Credit Rating Inflation and Firms' Investments

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Abstract

We analyze credit ratings' effects on firms' investments in a rational debt-rollover game that features a feedback loop. The credit rating agency (CRA) inflates the rating, providing a biased but informative signal to investors. Investors' response to the rating affects the firm's investment decision and credit quality, which is reflected in the rating. The CRA might reduce ex-ante economic efficiency, which results solely from the feedback effect of the rating: The CRA assigns more firms high ratings and allows them to gamble for resurrection. We derive empirical predictions on the determinants of rating standards and inflation and discuss policy implications.

Key Words: Credit rating agency, rating inflation, real effect, feedback effect, global game

JEL Classification: D82, D83, G24, G32

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1 Introduction

Credit rating agencies (CRAs) have been criticized for playing a central role in financial failures. Prominent examples include the collapse of companies like Enron and WorldCom in 2002, and the crisis of $2007-2009$ that led the Financial Crisis Inquiry Report to conclude that "the failures of credit rating agencies were essential cogs in the wheel of financial destruction." CRAs are often blamed for assigning overgenerous ratings, and this has been documented by several empirical studies.[1] These studies argue that the credit rating inflation can be attributed to the conflicts of interest caused by the "issuer-pays" business model, according to which CRAs are paid by the issuers they are assessing. The concern is then that inflated credit ratings might mislead investors, help bad investments get funded, and thus have negative real effects.

Thinking about these claims through the lens of models with rational investors, it is not clear why inflated credit ratings would have negative real effects. To mislead investors, credit ratings must provide some valuable information. Otherwise, the ratings would be ignored and CRAs would have no real effect. But, if CRAs are providing investors with informative (though potentially biased) signals, they should be able to promote, rather than hurt, economic efficiency, even if they do not reach the first-best outcome. The question then is whether CRAs with a motive to inflate ratings can have negative effects on economic efficiency in a world with rational investors.

In this paper, we provide a model to analyze this question. Our model is parsimonious, but rich enough to capture the essential elements of the interaction between a CRA, investors, and the issuing firm. First, we consider a CRA with an inherent motive to inflate the rating provided to the issuing firm, but who is also subject to a partial verifiability constraint. By such a constraint, the CRA will never assign a high rating to a firm who will surely fail. Hence, its rating is biased but contains valuable information. Second, the audience for the rating is a group of rational investors who have to make a decision whether to invest in the debt of the firm or not. The interaction between the investors constitutes a coordination game, as the benefit that each investor derives from investing in the debt depends in part on how many other investors do so. Third, the aggregate action taken by the investors affects the cost of capital of the firm, which affects the firm's investment decision and, through that, its credit quality. Fourth, when setting the initial rating, the CRA accounts for the effect of the rating on the actions of investors and the firm and their effects on the credit quality. Hence, there is a feedback loop, whereby the rating affects investors' behavior, which affects the behavior of the

[1] See, for example, Jiang, Stanford, and Xie (2012), Strobl and Xia (2012), and Cornaggia and Cornaggia (2013).

issuer and its credit quality, which in turn is reflected in the rating.

In our view, this feedback loop is central to understanding the effects of CRAs. After all, CRAs, given their market power, are in a unique position to provide information that ends up affecting the credit quality of the firms that they rate. Indeed, credit rating agencies claim that their ratings are forward-looking, emphasizing that they will assess the potential impact of foreseeable future events that include the impacts of the ratings themselves. For example, Moody's, in a document that explains its rating process, explicitly acknowledges "the effect of the rating action on the issuer, including the possible effect on issuer's market access or conditional obligations." It goes on to note that "the level of rating that Moody's assigns to an issuer that might experience potential changes in market access or conditional obligations will reflect Moody's assessment of the issuer's creditworthiness, including such considerations." As we argue below, this feedback loop is largely missing from the existing literature on credit rating agencies. We show that it plays a critical role in our results.

In our model, a high rating, even though potentially inflated, provides positive information to investors because it implies that the firm does not belong to a group of particularly low quality, for which the partial verifiability constraint binds.[2] Hence, a high rating makes creditors more optimistic and likely to invest in the debt, which reduces the firm's financial costs and changes its investment decision. This is how the rating ends up having a real effect. For some firms, for which financial costs are relatively high, the reduction in financial costs leads to inefficient risk taking. Lower financial costs enable them to gamble for resurrection and take an investment with negative net present value but high potential upside. For other firms, for which financial costs are relatively low, the reduction in financial costs provides more skin in the game encouraging a shift from risky negative net-present-value investments to less risky positive net-present-value investments. The implications for economic efficiency are negative in the first case and positive in the second. Hence, the overall effect of the CRA on economic efficiency depends on how strong these effects are expected to be relative to each other. This depends on model parameters.

Varying the parameters of the model, we can show that the overall expected real effects of the CRA can be positive or negative. A key result that we identify is that the CRA's expected real effect is more likely to be negative when the upside of the risky inefficient investment

[2]Hence, a high credit rating generated by the lax rating strategy is not a cheap talk as in Crawford and Sobel (1982). Due to the partial verifiability constraint, the high rating provides the creditors with a public signal about the firm. Such a public signal is endogenous and takes a different form from that in Morris and Shin (2002): It truncates the supports of creditors' interim beliefs from below.

is higher. A high upside makes gambling for resurrection more attractive and more likely to follow from a reduction in the cost of capital. To put this result in context, we can tie it to the conditions during the subprime boom, where many financial institutions, such as Bear Stearns and Lehman Brothers were actively issuing securities backed by subprime mortgages. These have high profit potential, but also feature high default risk. Hence, the introduction of subprime mortgage-backed securities to the financial system might have shifted the balance of the effects of the CRAs' rating inflation to a point where the overall expected effect on economic efficiency was negative, and so CRAs were subject to much more criticism in the wake of the crisis.[3]

An important insight of our model emerges when we decompose the CRA's ex-ante real effects into two components. The first one is the CRA's pure *informational effect* and the second one is the CRA's *feedback effect*. The informational effect is obtained when the CRA does not recognize the effect of its rating on the firm's investment and credit quality, and just provides the (biased) information that would pertain to the equilibrium without a CRA. The feedback effect is the additional effect coming from the fact that the CRA is strategic and takes into account how the rating affects the investors and the firm. It then assigns a rating that takes advantage of these responses in maximizing its objective function (which is to provide high rating subject to the partial verifiability constraint).

We show that the informational effect always increases economic efficiency. When the CRA acts in a reflecting way, just providing the biased information to the investors, it helps them achieve a more efficient outcome. The negative implications for economic efficiency thus come purely from the feedback effect: When taking into account the creditors' and the firm's responses to the ratings, the CRA finds it beneficial to assign high ratings to more firms, allowing them to gamble for resurrection. When they gamble for resurrection, the CRA can assign them a high rating without violating the partial verifiability constraint, and so achieve higher values of its objective function at the expense of lower economic efficiency. The CRA essentially uses its market power in information provision to shape economic outcomes, and because of the inflation motive, economic efficiency might be sacrificed. This proves that the introduction of feedback effects into models of credit ratings is indeed crucial for understanding the overall effects of CRAs.

We derive several empirical implications out of the model. First, a key insight that emerges out of our analysis is that lax rating standards and rating inflation are two different endoge-

[3]Note that we are referring here to the credit ratings issued to the financial institutions and not to the products themselves, although surely the latter also contributed to the overall negative real effect.

nous terms and they do not necessarily move in the same direction. Laxer rating standards correspond to a case where the CRA is more likely to give a high rating to a given firm with given fundamentals. However, this does not necessarily imply higher rating inflation. The reason is the feedback effect: When the CRA changes the rating policy, it also affects the credit quality of the firm, and so inflation, which is the difference between reported credit quality and actual credit quality, could change in either direction. This is an important point to consider in future empirical work. Second, we conduct several comparative static analyses, which demonstrate this point and provide new empirical predictions about CRAs' credit rating standards and credit rating inflation. In particular, a decrease in the firm's transparency,[4] an increase in upside returns of risky projects, and an increase in the market liquidity will all lead to laxer rating standards (assigning high ratings to more firms). However, these changes of economic environments do not necessarily cause higher rating inflation. Specifically, a decrease in the firm's transparency has an ambiguous effect on rating inflation, an increase in upside returns of risky projects will cause higher rating inflation, and an increase in the market liquidity leads to lower rating inflation.

We also derive policy implications. The inefficiencies from rating inflation highlighted in our paper can be avoided if the rating agencies face different structure of costs and benefits. For example, setting a cost for the rating agency when the issuer ends up defaulting and the rating was high would serve to tame the incentive to inflate. It should be noted, however, that the ratio of the incremental cost to the incremental benefit of a higher rating should not be too high, because then the CRA would have an incentive to deflate the rating, and this ends up having the same efficiency implications as an inflation motive. Therefore, in order to get to a truth telling CRA in equilibrium, the ratio of the incremental cost and the incremental benefit of a higher rating has to be set in a particular range. Unfortunately, this might be difficult for policymakers to calibrate.

The fact that rating inflation and rating deflation end up having the same efficiency implications highlights that the key effect of the CRA is that of pooling different firms together. This is intuitive, given that in our model, creditors are all rational and update based on the equilibrium strategy of the CRA. Hence, the label that is put on firms that are pooled together should not matter; the only thing that matters is which firms are pooled together. This is different from other models, where the effect of inflation is assumed via investors' irrationality or regulatory arbitrage. We stick to the focus on inflation rather than deflation because inflation is the

[4]This is related to Fong, Hong, Kacperczyk, and Kubik (2014), who find that higher analyst coverage provides investors with more information about firms, leading to stricter rating strategies.

phenomenon that has been discussed empirically and in policy circles. But, it is important to highlight that in a model where the real effect is driven by rational updating, inflation should not have different efficiency implications than deflation.

Finally, a question that arises concerning the real effects of credit ratings is whether they should be expected to persist in a framework where, in addition to credit ratings, there are various other signals available to investors in the corporate bond market. We argue that even if there are other public signals, the CRA's real effects are still significant given creditors' heterogeneous private signals about the firm's economic fundamentals and the strategic complementarities among them. By truncating their belief supports, some creditors' beliefs about the firm's investment choice are surely affected by the credit ratings. The strategic complementarities among creditors will then amplify such an effect to other creditors, leading to the significant real effects of the CRA. We demonstrate that this is true even when creditors have precise information from other sources.

The real effects of the CRAs have been documented empirically. For example, Sufi (2009), Bannier, Hirsch, and Wiemann (2012), and Almeida, Cunha, Ferreira, and Restrepo (2017) show credit ratings' causal effects on firm investment decisions. While such feedback effects are largely absent in models of credit ratings, several previous papers introduced different forms of feedback, in particular Boot, Milbourn, and Schmeits (2006), Manso (2013), Goel and Thakor (2015), and Holden, Natvik, and Vigier (2016). A key difference between our paper and these previous papers is that in our paper the feedback effect is a result of information transmission from the rating agency to investors, whereas in these papers it is a result of changing the focal point for equilibrium selection, or of contractual features that affect the firm when the rating changes, or of the CRA's incentives to balance the issuer's payoff and the social welfare. While we think these are interesting dimensions to explore, we believe that the informational role of the rating is fundamental, going back to the basic motivation of introducing ratings to begin with, and so we focus on it here. Another key difference is that in these other papers there is no or limited rating inflation and the CRA wants to provide accurate ratings. Our research question, on the other hand, centers on the positive and negative real effects of a CRA with an inflation motive. As we show, these effects are all driven by the information transmission, and would not arise in the frameworks of the other feedback papers.

There are also several theory papers that study rating inflation. Usually, they attribute credit rating inflation to investors' imperfect rationality (Bolton, Freixas, and Shapiro 2012; Skreta and Veldkamp 2009), or regulations tied to ratings (Opp, Opp, and Harris 2013).[5] Hence, in these

[5]One exception is Frenkel (2015) who shows credit rating inflation may be generated by CRAs' "double reputa-

models, inflated credit ratings are not informative signals to the investors who are naïve or have regulatory motives. Again, while we think that bounded rationality and regulatory constraints are important, our aim is to analyze the role of the CRA in a rational environment.

The coordination game played among investors in our model is a global game (Carlsson and van Damme 1993; Morris and Shin 2003), but with endogenous information provided by the CRA to the investors. The coordination aspect gives the information provided by the CRA prominence even when investors are very well informed. Indeed, we show that the CRA's real effects quickly cease to exist if the coordination aspect is eliminated. Several papers endogenized the information structure in different ways in a global game setting. Angeletos, Hellwig, and Pavan (2006) and Angeletos and Pavan (2013) model the signaling effects of the government's preemptive defending policies, which pools very strong governments and very weak ones together. Edmond (2013) discusses a dictator's costly private information manipulation, where all revolutionaries' interim beliefs have full supports. Hence, the belief updating in these models differs from that in our model. In fact, the belief updating in our model is closer to that in Angeletos, Hellwig, and Pavan (2007) and Huang (2017). Nevertheless, our model has a unique equilibrium, because the CRA's incentives to inflate credit ratings generate new dominant regions of not investing. Finally, our paper is also related to Goldstein and Huang (2016) who show how the government persuades investors not to attack a regime by committing to abandon the regime when it is below some cutoff level. Our current model is different in several ways, such as that the CRA cannot commit to a rating strategy, and the firm has moral hazard issues that interact with the rating policy.[6] As a result, unlike the government in Goldstein and Huang (2016), the CRA in the current paper may have negative ex-ante real effects.

2 A Model of Corporate Credit Ratings

We study a model of a CRA that is assigning credit ratings to a firm who needs to roll over its maturing debt. There are three dates, $t = 0, 1, 2$. At the beginning of date 0, the firm's existing debt matures, and so it has to repay \$1 to the existing debt holders. To finance \$1,

tion." However, one necessary condition in Frenkel (2015) is that the CRA has different possible behavioral types, "honest" or "corrupt." In contrast, in our paper, the conflicts of interest caused by the issuer-pays business model are commonly known by all creditors. This may be a better description of the credit market, especially given what happened in the subprime crisis.

[6]These features also make our model different from those in the literature on Bayesian persuasion, such as Kamenica and Gentzkow (2011).

the firm can issue new debt (with relatively low costs) or borrow through a non-debt channel (with relatively high costs). One possible non-debt channel is a predetermined bank credit line, which we use in this paper for ease of exposition.

At date 0, the CRA assigns credit ratings to the firm. Observing the rating, new creditors in the debt market (with the measure $1-\gamma$) simultaneously decide whether to buy the newly issued debt or not. Since the existing debt holders are completely passive in the model, we will call the new creditors in the debt market "creditors." At date 1, depending on the financial cost, the firm may choose to default or to continue investing. In the latter case, the cash flow is realized at date 2, and, if possible, creditors are paid in full.

We view the model as a model about issuer credit ratings. In practice, CRAs assign credit ratings to both "issuers" and specific "issues." While we focus on credit ratings to the firm, in the model they are consistent with credit ratings to the corporate bond, because the firm has only one bond issue in the model. We think that these ratings, however, are different from credit ratings to structured finance products, such as MBSs, whose real effects are not as salient as corporate credit ratings.

2.1 Firm Investment and Economic Efficiency

Following Boot, Milbourn, and Schmeits (2006), we assume that if the firm fully repays the existing debt, it can continue investments either in a viable (i.e., low-risk) project VP or a high-risk alternative HR at date 1. VP generates a cash flow $V > 0$ with probability $p \in (0, 1)$; however, it fails with probability $1 - p$. Similarly, HR generates a cash flow $H > V$ with probability $q \in (0, p)$ but fails with probability $1 - q$. The firm will receive a zero cash flow if the project fails. Since both VP and HR fail with positive probabilities, the firm's investment choice between VP and HR is unobservable and unverifiable.[7]

At date 1, instead of investing in VP or HR, the firm may choose to default. In such a case, the firm will not withdraw from the credit line, and its liquidation value is $B \in (0, \gamma]$. We assume that the liquidation value and the funds from the newly issued debt are used to repay the exiting debt, since the existing debt is senior.[8] If the firm defaults at date 1, the game ends,

[7]In practice, creditors may know the name of the project the firm invests in, but they usually lack the professional knowledge to judge whether the project is HR or VP. Therefore, the choice between VP and HR is unverifiable even ex post.

[8]We assume $B \leq \gamma$ for simplicity. By this assumption, when the firm defaults at date 1, the amount of funds available is at most \$1. Hence, any creditor who buys the newly issued debt will get nothing, which implies global strategic complementarities among creditors.

and thus its early default decision is publicly observable and verifiable.

