rating only if it would invest in VP or HR in the benchmark model without a CRA. Because a high rating reduces the firm's financial costs, in this case it is possible for a firm to switch from HR to VP but impossible for a firm to switch from efficient default to HR. Consequently, given a credit rating assigned by the reflecting CRA, firm credit quality is at least as high as that in the benchmark model.

We finally investigate the CRA's strategic effects. Similar to Proposition 3, we have two cases: $\theta_2^* \geq \widehat{\theta}_1$ and $\theta_2^* < \widehat{\theta}_1$. In both cases, the CRA's strategic effects have a negative component. Specifically, the strategic CRA knows that when it assigns the rating $R = p$, more creditors will buy the firm's bonds and the firm's financial costs will decrease, so it can issue the high rating $R = p$ to more firms. Thus, in equilibrium, the strategic CRA will employ the rating strategy with the threshold $\theta_1^* < \min\{\widehat{\theta}_1, \theta_2^*\}$. Such a manipulation leads firms with $\theta \in [\theta_1^*, \min\{\widehat{\theta}_1, \theta_2^*\})$ to gamble for resurrection and thus to adverse real effects.

In the case with $\theta_2^* \geq \widehat{\theta}_1$, the strategic CRA's real effects have another negative component. Because $\theta_1^* < \widehat{\theta}_1$, the rating $R = p$ assigned by the strategic CRA is less informative than the rating $R = p$ assigned by the reflecting CRA. So, with the strategic CRA, after the rating $R = p$, fewer creditors buy the bonds (compared to the case with the reflecting CRA) and thus the firm's financial costs increase. Hence, fewer firms (captured by $\theta_2^* - \widehat{\theta}_2$) switch from HR to VP. In sum, in the case with $\theta_2^* > \widehat{\theta}_1$, the strategic CRA's rating strategy will weaken its informational effects. The CRA's strategic effects in the case with $\theta_2^* \geq \tilde{\theta}_1$ are illustrated in Figure 4 (with the (light) green indicating positive real effects and the (dark) red indicating negative real effects).

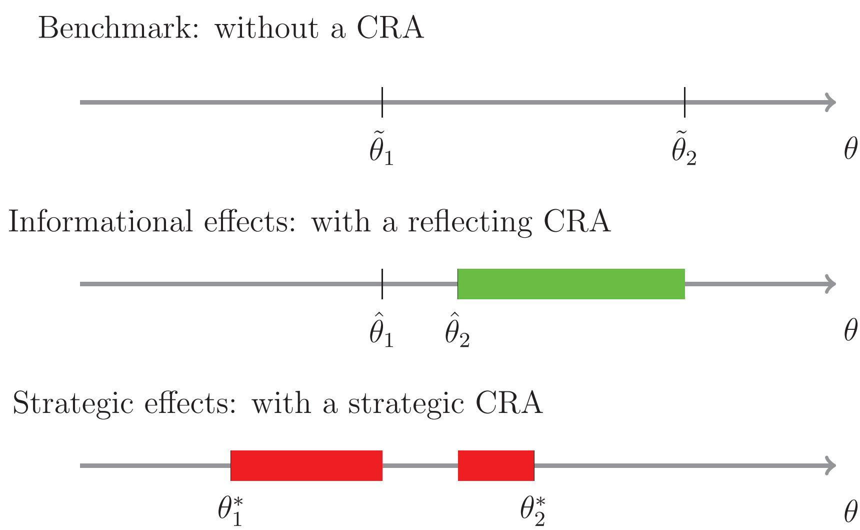

Figure 4. CRA's strategic effects when $\theta_2^* > \tilde{\theta}_1$. (Color figure can be viewed at wileyonlinelibrary.com)

In the other case with $\theta_2^* < \widehat{\theta}_1$, the second component of the strategic CRA's strategic effects is positive. This is because by assigning the rating $R = p$ to the firm with $\theta \in [\theta_1^*, \widehat{\theta}_1)$, it is possible for the firm to invest in VP. Indeed, when $\theta \in [\theta_2^*, \widehat{\theta}_2)$, the firm does invest in VP, implying a positive component of the CRA's strategic effects.

Proposition 5 summarizes the above arguments.

PROPOSITION 5: *The CRA's real effects can be decomposed into its informational effects and its strategic effects. The informational effects, which are captured by $(\tilde{\theta}_2 - \widehat{\theta}_2)(pV - qH)$, are always positive. When the parameters are such that $\theta_2^* \geq \widehat{\theta}_1$, the strategic effects are captured by*

$$(\tilde{\theta}_1 - \theta_1^*)(qH - L) + (\theta_2^* - \widehat{\theta}_2)(qH - pV),$$

which is negative, but when $\theta_2^* < \widehat{\theta}_1$, the strategic effects are captured by

$$(\theta_2^* - \theta_1^*)(qH - L) + (\widehat{\theta}_1 - \theta_2^*)(pV - L) + (\widehat{\theta}_2 - \widehat{\theta}_1)(pV - qH),$$

the sign of which is undetermined.

Proposition 5 implies that credit rating inflation itself does not necessarily lead to negative ex ante real effects. Because inflated ratings are informative signals, they do increase market efficiency and have positive real effects. Negative real effects, however, can arise from the CRA's strategic effects. Because the CRA knows that the rating will reduce the firm's financial costs and default likelihood, it will issue the high rating to more firms, providing them with opportunities to gamble for resurrection.

V. Empirical Predictions

The theory that we develop in this paper provides several new empirical predictions about CRAs' rating strategies and credit rating inflation. In this section, we examine how a CRA's rating strategy and rating inflation vary when the economic environment changes. In particular, we perform comparative statics analyses to obtain empirical predictions about CRAs' rating strategies and credit rating inflation.

From these comparative statics analyses, we show that laxer credit rating strategies are not necessarily accompanied by higher rating inflation. In our model, both the CRA's rating strategy (captured by θ_1^*) and credit rating inflation (captured by $\theta_2^* - \theta_1^*$) are endogenously determined. Thus, an exogenous change to the economic environment may lead to both a laxer rating strategy and lower financial costs to the firm. While the former effect may increase rating inflation, the latter effect may encourage the firm to invest in VP, which reduces rating inflation. Therefore, whether a laxer rating strategy is accompanied by higher rating inflation depends on which effect dominates. This tradeoff can shed light on recent empirical findings: Alp (2013) and Baghai, Servaes, and Tamayo (2014) find that CRAs become more conservative by using stricter rating standards, while Strobl and Xia (2012) show that stricter rating standards do not reduce credit rating inflation.

PROPOSITION 6: *When β is sufficiently large,*[12] *a decrease in β, an increase in H, and a decrease in γ will lead to a decrease in θ_1^*. However, a decrease in β has an ambiguous effect on $\theta_2^* - \theta_1^*$, an increase in H increases $\theta_2^* - \theta_1^*$, and a decrease in γ decreases $\theta_2^* - \theta_1^*$.*

First, because β proxies for the precision of creditors' private signals, it captures the firm's transparency. Proposition 6 shows that for more opaque firms, the CRA employs laxer rating strategies. By the properties of a truncated normal random variable's mean, when creditors' private signals become less precise, they infer that the firm is more likely to invest in VP. As a result, more creditors invest in the bonds and the firm's financial costs decrease, which allows the CRA to employ a laxer rating strategy. This is consistent with the recent empirical finding in Fong et al. (2014) that security analysts can discipline CRAs by providing creditors more information.

