Information Diversity and Complementarities in Trading and Information Acquisition

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Abstract

We analyze a model where different traders are informed of different fundamentals that affect the security value. We identify a source for strategic complementarities in trading and information acquisition: The aggressive trading on information about one fundamental reduces the uncertainty in trading on information about the other fundamental, encouraging more trading and information acquisition on that fundamental. This tends to amplify the effect of exogenous changes in the underlying information environment. Due to complementarities, greater diversity of information in the economy improves price informativeness. We discuss the relation between our model and recent financial phenomena and derive testable empirical implications.
Security prices reflect information available to traders about future payoffs. Uncertainty about these payoffs typically involves multiple dimensions. Obvious examples include multinational firms, for which there is uncertainty originating from the different countries where the firm operates, and conglomerates, for which there is uncertainty about the different industries the firm operates in. More generally, even focused firms are exposed to multiple dimensions of uncertainty: Cash flows depend on the demand for firms’ products and the technology that they develop; they depend on firms’ idiosyncratic developments and the way they are affected by the macroeconomy or the industry; they depend on the success of firms’ operations in traditional lines of business and in new speculative lines of business. In the modern world, information is so complex that traders tend to have a comparative advantage or specialize in different types of information. The price of the security is then based on the trading activities of the different types of informed traders, reflecting the overall expected value of the security given the information in the market.

A key question in understanding the workings of financial markets is what is the nature of interaction between the different types of informed traders. Suppose that people informed about one dimension of uncertainty trade more aggressively on their information or that there is more information produced on this dimension of uncertainty. Does this encourage people with expertise on other dimensions to trade more aggressively on their information and produce more information or does it deter them from doing so? If the former holds, then there is a strategic complementarity between the different groups in trading on their information and producing information. If the latter holds, then there is a strategic substitutability between these differentially informed groups. There is a long standing interest in the literature in understanding mechanisms for strategic complementarity vs. substitutability in financial markets, since complementarities are generally thought to create amplification of
shocks, whereas substitutabilities are thought to attenuate them.

In this paper, we study a model in the spirit of the seminal work of Grossman and Stiglitz (1980) to address these questions. We extend their setting to include two dimensions of uncertainty about the payoff from the traded security, say, the technology of the firm and the demand for its products. We first consider an economy where traders are endowed with different types of information; for example, some traders are informed about the technology and others are informed about the demand for the firm’s products. As in Grossman and Stiglitz (1980), traders are risk averse, and trade in a market with uninformed traders and with noisy supply. We then extend the model to endogenize the decision to produce information, and (in another extension) also allow traders to become informed about the two dimensions simultaneously.

Considering the trading stage, suppose that the size of the group of technology-informed traders increases, will this cause traders informed about product demand to trade more or less aggressively on their information? The presence of more technology-informed traders implies that more of their information will get into the price. When they trade, demand-informed traders condition on the price of the security (as is typically the case in the Grossman-Stiglitz framework), and hence the information in the price affects their trading decision. Then, the additional information about technology has two opposite effects on how demand-informed traders trade on their own information. First, the increased technology information in the price reduces the uncertainty that demand-informed traders have to face concerning technology issues when they trade, which causes demand-informed traders to trade more aggressively on what they know without being exposed to risks they do not understand. This is the “uncertainty reduction effect,” which favors strategic complementarities in trading on different types of information. Second, the additional information about technology
in the price also makes demand-informed traders use the price more to infer the level of the fundamental of technology. This will make demand-informed traders trade less aggressively on their own information. More specifically, suppose demand-informed traders have a positive signal about the demand for the firm’s product, which causes them to take a large long position. But, holding the price constant, when the price contains more information about technology, a strong demand fundamental implies a weak technology fundamental, and thus demand-informed traders will scale down their positions, trading less aggressively on their information. We call this effect the “inference augmentation effect,” and we note that it favors strategic substitutability in trading. Overall, when the uncertainty reduction effect dominates the inference augmentation effect, trading on one type of information is a complement to trading on the other type of information.

