# Stress Tests and Information Disclosure*

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**Abstract**

We study an optimal disclosure policy of a regulator that has information about banks (e.g., from conducting stress tests). We focus on the following tradeoff: Disclosing some information may be necessary to prevent a market breakdown, but disclosing too much information destroys risk-sharing opportunities (the Hirshleifer effect). We find that during normal times, no disclosure is optimal, but during bad times, some disclosure is necessary. We characterize its optimal form, e.g., under what conditions a simple cutoff rule is optimal. We relate our results to the Bayesian persuasion literature.

*Keywords*: Bayesian persuasion, optimal disclosure, stress tests

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# 1 Introduction

In the new era of financial regulation following the crisis of 2008, central banks around the world will conduct periodic stress tests for financial institutions to assess their ability to withstand future shocks. A key question that occupies policymakers and bankers is whether the results of the stress tests should be disclosed and, if so, at what level of detail. The debate over this question is summarized in a *Wall Street Journal* article.[1] In this article, Fed Governor Daniel Tarullo expresses support for wide disclosure, saying "The disclosure of stress-test results allows investors and other counterparties to better understand the profiles of each institution." But the Clearing House Association expresses the concern that making the additional information public "could have unanticipated and potentially unwarranted and negative consequences to covered companies and U.S. financial markets."

A classic concern about disclosure is based on the Hirshleifer effect (Hirshleifer, 1971). According to the Hirshleifer effect, greater disclosure might decrease welfare because it reduces risk-sharing opportunities for economic agents. This is indeed a relevant concern in the context of banks and stress tests. Vast literature (e.g., Allen and Gale, 2000) studies risk-sharing arrangements among banks. If banks are exposed to random liquidity shocks, they will create arrangements among themselves or with outside markets to insure against such shocks. More recently, banks are known to hedge their risks with various derivative contracts. If more information about the state of each individual bank and its ability to withstand future shocks is publicly disclosed, then such risk-sharing and hedging opportunities will be limited, generating a welfare loss.

While this concern may provide credible content to the "unwarranted and negative consequences" referred to in the previous quote from the Clearing House Association, it is hard to deny that greater disclosure that "allows investors and other counterparties to better understand the profiles of each institution" appears to be crucial at times. In particular, as was clear during the recent financial cri-

[1] See "Lenders Stress over Test Results," *Wall Street Journal*, March 5, 2012.

sis, when aggregate conditions seem bleak, the lack of disclosure might lead to a breakdown in financial activity. In the context of risk sharing and insurance, if the aggregate state of the financial sector is perceived to be weak, banks would not be able to insure themselves against undesirable outcomes (e.g., Leitner, 2005). In this case, some disclosure might be necessary to enable some risk sharing and its welfare-improving effects.

In this paper, we study a model to analyze these forces and provide guidance for optimal disclosure policy in light of these forces. The model can address the debate on the disclosure of stress test results, but it applies more generally to the issue of information disclosure even outside the stress-test arena. In the model, financial institutions suffer losses if their future capital falls below a certain level. Part of the future capital of a financial institution can be forecasted based on current analysis and will become clear to policymakers conducting stress tests. However, there are also future shocks that cannot be forecasted with such an analysis. Financial institutions can engage in risk-sharing arrangements to guarantee that their capital does not fall below the critical level.

These risk-sharing arrangements work well if the overall state of the financial industry is perceived to be strong. In this case, no disclosure by the regulator is needed. Consistent with the Hirshleifer effect, disclosure can even be harmful because it prevents optimal risk-sharing arrangements from taking place. However, if, on average, banks are perceived to have capital below the critical level, then risk-sharing arrangements that insure them against falling below that level cannot arise without some disclosure. In this case, partial disclosure generally emerges as the optimal solution.

To study optimal disclosure in bad times, we distinguish between two cases. First, we consider an environment in which the information discovered by the regulator in the stress test is not already known to the bank. This is a reasonable assumption if the regulator does not reveal the models that support the stress tests[2] or if the information involves an assessment of bank exposure to the state

[2]See "Fed says stress test models will stay a secret," *Market Watch*, June 25, 2015,

of other banks, which is known to the regulator, that analyzes many banks, but not to the individual banks themselves. In this case, we show that it is optimal to create two scores – a high score and a low score – and to give the high score to a group of banks whose average forecastable capital is equal to the critical level and the low score to other banks. This is similar to the Bayesian persuasion solution proposed by Kamenica and Gentzkow (2011).

By providing disclosure that separates banks into two groups, the regulator enables risk sharing among the banks that receive the high score. All banks whose forecasted capital is above the critical level receive the high score, but some banks with forecasted capital below the critical level also receive the high score. Importantly, for this to work, the regulator must not provide detailed information about banks receiving the high score, because with too much information, banks that are below the critical level would not be able to participate in risk sharing.

Interestingly, the optimal disclosure rule is not necessarily monotone; i.e., it is not always the case that banks below a certain threshold receive a low score and banks above the threshold receive a high score. There is a gain and a cost from giving a bank a high score. The gain is enabling the bank to participate in risk sharing, preventing a welfare-decreasing drop in capital. The cost is that giving a high score to one bank takes resources, thereby preventing other banks from receiving a high score. The allocation of banks into the high-score group depends on the gain-to-cost ratio, and this does not always generate a monotone rule; it depends on the distribution of shocks that banks are exposed to. We provide conditions under which the disclosure rule is monotone.

The second environment we consider is one where the information discovered by the regulator in the stress test is known to the bank itself but not to the outside market.[3] In this case, pooling banks into two groups generally will not work. Banks with a forecastable level of capital that is significantly above the critical

http://www.marketwatch.com/story/fed-says-stress-test-models-will-stay-a-secret-2015-06-25.

[3]For empirical evidence consistent with this assumption, see, e.g., Flannery, Hirtle and Kovner (2015); Morgan, Peristiani, and Savino (2014).

level will refuse to participate in a risk-sharing arrangement with a group that has an average forecastable capital that is just at the critical level. Hence, in this case, the optimal disclosure rule has multiple scores. As before, one score is reserved for banks that are revealed to be below the critical capital level, and these banks are shunned from risk-sharing arrangements. Other scores pool together banks below the critical level with a bank above the critical level to enable risk sharing. Different scores are required to accommodate the different reservation utilities of different banks above the critical level of capital.

Interestingly, in this environment, non-monotonicity becomes a general feature of optimal disclosure rules. When considering banks below the critical level of capital, it turns out that the stronger ones are pooled with a bank that has a level of capital only slightly above the critical level (hence receiving a moderate score), while the weaker ones are pooled with a bank that has a level of capital significantly above the critical level (hence receiving a high score). As we show in this paper, the increase in cost from pooling with a moderately strong bank to pooling with a very strong bank is not significant for the weakest banks but is significant for the moderately weak banks; this leads to the non-monotonicity result.

Finally, we explore whether non-monotonicity continues to be a feature of optimal disclosure rules if we enrich our model. A natural enrichment is to add a constraint that stronger banks must end up with higher equilibrium payoffs. For example, this would be the case if banks could affect the value of their assets by freely disposing assets (Innes, 1990). We show that the outcome in this case depends on whether the regulator can randomize; i.e., on whether he can use stochastic disclosure rules. If the regulator can randomize, then for some parameter values, the optimal disclosure rule continues to be non-monotone, but weaker banks that are pooled together with strong banks participate in risk sharing with a probability that is less than 1. If the regulator cannot randomize (i.e., when he must follow deterministic rules), the optimal disclosure rule becomes monotone and generally involves two cutoffs. The lower cutoff determines which banks participate in risk sharing. Banks whose forecastable capital is above the cutoff participate; those below the cutoff do not participate. The higher cutoff determines which banks are pooled together. Banks whose forecastable capital is between the two cutoffs obtain the same score, while banks that are above the higher cutoff must obtain a different score (or scores) to reflect their higher reservation utilities.

In summary, our paper generates the following results about optimal disclosure. First, if the overall state of the financial industry is perceived to be sufficiently strong ("normal times"), the regulator does not need to disclose anything. Otherwise, some disclosure is necessary, which generally takes the form of partial disclosure, pooling together banks of different levels of strength. Second, the number of scores increases as we move from the case in which banks do not already have the information revealed in the stress test to the case in which they do possess this information. Third, non-monotonicity appears to be a pervasive feature of optimal disclosure rules, such that weaker banks may receive higher scores than stronger banks. However, this non-monotonicity disappears if banks can freely dispose assets and the regulator must follow deterministic disclosure rules. In this case, the disclosure rule has two thresholds: Banks below the lower threshold are excluded from risk sharing, those in the middle are pooled together in a risk sharing arrangement that entails selling the assets at the average price, and those above the higher threshold sell their assets for their fair prices without subsidizing any weaker types.

**Related literature.** Our paper is related to the literature on Bayesian persuasion, going back to Kamenica and Gentzkow (2011). The solution for the first case in which the bank does not know its type is close to the solution in Kamenica and Gentzkow (2011). However, since we put more structure on the regulator's objective function in the context of the banking industry, we obtain more results. In particular, we show that disclosure should be based on the gain-to-cost ratio and provide conditions under which a simple cutoff rule is optimal. The second case in which the bank knows its type is completely new to this literature and provides new results, which could be applied in other settings of Bayesian persuasion (see Section 5).[4]

[4] In a different model of persuasion in the banking sector, Gick and Pausch (2014) study a

The literature on the disclosure of regulatory information is reviewed in Goldstein and Sapra (2013) and Leitner (2014). Morris and Shin (2002) show that disclosure might be bad if economic agents share strategic complementarities and wish to act like each other even though it is not socially optimal. Providing a public signal makes agents place too large of a weight on it because it provides information not only about fundamentals but also about what others know about the fundamentals. However, Angeletos and Pavan (2007) show that this conclusion may not hold when agents share strategic substitutes or when coordination is socially desirable. Leitner (2012) shows that disclosing information may reduce the regulator's ability to obtain information about contracts that banks enter with one another. In his setting, it is optimal to reveal whether a bank has reached some prespecified position limit, but not the actual position. The idea that disclosing information may reduce the regulator's ability to collect information from banks also appears in Prescott (2008). Bond and Goldstein (2015) show that disclosing information might harm the regulator's ability to learn from the market, so the regulator may want to disclose information only on variables on which it cannot learn from the market (e.g., his objective function). Increased disclosure might also be harmful due to the adverse effect it might have on the ex-ante incentives of bank managers, as in the traditional corporate-finance literature that emphasizes the tension between ex-post and ex-ante optimal actions (e.g., Burkart, Gromb, and Panunzi, 1997). Morrison and White (2013) and Shapiro and Skeie (2015) study how the regulator's disclosure policy is affected by reputational concerns. Our paper analyzes a different tradeoff involving risk-sharing opportunities, which are at the heart of financial activity: Disclosure may harm risk sharing arrangement among banks, but some disclosure may be necessary to prevent a market breakdown.

Andolfatto, Berentsen, and Waller (2014) study risk sharing in a monetary

game in which investors with heterogeneous priors can take one of two actions, and the regulator's objective is to get as close as possible to an outcome in which some predetermined fraction of investors take the first action. They show that in general, it is optimal for the regulator to choose a signal that is not too informative because full information induces investors to herd on the same action.

search model. In their model, information has no social value (the Hirshleifer effect), and it is optimal to disclose information only when this is done to prevent individuals from wastefully acquiring (this same information) on their own. Diamond (1985) studies risk sharing in a competitive rational expectations model. In his setting, optimal disclosure not only reduces the social cost of acquiring information but also prevents investors from acquiring different pieces of information on their own; so disclosure enhances trade by reducing information asymmetries among investors. Our tradeoff does not involve information production by market participants.

Dang, Gorton, Holmström, and Ordoñez (2014) use the insights from the Hirshleifer effect to explain bank opaqueness. They focus on the tension between the desire to keep information secret and the desire to produce information to enhance investment decisions. We focus on a different tradeoff. Our goal is not to explain bank opaqueness, but instead to characterize optimal disclosure rules, namely what should be disclosed. In fact, full disclosure can be optimal in our setting and can even emerge as the unique optimal outcome when the regulator must follow deterministic disclosure rules.

Bouvard, Chaigneau, and De Motta (2015) study how disclosure affects the possibility of bank runs. They show that during normal times (when the proportion of high-quality banks is sufficiently high), disclosing bank-specific information is undesirable because it can lead to bank runs, but during crises, disclosing information is preferred to no disclosure because some runs can be prevented. This result relates to one of our results but is based on completely different microfoundations. Moreover, most of our results on the design of optimal disclosure rules are absent in their setting because they assume that there are only two types of banks.[5] Bouvard, Chaigneau, and De Motta (2015) also show that when the regulator privately

[5] Castro, Martinez, and Philippon (2014) study a model of bank runs with two types of banks, in which the government can intervene to prevent inefficient runs. They analyze how the government's disclosure policy is affected by its fiscal capacity. Alvarez and Barlevy (2014) study the desirability of mandatory disclosure in a model of financial contagion with two types of banks and information spillovers.

observes aggregate conditions, he would like to deviate ex post from the ex-ante optimal rule, and this could lead to less disclosure during crisis. We focus on the case in which aggregate conditions are common knowledge, but in Section 5, we discuss an extension in which the regulator would not like to deviate ex post from the ex-ante optimal rule, even if he privately observes the aggregate state.

There is also extensive literature on information disclosure by firms.[6] One of the issues is whether mandatory disclosure is desirable (e.g., Admati and Pfleiderer, 2000; Fishman and Hagerty, 2003). Our paper adds to this literature by illustrating a case in which the regulator would like to restrict information flow from firms. A strong firm ignores the fact that revealing information destroys risk-sharing opportunities for weak firms, but the regulator takes this negative externality into account.[7] In the context of stress tests, our model suggests that the regulator might want to restrict banks from disclosing detailed information about the results of stress tests they conduct on their own.[8] Of course, if banks could precommit to act according to the regulator's optimal disclosure rule, a regulator would not be needed in our setting. However, as we discuss in Section 5, in many cases, the regulator's commitment arises more naturally than that of the bank. Moreover, the regulator and banks may have different objectives. The regulator may care about externalities that banks impose on the rest of society. Our model can incorporate such externalities in the regulator's objective function. We also discuss an extension in which the regulator provides funds to banks.

While we focus on disclosure by a regulator, we believe that our model can also be used as a benchmark to think of credit rating agencies. Within this literature, Lizzeri (1999) shows that to extract more rents, a monopolist intermediary will reveal only the minimum information that is required for an efficient exchange. Goel and Thakor (2015) show that credit ratings agencies may choose coarse ratings, even

[6] For surveys, see Verrecchia (2001) and Beyer, Cohen, Lys, and Walther (2010).

[7] In a different setting, Fishman and Hagerty (1990) show that when an informed seller can verifiably disclose some, but not all, of its information, it may be optimal to restrict the *type* of information that can be disclosed, so that the seller cannot cherry-pick positive information.

[8] The Dodd-Frank Act requires systemically important financial firms to conduct their own stress tests in addition to those conducted by the regulator.

though coarse ratings reduce welfare.[9] In these papers, full disclosure achieves the first-best outcome. Instead, in our setting, the first-best outcome typically involves pooling, and coarse ratings arise as a socially optimal outcome. Our model suggests that low types receiving high ratings may be a feature of a socially optimal outcome.[10]

In a different context, Marin and Rahi (2000) provide a theory of market incompleteness, which is based on the tradeoff between adverse selection and the Hirshleifer effect. Adverse selection favors an increase in the number of securities because it reduces information asymmetries among agents. The Hirshleifer effect favors a reduction in the number of securities. Our paper does not talk about security design but instead discusses how the regulator should pool banks into groups to enable risk sharing. Because the payoff function in our setting exhibits some convexity (a bank suffers a loss if its capital falls below a certain level), two groups may be necessary even when banks do not have private information. When banks have private information, more groups are necessary to accommodate the different reservation utilities of banks above the critical level.

Finally, the idea that risk-sharing arrangements may break down when aggregate conditions are bleak relates to Leitner (2005). He shows that in this case, if there are many banks, it is optimal for them to remain unlinked rather than form a financial network. In one interpretation of our model, we show how the disclosure policy affects the financial networks that banks form.

[9] Kartasheva and Yilmaz (2013) extend Lizzeri (1999) by adding outside options to firms as well as information asymmetries among potential buyers.

[10] DeHaan (2013) provides empirical evidence on nonmonotone relationship between credit rating scores and debt prices. If investors have access to the same sources of information that credit rating agencies do, debt prices might reflect what we refer to as the bank's type. A firm can benefit from a high rating because a high rating can help the firm persuade other economic agents, who may need to follow credit ratings more blindly, to take the action desired by the firm. For example, credit ratings are used to calculate risk-based capital requirements, and some funds cannot invest in low-graded assets.

# 2 A model

**Economic environment.** There is a bank, a regulator (i.e., a planner), and a perfectly competitive market. The bank has an asset, which yields a random cash flow $\tilde{\theta}+\tilde{\varepsilon}$, where $\tilde{\theta}$ is referred to as the bank's type and $\tilde{\varepsilon}$ is the bank's idiosyncratic risk, which is independent of type. The bank can sell its asset in the market for an amount $x$, which will be derived endogenously. Everyone is risk neutral, and the risk-free rate is normalized to be zero percent. Therefore, the price $x$ is the expected value of the asset $\tilde{\theta}+\tilde{\varepsilon}$, conditional on the information available to the market (the information will depend on the disclosure regime). We use $z$ to denote the bank's final cash holdings, and so $z = x$ if the bank sells the asset and $z = \tilde{\theta}+\tilde{\varepsilon}$ if the bank keeps the asset.

We assume that the bank derives the following final payoff as a function of $z$:

$$R(z) = \begin{cases} z & \text{if } z < 1 \\ z + r & \text{if } z \geq 1, \end{cases} \tag{1}$$

for a parameter $r > 0$. This payoff function captures the general idea that a bank derives some gains when its cash holdings are (weakly) above some threshold. One can think of several motivations: (1) The bank has a project that yields a positive net present value $r$ but requires a minimum level of investment. For various reasons (e.g., projects cash flows are nonverifiable), the bank cannot finance the project if it does not have sufficient cash in hand. (2) The bank has a debt liability of 1. Not paying it leads to loss of future income $r$. (3) The bank faces a run if its cash holdings fall below some threshold.

Note that our results do not depend on the particular specification for $R(z)$ above. For example, our results extend to the case in which $r$ depends on the bank's type (we discuss this more later). The results also extend to other payoff functions that exhibit discontinuity, such as assuming that the bank obtains $az$ for some $a \in [0, 1)$ if $z < 1$, and $z + r$ if $z \geq 1$ (where $r$ can be set to zero). The case $a = r = 0$ may best capture the idea that when the asset value falls below some threshold, there is a bank run and the bank is left with nothing. The key to all these specifications is the discontinuity in payoffs.[11]

The bank chooses whether to keep its asset or sell it in the market. The bank does so in a way that maximizes its expected final payoff $R(z)$, conditional on the information available to it. As will be clear later, the bank will have a motive to sell its asset at a price of at least 1. This essentially provides insurance that the bank's cash holdings do not fall below the threshold. More generally, selling the asset can be thought of as engaging in risk sharing. In our model, risk sharing takes a simple form: the bank replaces a random cash flow with a deterministic cash flow by selling its asset to the market.[12] The nature of our model continues to hold for other forms of risk sharing, including the case in which multiple banks share risk among themselves (see the discussion in Section 5).

The bank's type $\tilde{\theta}$ is drawn from a finite set $\Theta \subset \mathbb{R}$ according to a probability distribution function $p(\theta) = \Pr(\tilde{\theta} = \theta)$. The idiosyncratic risk $\tilde{\varepsilon}$ is drawn from a continuous cumulative distribution function $F$ that satisfies $E(\tilde{\varepsilon}) = 0$. The probability structure (i.e., the functions $p$ and $F$) is common knowledge.

The focus of this paper is on the optimal disclosure policy of a regulator who has information about the bank. For example, the regulator could obtain information by maintaining examination staff at the bank or by conducting stress tests. The regulator can disclose information to the market before the bank can sell its asset. Hence, disclosure affects the terms of trade and the bank's ability and incentive to engage in risk sharing.

Specifically, we assume that the regulator observes the realization of $\tilde{\theta}$, which we denote by $\theta$. The market does not observe $\theta$. As for the bank, we focus on two cases: (1) The bank does not observe $\theta$; (2) The bank observes $\theta$. In both cases, we assume that no one observes the realization of $\tilde{\varepsilon}$ (denoted by $\varepsilon$), which is residual noise.

The first case captures the idea that the regulator may have some information

[11] A similar discontinuity in payoffs appears in Leitner (2005) and in Elliott, Golub, and Jackson (2014).