While Proposition 6 implies that CRAs employ laxer rating strategies for more opaque firms, it does not imply that credit ratings assigned to more opaque firms are more inflated. Since creditors will decrease their bond-investment threshold when the firm is more opaque, the firm's financial costs are lower, which leads to a smaller θ_2^*, the firm's VP-investment threshold. Consequently, when creditors' signals are less precise, the CRA is more likely to assign the rating p to the firm, but the firm with a high rating is more likely to invest in VP. As a result, whether rating inflation for a more opaque firm is

[12] This is the condition for equilibrium uniqueness, which is critical for comparative statics analysis.

higher or lower depends on which of these two effects dominates. This depends in turn on other parameters of the model.

Second, cross-sectionally, firms differ in the upside returns of their available projects. Equation (22) suggests that the highest upside return among all available projects may determine the credit rating assigned to the firm. It is therefore interesting to consider how the firm's upside return from HR affects the CRA's rating strategy. An increase in H does not directly affect creditors' behavior because creditors' payoffs are determined solely by the bond face value, which does not involve the cash flow to the firm, conditional on the success of the investment. However, H directly affects both the firm's investment and the CRA's rating strategy. On the one hand, an increase in H increases the firm's incentives to invest in HR, because the expected return from HR is higher. On the other hand, an increase in H decreases the firm's incentives to default early, because the firm has limited liability. As a result, for fixed creditors' strategies, when H increases, the CRA's rating strategy will be laxer and the firm is more likely to invest in HR than VP, resulting in higher credit rating inflation.

Finally, consider an increase in bond market liquidity (i.e., a decrease in γ). The measure of total potential creditors, $1-\gamma$, will increase. The direct effect is that the firm's financial costs will surely decrease because the firm needs to raise less money from expensive nonbond sources, such as the bank credit line. In addition, a decrease in γ will lead more creditors to buy the firm's bonds due to the strategic complementarities among creditors. This further reduces the firm's financial costs. As a result, the firm's VP-investment threshold will decrease, implying that fewer firms will invest in HR given the CRA's credit rating strategy. At the same time, the firm's lower financial costs imply that fewer firms may default early. As a result, the CRA would want to employ a laxer rating strategy. Furthermore, as γ decreases, the measure of firms that shift from HR to VP due to the lower financial costs is greater than the measure of firms that gamble for resurrection because of the high credit rating, leading to lower credit rating inflation.

VI. The Role of Dispersed Beliefs

In this section, we discuss how belief dispersion among creditors impacts the CRA's real effects.[13] We analyze an environment in which all creditors, the firm, and the CRA share a common prior belief, $\theta \in \mathcal{N}(\theta_s, \alpha^{-1})$. We consider the case in which α is sufficiently large, and we focus on a symmetric pure-strategy equilibrium. Thus, creditors' bond-investment decisions directly affect the firm's financial costs. Specifically, the firm's total repayments at date 2 are

$$K(\theta) = \begin{cases} f(\theta) + (1-\gamma)F + \gamma M, & \text{if all creditors invest in the bonds,} \\ f(\theta) + M, & \text{if all creditors refrain from investing.} \end{cases}$$

[13] Since a continuum of creditors with a homogeneous belief is informationally equivalent to a single large creditor, the analysis in this section may shed light on the CRA's real effects when the firm borrows from a large creditor such as a bank.

Hence, when all creditors choose to invest in the bonds, the firm's optimal investment choice is

$$\begin{cases} \text{Default early}, & \text{if} \quad f(\theta) + (1-\gamma)F + \gamma M > H, \\ \text{HR}, & \text{if} \quad f(\theta) + (1-\gamma)F + \gamma M \in \left(\frac{pV-qH}{p-q}, H\right], \\ \text{VP}, & \text{if} \quad f(\theta) + (1-\gamma)F + \gamma M \leq \frac{pV-qH}{p-q}. \end{cases}$$

Denote by y_1 the solution to the equation $f(\theta) + (1-\gamma)F + \gamma M = H$ and by y_2 the solution to the equation $f(\theta) + (1-\gamma)F + \gamma M = (pV - qH)/(p-q)$. The firm's optimal investment choice when all creditors invest in the bonds can then be written as

$$\begin{cases} \text{Default early}, & \text{if} \quad \theta < y_1, \\ \text{HR}, & \text{if} \quad \theta \in [y_1, y_2), \\ \text{VP}, & \text{if} \quad \theta \geq y_2. \end{cases}$$

Similarly, denote by y_1' the solution to the equation $f(\theta) + M = H$ and by y_2' the solution to the equation $f(\theta) + M = (pV - qH)/(p-q)$. The firm's optimal investment choice when all creditors choose not to invest in the bonds can be written as

$$\begin{cases} \text{Default early}, & \text{if} \quad \theta < y_1', \\ \text{HR}, & \text{if} \quad \theta \in [y_1', y_2'), \\ \text{VP}, & \text{if} \quad \theta \geq y_2'. \end{cases}$$

Because $M > (1-\gamma)F + \gamma M$ for any θ, we have $y_1 < y_1'$ and $y_2 < y_2'$. When α is sufficiently large, creditors rely mainly on the public signal to make the bond-investment decision. Because creditors' behavior determines the firm's investment choice, the public signal and creditors' behavior determine the CRA's credit rating. Proposition 7 summarizes the equilibrium credit rating strategy in this extension with homogeneous beliefs among creditors.

PROPOSITION 7: *There exists $\bar{\alpha} > 0$ such that, for all $\alpha > \bar{\alpha}$, the public signal determines the CRA's equilibrium rating strategy. Specifically:*

(1) When $\theta_s \geq y_2'$, the CRA will employ the rating strategy $\theta_1^* = y_1$.
(2) When $\theta_s < y_2$, the CRA will employ the rating strategy $\theta_1^* = y_1'$.
(3) When $\theta_s \in [y_2, y_2')$, the CRA will set $\theta_1^* = y_1$ if all creditors invest in the bonds after $R = p$, while the CRA will set $\theta_1^* = y_1'$ if all creditors refrain from investing in the bonds after $R = p$.

Proposition 7 indicates that when the public signal is very positive (i.e., $\theta_s \geq y_2'$), the CRA employs a laxer rating strategy, meaning that the good rating is a less positive signal. When the public signal is very negative (i.e., $\theta_s < y_2'$), the CRA employs a stricter rating strategy, meaning that the good rating is a more positive signal. Such a "substitution" results from the fact that creditors rely on the public signal more when making the bond-investment decision. When the public signal is in the medium range, multiple equilibria exist: if all creditors invest in the bonds, the CRA will employ a more inflated credit rating strategy, whereas if all creditors refrain from investing in the bonds, the CRA will employ a more conservative rating strategy.