The economic efficiency is ranked by the firm's expected net present value (NPV), which is determined by the firm's investment decision. This notion of economic efficiency is equivalent to the sum of all agents' ex-ante payoffs at date 0 (except the CRA). Specifically, the firm's investment cost is 1, which consists of the bond investments of the new creditors and the funds withdrawn from the bank credit line. (If the firm chooses to default early, the total economic loss is also 1, which is from the loss of some existing creditors and the new creditors who invest.) The ex-post repayments, however, are all transfers from one group of agents to another, provided that the firm does not default. Therefore, the economic efficiency is equal to the firm's expected net present value, conditional on the firm's investment choice.

When the firm invests in VP, its expected NPV is $pV - 1$; when the firm invests in HR, its expected NPV is $qH - 1$; and if the firm defaults at date 1, the NPV is $B - 1$. We assume that it is very unlikely for HR to generate a positive cash flow. Specifically,

$$pV > 1 > B > qH. \tag{1}$$

Therefore, VP has a positive expected NPV and thus leads to the highest economic efficiency. Inequality (1) also implies that, while both HR and early default lead to negative expected NPVs, an investment in HR leads to an even lower economic efficiency than the early default does.

2.2 Financing

There is a continuum of creditors with measure $1 - \gamma$ in the debt market, each having \$1. Here, γ measures the liquidity of the debt market, with a larger γ meaning a lower liquidity level of the debt market. We assume that $\gamma \in (0, 1)$, and so it is impossible for the firm to finance by just issuing new debt.

The new debt is a zero-coupon bond with the face value $F > 1$. It matures at date 2. So long as the firm does not default either endogenously at date 1 or exogenously at date 2, the creditors who buy the new debt will get full repayment. Here, in order to focus on the role of the credit rating agency, we follow He and Xiong (2012) to assume that F, the face value of the new debt contract, is exogenously given. This assumption does not change the insights about credit ratings' real effects. Indeed, the key mechanism by which credit ratings affect the firm's investment decisions is through their effects on the firm's total cost of financing in the debt market, rather than through the face value of the newly issued debt. In our model, credit ratings do affect the firm's total cost of financing in the debt market (by affecting the measure of creditors who invest in the debt), even if the face value of the new debt contract is exogenous. Therefore, endogenizing F should only strengthen the results.

We assume $pF > 1$, and so if any creditor i knows that the firm will invest in VP, he will buy the new debt. On the other hand, the probability that HR is successful is so low ($qF < qH < B < 1$) that creditor i will not buy the new debt, if he knows that the firm will surely invest in HR. Obviously, if creditor i knows that the firm will default early, he does not buy the new debt either. We denote by $a_i \in \{0, 1\}$ creditor i's debt-investment decision, where $a_i = 1$ means creditor i buys the new debt, while $a_i = 0$ means creditor i does not buy.

We denote by W the measure of creditors who buy the new debt, and so the firm needs to finance $1 - W$ from the bank credit line. The firm can withdraw up to \$1 from the credit line with the constant marginal cost $f(\theta)$. Here, θ represents the firm's capacity to manage liquidity and is drawn by nature from the real line $\mathbb{R}$, according to a common improper uniform prior. (We also call θ the fundamentals of the firm, and call a firm with the fundamentals θ the "θ-firm.") The function $f(\cdot)$ is differentiable and strictly decreasing. When the firm's fundamentals are extremely good, the marginal cost of the credit line financing approaches the face value of the newly issued debt; that is, $\lim_{\theta \to +\infty} f(\theta) = F$. However, when the firm's fundamentals are extremely bad, borrowing from the credit line is extremely expensive, so $\lim_{\theta \to -\infty} f(\theta) = +\infty$. Therefore, if the firm decides to invest in either VP or HR, the firm's financial cost is

$$K(\theta) = WF + (1 - W)f(\theta). \tag{2}$$

One feature of this framework is that the firm's fundamentals determine the marginal cost of the firm's non-bond financing (e.g., the bank credit line) rather than the quality of the investment projects. This helps simplify the analysis. Also, it emphasizes an important factor that may determine a firm's credit ratings — the firm's liquidity management ability. Indeed, the bank credit line is one of the most important methods for a firm to manage its liquidity (Almeida, Campello, Cunha, and Weisbach 2014).

While we interpret the firm's economic fundamentals as the firm's liquidity management ability, the main mechanism and the conclusions of the paper remain unchanged if we model the firm's economic fundamentals as its investment profitability (conditional on the success of the investment). Specifically, suppose that if the firm invests in VP, the profit (conditional on the success) is $V - f(\theta)$; if it invests in HR, the profit (conditional on the success) is $H - f(\theta)$. Here, θ determines the firm's operational efficiency and is privately known to the firm and the CRA only. As before, we also assume that the prior distribution of θ is improper uniform, and the function $f(\theta)$ is differentiable and strictly decreasing. In addition, $\lim_{\theta \to +\infty} f(\theta) = 0$ and $\lim_{\theta \to -\infty} f(\theta) = +\infty$.

In order to make the investment, the firm needs to finance \$1 to repay the existing debt holders. It can finance in the public bond market where there is a continuum of new creditors with measure $1 - \gamma$. The public bond is sold at the price \$1 and has the face value $F > 1$. The firm can also finance through another channel (for example, a predetermined bank credit line) with the marginal cost $M > F$. Therefore, if there are W measure of creditors who invest in the firm's public bond, and the firm's fundamentals are θ, the sum of the firm's financial and operational costs is

$$K(\theta, W) = f(\theta) + WF + (1 - W)M.$$

Then, under the assumption that $M < \frac{pV - qH}{p - q}$, such a model is isomorphic to our core model.

2.3 Firm's Payoff

The firm has limited liability. If it defaults, whether endogenously at date 1 or exogenously at date 2 (when the project fails), its payoff is zero. If the firm generates a positive cash flow at date 2, the firm needs to repay the creditors according to the new debt contract. Therefore, the firm's payoff U depends on its own investment choice and its financial cost:

$$U = \begin{cases} 0, & \text{if the firm defaults at date 1;} \\ p\left[V - WF - (1 - W)f(\theta)\right], & \text{if the firm invests in VP;} \\ q\left[H - WF - (1 - W)f(\theta)\right], & \text{if the firm invests in HR.} \end{cases} \tag{3}$$

2.4 Information Structure

The firm's liquidity management ability, θ, is the firm's private information, which remains unknown to creditors. We assume that the firm gets to know θ after it decides to issue the new debt, so that the fact that the firm is issuing the new debt is not informative about θ to the creditors. Before deciding whether to buy the new debt, each creditor i observes a private signal $x_i = \theta + \xi_i$, where $\xi_i \sim \mathcal{N}(0, \beta^{-1})$ is independent of θ and independent across all creditors. Since we aim to analyze credit ratings' effects on rational, well-informed creditors, in this paper, we focus on the case when β is sufficiently large. Besides their private signals, creditors also observe a public credit rating by a CRA.

2.5 Credit Rating Agency

The CRA assigns the firm a credit rating R. Following Boot, Milbourn, and Schmeits (2006), We restrict the space of ratings to $\{0, q, p\}$, because these are the only possible credit qualities of the firm: Early default at date 1 means the firm will certainly default, and thus the firm's credit quality is 0; similarly, the firm investing in HR has a credit quality q, and the firm investing in VP has a credit quality p. This assumption is without loss of generality. Because the CRA cannot commit to a rating rule in our model, even if the CRA is allowed to assign ratings in a flexible space, the number of effective ratings cannot be strictly more than three in equilibrium. For example, if the CRA is announcing a θ' directly, it will announce the highest θ' that will lead to the same credit quality of the firm.

We assume that the CRA knows θ perfectly. In addition, we consider pure strategies. Hence, the CRA can perfectly predict the firm's choice and its corresponding default probability at date 0. Our model captures an important feature of credit ratings —- forward-looking. The ratings take into account the effect they have on the firm's action and success.

Due to the "issuer-pays" business model, the CRA always has incentives to assign the firm a high credit rating, in order to please issuers. The CRA's incentives to assign high credit ratings may also come from issuers' rating shopping (Bolton, Freixas, and Shapiro 2012), or the CRA's reputation for being nice to issuers (Frenkel 2015). Therefore, we assume that for each θ, the CRA wants to maximize the nominal rating.

On the other hand, a partial verifiability condition constrains the CRA's rating. The CRA wants to avoid lawsuits resulting from verifiable fraud; that is, the CRA never wants to be caught lying. Consequently, for any given θ, the CRA wants to assign the firm the highest possible nominal rating, provided that it cannot be verified as wrong (White 2013).[9]

Note that the CRA's objective to maximize the nominal rating subject to the partial verifiability constraint is a stark representation of incentives discussed in the literature, assumed here to simplify the analysis and demonstrate the insights in a clear way. These assumptions, however, can be generalized. In section 7.1, we formally model the CRA's revenue and its cost (conditional on firm failure) as increasing functions of credit ratings the CRA assigns. We discuss when our results go through and provide implications for optimal incentive schemes.

[9]Credit ratings are viewed as CRAs' "free speech." So, protected by the First Amendment, CRAs are not liable for any losses incurred by the inaccuracy of their ratings, unless it is proven that they know the ratings were false.

2.6 Timeline and Equilibrium

We summarize the model's timeline in Figure 1 below. The CRA's rating strategy, denoted by $\mathcal{R}$, maps the firm's fundamentals to the rating space $\{0, q, p\}$; the firm's strategy maps its fundamentals, the CRA's rating, and the measure of creditors investing in the new debt to project choices; and creditors' strategies map their own private signals and the CRA's rating to their debt-investment decisions.

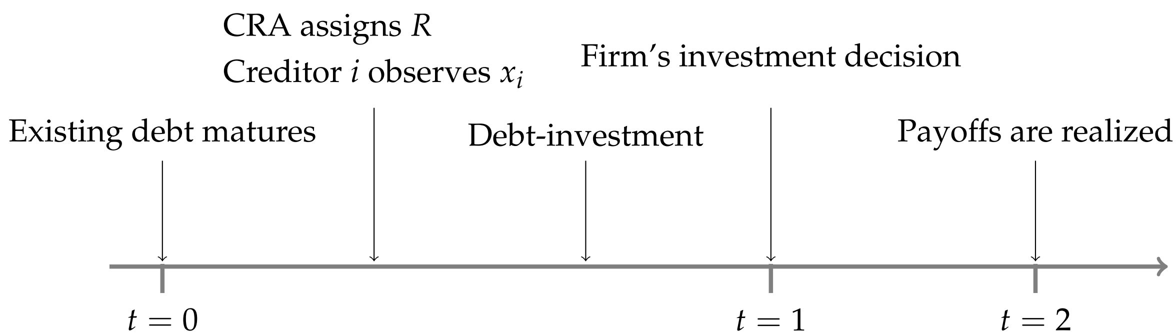

Figure 1: **Timing**

The solution concept of the model is a *monotone Bayesian Perfect equilibrium*.

Definition 1 *The CRA's rating strategy, the firm's investment strategy, and the creditors' debt-investment strategies constitute an equilibrium, if*

1. *given the firm's investment strategy and the creditors' debt-investment strategies, the CRA maximizes the nominal rating $\mathcal{R}(\theta)$ for all $\theta \in \mathbb{R}$ subject to the partial verifiability constraint;*

2. *given financial costs in equation (2), the firm's investment strategy will maximize the firm's expected profits;*

3. *given the CRA's rating strategy, the firm's investment strategy, and other creditors' strategies, any creditor i's strategy is monotonic in his private signal x_i and will maximize his expected payoff;*

4. *and, creditors use Bayes' rule to update their beliefs.*

3 The Benchmark: No CRA

In order to analyze the CRA's real effects, we set up a benchmark that excludes the CRA. In such a benchmark, when deciding whether to buy the newly issued debt, all creditors make choices solely based on their own private information. After observing the measure of creditors who invest in the debt, the firm makes its investment choice. Such a benchmark is similar to the debt-run model by Morris and Shin (2004), with the key difference being that the firm's choice is not binary.

We first analyze the firm's behavior in this benchmark model. Because of the law of large numbers, given the creditors' strategies, the measure of the creditors who buy the debt is a deterministic function $W(\theta)$. Hence, any θ-firm's financial cost is deterministic:

$$W(\theta)F + (1 - W(\theta))\, f(\theta).$$

Since $H > V$, the θ-firm will default early, if and only if,[10]

$$W(\theta)F + (1 - W(\theta))f(\theta) > H. \tag{4}$$

Conditional on that the θ-firm decides to continue investing, it invests in VP rather than HR, if and only if,

$$\begin{aligned} & p\left[V - W(\theta)F - (1 - W(\theta))f(\theta)\right] \geq q\left[H - W(\theta)F - (1 - W(\theta))f(\theta)\right] \\ \Rightarrow\quad & W(\theta)F + (1 - W(\theta))f(\theta) \leq \frac{pV - qH}{p - q}. \end{aligned} \tag{5}$$

The firm's choice between VP and HR is the same as in Boot, Milbourn, and Schmeits (2006). To keep the model interesting, we assume that if the firm's fundamentals are extremely good (so the firm's financial cost is arbitrarily close to F), the firm will choose VP. That is, we maintain the assumption that

$$F < \frac{pV - qH}{p - q}.$$

As a result, given the creditors' strategies, the θ-firm's optimal investment strategy is

$$\begin{cases} \text{early default,} & \text{if} \quad W(\theta)F + (1 - W(\theta))f(\theta) > H; \\ \text{HR,} & \text{if} \quad W(\theta)F + (1 - W(\theta))f(\theta) \in \left(\frac{pV - qH}{p - q}, H\right]; \\ \text{VP,} & \text{if} \quad W(\theta)F + (1 - W(\theta))f(\theta) \leq \frac{pV - qH}{p - q}. \end{cases} \tag{6}$$

Since the measure of creditors in the debt market is only $1 - \gamma$, the firm has to withdraw from the credit line, if it decides to invest in either VP or HR. Then, from the properties of the function $f(\cdot)$, when the firm's fundamentals are extremely good ($\theta \to +\infty$), $f(\theta)$ is very

[10] We assume that the firm will default at date 1, if its financial cost is larger than the highest possible cash flow the firm can generate. This can reflect some very small cost incurred by the manager in case he continues.

close to the face value of the firm's new debt, F; then $WF + (1-W)f(\theta)$ is strictly less than $(pV - qH)/(p-q)$, implying that the firm will invest in VP. When the firm's fundamentals are extremely bad ($\theta \to -\infty$), $f(\theta)$ is extremely large, so that $WF + (1-W)f(\theta) > H$, implying that the firm will default early.

Hence, as shown in the global games literature, in such a benchmark model, all creditors have both the *dominant region of not investing* and the *dominant region of investing*. That is, when creditor i's private signal x_i is extremely negative, he believes that the firm's fundamentals are weak, so that even if all other creditors invest in the debt, the firm's financial cost of investing in the project is beyond its highest possible cash flow H, and thus the firm will default at date 1. Therefore, creditor i will refrain from investing, even when all other creditors invest. This establishes creditors' dominant region of not investing. Conversely, the creditors also have a dominant region of investing. If creditor i's private signal x_i is extremely positive, he believes that the firm's fundamentals are extremely good, and thus the firm will choose VP; as a result, creditor i will invest in the debt, even when all other creditors do not. Therefore, as in other global games models, in a monotone equilibrium, any creditor employs a cutoff strategy with the threshold $\tilde{x}$, such that he invests in the debt, if and only if $x_i \geq \tilde{x}$.

Given θ and the creditors' cutoff strategy, the measure of creditors who invest is

$$W(\theta) = (1-\gamma)\Pr\left(x \geq \tilde{x} | \theta\right) = (1-\gamma)\left\{1 - \Phi\left[\sqrt{\beta}(\tilde{x} - \theta)\right]\right\},$$

where $\Phi(\cdot)$ is the CDF of the standard normal distribution. Then, the θ-firm's financial cost is

$$\begin{aligned} K(\theta) &= (1-\gamma)\left\{1 - \Phi\left[\sqrt{\beta}(\tilde{x} - \theta)\right]\right\} F + \left[\gamma + (1-\gamma)\Phi[\sqrt{\beta}(\tilde{x} - \theta)]\right] f(\theta) \\ &= [(1-\gamma)F + \gamma f(\theta)] + (1-\gamma)\Phi\left[\sqrt{\beta}(\tilde{x} - \theta)\right] (f(\theta) - F). \end{aligned} \tag{7}$$

The first term in equation (7) is the financial cost resulting from insufficient liquidity in the debt market, whereas the second term in equation (7) is the endogenous financial cost resulting from creditors' strategic complementarities.