[12] Unlike the bank, the market is not affected by the discontinuity in payoffs and just gets $\theta + \varepsilon$ if it buys the asset. Hence, this transfer of risk can increase surplus.

advantage relative to banks, as motivated in the introduction. The second case captures the idea that the regulator and the bank share the same information, which is unobservable to other market participants. For example, the bank may know its ability to withstand future liquidity shocks, and the regulator can find out this information by conducting stress tests. Throughout most of the analysis, we assume that the bank cannot affect what the regulator observes (i.e., $\theta$ is given), but in the second case, we also analyze a situation in which the bank observes $\theta$ before the regulator and can freely (and secretly) dispose assets, i.e., reduce $\theta$ (see Section 4.5).

Denote the types in $\Theta$ by $\theta_{\max} = \theta_1 > \theta_2 > ... > \theta_m = \theta_{\min}$. We assume that there are $k \geq 1$ types at or above 1. If information on $\theta$ was publicly available, these types could sell the asset at a price that guarantees their cash holdings to end up above the threshold of 1. We also assume that:

**Assumption 1:** $F(1 - \theta_{\min}) < 1$ and $F(1 - \theta_{\max}) > 0$.

Assumption 1 implies that even for the lowest type, there is a positive probability that the asset cash flow will be above 1, and even for the highest type, there is a positive probability that the asset cash flow will be below 1.

**Disclosure rules.** Before finding out the realization of $\tilde{\theta}$, the regulator chooses and announces a disclosure rule to maximize expected total surplus. Since the market breaks even on average, maximizing expected total surplus is the same as maximizing the bank's expected payoff across the different types. The regulator has the ability to commit to the chosen disclosure rule.

Formally, a *disclosure rule* is a set of "scores" $S$ and a function that maps each type to a distribution over scores. In our setting, the optimal disclosure rule can be implemented with a finite number of scores. Hence, there is no loss of generality in assuming that $S$ is finite (or countable). We use $g(s|\theta)$ to denote the probability, according to the disclosure rule, that the regulator assigns a score $s \in S$ when he observes type $\theta$. That is, $g(s|\theta) = \Pr(\tilde{s} = s|\tilde{\theta} = \theta)$. Of course, for every $\theta \in \Theta$, the following has to hold: $\sum_{s \in S} g(s|\theta) = 1$.

To gain intuition on how disclosure rules work, note that full disclosure is obtained when for every type $\theta$, the regulator assigns some score $s_\theta \in S$ with probability 1, such that $s_\theta \neq s_{\theta'}$ if $\theta \neq \theta'$. No disclosure is obtained when the regulator assigns the same distribution over scores to all types (e.g., each type obtains the same score).

For use below, denote $\mu(s) = E[\tilde{\theta} + \tilde{\varepsilon} | \tilde{s} = s]$. This is the expected value of the bank's asset conditional on the bank obtaining score $s$. Since $\tilde{\varepsilon}$ is independent of $\tilde{\theta}$ (and hence, $\tilde{s}$), and since $E(\tilde{\varepsilon}) = 0$, we obtain that

$$\mu(s) = E[\tilde{\theta} | \tilde{s} = s] = \sum_{\theta \in \Theta} \theta \Pr(\tilde{\theta} = \theta | \tilde{s} = s) = \frac{\sum_{\theta \in \Theta} \theta p(\theta) g(s|\theta)}{\sum_{\theta \in \Theta} p(\theta) g(s|\theta)}, \tag{2}$$

where the last equality follows from Bayes' rule.

**Sequence of events.** The sequence of events is as follows:

1. The regulator chooses a disclosure rule $(S, g)$ and publicly announces it.

2. The bank's type $\theta$ is realized and observed by the regulator. (In case 2, $\theta$ is also observed by the bank.)

3. The regulator assigns the bank a score $s$ according to the disclosure rule and publicly announces $s$.

4. The market offers to purchase the asset at a price $x(s)$.

5. The bank chooses whether to keep its asset or sell it for a price $x(s)$.

6. The residual noise $\varepsilon$ is realized. As a result, the bank's cash holdings $z$ and the bank's final payoff $R(z)$ are determined. The market's payoff is $\theta + \varepsilon - x(s)$ if it purchases the asset, and zero otherwise.

The regulator's disclosure rule and assigned score specify a game between the bank and the market. Essentially, a score is just a price recommendation to the market. We focus on the regulator's preferred perfect Bayesian equilibria of this game. Specifically, the bank chooses whether to sell or keep the asset to maximize its expected payoff conditional on its information, and the market chooses a price $x(s)$ that equals the expected value of the asset conditional on the publicly announced score, taking as given the bank's equilibrium strategy (i.e., whether the bank sells at this price or not); formally, we assume Bertrand competition among at least two market participants. We assume that if the bank is indifferent between selling and not selling, it sells, and if the market is indifferent between two prices, it offers the highest price. The regulator chooses a disclosure rule that maximizes the bank's expected payoff across the different types, taking as given the equilibrium strategies of the market and the bank. We discuss the assumption that the regulator can commit to a disclosure rule as well as other possible regulator's objective functions in Section 5.

# 3 Bank does not observe its type

We start with the case in which the bank does not observe $\theta$. So, the bank observes only the score $s$ assigned to it by the regulator. We solve the game backward. One observation that simplifies the analysis is that the bank's decision of whether to sell the asset depends on $s$ but not on $\theta$ or $\varepsilon$, which are unobservable to the bank. Hence, the decision of the bank to sell does not convey any additional information to the market. Consequently, the market sets a price $x(s) = \mu(s)$. It then follows from the payoff structure in (1) that:

**Lemma 1** *In equilibrium, the bank sells the asset if and only if it obtains a score $s$ such that $\mu(s) \geq 1$.*

The proof of Lemma 1 and all other proofs are in the Appendix. The idea behind Lemma 1 is simple. If $\mu(s) \geq 1$, selling guarantees that the bank's cash holding will not fall below 1. Because of the penalty in the payoff structure when cash holdings fall below 1, the bank acts like a risk-averse agent and is happy to replace the asset's random cash flow with its expected value. If instead, $\mu(s) < 1$, the bank prefers to keep the asset because if the bank sells the asset at a price below 1, the bank's cash holdings will surely be below 1, but if the bank keeps the asset, there is a positive probability that the asset's cash flow will turn out to be more than 1 (by Assumption 1). In this case, the bank acts like a risk-loving agent.

The expected payoff for a bank of type $\theta$, given disclosure rule $(S, g)$, is then

$$u(\theta) \equiv \sum_{s:\mu(s)<1} [\theta + r \Pr(\tilde{\varepsilon} \geq 1 - \theta)] g(s|\theta) + \sum_{s:\mu(s)\geq 1} [\mu(s) + r] g(s|\theta). \tag{3}$$

The first term represents the cases in which the bank keeps the asset, and the second term represents the cases in which the bank sells the asset. The regulator's problem is to choose a disclosure rule $(S, g)$ to maximize the bank's ex-ante expected payoff $\sum_{\theta\in\Theta} p(\theta)u(\theta)$.

From the law of iterated expectations, we obtain that

**Lemma 2** *The regulator's problem reduces to choosing a disclosure rule $(S, g)$ to maximize*

$$\sum_{\theta\in\Theta} p(\theta) \Pr(\tilde{\varepsilon} < 1 - \theta) \sum_{s:\mu(s)\geq 1} g(s|\theta). \tag{4}$$

The term $\sum_{s:\mu(s)\geq 1} g(s|\theta)$ in the objective function (4) is the probability that a bank of type $\theta$ sells its asset (or, more broadly, engages in risk sharing). The term $\Pr(\tilde{\varepsilon} < 1 - \theta)$ represents the social gain from having type $\theta$ sell its asset: type $\theta$ can guarantee that its cash holdings are at least 1 even if the asset cash flow turns out to be less than 1 (when $\tilde{\varepsilon} < 1 - \theta$).

**Lemma 3** *The probability that type $\theta$ sells its asset is the same under a general disclosure rule $(S, g)$ and a disclosure rule that assigns only two scores $s_1$ and $s_0$ with probabilities $\sum_{s:\mu(s)\geq 1} g(s|\theta)$ and $1 - \sum_{s:\mu(s)\geq 1} g(s|\theta)$, respectively. The value of the regulator's objective function is also the same under both rules.*[13]

According to Lemma 3, we can focus, without loss of generality, on disclosure rules that assign at most two scores, $s_1$ and $s_0$, such that $\mu(s_1) \geq 1$ and $\mu(s_0) < 1$. The bank sells its asset if and only if it obtains score $s_1$. We refer to scores $s_1$ and $s_0$ as "high" and "low", respectively, and denote the probability that type $\theta$ obtains the high score $s_1$ by $h(\theta)$.

[13]Note, however, that the expected utility for type $\theta$ need not be the same in both cases.

**Lemma 4** *The regulator's problem reduces to finding a function $h : \Theta \to [0, 1]$ to maximize*

$$\sum_{\theta \in \Theta} p(\theta) \Pr(\tilde{\varepsilon} < 1 - \theta) h(\theta), \tag{5}$$

*subject to*

$$\sum_{\theta \in \Theta} p(\theta)(\theta - 1) h(\theta) \geq 0. \tag{6}$$

The objective function (5) follows from Lemma 2. Constraint (6) follows since $\mu(s_1) \geq 1$, using equation (2). The constraint says that the average $\theta$ conditional on obtaining the high score must be at least 1.

We can think of constraint (6) as a persuasion constraint. The regulator wants to persuade the market to purchase the bank's asset at a price that is at least 1. For this, the average type that sells the asset must be at least 1. Essentially, by giving a high score, the regulator implements a cross subsidy from types with $\theta > 1$ to types with $\theta < 1$, so a high type sells its asset for less than what the asset is truly worth, and a low type sells its asset for more than what the asset is worth. This is beneficial because more types can ensure that their cash holdings are at least 1.

While there is no real transfer of resources in our basic setting, it might be useful to think of constraint (6) also as a resource constraint, where high types provide resources to low types. For example, this would be the case if multiple banks share risk among themselves, as discussed in Section 5. Then banks that ended up with high realizations of cash flows would indeed transfer resources to those that ended up with low realizations.

The solution to the regulator's problem is as follows. If $E(\tilde{\theta}) \geq 1$, assigning $h(\theta) = 1$ for every $\theta \in \Theta$ satisfies constraint (6) and hence is optimal. Otherwise, if $E(\tilde{\theta}) < 1$, it is impossible to assign $h(\theta) = 1$ for every $\theta \in \Theta$ and so constraint (6) is binding, such that the average type getting the high score is exactly 1. The optimal disclosure rule then has to determine the probability with which each type gets the high score. This depends on comparing the "gain-to-cost ratio" from increasing $h(\theta)$ for different types. The gain from increasing $h(\theta)$ for a bank of type $\theta$ is the term $\Pr(\tilde{\varepsilon} < 1-\theta)$ in the objective function (5). The cost is that type $\theta$ requires resources in the amount $1-\theta$, as in equation (6). So the gain-to-cost ratio for type $\theta$ is

$$G(\theta) \equiv \frac{\Pr(\tilde{\varepsilon} < 1-\theta)}{1-\theta}. \tag{7}$$

For types with $\theta \geq 1$, it is optimal to assign $h(\theta) = 1$ because there is no cost; these types provide resources. For types with $\theta < 1$, it follows from the linearity of the problem that it is optimal to set a cutoff $G^*$ such that types with a gain-to-cost ratio above the cutoff are assigned $h(\theta) = 1$, and types with a gain-to-cost ratio below the cutoff are assigned $h(\theta) = 0$. The optimal $G^*$ is the lowest cutoff possible that satisfies constraint (6). For types with a gain-to-cost ratio that equals $G^*$, the probability of obtaining the high score can be between 0 and 1 and is set such that constraint (6) is satisfied with equality.

The following proposition summarizes the optimal disclosure rule, namely, the probability that each type obtains the high score.

**Proposition 1** *When a bank does not observe its type, the optimal disclosure rule is such that*

1. *If $E(\tilde{\theta}) \geq 1$, then $h(\theta) = 1$ for every $\theta \in \Theta$.*
2. *If $E(\tilde{\theta}) < 1$, then*

$$h(\theta) = \begin{cases} 1 \text{ if } \theta \geq 1 \text{ or if } \theta < 1 \text{ and } G(\theta) > G^* \\ 0 \text{ if } \theta < 1 \text{ and } G(\theta) < G^*, \end{cases} \tag{8}$$

*where $G^*$ is the lowest $G \in \{G(\theta)\}_{\theta<1}$ that satisfies $\sum_{\theta\geq 1} p(\theta)(\theta-1)+\sum_{\theta<1:G(\theta)>G} p(\theta)(\theta-1) \geq 0$. If $G(\theta) = G^*$, then $h(\theta) \in [0,1)$.*

An interesting question is whether and when full disclosure is optimal, and whether and when no disclosure is optimal. If $E(\tilde{\theta}) \geq 1$, we know from Proposition 1 that every type must sell its asset with probability 1. The regulator can implement this by giving all types the same score, i.e., with no disclosure. There are other ways to implement the optimal outcome, assigning more than one score such that the average $\theta$ of types receiving each score is at least 1. In the special case $\theta_{\min} \geq 1$, the regulator can even assign a different score to each type, i.e., provide full disclosure.[14] In contrast, if $E(\tilde{\theta}) < 1$, the regulator must assign at least two scores. Some disclosure is necessary because without disclosure, the price would be less than 1 and no type would sell its asset. Yet, full disclosure is suboptimal because under full disclosure, only types above 1 sell their assets, while under the optimal disclosure rule, some types that are below 1 also sell their assets. Hence, partial disclosure that pools together types above 1 with some types below 1 is the only way to achieve the optimal outcome. The following result, characterizing circumstances under which full disclosure or no disclosure achieve the optimal rule, follows:

**Corollary 1** *Full disclosure achieves the optimal outcome if and only if $\theta_{\min} \geq 1$. No disclosure achieves the optimal outcome if and only if $E(\tilde{\theta}) \geq 1$. If $E(\tilde{\theta}) < 1$, then partial disclosure is the only way to achieve the optimal outcome.*

Interestingly, in the case of $E(\tilde{\theta}) < 1$ (summarized in the second part of Proposition 1), the types that obtain the low score are not necessarily the lowest. So, a simple cutoff rule that assigns the high score to high types and the low score to low types is not necessarily optimal. Intuitively, the gain from giving the high score is higher for lower types because low types are more likely to end up with low realizations of cash flow. That is, the numerator of (7) is decreasing in $\theta$. But the cost of giving the high score to low types is also higher because low types require more resources. That is, the denominator of (7) is also decreasing in $\theta$. Hence, it is unclear whether $G(\theta)$ is increasing or decreasing, or whether it is even monotone. The function $G(\theta)$, and hence the optimal disclosure rule, depends on the distribution of the idiosyncratic risk $\tilde{\varepsilon}$.

The optimal rule will involve a simple cutoff with respect to $\theta$ when $G(\theta)$ is increasing when $\theta < 1$. In this case, types above the cutoff obtain the high score with probability 1, and types below the cutoff obtain the low score with probability 1. A simple example in which this happens is when there is no idiosyncratic risk

[14] In fact, any disclosure rule is optimal in this special case; but see also Section 4.5.2.

(i.e., $\varepsilon = 0$). Then, the gain-to-cost ratio when $\theta < 1$ is simply $G(\theta) = \frac{1}{1-\theta}$.[15] More generally, a sufficient condition for obtaining a cutoff rule is that the cumulative distribution function of $\tilde{\varepsilon}$ satisfies Condition 1 below. This condition is satisfied by any cumulative distribution function that is concave on the positive region. Examples include a normal distribution and a uniform distribution (both with mean zero).

**Condition 1** *$F(\varepsilon)/\varepsilon$ is decreasing when $\varepsilon > 0$.*

**Corollary 2** *If $E(\tilde{\theta}) < 1$ and Condition 1 holds, the optimal disclosure rule involves a cutoff such that types below the cutoff obtain a low score and types above the cutoff obtain a high score.*

Another case in which the optimal rule involves a simple cutoff with respect to $\theta$ is when $r$ in the payoff function (1) depends on $\theta$ according to some function $r(\theta)$, which is increasing in $\theta$ sufficiently strongly. This has a simple and intuitive economic interpretation: good banks have better investment opportunities in addition to having better assets in place. In this case, the gain from giving the high score is $r(\theta)\Pr(\tilde{\varepsilon} < 1 - \theta)$ and the gain-to-cost ratio is $r(\theta)G(\theta)$. So, no matter what shape $G(\theta)$ has, if $r(\theta)$ is increasing sufficiently strongly, the gain-to-cost ratio will be monotonically increasing, and the disclosure rule will look like a cutoff rule. For example, if $r(\theta) = \frac{1}{\Pr(\tilde{\varepsilon}<1-\theta)}$, then $r(\theta)G(\theta) = \frac{1}{1-\theta}$, which is increasing in $\theta$.

Finally, an example in which the optimal disclosure rule does not involve a simple cutoff as in Corollary 2 is when $G(\theta)$ is decreasing when $\theta \leq \theta_{k+1}$. In this case, the optimal disclosure rule is nonmonotone in type. It includes a cutoff such that types below the cutoff and types above 1 obtain the high score, while types in the middle obtain the low score. A sufficient condition for this to happen is that $F(\varepsilon)/\varepsilon$ is increasing when $\varepsilon > 1 - \theta_{k+1}$.[16]

[15] The case $\varepsilon = 0$ is isomorphic to an example in Kamenica and Gentzkow (2011), in which a firm provides information to consumers to help them learn about the match quality between their preferences and the characteristics of the firm's products.

[16] An example of a probability distribution function that satisfies the condition above is a truncated Cauchy distribution (Nadarajah and Kotz, 2006) on the interval $[-A, 0]$ minus its

# 4 Bank observes its type

So far, we assumed that the bank does not observe its type. We showed that it is possible to implement the optimal disclosure rule with two scores, such that the regulator pools all the types that sell under the same score. In this section, we show that this conclusion may no longer be true when the bank observes its type. The difference is that now each type has a "reservation price," i.e., a minimum price at which it is willing to sell. When different types have different reservation prices, the regulator may need to assign more than two scores to distinguish among them. We also discuss how the regulator should assign these multiple scores to low types that are pooled with high types.

## 4.1 Derivation of the regulator's problem

We first derive banks' reservation prices. Define

$$\rho(\theta) = \begin{cases} \max\{1, \theta - r \Pr(\tilde{\varepsilon} < 1 - \theta)\} \text{ if } \theta \geq 1 \\ \min\{1, \theta + r \Pr(\tilde{\varepsilon} \geq 1 - \theta)\} \text{ if } \theta < 1. \end{cases} \tag{9}$$

**Lemma 5** *In equilibrium, a bank of type $\theta$ agrees to sell at price $x$ if and only if $x \geq \rho(\theta)$.*

Lemma 5 is derived as follows. If type $\theta$ keeps its asset, its expected payoff is $E[R(\theta + \tilde{\varepsilon})] = \theta + r \Pr(\tilde{\varepsilon} \geq 1 - \theta)$. If type $\theta$ sells at price $x$, its payoff is $R(x)$, i.e., it is $x$ when $x < 1$ and $x + r$ when $x \geq 1$. Hence, type $\theta$ agrees to sell if and only if $R(x) \geq \theta + r \Pr(\tilde{\varepsilon} \geq 1 - \theta)$. This reduces to $x \geq \rho(\theta)$.

We refer to $\rho(\theta)$ as type $\theta$'s reservation price and denote $\rho_i = \rho(\theta_i)$. As illustrated in Figure 1, the reservation price satisfies two properties, which we use later. First, $\rho(\theta)$ is increasing in $\theta$. Second, $\rho(\theta) < \theta$ when $\theta > 1$. The intuition for the second property is that types above 1 are willing to sell below their true valuation (i.e., at a discount) to guarantee that their cash holdings do not fall below 1. This

mean, where the lower bound $A$ depends on the model parameters. Intuitively, for the sufficient condition above to hold, the probability distribution of $\tilde{\varepsilon}$ must put low weight on low values; that is, $F(1-\theta_{k+1}) < \frac{1-\theta_{k+1}}{1-\theta_{\min}} F(1-\theta_{\min}) < \frac{1-\theta_{k+1}}{1-\theta_{\min}}$. So, when $\theta_{k+1}$ is close to 1, the distribution must have a fat tail to satisfy $E(\tilde{\varepsilon}) = 0$.

is the insurance motive. Figure 1 also shows that a very low type will agree to sell its asset for less than 1, but only if the price is above $\theta$. Intuitively, the bank will demand compensation for losing the option value of ending up with cash holdings above 1. As emphasized previously, in this range, the bank is essentially risk loving. However, as we show below, in equilibrium, such transactions will not happen because the market is not willing to pay a price above the expected type. Overall, as in the previous section, a bank never sells in equilibrium for a price below 1.