It follows from Proposition 7 that, when all creditors share an accurate common belief, the CRA has little real effect—creditors will ignore the information extracted from credit ratings. By contrast, in the model presented in Section I, even if we allow for a public signal or an informative prior, if creditors' private signals are sufficiently precise (β is large), credit ratings will surely affect a positive measure of creditors' decisions. This is because the continuum of creditors have dispersed beliefs due to their conditionally independent private signals. Since credit ratings affect creditors' beliefs by truncating their belief supports, some creditors' beliefs about the firm's investment choice are surely affected. These creditors will change their bond-investment decision, which will affect the firm's financial costs, investment choice, and credit quality. As a result, the bond-investment decisions of other creditors whose beliefs are little affected by credit ratings directly will also be affected, leading the CRA to have significant real effects. Such a comparison shows the importance of creditors' belief dispersion in our core model. This analysis also demonstrates that the strategic effects of CRAs will remain large even if other sources of accurate public information exist.

VII. Conclusion

We study CRAs' effects on firm investment. We show that high credit ratings, although commonly known to be potentially inflated, exclude extremely bad firms from creditor belief support. Therefore, high ratings make creditors more optimistic, which reduces the firm's financial costs and changes its investment decisions. That is, even in an environment with perfectly rational and well-informed creditors, inflated ratings have significant real effects.

Such real effects, however, could be positive or negative. With high ratings, some firms take risky projects rather than default efficiently, implying that CRAs have adverse real effects, but other firms switch from risky inefficient investments to safe efficient investments, implying that CRAs have positive real effects. CRAs' overall ex ante real effects thus depend on the economic environment. Specifically, when the upside return of a risky project is high, CRAs' overall ex ante real effects are negative.

To better understand why the CRA may have negative ex ante real effects, even though it provides informative signals to the corporate bond market, we decompose its real effects into its informational effects and its strategic effects. We show that credit ratings that act as new informative signals positively affect firms' investment efficiency. Hence, the CRA's negative real effects arise solely from its strategic effects. Indeed, the CRA takes advantage of the feedback between credit ratings and firm investments to assign high ratings to more firms, providing those firms with a chance to gamble for resurrection. Such a manipulation leads to negative real effects.

We emphasize that credit rating standards and credit rating inflation are two distinct concepts, and that they are both endogenously determined.

Furthermore, changes in economic environment that lead to laxer rating strategies do not necessarily cause higher rating inflation.

Our paper offers both applied and theoretical contributions. From the applied perspective, we develop a rational framework that allows us to analyze credit rating inflation and CRAs' real effects in a model with feedback effects. While in this paper we focus on the credit ratings assigned to a firm, our model can also be applied to sovereign ratings. In fact, the assumption of the unverifiability of the firm's investment (either VP or HR) may be well suited to the scenario of sovereign ratings: because there are fewer observations of sovereign ratings, inflated ratings may be harder to detect. Our model also generates several testable empirical predictions and some policy implications.

While we focus on the real effects of credit ratings in this paper, the intuition and key mechanism in this paper may be more broadly applicable. For example, in the context of bank stress tests (Goldstein and Leitner, 2018; Inostroza and Pavan, 2018), regulators may want to declare a larger number of banks solvent than is the case. Such a policy may be efficiency enhancing for some banks (because of the reduced financial costs) but lead other banks to take risky projects. In addition, the economic environment that we set up can be applied to many other scenarios, such as financial advising, firm disclosure, auditing, marketing, and academic grading and recommendations.

From a theoretical perspective, we analyze an expert information disclosure model with multiple audiences who have dispersed beliefs. More importantly, the expert's message endogenously affects the fundamentals signaled by the message. This design can motivate new research on general disclosure models.

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Appendix A: Proofs

In this Appendix, we present proofs.

PROOF OF PROPOSITION 1: To show there is a unique equilibrium in this benchmark model, we only need to show that there is a unique solution $(\tilde{\theta}_1, \tilde{\theta}_2, \tilde{x})$ to equations (10), (11), and (12).

We first solve $\tilde{x}$ from equation (11). Define

$$\widetilde{\Delta} = \frac{\frac{pV-qH}{p-q} - \left[(1-\gamma)F + \gamma M\right] - f(\tilde{\theta}_2)}{(1-\gamma)(M-F)}.$$

Because $f(\theta)$ is strictly decreasing, $\widetilde{\Delta}$ is strictly increasing in $\tilde{\theta}_2$. We thus have

$$\tilde{x} = \tilde{\theta}_2 + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\widetilde{\Delta}), \tag{A.1}$$

which is strictly increasing in $\tilde{\theta}_2$.

Plugging $\tilde{x}$ as a function of $\tilde{\theta}_2$ into equation (12), we have

$$\widetilde{\Delta}(pF - qF) + \Phi\left[\sqrt{\beta}\left(\tilde{\theta}_2 - \tilde{\theta}_1 + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\widetilde{\Delta})\right)\right]qF = 1. \tag{A.2}$$

The left-hand side of equation (A.2) is strictly increasing in $\tilde{\theta}_2$ and strictly decreasing in $\tilde{\theta}_1$. We therefore have $\partial\tilde{\theta}_2/\partial\tilde{\theta}_1 > 0$ and $\partial\tilde{x}/\partial\tilde{\theta}_1 > 0$.Finally, we consider equation (10). The derivative of the left-hand side of equation (10) is

$$\frac{\partial K}{\partial\tilde{\theta}_1} + \frac{\partial K}{\partial\tilde{x}}\frac{\partial\tilde{x}}{\partial\tilde{\theta}_1},$$

where

$$\frac{\partial K}{\partial\tilde{\theta}_1} = f'(\tilde{\theta}_1) - (1-\gamma)(M-F)\sqrt{\beta}\varphi\left(\sqrt{\beta}(\tilde{x} - \tilde{\theta}_1)\right) < 0$$
$$\frac{\partial K}{\partial\tilde{x}}\frac{\partial\tilde{x}}{\partial\tilde{\theta}_1} = (1-\gamma)(M-F)\sqrt{\beta}\varphi\left(\sqrt{\beta}(\tilde{x} - \tilde{\theta}_1)\right)\frac{\partial\tilde{x}}{\partial\tilde{\theta}_1} > 0.$$

Note that $\widetilde{\Delta}$ is between zero and one. From equation (A.2), we have that as $\beta \to +\infty$, and thus $\widetilde{\Delta}$ is bounded away from both zero and one. To see this, suppose first that $\widetilde{\Delta} \to 1$. Then, the left-hand side of equation (A.2) goes to pF, which is greater than one , the right-hand side of equation (A.2). Similarly, if $\widetilde{\Delta} \to 0$, the left-hand side of equation (A.2) is strictly less than one.