As θ increases, that is, as the firm's fundamentals improve, the cost of withdrawing from the credit line decreases (since $f(\theta)$ is strictly decreasing), and the measure of creditors investing increases. Therefore, given the creditors' cutoff strategies, the firm's financial cost strictly decreases in its fundamentals. In contrast with classical global games, in this benchmark model the firm has two indifference conditions. First, given the creditors' strategies, the firm will choose to default early if and only if $\theta < \tilde{\theta}_1$. This implies that

$$K(\tilde{\theta}_1) = \left[(1-\gamma)F + \gamma f(\tilde{\theta}_1)\right] + (1-\gamma)\Phi\left[\sqrt{\beta}(\tilde{x} - \tilde{\theta}_1)\right] \left(f(\tilde{\theta}_1) - F\right) = H. \tag{8}$$

Because $K(\theta)$ is strictly decreasing, for any $\theta < \tilde{\theta}_1$, the firm's financial cost will be greater than H, the upside cash flow of HR; as a result, the firm would default at date 1. But if $\theta \geq \tilde{\theta}_1$, the firm can at least choose HR in order to receive a non-negative expected payoff due to its limited liability, and thus the firm will not default early.

When $\theta \geq \tilde{\theta}_1$, the firm needs to choose between VP and HR. From equation (6) and the fact that $K(\theta)$ is strictly decreasing in θ, there must be a $\tilde{\theta}_2 > \tilde{\theta}_1$, such that the firm will choose VP if and only if $\theta \geq \tilde{\theta}_2$. Hence,

$$K(\tilde{\theta}_2) = \left[(1-\gamma)F + \gamma f(\tilde{\theta}_2)\right] + (1-\gamma)\Phi\left[\sqrt{\beta}(\tilde{x} - \tilde{\theta}_2)\right]\left(f(\tilde{\theta}_2) - F\right) = \frac{pV - qH}{p - q}. \tag{9}$$

Following the above arguments, in a monotone equilibrium, the firm will default early if $\theta < \tilde{\theta}_1$, invest in HR if $\theta \in \left[\tilde{\theta}_1, \tilde{\theta}_2\right)$, and invest in VP if $\theta \geq \tilde{\theta}_2$.

Any creditor i, receiving a private signal x_i about θ, first updates his belief about θ according to Bayes' rule:

$$\theta | x_i \sim \mathcal{N}\left(x_i, \frac{1}{\beta}\right).$$

Then, given the firm's strategy described above, creditor i calculates his return from investing in the debt:

$$\left\{\Phi\left[\sqrt{\beta}(\tilde{\theta}_2 - x_i)\right] - \Phi\left[\sqrt{\beta}(\tilde{\theta}_1 - x_i)\right]\right\} qF + \left\{1 - \Phi\left[\sqrt{\beta}(\tilde{\theta}_2 - x_i)\right]\right\} pF.$$

Given the dominant regions of investing and not investing, there must be a marginal creditor who is indifferent between investing and not investing in equilibrium. Because any creditor will receive the payoff 1 if he does not invest, and his expected payoff from investing is strictly increasing in his private signal, the marginal creditor must have the private signal $\tilde{x}$ that makes his indifference condition hold:

$$\left\{\Phi\left[\sqrt{\beta}(\tilde{\theta}_2 - \tilde{x})\right] - \Phi\left[\sqrt{\beta}(\tilde{\theta}_1 - \tilde{x})\right]\right\} qF + \left\{1 - \Phi\left[\sqrt{\beta}(\tilde{\theta}_2 - \tilde{x})\right]\right\} pF = 1. \tag{10}$$

Proposition 1 below characterizes the equilibrium of the benchmark model.

Proposition 1 (The Unique Equilibrium in the Benchmark Model) *There exists a $\tilde{\beta} > 0$, such that for all $\beta > \tilde{\beta}$, the benchmark model without a CRA has a unique equilibrium described by $\left(\tilde{\theta}_1, \tilde{\theta}_2, \tilde{x}\right)$, where $\tilde{\theta}_1 < \tilde{\theta}_2$. In particular,*

1. *the firm's investment strategy is*

$$\begin{cases} VP, & \text{if } \theta \geq \tilde{\theta}_2; \\ HR, & \text{if } \theta \in \left[\tilde{\theta}_1, \tilde{\theta}_2\right); \\ \text{early default}, & \text{if } \theta < \tilde{\theta}_1; \end{cases}$$

2. *and, any creditor i invests in the newly issued debt if and only if $x_i \geq \tilde{x}$.*

4 Credit Rating Inflation

We now consider our core model where the CRA strategically designs the rating rule. As a first step to solve an equilibrium, we discuss the informativeness of credit ratings. Although the CRA always has incentives to assign overgenerous ratings, its rating strategy is subject to the partial verifiability constraint: The event of the firm's early default is publicly observable and thus verifiable. As a result, in equilibrium, the CRA will assign a high rating if and only if the firm does not default at date 1. This partial verifiability constraint plays a critical role in determining the informativeness of the CRA's ratings.

We then solve the CRA's equilibrium rating strategy. Importantly, when assigning credit ratings, the CRA will take into account the effects of the ratings on the creditors' debt-investment decisions and thus the firm's investment choices. As we show later in Section 5, this feature, as well as the informativeness of credit ratings, is the key to understand the credit ratings' real effects.

4.1 Informativeness of a Rating Strategy

We first argue that an equilibrium rating strategy must be monotonic. That is, the firm with better fundamentals will be assigned a (weakly) higher credit rating. Consider a rating strategy $\mathcal{R}(\theta)$ that assigns the rating p to θ'-firm and the rating q to θ''-firm, where $\theta'' > \theta'$. We then claim that the CRA can profitably deviate to assign the rating p to θ''-firm. To see this, first note that creditors are more likely to buy the newly issued debt when their private signals are higher. Such monotonic debt-investment strategies then imply that among firms in the same rating category, the ones with better fundamentals have lower financial costs, because firms with better fundamentals have both lower non-debt financing costs and more creditors investing in the newly issued debt. Hence, with the rating p, θ''-firm will have a lower financial cost than θ'-firm; then, the fact that θ'-firm does not default early (because it receives the rating p in the strategy profile under consideration) implies that after receiving the rating p, θ''-firm will not default early either. As a result, such a deviation is profitable, and thus, the rating strategy $\mathcal{R}(\theta)$ under consideration cannot be part of an equilibrium. Similarly, in equilibrium, the CRA will not assign the rating 0 to θ-firm, if it assigns the rating p to θ'-firm with $\theta' < \theta$. Therefore, an equilibrium rating strategy is increasing in the firm's fundamentals.

Furthermore, the rating q will not be assigned in equilibrium rating strategy. Suppose that the CRA assigns the rating q to θ-firm; because the rating strategy is monotonic, it has worse fundamentals than those receiving the rating p. (Because it is strictly dominant for the firm to invest in VP when θ is sufficiently large, the CRA will always assign some firms the rating p in equilibrium.) Then, if the CRA deviates to assign the rating p to θ-firm, more creditors will buy its newly issued debt. As a result, θ-firm's financial cost decreases and thus will not default early. Therefore, the CRA's deviation is also profitable, which implies that the CRA will not assign the rating q in an equilibrium.[11]

These arguments lead to Lemma 1 below, which characterizes all possible equilibrium rating strategies and simplifies our analysis much.

Lemma 1 (Cutoff Rating Strategy) *In any monotone equilibrium (if it exists), the CRA's rating strategy can be described by a threshold θ_1^*, such that*

$$\mathcal{R}(\theta) = \begin{cases} 0, & \text{if} \quad \theta < \theta_1^*; \\ p, & \text{if} \quad \theta \geq \theta_1^*. \end{cases} \tag{11}$$

From Lemma 1, when θ_1^* decreases, the CRA assigns more firms with the high rating p. So for two rating strategies $\mathcal{R}_1$ with the threshold θ_1^* and $\mathcal{R}_2$ with the threshold θ_2^*, we say that the rating strategy $\mathcal{R}_2$ is *laxer* than the rating strategy $\mathcal{R}_1$ if and only if $\theta_2^* < \theta_1^*$. However, the laxer rating strategy $\mathcal{R}_2$ may not lead to higher *credit rating inflation*, which refers to the fact that the nominal rating is strictly higher than the real credit quality. Formally:

Definition 2 *A credit rating assigned to a θ-firm is inflated, if in equilibrium, the θ-firm chooses HR and thus has the credit quality q, but the CRA assigns the rating p. In addition, a rating strategy is inflated, if credit ratings assigned according to the rating strategy are inflated for a non-negligible subset of fundamentals; and a credit rating strategy is more inflated, if for a larger measure of fundamentals, credit ratings assigned according to the rating strategy are inflated.*

In equilibrium, the firm receiving the rating p does not default at date 1, due to the partial verifiability constraint. However, the rating p cannot guarantee that the firm will invest in VP. Indeed, if all creditors believe that the firm with the rating p will surely invest in VP, they will all buy the debt, leading to the lowest possible financial cost to any θ-firm. Then, the

[11] On the off-equilibrium path following $R = q$, the creditors form the belief that the firm will choose to continue to invest in HR. In Section 7.1, we analyze a self-disciplined CRA, where the rating $R = q$ may appear in some equilibria.

assumption that the θ_1^*-firm will invest in VP implies that θ_1^*-firm's financial cost is strictly less than H. In consequence, the firms with the fundamentals slightly lower than θ_1^* will not default early, if they receive the rating p, which provides the CRA with incentives to deviate to assign the rating p to such firms. Therefore, in equilibrium, some firms with the rating p will invest in HR, implying credit rating inflation in equilibrium. Formally:

Lemma 2 (Rating Inflation) *There is no monotone equilibrium in which all θ-firms receiving a rating $R = p$ invest in VP.*

While rating inflation inevitably appears in equilibrium, credit ratings are still informative to creditors. Lemma 1 implies that if $R = p$, all creditors know that $\theta \geq \theta_1^*$. So, the rating p guarantees creditors that the firm's fundamentals are not extremely bad.

Corollary 1 (Creditors' belief supports following $R = p$) *Following the credit rating $R = p$, regardless of his private signal x_i, the support of any creditor i's interim belief about θ is truncated from below by θ_1^*.*

4.2 Firm's Investment after the Rating p

As shown in Lemma 1, only the rating 0 and the rating p may appear in equilibrium. Because the CRA tries to maximize the nominal rating, it will assign the rating 0 only if it knows that the firm will default early even with the rating p. Therefore, when creditors observe the rating 0, they all believe that the firm will default early, and so they refrain from investing. Hence, following $R = 0$, there is a unique equilibrium in which no creditor invests in the newly issued debt, and the firm defaults at date 1. Since the rating strategy assigns the rating 0 to the firm if and only if $\theta < \theta_1^*$, we must have $K(\theta) = f(\theta) > H$, $\forall \theta < \theta_1^*$. Then, by the continuity of $f(\cdot)$, we have the first condition for the equilibrium below:

$$f(\theta_1^*) \geq H. \tag{12}$$

We now focus on the subgame following the rating p. Given the rating strategy, after observing the rating p, all creditors believe that the firm's true fundamentals are above θ_1^*. However, as shown in Lemma 2, creditors are not sure whether the firm will invest in HR or VP. In particular, because the firm will invest in HR when θ is slightly above θ_1^*, the creditors with extremely negative signals believe that the firm will invest in HR and thus choose not to buy the debt as a dominant action. As a result, in equilibrium, given the belief about the CRA's rating strategy described by θ_1^*, after observing the rating p, any creditor i will invest in the newly issued debt if and only if x_i lands above some threshold x^*. Then, as in the benchmark model, because the firm's financial cost strictly decreases in its fundamentals, the firm will invest in HR if and only if θ is less than a threshold θ_2^*. Hence, given a possible equilibrium rating strategy θ_1^*, a monotone equilibrium following the rating p could be described by (x^*, θ_2^*), such that

1. $\theta_2^* > \theta_1^*$;
2. creditor i invests in the newly issued debt if and only if $x_i \geq x^*$; and
3. θ-firm chooses VP if $\theta \in [\theta_2^*, +\infty)$, and it chooses HR if $\theta \in [\theta_1^*, \theta_2^*)$.

Given the creditors' cutoff strategy with the threshold x^*, $(1-\gamma)\left[1-\Phi(\sqrt{\beta}(x^*-\theta))\right]$ measure of creditors will invest in the debt, for any $\theta \geq \theta_1^*$. Consequently, if the θ-firm decides to invest in either VP or HR, its financial cost is

$$\begin{aligned} & K(\theta) \\ = & (1-\gamma)\left[1-\Phi(\sqrt{\beta}(x^*-\theta))\right]F + \left[\gamma + (1-\gamma)\Phi(\sqrt{\beta}(x^*-\theta))\right]f(\theta) \\ = & [(1-\gamma)F + \gamma f(\theta)] + (1-\gamma)\Phi(\sqrt{\beta}(x^*-\theta))(f(\theta)-F). \end{aligned}$$

This is precisely the same as equation (7). Because the firm invests in VP if and only if $K(\theta) \geq (pV - qH)/(p-q)$, and θ_2^*-firm is indifferent between HR and VP, the firm's indifference condition, given the creditors' strategy, is

$$(1-\gamma)\left[1-\Phi(\sqrt{\beta}(x^*-\theta_2^*))\right]F + \left[\gamma + (1-\gamma)\Phi(\sqrt{\beta}(x^*-\theta_2^*))\right]f(\theta_2^*) = \frac{pV-qH}{p-q}. \tag{13}$$

Let's consider a creditor i's decision. With his private signal x_i, creditor i's interim belief about θ given the CRA's rating strategy θ_1^* would be a normal distribution with mean x_i and precision β, truncated from below by θ_1^*. This truncation is due to creditors' belief about the CRA's rating strategy that $\mathcal{R}(\theta) = p$ if and only if $\theta \geq \theta_1^*$. Then, given the firm's strategy, creditor i's expected payoff from investing is

$$\frac{\Phi[\sqrt{\beta}(\theta_2^*-x_i)] - \Phi[\sqrt{\beta}(\theta_1^*-x_i)]}{1-\Phi[\sqrt{\beta}(\theta_1^*-x_i)]}qF + \frac{1-\Phi[\sqrt{\beta}(\theta_2^*-x_i)]}{1-\Phi[\sqrt{\beta}(\theta_1^*-x_i)]}pF.$$

Because refraining from investing in the debt always brings a creditor a payoff 1, a marginal creditor with the private signal x^* must have

$$\frac{\Phi[\sqrt{\beta}(\theta_2^*-x^*)] - \Phi[\sqrt{\beta}(\theta_1^*-x^*)]}{1-\Phi[\sqrt{\beta}(\theta_1^*-x^*)]}qF + \frac{1-\Phi[\sqrt{\beta}(\theta_2^*-x^*)]}{1-\Phi[\sqrt{\beta}(\theta_1^*-x^*)]}pF = 1. \tag{14}$$

Lemma 3 (Debt Financing Following $R = p$) *There exists a $\beta^* > 0$, such that for any $\beta > \beta^*$, if an equilibrium exists, given the CRA's rating strategy θ_1^*, following the rating p, there is a unique solution (θ_2^*, x^*) with $\theta_2^* > \theta_1^*$ to equation (13) and equation (14).*

In the analysis of the interaction between the firm and the creditors above, the CRA's rating strategy θ_1^* is given. Lemma 4 below shows how the CRA's rating strategy affects the creditors' debt investment decisions and the firm's moral hazard.

Lemma 4 (Laxer Rating Strategy) *For any $\beta > \beta^*$, both x^* and θ_2^* are strictly decreasing in θ_1^*.*

When θ_1^* is lower, the CRA's rating strategy is laxer. In this scenario, the creditors discount the positive information conveyed by the good rating by increasing their debt-investment threshold. Because more creditors refrain from investing in the debt, the firm's financial cost is higher for any θ; as a result, the threshold that the firm chooses VP is also higher.