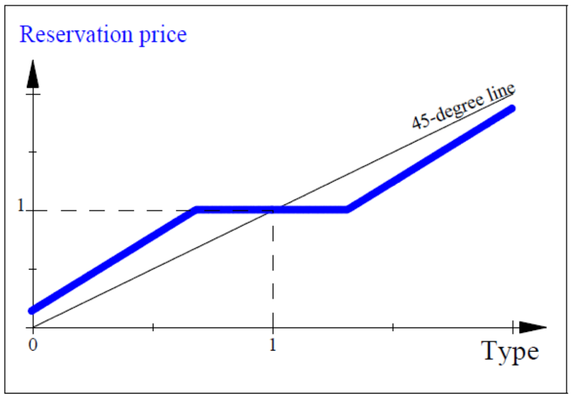


Figure 1.

We now derive some properties that must hold under an optimal disclosure rule.[17]

**Lemma 6** *Under an optimal disclosure rule:*

1. *Every type* $\theta_i \geq 1$ *sells its asset with probability* 1.
2. *Whenever type* $\theta_i \geq 1$ *receives score* $s$*, the price is* $x(s) = \mu(s)$.
3. *If the highest type that obtains score* $s$ *is below* 1*, then every type keeps its asset upon obtaining score* $s$.

The first part in Lemma 6 follows because if a type $\theta \geq 1$ did not sell its asset, the regulator could strictly increase type $\theta$'s payoff, without affecting the payoffs of other types, by fully revealing $\theta$'s type. Then, the market would offer to buy type $\theta$'s asset at a price $\theta$, and type $\theta$ would accept the offer.

[17] We establish the existence of an optimal disclosure rule below.

The second part follows from the first part and the observation that the reservation price is increasing in $\theta$. These imply that every type sells its asset upon obtaining score $s$, as long as score $s$ is also obtained by type $\theta_i \geq 1$. Hence, selling at this score does not convey any additional information to the market, and the market prices the asset at the expected value given the score: $x(s) = \mu(s)$.

The third part reflects the fact that if no type above 1 obtains score $s$, the price $x(s)$ must be less than 1. Then, the bank will sell only if the price is strictly above the true value. But this cannot be an equilibrium outcome because the market would overpay in expectation. The third part holds under any disclosure rule, not only an optimal one.

It follows from Lemmas 5 and 6 that as in Section 3,

**Lemma 7** *Under an optimal disclosure rule, a bank sells its asset upon receiving score $s \in S$ if and only if $\mu(s) \geq 1$.*

Hence, under an optimal rule, the equilibrium payoff for type $\theta$ is $u(\theta)$, as in equation (3).

It also follows that if the highest type that obtains score $s$ is type $\theta_i \geq 1$, then $\mu(s) \geq \rho_i$, so that $\theta_i$ agrees to sell. Formally, denote by $S_i$ the set of scores that type $\theta_i \geq 1$ obtains with a positive probability but higher types do not obtain; that is, $S_i = \{s \in S : g(s|\theta_i) > 0 \text{ and } g(s|\theta') = 0 \text{ for every } \theta' > \theta\}$. Then

$$\mu(s) \geq \rho_i \text{ for all } s \in S_i \text{ and } i \in \{1, ..., k\}. \tag{10}$$

(Recall that types $\theta_1$ through $\theta_k$ are above 1.)

The regulator's problem reduces then to finding a disclosure rule $(S, g)$ to maximize the expected payoff across types $\sum_{\theta \in \Theta} p(\theta) u(\theta)$, just as in the previous section, such that equation (10) holds. This equation is a generalization of the condition for selling $\mu(s) \geq 1$ in Lemma 1, but now to satisfy the reservation prices of different types, there are different conditions for different scores. In particular, now the disclosure rule determines not only whether a bank sells its asset, but also the sale price, which depends on the reservation price of the highest type the bank is matched with.

Since there are $k$ types above 1, we can focus, without loss of generality, on disclosure rules that assign at most $k+1$ scores. As in Section 3, one score, which we denote by $s_0$, is reserved for types that do not sell their asset. The other $k$ scores, which we denote by $s_1, ...s_k$, are reserved for types that sell their asset. For every $i \in \{1, ..., k\}$, the highest type that obtains score $s_i$ is $\theta_i$. Formally,

**Lemma 8** *Suppose $(S, g)$ is an optimal disclosure rule. Then a disclosure rule that assigns $k+1$ scores $s_0, s_1, ..., s_k$, such that for each $i \in \{1, ..., k\}$, the probability of obtaining score $s_i$ is $\sum_{s \in S_i} g(s|\theta)$, is also optimal.*

For $i \in \{1, ..., k\}$, denote the probability that type $\theta$ obtains score $s_i$ by $h_i(\theta)$. The probability that a bank sells its asset is then $\sum_{i=1}^{k} h_i(\theta)$. We can write down the regulator's problem as follows:

**Lemma 9** *When the bank observes its type, the regulator's problem reduces to choosing a set of functions $\{h_i : \Theta \longrightarrow [0,1]\}_{i=1,...,k}$ to maximize*

$$\sum_{\theta \in \Theta} p(\theta) \Pr(\tilde{\varepsilon} < 1 - \theta) \sum_{i=1}^{k} h_i(\theta), \tag{11}$$

*such that (12) – (14) holds:*

$$\sum_{\theta \in \Theta} p(\theta)(\theta - \rho_i) h_i(\theta) \geq 0 \text{ for every } i \in \{1, ..., k\}, \tag{12}$$

$$\sum_{i=1}^{k} h_i(\theta) \leq 1 \text{ for every } \theta \in \Theta, \tag{13}$$

$$h_i(\theta) = 0 \text{ for every } i \in \{1, ..., k\} \text{ and } \theta > \theta_i. \tag{14}$$

The derivation of Lemma 9 follows directly from the discussion above and the analysis in the previous section. The objective function (11) is as in Lemma 4, noting that the probability that type $\theta$ sells its asset is $\sum_{i=1}^{k} h_i(\theta)$ rather than $h(\theta)$. Again, the regulator wants to maximize the expected gain from giving banks a score that enables them to sell at a price above 1 when otherwise they would end up with cash holdings below 1. Equation (12) is a generalization of constraint (6). Now there is a constraint for every score $s_i$; the constraint for score $s_i$ follows since $\mu(s_i) \geq \rho_i$ (and (2)). Equation (13) simply says that the probability that type $\theta$ sells its asset is at most 1. Equation (14) says that the highest type that obtains score $s_i$ is type $\theta_i$, by definition.

The problem in Lemma 9 is a linear programming problem. Because the feasible region is bounded ($h_i(\theta) \in [0,1]$) and nonempty,[18] a solution exists.

As in Corollary 1, full disclosure achieves an optimal outcome if, and only if, there are no types below 1. No disclosure achieves an optimal outcome if, and only if, $E(\tilde{\theta})$ is sufficiently high. However, the condition for no disclosure to be optimal is stricter than in the previous section, reflecting the reservation price of the highest type.

**Corollary 3** *When the bank observes its type, no disclosure achieves the optimal outcome if and only if $E(\tilde{\theta}) \geq \rho_1$.*

We can interpret the condition for no disclosure in Corollary 3 as "normal" times. (So $E(\tilde{\theta}) < \rho_1$ would represent "bad" times.) In particular, in normal times banks not only have good assets in place $E(\tilde{\theta})$ but they also have good investment opportunities $r$ in the payoff function (1). When $r$ is higher, the reservation price $\rho_1$ is lower, and it is easier to satisfy the condition for no disclosure, because the highest type has more to lose by not participation in risk sharing. When $r$ is sufficiently high, we obtain that $\rho_1 = 1$, and the condition for no disclosure is the same as in Corollary 1.

## 4.2 Properties of optimal disclosure rules

In this section, we derive two properties of optimal disclosure rules, which must hold when "resources are scarce." We say that resources are scarce if it is impossible to implement an outcome in which every type sells its asset with probability 1. A

[18] Setting $h_i(\theta) = 1$ if $\theta = \theta_i$, and $h_i(\theta) = 0$ if $\theta \neq \theta_i$, satisfies all the constraints.

sufficient condition for this to happen is that $E(\tilde{\theta}) < 1$.[19] If resources are not scarce, there is always an optimal disclosure rule that satisfies the two properties below, but there are also optimal disclosure rules that do not satisfy these properties.

As a preliminary, observe that when resources are scarce, all $k$ constraints in (12) are binding. Hence, the price for score $s_i$ must be $\rho_i$. In other words, the sale price equals to the reservation price of the highest type obtaining the score.

**Lemma 10** *If resources are scarce, then for every $i \in \{1, ..., k\}$, $x(s_i) = \rho_i$.*

We say that a score is higher than another score, if it induces a higher price. So score $s_1$ is the highest, score $s_2$ is the second highest, and so on. As noted earlier, assigning a score is equivalent to recommending a price to the market, and the price is such that the market breaks even, on average.

The first property is as follows:

**Proposition 2** *Suppose resources are scarce. Under an optimal disclosure rule, types above* 1 *that have different reservation prices obtain different scores (and sell for different prices).*

The intuition for Proposition 2 is as follows. If two types above 1 have different reservation prices but the same score, the sale price equals the reservation price of the higher type. This means that the lower type sells for more than its reservation price and, therefore, ends up with more resources than it requires. But this is a waste of resources without any gain. The regulator can do better by assigning the lower type its own score so that this type ends up with fewer resources. This frees up resources that can be used to cross subsidize types below 1. This, in turn, increases the probability that low types will end up with cash holdings that are at least 1.

An immediate corollary is that

[19] See more details in the Appendix. A necessary and sufficient condition is that $a_k$ in Lemma A-1 in the Appendix is well defined.

**Corollary 4** *If resources are scarce and $\rho_1 > \rho_2 > ... > \rho_k$, the regulator must assign at least $k+1$ scores.*

Proposition 2 implies that if we focus on disclosure rules with $k+1$ scores (as in Lemma 8), then for each $i \in \{1, ..., k\}$, type $\theta_i > 1$ obtains score $s_i$ with probability 1. Next, we discuss the allocation of those $k+1$ scores to types below 1.

The second property says that among the types below 1 that obtain a score (or scores) that induce selling, lower types obtain higher scores. Saying it differently, among the types below 1 that are pooled with types above 1, the lowest types below 1 are pooled with the highest types above 1. Formally,

**Proposition 3** *Suppose resources are scarce and $\Theta$ contains at least four types, $\theta_v < \theta_w < 1 < \theta_i < \theta_j$, such that $\rho_i < \rho_j$. Under an optimal disclosure rule, if a positive probability exists that $\theta_v$ is pooled together with $\theta_i$, then $\theta_w$ cannot be pooled together with $\theta_j$. That is, the solution to Lemma 9 is such that if $h_i(\theta_v) > 0$ (i.e., $\theta_v$ obtains score $s_i$ with a positive probability), then $h_j(\theta_w) = 0$ (i.e., $\theta_w$ does not obtain the higher score $s_j$).*

This non-monotonicity result seems surprising. To understand the intuition, recall that in the previous section, when the bank did not know its type, the regulator allocated scores to types below 1 based on a gain-to-cost ratio (equation (7)). In the case studied in this section, the regulator will also use a gain-to-cost ratio to allocate scores, but now there will be a different gain-to-cost ratio for every score. In particular, for $i \in \{1, ..., k\}$, the gain from assigning score $s_i$ to type $\theta < 1$ is the term $\Pr(\tilde{\varepsilon} < 1 - \theta)$ in the objective function (11). The cost is $\rho_i - \theta$, which follows from constraint $i$ in (12). So the the gain-to-cost ratio from assigning score $s_i$ to type $\theta$ is

$$G_i(\theta) \equiv \frac{\Pr(\tilde{\varepsilon} < 1 - \theta)}{\rho_i - \theta}. \tag{15}$$

While the gain does not depend on the specific score and is exactly the same as in the previous section, the cost is different. It is more costly to assign a higher score because the bank sells for a higher price and ends up with more resources. The non-monotonicity result follows because the increase in cost from assigning a high score rather than a low score is less significant for the lower types. That is, when $\rho_i < \rho_j$, the ratio $\frac{\rho_j - \theta}{\rho_i - \theta}$ is increasing in $\theta$.

Another way to see the intuition for the non-monotonicity result in Proposition 3 is by separating the cost $\rho_i - \theta$ into two components $\rho_i - 1$ and $1 - \theta$. The latter is the cost of bringing the bank up to the threshold of 1, and the former is the cost of bringing it up further from 1 to the price $\rho_i$, which is associated with the score. For types that are slightly below 1, the second component is negligible, while the first component is first order. In contrast, for very low types, the second component is first order. Hence, to save on resources, it is more beneficial to reduce the first component for the types that are closer to 1. This can be done by giving these types lower scores, i.e., pooling them with the lower types that are above 1.

An immediate corollary of Proposition 3 is that among the types below 1 that sell their assets, lower types sell for higher prices. Formally,

**Corollary 5** *Suppose resources are scarce, and $\Theta$ contains at least two types below 1, $\theta_v < \theta_w < 1$. Under an optimal disclosure rule, if a positive probability exists that $\theta_v$ sells at price $x'$ and $\theta_w$ sells at price $x''$, then $x'' \leq x'$.*

Corollary 5 implies that the sale price is non-monotone in type. Among the types below 1 that sell their assets, lower types sell for higher prices. However, among the types above 1, the opposite is true, as these types end up selling exactly for their reservation price, which is increasing in type.

## 4.3 Closed-form solutions and examples

The results in the previous subsection provide general properties of the optimal disclosure rule as well as a general algorithm that can be used to determine the optimal disclosure rule for every set of parameters and distribution functions covered by our model. To get a better idea of how the disclosure rule works, in this subsection, we illustrate the optimal disclosure rule in some special cases.

**Case 1.** No idiosyncratic risk, i.e., $\tilde{\varepsilon} = 0$: Here, the regulator cannot implement cross-subsidies from high types to low types, and type $\theta \in \Theta$ ends up with a payoff $R(\theta)$ independently of the disclosure rule. Hence, every disclosure rule leads to the same outcome, and so every disclosure rule is optimal.

**Case 2.** $\rho_1 = 1$: Here, the highest reservation price is 1. As we know from (9), this can be consistent with having multiple types above 1, but either $r$ is sufficiently high or $\theta_{\max}$ is sufficiently low, so they are willing to sell the asset at a price of 1. It follows immediately from Proposition 1 and from the analysis in the previous subsection that in this case, the optimal disclosure rule is essentially identical to the one when the bank does not observe its type, as in Proposition 1.

**Case 3.** $k = 1$: Here, there is only one type above 1. It can be shown easily that in this case the optimal disclosure rule is similar to that in Proposition 1 (when the bank does not know its type), except that the gain-to-cost ratio is $G_1(\theta)$ instead of $G(\theta)$. Then, Corollary 2 describing when the disclosure rule features monotonicity holds only if $\rho_1$ is sufficiently small. Otherwise, $G_1(\theta)$ is decreasing when $\theta < 1$ even if Condition 1 is satisfied.[20] Intuitively, when $\rho_1$ is very high, the cost $(\rho_1 - \theta)$ of giving the high score to type $\theta < 1$ is very high no matter how high $\theta$ is, and so the dominant factor in deciding which types should be included in risk sharing is that the gain $\Pr(\tilde{\varepsilon} < 1 - \theta)$ is decreasing in $\theta$. So, when $\rho_1$ is sufficiently high, low types and the type above 1 obtain the high score, while types in the middle obtain the low score.

**Case 4.** $k \geq 2$ and $G_i(\theta)$ is increasing in $\theta$ for every $\theta < 1$ and every $i \in \{1, ..., k\}$:[21] Using similar logic as in Section 3, one can show that the lowest types are excluded from risk sharing. The optimal disclosure rule can be found by applying Propositions 2 and 3. First, pool the lowest type above 1 (type $\theta_k$) with the highest types below 1 until all the resources from type $\theta_k$ are exhausted (that is, until the average $\theta$ for the group equals $\rho_k$). Next, pool the second lowest type

[20] Formally, $G_1(\theta)$ is increasing when $\theta < 1$ if and only if $-F'(1-\theta)(\rho_1 - \theta) + F(1-\theta) \geq 0$. This reduces to $\rho_1 \leq \max_{\theta:\theta<1}\{\theta + \frac{F(1-\theta)}{F'(1-\theta)}\}$.

[21] A sufficient condition for this to happen is that Condition 1 holds and $\rho_1 < \max_{\theta:\theta<1}\{\theta + \frac{F(1-\theta)}{F'(1-\theta)}\}$.

above 1 (type $\theta_{k-1}$) with the remaining highest types below 1 until the resources from type $\theta_{k-1}$ are exhausted. And so on, until we exhaust the resources from the highest type $\theta_1$.[22] The following example illustrates the solution:

**Example 1** There are five types $\theta_1 > \theta_2 > 1 > \theta_3 > \theta_4 > \theta_5$ and $\rho_1 > \rho_2$. So, without loss of generality, there are three scores $s_0, s_1, s_2$. Suppose $G_i(\theta)$ is increasing in $\theta$ for every $\theta < 1$ and $i \in \{1, 2\}$. Suppose in addition that:

$$p(\theta_2)(\theta_2 - \rho_2) = p(\theta_3)(\rho_2 - \theta_3) \tag{16}$$

$$p(\theta_1)(\theta_1 - \rho_1) = p(\theta_4)(\rho_1 - \theta_4). \tag{17}$$

Then, the optimal disclosure rule is as follows (an element in the table is the probability that type $\theta_j$ obtains score $s_i$):

| | $\theta_5$ | $\theta_4$ | $\theta_3$ | $\theta_2$ | $\theta_1$ |
|---|---|---|---|---|---|
| score $s_1$ (sell at price $\rho_1$) | | 1 | | | 1 |
| score $s_2$ (sell at price $\rho_2$) | | | 1 | 1 | |
| score $s_0$ (keep asset) | 1 | | | | |

In particular, equation (16) implies that $\theta_3$ gets score $s_2$ with probability 1, and equation (17) implies that $\theta_4$ gets score $s_1$ with probability 1, so that the two constraints in (12) are satisfied with equality. As we can see, $\theta_1$ and $\theta_4$ are pooled together at the highest price, $\theta_2$ and $\theta_3$ are pooled together at the lower price, and $\theta_5$ does not sell and does not participate in risk sharing.[23]

**Case 5.** $k \geq 2$ and $G_i(\theta)$ is decreasing in $\theta$ for every $\theta < 1$ and every $i \in \{1, ..., k\}$:[24] In this case, types in the "middle" are excluded from risk sharing, and the optimal disclosure rule can be found as follows: Pool the highest type $\theta_1$ with

[22] Proposition A-1 in the Appendix provides a closed-form solution. Note that, in general, when resources are scarce, the optimal disclosure rule uniquely determines for each $i \in \{1, ..., k\}$ and $\theta \in \Theta$, the probability $h_i(\theta)$ of obtaining score $s_i$ (and hence the sale price $\rho_i$). However, as suggested by Lemma 8, there is more than one way to implement these probabilities.

[23] In general, a type below 1 will be pooled with more than one type above 1. For example, if we changed equations (16) and (17) so that $p(\theta_2)(\theta_2 - \rho_2) = \frac{1}{3}p(\theta_3)(\rho_2 - \theta_3)$ and $p(\theta_1)(\theta_1 - \rho_1) = \frac{2}{3}p(\theta_3)(\rho_2 - \theta_3) + \frac{4}{5}p(\theta_4)(\rho_1 - \theta_4)$, then $\theta_3$ will obtain score $s_1$ with probability $\frac{2}{3}$ and score $s_2$ with probability $\frac{1}{3}$, and $\theta_4$ will obtain $s_1$ with probability $\frac{4}{5}$ (and $s_0$ with probability $\frac{1}{5}$).

[24] A sufficient condition for this to happen is that $\rho_k > \max_{\theta:\theta<1}\{\theta + \frac{F(1-\theta)}{F'(1-\theta)}\}$.

the lowest types until all the resources from type $\theta_1$ are exhausted. Next, pool the second highest type $\theta_2$ with the remaining lowest types, and so on, until all the resources of types above 1 are exhausted.

**Case 6.** $k \geq 2$ and there exists $\hat{k} \in \{1, ..., k\}$ such that for every $\theta < 1$, $G_i(\theta)$ is decreasing in $\theta$ if $i \in \{1, ..., \hat{k}\}$ and increasing in $\theta$ if $i \in \{\hat{k}+1, ..., k\}$:[25] In this case, the optimal disclosure rule can by found be combining the procedures in cases 4 and 5. We illustrate in the example below.