Hence, from equation (A.1), we have $\tilde{x} \to \tilde{\theta}_2$. In addition, as $\beta \to +\infty$, $\tilde{\theta}_2$ cannot converge to $\tilde{\theta}_1$, as otherwise equations (10) and (11) cannot hold simultaneously. Therefore, as $\beta \to +\infty$, $\tilde{x} - \tilde{\theta}_1$ is bounded away from zero. This implies that

$$\lim_{\beta\to+\infty}\sqrt{\beta}\varphi\left(\sqrt{\beta}(\tilde{x} - \tilde{\theta}_1)\right) = 0,$$

which further implies that $\frac{\partial\tilde{\theta}_2}{\partial\tilde{\theta}_1}$ and $\frac{\partial\tilde{x}}{\partial\tilde{\theta}_1}$ converge to zero as $\beta \to +\infty$. Therefore, although $\frac{\partial K}{\partial\tilde{x}}\frac{\partial\tilde{x}}{\partial\tilde{\theta}_1} > 0$, when β is large enough, $\frac{\partial K}{\partial\tilde{x}}\frac{\partial\tilde{x}}{\partial\tilde{\theta}_1}$ is very close to zero. For the term $\frac{\partial K}{\partial\tilde{\theta}_1}$, it will not go to zero as β goes to ∞, because $f'(\tilde{\theta}_1) < 0$. Therefore, there exists a $\tilde{\beta} > 0$ such that, for all $\beta > \tilde{\beta}$, the left-hand side of equation (10) is strictly decreasing in $\tilde{\theta}_1$. The left-hand side of equation (10) will be less than M and thus less than H as $\tilde{\theta}_1$ goes to $+\infty$, and will diverge to $+\infty$ when $\tilde{\theta}_1$ goes to $-\infty$. Therefore, by the continuity of function $f(\cdot)$, there exists a unique $\tilde{\theta}_1$. We thus have that there is a unique solution to equations (10), (11), and (12). ■

PROOF OF LEMMA 1: We prove each part of this lemma.

(1) The case $\frac{V^p - V^q}{C^p - C^q} \geq 1 - q$.
Because $p > q$, this implies

$$\frac{V^p - V^q}{C^p - C^q} \geq 1 - q > 1 - p. \tag{A.3}$$

We first show that the CRA will not assign the rating q in equilibrium. Suppose that there is an equilibrium in which the CRA assigns the rating q to a θ-firm when $\theta \in (\theta_1, \theta_2)$. There are two cases. In the first case, a firm with rating $R = p$ is believed to have better economic fundamentals than a firm with rating $R = q$. Consider that the CRA deviates to assign the rating $R = p$ to the θ-firm. Then, observing the rating p, creditors are more optimistic about the firm's fundamentals and so more creditors buy the bonds. Consequently, the firm will have lower financial costs. It follows that after such a deviation by the CRA, the firm will not default at date 1. The condition $\frac{V^p - V^q}{C^p - C^q} \geq 1 - q > 1 - p$ then implies that if the firm invests in HR, we have $V^p - (1-q)C^p \geq V^q - (1-q)C^q$, and if the firm invests in VP, we have $V^p - (1-p)C^p > V^q - (1-p)C^q$. These arguments imply that such a deviation is profitable to the CRA.

Now consider the second case, in which a firm with rating q is believed to have better economic fundamentals than a firm with rating p. Then there must exist intervals $[\theta_3, \theta_4)$ and (θ_1, θ_2), where $\theta_3 < \theta_4 < \theta_1 < \theta_2$, such that the rating strategy specifies $\mathcal{R}(\theta) = p$ when $\theta \in [\theta_3, \theta_4)$ and $\mathcal{R}(\theta) = q$ when $\theta \in (\theta_1, \theta_2)$. Since the firms with economic fundamentals in both (θ_1, θ_2) and $[\theta_3, \theta_4)$ will not default at date 1, the θ_3-firm does not default early. Thus, if the CRA assigns the rating p to any θ-firm for $\theta \in (\theta_1, \theta_2)$, the θ-firm's financial costs are lower than the θ_3-firm's. Since such a θ-firm will have a lower operation cost than the θ_3-firm, the θ-firm does not default at date 1 after being assigned the rating p. It therefore follows from the condition $\frac{V^p - V^q}{C^p - C^q} \geq 1 - q$ that such a deviation is profitable to the CRA. These arguments show that the CRA will not assign the rating q in equilibrium.

We next prove that the rating strategy must be monotonic. Suppose that there exists an equilibrium in which a θ-firm does not default at date 1 if it is assigned the rating p. The CRA should respond by assigning the rating $\mathcal{R}(\theta) = p$, because by assigning the rating $R = 0$, the CRA can get zero profit only. Let $W(p)$ be the measure of creditors who choose to buy the bonds, after observing the credit rating $\mathcal{R}(\theta)$ and their own private signals. Then the assumption that the θ-firm does not default early implies

$$K(\theta) = f(\theta) + W(p, \theta)F + \big(1 - W(p, \theta)\big)M < H.$$

Now consider any θ'-firm with $\theta' > \theta$. Again, the CRA can only get zero payoff by assigning the rating $R = 0$, so it will assign the rating $R = p$ to the θ'-firm if the θ'-firm does not default at date 1. In a monotone equilibrium, any creditor i's strategy is monotonic in his private signal x_i, and any creditor's private signal conditional on θ' first-order stochastic dominates that conditional on θ. Thus, $W(p, \theta') > W(p, \theta)$, and we then have

$$K(\theta') = f(\theta') + W(p, \theta')F + \big(1 - W(p, \theta')\big)M$$

$$< f(\theta) + W(p,\theta)F + \big(1 - W(p,\theta)\big)M$$
$$< H.$$

Therefore, the θ'-firm does not default at date 1 either, implying that $R(\theta') = p$ in equilibrium.

Furthermore, independent of creditors' decisions, when θ is very negative, the firm will default early, and when θ is very positive, the firm will not default early. As a result, in any equilibrium (if one exists), the CRA's rating strategy must be of the form described by equation (14).

(2) The case $\frac{V^p - V^q}{C^p - C^q} \leq 1 - p$.

This case is same as the previous one, except that we replace the rating p by the rating q.

(3) The case $\frac{V^p - V^q}{C^p - C^q} \in (1 - p, 1 - q)$.

In this case, we have

$$V^p - (1-p)C^p > V^q - (1-p)C^q > V^q - (1-q)C^q > V^p - (1-q)C^p.$$

Therefore, if a θ-firm chooses VP (no matter whether it is assigned the rating p or the rating q), the CRA will assign the rating p. Similarly, if the θ-firm chooses HR (no matter whether it is assigned the rating p or the rating q), the CRA will assign the rating q. In addition, when the θ-firm chooses VP following the rating p and HR following the rating q, the CRA will assign the rating p. Furthermore, because for any particular θ-firm, the financial cost following the rating p is lower than the financial cost following the rating q, if the firm invests in HR following the rating p, it will invest in HR following the rating q.

Hence, the rating strategy must be of the form as in equation (16). Whether the rating q can appear in the equilibrium depends on the model's parameters, so θ^q could be equal to θ^p, in which case the CRA will not assign the rating q in equilibrium.

■

PROOF OF LEMMA 2: Suppose that there exists an equilibrium in which the firm invests in VP for all θ such that $\mathcal{R}(\theta) = p$. All creditors will invest in the bonds, leading the firm's financial costs to equal $(1-\gamma)F + \gamma M$. For the firm to choose VP if and only if $\theta \geq \theta_1^*$, we must have

$$f(\theta_1^*) + (1-\gamma)F + \gamma M \leq \frac{pV - qH}{p - q} < H.$$

But because $f(\cdot)$ is continuous and strictly decreasing, there exists $\widehat{\theta}_1^* < \theta_1^*$ such that

$$\frac{pV - qH}{p - q} < f(\widehat{\theta}_1^*) + (1-\gamma)F + \gamma M < H.$$

That is, there is a subset of θ's with positive measure that are greater than $\widehat{\theta}_1^*$ but very close to $\widehat{\theta}_1^*$ such that the firm will invest in HR. Since the firm's investment choice HR is unverifiable, a deviation to the rating strategy with $\widehat{\theta}_1^*$ is profitable to the CRA. Therefore, the rating strategy with θ_1^* such that $f(\theta_1^*)+(1-\gamma)F+\gamma M \leq \frac{pV-qH}{p-q}$ cannot be part of an equilibrium. Therefore, in any monotone equilibrium (if it exists), the rating strategy must be inflated. ∎

PROOF OF PROPOSITION 2: We first show that given the CRA's rating strategy θ_1^*, there is a unique equilibrium play of the firm and the creditors following the rating $R = p$. This is formally presented in Lemma A1 below.