4.3 Equilibrium Rating Strategy

Lemma 3 shows that creditors' belief about the CRA's rating strategy θ_1^* determines the measure of creditors investing and thus any θ-firm's financial cost. On the other hand, when the CRA assigns the rating to θ-firm, the CRA, based on the knowledge of θ and the creditors' responses to the ratings, can perfectly predict whether the firm will default early or not. Hence, in equilibrium, θ_1^*-firm must be indifferent between early default and HR. Because of the firm's limited liability, the firm will choose to default early only if the financial cost is higher than H, the upside return from investing in HR. Therefore, the θ_1^*-firm's indifference condition implies

$$(1-\gamma)\left[1-\Phi(\sqrt{\beta}(x^*-\theta_1^*))\right]F+\left[\gamma+(1-\gamma)\Phi(\sqrt{\beta}(x^*-\theta_1^*))\right]f(\theta_1^*)=H. \qquad (15)$$

Proposition 2 below shows that the model has a unique equilibrium in which the CRA's rating, the firm's investment decision, and the creditors' debt-investment decisions interact with one another.

Proposition 2 (Unique Equilibrium) *There is a $\beta^* > 0$, such that when $\beta > \beta^*$, the model has a unique equilibrium. The equilibrium is characterized by $(\theta_1^*, \theta_2^*, x^*)$ with $\theta_2^* > \theta_1^*$, such that*

1. *the CRA will assign a rating $R = p$, if the firm's fundamentals $\theta \in [\theta_1^*, +\infty)$; and it will assign a rating $R = 0$, if the firm's fundamentals $\theta < \theta_1^*$;*

2. *if $R = 0$, no creditor buys the newly issued debt, and the firm defaults at date 1;*

3. *if $R = p$, a creditor invests in the debt if and only if his private signal lands above x^*, and the firm will choose HR if $\theta \in [\theta_1^*, \theta_2^*)$ and VP if $\theta \in [\theta_2^*, +\infty)$; and*

4. *$(\theta_1^*, \theta_2^*, x^*)$ solves equations* (13), (14), *and* (15).

The equilibrium uniqueness arises from creditors' new dominant region of not investing, generated by the credit rating p. From Lemma 2, because the CRA aims to maximize the nominal rating, it will assign the rating p to the firm that has the fundamentals just above θ_1^* and thus will invest in HR. Consequently, when creditors receive very negative signals, they believe that the firm has fundamentals landing within this region and thus invests in HR, so they refrain from investing in the debt. Hence, it is impossible for all creditors to buy the debt in equilibrium, and thus, the creditors have a unique best response to the rating p.

Proposition 2 provides us with a clear measure of equilibrium rating inflation. When $\theta < \theta_1^*$, the CRA will assign the rating 0 to the firm. Since the firm will default early, the credit rating truly reflects the firm's credit quality. When $\theta \geq \theta_2^*$, the firm's fundamentals are sufficiently good, so it will invest in VP. In this case, the credit rating p also equals the firm's credit quality. However, when $\theta \in [\theta_1^*, \theta_2^*)$, the firm invests in HR and thus has the credit quality q, but it receives the high rating p. So the credit ratings assigned to such firms are inflated. Hence, the rating inflation can be measured by $\theta_2^* - \theta_1^*$.

5 The CRA's Real Effects

We are now in a position to analyze the CRA's real effects. For a given θ-firm, if the assigned credit rating changes its expected NPV (comparing to its investment in the benchmark model without a CRA), the CRA has effects on the economic efficiency. In such a case, we say that the CRA has *real effects* on the θ-firm. Such effects are *positive*, if the CRA leads to higher economic efficiency; conversely, if the CRA leads to lower economic efficiency, the CRA's real effects on the θ-firm are *negative*. The CRA's ex-ante real effects are then measured by the average change of the economic efficiency. Hence, the ex-ante real effects of the CRA are positive (negative) if the average change of the economic efficiency is higher (lower) with the CRA.

From Proposition 2, we can see that the CRA affects a firm's investment decision through two interacting channels. On the one hand, by assigning the rating $R = p$, the CRA separates firms with fundamentals above a threshold from those with the fundamentals below the threshold. Hence, the rating $R = p$ provides the creditors with new information about the firm's fundamentals. Such new information affects the creditors' debt-investment decisions, and thus the firm's financial cost and investment. We call such effects the CRA's *informational effects*.

On the other hand, the CRA strategically chooses θ_1^* to pool firms investing in HR with those investing in VP. Hence, the set of types of the firm that invest in either HR or VP may differ in cases with and without a CRA. This also affects firms' investment decisions. We call such effects the CRA's *feedback effects*, since the CRA, when choosing θ_1^*, takes into account the creditors' and the firm's best responses to the ratings.

In this section, we first compare the firm's equilibrium investment decision in the model with the CRA to those in the benchmark model. Such a comparison shows the CRA's real effects. And then, we decompose the CRA's real effects into its informational effects and its feedback effects to get a full understanding of its real effects.

5.1 The CRA's Real Effects

Lemma 5 below shows that, with the CRA, both the firm's early default threshold and VP-investing threshold are lower than those in the benchmark model without a CRA.

Lemma 5 *Comparing the equilibrium of the model with a CRA (described in Proposition 2) to that of the benchmark model without a CRA (described in Proposition 1), we have $\theta_1^* < \tilde{\theta}_1$, $\theta_2^* < \tilde{\theta}_2$, and $x^* < \tilde{x}$. However, the sign of $\theta_2^* - \tilde{\theta}_1$ is undetermined.*

We illustrate the CRA's real effects in the case with $\theta_2^* > \tilde{\theta}_1$ in Figure 2 below.

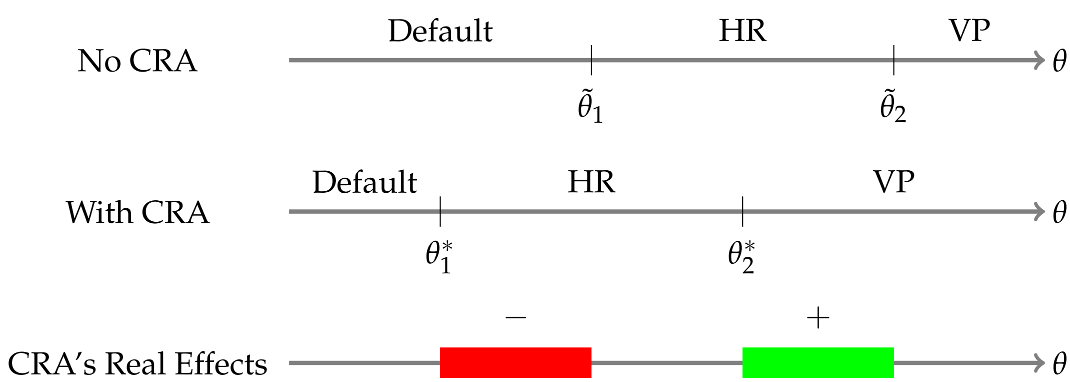

Figure 2: **CRA's real effects when $\theta_2^* > \tilde{\theta}_1$**

When $\theta_2^* > \tilde{\theta}_1$, there are two cases of the CRA's real effects. First, when $\theta \in [\theta_1^*, \tilde{\theta}_1)$, without the CRA, the firm's financial cost is so high that it will default early; but when the CRA is present, it will assign the firm the inflated rating p, leading to lower financial costs to the firm. Such a decrease in the financial costs encourages the firm to gamble for resurrection, rather than default early, which shows the CRA's negative real effects. Second, when $\theta \in [\theta_2^*, \tilde{\theta}_2)$, because the high rating p helps the firm reduce financial costs, the firm switches from HR to VP, which implies positive real effects.

When $\theta_2^* \leq \tilde{\theta}_1$, the CRA's real effects are similar, except that the range for the negative real effect is different. Proposition 3 below formally summarizes the CRA's real effects.

Proposition 3 *There are two cases of the analysis of the CRA's real effects.*

1. *If $\theta_2^* > \tilde{\theta}_1$, the CRA has positive real effects when $\theta \in [\theta_2^*, \tilde{\theta}_2)$ and has negative real effects when $\theta \in [\theta_1^*, \tilde{\theta}_1)$; hence, the CRA's ex-ante real effects are*

$$(\tilde{\theta}_2 - \theta_2^*)(pV - qH) + (\tilde{\theta}_1 - \theta_1^*)(qH - B).$$

2. *If $\theta_2^* \leq \tilde{\theta}_1$, the CRA has positive real effects when $\theta \in [\theta_2^*, \tilde{\theta}_2)$ and has negative real effects when $\theta \in [\theta_1^*, \theta_2^*)$; hence, the CRA's ex-ante real effects are*

$$(\tilde{\theta}_2 - \tilde{\theta}_1)(pV - qH) + (\tilde{\theta}_1 - \theta_2^*)(pV - B) + (\theta_2^* - \theta_1^*)(qH - B).$$

Importantly, Proposition 3 shows that the CRA who employs an inflated rating strategy may have positive or negative real effects, depending on the firm's fundamentals. The CRA's ex-ante real effects then depend on the model's parameters. In Figure 3 below, we numerically show the CRA's ex-ante real effects as a function of the upside return of the risky project, H.

In particular, Figure 3 shows that when the upside return of the risky project is relatively high, the CRA's ex-ante real effects are negative. This is because, as shown in Proposition 6, when H is large, the firm has stronger incentives to take risks by investing in HR and thus is less likely to default early. The CRA then will assign more firms the high rating $R = p$, which allows those firms to gamble for resurrection, and so have negative ex-ante real effects. When H is relatively small, the CRA encourages more firms to switch from HR to VP and thus has positive ex-ante real effects.

The fact that as the upside return of the risky project increases beyond a threshold, the CRA's ex-ante real effects become negative may help explaining why the conflicts of interest caused by the issuer-pays business model attract much more attention after the recent subprime

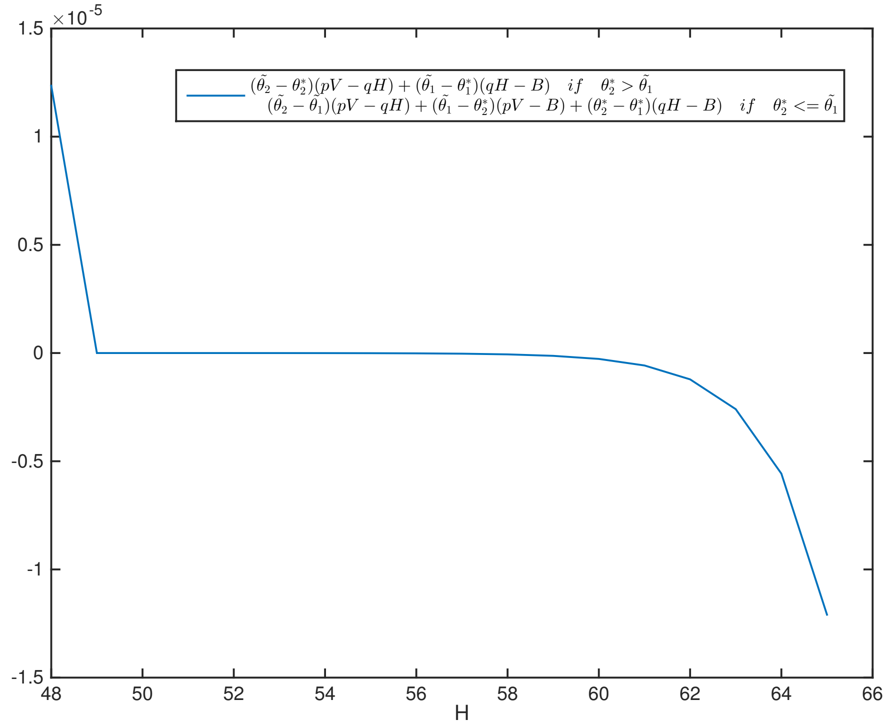

Figure 3: **The CRA's Real Effects as a Function of H.** The parameters used in this figure are $F = 1.2$, $V = 2$, $p = 0.9$, $q = 0.01$, $\gamma = 0.7$, $B = 0.7$, $\beta = 10$, and $f(\theta) = e^{-\theta} + 1.2$.

crisis. Subprime mortgages provided financial institutions with opportunities to invest in risky projects with large upside. They bought and pooled subprime mortgages, and then issued securities backed by such subprime mortgages, which are named subprime MBSs. Issuing subprime MBSs potentially has high profits because subprime mortgages are cheap. On the other hand, however, subprime borrowers feature high default probabilities, leading to the high risks of subprime MBSs. As a result, leading to the crisis, the effects of the CRAs' rating inflation might have been shifted to a point where the overall expected effect on economic efficiency was negative due to the greater temptation to engage in risky projects fueled by the high rating.

5.2 Informational Effects and Feedback Effects

As we have argued, the CRA's real effects can be decomposed into two components: the informational effects, because the CRA provides the creditors with new information, and the feedback effects, because the CRA strategically chooses its rating rule, taking advantage of the feedback between credit ratings and firms' investments. We now analyze how these two effects interact to determine the CRA's real effects.

We first analyze the CRA's informational effects. Let's consider the case in which the CRA commits to the following rating strategy:

$$\mathcal{R}(\theta) = \begin{cases} 0, & \text{if } \theta < \hat{\theta}_1 \equiv \tilde{\theta}_1; \\ p, & \text{if } \theta \geq \hat{\theta}_1. \end{cases} \tag{16}$$

Here, $\tilde{\theta}_1$, which is characterized in Proposition 1, is the early-default cutoff of the firm when there is no CRA.

In such a case, the CRA does not behave strategically, though such a rating strategy may still be inflated. Indeed, the committed rating strategy just reflects the firm's investment decision in the benchmark model without a CRA. So, for ease of exposition, we call such a CRA a *reflecting CRA* and the CRA analyzed in Section 4 a *strategic CRA*. Importantly, a reflecting CRA does not have feedback effects, because it does not take into account its effects on the firm's investment decision when committing to its rating strategy. Therefore, the real effects of the reflecting CRA is just the informational effects of the strategic CRA. Then, by comparing the strategic CRA's real effects with the reflecting CRA's real effects, we can find the strategic CRA's feedback effects.

Proposition 4 shows the firm's equilibrium investment decision in the case with the reflecting CRA.

Proposition 4 *Given the committed rating strategy in equation* (16)*, the generated credit ratings lead to two subgames. In particular,*

1. *in the subgame following $R = 0$, there is a unique equilibrium in which the firm defaults at date 1; and,*

2. *in the subgame following $R = p$, in any equilibrium, the θ-firm invests in VP if $\theta \geq \hat{\theta}_2$ and invests in HR if $\theta \in [\hat{\theta}_1, \hat{\theta}_2)$. Furthermore, if $\theta_2^* > \tilde{\theta}_1$, we have $\hat{\theta}_2 < \theta_2^*$.*

Now, let's analyze the reflecting CRA's real effects. Proposition 4 shows that with the rating p assigned by a reflecting CRA, if $\theta > \tilde{\theta}_2$ (which is strictly greater than $\hat{\theta}_2$), the firm invests in VP, in both the case with a reflecting CRA and the case without a CRA. Therefore, for any $\theta > \tilde{\theta}_2$, the reflecting CRA does not have real effects. Similarly, the reflecting CRA does not have any real effects when $\theta \in [\hat{\theta}_1, \hat{\theta}_2)$

However, the reflecting CRA will change the firm's investment decision when $\theta \in [\hat{\theta}_2, \tilde{\theta}_2)$. In particular, without a CRA, the firm invests in HR, but with a reflecting CRA, the firm will invest in VP. Therefore, the reflecting CRA has positive real effects, which are measured by $(\tilde{\theta}_2 - \hat{\theta}_2)(pV - qH)$. That is, the strategic CRA's informational effects are always positive, precisely because its rating $R = p$, though potentially inflated, provides the creditors with an informative signal and thus correctly guides creditors' debt-investment decisions and with that influencing the firm's investment.

The CRA has informational effects because credit ratings are informative public signals to creditors. Generally, public signals may have negative effects, as analyzed by Morris and Shin (2002) and others.[12] In our model, however, the CRA's informational effects are always positive. Given the credit rating rule committed by the reflecting CRA, a firm can receive a high rating only if it would invest in either HR or VP in the benchmark model without a CRA. Because a high rating reduces the firm's financial costs, in such a case, it is possible for a firm to switch from HR to VP, but it is impossible for a firm to switch from efficient default to HR. Consequently, given a credit rating assigned by the reflecting CRA, the resulting firm credit quality is at least as high as that in the benchmark model.