**Example 2** In Example 1, suppose that $G_1(\theta)$ is decreasing when $\theta < 1$ (rather than increasing) and $p(\theta_1)(\theta_1 - \rho_1) = p(\theta_5)(\rho_1 - \theta_5)$. The optimal disclosure rule is as follows:

| | $\theta_5$ | $\theta_4$ | $\theta_3$ | $\theta_2$ | $\theta_1$ |
|---|---|---|---|---|---|
| score $s_1$ (sell at price $\rho_1$) | 1 | | | | 1 |
| score $s_2$ (sell at price $\rho_2$) | | | 1 | 1 | |
| score $s_0$ (keep asset) | | 1 | | | |

## 4.4 Discussion of non-monotonicity

Optimal disclosure rules may exhibit two forms of non-monotonicity. First, the probability of selling the asset may be nonmonotone in type (Example 2). Second, the sale price may be nonmonotone in type (Examples 1 and 2).

The first form of non-monotonicity arises when the gain-to-cost ratio is decreasing in $\theta$. A necessary condition for this is that the gain is decreasing in $\theta$. That is, for a given cost, the regulator has a preference for helping low types. In our model this happens because every type may end up with cash holdings above 1 and obtain $r$ even without selling its asset, but lower types are less likely to be in that position. So, the gain from having a low type sell its asset is higher. This will not be true in a variation of our model in which the bank's final payoff is $R(z)$ if it sells the asset and $z$ otherwise. For example, the bank may have an investment opportunity that expires before the asset cash flows are obtained. In this case, the gain

[25] A sufficient condition for this to happen is that Condition 1 holds and $\rho_{\hat{k}} \geq \max_{\theta:\theta<1}\{\theta + \frac{F(1-\theta)}{F'(1-\theta)}\} > \rho_{\hat{k}+1}$.

from selling at price $x \geq 1$ is the same for all types, and the regulator's objective becomes $\sum_{\theta \in \Theta} p(\theta) \sum_{i=1}^{k} h_i(\theta)$. Then, the gain-to-cost ratio becomes $\frac{1}{\rho_i - \theta}$, which is increasing in $\theta$.[26]

The second form of non-monotonicity follows from the payoff function (1), which induces pooling between types above 1 and types below 1, and because the sale price reflects the reservation price of the highest type in the pool, meaning that pooling with a higher type is more costly. As a result, pooling between types above 1 and types below 1 is nonmonotone, and types below 1 can sell at a price above the price obtained by some types above 1. This non-monotonicity continues to hold under different regulator's objective functions, e.g., when the regulator maximizes $\sum_{\theta \in \Theta} p(\theta) \sum_{i=1}^{k} h_i(\theta)$, as previously discussed, or when the regulator cares about externalities, as in Section 5. However, this non-monotonicity disappears under some model enrichments (see Section 4.5) .

### 4.5 What if banks can freely dispose assets?

The two forms of non-monotonicity can lead to optimal disclosure schemes in which low types end up with higher expected payoffs than high types. For example, in Example 1, type $\theta_4$ ends up with a higher expected payoff than type $\theta_3$ because both types sell with probability 1, but type $\theta_4$ sells for a higher price. Such an outcome is plausible if the bank and the regulator learn $\theta$ at the same time and the bank cannot affect $\theta$. However, if the bank learns its $\theta$ before the regulator and can freely (and secretly) dispose of assets, the equilibrium above breaks down because a high type has strong incentives to increase its equilibrium payoff by destroying assets.

In this section, we discuss optimal disclosure rules that are not exposed to free disposal when such free disposal is a possibility. Specifically, we solve the regulator's problem with an additional monotonicity constraint, namely that the bank's expected equilibrium payoff is weakly increasing in type. We show that the optimal

[26] In the alternative model, type $\theta$'s reservation price changes to $\rho(\theta) = \max\{1, \theta - r\}$ if $\theta \geq 1$, and $\rho(\theta) = \theta$, if $\theta < 1$. Propositions 2 and 3 continue to hold.

disclosure rule depends on whether the regulator can randomize; namely, whether the probability of assigning scores $g(s|\theta)$ can take any value on $[0, 1]$ (stochastic rules) or only the values 0 and 1 (deterministic rules).

### 4.5.1 Regulator can randomize (stochastic rules)

In this case, we show in an online appendix that, for some parameter values, lower types continue to sell at higher prices. However, to satisfy the monotonicity constraint so that high types do not have an incentive to destroy assets, low types sell with probability that is less than 1. We also show that for some parameter values, it is no longer optimal that types above 1 with different reservation prices obtain different scores. For example, for some parameter values in Example 1, it is optimal to pool type $\theta_2$ with type $\theta_1$ so that type $\theta_2$ sells its asset at a price above its reservation price. This increases the payoff for type $\theta_2$, which is beneficial because it relaxes the monotonicity constraint for lower types, thereby allowing them to sell with a higher probability.

### 4.5.2 Regulator cannot randomize (deterministic rules)

In this case (the appendix provides mode details), the optimal disclosure rule becomes monotone, and it involves two cutoffs $z_1, z_2$, where $z_1 < z_2 \leq \theta_1$. The role of $z_1$ is to determine which types sell their assets. Types below $z_1$ sell with probability 0, and all other types sell with probability 1. The role of $z_2$ is to determine which types are pooled together. Types on the interval $[z_1, z_2]$ obtain the same score, and they all sell at a price which is at least as high as type $z_2$'s reservation price. Types above $z_2$ sell at higher prices to reflect their reservation prices, and without loss of generality, each type above $z_2$ obtains its own score.

Intuitively, if the regulator has to use deterministic rules, he can no longer adjust probabilities of scores to satisfy the monotonicity constraint. Hence, the only way to make sure that types receive monotone expected payoffs is to have prices that are monotone in types. Generally, this implies that very high types can no longer subsidize lower types. Hence, they sell at high prices that correspond to the fair value of their assets. Pooling will occur at an intermediate price for types in the middle. Finally, very low types will be excluded from risk sharing as before. Interestingly, if $\theta_{\min} \geq 1$, then for some parameter values[27], the only way to achieve the optimal outcome is for each type to have its own score, i.e., to have full disclosure.

For a given $z_2$, the regulator would like to set $z_1$ as low as possible subject to the constraint that the average cash flow for types on $[z_1, z_2]$ is at least $\rho(z_2)$. This constraint reduces to $z_1 = \hat{z}_1(z_2)$, where

$$\hat{z}_1(z_2) = \min\{z \in \Theta : \sum_{\theta \in [z, z_2]} p(\theta)[\theta - \rho(z_2)] \geq 0\}. \tag{18}$$

The optimal $z_2$ is the one that minimizes $\hat{z}_1(z_2)$. Denote $z_2^* = \arg\min_{z_2 \in \Theta} \hat{z}_1(z_2)$, and let $\mu^* = E[\tilde{\theta}|\tilde{\theta} \in [\hat{z}_1(z_2^*), z_2^*]]$. The optimal outcome is summarized in the next proposition.

**Proposition 4** *Suppose banks can freely dispose assets, and the regulator must follow a deterministic disclosure rule. Then under an optimal disclosure rule, types below $\hat{z}_1(z_2^*)$ do not sell their assets, types that belong to the interval $[\hat{z}_1(z_2^*), z_2^*]$ sell at the same price $\mu^* \geq \rho(z_2^*)$, and types above $z_2^*$ sell at prices that are above $\mu^*$.*

When $z_2^* = \theta_1$, the optimal disclosure rule can be implemented with only two scores. However, when $z_2^* < \theta_1$, at least three scores are needed.[28] Example 3 in the appendix illustrates both cases. Essentially, the choice of $z_2^*$ involves a tradeoff. A higher $z_2$ increases the resources that are available to cross subsidize types below 1, but it also increases the resources that each type ends up with.

# 5 Discussion

In this section, we discuss some of the assumptions, interpretations, and possible extensions of the model.

[27]For example, if for every $\theta$, $\hat{z}_1(\theta) = \theta$, where $\hat{z}_1$ is defined in equation (18) below.

[28]Three scores are sufficient if the average $\theta$ for types above $z_2$ is at least $\rho_1$. Otherwise, more than three scores are necessary because giving the same score to all the types above $z_2$ will violate type $\theta_1$'s participation constraint.

1. In our model, risk sharing takes a simple form: a bank sells its asset to a competitive market. We obtain similar results if we assume instead that the bank can enter into more complicated derivative contracts, under which the bank replaces a random cash flow with a deterministic cash flow. Such derivative contracts are quite common in today's banking industry. The nature of our results also remains the same if banks can create risk-sharing arrangements among themselves such as in the traditional interbank market (see more below).

2. In our model, the regulator discloses information about the value of existing assets. In practice, stress tests are designed to assess how the bank would perform under some hypothetical stress scenario. As we argue below, information from stress tests could still have a significant impact on current values. First, if the probability of the stress scenario is sufficiently high, the results from stress tests will clearly affect current asset values. Second, stress tests results will also have a significant impact on current values if the regulator requires the bank to take some action (e.g., increase capital), which is costly to the bank, say, by preventing the bank from implementing its desired investment strategy. Our model suggests that rather than provide detailed information about how each bank would perform under the stress scenario, it might be optimal to just say what action each bank is required to take. Moreover, it might be optimal to require banks of different levels of strength to take the same action (see more below).

3. An interesting extension of our model would allow the regulator to provide funds to banks. Such an extension would suggest that in some cases, it is optimal to inject money not only to weak banks but also to strong banks so that the market cannot distinguish among them.[29] For example, suppose there are two banks: strong ($\theta_1 = 1.2$, $\rho_1 = 1$) and weak ($\theta_2 = 0.4$), and the regulator has a bailout

[29] Indeed, one of the first uses of the Troubled Asset Relief Program (TARP) funds was providing capital to nine major financial institutions as part of the Capital Purchase Program (CPP), a program designed to infuse capital to "healthy" banks. During the audit, former Federal Reserve Chair Ben Bernanke told the Special Inspector General for the TARP (SIGTARP) that "there were differences in the nine banks in terms of strength and weakness, but that the selection was generalized in order to avoid stigmatizing any one bank as being a weak bank and creating panic." (See SIGTARP report 10-001, "Emergency capital injections provided to support the viability of Bank of America, other major banks, and the U.S. financial system," October 2009)

fund in the amount of 0.4. Suppose that the regulator would like each bank to end up with at least one dollar, and banks can raise cash by issuing equity. Giving all the money to the weak bank identifies the bank as weak. Because the value of the weak bank after the cash injection is 0.8, it will not be able to raise one dollar by issuing equity. Splitting the money equally between the two banks leads to a better outcome. Now, after the cash injection, the value of the strong bank is 1.4, and the value of the weak bank is 0.6. Since the market cannot distinguish between the two banks, each bank will be able to sell its equity for a price of 1, which is the average value of both banks. Then, both banks could guarantee cash holdings of 1.[30] Note that the strong bank would prefer an outcome in which only the weak bank gets a cash injection. The regulator might be able to persuade the strong bank to participate by threatening not to inject money to anyone, which will lead to a worse outcome to the strong bank, or by threatening not to support the strong bank if it needs aide in the future.

4. In our model, all the economic surplus is captured by the banking sector, and so the regulator sets a disclosure rule aiming to maximize the surplus in the banking sector. However, our model can also easily capture externalities imposed by banks on the rest of society. Suppose, for example, that when a bank of type $\theta$ fails (i.e., when $\theta + \varepsilon < 1$), society suffers some exogenous loss $l(\theta)$. Then, the social gain from having a bank sell its asset is higher by $l(\theta)$. We can include this gain in the regulator's objective function and take it into account in the design of the disclosure rule. Specifically, we can add $l(\theta)$ to the coefficient of $h_i(\theta)$ in the regulator's objective function (11). Our main results continue to hold in this case. Clearly, now the regulator will have a stronger motive to help banks with a high $l(\theta)$. This may capture the familiar "too big to fail" argument, whereby

[30] We can extend the example to show that in some cases, it is optimal to create two groups of banks: one group contains intermediate banks, and the other group pools together strong and weak banks; the planner injects money only to banks in the first group. For example, if there are four banks: $\theta_1 = 2.2$, $\theta_2 = 1.2$, $\theta_3 = 0.4$, $\theta_4 = 0.2$, with $\rho_1 = 1.2$ and $\rho_2 = 1$, it is optimal to have $\theta_2$ and $\theta_3$ in one group, as before, and $\theta_1$ and $\theta_4$ in another group, and it is optimal to split the bailout money (0.4) equally only among banks in the first group. Then all banks can engage in risk sharing.

regulators try to save institutions whose failure will cause a substantial damage to the economy.

5. As in any standard mechanism design problem, we assumed that the regulator can commit to assigning scores according to the announced disclosure rule. In many cases, this commitment would arise endogenously. For example, if banks create risk-sharing arrangements among themselves rather than with a third party, the regulator cannot gain by deviating from the optimal ex-ante disclosure rule, e.g., saying things are better than they are, since then banks will have insufficient funds to honor the agreements and they will all fail.[31] Similarly, even if risk sharing is done with the market, as is explicitly the case in our model, the regulator cannot gain by deviating ex post in case there is a continuum of banks and the probability $p(\theta)$ of being a type $\theta$ represents the proportion of banks of this type in the continuum. In this case, maximizing the bank's ex-ante expected payoff is the same as maximizing the sum of banks' ex-post payoffs. Since the regulator is interested in that, he has no incentive to deviate ex post and say that some banks are doing better because this will come at the expense of other banks (remember the market knows the proportion of banks of each type). In this sense, the regulator is different from a single bank. The bank cares only about its own payoff and will have strong incentive to deviate and disclose a better type ex post. Moreover, a strong bank will have incentive to reveal itself as strong, even when it is socially optimal that the bank is pooled together with weaker banks.[32]

6. The discussion above suggests that studying disclosure by the regulator and not by individual banks is very natural in the context of our model. First, ex ante, the regulator and banks may have different objective functions because the regulator cares about externalities that banks impose on the rest of society. Second, ex-post, the regulator's commitment to a disclosure rule arises more naturally than that of an individual bank.

[31] This would be true, even if the regulator privately observed the aggregate state.

[32] For empirical evidence consistent with the view that banks disclose good news but look to hide bad news, see, e.g., Berger and Davies (1994).

7. In our model, assigning a score is equivalent to recommending a price to the market. The regulator does not lie in our model. A high score does not necessarily mean that the bank is strong. It only means that the average cash flow conditional on obtaining the score equals the recommended price. One can also think of scores more broadly. Scores separate banks into groups, and assigning scores is isomorphic to recommending to banks which groups to form. For example, one can think of scores as suggesting mergers among banks or joint liability arrangements as in Leitner (2005). We solved for the optimal design of groups under the constraint that each bank prefers to join the recommended group rather than stay in autarky and under the assumption that idiosyncratic risk is fully diversified within a group. This might be the case if there is a continuum of banks of each type, or more realistically, if the regulator provides insurance against idiosyncratic risk within a group.[33]

8. As noted in the introduction, we believe that our model can be used as a benchmark to think of credit rating agencies. An interesting question is what the optimal disclosure rule looks like when the regulator faces competition from credit rating agencies or whether it is possible to implement risk sharing when the regulator and credit rating agencies have different objectives.

9. We could interpret our paper as a paper about the design of stress tests rather than disclosure. In particular, choosing a disclosure rule is the same as designing an experiment (e.g., stress tests) that provides a public signal $s \in S$ according to some distribution $g(s|\theta)$. With this interpretation, the regulator's commitment boils down to committing not to manipulate the public signal.

10. Finally, our results could also be applied to other settings of Bayesian persuasion. The novelty in our setting relative to the broad Bayesian persuasion literature (aside from the microfoundations for the banking context) is that agents, whose types are being disclosed by a regulator, know their types and so have different reservation prices. This generates interesting implications for the optimal

[33] We do abstract, however, from other issues of group formation, such as whether a bank receiving one score will attempt to form a link with a bank receiving a different score.

disclosure rule, which are explored in Section 4. This is applicable for many other settings studied with Bayesian persuasion tools. For example, consider schools that grade students with different abilities and potential employers who care about the average ability of students they hire. Suppose that students know their own abilities, and students can open their own businesses instead of getting hired. In this case, a student's reservation price is the benefit from opening his own business. Our analysis can shed light on the way schools will communicate information about students.[34]

# 6 Conclusion

We provide a model of an optimal disclosure policy of a regulator that has information about banks (e.g., from conducting stress tests). The disclosure policy affects whether banks can take corrective actions, particularly whether banks can engage in risk sharing to ensure that their capital does not fall below some critical level.

We show that if the average forecasted capital $E(\theta)$ is sufficiently high, no disclosure is necessary. Otherwise, some disclosure is needed, and disclosure takes the form of different scores pooling together banks of different levels of strength. Two scores are sufficient if banks do not have the information that the regulator has. In this case, the optimal disclosure rule may take a simple form, such that banks whose forecasted capital is below some threshold obtain the low score and banks whose forecasted capital is above the threshold obtain the high score. More than two scores may be needed if a bank shares the same information that the regulator has about the bank. In this case, the optimal disclosure rule is non-monotone. Among the strong banks, the stronger banks obtain higher scores, which reflect a higher asset value, on average, but among the weak banks that are pooled with strong banks, the weaker banks obtain higher scores when they are pooled with strong banks. However, this non-monotonicity disappears if banks can freely dispose assets and the regulator must follow a deterministic (vs. stochastic)

[34] Ostrovsky and Schwarz (2010) study a similar problem but without such reservation prices.

disclosure rule.

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# Appendix

**Proof of Lemma 1.** From the text, the equilibrium price is $x(s) = \mu(s)$. Conditional on the bank's information, the bank's expected payoff is $E[R(\tilde{\theta} + \tilde{\varepsilon}|\tilde{s} = s)] = \mu(s) + r\Pr(\tilde{\theta} + \tilde{\varepsilon} \geq 1|\tilde{s} = s)$ if it keeps the asset and $R(\mu(s))$ if it sells. Hence, if $\mu(s) \geq 1$, it is optimal for the bank to sell because $R(\mu(s)) = \mu(s) + r > E[R(\tilde{\theta} + \tilde{\varepsilon}|\tilde{s} = s)]$. If $\mu(s) < 1$, it is optimal to keep the asset because $R(\mu(s)) = \mu(s) < E[R(\tilde{\theta} + \tilde{\varepsilon}|\tilde{s} = s)]$. The strict inequality follows from Assumption 1.

**Proof of Lemma 2.** The regulator's problem is to choose a disclosure rule $(S, g)$ to maximize the bank's ex-ante payoff $\sum_{\theta \in \Theta} p(\theta)u(\theta)$. From the law of iterated expectations, $\sum_{\theta \in \Theta} p(\theta) \sum_{s:\mu(s) \geq 1} \mu(s)g(s|\theta) = \sum_{\theta \in \Theta} p(\theta) \sum_{s:\mu(s) \geq 1} \theta g(s|\theta)$. Hence,

$$\begin{aligned}\sum_{\theta \in \Theta} p(\theta)u(\theta) &= \sum_{\theta \in \Theta} p(\theta) \sum_{s:\mu(s)<1} [\theta + r\Pr(\tilde{\varepsilon} \geq 1 - \theta)]g(s|\theta) + \sum_{\theta \in \Theta} p(\theta) \sum_{s:\mu(s) \geq 1} [\theta + r]g(s|\theta) \\ &= \sum_{\theta \in \Theta} p(\theta)[\theta + r\Pr(\tilde{\varepsilon} \geq 1 - \theta)] \sum_{s:\mu(s)<1} g(s|\theta) + \sum_{\theta \in \Theta} p(\theta)[\theta + r] \sum_{s:\mu(s) \geq 1} g(s|\theta).\end{aligned}$$

Since $\sum_{s:\mu(s)<1} g(s|\theta) = 1 - \sum_{s:\mu(s) \geq 1} g(s|\theta)$, we obtain that

$$\sum_{\theta \in \Theta} p(\theta)u(\theta) = \sum_{\theta \in \Theta} p(\theta)[\theta + r\Pr(\tilde{\varepsilon} \geq 1 - \theta)] + r\sum_{\theta \in \Theta} p(\theta)\Pr(\tilde{\varepsilon} < 1 - \theta) \sum_{s:\mu(s) \geq 1} g(s|\theta).$$

In the right-hand side of the equation above, only the second term is affected by the disclosure rule. Hence, maximizing $\sum_{\theta \in \Theta} p(\theta)u(\theta)$ is the same as maximizing $\sum_{\theta \in \Theta} p(\theta)\Pr(\tilde{\varepsilon} < 1 - \theta) \sum_{s:\mu(s) \geq 1} g(s|\theta)$.