LEMMA A1: *There exists a $\beta^* > 0$ such that, for any $\beta > \beta^*$ and given the CRA's rating strategy θ_1^*, following the rating p, there is a unique solution (θ_2^*, x^*), where $\theta_2^* > \theta_1^*$, to equation (20) and equation (21).*

PROOF OF LEMMA A 1: For a given $x^* \in \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$, the left-hand side (LHS) of equation (20) is strictly decreasing in θ. When $\theta \to +\infty$, the LHS of equation (20) goes to $(1-\gamma)F + \gamma M$, which is strictly less than $\frac{pV-qH}{p-q}$, since $\frac{pV-qH}{p-q} > M > F$. However, when $\theta = \theta_1^*$, if the LHS is still less than $\frac{pV-qH}{p-q}$, then the firm will always choose VP after the rating $R = p$. This contradicts Lemma 2. Therefore, for a given $x^* \in \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$, there exists a unique $\theta_2^* > \theta_1^*$ such that equation (20) holds. We can therefore solve for x^* from equation (20):

$$x^* = \theta_2^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}\left[\frac{\frac{pV-qH}{p-q} - \left[f(\theta_2^*) + (1-\gamma)F + \gamma M\right]}{(1-\gamma)\left[M-F\right]}\right].$$

Define

$$\Delta = \frac{\frac{pV-qH}{p-q} - \left[f(\theta_2^*) + (1-\gamma)F + \gamma M\right]}{(1-\gamma)\left[M-F\right]},$$

so $x^* = \theta_2^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)$. Because $f(\cdot)$ is strictly decreasing, Δ is strictly increasing in θ_2^*, and thus x^* is strictly increasing in θ_2^*.

Next, plugging x^* as a function of θ_2^* into equation (21), we have

$$\frac{\Delta}{\Phi\left[\sqrt{\beta}\left(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)\right)\right]}(pF - qF) = 1 - qF. \tag{A.4}$$

Differentiating the LHS of equation (A.4), the sign of this derivative would be the same as the sign of

$$\frac{\partial\Delta}{\partial\theta_2^*}\frac{\Phi\left[\sqrt{\beta}\left(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)\right)\right]}{\Delta}$$
$$-\varphi\left[\sqrt{\beta}\left(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)\right)\right]\left(\sqrt{\beta} + \frac{1}{\varphi(\Delta)}\frac{\partial\Delta}{\partial\theta_2^*}\right)$$

$$= \frac{\partial \Delta}{\partial \theta_2^*} \frac{pF - qF}{1 - qF} - \varphi\left[\sqrt{\beta}\left(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)\right)\right]\left(\sqrt{\beta} + \frac{1}{\varphi(\Delta)}\frac{\partial \Delta}{\partial \theta_2^*}\right).$$

The first term is positive for any β, because Δ is not a function of β, and θ_2^* is bounded (and so $f'(\theta_2^*)$ is bounded away from zero).

The second term, although negative, will converge to zero as $\beta \to +\infty$. This is because $\varphi[\sqrt{\beta}(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta))]$ will converge to zero higher order faster than $\sqrt{\beta}$. We need to consider three cases to prove this argument. First, as $\beta \to +\infty$, Δ must be bounded away from one; otherwise, the LHS of equation (A.4) converges to $pF - qF$, which is strictly greater than $1 - qF$, the right-hand side (RHS) of equation (A.4). Second, suppose that as $\beta \to +\infty$, Δ is also bounded away from zero. Then $x^* - \theta_2^* \to 0$. But it follows from equation (A.4) that $\theta_2^* - \theta_1^*$ must be positive and bounded away from zero; otherwise, the LHS of equation (A.4) converges to $pF - qF$, which is strictly greater than the RHS of equation (A.4). Hence, $\varphi[\sqrt{\beta}(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta))]\sqrt{\beta} = \varphi[\sqrt{\beta}(x^* - \theta_1^*)]\sqrt{\beta}$ must converge to zero. Finally, as $\beta \to +\infty$, $\Delta \to 0$. Thus, from equation (A.4) we must have $\Phi[\sqrt{\beta}(x^* - \theta_1^*)] \to 0$ and leave $\sqrt{\beta}(x^* - \theta_1^*) \to -\infty$ as $\beta \to +\infty$. By L'Hôpital's rule, we have

$$\lim_{\beta \to +\infty} \frac{1}{\sqrt{\beta}(x^* - \theta_1^*)} = \lim_{\beta \to +\infty} \frac{\beta^{-\frac{1}{2}}}{(x^* - \theta_1^*)} = \lim_{\beta \to +\infty} \frac{1}{2\beta^{\frac{3}{2}} \frac{\partial x^*}{\partial \beta}} = 0.$$

Therefore, $\lim_{\beta \to +\infty} 2\beta^{\frac{3}{2}} \frac{\partial x^*}{\partial \beta} = +\infty$. Simple algebra can lead to the result that $\varphi[\sqrt{\beta}(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta))]\sqrt{\beta}$ converges to zero, as $\beta \to +\infty$. We therefore have that there exists a β^* such that when $\beta > \beta^*$, the LHS of equation (A.4) is strictly increasing in θ_2^*.

Note that definition, Δ must be a number in $[0, 1]$. Therefore, there exist $\bar{\theta}$ and $\underline{\theta}$ such that $\theta_1^* < \underline{\theta} < \bar{\theta} < +\infty$, $\Delta(\bar{\theta}) = 1$, and $\Delta(\underline{\theta}) = 0$. When $\theta_2^* \to \bar{\theta}$, the LHS of equation (A.4) is strictly greater than $1 - qF$; when $\theta_2^* \to \underline{\theta}$, the LHS of equation (A.4) is close to zero and thus strictly smaller than $1 - qF$.

Therefore, there exists a unique θ_2^*, and thus there exists a unique x^*. ■

We now argue that the creditor's threshold x^* and the firm's VP-investment threshold θ_2^* are both strictly increasing in θ_1^*. Intuitively, when the CRA employs a laxer rating strategy (a smaller θ_1^*), the creditors will discount the positive information conveyed by the rating $R = p$ and so are less likely to buy the bonds (x^* increases). This increases the firm's financial costs, and so the firm is less likely to invest in VP (θ_2^* increases). This argument is presented in Lemma A2 below.