Finally, let's investigate the strategic CRA's feedback effects. Similarly to Proposition 3, there are two cases: $\theta_2^* \geq \hat{\theta}_1$ and $\theta_2^* < \hat{\theta}_1$. In both cases, the strategic CRA's feedback effects have a negative component. Because the strategic CRA knows that when it assigns the rating $R = p$, more creditors will buy the debt and the firm's financial cost will decrease, it can assign more types of the firm the high rating $R = p$. That is, in equilibrium, the strategic CRA will employ the rating strategy with the threshold $\theta_1^* < \min\{\hat{\theta}_1, \theta_2^*\}$. Such a manipulation leads firms with $\theta \in [\theta_1^*, \min\{\hat{\theta}_1, \theta_2^*\})$ to gamble for resurrection, and thus leads to adverse real effects.

In the case with $\theta_2^* \geq \hat{\theta}_1$, the strategic CRA's real effects have another negative component. Because $\theta_1^* < \hat{\theta}_1$, the rating $R = p$ assigned by the strategic CRA is less informative than the rating $R = p$ assigned by the reflecting CRA. So, with the strategic CRA, after the rating $R = p$, fewer creditors buy the debt (comparing to the case with the reflecting CRA), the firm's financial cost increases, and thus fewer firms (measured by $\hat{\theta}_2 - \theta_2^*$) switch from HR to VP. That is, in the case with $\theta_2^* > \hat{\theta}_1$, the strategic CRA's rating strategy will weaken its information effects. The CRA's feedback effects in the case with $\theta_2^* \geq \tilde{\theta}_1$ are then illustrated in Figure 4 (with the green indicating positive real effects and the red indicating negative real effects).

[12] In the context of ratings, Daley, Green, and Vanasco (2017) recently show that credit ratings, as a source of public information, could reduce lending standards and lead to an oversupply of credit.

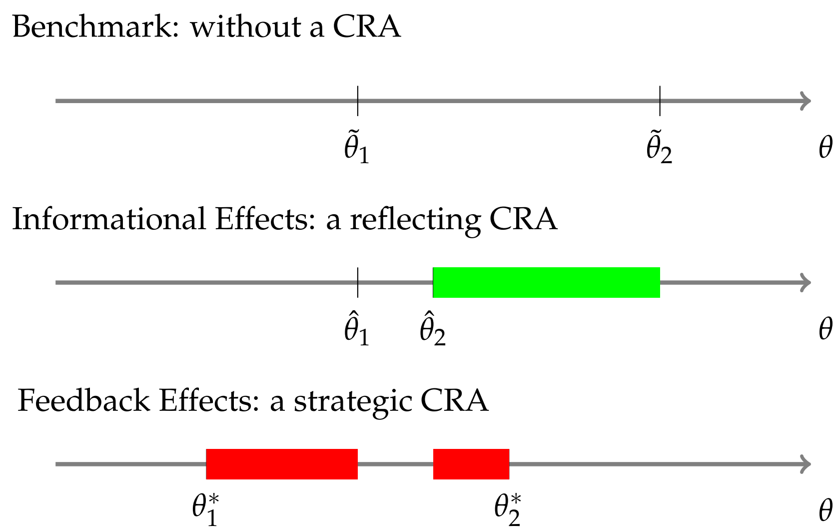

Figure 4: **CRA's feedback effects when $\theta_2^* > \tilde{\theta}_1$**

In the other case with $\theta_2^* < \hat{\theta}_1$, the second component of the strategic CRA's feedback effects is positive. This is because by assigning the rating $R = p$ to the firm with $\theta \in [\theta_1^*, \hat{\theta}_1)$, it is possible for the firm to invest in VP. Indeed, when $\theta \in [\theta_2^*, \hat{\theta}_2)$, the firm invests in VP, implying that the strategic CRA has positive feedback effects.

The above arguments are summarized in Proposition 5.

Proposition 5 *The strategic CRA's real effects can be decomposed into its informational effects and its feedback effects. The strategic CRA's informational effects, which are measured by $(\tilde{\theta}_2 - \hat{\theta}_2)(pV - qH)$, are always positive. When the set of parameters are such that $\theta_2^* \geq \hat{\theta}_1$, the strategic CRA's feedback effects, measured by*

$$(\tilde{\theta}_1 - \theta_1^*)(qH - B) + (\theta_2^* - \hat{\theta}_2)(qH - pV),$$

are negative; but given the set of parameters such that $\theta_2^* < \hat{\theta}_1$, the strategic CRA's feedback effects are measured by

$$(\theta_2^* - \theta_1^*)(qH - B) + (\hat{\theta}_1 - \theta_2^*)(pV - B) + (\hat{\theta}_2 - \hat{\theta}_1)(pV - qH),$$

whose sign is undetermined.

Proposition 5 implies that credit rating inflation itself does not necessarily lead to negative ex-ante real effects. Because inflated ratings are informative signals, they do increase the market's efficiency and lead to positive real effects. The negative real effects, however, arise from the CRA's feedback effects. Because the CRA knows that the rating will reduce the firm's financial costs and default likelihood, it will assign more firms the high rating, providing them with the opportunities to gamble for resurrection.

6 Empirical Predictions

The theory we develop in this paper provides several new empirical predictions about CRAs' rating strategies and credit rating inflation. In this section, we analyze how a CRA's rating strategy and the rating inflation vary when the economic environment changes. That is, we perform comparative static analysis to provide empirical predictions about CRAs' rating strategies and credit rating inflation.

From these comparative static analysis, we show that laxer credit rating strategies are not necessarily accompanied by higher rating inflation. In our model, both the CRA's rating strategy (measured by θ_1^*) and the credit rating inflation (measured by $\theta_2^* - \theta_1^*$) are endogenously determined. Then, an exogenous economic environment change may lead to a laxer rating strategy and a lower financial cost to the firm at the same time. While the former effect may increase the rating inflation, the latter effect may encourage the firm to invest in VP, which reduces the rating inflation. Therefore, whether a laxer rating strategy is accompanied by higher rating inflation depends on which effect dominates. This can help interpreting recent empirical findings: Alp (2013) and Baghai, Servaes, and Tamayo (2014) find that CRAs become more conservative by using stricter rating standards, but Strobl and Xia (2012) show that the stricter rating standards do not reduce credit rating inflation.

Proposition 6 *When β is sufficiently large,*[13] *a decrease in β, an increase in H, and a decrease in γ will all lead to a decrease in θ_1^*. However, a decrease in β has ambiguous effects on $\theta_2^* - \theta_1^*$, an increase in H increases $\theta_2^* - \theta_1^*$, and a decrease in γ decreases $\theta_2^* - \theta_1^*$.*

First, β is the precision of creditors' private signals, so it measures the firm's transparency. Proposition 6 shows that for more opaque firms, the CRA employs laxer rating strategies. By the properties of the truncated normal random variable's mean, when creditors' private signals become less precise, they believe that the firm is more likely to invest in VP. As a result, more creditors invest in the debt, and the firm's financial cost decreases, which allows the CRA to employ a laxer rating strategy. This is consistent with a recent empirical finding in Fong, Hong,

[13]This is the condition for the equilibrium uniqueness, which is critical for comparative static analysis.

Kacperczyk, and Kubik (2014) that security analysts can discipline CRAs by providing creditors with more information.

While Proposition 6 implies that CRAs employ laxer rating strategies for more opaque firms, it does not imply that credit ratings assigned to more opaque firms are more inflated. Since creditors will decrease their debt-investment threshold when the firm is more opaque, the firm's financial cost is lower, causing a smaller θ_2^*, which is the firm's VP investment threshold. Consequently, when creditors' signals are less precise, the CRA is more likely to assign the rating p to the firm, but the firm with a high rating is more likely to invest in VP. As a result, whether the rating inflation for a more opaque firm is higher or lower depends on which of these two effects dominates. This in turn depends on other parameters of the model.

Second, cross-sectionally, firms differ in the upside returns of their available projects. Equation (15) suggests that the highest upside return among all available projects may determine the credit rating assigned to the firm. Hence, it is interesting to consider how the firm's upside return from HR affects the CRA's rating strategy. An increase in H does not directly affect the creditors' behavior, because the creditors' payoffs are solely determined by the debt contract, which does not involve the cash flow to the firm, conditional on the success of the investment. Yet, H has direct effects on both the firm's investment and the CRA's rating strategy. On the one hand, an increase in H increases the firm's incentives to invest in HR, because the expected return from the HR is higher. On the other hand, an increase in H decreases the firm's incentives to default early, because the firm has limited liability. As a result, for fixed creditors' strategies, when H increases, the CRA's rating strategy will be laxer, and the firm is more likely to invest in HR rather than VP, resulting in higher credit rating inflation.

Finally, consider an increase in the debt market liquidity (formally, a decrease in γ). Then, the measure of total potential new creditors, $1-\gamma$, increases. The direct effect is that the firm's financial cost will surely decrease, because the firm needs to finance less money from the expensive non-debt sources, such as the bank credit line. In addition, a decrease in γ will lead more creditors to buy the debt, due to the strategic complementarities among the creditors. This further reduces the firm's financial cost. Then, the firm's threshold of investing in VP will decrease, implying that fewer firms will invest in HR given the CRA's credit rating strategy. In the meanwhile, the lower financial cost of the firm means that fewer firms may default early. As a result, the CRA would want to employ a laxer rating strategy. Furthermore, as γ decreases, the measure of firms that shift from HR to VP due to the reduced financial cost is greater than the measure of firms that gamble for resurrection because of the high credit rating, leading to lower credit rating inflation.

7 Discussions

In our model, the CRA maximizes the nominal rating for each θ-firm, and the credit ratings are only regulated by the partial verifiability constraint. That is, unless the firm defaults early, the CRA that assigns a positive rating to the firm will not be punished. In this section, we first analyze how a rating-dependent cost scheme (for example, reputation costs) affects the CRA's incentives and thus its real effects. Specifically, we assume that the CRA will incur a cost if the firm defaults, and such a cost is higher if the rating assigned is higher. This analysis not only generalizes our results in the core model, but also provides an important potential policy to regulate the credit rating industry.

We have also shown that the coordination and heterogeneous information among creditors play critical roles in the inflated credit rating's real effects. In this section, we further study the role of coordination and heterogeneous information among creditors by assuming that there is only one creditor who has $\$(1-\gamma)$. We assume that such a "large" creditor does not have a private signal, but all players observe a public signal. We find that when the public signal is sufficiently precise, the CRA barely has any real effect, demonstrating the important role of creditors' coordination in the CRA's real effects.

7.1 Reputation Costs when the Firm's Investment Fails

In this subsection, we consider the CRA's reputation cost conditional on the failure of the firm's investment. We assume that the CRA's benefits by providing the rating services are increasing in the ratings, due to the conflicts of interest. Therefore, $V^p > V^q > V^0 = 0$, where V^R is the CRA's benefit from assigning the rating R, and we normalize the benefit from assigning the rating $R = 0$ to be zero. If the firm does not default (either at date 1 or at date 2), the CRA will not incur any cost. However, if the firm defaults early, the CRA will incur a cost greater than V^p, because the early default is verifiable. This is indeed a general form of the partial verifiability constraint assumed in the core model. Importantly, if the firm invests, and the investment fails at date 2, the CRA incurs a cost. Such a cost, denoted by C^R, depends on the rating R assigned to the firm, and $C^p > C^q > 0$. Here, we assume that such reputation costs are exogenously designed by the government regulation. They may, however, arise from the repeated interactions between the creditors and the CRA. As shown by Mathis, McAndrews, and Rochet (2009), reputation cycles emerge in an infinite-horizon setup: Initially, the CRA's reputation cost is high, and so it engages in reputation building; when the CRA's reputation is sufficiently high, its reputation cost becomes lower, and it will inflate the rating.

Given the creditors' belief about the CRA's rating strategy and their best responses to credit ratings, the CRA can perfectly predict the firm's investment. Lemma 6 below shows that, the CRA's equilibrium rating strategy depends on the ratio of the benefit increment to the cost increment due to the rating upgrading from q to p.

Lemma 6 *The CRA's equilibrium rating strategy depends on the ratio $\left(V^p - V^q\right) / \left(C^p - C^q\right)$. There are three cases.*

1. *if $\frac{V^p - V^q}{C^p - C^q} \geq 1 - q$, the equilibrium rating strategy is in the form*

$$\mathcal{R}(\theta) = \begin{cases} p, & \text{if } \theta \geq \theta^I; \\ 0, & \text{if } \theta < \theta^I. \end{cases} \tag{17}$$

2. *if $\frac{V^p - V^q}{C^p - C^q} \leq 1 - p$, the equilibrium rating strategy is in the form*

$$\mathcal{R}(\theta) = \begin{cases} q, & \text{if } \theta \geq \theta^D; \\ 0, & \text{if } \theta < \theta^D. \end{cases} \tag{18}$$

3. *if $\frac{V^p - V^q}{C^p - C^q} \in (1 - p, 1 - q)$, the equilibrium rating strategy is in the form*

$$\mathcal{R}(\theta) = \begin{cases} p, & \text{if } \theta \geq \theta^p; \\ q, & \text{if } \theta \in \left[\theta^q, \theta^p\right); \\ 0, & \text{if } \theta < \theta^q, \end{cases} \tag{19}$$

   where $\theta^q \leq \theta^p$.

Lemma 6 shows that if the benefit of upgrading the rating from q to p is high enough (relative to the change of the cost), the CRA will employ an inflated rating strategy in equilibrium. This is actually the case in our core model, so the equilibrium will be the same as described in Proposition 2, and credit ratings' real effects are the same as in Proposition 3.

When the benefit of upgrading the rating from q to p (relative to the cost) is low enough, the CRA will employ a *deflated* rating strategy in equilibrium. Since such a rating strategy will not change creditors' belief (following the rating q), the equilibrium characterization is very similar to that in Proposition 2, and credit ratings' real effects are the same as in Proposition 3.

When the ratio of the benefit increment to the cost increment due to the rating upgrading from q to p is between $1 - p$ and $1 - q$, the CRA will assign a rating, which reflects the firm's true investment choice. In this case, the CRA is "self-disciplined." We now analyze a self-disciplined CRA's equilibrium rating behavior in more details.

Suppose that the CRA's rating is $R = 0$. Since the CRA is self-disciplined, all creditors believe that the firm will surely default early. Then, after $R = 0$, no creditor buys the debt, leading to the firm's financial cost $K(\theta) = f(\theta)$. Thus, the firm chooses to default early following $R = 0$ if and only if

$$f(\theta) \geq H. \tag{20}$$

Let z_0 be the solution to equation (20). Then, following the rating $R = 0$, the firm will default early if $\theta \leq z_0$.

Suppose that $R = q$. Then, in the equilibrium, all creditors believe that the firm will invest in HR. Because $qF < 1$, no creditor invests in the debt. Thus, the firm's financial cost $K(\theta) = f(\theta)$. For the firm to choose HR, we must have

$$\frac{pV - qH}{p - q} \leq f(\theta) < H. \tag{21}$$

Denote the solution to the equation $\frac{pV-qH}{p-q} = f(\theta)$ by z_q. Because $f(\theta)$ is decreasing, and $H > \frac{pV-qH}{p-q}$, we have $z_q > z_0$. Then, the firm invests in HR following the rating $R = q$, if and only if $\theta \in \left(z_0, z_q\right]$.

Finally, suppose that $R = p$. Then, in the equilibrium, all creditors invest in the debt, because they believe that the firm will invest in VP. This implies that the firm's financial cost is

$$K(\theta) = (1 - \gamma)F + \gamma f(\theta).$$

Denote the solution to the equation $(1 - \gamma)F + \gamma f(\theta) = \frac{pV-qH}{p-q}$ by z_p. Then, the firm invests in VP following the rating $R = p$, if and only if $\theta \geq z_p$. Note that $(1 - \gamma)F + \gamma f(\theta) < f(\theta)$ for all θ. Therefore, $z_q > z_p$.

The arguments above prove Proposition 7 below, which characterizes the equilibrium of the model with a self-disciplined CRA.