**Proof of Lemma 3.** Consider a disclosure rule $(S, g)$. From Lemma 1, under $(S, g)$, type $\theta$ sells with probability $\sum_{s:\mu(s) \geq 1} g(s|\theta)$. Define a disclosure rule $(\hat{S}, \hat{g})$ by $\hat{S} = \{s_0, s_1\}$, $\hat{g}(s_1|\theta) = \sum_{s:\mu(s) \geq 1} g(s|\theta)$, and $\hat{g}(s_0|\theta) = 1 - \sum_{s:\mu(s) \geq 1} g(s|\theta)$. For $s \in \hat{S}$, denote $\hat{\mu}(s) = E[\tilde{\theta}|\tilde{s} = s]$, where the expectation is calculated under $\hat{g}$. From Bayes' rule and the law of iterated expectations,

$$\hat{\mu}(s_1) = \frac{\sum_{\theta \in \Theta} \theta p(\theta)\hat{g}(s_1|\theta)}{\sum_{\theta \in \Theta} p(\theta)\hat{g}(s_1|\theta)} = \frac{\sum_{\theta \in \Theta} p(\theta) \sum_{s:\mu(s) \geq 1} \mu(s)g(s|\theta)}{\sum_{\theta \in \Theta} p(\theta) \sum_{s:\mu(s) \geq 1} g(s|\theta)}.$$

Hence, $\hat{\mu}(s_1) \geq 1$. Similarly, $\hat{\mu}(s_0) < 1$. The results then follow from Lemmas 1 and 2.

**Proof of Proposition 1.** 1. Setting $h(\theta) = 1$ for every $\theta \in \Theta$ achieves the maximal attainable value for (5) and satisfies constraint (6). Any other $h : \Theta \to [0, 1]$ reduces the value of (5) by Assumption 1.

2. Suppose $h$ solves the problem in Lemma 4. By Assumption 1, $h(\theta) = 1$ for every $\theta \geq 1$. Now consider a type $\hat{\theta} < 1$ s.t. $h(\hat{\theta}) > 0$. We must have $h(\theta) = 1$ for every $\theta \in \Theta$ s.t. $G(\theta) > G(\hat{\theta})$ because if a type $\theta_i < 1$ exists s.t. $h(\theta_i) < 1$ and $G(\theta_i) > G(\hat{\theta})$, we obtain a contradiction to the optimality of $h$ by defining an alternate solution $\tilde{h}(\theta) = \begin{cases} h(\theta) \text{ if } \theta \notin \{\theta_i, \hat{\theta}\} \\ h(\theta) + \Delta \text{ if } \theta = \theta_i \\ h(\theta) - \frac{p(\theta_i)(1-\theta_i)}{p(\hat{\theta})(1-\hat{\theta})}\Delta \text{ if } \theta = \hat{\theta} \end{cases}$. In particular, if $\Delta > 0$ is sufficiently small, $\tilde{h}$ is a function from $\Theta$ to $[0, 1]$, which satisfies (6) and increases the value of (5) by $p(\theta_i)\Pr(\tilde{\varepsilon} < 1 - \theta_i)\Delta - p(\hat{\theta})\Pr(\tilde{\varepsilon} < 1 - \hat{\theta})\frac{p(\theta_i)(1-\theta_i)}{p(\hat{\theta})(1-\hat{\theta})}\Delta$, which equals $\Delta p(\theta_i)(1 - \theta_i)[G(\theta_i) - G(\hat{\theta})] > 0$. Consequently, if $h(\hat{\theta}) > 0$, we must have $\sum_{\theta \geq 1} p(\theta)(\theta - 1) + \sum_{\theta < 1 : G(\theta) > G(\hat{\theta})} p(\theta)(\theta - 1) \geq 0$ to satisfy (6). Since the coefficient of $h(\theta)$ in (5) is positive, $h$ is given by (8).

**Proof of Corollary 1.** Under full disclosure, $\mu(s) = \theta$ for every $s \in S$ such that $g(s|\theta) > 0$. Hence, the probability that type $\theta$ sells its asset is 1, if $\theta \geq 1$, and 0, otherwise. By Proposition 1, this outcome is optimal if $\theta_{\min} \geq 1$ and suboptimal if $\theta_{\min} < 1$. Under no disclosure, $\mu(s) = E(\tilde{\theta})$ for every $s \in S$. If $E(\tilde{\theta}) \geq 1$, the outcome is that every type sells with probability 1, which is optimal. If $E(\tilde{\theta}) < 1$, the outcome is that every type sells with probability 0, which is suboptimal. Finally, when $E(\tilde{\theta}) < 1$, neither full disclosure nor no disclosure is optimal. Hence, partial disclosure is the only way to achieve the optimal outcome.

**Proof of Corollary 2.** Let $\theta_i < \theta_j < 1$. Then $1 - \theta_i > 1 - \theta_j > 0$. From condition 1, $G(\theta_i) < G(\theta_j)$. The result then follows from Proposition 1.

**Proof of Lemma 5.** The proof is in the text.

**Proof of Lemma 6.** 1. Consider an optimal disclosure rule $(S, g)$, a type $\theta' \geq 1$, and a score $s' \in S$ s.t. $g(s'|\theta') > 0$. Suppose to the contrary that $\theta'$ does not sell its asset upon obtaining $s'$. Consider an alternate rule defined by $\tilde{S} = S \cup \{\tilde{s}\}$, $\tilde{g}(s|\theta') = \begin{cases} g(s'|\theta') \text{ if } s = \tilde{s} \\ 0 \text{ if } s = s' \\ g(s|\theta') \text{ if } s \notin \{s', \tilde{s}\} \end{cases}$, and for $\theta \neq \theta'$, $\tilde{g}(s|\theta) = \begin{cases} g(s|\theta) \text{ if } s \neq \tilde{s} \\ 0 \text{ if } s = \tilde{s} \end{cases}$. Under $(\tilde{S}, \tilde{g})$, the only type that obtains score $\tilde{s}$ is $\theta'$. So, the equilibrium price for score $\tilde{s}$ is $\theta'$. By Lemma 5, $\theta'$ sells upon obtaining score $\tilde{s}$. Equilibrium prices for all other scores remain unchanged. Hence, the alternate rule increases the expected payoff for type $\theta'$ by $rg(s'|\theta') \Pr(\tilde{\varepsilon} < 1 - \theta')$, while keeping the payoffs for all other types unchanged.

2. Consider an optimal disclosure rule $(S, g)$, a type $\theta \geq 1$, and a score $s \in S$ s.t. $g(s|\theta) \geq 0$. From part 1, type $\theta$ sells upon obtaining score $s$. So, by Lemma 5, $\rho(\theta) \leq x(s)$. To show $x(s) = \mu(s)$, we show that every type $\theta'$ s.t. $g(s|\theta') > 0$ sells upon obtaining score $s$, so selling does not convey additional information to the market. If $\theta' > \theta$, type $\theta'$ sells from part 1. If $\theta' < \theta$, then $\rho(\theta') \leq \rho(\theta) \leq x(s)$, and type $\theta'$ sells by Lemma 5.

3. Consider a disclosure rule $(S, g)$, not necessarily an optimal one, such that $g(s|\theta) = 0$ for every $\theta \geq 1$. Suppose to the contrary that in the equilibrium induced by $(S, g)$, some types sell upon obtaining score $s$. Suppose the highest type that sells is $\tilde{\theta} < 1$. So $\tilde{\theta} < \rho(\tilde{\theta})$. Since the market does not expect to lose money, the price must satisfy $x(s) \leq \tilde{\theta}$. But then $x(s) < \rho(\tilde{\theta})$, which contradicts Lemma 5.

**Proof of Lemma 7.** If $\mu(s) \geq 1$, there must be a type $\theta' \geq 1$, s.t. $g(s|\theta') \geq 0$. Hence, from the proof of the second part of Lemma 6 every type sells upon receiving score $s$. If $\mu(s) < 1$, the highest type that obtains score $s$ is below 1 because if $g(s|\theta') > 0$ for some $\theta' \geq 1$, then from Lemma 6, $\theta'$ sells at price $\mu(s)$, which contradicts Lemma 5. It then follows from the third part of Lemma 5 that every type keeps its asset upon obtaining score $s$.

**Proof of Lemma 8.** Consider an optimal disclosure rule $(S, g)$. Under $(S, g)$, type $\theta$ sells with probability $\sum_{s:\mu(s)\geq 1} g(s|\theta)$, which can be rewritten as

$\sum_{s\in\cup_{i=1}^k S_i} g(s|\theta)$. Define a disclosure rule $(\hat{S},\hat{g})$ by $\hat{S}=\{s_0,s_1,...,s_k\}$ and $\hat{g}(s_i|\theta)=\sum_{s\in S_i} g(s|\theta)$, for each $i\in\{1,...,k\}$. So the highest type that obtains score $s_i\neq s_0$ is $\theta_i$. For $s\in\hat{S}$, denote $\hat{\mu}(s)=E[\tilde{\theta}|\tilde{s}=s]$, where the expectation is calculated under $\hat{g}$. From Bayes' rule, the law of iterated expectations, and (10), it follows that for each $i\in\{1,...,k\}$,

$$\hat{\mu}(s_i)=\frac{\sum_{\theta\in\Theta}\theta p(\theta)\hat{g}(s_i|\theta)}{\sum_{\theta\in\Theta}p(\theta)\hat{g}(s_i|\theta)}=\frac{\sum_{\theta\in\Theta}p(\theta)\sum_{s\in S_i}\mu(s)g(s|\theta)}{\sum_{\theta\in\Theta}p(\theta)\sum_{s\in S_i}g(s|\theta)}\geq\rho_i. \tag{A-1}$$

Hence, every type sells its asset upon obtaining score $s_i$. Similarly, we can show that $\hat{\mu}(s_0)<1$. So no type sells upon obtaining score $s_0$. Hence, the probability that type $\theta$ sells the asset under $(\hat{S},\hat{g})$ is $\sum_{i=1}^k\hat{g}(s_i|\theta)=\sum_{s\in\cup_{i=1}^k S_i}g(s|\theta)=\sum_{s:\mu(s)\geq 1}g(s|\theta)$. Hence, using the observation in Lemma 4, $(\hat{S},\hat{g})$ is optimal.

**Proof of Corollary 3.** Under no disclosure, every type obtains score $s_1$ with probability 1; that is $h_1(\theta)=1$ for every $\theta\in\Theta$. If $E(\tilde{\theta})\geq\rho_1$, this satisfies constraint (12), and is clearly optimal because it achieves the maximal attainable value for (11) given constraint (13). If $E(\tilde{\theta})<\rho_1$, constraint (12) is violated, and so no disclosure cannot be optimal.

**Remarks for footnote 19.** We show that $E(\tilde{\theta})<1$ implies that resources are scarce, as follows. Summing up all $k$ constraints in (12) and changing the order of summation, we obtain $\sum_{\theta\in\Theta}\sum_{i=1}^k p(\theta)(\theta-\rho_i)h_i(\theta)\geq 0$. Since $\rho_i\geq 1$ for every $i\in\{1,...,k\}$, it follows that $\sum_{\theta\in\Theta}p(\theta)(\theta-1)\sum_{i=1}^k h_i(\theta)\geq 0$. Since $E(\tilde{\theta})<1$, $\sum_{\theta\in\Theta}p(\theta)(\theta-1)<0$. Hence, a type $\theta\in\Theta$ exists for which $\sum_{i=1}^k h_i(\theta)<1$.

**Proof of Lemma 10.** From Lemma 6, $x(s_i)=\mu(s_i)$. Hence, from (2), $x(s_i)=\rho_i$ if and only if $\sum_{\theta\in\Theta}p(\theta)(\theta-\rho_i)h_i(\theta)=0$. Suppose to the contrary that $\{h_i\}_{i=1,...,k}$ solves the problem in Lemma 9 and there exists $i\in\{1,...,k\}$ s.t. $\sum_{\theta\in\Theta}p(\theta)(\theta-\rho_i)h_i(\theta)>0$. Since resources are scarce, a type $\tilde{\theta}<1$ exists such that $h(\tilde{\theta})<1$. Let $\Delta\in(0,1-h(\tilde{\theta})]$ and $\tilde{h}_j(\theta)=\begin{cases} h_j(\theta)+\Delta \text{ if } j=i \text{ and } \theta=\tilde{\theta} \\ h_j(\theta) \text{ otherwise}\end{cases}$. Then $\{\tilde{h}_i\}_{i=1,...k}$ satisfies the constraints in Lemma 9 and strictly increases the value of the objective function, leading to a contradiction to the optimality of $\{h_i\}_{i=1,...,k}$.

**Proof of Proposition 2.** Consider an optimal disclosure rule $(S, g)$ and two types $\theta_j > \theta_i > 1$ s.t. $\rho_j > \rho_i$. Let $\{h_z(\theta)\}_{z\in\{1,...,k\}}$ be the corresponding solution to Lemma 9. That is, for every $z \in \{1, ..., k\}$, $h_z(\theta) = \sum_{s\in S_z} g(s|\theta)$. Suppose to the contrary that a score $s' \in S$ exists such that $g(s'|\theta_i) > 0$ and $g(s'|\theta_j) > 0$. Without loss of generality, $g(s'|\theta) = 0$ for every $\theta > \theta_j$. Then $h_j(\theta_i) > 0$ and $h_j(\theta_j) > 0$. From Lemma 10, $\sum_{\theta\in\Theta} p(\theta)(\theta - \rho_z)h_z(\theta) = 0$ for every $z \in \{1, ..., k\}$. We obtain a contradiction to Lemma 10 by constructing an alternate solution $\{\tilde{h}_z\}_{z=1,...,k}$ to the problem in Lemma 9, such that at least one of the $k$ constraints in (12) is satisfied with strict inequality.

Specifically, if $\rho_j \geq \theta_i$, construct $\tilde{h}_z(\theta) = \begin{cases} h_z(\theta) + \Delta \text{ if } z = i \text{ and } \theta = \theta_i \\ h_z(\theta) - \Delta \text{ if } z = j \text{ and } \theta = \theta_i \\ h_z(\theta) \text{ otherwise} \end{cases}$. It is easy to verify that if $\Delta$ is sufficiently small, $\tilde{h}_z(\theta)$ solves the problem in Lemma 9. In particular, constraint $i$ is satisfied because $\sum_{\theta\in\Theta} p(\theta)(\theta - \rho_i)h_i(\theta) = 0$ and $\theta_i \geq \rho_i$ imply that $\sum_{\theta\in\Theta} p(\theta)(\theta - \rho_i)\tilde{h}_i(\theta) = \Delta p(\theta_i)(\theta_i - \rho_i) \geq 0$. Constraint $j$ is satisfied because $\sum_{\theta\in\Theta} p(\theta)(\theta - \rho_j)\tilde{h}_j(\theta) = -\Delta p(\theta_i)(\theta_i - \rho_j) \geq 0$. Moreover, either $\theta_i > \rho_i$ or $\theta_i = \rho_i < \rho_j$. So, at least one resource constraint is satisfied with strict inequality. If, instead, $\rho_j < \theta_i$, then since $\sum_{\theta\in\Theta} p(\theta)(\theta - \rho_j)h_j(\theta) = 0$, a type $\tilde{\theta} < \rho_j$ exists such that $h_j(\tilde{\theta}) > 0$. The alternate solution is similar to $\tilde{h}$ but for type $\tilde{\theta}$, $\tilde{h}_i(\tilde{\theta})$ changes to $\tilde{h}_i(\tilde{\theta}) + \tilde{\Delta}$ and $\tilde{h}_j(\tilde{\theta})$ changes to $\tilde{h}_j(\tilde{\theta}) - \tilde{\Delta}$, where $\tilde{\Delta} = \frac{p(\theta_i)(\theta_i - \rho_j)}{p(\tilde{\theta})(\rho_j - \tilde{\theta})}\Delta > 0$. Again, it is easy to verify that the alternate solution solves the problem in Lemma 9. Constraint $j$ continues to be binding because $\sum_{\theta\in\Theta} p(\theta)(\theta - \rho_j)\tilde{h}_j(\theta) = -p(\theta_i)(\theta_i - \rho_j)\Delta - p(\tilde{\theta})(\tilde{\theta} - \rho_j)\tilde{\Delta} = 0$. Constraint $i$ is satisfied with strict inequality because $\sum_{\theta\in\Theta} p(\theta)(\theta - \rho_i)\tilde{h}_i(\theta) = p(\theta_i)(\theta_i - \rho_i)\Delta + p(\tilde{\theta})(\tilde{\theta} - \rho_i)\tilde{\Delta} > p(\theta_i)(\theta_i - \rho_i)\Delta + p(\tilde{\theta})(\tilde{\theta} - \rho_j)\tilde{\Delta} = p(\theta_i)(\rho_j - \rho_i)\Delta > 0$.

**Proof of Corollary 4.** From Proposition 2, we need at least $k$ scores so that each type above 1 obtains a different score. Since resources are scarce, we need another score for types below 1 that do not sell their assets.

**Proof of Proposition 3.** Consider an optimal disclosure rule $(S, g)$ and four types $\theta_v < \theta_w < 1 < \theta_i < \theta_j$, such that $\rho_i < \rho_j$. (Clearly, $\rho_i \geq 1$.) Let

$\{h_z(\theta)\}_{z\in\{1,...,k\}}$ be the corresponding solution to Lemma 9. That is, for every $z \in \{1,...,k\}$, $h_z(\theta) = \sum_{s\in S_z} g(s|\theta)$. From Proposition 2, if $\theta_v$ is pooled together with type $\theta_i$, then $h_i(\theta_v) > 0$. Suppose to the contrary that $\theta_w$ is pooled together with type $\theta_j$. So from Proposition 2, $h_j(\theta_w) > 0$. From Lemma 10, $\sum_{\theta\in\Theta} p(\theta)(\theta - \rho_i)h_i(\theta) = 0$ for every $i \in \{1,...,k\}$. We obtain a contradiction to Lemma 10 by constructing an alternate solution $\{\tilde{h}\}_{i=1,...,k}$ to the problem in Lemma 9 that satisfies at least one of the constraints in (12) with strict inequality. Specifically, construct $\tilde{h}$ from $h$ as follows. For type $\theta_w$, reduce $h_j(\theta_w)$ and increase $h_i(\theta_w)$, both by a small $\Delta > 0$. For type $\theta_v$, reduce $h_i(\theta_v)$ and increase $h_j(\theta_v)$, both by $\Delta_1 = \frac{p(\theta_w)(\theta_w-\rho_j)}{p(\theta_v)(\theta_v-\rho_j)}\Delta > 0$. Clearly, $\{\tilde{h}\}_{i=1,...,k}$ keeps the value of the objective function unchanged. Constraint $j$ continues to be binding because $-\Delta p(\theta_w)(\theta_w-\rho_j)+\Delta_1 p(v)(\theta_v-\rho_j) = 0$. Constraint $i$ is loosened because by simple algebra, $\Delta p(\theta_w)(\theta_w - \rho_i) - \Delta_1 p(\theta_v)(\theta_v - \rho_i) = \Delta p(\theta_w)\frac{(\rho_j-\rho_i)(\theta_v-\theta_w)}{(\theta_v-\rho_j)} > 0$. Clearly, $\tilde{h}$ also satisfies the other constraints in Lemma 9.

**Proof of Corollary 5.** From Lemma 6 and Proposition 2, we know that there are types $\theta_j \geq 1$ and $\theta_i \geq 1$ with reservation prices $\rho_i = x'$ and $\rho_j = x''$, such that $h_i(\theta_v) > 0$ and $h_j(\theta_w) > 0$. We must have $x'' \leq x'$, because otherwise $\theta_i < \theta_j$ and Proposition 3 implies that $h_j(\theta_w) = 0$, which is a contradiction.

**Lemma A-1** *If resources are scarce, the following expressions are well defined:* $a_1 = \max\{\theta \in \Theta : \sum_{\theta}^{\theta_{k+1}} p(\theta)(\rho_k-\theta) > p(\theta_k)(\theta_k-\rho_k)\}$, $b_1 = \frac{p(\theta_k)(\theta_k-\rho_k)-\sum_{\theta:\theta\in(a_1,\theta_{k+1}]}(\rho_k-\theta)}{p(a_1)(\rho_k-a_1)}$, $c_1 = 1-b_1$*; and for $i > 1$, define recursively $a_i$ to be the largest type $\theta' \leq a_{i-1}$, such that* $\sum_{\theta:[\theta',a_{i-1})} p(\theta)(\rho_{k-i} - \theta) + c_{i-1}p(a_{i-1})(\rho_{k-i} - a_{i-1}) > p(\theta_{k-i})(\theta_{k-i} - \rho_{k-i})$,

$$b_i = \frac{p(\theta_{k-i})(\theta_{k-i} - \rho_{k-i}) - \sum_{\theta:\theta\in(a_i,a_{i-1})} p(\theta)(\rho_{k-i} - \theta) - c_{i-1}p(a_{i-1})(\rho_{k-i} - a_{i-1})}{p(a_i)(\rho_{k-i} - a_i)},$$

*and* $c_i = \begin{cases} c_{i-1} - b_i & \text{if } a_i = a_{i-1} \\ 1 - b_i & \text{otherwise} \end{cases}$.