LEMMA A2: *For any $\beta > \beta^*$, both x^* and θ_2^* are strictly decreasing in θ_1^*.*

PROOF OF LEMMA A 2: The LHS of equation (A.4) is strictly increasing in θ_1^*, fixing θ_2^*. It also follows from the proof of Lemma A1 that the LHS of equation (A.4) is strictly increasing in θ_2^*. The Implicit Function Theorem then implies that θ_2^* is strictly decreasing in θ_1^*. Since x^* is strictly increasing in θ_2^*, x^* is strictly decreasing in θ_1^* (given θ_2^*, x^* is determined by equation $x^* = \theta_2^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}[\Delta]$). ■

Given Lemma A1 and Lemma A2, to prove Proposition 2, we only need to show that there exists a unique θ_1^* such that equation (22) holds, given x^* as a function of θ_1^*. When β is sufficiently large, by Lemma A2, x^* is strictly decreasing in θ_1^*. Then the derivative of the LHS of equation (22) with respect to θ is

$$f'(\theta) - (1-\gamma)\varphi[\sqrt{\beta}(x^* - \theta)]\sqrt{\beta}(M - F)$$
$$+(1-\gamma)(M-F)\varphi[\sqrt{\beta}(x^* - \theta)]\sqrt{\beta}\frac{\partial x^*}{\partial \theta} < 0.$$

We know that when $\theta \to +\infty$, $f(\theta) \to 0$, and the LHS of equation (22) converges to $(1-\gamma)F + \gamma M$, which is less than H; when $\theta \to -\infty$, $f(\theta) \to +\infty$, and the LHS of equation (22) diverges to $+\infty$, which is greater than H. Therefore, the solution to equation (22) exists and is unique.

Because $H > \frac{pV - qH}{p - q}$, equations (22) and (20) imply that $\theta_2^* > \theta_1^*$. In addition, equation (22) also implies that $f(\theta_1^*) + M > H$, because $M > F$. This completes the proof of the uniqueness of the equilibrium of the model. ■

PROOF OF LEMMA 3: Recall that the three equations determining the equilibrium of the model without the CRA are

$$f(\theta_1) + \left[(1-\gamma)F + \gamma M\right] + (1-\gamma)\Phi\left[\sqrt{\beta}(x - \theta_1)\right](M - F) = H, \tag{A.5}$$

$$f(\theta_2) + \left[(1-\gamma)F + \gamma M\right] + (1-\gamma)\Phi\left[\sqrt{\beta}(x - \theta_2)\right](M - F) = \frac{pV - qH}{p - q}, \tag{A.6}$$

$$\left\{\Phi\left[\sqrt{\beta}(\theta_2 - x)\right] - \Phi\left[\sqrt{\beta}(\theta_1 - x)\right]\right\}qF + \left\{1 - \Phi\left[\sqrt{\beta}(\theta_2 - x)\right]\right\}pF = 1; \tag{A.7}$$

and the three equations determining the equilibrium of the model with the CRA are

$$f(\theta_1) + \left[(1-\gamma)F + \gamma M\right] + (1-\gamma)\Phi\left[\sqrt{\beta}(x - \theta_1)\right](M - F) = H, \tag{A.8}$$

$$f(\theta_2) + \left[(1-\gamma)F + \gamma M\right] + (1-\gamma)\Phi\left[\sqrt{\beta}(x - \theta_2)\right](M - F) = \frac{pV - qH}{p - q}, \tag{A.9}$$

$$\frac{\Phi[\sqrt{\beta}(\theta_2 - x)] - \Phi[\sqrt{\beta}(\theta_1 - x)]}{1 - \Phi[\sqrt{\beta}(\theta_1 - x)]}qF + \frac{1 - \Phi[\sqrt{\beta}(\theta_2 - x)]}{1 - \Phi[\sqrt{\beta}(\theta_1 - x)]}pF = 1. \tag{A.10}$$

The difference between the equilibrium in the benchmark model and that in the model with the CRA stems from the difference between equations (A.7) and (A.10). That is, the creditors' indifference conditions differ. If we change equation (A.7) by dividing both sides by the term $1 - \Phi[\sqrt{\beta}(\theta_1 - x)]$, we have

$$\begin{aligned} &\frac{\Phi[\sqrt{\beta}(\theta_2 - x)] - \Phi[\sqrt{\beta}(\theta_1 - x)]}{1 - \Phi[\sqrt{\beta}(\theta_1 - x)]} qF + \frac{1 - \Phi[\sqrt{\beta}(\theta_2 - x)]}{1 - \Phi[\sqrt{\beta}(\theta_1 - x)]} pF \\ &= \frac{1}{1 - \Phi[\sqrt{\beta}(\theta_1 - x)]} \\ &> 1. \end{aligned} \tag{A.11}$$

Solve x as a function of θ_2 from equation (A.6) or equation (A.9), and plug it into equations (A.11) and (A.10). Then, for the same θ_1, θ_2 in equation (A.11) is greater than that in equation (A.10) because the LHSs of these two equations are strictly increasing in θ_2. Hence, x in equation (A.11) is greater than x in equation (A.10). Furthermore, because θ_1 in the benchmark model is positively correlated with θ_2, while θ_1 in the model with the CRA is negatively correlated with θ_2, we know that $\theta_1^* < \tilde{\theta}_1$. Moreover, we have $\tilde{\theta}_2 > \theta_2^*$ and $\tilde{x} > x^*$.

However, the sign of $\theta_2^* - \tilde{\theta}_1$ is undetermined. Consider equations (A.5) and (A.9). Both $\tilde{\theta}_1$ and θ_2^* are strictly increasing functions of x. While we have shown that $\tilde{x} > x^*$, the RHS of equation (A.5) is greater than that of equation (A.9). Therefore, without specifying parameters' values, we cannot determine the sign of $\theta^*{}_2 - \tilde{\theta}_1$. ■

PROOF OF PROPOSITION 3: We denote by Ω and $\tilde{\Omega}$ the equilibrium economic efficiency in the model with the CRA and without the CRA, respectively. We first consider the firm with $\theta \geq \tilde{\theta}_2$. It follows from Proposition 2 that the CRA assigns the firm the rating p, and hence the firm will invest in VP. In the model without the CRA, the firm will also invest in VP, and thus, $\Omega = \tilde{\Omega}$. Hence, the CRA has no effect on the expected net present value (NPV). Similarly, when $\theta < \theta_1^*$, the firm defaults at date 1 with or without the CRA. Therefore, $\Omega = \tilde{\Omega}$. Hence, when $\theta < \theta_1^*$, the CRA has no effect on economic efficiency either.