Proposition 7 *When $\frac{V^P-V^q}{C^p-C^q} \in (1-p, 1-q)$, the model has a unique equilibrium. In particular, if $z_q > z_p > z_0$, the equilibrium rating strategy is*

$$\mathcal{R}(\theta) = \begin{cases} 0, & \text{if} \quad \theta \leq z_0; \\ q, & \text{if} \quad \theta \in \left(z_0, z_p\right]; \\ p, & \text{if} \quad \theta \in \left(z_p, +\infty\right). \end{cases} \tag{22}$$

If $z_p < z_0 < z_q$, the equilibrium rating strategy is

$$\mathcal{R}(\theta) = \begin{cases} 0, & \textit{if} \quad \theta \leq z_p; \\ p, & \textit{if} \quad \theta > z_p. \end{cases} \tag{23}$$

Proposition 7 can have policy implications. Because both the credit rating and the firm's investment outcome are verifiable, the government can design a cost scheme (C^p, C^q), such that $\frac{V^p - V^q}{C^p - C^q} \in (1-p, 1-q)$, to punish the CRA when the firm's investment fails. Such a cost scheme can make the CRA "self-disciplined" and lead to unbiased credit ratings. However, to design the appropriate cost scheme (C^p, C^q), the government has to know the CRA's benefits from ratings, (V^p, V^q). While such information may not be available to the government, Lemma 6 at least shows that imposing too high costs is not a correct policy, because this may lead the CRA to employ a deflated rating strategy, which will have the same real effects as those of an inflated rating strategy.

7.2 The Role of Coordination and Belief Dispersion

In this section, we show how creditors' coordination incentives and belief dispersion play a role in determining the CRA's real effects. We analyze an environment with a large creditor in the debt market who has $\$(1-\gamma)$. In our model with a continuum of creditors, given the creditors' strategies, the CRA can perfectly predict the measure of creditors who buy the debt and thus the firm's financial cost and investment decision. Hence, to make a clear comparison, we assume that the large creditor in this extension does not possess a private signal about θ; instead, the creditor, the CRA, and the firm all observe a common precise public signal.

To formalize the idea, let's assume that the public signal leads to the belief $\theta \sim \mathcal{N}(\theta_s, \alpha^{-1})$. We will consider the case when α is sufficiently large. The key difference between our model with a continuum of creditors and this extension with a large creditor is that the large creditor's debt-investment decision will directly determine the firm's financial cost. Specifically,

$$K(\theta) = \begin{cases} (1-\gamma)F + \gamma f(\theta), & \text{if the large creditor invests in the debt;} \\ f(\theta), & \text{if the large creditor refrains from investing.} \end{cases}$$

Hence, when the large creditor chooses to invest in the debt, the firm's optimal investment choice is

$$\begin{cases} \text{Default early,} & \text{if} \quad (1-\gamma)F + \gamma f(\theta) > H; \\ \text{HR,} & \text{if} \quad (1-\gamma)F + \gamma f(\theta) \in \left(\frac{pV-qH}{p-q}, H\right]; \\ \text{VP,} & \text{if} \quad (1-\gamma)F + \gamma f(\theta) \le \frac{pV-qH}{p-q}. \end{cases}$$

Denote the solution to the equation $(1-\gamma)F + \gamma f(\theta) = H$ by y_1 and that to the equation $(1-\gamma)F + \gamma f(\theta) = (pV - qH)/(p-q)$ by y_2. The firm's optimal investment choice when the large creditor invests in the debt can be written as

$$\begin{cases} \text{Default early,} & \text{if} \quad \theta < y_1; \\ \text{HR,} & \text{if} \quad \theta \in [y_1, y_2); \\ \text{VP,} & \text{if} \quad \theta \ge y_2. \end{cases}$$

Similarly, we denote the solution to the equation $f(\theta) = H$ by y_1' and that to the equation $f(\theta) = (pV - qH)/(p-q)$ by y_2'. The firm's optimal investment choice when the large creditor chooses not to invest in the debt can be written as

$$\begin{cases} \text{Default early,} & \text{if} \quad \theta < y_1'; \\ \text{HR,} & \text{if} \quad \theta \in [y_1', y_2'); \\ \text{VP,} & \text{if} \quad \theta \ge y_2'. \end{cases}$$

Because $f(\theta) > (1-\gamma)F + \gamma f(\theta)$ for any θ, we have $y_1 < y_1'$ and $y_2 < y_2'$. When α is sufficiently large, the large creditor mainly relies on the public signal to make the debt-investment decision. Because the large creditor's behavior determines the firm's investment choice, the public signal and the large creditor's behavior determine the CRA's credit rating. Proposition 8 below shows the equilibrium credit rating strategy in this extension with a large creditor.

Proposition 8 (Credit Ratings Affected by Public Signals) *There exists $\bar{\alpha} > 0$, such that for all $\alpha > \bar{\alpha}$, the public signal determines the CRA's equilibrium rating strategy. Specifically,*

1. *when $\theta_s \ge y_2'$, the CRA will employ the rating strategy $\theta_1^* = y_1$;*
2. *when $\theta_s < y_2$, the CRA will employ the rating strategy $\theta_1^* = y_1'$; and*
3. *when $\theta_s \in [y_2, y_2')$, the CRA will set $\theta_1^* = y_1$ if the large creditor invests in the debt after $R = p$, while the CRA will set $\theta_1^* = y_1'$ if the large creditor refrain from investing in the debt after $R = p$.*

We observe in Proposition 8 that, when the public signal is very positive ($\theta_s \geq y_2'$), the CRA employs a laxer rating strategy, meaning that the good rating is a less positive signal. When the public signal is very negative ($\theta_s < y_2'$), the CRA employs a stricter rating strategy, meaning that the good rating is a more positive signal. Such a "substitution" results from the fact that the large creditor relies more on the public signal when making the debt-investment decision. When the public signal is in the medium range, there will be multiple equilibria: If the large creditor invests in the debt, the CRA will employ a more inflated credit rating strategy; and if the large creditor refrains from investing in the debt, the CRA will employ a more conservative rating strategy.

It follows from Proposition 8 that, when there is a large creditor (and so no coordination problem or belief dispersion) and the public signal is sufficiently precise, the CRA hardly has any real effect; the large creditor will ignore the information extracted from the credit ratings. By contrast, in the model presented in Section 2, even if we allow for a public signal or an informative prior, if creditors' private signals are sufficiently precise (β is extremely large), the credit ratings will surely affect a positive measure of creditors' decisions. This is because the continuum of creditors have dispersed beliefs caused by their conditionally independent private signals. Since credit ratings affect creditors' beliefs by truncating their belief supports, some creditors' beliefs about the firm's investment choice are surely affected. Then, the strategic complementarities among creditors will amplify such an effect, expanding it to other creditors who are not directly affected, leading to the CRA's significant real effects. Such a comparison shows the importance of creditors' coordination and belief dispersion in our core model. This analysis also demonstrates that the feedback effect of credit rating agencies will stay large even if other sources of precise public information emerge.

8 Conclusion

We study credit rating agencies' effects on firm investments. We show that high credit ratings, though commonly known to be potentially inflated, exclude extremely bad firms from creditor belief support. Therefore, high ratings make the creditors more optimistic, reduce the firm's financial costs, and thus change its investments. That is, even in an environment with perfectly rational and well-informed creditors, inflated ratings still have significant real effects.

Such real effects, however, could be positive or negative. With the high ratings, some firms take risky projects instead of default efficiently, implying CRAs' adverse real effects; but some other firms will switch from risky inefficient investments to safe efficient investments, implying

CRAs' positive real effects. CRAs' overall ex-ante real effects then depend on the economic environment. Specifically, when the upside return of the risky project is high, CRAs' overall ex-ante real effects are negative.

In order to better understand why the CRA may have negative ex-ante real effects, even though it provides informative signals to the corporate bond market, we further decompose its real effects into its informational effects and its feedback effects. We show that credit ratings that act as new informative signals do positively affect firms' investment efficiency. Hence, the CRA's negative real effects arise from its feedback effects. Indeed, the CRA takes advantage of the feedback between credit ratings and firm investments to assign high ratings to more firms, providing a chance for those firms to gamble for resurrection. Such a manipulation leads to negative real effects.

We emphasize that credit rating standards and credit rating inflation are two different concepts, and they are both endogenously determined. Furthermore, changes of economic environments that lead to laxer rating strategies do not necessarily cause higher rating inflation.

Our paper offers applied and theoretical contributions. From the applied perspective, we provide a rational framework, enabling us to analyze credit rating inflation and credit rating agencies' real effects in a model of feedback. While we focus on the credit ratings assigned to a firm in the paper, our model can also be applied to sovereign ratings. In fact, the assumption of the credit ratings' partial verifiability constraints may be very appropriate in the scenario of sovereign ratings: Because there are fewer data points of sovereign ratings, the inflated ratings are harder to be detected. Our model also generates several testable empirical predictions and some policy suggestions.

While we focus on credit ratings' real effects in this paper, the intuition and key mechanism in this paper may be more broadly applicable. For example, in the bank stress tests, regulators may want to declare a larger number of banks solvent than truly are. This may be efficiency-enhancing for some banks (because of the reduced financial costs), but may lead some other banks to take risky projects. In addition, the economic environment we set up can be applied to many other scenarios, such as financial advising, firm disclosure, auditing, marketing, and academic grading and recommendation.

From the theoretical perspective, we analyze an expert information disclosure model with multiple audiences, who have coordination incentives and dispersed beliefs. More importantly, the expert's message will endogenously affect the fundamentals signaled by the message. This can motivate new research on general disclosure models.

A Proofs of lemmas and propositions

Proof of Proposition 1:

To show there is a unique equilibrium in this benchmark model, we only need to show that there is a unique solution $(\tilde{\theta}_1, \tilde{\theta}_2, \tilde{x})$ to equations (8), (9), and (10).

We first solve $\tilde{x}$ from equation (9). Define

$$\tilde{\Delta} = \frac{\frac{pV-qH}{p-q} - \left[(1-\gamma)F + \gamma f(\tilde{\theta}_2)\right]}{(1-\gamma)\left(f(\tilde{\theta}_2) - F\right)}.$$

Because $f(\theta)$ is strictly decreasing, $\tilde{\Delta}$ is strictly increasing in $\tilde{\theta}_2$. Then we have

$$\tilde{x} = \tilde{\theta}_2 + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\tilde{\Delta}), \tag{24}$$

and $\tilde{x}$ is strictly increasing in $\tilde{\theta}_2$.

Plugging $\tilde{x}$ as a function of $\tilde{\theta}_2$ into equation (10), we have

$$\tilde{\Delta}(pF - qF) + \Phi\left[\sqrt{\beta}\left(\tilde{\theta}_2 - \tilde{\theta}_1 + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\tilde{\Delta})\right)\right]qF = 1. \tag{25}$$

The left-hand side of equation (25) strictly increases in $\tilde{\theta}_2$ and strictly decreases in $\tilde{\theta}_1$. So, we have $\partial\tilde{\theta}_2/\partial\tilde{\theta}_1 > 0$ and $\partial\tilde{x}/\partial\tilde{\theta}_1 > 0$.

Let's finally consider equation (8). The derivative of the left-hand side of equation (8) is

$$\frac{\partial K}{\partial \tilde{\theta}_1} + \frac{\partial K}{\partial \tilde{x}}\frac{\partial \tilde{x}}{\partial \tilde{\theta}_1},$$

where

$$\begin{aligned}
\frac{\partial K}{\partial \tilde{\theta}_1} &= \left[\gamma + (1-\gamma)\Phi\left(\sqrt{\beta}(\tilde{x} - \tilde{\theta}_1)\right)\right] f'(\tilde{\theta}_1) \\
&\qquad - (1-\gamma)\sqrt{\beta}\phi\left(\sqrt{\beta}(\tilde{x} - \tilde{\theta}_1)\right)\left(f(\tilde{\theta}_1) - F\right) < 0; \\
\frac{\partial K}{\partial \tilde{x}}\frac{\partial \tilde{x}}{\partial \tilde{\theta}_1} &= (1-\gamma)\sqrt{\beta}\phi\left(\sqrt{\beta}(\tilde{x} - \tilde{\theta}_1)\right)\left(f(\tilde{\theta}_1) - F\right)\frac{\partial \tilde{x}}{\partial \tilde{\theta}_1} > 0.
\end{aligned}$$

Note that $\tilde{\Delta}$ is between 0 and 1. From equation (25), we have $\beta \to +\infty$, $\tilde{\Delta}$ is bounded away from both 0 and 1. To see this, suppose $\tilde{\Delta} \to 1$ first. Then, the left-hand side of equation (25) goes to pF, which is greater than 1, the right-hand side of equation (25). Similarly, if $\tilde{\Delta} \to 0$, the left-hand side of equation (25) is strictly less than 1.

Hence, from equation (24), $\tilde{x} \to \tilde{\theta}_2$. In addition, as $\beta \to +\infty$, $\tilde{\theta}_2$ cannot converge to $\tilde{\theta}_1$; otherwise, equation (8) and equation (9) cannot hold at the same time. Therefore, as $\beta \to +\infty$, $\tilde{x} - \tilde{\theta}_1$ is bounded away from 0. This implies that

$$\lim_{\beta \to +\infty} \sqrt{\beta}\phi\left(\sqrt{\beta}(\tilde{x} - \tilde{\theta}_1)\right) = 0.$$

Therefore, though $\frac{\partial K}{\partial \tilde{x}} \frac{\partial \tilde{x}}{\partial \tilde{\theta}_1} > 0$, when β is large enough, $\frac{\partial K}{\partial \tilde{x}} \frac{\partial \tilde{x}}{\partial \tilde{\theta}_1}$ is very close to 0. For the term of $\frac{\partial K}{\partial \tilde{\theta}_1}$, it will not go to 0 as β goes to ∞, because $f'(\tilde{\theta}_1) < 0$. Therefore, there is a $\tilde{\beta} > 0$, such that for all $\beta > \tilde{\beta}$, the left-hand side of equation (8) strictly decreases in $\tilde{\theta}_1$. The left-hand side of equation (8) converges to F when $\tilde{\theta}_1$ goes to $+\infty$ and diverges to $+\infty$ when $\tilde{\theta}_1$ goes to $-\infty$. Then by the continuity of function $f(\cdot)$, there is a unique $\tilde{\theta}_1$. Then, there is a unique solution to equation (8), (9), and (10).

Q.E.D.

Proof of Lemma 1:

We first show that the CRA will not assign the rating q in equilibrium. Suppose there is an equilibrium, in which the CRA assigns the rating q to θ-firm when $\theta \in (\theta_1, \theta_2)$. There are two cases. In the first case, there is no $\theta < \theta_1$ such that θ-firm receives the rating p. Then, observing the rating p, creditors are more optimistic about the firm's fundamentals. Since f is strictly decreasing in θ, if the CRA deviates to assign the rating p to θ-firm when $\theta \in (\theta_1, \theta_2)$, more creditors will invest in the debt, reducing θ-firm's financial cost. Hence, because θ-firm does not default early following the rating q, it does not default early following the rating p either. So, a rating strategy that assigns the rating q to θ-firm for $\theta \in (\theta_1, \theta_2)$ but does not assign the rating p to a positive measure of firms with θ below θ_1 cannot be an equilibrium rating strategy.

Now, consider the second case in which the rating strategy specifies $\mathcal{R}(\theta) = p$ when $\theta \in [\theta_3, \theta_4)$ and $\mathcal{R}(\theta) = p$ when $\theta \in (\theta_1, \theta_2)$, where $\theta_3 < \theta_4 < \theta_1$. Due to the partial verifiability constraint, θ_3-firm does not default early. Then, if the CRA assigns the rating p to θ-firm for $\theta \in (\theta_1, \theta_2)$, θ-firm's financial cost is lower than θ_3-firm's. Therefore, θ-firm does not default early, and the deviation is profitable. These arguments show that the CRA will not assign the rating q in equilibrium.

Suppose there is an equilibrium, in which θ-firm does not default early. So the CRA will assign the rating $\mathcal{R}(\theta) = p$, because the CRA wants to maximize the nominal rating $\mathcal{R}(\theta)$. Let $W(p)$ be the measure of creditors who choose to buy the debt, after observing the credit rating

$\mathcal{R}(\theta)$ and their own private signals. Then the assumption that θ-firm does not default early implies

$$K(\theta) = W(p,\theta)F + (1 - W(p,\theta))\, f(\theta) < H.$$

Now, let's consider any θ'-firm with $\theta' > \theta$. Again, because the CRA wants to maximize the nominal rating $R(\theta')$, if and only if the θ'-firm does not default early, the CRA will assign $R(\theta') = p$. In a monotone equilibrium, any creditor i's strategy is monotonic in his private signal x_i, and any creditor's private signal conditional on θ' First-order Stochastic dominates that conditional on θ. So $W(p,\theta') > W(p,\theta)$. Recalling that $f(\theta) > F$ for all θ, we have

$$\begin{aligned}
K(\theta') &= W(p,\theta')F + \left(1 - W(p,\theta')\right) f(\theta') \\
&< W(p,\theta)F + (1 - W(p,\theta))\, f(\theta') \\
&< W(p,\theta)F + (1 - W(p,\theta))\, f(\theta) \\
&< H.
\end{aligned}$$

Therefore, if θ-firm does not default early, θ'-firm does not default early either, implying that in the equilibrium, $R(\theta') = p$.