**Proposition A-1** *If resources are scarce and $G_i(\theta)$ is increasing in $\theta$ for every $\theta < 1$ and $i \in \{1, ..., k\}$, the optimal disclosure rule is such that for every $i \in \{1, ..., k\}$,*

$$h_{k+1-i}(\theta) = \begin{cases} 1 & if \quad \theta = \theta_i \text{ or } \theta \in (a_i, a_{i-1}) \\ c_{i-1} & if \quad \theta = a_{i-1} \text{ and } a_i < a_{i-1} \\ b_i & if \quad \theta = a_i \\ 0 & if \quad \theta < a_i \end{cases},$$

*where $a_i, b_i,$ and $c_i$ are defined in Lemma A-1.*

**Proof of Proposition 4.** Consider an optimal disclosure rule $(S, g)$ that satisfies the monotonicity constraint, and suppose $g(s|\theta) \in \{0, 1\}$ for every $s \in S$ and $\theta \in \Theta$, so the regulator does not randomize. Then for each type $\theta$, there exists some score $s \in S$, such that $g(s|\theta) = 1$. Since $\theta_{\max} \geq 1$, at least one type sells because otherwise, the regulator could increase the value of the objective by giving each type its own score. Suppose the lowest type that sells under $(S, g)$ is $\check{\theta}$, and suppose that $\check{\theta}$ obtains score $s'$ with probability 1.

We first show that all types above $\check{\theta}$ sell their assets. If $\check{\theta} \geq 1$, the result follows because if there was a type above $\check{\theta}$ that did not sell its asset, the regulator could increase the value of the objective function by giving each type its own score, so that all types above 1 sell their assets. If instead $\check{\theta} < 1$, we know from the third part in Lemma 6 (we can apply the original proof to show that it continues to hold in the case under consideration) that a type $\theta_j > 1$ exists such that $\theta_j$ sells its asset and $\check{\theta}$ and $\theta_j$ obtain the same score $s'$. Denote the highest type that obtains score $s'$ by $\theta_i$. Then types $\check{\theta}$ and $\theta_i$ end up with expected payoff $x(s') + r$, and from the monotonicity constraint, all types between $\check{\theta}$ and $\theta_i$ also end up with $x(s') + r$, so if they sell, the price must be $x(s')$. From Lemma 5, $x(s') \geq \rho(\theta_i) \geq 1$. To show that all types between $\check{\theta}$ and $\theta_i$ indeed sell, note that if a type $\theta \in (\check{\theta}, \theta_i)$ did not sell, it would end up with $\theta + r \Pr(\tilde{\varepsilon} \geq 1 - \theta)$, which is less than $\theta_i + r \Pr(\tilde{\varepsilon} \geq 1 - \theta_i)$, which is less than or equal to $\rho(\theta_i) + r$, and hence, $x(s') + r$. All types above $\theta_i$ also sell because otherwise, the regulator could increase the value of the objective function by giving each type $\theta > \theta_i$ its own score, so that each $\theta$ would sell for price $\theta$; since $x(s') \leq \theta_i$, monotonicity would be preserved.

Clearly, to preserve monotonicity, any type $\theta > \theta_i$ must sell at price above $x(s')$. Hence, we have established that there exists a cutoff $z_2$, such that all types on $[\check{\theta}, z_2]$ sell for the same price, and types above $z_2$ sell for higher prices. The optimal cutoff must be as described in the proposition, and in particular, we must have $\check{\theta} = \hat{z}_1(z_2^*)$, because otherwise, the regulator could increase the value of the objective function by giving all types on $[\hat{z}_1(z_2^*), z_2^*]$ the same score, and giving all other types their own scores.

**Example 3** Suppose there are five types $\theta_1 > \theta_2 > 1 > \theta_3 > \theta_4 > \theta_5$, equation (16) holds, and $\rho_1 > \rho_2$. From (16) and (18), $\hat{z}_1(\theta_2) = \theta_3$. There are three cases:

Case 1: $\sum_{i=1}^{3} p(\theta_i)(\theta_i - \rho_1) > 0$. In this case, $\hat{z}_1(\theta_1) < \theta_3$, and it is optimal to set $z_2 = \theta_1$ and $z_1 < \theta_3$. Types below $z_1$ obtain score $s_0$ and keep their assets. All other types obtain score $s_1$ and sell at price $\rho_1$.

Case 2: $\sum_{i=1}^{3} p(\theta_i)(\theta_i - \rho_1) < 0$. In this case, $\hat{z}_1(\theta_1) > \theta_3$, and it is optimal to set $z_2 = \theta_2$ and $z_1 = \theta_3$. Types below $\theta_3$ keep their assets; type $\theta_1$ sells at price $\rho_1$; types $\theta_2$ and $\theta_3$ sell at price $\rho_2$.

Case 3: $\sum_{i=1}^{3} p(\theta_i)(\theta_i - \rho_1) = 0$. In this case, $\hat{z}_1(\theta_1) = \theta_3$, and the regulator is indifferent between setting $z_2 = \theta_1$ and $z_2 = \theta_2$. In both cases, $z_1 = \theta_3$.

To get some intuition, note that the condition $\sum_{i=1}^{3} p(\theta_i)(\theta_i - \rho_1) > 0$ is equivalent to $p(\theta_1)(\theta_1 - \rho_1) > (\rho_1 - \rho_2) \sum_{i=2}^{3} p(\theta_i)$. The term $p(\theta_1)(\theta_1 - \rho_1)$ represents the benefits of choosing a higher $z_2$, and the term $(\rho_1 - \rho_2) \sum_{i=2}^{3} p(\theta_i)$ represents the cost.

# Online appendix

In this appendix, we analyze the regulator's problem from Section 4 with the additional constraint that the bank's equilibrium payoff is weakly increasing in type (as motivated in Section 4.5). We refer to this constraint as the monotonicity constraint and to the solution to the constrained problem as an optimal monotone rule.

## Regulator's problem with monotonicity constraint

We first establish that (all proofs are at the end of this appendix):

**Lemma B-1** *Lemma 6 continues to hold when we restrict attention to monotone rules.*

**Lemma B-2** *Suppose* $E(\tilde{\theta}) < 1$. *Under an optimal monotone rule, if* $s' \in S_j$ *and* $j \in \{1, ..., k\}$, *then* $x(s') = \rho_j$.

From Lemma B-1, type $\theta$'s expected payoff is $u(\theta)$, as in equation (3). The monotonicity constraint is that for every two types $\theta' < \theta$,

$$u(\theta') \leq u(\theta). \tag{B-1}$$

From Lemma B-2, $u(\theta)$ reduces to

$$u(\theta) = \theta + r\Pr(\tilde{\varepsilon} \geq 1 - \theta) + \sum_{i=1}^{k} [\rho_i - \theta + r\Pr(\tilde{\varepsilon} < 1 - \theta)] h_i(\theta), \tag{B-2}$$

which is a linear combination of $\{h_i(\theta)\}_{i \in \{1,\ldots,k\}}$. The term $\theta + r\Pr(\tilde{\varepsilon} \geq 1 - \theta)$ in (B-2) is the payoff that type $\theta$ obtains without selling its asset, and the coefficient of $h_i(\theta)$ is the extra payoff from selling at price $\rho_i$.

Hence, the regulator's problem reduces to the problem in Lemma 9, with the additional constraint (B-1), where $u(\theta)$ is given by (B-2). This is a linear programming problem. As in Section 4.1, a solution exists and can be implemented with $k + 1$ scores.

## Optimal monotone rules

In the examples that follow, we illustrate optimal monotone rules for two special cases. We show that for some parameter values, optimal monotone rules continue to exhibit the two forms of non-monotonicity discussed in Section 4.4. We also show that for some parameter values, optimal monotone rules do not satisfy the property in Proposition 2.

In the first example, we use the following lemma, which is similar to Proposition 3 but holds under stricter conditions.

**Lemma B-3** *Suppose $E(\tilde{\theta}) < 1$, and $G_i(\theta)$ is increasing in $\theta$ when $\theta < 1$ for every $i \in \{1, ..., k\}$. Under an optimal monotone rule, if $\theta'' < 1$, $h(\theta'') < 1$, and type $\theta''$ ever sells its asset at price $x$, then lower types never sell at prices below $x$.*

The idea behind Lemma B-3 is that if type $\theta''$ sells at price $x$ and type $\theta' < \theta''$ sells at price $x' < x$, the regulator can increase the value of the objective function, as in Proposition 3. This could violate monotonicity because the payoff of type $\theta''$ falls and that of type $\theta'$ increases. But because $G_i(\theta)$ is increasing in $\theta$ when $\theta < 1$, the regulator can restore monotonicity (and increase the value of the objective function even further) by transferring resources from type $\theta'$ to type $\theta''$.

**Example B-1** Suppose there are two types above 1, as in Example 1. We show below that if $p(\theta_5)$ is sufficiently large, there exists a scalar $\bar{\alpha} > 0$ and functions $\gamma(\alpha)$, $\hat{\beta}(\alpha)$, $\Gamma(\alpha)$, which depend on the model parameters, such that the optimal monotone rule is given by

| | $\theta_5$ | $\theta_4$ | $\theta_3$ | $\theta_2$ | $\theta_1$ |
|---|---|---|---|---|---|
| score $s_1$ (sell at price $\rho_1$) | $\gamma(\alpha^*)$ | $\hat{\beta}(\alpha^*)$ | $\alpha^*$ | $\alpha^*$ | 1 |
| score $s_2$ (sell at price $\rho_2$) | | | $1-\alpha^*$ | $1-\alpha^*$ | |
| score $s_0$ (keep asset) | $1-\gamma(\alpha^*)$ | $1-\hat{\beta}(\alpha^*)$ | | | |

where $\alpha^* = \begin{cases} 0 \text{ if } \Gamma(0) > \Gamma(\bar{\alpha}) \\ \bar{\alpha} \text{ if } \Gamma(0) < \Gamma(\bar{\alpha}) \end{cases}$. (If $\Gamma(0) = \Gamma(\bar{\alpha})$, both $\alpha^* = 0$ and $\alpha^* = \bar{\alpha}$ are optimal.) Moreover, $0 < \hat{\beta}(0) < \hat{\beta}(\bar{\alpha}) < 1$ and $0 < \gamma(\bar{\alpha}) < \gamma(0) < 1$.

Specifically, let

$$
\begin{aligned}
\hat{\beta}(\alpha) &\equiv \frac{\rho_2 + \alpha(\rho_1 - \rho_2) - [\theta_4 + r\Pr(\tilde{\varepsilon} \geq 1 - \theta_4)]}{\rho_1 - \theta_4 + r\Pr(\tilde{\varepsilon} < 1 - \theta_4)}, \\
\check{\beta}(\alpha) &\equiv \frac{p(\theta_1)(\theta_1 - \rho_1) - \alpha[p(\theta_2)(\rho_1 - \theta_2) + p(\theta_3)(\rho_1 - \theta_3)]}{p(\theta_4)(\rho_1 - \theta_4)}, \\
\gamma(\alpha) &\equiv \frac{p(\theta_1)(\theta_1 - \rho_1) - \alpha[p(\theta_2)(\rho_1 - \theta_2) + p(\theta_3)(\rho_1 - \theta_3)] - \min\{\hat{\beta}(\alpha), \check{\beta}(\alpha)\}p(\theta_4)(\rho_1 - \theta_4)}{p(\theta_5)(\rho_1 - \theta_5)}, \\
\hat{\gamma}(\alpha) &\equiv \frac{\rho_2 + \alpha(\rho_1 - \rho_2) - [\theta_5 + r\Pr(\tilde{\varepsilon} \geq 1 - \theta_5)]}{\rho_1 - \theta_5 + r\Pr(\tilde{\varepsilon} < 1 - \theta_5)}, \\
\Gamma(\alpha) &\equiv p(\theta_4)\Pr(\tilde{\varepsilon} < 1 - \theta_4)\hat{\beta}(\alpha) + p(\theta_5)\Pr(\tilde{\varepsilon} < 1 - \theta_5)\gamma(\alpha).
\end{aligned}
$$

Let $\breve{\alpha}$ be the unique solution to $\hat{\beta}(\alpha) = \check{\beta}(\alpha)$ and $\bar{\alpha} = \min\{1, \breve{\alpha}\}$. Assume $\gamma(0) \leq \hat{\gamma}(0)$ (e.g., $p(\theta_5)$ is sufficiently large). Observe that $\bar{\alpha} > 0$, $\hat{\beta}(0) < \hat{\beta}(\bar{\alpha}) < 1$ and $\gamma(\bar{\alpha}) < \gamma(0) < 1$, where the last inequality follows from equation (17).

The derivation of the optimal monotone rule is as follows:

Since Lemma 6 continues to hold, an $\alpha \in [0,1]$ exists such that $h_1(\theta_1) = 1$, $h_1(\theta_2) = \alpha$, and $h_2(\theta_2) = 1 - \alpha$. From the resource constraint for score $s_2$, $h_2(\theta_2)p(\theta_2)(\theta_2 - \rho_2) \geq h_3(\theta_3)p(\theta_3)(\rho_2 - \theta_3)$. From equation (16), $h_2(\theta_3) \leq 1 - \alpha$.

It is suboptimal to set $h_2(\theta_3) < 1 - \alpha$, as follows. Suppose to the contrary that $h_2(\theta_3) < 1 - \alpha$. To satisfy $u(\theta_3) \leq u(\theta_2)$, we must have $h_1(\theta_3) + h_2(\theta_3) < 1$, and from the resource constraint for score $s_2$, $h_2(\theta) > 0$ for some $\theta < \theta_3$. Hence, from Lemma B-3, $h_1(\theta_3) = 0$. That is, if type $\theta < \theta_3$ sells at price $\rho_2$, type $\theta_3$ cannot sell at price $\rho_1 > \rho_2$. But then $u(\theta_3) < u(\theta_2)$, and since the gain-to-cost ratio is increasing, the regulator can increase the value of the objective by transferring resources from the lowest type that sells with a positive probability to type $\theta_3$. Hence, a contradiction.

Consequently, $h_2(\theta_3) = 1 - \alpha$, and from equation (16), types $\theta_4$ and $\theta_5$ can obtain only scores $s_0$ and $s_1$. Since the gain-to-cost ratio is increasing, it is optimal to set $h_1(\theta_3) = \alpha$. As for $h_1(\theta_4)$, the regulator would like to set it as high as possible, subject to the monotonicity constraint $u(\theta_4) \leq u(\theta_3)$ and the resource constraint for score $s_1$. The monotonicity constraint reduces to $h_1(\theta_4) \leq \hat{\beta}(\alpha)$. The resource constraint reduces to $h_1(\theta_4) \leq \check{\beta}(\alpha)$. Hence, $h_1(\theta_4) = \min\{\hat{\beta}(\alpha), \check{\beta}(\alpha)\}$. All remaining resources from type $\theta_1$ are allocated to type $\theta_5$ so that the resource constraint for type $\theta_1$ is satisfied with equality. Hence, $h_1(\theta_5) = \gamma(\alpha)$. The monotonicity constraint $u(\theta_5) \leq u(\theta_4)$ reduces to $\gamma(\alpha) \leq \hat{\gamma}(\alpha)$ and is not binding, from the assumption $\gamma(0) \leq \hat{\gamma}(0)$ and the observation that $\gamma(\alpha)$ is decreasing in $\alpha$ and $\hat{\gamma}(\alpha)$ is increasing.

Hence, the regulator's problem reduces to choosing $\alpha \in [0,1]$ to maximize $p(\theta_4)\Pr(\tilde{\varepsilon} < 1-\theta_4)h_1(\theta_4)+p(\theta_5)\Pr(\tilde{\varepsilon} < 1-\theta_5)\gamma(\alpha)$, such that $h_1(\theta_4) = \min\{\hat{\beta}(\alpha), \breve{\beta}(\alpha)\}$. Since $h_1(\theta_4)$ decreases in $\alpha$ when $\alpha > \breve{\alpha}$, it follows from the linearity of the problem that it is optimal to choose either $\alpha = 0$ or $\alpha = \bar{\alpha}$. The result follows. ■

Example B-1 illustrates two properties of optimal monotone rules.[35] First, for some parameter values ($\Gamma(0) > \Gamma(\bar{\alpha})$), lower types continue to sell at higher prices (types $\theta_4$ and $\theta_5$ sell at a price above the one obtained by types $\theta_2$ and $\theta_3$). However, to satisfy the monotonicity constraint so that high types do not have an incentive to destroy assets, the low types sell with probability that is less than 1. Second, for other parameter values ($\Gamma(0) < \Gamma(\bar{\alpha})$), it is no longer optimal that types above 1 with different reservation prices obtain different scores. Instead, it is optimal to pool type $\theta_2$ with type $\theta_1$ so that type $\theta_2$ sells its asset at a price above its reservation price. This increases the payoff for type $\theta_2$, which is beneficial because it relaxes the monotonicity constraint for lower types. In the extreme case $\bar{\alpha} = 1$, all types that sell obtain the same score.

In the next example, optimal monotone rules exhibit the first type of nonmonotonicity (in probability of sale).

**Example B-2** Suppose there is only one type above 1 and the gain-to-cost ratio is decreasing in type. We show in Proposition B-1 that follows that under the optimal monotone rule, the probability of selling the asset continues to be nonmonotone in $\theta$: Lower types sell with higher probability than middle types. Relative to the case in which we do not impose the monotonicity constraint, the probability that low

[35] The result extends to a more general case in which there are two types above 1 and $G_i(\theta)$ is increasing in $\theta$ for every $\theta < 1$ and $i \in \{1, 2\}$.

types sell is lower, while the probability that types in the middle sell is higher. In other words, the increase in sale probability as type decreases is moderated in order to satisfy the monotonicity constraint, but overall non-monotonicity in probability of sale remains part of the solution.

**Proposition B-1** *Suppose there is only one type above 1, $E(\tilde{\theta}) < \rho_1$, and $G(\theta)$ is decreasing in $\theta$ when $\theta < 1$. Let*

$$\delta_\theta(\alpha) \equiv \frac{\theta_m + r\Pr(\tilde{\varepsilon} \geq 1-\theta_m) + \alpha[\rho_1 - \theta_m + r\Pr(\tilde{\varepsilon} < 1-\theta_m)] - [\theta + r\Pr(\tilde{\varepsilon} \geq 1-\theta)]}{\rho_1 - \theta + r\Pr(\tilde{\varepsilon} < 1-\theta)}$$

*and $\alpha^*$ be the (unique) $\alpha$ that solves $p(\theta_1)(\theta_1 - \rho_1) = \alpha p(\theta_m)(\rho_1 - \theta_m) + \sum_{i=2}^{m-1} p(\theta_i)(\rho_1 - \theta_i)\max\{0, \delta_\theta(\alpha)\}$.*

*(i) Under the optimal monotone rule, type $\theta_1$ sells with probability 1, type $\theta_m$ sells with probability $\alpha^*$, and type $\theta \in (\theta_m, \theta_2)$ sells with probability $\max\{0, \delta_\theta(\alpha^*)\}$, which is decreasing in $\theta$.*

*(ii) The probability that low types sell is lower relative to the unconstrained benchmark (i.e., the problem without the monotonicity constraint), while the probability that high types (below 1) sell is higher.*

## Proofs

**Proof of Lemma B-1.** Suppose $(S, g)$ is an optimal monotone rule with equilibrium prices $x(s)$. By Lemma 5, type $\theta$'s expected payoff is

$$\bar{V}(\theta) \equiv \sum_{s:x(s)<\rho(\theta)} [\theta + r\Pr(\tilde{\varepsilon} \geq 1-\theta)]g(s|\theta) + \sum_{s:x(s)\geq\rho(\theta)} [x(s) + r]g(s|\theta). \qquad \text{(B-3)}$$

The probability that type $\theta$ sells its asset is $\bar{h}(\theta) = \sum_{s:x(s)\geq\rho(\theta)} g(s|\theta)$. The price equals the expected cash flow of types purchasing the asset conditional on obtaining the score. Hence, $\sum_{\theta:\rho(\theta)\leq x(s)} p(\theta)[\theta - x(s)]g(s|\theta) = 0$ for every $s \in S$. The monotonicity constraint is that for every two types $\theta' < \theta$, $\bar{V}(\theta') \leq \bar{V}(\theta)$. We prove part 1 below. Parts 2 and 3 follow as in Lemma 6.

Consider a type $\theta_i > 1$. Suppose to the contrary that $\bar{h}(\theta_i) < 1$. So by Lemma 5, a score $s'$ exists such that $g(s'|\theta_i) > 0$ and $x(s') < \rho_i$. Without loss,

$\theta_i = \max\{\theta : \bar{h}(\theta) < 1\}$. If $\theta_i = \theta_1$ or $\bar{V}(\theta_i) < \bar{V}(\theta_{i-1})$, apply the proof of Lemma 6, but the probability that $\theta_i$ gets its own score must be sufficiently low so that monotonicity is preserved.