When $\theta \in [\theta_1^*, \tilde{\theta}_2)$, the CRA's effect depends on the parameters of the model. In the first case where $\theta_2^* > \tilde{\theta}_1$, for $\theta \in [\theta_1^*, \tilde{\theta}_1)$, $\Omega - \tilde{\Omega} = qH - L < 0$, because qH is assumed to be less than L. For $\theta \in [\theta_2^*, \tilde{\theta}_2)$, $\Omega - \tilde{\Omega} = pV - qH > 0$. For all $\theta \in [\tilde{\theta}_1, \theta_2^*)$, the firm invests in HR with or without the CRA and thus the CRA has no effect on economic efficiency. In this case, the CRA's ex ante real effects are

$$(\tilde{\theta}_2 - \theta_2^*)(pV - qH) + (\tilde{\theta}_1 - \theta_1^*)(qH - L).$$

In the second case where $\theta_2^* \leq \tilde{\theta}_1$, for $\theta \in [\theta_1^*, \theta_2^*)$, $\Omega - \tilde{\Omega} = qH - L$. For $\theta \in [\theta_2^*, \tilde{\theta}_1)$, $\Omega - \tilde{\Omega} = pV - L$. Finally, for $\theta \in [\tilde{\theta}_1, \tilde{\theta}_2)$, $\Omega - \tilde{\Omega} = pV - qH$. Therefore, in this case, the CRA's ex ante real effects are

$$(\tilde{\theta}_2 - \tilde{\theta}_1)(pV - qH) + (\tilde{\theta}_1 - \theta_2^*)(pV - L) + (\theta_2^* - \theta_1^*)(qH - L).$$

■

PROOF OF PROPOSITION 4: Part 1: We first consider the continuation play following the rating $R = 0$. It then follows from equation (23) that $\theta < \hat{\theta}_1 = \tilde{\theta}_1$. Suppose that all creditors refrains from investing. Then if the θ-firm continues to make investments, its total repayments at date 2 are

$$\begin{aligned} K(\theta) &= f(\theta) + M \\ &> f(\theta) + \big[(1-\gamma)F + \gamma M\big] + (1-\gamma)\Phi\Big(\sqrt{\beta}(\tilde{x} - \theta)\Big)(M - F) \\ &> f(\tilde{\theta}_1) + \big[(1-\gamma)F + \gamma M\big] + (1-\gamma)\Phi\Big(\sqrt{\beta}(\tilde{x} - \tilde{\theta}_1)\Big)(M - F) \\ &= H. \end{aligned}$$

Hence, if all creditors refrains from investing, the θ-firm will default at date 1. On the other hand, given that any θ-firm will default early, no creditor will invest in the bonds, implying that there is an equilibrium in which the firm will default early when receiving the rating $R = 0$ from the reflecting CRA.

We now show that in the continuation play following $R = 0$, the firm will not continue to invest in either HR or VP. Suppose that there is a (monotone) equilibrium in which a creditor with the private signal x_i invests in the bonds if and only if $x_i \geq x'$ when the rating is $R = 0$. Here, $x' \in \mathbb{R}$. Since some creditors are willing to invest, they must believe that any θ-firm will invest in VP if $\theta \in [\theta_2', \hat{\theta}_1)$, and that by the continuity of the firm's financial costs, any θ-firm will invest in HR if $\theta \in [\theta_1', \theta_2')$, where $\theta_1', \theta_2' \in \mathbb{R}$ and $\theta_1' < \theta_2' < \hat{\theta}_1$. Such an equilibrium can be characterized by the following system of equations:

$$f(\theta_1') + \big[(1-\gamma)F + \gamma M\big] + (1-\gamma)\Phi\left[\sqrt{\beta}(x' - \theta_1')\right](M - F) = H, \tag{A.12}$$

$$f(\theta_2') + \big[(1-\gamma)F + \gamma M\big] + (1-\gamma)\Phi\left[\sqrt{\beta}(x' - \theta_2')\right](M - F) = \frac{pV - qH}{p - q}, \tag{A.13}$$

$$\frac{\Phi[\sqrt{\beta}(\theta_2' - x')] - \Phi[\sqrt{\beta}(\theta_1' - x')]}{\Phi[\sqrt{\beta}(\widehat{\theta} - x')]} qF + \frac{\Phi[\sqrt{\beta}(\widehat{\theta} - x')] - \Phi[\sqrt{\beta}(\theta_2' - x')]}{\Phi[\sqrt{\beta}(\widehat{\theta} - x')]} pF = 1. \tag{A.14}$$

Comparing equation (A.13) with equation (11), we can see that since $\theta_2' < \widehat{\theta}_1 = \tilde{\theta}_1 < \tilde{\theta}_2$, x' must be strictly less than $\widehat{x}$.Note that for any $\widehat{\theta}_1$ (which may not be $\tilde{\theta}_1$), given the committed rating rule (equation (23)), a monotone equilibrium with some θ-firm investing in VP or HR must be characterized by the system of equations (A.12), (A.13), and (A.14). The following argument therefore relies on the comparative statics analysis of the solution to such a system of equations.

Solving x' as a function of θ_2' from equation (A.13) and substituting it into equations (A.12) and (A.14), we get

$$f(\theta_1') + (1-\gamma)(M - F)\Phi(\Psi) = H - [(1-\gamma)F + \gamma M]$$

$$\Delta(pF - qF) + qF\Phi(\Psi) - (pF - 1)\Phi\left(\Psi'\right) = 1,$$

where $\Delta = \frac{\frac{pV-qH}{p-q} - [(1-\gamma)F + \gamma M] - f(\theta_2')}{(1-\gamma)(M-F)}$, $\Psi = \sqrt{\beta}(\theta_2' - \theta_1') + \Phi^{-1}(\Delta)$, and $\Psi' = \sqrt{\beta}(\theta_2' - \widehat{\theta}_1) + \Phi^{-1}(\Delta)$. Totally differentiating the above system of equations, we get

$$A \begin{bmatrix} \dfrac{\partial\theta_1'}{\partial\widehat{\theta}_1} \\ \dfrac{\partial\theta_2'}{\partial\widehat{\theta}_1} \end{bmatrix} = \begin{bmatrix} 0 \\ (pF - 1)\varphi(\Psi')\dfrac{\partial\Psi'}{\partial\widehat{\theta}_1} \end{bmatrix}, \tag{A.15}$$

where

$$A = \begin{bmatrix} f'(\theta_1') + (1-\gamma)(M-F)\varphi(\Psi)\frac{\partial\Psi}{\partial\theta_1'} & (1-\gamma)(M-F)\varphi(\Psi)\frac{\partial\Psi}{\partial\theta_2'} \\ qF\varphi(\Psi)\frac{\partial\Psi}{\partial\theta_1'} & (p-q)F\frac{\partial\Delta}{\partial\theta_2'} + qF\varphi(\Psi)\frac{\partial\Psi}{\partial\theta_2'} - (pF-1)\varphi(\Psi')\frac{\partial\Psi'}{\partial\theta_2'} \end{bmatrix}. \tag{A.16}$$

Note that when β is sufficiently large, $|A| < 0$. We show that $\partial\theta_1'/\partial\widehat{\theta}_1 < 0$ and $\partial\theta_2'/\partial\widehat{\theta}_1 < 0$. Therefore, when $\widehat{\theta}$ goes to $+\infty$, the equilibrium converges to an equilibrium with $x' < \widehat{x}$. However, when $\widehat{\theta}$ goes to $+\infty$, equation (A.14) goes to equation (12). This implies that the benchmark model has another equilibrium with $x' < \widehat{x}$. This contradicts the conclusion in Proposition 1 that the benchmark model has a unique equilibrium. Therefore, in the case with a reflecting CRA, the subgame following $R = 0$ has a unique equilibrium in which the firm defaults early and all creditors run.

Part 2: Similar to the proof of Part 1, in the continuation play following $R = p$, there cannot be an equilibrium in which a positive measure of firm types default early. Otherwise, when $\widehat{\theta}_1$ goes to $-\infty$, we show that the benchmark model will have two different equilibria, violating the equilibrium uniqueness result.