Furthermore, independent of creditors' decisions, when θ is very negative, the firm will default early, and when θ is very positive, the firm will not default early. As a result, in any equilibrium (if exists), the CRA's rating strategy must be in the form described by equation (11).

Q.E.D.

Proof of Lemma 2:

Suppose there is an equilibrium in which the firm invests in VP for all θ such that $\mathcal{R}(\theta) = p$. All creditors will invest in the debt, leading to the firm's financial cost

$$\gamma f(\theta) + (1 - \gamma)F.$$

For the firm to choose VP if and only if $\theta \geq \theta_1^*$, we must have

$$(1 - \gamma)F + \gamma f(\theta_1^*) \leq \frac{pV - qH}{p - q} < H.$$

But because $f(\dot{})$ is continuous and strictly decreasing, there exists $\hat{\theta}_1^* < \theta_1^*$ such that,

$$\frac{pV - qH}{p - q} < \gamma f(\hat{\theta}_1^*) + (1 - \gamma)F < H.$$

That is, there is a positive measure subset of θ's that are greater than $\hat{\theta}_1^*$ but very close to $\hat{\theta}_1^*$, the firm will invest in HR. Since the firm's investment choice HR is unverifiable, a deviation to the rating strategy with $\hat{\theta}_1^*$ is profitable to the CRA. Therefore, the rating strategy with θ_1^* such that $(1-\gamma)F + \gamma f(\theta_1^*) \leq \frac{pV-qH}{p-q}$ cannot be part of an equilibrium. Therefore, in any monotone equilibrium (if it exists), the rating strategy must be inflated.

Q.E.D.

Proof of Lemma 3:

For a given $x^* \in \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$, the left-hand side of equation (13) is strictly decreasing in θ. When $\theta \to +\infty$, the LHS of equation (13) goes to F, which is assumed strictly less than $\frac{pV-qH}{p-q}$. However, if when $\theta = \theta_1^*$, the LHS is still less than $\frac{pV-qH}{p-q}$, the firm will always choose VP after the rating $R = p$. This contradicts Lemma 2. Therefore, for a given $x^* \in \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$, there is a unique $\theta_2^* > \theta_1^*$, such that equation (13) holds. Then we can solve for x^* from equation (13)

$$x^* = \theta_2^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}\left[\frac{\frac{pV-qH}{p-q} - ((1-\gamma)F + \gamma f(\theta_2^*))}{(1-\gamma)\left[f(\theta_2^*) - F\right]}\right].$$

Denote

$$\Delta = \frac{\frac{pV-qH}{p-q} - ((1-\gamma)F + \gamma f(\theta_2^*))}{(1-\gamma)\left[f(\theta_2^*) - F\right]},$$

so $x^* = \theta_2^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)$. Because $f(\cdot)$ is strictly decreasing, Δ is strictly increasing in θ_2^*, and thus x^* is strictly increasing in θ_2^*.

Then, plugging x^* as a function of θ_2^* into equation (14), we have

$$\frac{\Delta}{\Phi\left[\sqrt{\beta}\left(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)\right)\right]}(pF - qF) = 1 - qF. \tag{26}$$

Differentiating the left-hand side of equation (26), the sign of this derivative would be the same as the sign of

$$\begin{aligned}
&\frac{\partial \Delta}{\partial \theta_2^*}\frac{\Phi\left[\sqrt{\beta}\left(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)\right)\right]}{\Delta} \\
&\qquad - \phi\left[\sqrt{\beta}\left(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)\right)\right]\left(\sqrt{\beta} + \frac{1}{\phi(\Delta)}\frac{\partial \Delta}{\partial \theta_2^*}\right) \\
= {} & \frac{\partial \Delta}{\partial \theta_2^*}\frac{pF - qF}{1 - qF} - \phi\left[\sqrt{\beta}\left(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)\right)\right]\left(\sqrt{\beta} + \frac{1}{\phi(\Delta)}\frac{\partial \Delta}{\partial \theta_2^*}\right).
\end{aligned}$$

The first term is positive for any β. The second, though is negative, will converge to 0 as $\beta \to +\infty$. This is because $\phi\left[\sqrt{\beta}\left(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)\right)\right]$ will converge to 0 higher order faster than $\sqrt{\beta}$. We need to consider three cases to prove this argument. First, as $\beta \to +\infty$, $\Delta \to 1$. In this case, it is trivially that $\phi\left[\sqrt{\beta}\left(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)\right)\right]\sqrt{\beta}$ converges to 0. Second, as $\beta \to +\infty$, Δ is bounded away from both 0 and 1. Then $x^* - \theta_2^* \to 0$. But because $\theta_2^* - \theta_1^*$ is positive and bounded away from 0, $\phi\left[\sqrt{\beta}\left(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)\right)\right]\sqrt{\beta} = \phi\left[\sqrt{\beta}(x^* - \theta_1^*)\right]\sqrt{\beta}$ must converge to 0. Finally, as $\beta \to +\infty$, $\Delta \to 0$. Then from equation (26), we must have $\Phi\left[\sqrt{\beta}(x^* - \theta_1^*)\right] \to 0$ and thus $\sqrt{\beta}(x^* - \theta_1^*) \to -\infty$ as $\beta \to +\infty$. By L'Hôpital's rule, we have

$$\lim_{\beta \to +\infty} \frac{1}{\sqrt{\beta}(x^* - \theta_1^*)} = \lim_{\beta \to +\infty} \frac{\beta^{-\frac{1}{2}}}{(x^* - \theta_1^*)} = \lim_{\beta \to +\infty} \frac{1}{2\beta^{\frac{3}{2}}\frac{\partial x^*}{\partial \beta}} = 0$$

Therefore, $\lim\limits_{\beta \to +\infty} 2\beta^{\frac{3}{2}}\frac{\partial x^*}{\partial \beta} = +\infty$. Then, simple algebra can lead to the result that $\phi\left[\sqrt{\beta}\left(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)\right)\right]\sqrt{\beta}$ converges to 0, as $\beta \to +\infty$. Therefore, there is a β^* such that when $\beta > \beta^*$, the left-hand side of equation (26) is strictly increasing in θ_2^*.

Note by definition, Δ must be a number in $[0,1]$. Therefore, there are $\bar{\theta}$ and $\underline{\theta}$ such that, $\theta_1^* < \underline{\theta} < \bar{\theta} < +\infty$, $\Delta(\bar{\theta}) = 1$, and $\Delta(\underline{\theta}) = 0$. Then when $\theta_2^* \to \bar{\theta}$, the left hand side of equation (26) is strictly greater than $1 - qF$; when $\theta_2^* \to \underline{\theta}$, the left hand side of equation (26) is close to 0 and thus strictly smaller than $1 - qF$.

Therefore, there is a unique θ_2^*, and thus there is a unique x^*.

Q.E.D.

Proof of Lemma 4:

The left-hand side of equation (26) is strictly increasing in θ_1^*, fixing θ_2^*. Combined with the fact that the left-hand side of equation (26) is strictly increasing in θ_2^*, the Implicit Function Theorem implies that θ_2^* is strictly decreasing in θ_1^*. Since x^* is strictly increasing θ_2^*, x^* is strictly decreasing in θ_1^* (given θ_2^*, x^* is determined by equation $x^* = \theta_2^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}[\Delta]$).

Q.E.D.

Proof of Proposition 2:

When β is sufficiently large, Lemma 3 shows that, for a fixed θ_1^*, there is a unique solution, (x^*, θ_2^*), to equation (13) and equation (14) following $R = p$. Then, by Lemma 4, we only need to show that there is a unique θ_1^* such that equation (15) holds, given x^* as a function of θ_1^*. When β is sufficiently large, by Lemma 4, x^* is strictly decreasing in θ_1^*. Then, the derivative of the left hand side of equation (15) with respect to θ is

$$\begin{aligned}&\left[\gamma + (1-\gamma)\Phi[\sqrt{\beta}(x^* - \theta)]\right] f'(\theta)\\ &\quad -(1-\gamma)\phi[\sqrt{\beta}(x^* - \theta)]\sqrt{\beta}(f(\theta) - F) + (1-\gamma)\phi[\sqrt{\beta}(x^* - \theta)]\sqrt{\beta}\frac{\partial x^*}{\partial \theta}(f(\theta) - F) < 0.\end{aligned}$$

We know that when $\theta \to +\infty$, $f(\theta) \to F$, the left hand side of equation (15) converges to F, which is less than H; when $\theta \to -\infty$, $f(\theta) \to +\infty$, the left hand side of equation (15) diverges to $+\infty$, which is greater than H. Therefore, the solution to equation (15) exists and is unique.

Because $H > \frac{pV - qH}{p-q}$, equation (15) and equation (13) imply that $\theta_2^* > \theta_1^*$. In addition, equation (15) also implies that $f(\theta_1^*) > H$, because $f(\theta) > F$ for all $\theta \in \mathbb{R}$. These complete the proof of the uniqueness of the equilibrium of the model.

Q.E.D.

Proof of Lemma 5:

Recall that the three equations determining the equilibrium of the benchmark model are

$$[(1-\gamma)F + \gamma f(\theta_1)] + (1-\gamma)\Phi\left[\sqrt{\beta}(x - \theta_1)\right](f(\theta_1) - F) = H \tag{27}$$

$$[(1-\gamma)F + \gamma f(\theta_2)] + (1-\gamma)\Phi\left[\sqrt{\beta}(x - \theta_2)\right](f(\theta_2) - F) = \frac{pV - qH}{p-q} \tag{28}$$

$$\left\{\Phi\left[\sqrt{\beta}(\theta_2 - x)\right] - \Phi\left[\sqrt{\beta}(\theta_1 - x)\right]\right\} qF + \left\{1 - \Phi\left[\sqrt{\beta}(\theta_2 - x)\right]\right\} pF = 1; \tag{29}$$

and the three equations determining the equilibrium of the model with the CRA are

$$[(1-\gamma)F + \gamma f(\theta_1)] + (1-\gamma)\Phi\left[\sqrt{\beta}(x - \theta_1)\right](f(\theta_1) - F) = H \tag{30}$$

$$[(1-\gamma)F + \gamma f(\theta_2)] + (1-\gamma)\Phi\left[\sqrt{\beta}(x - \theta_2)\right](f(\theta_2) - F) = \frac{pV - qH}{p-q} \tag{31}$$

$$\frac{\Phi[\sqrt{\beta}(\theta_2 - x)] - \Phi[\sqrt{\beta}(\theta_1 - x)]}{1 - \Phi[\sqrt{\beta}(\theta_1 - x)]} qF + \frac{1 - \Phi[\sqrt{\beta}(\theta_2 - x)]}{1 - \Phi[\sqrt{\beta}(\theta_1 - x)]} pF = 1 \tag{32}$$

The difference between the equilibrium in the benchmark model and that in the model with the CRA stems from the difference between equation (29) and equation (32). That is, the creditors' indifference conditions differ. If we change equation (29) by dividing both sides by the term $1-\Phi[\sqrt{\beta}(\theta_1-x)]$, we have

$$
\begin{aligned}
& \frac{\Phi[\sqrt{\beta}(\theta_2-x)]-\Phi[\sqrt{\beta}(\theta_1-x)]}{1-\Phi[\sqrt{\beta}(\theta_1-x)]}qF+\frac{1-\Phi[\sqrt{\beta}(\theta_2-x)]}{1-\Phi[\sqrt{\beta}(\theta_1-x)]}pF \\
= & \frac{1}{1-\Phi[\sqrt{\beta}(\theta_1-x)]} \\
> & 1.
\end{aligned} \tag{33}
$$

Solve x as a function of θ_2 from equation (28) or equation (31), and plug it into equation (33) and equation (32). Then, for a same θ_1, θ_2 in equation (33) is greater than that in equation (32), because the left-hand sides of these two equations are strictly increasing in θ_2. Hence, x in equation (33) is greater than x in equation (32). Furthermore, because θ_1 in the benchmark model is positively correlated to θ_2, while θ_1 in the model with the CRA is negatively correlated to θ_2, we know $\theta_1^* < \tilde{\theta}_1$. Moreover, we have $\tilde{\theta}_2 > \theta_2^*$ and $\tilde{x} > x^*$.

However, the sign of $\theta_2^* - \tilde{\theta}_1$ is undetermined. Consider equation (27) and equation (31). Both $\tilde{\theta}_1$ and θ_2^* are strictly increasing functions of x. While we have shown that $\tilde{x} > x^*$, the right-hand side of equation (27) is greater than that of equation (31). Therefore, without specifying parameters' values, we cannot determine the sign of $\theta_2^* - \tilde{\theta}_1$.

Q.E.D.

Proof of Proposition 3:

Suppose that $\theta \geq \tilde{\theta}_2$. It follows from Proposition 2 that the CRA assigns the rating p to the firm, and the firm will invest in VP. Hence, the firm's expected NPV is $\Omega = pV - 1$. Denote by $\tilde{\Omega}$ the expected NPV without the CRA. As we show in Proposition 1, without the CRA, when $\theta \geq \tilde{\theta}_2$, the firm invests in VP, and thus $\tilde{\Omega} = pV - 1$. Therefore, in this case, $\Omega = \tilde{\Omega}$, and hence, the CRA has no effect on the expected NPV.

Similarly, when $\theta < \theta_1^*$, the firm defaults early with or without the CRA. Therefore, $\Omega = \tilde{\Omega} = B$. Hence, for this range of θ's, the CRA has no effect on the expected NPV either.

When $\theta \in [\theta_1^*, \tilde{\theta}_2)$, the CRA's effect depends on the parameters of the model. In the first case where $\theta_2^* > \tilde{\theta}_1$, for $\theta \in [\theta_1^*, \tilde{\theta}_1)$, $\Omega = qH - 1 < B = \tilde{\Omega}$, because qH is assumed to be less than B. For $\theta \in [\theta_2^*, \tilde{\theta}_2)$, $\Omega = pV - 1 > qH - 1 = \tilde{\Omega}$. For all $\theta \in [\tilde{\theta}_1, \theta_2^*)$, the firm invests in HR with or without the CRA; thus, CRA has no effect on the expected NPVs of such firms. Therefore, in such a case, the CRA's ex-ante real effects are

$$(\tilde{\theta}_2 - \theta_2^*)(pV - qH) + (\tilde{\theta}_1 - \theta_1^*)(qH - B).$$

In case 2 where $\theta_2^* \leq \tilde{\theta}_1$, for $\theta \in [\theta_1^*, \theta_2^*)$, $\Omega = qH - 1 < B = \tilde{\Omega}$. For $\theta \in \left[\theta_2^*, \tilde{\theta}_1\right)$, $\Omega = pV - 1 > B = \tilde{\Omega}$. Finally, for $\theta \in \left[\tilde{\theta}_1, \tilde{\theta}_2\right)$, $\Omega = pV - 1 > qH - 1 = \tilde{\Omega}$ Therefore, in this case, the CRA's ex-ante real effects are

$$(\tilde{\theta}_2 - \tilde{\theta}_1)(pV - qH) + (\tilde{\theta}_1 - \theta_2^*)(pV - B) + (\theta_2^* - \theta_1^*)(qH - B).$$

Q.E.D.

Proof of Proposition 4:

Part 1: We first consider the subgame following the rating $R = 0$. It then follows from equation (16) that $\theta < \hat{\theta}_1 = \tilde{\theta}_1$. Suppose that all creditors refrain from investing. Then the θ-firm's financial cost is

$$\begin{aligned}
K(\theta) &= f(\theta) \\
&> [(1-\gamma)F + \gamma f(\theta)] + (1-\gamma)\Phi\left(\sqrt{\beta}(x^* - \theta)\right)(f(\theta) - F) \\
&> \left[(1-\gamma)F + \gamma f(\hat{\theta}_1)\right] + (1-\gamma)\Phi\left(\sqrt{\beta}(x^* - \hat{\theta}_1)\right)\left(f(\hat{\theta}_1) - F\right) \\
&> \left[(1-\gamma)F + \gamma f(\tilde{\theta}_1)\right] + (1-\gamma)\Phi\left(\sqrt{\beta}(\tilde{x} - \tilde{\theta}_1)\right)\left(f(\tilde{\theta}_1) - F\right) \\
&= H.
\end{aligned}$$

Hence, if all creditors refrains from investing, the θ-firm will default early. On the other hand, given that any θ-firm will default early, no creditor will invest in the debt, implying that there is an equilibrium in which the firm will default early when receiving the rating $R = 0$ assigned by the reflecting CRA.