The remainder of the proof applies when $\theta_i < \theta_1$ and $\bar{V}(\theta_i) = \bar{V}(\theta_{i-1})$. Let $\theta_j = \max\{\theta : \bar{V}(\theta) = \bar{V}(\theta_i)\}$. Let $x_{\min} = \min\{x(s) : g(s|\theta_j) > 0\}$. Since $\bar{h}(\theta_j) = 1$, $\bar{V}(\theta_j) \geq x_{\min} + r$. Observe that $x_{\min} + r \geq \rho(\theta_j) + r \geq \theta_j + r\Pr(\tilde{\varepsilon} \geq 1 - \theta_j) > \theta_i + r\Pr(\tilde{\varepsilon} \geq 1 - \theta_i)$. Hence, to satisfy $\bar{V}(\theta_i) = \bar{V}(\theta_j)$, there must be scores $s_i$ and $s_j$, such that $g(s_i|\theta_i) > 0$, $g(s_j|\theta_j) > 0$, and $x(s_i) > x(s_j) \geq \rho_j > \rho_i$, because otherwise $\bar{V}(\theta_i) < \bar{V}(\theta_j)$.

Case 1: $\theta_i \geq x(s_i)$. Apply the logic from Lemma 6, but to satisfy the monotonicity constraint, reduce $g(s_i|\theta_i)$ and increase $g(s_j|\theta_i)$ so that $\theta_i$'s payoff is unchanged. To keep prices unchanged, increase $g(s_i|\theta_j)$ and reduce $g(s_j|\theta_j)$, so that the resources that type $\theta_j$ does not provide for score $s_i$ are provided by type $\theta_i$, and the resources that $\theta_j$ does not provide for score $s_j$ are provided by $\theta_i$. Formally, for a given $\Delta > 0$, let $\Delta_1 = \frac{p(\theta_j)}{p(\theta_i)}\frac{\theta_j - x(s_i)}{\theta_i - x(s_i)}\Delta$, $\Delta_2 = \frac{p(\theta_j)}{p(\theta_i)}\frac{\theta_j - x(s_j)}{\theta_i - x(s_j)}\Delta$, $\Delta_3 = \frac{\Delta_1[x(s_i)-\theta_i] - \Delta_2[x(s_j)-\theta_i] + (\Delta_1 - \Delta_2)[r\Pr(\tilde{\varepsilon}<1-\theta_1)]}{r\Pr(\tilde{\varepsilon}<1-\theta_1)}$. Observe that $\Delta_1 > \Delta_2 > 0$ and $\Delta_3 > 0$.

Consider $(\tilde{S}, \tilde{g})$ defined by $\tilde{S} = S \cup \{\tilde{s}\}$, $\tilde{g}(s|\theta_i) = \begin{cases} g(s|\theta_i) - \Delta_1 \text{ if } s = s_i \\ g(s|\theta_i) + \Delta_2 \text{ if } s = s_j \\ \Delta_3 \text{ if } s = \tilde{s} \\ g(s|\theta_i) + \Delta_1 - \Delta_2 - \Delta_3 \text{ if } s = s' \\ g(s|\theta_i) \text{ if } s \notin \{s_i, s_j, s', \tilde{s}\} \end{cases}$,

$\tilde{g}(s|\theta_j) = \begin{cases} g(s|\theta_j) + \Delta \text{ if } s = s_i \\ g(s|\theta_j) - \Delta \text{ if } s = s_j \\ 0 \text{ if } s = \tilde{s} \\ g(s|\theta_j) \text{ if } s \notin \{s_i, s_j\} \end{cases}$, and for $\theta \notin \{\theta_i, \theta_j\}$, $\tilde{g}(s|\theta) = \begin{cases} g(s|\theta) \text{ if } s \neq \tilde{s} \\ 0 \text{ if } s = \tilde{s} \end{cases}$.

If $\Delta$ is sufficiently small, $(\tilde{S}, \tilde{g})$ is a disclosure rule. Clearly, prices for scores $s \notin \{s', \tilde{s}, s_i, s_j\}$ are the same under $(S, g)$ and $(\tilde{S}, \tilde{g})$. Prices for scores $s_i$ and $s_j$ are also the same under both rules because the average cash flow conditional on obtaining each score and purchasing the asset remains unchanged. Formally, since $-p(\theta_i)[\theta_i - x(s_i)]\Delta_1 + p(\theta_j)[\theta_j - x(s_i)]\Delta = 0$, it follows that $\sum_{\theta:\rho(\theta)\leq x(s_i)} p(\theta)[\theta - x(s_i)]\tilde{g}(s|\theta) = \sum_{\theta:\rho(\theta)\leq x(s_i)} p(\theta)[\theta - x(s_i)]g(s|\theta) = 0$, and since $p(\theta_i)[\theta_i - x(s_j)]\Delta_2 - p(\theta_j)[\theta_j - x(s_j)]\Delta = 0$, it follows that $\sum_{\theta:\rho(\theta)\leq x(s_j)} p(\theta)[\theta - x(s_j)]\tilde{g}(s|\theta) = \sum_{\theta:\rho(\theta)\leq x(s_j)} p(\theta)[\theta -$

$x(s_j)]g(s|\theta) = 0$. The price for score $s'$ remains $x(s')$ because if $\Delta$ is sufficiently small, the average cash flow for score $s'$ remains below $\rho_i$ even if we include type $\theta_i$, and so type $\theta_i$ continues not to sell upon obtaining $s'$. The price for score $\tilde{s}$ is $\Delta_3$. Type $\theta_j$'s payoff increases by $\Delta[x(s_i) - x(s_j)]$, but if $\Delta$ is sufficiently small, monotonicity is preserved. Type $\theta_i$'s payoff remains unchanged from the definition of $\Delta_3$. Clearly, payoffs for all other types remain unchanged. Hence, a contradiction to the optimality of $(S, g)$.

Case 2: $\theta_i < x(s_i)$. Now type $\theta_i$ takes resources from $s_i$, so a higher type exists that provides resources. To satisfy the monotonicity constraint for type $\theta_i$, we reduce $g(s_i|\theta_i)$. To keep the price for $s_i$ unchanged, we reduce the probability that the higher type obtains score $s_i$. Formally, let $\theta_z = \max\{\theta : g(s_i|\theta) > 0\}$. So $\theta_z > x(s_i)$, $g(s'|\theta_z) = 0$, and $x(s_i) \geq \rho_z \geq \theta_z - r\Pr(\tilde{\varepsilon} < 1 - \theta_z) > \theta_i - r\Pr(\tilde{\varepsilon} < 1 - \theta_i)$. For a given $\Delta' > 0$, let $\Delta_4 = \frac{p(\theta_i)[x(s_i)-\theta_i]}{p(\theta_z)[\theta_z - x(s_i)]}\Delta'$, $\Delta_5 = \frac{r\Pr(\tilde{\varepsilon}<1-\theta_i)+x(s_i)-\theta_i}{r\Pr(\tilde{\varepsilon}<1-\theta_i)}\Delta'$, $\Delta_6 = \frac{r\Pr(\tilde{\varepsilon}<1-\theta_z)+x(s_i)-\theta_z}{r\Pr(\tilde{\varepsilon}<1-\theta_z)}\Delta_4$. Then $\Delta_4 > \Delta_6 > 0$, $\Delta' > \Delta_5 > 0$. Consider $(\hat{S}, \hat{g})$ defined by $\hat{S} = S \cup \{\tilde{s}_i, \tilde{s}_z\}$, $\hat{g}(s|\theta_i) = \begin{cases} g(s|\theta_i) - \Delta' \text{ if } s = s_i \\ \Delta_5 \text{ if } s = \tilde{s}_i \\ g(s|\theta_i) + \Delta' - \Delta_5 \text{ if } s = s' \\ 0 \text{ if } s = \tilde{s}_z \\ g(s|\theta_i) \text{ if } s \notin \{s_i, \tilde{s}_i, s'\} \end{cases}$, $\hat{g}(s|\theta_z) = \begin{cases} g(s|\theta_z) - \Delta_4 \text{ if } s = s_i \\ \Delta_6 \text{ if } s = \tilde{s}_z \\ \Delta_4 - \Delta_6 \text{ if } s = s' \\ 0 \text{ if } s = \tilde{s}_i \\ g(s|\theta_z) \text{ if } s \notin \{s_i, \tilde{s}_z, s'\} \end{cases}$, and for $\theta \notin \{\theta_i, \theta_z\}$, $\hat{g}(s|\theta) = \begin{cases} g(s|\theta) \text{ if } s \notin \{\tilde{s}_i, \tilde{s}_z\} \\ 0 \text{ if } s \in \{\tilde{s}_i, \tilde{s}_z\} \end{cases}$.
If $\Delta'$ is sufficiently small, $(\hat{S}, \hat{g})$ is a disclosure rule. The cash flow conditional on score $s'$ remains below $\rho_i$ even if we include type $\theta_i$ and $\theta_z$, so these types continue not to sell upon obtaining score $s'$, and the price remains $x(s')$. The price for score $\tilde{s}_i$ is $\theta_i$, and the price for $\tilde{s}_z$ is $\theta_z$. The prices for all other scores are the same under $(S, g)$ and $(\hat{S}, \hat{g})$. For score $s_i$, this follows because $-p(\theta_i)[\theta_i - x(s_i)]\Delta' - p(\theta_z)[\theta_z - x(s_i)]\Delta_4 = 0$. Type $\theta_i$'s payoff remains unchanged because $-\Delta'[x(s_i) - \theta_i + r\Pr(\tilde{\varepsilon} < 1 - \theta_i)] + \Delta_5 r\Pr(\tilde{\varepsilon} < 1 - \theta_i) = 0$. Type $\theta_z$'s payoff remains unchanged because $-\Delta_4[x(s_i) - \theta_z + r\Pr(\tilde{\varepsilon} < 1 - \theta_i)] + \Delta_6 r\Pr(\tilde{\varepsilon} < 1 - \theta_z) = 0$. Clearly, payoffs for all types also remain unchanged. The probability that $\theta_z$ sells

its asset is less than 1 because $\hat{g}(s'|\theta_z) > 0$. Restart the proof of this lemma for the problem in which $\theta_z$ is the highest type above 1 that sells with probability less than 1. Since there is a finite number of types, the process ends in a finite number of steps leading a contradiction to the optimality of $(S, g)$.

**Lemma B-4** *Suppose $E(\tilde{\theta}) < 1$. Under an optimal monotone rule, a type $\theta_i < 1$ exists such that $h(\theta_i) < 1$ and $u(\theta_i) < u(\theta_{i-1})$.*

**Proof.** Suppose to the contrary that for every type $\theta_i < 1$, either $h(\theta_i) = 1$ or $u(\theta_i) = u(\theta_{i-1})$. From Lemma B-1, $h(\theta_i) = 1$ for every $i \in \{1, ..., k\}$. By induction on $i$, $u(\theta_i) \geq 1 + r$ for every $i \in \{1, .., m\}$. Hence, $\sum_{\theta \in \Theta} p(\theta)u(\theta) \geq 1 + r$. But since the market breaks even, $\sum_{\theta \in \Theta} p(\theta)u(\theta) \leq E(\tilde{\theta}) + r < 1 + r$. ■

**Proof of Lemma B-2.** Consider an optimal monotone rule $(S, g)$ and a score $s' \in S_j$, where $j \in \{1, ..., k\}$. From Lemmas 5 and B-1, $x(s') = \mu(s') \geq \rho_j$. Suppose to the contrary that $\mu(s') > \rho_j$. Let $\theta_i = \min\{\theta \in \Theta : g(s'|\theta) > 0\}$. Without loss, $g(s'|\theta) = 0$ if $\theta \notin \{\theta_i, \theta_j\}$. Hence,

$$p(\theta_i)g(s'|\theta_i)[\mu(s') - \theta_i] = p(\theta_j)g(s'|\theta_j)[\theta_j - \mu(s')]. \tag{B-4}$$

Since $E(\tilde{\theta}) < 1$, a type $\theta_z < 1$ exists such that $h(\theta_z) < 1$ and $V(\theta_z) < V(\theta_{z-1})$ (Lemma B-4). Hence, a score $s_0 \in S$ exists such that $g(s_0|\theta_z) > 0$ and $x(s_0) < \rho(\theta_z)$, so type $\theta_z$ does not sell upon obtaining $s_0$.

Case 1. $h(\theta_i) < 1$. Then there exists a score $\tilde{s}_0 \in S$, such that $g(\tilde{s}_0|\theta_i) > 0$ and $x(\tilde{s}_0) < \rho_i$. From Lemma B-1, $\theta_i < 1 < \theta_j$. We construct an alternate monotone rule that increases type $\theta_z$'s payoff and keeps the payoffs of all other types unchanged. Under the alternate rule, the price for score $s'$ drops to $x(s') - \varepsilon$, and $g(s'|\theta_z)$ increases. To keep $\theta_i$'s payoff unchanged, we increase $g(s'|\theta_i)$, and to keep $\theta_j$'s payoff unchanged, we assign it its own score. Formally, for a given $\varepsilon > 0$, let $\Delta$ solve

$$[g(s'|\theta_j) - \Delta][\theta_j - \mu(s') + \varepsilon] = g(s'|\theta_j)[\theta_j - \mu(s')], \tag{B-5}$$

$\Delta_1$ solve

$$[g(s'|\theta_i) + \Delta_1][\mu(s') - \varepsilon - \theta_i] + \Delta_1 rF(1-\theta_i) = g(s'|\theta_i)[\mu(s') - \theta_i], \tag{B-6}$$

and $\tilde{\Delta}$ solve

$$\begin{aligned} & p(\theta_i)[g(s'|\theta_i) + \Delta_1][\mu(s') - \varepsilon - \theta_i] + \tilde{\Delta} p(\theta_z)[\mu(s') - \varepsilon - \theta_z] \\ = \ & p(\theta_j)[g(s'|\theta_j) - \Delta][\theta_j - \mu(s') + \varepsilon]. \end{aligned} \tag{B-7}$$

Then $\Delta = \frac{g(s'|\theta_j)\varepsilon}{\theta_j - \mu(s') + \varepsilon} > 0$, $\Delta_1 = \frac{g(s'|\theta_i)\varepsilon}{\mu(s') - \varepsilon - \theta_i + rF(1-\theta_i)}$, and $\tilde{\Delta} = \frac{p(\theta_i)\Delta_1 rF(1-\theta_i)}{p(\theta_z)[\mu(s') - \varepsilon - \theta_z]}$.[36] Consider an alternate rule $(\tilde{S}, \tilde{g})$, defined by $\tilde{S} = \{S, \tilde{s}_j\}$, $\tilde{g}(s'|\theta) = \begin{cases} g(s'|\theta) + \Delta_1 \text{ if } \theta = \theta_i \\ \tilde{\Delta} \text{ if } \theta = \theta_z \\ g(s'|\theta) - \Delta \text{ if } \theta = \theta_j \\ 0 \text{ if } \theta \notin \{\theta_i, \theta_j, \theta_z\} \end{cases}$, $\tilde{g}(s_0|\theta) = \begin{cases} g(s_0|\theta) - \tilde{\Delta} \text{ if } \theta = \theta_z \\ g(s_0|\theta) \text{ if } \theta \neq \theta_z \end{cases}$, $\tilde{g}(\tilde{s}_0|\theta) = \begin{cases} g(\tilde{s}_0|\theta) - \Delta_1 \text{ if } \theta = \theta_i \\ g(\tilde{s}_0|\theta) \text{ if } \theta \neq \theta_i \end{cases}$, $\tilde{g}(\tilde{s}_j|\theta) = \begin{cases} \Delta \text{ if } \theta = \theta_j \\ 0 \text{ if } \theta \neq \theta_j \end{cases}$, and for $s \notin \{s, s_0, \tilde{s}_0, \tilde{s}_j\}$, $\tilde{g}(s|\theta) = g(s|\theta)$ for every $\theta \in \Theta$. If $\varepsilon$ is sufficiently small, $\Delta_1 > 0$, $\tilde{\Delta} > 0$, and $(\tilde{S}, \tilde{g})$ is a disclosure rule. From (B-7), the expected cash flow conditional on score $s'$ is $\mu(s') - \varepsilon > \rho_j$. Hence, types $\theta_i, \theta_j, \theta_z$ sell upon obtaining score $s'$, and the price is $\mu(s') - \varepsilon$. The price for score $\tilde{s}_j$ is $\Delta$. Clearly, prices for all other scores are the same as under $(S, g)$. Type $\theta_z$'s payoff increases by $\tilde{\Delta}[\mu(s') - \varepsilon + rF(1-\theta_z)]$, but if $\varepsilon$ is sufficiently small, monotonicity is preserved. The payoffs for types $\theta_i$ and $\theta_j$ remain unchanged by equations (B-6) and (B-5), respectively.

Case 2. $h(\theta_i) = 1$ and $u(\theta_i) > u(\theta_{i+1})$. Since maximizing $\sum_{\theta \in \Theta} p(\theta)u(\theta)$ is the same as maximizing $\sum_{\theta \in \Theta} p(\theta) \Pr(\tilde{\varepsilon} < 1 - \theta) h(\theta)$, to obtain a contradiction, it is sufficient to construct an alternate monotone rule that increases $h(\theta_z)$ and for every $\theta \neq \theta_z$, keeps $h(\theta)$ unchanged. If $\theta_j = \theta_i$, then $\theta_i > 1$, and the alternate rule assigns to type $\theta_z$ score $s'$ instead of $s_0$, with a small probability $\varepsilon$. Type $\theta_i$'s payoff drops by $\varepsilon g(s'|\theta_i)$, but if $\varepsilon$ is sufficiently small, monotonicity is preserved. If

[36] To derive $\tilde{\Delta}$, observe that from (B-6), $p(\theta_i)[g(s'|\theta_i) + \Delta_1][\mu(s') - \varepsilon - \theta_i] = p(\theta_i)g(s'|\theta_i)[\mu(s') - \theta_i] - p(\theta_i)\Delta_1 rF(1-\theta_i)$, and from (B-4) and (B-5), $p(\theta_j)[g(s'|\theta_j) - \Delta][\theta_j - \mu(s') + \varepsilon] = p(\theta_j)g(s'|\theta_j)[\theta_j - \mu(s')] = p(\theta_i)g(s'|\theta_i)[\mu(s') - \theta_i]$.

instead $\theta_j > \theta_i$, define $(\tilde{S}, \tilde{g})$ as in case 1 but set $\Delta_1 = 0$ and $\tilde{\Delta} = \frac{p(\theta_i)g(s'|\theta_i)\varepsilon}{p(\theta_z)[\mu(s')-\varepsilon-\theta_z]}$. Again, type $\theta_i$'s payoff drops but monotonicity is preserved.

Case 3. $h(\theta_i) = 1$ and $u(\theta_i) = u(\theta_{i+1})$. Let $\theta' = \min\{\theta : u(\theta) = u(\theta_i)\}$. Suppose the lowest score (that with lowest price) that $\theta_i$ obtains is $s''$ and the highest score that $\theta'$ obtains is $s'''$.[37] We must have $\mu(s''') \geq \mu(s'')$ because $\mu(s''') < \mu(s'')$ implies $u(\theta') < u(\theta_i)$.

Case 3.1. $\mu(s'') < \mu(s')$, $\theta_i > \mu(s'')$. Then a type $\theta'' < \mu(s'')$ exists such that $g(s''|\theta'') > 0$. Without loss of generality, $g(s''|\theta) = 0$ for $\theta \notin \{\theta_i, \theta''\}$. Hence,

$$p(\theta_i)g(s''|\theta_i)[\theta_i - \mu(s'')] = p(\theta'')g(s''|\theta'')[\mu(s'') - \theta'']. \tag{B-8}$$

As before, construct an alternate monotone rule that reduces $x(s')$. To keep $\theta_i$'s payoff unchanged, increase $g(s'|\theta_i)$ and reduce $g(s''|\theta_i)$. To keep the price for $s''$ unchanged, reduce $g(s''|\theta'')$. To keep the payoff of $\theta''$ unchanged, increase $g(s'|\theta'')$. We focus on the case in which $\theta_j > \theta_i$. If $\theta_j = \theta_i$, apply the same as if $\theta_j$ does not exist, that is, set $p(\theta_j) = 0$.