Now, suppose that $\theta_2^* > \tilde{\theta}_1$. Also, suppose that there exists $\widehat{\theta}_2 \in [\widehat{\theta}_1, +\infty)$ such that with the reflecting CRA, in the subgame following the rating $R = p$, the firm invests in VP if $\theta \le \widehat{\theta}_2$ and in HR if $\theta \in [\widehat{\theta}_1, \widehat{\theta}_2)$. It follows from Lemma A2 that $\widehat{\theta}_2$ and $\widehat{x}$ are both strictly decreasing in $\widehat{\theta}_1$. If $\widehat{\theta}_2 > \theta_2^*$ is part of an equilibrium, then when the reflecting CRA has the rating strategy $\widehat{\theta}_1 = \theta_1^*$, there is an equilibrium in which $\widehat{\theta}_2 > \theta_2^*$, violating the equilibrium uniqueness conclusion in Proposition 2. Therefore, in an equilibrium of the subgame following $R = p$, the firm invests in VP if $\theta \le \widehat{\theta}_2$ and in HR if $\theta \in [\widehat{\theta}_1, \widehat{\theta}_2)$. Furthermore, if $\theta_2^* > \tilde{\theta}_1$, we have $\widehat{\theta}_2 < \theta_2^*$. ■

PROOF OF PROPOSITION 6: We first show the comparative statics analysis with respect to β. Recall that

$$x^* = \theta_2^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}[\Delta],$$

where

$$\Delta = \frac{\frac{pV-qH}{p-q} - \left[(1-\gamma)F + \gamma M\right] - f(\theta_2^*)}{(1-\gamma)\left[M-F\right]},$$

so $\partial\Delta/\partial\theta_2^* > 0$.

Substitute x^* as a function of θ_2^* into equations (21) and (22), and define $\sqrt{\beta}(\theta_2^* - \theta_1^*) + \Phi^{-1}(\Delta) = \Psi$ for simplicity. We then have

$$\Delta(pF - qF) - \Phi\left[\Psi\right](1 - qF) = 0, \tag{A.17}$$

$$f(\theta_1^*) + (1-\gamma)(M-F)\Phi\left[\Psi\right] = H - [(1-\gamma)F + \gamma M]. \tag{A.18}$$

Total differentiation of the two equations above with respect to θ_2^*, θ_1^*, and β, we have

$$A\begin{bmatrix} \dfrac{\partial\theta_2^*}{\partial\beta} \\ \dfrac{\theta_1^*}{\partial\beta} \end{bmatrix} = \begin{bmatrix} \varphi(\Psi)(1-qF)\dfrac{\theta_2^*-\theta_1^*}{2\sqrt{\beta}} \\ -(1-\gamma)(M-F)\varphi(\Psi)\dfrac{\theta_2^*-\theta_1^*}{2\sqrt{\beta}} \end{bmatrix}, \tag{A.19}$$

where

$$A = \begin{bmatrix} \frac{\partial\Delta}{\partial\theta_2^*}(pF-qF) - \varphi(\Psi)\left(\sqrt{\beta} + \frac{1}{\varphi(\Delta)}\frac{\partial\Delta}{\partial\theta_2^*}\right)(1-qF) & \varphi(\Psi)\sqrt{\beta}(1-qF) \\ (1-\gamma)(M-F)\varphi(\Psi)\left(\sqrt{\beta} + \frac{1}{\varphi(\Delta)}\frac{\partial\Delta}{\partial\theta_2^*}\right) & f'(\theta_1^*) - (1-\gamma)(M-F)\varphi(\Psi)\sqrt{\beta} \end{bmatrix}. \tag{A.20}$$

As we have shown in the proof of Lemma A1, when β is large enough, $\varphi(\Psi)\sqrt{\beta}$ is very close to zero. Therefore, when β is sufficiently large, the determinant of the matrix A is close to

$$\frac{\partial\Delta}{\partial\theta_2^*}(pF-qF)f'(\theta_1^*) < 0,$$

because $f'(\theta_1^*) < 0$.

Further algebra shows that when β is sufficiently large, the sign of $\partial\theta_1^*/\partial\beta$ is the same as that of

$$(1-\gamma)(M-F)(pF-qF)\frac{\partial\Delta}{\partial\theta_2^*},$$

which is positive. Therefore, θ_1^* is strictly increasing in β.

Now let us consider the comparative statics analysis with respect to H. After total differentiation of equations (A.17) and (A.18) with respect to θ_2^*, θ_1^*, and

H, we have

$$A\begin{bmatrix}\dfrac{\partial\theta_2^*}{\partial H}\\ \dfrac{\partial\theta_1^*}{\partial H}\end{bmatrix}=\begin{bmatrix}-(pF-qF)\dfrac{\partial\Delta}{\partial H}+\varphi(\Psi)\dfrac{1}{\varphi(\Delta)}\dfrac{\partial\Delta}{\partial H}(1-qF)\\ 1-(1-\gamma)(M-F)\varphi(\Psi)\dfrac{1}{\varphi(\Delta)}\dfrac{\partial\Delta}{\partial H}\end{bmatrix},$$

where A is defined in equation (A.20).

Note that $\varphi(\Psi)\sqrt{\beta}$ is very close to zero when β is sufficiently large. We have

$$\text{sign}\begin{bmatrix}\dfrac{\partial\theta_2^*}{\partial H}\\ \dfrac{\partial\theta_1^*}{\partial H}\end{bmatrix}=\text{sign}\begin{bmatrix}(pF-qF)f'(\theta_1^*)\dfrac{\partial\Delta}{\partial H}\\ -(pF-qF)\dfrac{\partial\Delta}{\partial\theta_2^*}\end{bmatrix}.$$

Because $f'(\theta_1^*)<0$, $\partial\Delta/\partial H<0$, and $\partial\Delta/\partial\theta_2^*>0$, we have $\partial\theta_1^*/\partial H<0$ and $\partial\theta_2^*/\partial H>0$. Therefore, $\partial(\theta_2^*-\theta_1^*)/\partial H>0$.

We finally show the comparative statics analysis with respect to γ. Similar to the case for H, after total differentiation of equations (A.17) and (A.18) with respect to θ_2^*, θ_1^*, and γ, we have

$$A\begin{bmatrix}\dfrac{\partial\theta_2^*}{\partial\gamma}\\ \dfrac{\partial\theta_1^*}{\partial\gamma}\end{bmatrix}=\begin{bmatrix}-(pF-qF)\dfrac{\partial\Delta}{\partial\gamma}+\varphi(\Psi)\dfrac{1}{\varphi(\Delta)}\dfrac{\partial\Delta}{\partial\gamma}(1-qF)\\ (M-F)\left[\Phi(\Psi)-1-(1-\gamma)\varphi(\Psi)\dfrac{1}{\varphi(\Delta)}\dfrac{\partial\Delta}{\partial\gamma}\right]\end{bmatrix},$$

where A is defined in equation (A.20).Note that $\varphi(\Psi)\sqrt{\beta}$ is very close to zero when β is sufficiently large. In addition, $\partial\Delta/\partial\gamma>0$ and $(1-\Phi(\Psi))/\varphi(\Psi)\to 0$ when $\beta\to+\infty$. Simple algebra shows that $\partial\theta_1^*/\partial\gamma>0$ and $\partial\theta_2^*/\partial\gamma>0$. Further algebra shows that $\partial(\theta_2^*-\theta_1^*)/\partial\gamma>0$. ■

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix.
Replication code.