We now show that the subgame following $R = 0$ has no equilibrium in which the firm will continue to invest in either HR or VP. Suppose there is a (monotone) equilibrium in which a creditor with the private signal x_i invests in the debt if and only if $x_i \geq x'$, when the rating is $R = 0$. Here, $x' \in \mathbb{R}$. Since some creditors are willing to invest, they must believe that any θ-firm will invest in VP if $\theta \in \left[\theta_2', \hat{\theta}_1\right)$, and that by the continuity of the firm's financial cost, any θ-firm will invest in HR if $\theta \in [\theta_1', \theta_2')$, where $\theta_1', \theta_2' \in \mathbb{R}$ and $\theta_1' < \theta_2' < \hat{\theta}_1$. Therefore, such an equilibrium can be characterized by the following system of equations

$$\left[(1-\gamma)F + \gamma f(\theta_1')\right] + (1-\gamma)\Phi\left[\sqrt{\beta}(x' - \theta_1')\right]\left(f(\theta_1') - F\right) = H \tag{34}$$

$$\left[(1-\gamma)F + \gamma f(\theta_2')\right] + (1-\gamma)\Phi\left[\sqrt{\beta}(x' - \theta_2')\right]\left(f(\theta_2') - F\right) = \frac{pV - qH}{p - q} \tag{35}$$

$$\frac{\Phi[\sqrt{\beta}(\theta_2' - x)] - \Phi[\sqrt{\beta}(\theta_1' - x')]}{\Phi[\sqrt{\beta}(\hat{\theta} - x')]} qF + \frac{\Phi[\sqrt{\beta}(\hat{\theta} - x')] - \Phi[\sqrt{\beta}(\theta_2' - x')]}{\Phi[\sqrt{\beta}(\hat{\theta} - x')]} pF = 1 \tag{36}$$

Comparing equation (35) with equation (9), we can see that since $\theta_2' < \hat{\theta}_1 = \tilde{\theta}_1 < \tilde{\theta}_2$, x' must be strictly less than $\hat{x}$.

Note that for any $\hat{\theta}_1$ (which may not be $\tilde{\theta}_1$), given the committed rating rule (equation (16)), a monotone equilibrium with some θ-firm investing in VP or HR must be characterized by the system of equations (34), (35), and (36). The following argument therefore relies on the comparative static analysis of the solution to such a system of equations.

Solving x' as a function of θ_2' from equation (35) and substituting it into equation (34) and equation (36), we get

$$
\begin{aligned}
(1-\gamma)\Phi\left(\Psi\right)\left(f(\theta_1')-F\right)+\gamma f(\theta_1') &= H-(1-\gamma)F \\
\Delta(pF-qF)+qF\Phi\left(\Psi\right)-(pF-1)\Phi\left(\Psi'\right) &= 1,
\end{aligned}
$$

where $\Delta = \frac{\frac{pV-qH}{p-q}-\left[(1-\gamma)F+\gamma f(\theta_2')\right]}{(1-\gamma)\left(f(\theta_2')-F\right)}$, $\Psi = \sqrt{\beta}(\theta_2'-\theta_1')+\Phi^{-1}(\Delta)$, and $\Psi' = \sqrt{\beta}(\theta_2'-\hat{\theta}_1)+\Phi^{-1}(\Delta)$.

Totally differentiating the above system of equations, we get

$$
A\begin{bmatrix}\frac{\partial\theta_1'}{\partial\hat{\theta}_1} \\ \frac{\partial\theta_2'}{\partial\hat{\theta}_1}\end{bmatrix} = \begin{bmatrix} 0 \\ (pF-1)\phi(\Psi')\frac{\partial\Psi'}{\partial\hat{\theta}_1}\end{bmatrix}, \tag{37}
$$

where

$$
A = \begin{bmatrix} \left[(1-\gamma)\Phi(\Psi)+\gamma\right]f'(\theta_1')+(1-\gamma)\left(f(\theta_1')-F\right)\phi(\Psi)\frac{\partial\Psi}{\partial\theta_1'} & (1-\gamma)\left(f(\theta_1')-F\right)\phi(\Psi)\frac{\partial\Psi}{\partial\theta_2'} \\ qF\phi(\Psi)\frac{\partial\Psi}{\partial\theta_1'} & (p-q)F\frac{\partial\Delta}{\partial\theta_2'}+qF\phi(\Psi)\frac{\partial\Psi}{\partial\theta_2'}-(pF-1)\phi(\Psi')\frac{\partial\Psi'}{\partial\theta_2'}\end{bmatrix}. \tag{38}
$$

Note that, when β is sufficiently large, $|A| < 0$. We show that $\partial\theta_1'/\partial\hat{\theta}_1 < 0$ and $\partial\theta_2'/\partial\hat{\theta}_1 < 0$. Therefore, when $\hat{\theta}$ goes to $+\infty$, the equilibrium converges to an equilibrium with $x' < \hat{x}$. However, when $\hat{\theta}$ goes to $+\infty$, equation (36) goes to equation (10). This implies that the benchmark model has another equilibrium with $x' < \hat{x}$. This contradicts the conclusion in Proposition 1 that the benchmark model has a unique equilibrium. Therefore, in the case with a reflecting CRA, the subgame following $R = 0$ has a unique equilibrium in which the firm defaults early and all creditors run.

Part 2: Similarly to the proof of Part 1, in the subgame following $R = p$, there cannot be an equilibrium in which a positive measure of types of the firm defaults early. Otherwise, when $\hat{\theta}_1$ goes to $-\infty$, we show that the benchmark model will have two different equilibria, violating the equilibrium uniqueness result.

Now, suppose that $\theta_2^* > \tilde{\theta}_1$. Also, suppose that there is $\hat{\theta}_2 \in \left[\hat{\theta}_1, +\infty\right)$, such that with the reflecting CRA, in the subgame following the rating $R = p$, the firm invests in VP if $\theta \leq \hat{\theta}_2$ and in HR if $\theta \in [\hat{\theta}_1, \hat{\theta}_2)$. It follows from Lemma 4 that $\hat{\theta}_2$ and $\hat{x}$ are both strictly decreasing in $\hat{\theta}_1$. If $\hat{\theta}_2 > \theta_2^*$ is part of an equilibrium, when the reflecting CRA has the rating strategy $\hat{\theta}_1 = \theta_1^*$, there is an equilibrium in which $\hat{\theta}_2 > \theta_2^*$, violating the equilibrium uniqueness conclusion in Proposition 2. Therefore, in an equilibrium of the subgame following $R = p$, the firm invests in VP if $\theta \leq \hat{\theta}_2$ and in HR if $\theta \in [\hat{\theta}_1, \hat{\theta}_2)$. Furthermore, if $\theta_2^* > \tilde{\theta}_1$, we have $\hat{\theta}_2 < \theta_2^*$.

Q.E.D.

Proof of Proposition 6:

We first show the comparative static analysis with respect to β. Recall that

$$x^* = \theta_2^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}[\Delta],$$

where

$$\Delta = \frac{\frac{pV-qH}{p-q} - [(1-\gamma)F + \gamma f(\theta_2^*)]}{(1-\gamma)\left[f(\theta_2^*) - F\right]},$$

so $\partial\Delta/\partial\theta_2^* > 0$.

Substitute x^* as a function of θ_2^* into equation (14) and equation (15), and denote $\sqrt{\beta}(\theta_2^* - \theta_1^*) + \Phi^{-1}(\Delta) = \Psi$ for simplicity, we have

$$\begin{aligned} \Delta(pF - qF) - \Phi\left[\sqrt{\beta}(\theta_2^* - \theta_1^*) + \Phi^{-1}(\Delta)\right](1 - qF) &= 0 \quad (39)\\ (1-\gamma)\Phi\left[\sqrt{\beta}(\theta_2^* - \theta_1^*) + \Phi^{-1}(\Delta)\right][f(\theta_1^*) - F] + \gamma f(\theta_1^*) &= H - (1-\gamma)F \quad (40) \end{aligned}$$

Total differentiation of the above two equations with respect to θ_2^*, θ_1^*, and β, we have

$$A\begin{bmatrix} \frac{\partial\theta_2^*}{\partial\beta} \\ \frac{\theta_1^*}{\partial\beta} \end{bmatrix} = \begin{bmatrix} \phi(\Psi)(1-qF)\frac{\theta_2^*-\theta_1^*}{2\sqrt{\beta}} \\ -(1-\gamma)\phi(\Psi)\frac{\theta_2^*-\theta_1^*}{2\sqrt{\beta}}\left[f(\theta_1^*) - F\right] \end{bmatrix}, \quad (41)$$

where

$$A = \begin{bmatrix} \frac{\partial\Delta}{\partial\theta_2^*}(pF-qF) - \phi(\Psi)\left(\sqrt{\beta} + \frac{1}{\phi(\Delta)}\frac{\partial\Delta}{\partial\theta_2^*}\right)(1-qF) & -\phi(\Psi)\sqrt{\beta}(1-qF) \\ (1-\gamma)\phi(\Psi)\left[f(\theta_1^*) - F\right]\left(\sqrt{\beta} + \frac{1}{\phi(\Delta)}\frac{\partial\Delta}{\partial\theta_2^*}\right) & -(1-\gamma)\phi(\Psi)\left[f(\theta_1^*) - F\right]\sqrt{\beta} + \left[\gamma + (1-\gamma)\Phi(\Psi)\right]f'(\theta_1^*) \end{bmatrix} \quad (42)$$

As we have shown in the proofs of Lemma 3, when β is large enough, $\phi(\Psi)\sqrt{\beta}$ is very close to 0. Therefore, when β is sufficiently large, the determinant of the matrix A is close to

$$\frac{\partial\Delta}{\partial\theta_2^*}(pF - qF)\left[\gamma + (1-\gamma)\Phi(\Psi)\right]f'(\theta_1^*) < 0,$$

because $f'(\theta_1^*) < 0$.

Then further algebra shows that when β is sufficiently large, the sign of $\partial\theta_1^*/\partial\beta$ is the same as that of

$$\frac{\partial\Delta}{\partial\theta_2^*}\frac{pF-qF}{1-qF},$$

which is positive. Therefore, θ_1^* is strictly increasing in β.

Now, let us consider the comparative static analysis with respect to H. Total differentiation of equation (39) and equation (40) with respect to θ_2^*, θ_1^*, and H, we have

$$A\begin{bmatrix}\frac{\partial\theta_2^*}{\partial H}\\ \frac{\theta_1^*}{\partial H}\end{bmatrix} = \begin{bmatrix}-(pF-qF)\frac{\partial\Delta}{\partial H}+\phi(\Psi)\frac{1}{\phi(\Delta)}\frac{\partial\Delta}{\partial H}(1-qF)\\ 1-(1-\gamma)\phi(\Psi)\frac{1}{\phi(\Delta)}\frac{\partial\Delta}{\partial H}\left[f(\theta_1^*)-F\right]\end{bmatrix},$$

where A is defined in equation (42).

Note that $\phi(\Psi)\sqrt{\beta}$ is very close to 0 when β is sufficiently large, we have

$$\text{sign}\begin{bmatrix}\frac{\partial\theta_2^*}{\partial H}\\ \frac{\theta_1^*}{\partial H}\end{bmatrix} = \text{sign}\begin{bmatrix}\left[\gamma+(1-\gamma)\Phi(\Psi)\right]f'(\theta_1^*)\frac{\partial\Delta}{\partial H}\\ -\frac{\partial\Delta}{\partial\theta_2^*}\end{bmatrix}.$$

Because $f_1^*) < 0$, $\partial\Delta/\partial H < 0$, and $\partial\Delta/\partial\theta_2^* > 0$, we have $\partial\theta_1^*/\partial H < 0$ and $\partial\theta_2^*/\partial H > 0$. Therefore, $\partial(\theta_2^*-\theta_1^*)/\partial H > 0$.

We finally show the comparative static analysis with respect to γ. Similar to that about H, the total differentiation of of equation (39) and equation (40) with respect to θ_2^*, θ_1^*, and γ, we have

$$A\begin{bmatrix}\frac{\partial\theta_2^*}{\partial\gamma}\\ \frac{\theta_1^*}{\partial\gamma}\end{bmatrix} = \begin{bmatrix}-(pF-qF)\frac{\partial\Delta}{\partial\gamma}+\phi(\Psi)\frac{1}{\phi(\Delta)}\frac{\partial\Delta}{\partial\gamma}(1-qF)\\ -\left[1-\Phi(\Psi)+(1-\gamma)\phi(\Psi)\frac{1}{\phi(\Delta)}\frac{\partial\Delta}{\partial\gamma}\right](f(\theta_1^*)-F)\end{bmatrix},$$

where A is defined in equation (42).

Note that $\phi(\Psi)\sqrt{\beta}$ is very close to 0 when β is sufficiently large. In addition, $\partial\Delta/\partial\gamma > 0$ and $(1-\Phi(\Psi))/\phi(\Psi) \to 0$ when $\beta \to +\infty$. Then, simple algebra will show that $\partial\theta_1^*/\partial\gamma > 0$ and $\partial\theta_2^*/\partial\gamma > 0$. Furthermore, because when β is sufficiently large, $\partial\theta_1^*/\partial\gamma$ is close to 0, while $\partial\theta_2^*/\partial\gamma > 0$ is bounded away from 0, we have $\partial(\theta_2^*-\theta_1^*)/\partial\gamma > 0$.

Q.E.D.

Proof of Lemma 6:

We prove each part of this lemma.

1. The case $\frac{V^p - V^q}{C^p - C^q} \geq 1 - q$.

   Because $p > q$, this also implies

   $$\frac{V^p - V^q}{C^p - C^q} \geq 1 - q > 1 - p. \tag{43}$$

   Suppose there is an equilibrium in which the rating strategy $\mathcal{R}(\theta)$ assigns the rating q when $\theta \in (\theta_1, \theta_2)$. Then, as in the proof of Lemma 1, if the CRA deviates to assign the rating p to θ-firm for all $\theta \in (\theta_1, \theta_2)$, the firm will not default early.

   Now, consider any θ-firm with $\mathcal{R}(\theta) > 0$. equation (43) implies that

   $$V^p - (1-p)C^p \quad > \quad V^q - (1-p)C^q, \tag{44}$$
   $$V^p - (1-q)C^p \quad > \quad V^q - (1-q)C^q. \tag{45}$$

   equation (44) implies that if θ-firm invests in VP whether it receives the rating p or q, the CRA wants to assign the rating p; equation (45) implies that if θ-firm invests in HR whether it receives the rating p or q, the CRA wants to assign the rating p; and equation (44) and equation (45) together imply that if θ-firm invests in VP after receiving the rating p and invests in HR after receiving the rating q, the CRA wants to assign the rating p. These arguments also suggest that θ-firm has a lower financial cost after receiving the rating p than after receiving the rating q, so it is impossible for the firm to invest in HR following the rating p and to invest in VP following the rating q.

   Therefore, the CRA's deviation of assigning the rating p to θ-firm for all $\theta \in (\theta_1, \theta_2)$ is profitable. As a result, if the firm does not default early, the CRA will assign the rating p. Then, because the firm's financial cost is monotonic in θ, the equilibrium rating strategy must be in the form of equation (17).

2. The case $\frac{V^p - V^q}{C^p - C^q} \leq 1 - p$.

   This case is same as the previous one, except that we replace the rating p by the rating q.

3. The case $\frac{V^p - V^q}{C^p - C^q} \in (1 - p, 1 - q)$.

   In this case, we have

   $$V^p - (1-p)C^p > V^q - (1-p)C^q > V^q - (1-q)C^q > V^p - (1-q)C^p.$$

Therefore, when θ-firm chooses VP, no matter the rating is p or q, the CRA will assign the rating p; when θ-firm chooses HR, no matter the rating is p or q, the CRA will assign the rating q; when θ-firm chooses VP following the rating p and HR following the rating q, the CRA will assign the rating p. Furthermore, because for any particular θ-firm, the financial cost following the rating p is lower than the financial cost following the rating q, if the firm invest in HR following the rating p, it will invest in HR following the rating q.

Hence, the rating strategy must be in the form as in equation (19). Whether the rating q can appear in the equilibrium depends on the model's parameters, so θ^q could be equal to θ^p, in which case the CRA will not assign the rating q in equilibrium.

Q.E.D.

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