Formally, for a given $\varepsilon > 0$, let $\Delta_6 = \frac{\varepsilon g(s'|\theta_i)}{\mu(s')-\varepsilon-\mu(s'')}$, $\Delta_7 = \frac{p(\theta_i)[\theta_i-\mu(s'')]}{p(\theta'')[\mu(s'')-\theta'']}\Delta_6$, $\Delta_8 = \frac{\mu(s'')-\theta''+rF(1-\theta'')}{\mu(s')-\varepsilon-\theta''+rF(1-\theta'')}\Delta_7$, and $\Delta_9$ solve

$$\begin{aligned} & p(\theta_i)[g(s'|\theta_i) + \Delta_6][\mu(s') - \varepsilon - \theta_i] \\ & +\Delta_8 p(\theta'')[\mu(s') - \varepsilon - \theta''] + \Delta_9 p(\theta_z)[\mu(s') - \varepsilon - \theta_z] \\ = \; & p(\theta_j)[g(s'|\theta_j) - \Delta][\theta_j - \mu(s') + \varepsilon], \end{aligned} \tag{B-9}$$

Let $\tilde{S} = \{S, \tilde{s}_j\}$, $\tilde{g}(s'|\theta) = \begin{cases} g(s'|\theta) - \Delta \text{ if } \theta = \theta_j \\ g(s'|\theta) + \Delta_6 \text{ if } \theta = \theta_i \\ g(s'|\theta) + \Delta_8 \text{ if } \theta = \theta'' \\ \Delta_9 \text{ if } \theta = \theta_z \\ g(s'|\theta) \text{ if } \theta \notin \{\theta_j, \theta_i, \theta_z\} \end{cases}$, $\tilde{g}(s_0|\theta) = \begin{cases} g(s_0|\theta) - \Delta_9 \text{ if } \theta = \theta_z \\ g(s_0|\theta) + \Delta_7 - \Delta_8 \text{ if } \theta = \theta'' \\ g(s_0|\theta) \text{ if } \theta \notin \{\theta_z, \theta''\} \end{cases}$,

$\tilde{g}(\tilde{s}_j|\theta) = \begin{cases} \Delta \text{ if } \theta = \theta_j \\ 0 \text{ if } \theta \neq \theta_j \end{cases}$, $\tilde{g}(s''|\theta) = \begin{cases} g(s''|\theta) - \Delta_6 \text{ if } \theta = \theta_i \\ g(s''|\theta) - \Delta_7 \text{ if } \theta = \theta'' \\ g(s''|\theta) \text{ if } \theta \notin \{\theta_i, \theta''\} \end{cases}$, and for $s \notin \{s', s_0, \tilde{s}_j, s''\}$, $\tilde{g}(s|\theta) = g(s|\theta)$. Consider $(\tilde{S}, \tilde{g})$. If $\varepsilon$ is sufficiently small, $\Delta_6 > 0$, $\Delta_7 > \Delta_8 > 0$,

[37] Formally, $g(s''|\theta_i) > 0$, $g(s'''|\theta') > 0$, $x(s) \geq x(s'')$ for every $s \in S$ s.t. $g(s|\theta_i) > 0$, and $x(s) \leq x(s''')$ for every $s \in S$ s.t. $g(s|\theta') > 0$.

and $\Delta_9 > 0$.[38] Hence, $(\tilde{S}, \tilde{g})$ is a disclosure rule. From equation (B-9), the expected cash flow conditional on score $s'$ is $\mu(s') - \varepsilon > \rho_j$. Hence, types $\theta_i, \theta_j, \theta_z$ sell upon obtaining score $s'$, and the price is $\mu(s') - \varepsilon$. The price for score $\tilde{s}_j$ is $\Delta$. The price for score $s''$ remains $\mu(s'')$ because $\Delta_7 p(\theta'')[\mu(s'') - \theta''] = \Delta_6 p(\theta_i)[\theta_i - \mu(s'')]$. Clearly, prices for all other scores remain the same. Type $\theta_z$'s payoff increases by $\Delta_9[\mu(s') - \varepsilon + rF(1-\theta_z)]$, but if $\varepsilon$ is sufficiently small, monotonicity is preserved. The payoffs for types $\theta_i$ and $\theta''$ remain unchanged from the definition of $\Delta_6$ and $\Delta_8$, respectively. Type $\theta_j$'s payoff remains unchanged by equation (B-5).

Case 3.2. $\mu(s'') < \mu(s')$, $\theta_i \leq \mu(s'')$. The alternate rule is similar to that in case 3.1, but now type $\theta_i$ takes resources from score $s''$, so to keep the price for $s''$ unchanged, increase $g(s''|\theta_z)$. Formally, let $\Delta_{10} = \frac{p(\theta_i)[\mu(s'')-\theta_i]}{p(\theta'')[\mu(s'')-\theta_z]}\Delta_6$, and $\Delta_{11}$ solve

$$\begin{aligned} & p(\theta_i)[g(s'|\theta_i) + \Delta_6][\mu(s') - \varepsilon - \theta_i] + \Delta_{11} p(\theta_z)[\mu(s') - \varepsilon - \theta_z] \\ = \;& p(\theta_j)[g(s'|\theta_j) - \Delta][\theta_j - \mu(s') + \varepsilon]. \end{aligned}$$

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[38] $\Delta_9 > 0$, as follows. From (B-4) and (B-5), $p(\theta_j)[g(s'|\theta_j) - \Delta][\theta_j - \mu(s') + \varepsilon] = p(\theta_i)g(s'|\theta_i)[\mu(s') - \theta_i]$. Hence, (B-9) reduces to

$$\begin{aligned} & p(\theta_i)\Delta_6[\mu(s') - \varepsilon - \theta_i] + \Delta_9 p(\theta_z)[\mu(s') - \varepsilon - \theta'] \\ & + \Delta_8 p(\theta'')[\mu(s') - \varepsilon - \theta''] = p(\theta_i)g(s'|\theta_i)\varepsilon. \end{aligned} \tag{B-10}$$

From the definition of $\Delta_6$,

$$\varepsilon g(s'|\theta_i) = \Delta_6[\mu(s') - \varepsilon - \theta_i] + \Delta_6[\theta_i - \mu(s'')].$$

So

$$\Delta_9 p(\theta_z)[\mu(s') - \varepsilon - \theta'] = \Delta_6 p(\theta_i)[\theta_i - \mu(s'')] - \Delta_8 p(\theta'')[\mu(s') - \varepsilon - \theta''].$$

To show that $\Delta_9 > 0$, we need to show that

$$\Delta_6 p(\theta_i)[\theta_i - \mu(s'')] > \Delta_8 p(\theta'')[\mu(s') - \varepsilon - \theta''].$$

This reduces to

$$\Delta_7 p(\theta'')[\mu(s'') - \theta''] > p(\theta'')[\mu(s') - \varepsilon - \theta''] \frac{\mu(s'') - \theta'' + rF(1-\theta'')}{\mu(s') - \varepsilon - \theta'' + rF(1-\theta'')} \Delta_7,$$

or equivalently,

$$\frac{\mu(s'') - \theta''}{\mu(s') - \varepsilon - \theta''} > \frac{\mu(s'') - \theta'' + rF(1-\theta'')}{\mu(s') - \varepsilon - \theta'' + rF(1-\theta'')};$$

which follow since $\mu(s') - \varepsilon > \mu(s'')$.

If $\theta_j > \theta_i$, the alternate rule is defined by $\tilde{S} = \{S, \tilde{s}_j\}$, $\tilde{g}(s'|\theta) = \begin{cases} g(s'|\theta) - \Delta \text{ if } \theta = \theta_j \\ g(s'|\theta) + \Delta_6 \text{ if } \theta = \theta_i \\ \Delta_{11} \text{ if } \theta = \theta_z \\ g(s'|\theta) \text{ if } \theta \notin \{\theta_j, \theta_i, \theta_z\} \end{cases}$, $\tilde{g}(s_0|\theta) = \begin{cases} g(s_0|\theta) - \Delta_{10} - \Delta_{11} \text{ if } \theta = \theta_z \\ g(s_0|\theta) \text{ if } \theta \notin \{\theta_z, \theta''\} \end{cases}$, $\tilde{g}(\tilde{s}_j|\theta) = \begin{cases} \Delta \text{ if } \theta = \theta_j \\ 0 \text{ if } \theta \neq \theta_j \end{cases}$, $\tilde{g}(s''|\theta) = \begin{cases} g(s''|\theta) - \Delta_6 \text{ if } \theta = \theta_i \\ g(s''|\theta) + \Delta_{10} \text{ if } \theta = \theta_z \\ g(s''|\theta) \text{ if } \theta \notin \{\theta_i, \theta_z\} \end{cases}$, and for $s \notin \{s', s_0, \tilde{s}_j, s''\}$, $\tilde{g}(s|\theta) = g(s|\theta)$. If $\theta_j = \theta_i$, ignore type $\theta_j$; that is, set $p(\theta_j) = 0$.

Case 3.3. $\mu(s'') = \mu(s') = \mu(s''')$. First, combine scores $s'$ and $s'''$ into one score $\bar{s}$. That is, create a rule $(S, \bar{g})$, where $\bar{g}(s|\theta) = \begin{cases} g(s'|\theta) + g(s'''|\theta) \text{ if } s = \bar{s} \\ 0 \text{ if } s \in \{s', s'''\} \\ g(s|\theta) \text{ if } s \notin \{\bar{s}, s', s'''\} \end{cases}$. Clearly, $(S, \bar{g})$ is an optimal monotone rule, and the average cash flow for score $\bar{s}$ is $\mu(s')$. Since $\theta' \le \mu(s') \le \theta_j$, there is an optimal monotone rule $(\bar{S}, \bar{g}')$ and a score $s \in \bar{S}$ with price $\mu(s')$, such that the only types that obtain that score are $\theta'$ and $\theta_j$. We can then apply case 1 or case 2 to obtain a contradiction.

Case 3.4. $\mu(s'') = \mu(s') < \mu(s''')$. We construct an alternate monotone rule under which the price for score $s'$ drops to $x(s') - \varepsilon$, and $g(s'|\theta_z)$ increases, as in case 1. To keep $\theta_i$'s payoff unchanged, we increase $g(s'''|\theta_i)$ and reduce $g(s''|\theta_i)$. To keep prices unchanged, we reduce $g(s'''|\theta')$ and increase $g(s''|\theta')$. Formally, for a given $\varepsilon > 0$, let $\Delta_2 = \frac{\varepsilon g(s'|\theta_i)}{\mu(s''') - \mu(s'')}$, $\Delta_3 = \frac{p(\theta_i)[\mu(s''') - \theta_i]}{p(\theta')[\mu(s''') - \theta']}\Delta_2$, $\Delta_4 = \frac{p(\theta_i)[\mu(s'') - \theta_i]}{p(\theta')[\mu(s'') - \theta']}\Delta_2$, and $\Delta_5$ solve

$$\begin{aligned} & (\Delta_3 - \Delta_4)p(\theta')[\mu(s') - \varepsilon - \theta'] + \Delta_5 p(\theta_z)[\mu(s') - \varepsilon - \theta'] \\ & + p(\theta_i)g(s'|\theta_i)[\mu(s') - \varepsilon - \theta_i] \\ = \; & p(\theta_j)[g(s'|\theta_j) - \Delta][\theta_j - \mu(s') + \varepsilon]. \end{aligned} \tag{B-11}$$

Then $\Delta_2 > 0$, $\Delta_3 > \Delta_4 > 0$, and $\Delta_5 > 0$.[39] Consider an alliterate rule $(\hat{S}, \hat{g})$, where

[39] To see why $\Delta_5 > 0$, observe that from (B-4) and (B-5),

$$p(\theta_j)[g(s'|\theta_j) - \Delta][\theta_j - \mu(s') + \varepsilon] = p(\theta_j)g(s'|\theta_j)[\theta_j - \mu(s')] = p(\theta_i)g(s'|\theta_i)[\mu(s') - \theta_i].$$

Hence,

$$\Delta_5 = \frac{p(\theta_i)\varepsilon g(s'|\theta_i) - (\Delta_3 - \Delta_4)p(\theta')[\mu(s') - \varepsilon - \theta']}{p(\theta_z)[\mu(s') - \varepsilon - \theta']}.$$

$$\hat{S}=\{S,\tilde{s}_j\}, \hat{g}(s'|\theta)=\begin{cases} g(s'|\theta)-\Delta \text{ if } \theta=\theta_j \\ \Delta_3-\Delta_4 \text{ if } \theta=\theta' \\ \Delta_5 \text{ if } \theta=\theta_z \\ g(s'|\theta) \text{ if } \theta\notin\{\theta_j,\theta',\theta_z\} \end{cases}, \hat{g}(s_0|\theta)=\begin{cases} g(s_0|\theta)-\Delta_5 \text{ if } \theta=\theta_z \\ g(s_0|\theta) \text{ if } \theta\neq\theta_z \end{cases},$$

$$\hat{g}(\tilde{s}_j|\theta)=\begin{cases} \Delta \text{ if } \theta=\theta_j \\ 0 \text{ if } \theta\neq\theta_j \end{cases}, \hat{g}(s''|\theta)=\begin{cases} g(s''|\theta)-\Delta_2 \text{ if } \theta=\theta_i \\ g(s''|\theta)+\Delta_4 \text{ if } \theta=\theta' \\ g(s''|\theta) \text{ if } \theta\notin\{\theta_i,\theta'\} \end{cases}, \hat{g}(s'''|\theta)=\begin{cases} g(s'''|\theta)+\Delta_2 \text{ if } \theta=\theta_i \\ g(s'''|\theta)-\Delta_3 \text{ if } \theta=\theta' \\ g(s'''|\theta) \text{ if } \theta\notin\{\theta_i,\theta'\} \end{cases},$$

and for $s\notin\{s',s_0,\tilde{s}_j,s'',s'''\}$, $\hat{g}(s|\theta)=g(s|\theta)$. If $\varepsilon$ is sufficiently small, $(\hat{S},\hat{g})$ is a disclosure rule. From (B-9), the expected cash flow conditional on score $s'$ is $\mu(s')-\varepsilon>\rho_j$. Hence, the price for score $s'$ is $\mu(s')-\varepsilon$. The price for score $\tilde{s}_j$ is $\Delta$. The prices for all other scores under $(\hat{S},\hat{g})$ are the same as under $(S,g)$. For score $s''$, this follows because $\Delta_2 p(\theta_\iota)[\mu(s'')-\theta_i]=\Delta_4 p(\theta')[\mu(s'')-\theta']$ and because $\theta'$ agrees to sell at price $\mu(s'')$. For $s'''$, this follows because $\Delta_2 p(\theta_\iota)[\mu(s''')-\theta_i]=\Delta_3 p(\theta')[\mu(s''')-\theta']$ and because $\theta_i$ agrees to sell at price $\mu(s''')$. Relative to $(S,g)$, under $(\hat{S},\hat{g})$, the payoff for type $\theta'$ falls by $\Delta_3[\mu(s''')-\mu(s'')]$, and the payoff for type $\theta_z$ increases by $\Delta_5[\mu(s')-\varepsilon-\theta_5+rF(1-\theta_5)]$, but if $\varepsilon$ is sufficiently small, monotonicity is preserved. The payoffs for all other types remain unchanged. For $\theta_i$ this follows because $\varepsilon g(s'|\theta_i)=\Delta_2[\mu(s''')-\mu(s'')]$. For $\theta_j$, this follows from (B-5). The value of the regulator's objective function increases because the probability of selling the asset weakly increases for every type and strictly increases for $\theta_z$.

**Proof of Lemma B-3.** Consider an optimal monotone rule $(S,g)$, and two types $\theta'<\theta''<1$, such that $h(\theta'')<1$. Since $E(\tilde{\theta})<1$, we know from Lemma B-4 that a type $\theta_z<1$ exists such that $h(\theta_z)<1$ and $u(\theta_z)<u(\theta_{z-1})$. It also follows immediately that all resource constraints are binding. Suppose that with a positive probability, $\theta''$ sells its asset at price $x$ upon obtaining score $s\in S_j$, and $\theta'$ sells at price $x'$ upon obtaining score $s'\in S_i$. From Lemma B-2, $x=\rho_j$ and $x'=\rho_i$. Suppose to the contrary that $x'<x$ (i.e., $\rho_i<\rho_j$). We obtain a contradiction to

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$\Delta_5>0$ follows because

$$\begin{aligned} & (\Delta_3-\Delta_4)p(\theta')[\mu(s')-\varepsilon-\theta']<\Delta_3 p(\theta')[\mu(s''')-\theta']-\Delta_4 p(\theta')[\mu(s'')-\theta'] \\ = \; & \Delta_2 p(\theta_i)[\mu(s''')-\theta_i]-\Delta_2 p(\theta_i)[\mu(s'')-\theta_i]=\Delta_2 p(\theta_i)[\mu(s''')-\mu(s'')]=p(\theta_i)\varepsilon g(s'|\theta_i)>0 \end{aligned}$$

the optimality of $(S, g)$ by constructing an alternate monotone rule that increases the value of the objective function. We construct the alternate rule in three steps:

Step 1. For type $\theta''$, reduce $h_j(\theta'')$ and increase $h_i(\theta'')$, both by a small $\Delta > 0$. For type $\theta'$, reduce $h_i(\theta')$ and increase $h_j(\theta')$, both by $\Delta_1 = \frac{p(\theta'')(\theta''-\rho_j)}{p(\theta')(\theta'-\rho_j)}\Delta > 0$. From the proof of Proposition 3, we know that resource constraint $i$ is loosened, while all other resource constraints remain binding. Increase $h_i(\theta_z)$ until resource constraint $i$ is binding again. Overall, after these changes, the value of the regulator's objective function increases. The payoff for $\theta_z$ increases, but if $\Delta$ is sufficiently small, the monotonicity constraint for type $\theta_z$ is preserved. However, because the expected payoff for $\theta''$ falls by $\Delta(\rho_j - \rho_i)$ and the payoff for $\theta'$ rises by $\Delta_1(\rho_j - \rho_i)$, the monotonicity constraint for these types may be violated. If so, proceed to step 2.

Step 2. Reduce $h_i(\theta')$ by $\frac{\Delta_1(\rho_j-\rho_i)}{\rho_i-\theta'+r\Pr(\tilde{\varepsilon}<1-\theta')}$ so that the expected payoff for $\theta'$ returns to where it was before step 1. This loosens constraint $i$. Increase $h_i(\theta'')$ as much as possible until either (i) resource constraint $i$ is binding again or (ii) the expected payoff for $\theta''$ returns to where it was before step 1. (Recall that $h(\theta'') < 1$.) If (ii) happens first and resource constraint $i$ remains loose, increase $h_i(\theta_z)$ until it is binding again. In this case, we are done because we created an alternate rule that increases the payoff for $\theta_z$, without violating monotonicity, and keeps the payoffs for all other types unchanged. If (i) happens first, move to step 3. In this case, we know that since $G_i(\theta)$ is increasing in $\theta$ when $\theta < 1$, the value of the objective function increases (using similar arguments as in the proof of Proposition 1).

Step 3. Increase the payoff for type $\theta''$ to where it was before step 1 by moving resources from the lowest type that sells to $\theta''$. Specifically, if the lowest type with $h(\theta) > 0$ is $\hat{\theta}$, we know that $h_l(\hat{\theta}) > 0$ for some $l \in \{1, ..., k\}$. Increase $h_l(\theta'')$ by $\Delta_2 = \frac{\Delta(\rho_j-\rho_i)}{\rho_l-\theta''+r\Pr(\tilde{\varepsilon}<1-\theta'')}$ and reduce $h_l(\hat{\theta})$ by $\frac{p(\theta'')(\rho_l-\theta'')}{p(\hat{\theta})(\rho_l-\hat{\theta})}\Delta_2$ so that resource constraint $l$ remains binding. Again, since $G_i(\theta)$ is increasing in $\theta$ when $\theta < 1$, the value of the objective function increases. So, overall, after all three steps, the value of the objective function increases. If $\Delta$ is sufficiently small, monotonicity is preserved because the payoff of $\theta_z$ has slightly increased, the payoff of the lowest type $\hat{\theta}$ has slightly fallen, and the payoffs of all other types have remain unchanged.

**Proof of Proposition B-1.** From Lemmas B-1 and B-2, type $\theta_1$ sells with probability 1, and the sale price is $\rho_1$. If the lowest type $\theta_m$ sells with probability $\alpha$, the monotonicity constraint (B-1) implies that type $\theta \in (\theta_m, \theta_2)$ sells with probability of at least $\delta_\theta(\alpha)$. Since $E(\tilde{\theta}) < \rho_1$ (i.e., resources are scarce) and $G(\theta)$ is decreasing in $\theta$ when $\theta < 1$, it is optimal that $\theta \in (\theta_m, \theta_2)$ sells with probability $\max\{0, \delta_\theta(\alpha)\}$. The optimal $\alpha$ satisfies the resource constraint with equality and is given by $\alpha^*$. Part (ii) follows because in the problem without constraint (B-1), the optimal rule involves a cutoff, such that types below the cutoff and types above 1 sell with probability 1, and types in the middle sell with probability 0.