It is important to note that strategic complementarities do not arise naturally in most models of financial markets. In particular, the “uncertainty reduction effect” that we identify here as a source of strategic complementarities is muted in the traditional models in the literature that have one dimension of uncertainty (these models go back to Grossman and Stiglitz (1980) and Hellwig (1980)). Indeed, it appears that such an effect is consistent with many financial phenomena in recent years. For example, as the uncertainty about the macroeconomy and government policy was growing, commentators have argued that traders pull out of the market, attempting to limit their exposure to risks, which are not within their range of expertise. Similarly, traders, who traditionally trade on information concerning traditional aspects of the activities of financial institutions and other firms, might have become concerned that there are other dimensions that might be driving these institutions’ cash flows, such as their exposure to more exotic assets and to counterparty risks (as in the motivation for the recent paper by Caballero and Simsek (2013)). Since these are aspects
that these traders do not know much about, the uncertainty involved with trading the securities of these institutions became too large and they pulled out of the market, not even trading on the information they may have. Overall, as we highlight here, once it becomes more difficult to glean information on one dimension of uncertainty from the price, traders reduce their trading on information they may have involving other dimensions of uncertainty. This is the complementarity that might lead to large effects of changes in the underlying informational environment.

We show that the result of trading complementarities has important implications for market outcomes in our model. For example, in equilibrium, trading on the two types of information is complementary to each other when the trading intensity on one dimension of uncertainty is relatively close to the intensity on the other dimension of uncertainty. As a result, we find that the diversity of information in the economy improves the overall amount of information revealed by prices. That is, fixing the total mass of informed traders, an economy with a diverse information structure – i.e., where there is a greater balance between the two groups of informed traders – will exhibit a higher level of price informativeness than an economy with a concentrated information structure – i.e., where there are many more traders informed about one dimension than about the other.

We then extend our analysis to study the decision of traders on information acquisition. We start by allowing traders to become informed about one of the two dimensions of uncertainty. We again analyze the interaction between the two types of information. Suppose that there are more technology-informed traders in the market, what will be the effect on the incentives of agents to acquire information about demand? On the one hand, a variant of the traditional “Grossman-Stiglitz effect” exists in our model, reducing the incentive to produce information about demand when there are more technology-informed traders in the
market. On the other hand, the uncertainty reduction effect mentioned above creates a strategic complementarity in the information-acquisition stage as well: Knowing that more technology information will go into the price, traders know they will face less uncertainty when trading on demand information, and hence have a stronger incentive to produce information about product demand. We identify conditions under which this effect dominates, so that acquiring two types of information is complementary. Finally, we present an extension where traders can also acquire the two types of information simultaneously. Assuming a convex cost structure in information acquisition or that different agents have comparative advantage in acquiring different kinds of information, we show that our results do not change qualitatively, despite the much greater complexity in the analysis.

Our theory provides many empirical implications. We review some of them in Sections II.D. and III.D. These implications pertain to settings where it is natural to think of the firm’s activities as containing multiple dimensions of uncertainty. One can thus relatively easily test these implications in the context of multinational firms – analyzing the effect of changes in the information structure or investor base in one country on trading and information production in the other country – or firms operating in multiple industries. More generally, however, most firms are subject to multiple dimensions of uncertainty even if these are less easy to separate. As mentioned above, firms’ cash flows depend on technology and product demand; they depend on sensitivity to macroeconomic shocks and to idiosyncratic shocks. In these settings, it is natural to say that different types of investors specialize in different types of information, and then one can test our hypotheses using measures of the sizes of different investor bases and their trading activities. Overall, our theory highlights the importance of looking at the interaction across types of investors and types of information to understand the overall efficiency of the financial market in generating and processing information.
The remainder of this paper is organized as follows. Section I. presents the model and the characterization of the equilibrium in the trading stage. In Section II., we analyze the interaction between trading on the two types of information and provide a full characterization of when our model features complementarity vs. substitutability in trading. In this section, we also highlight our implications for the effect of information diversity. In Section III., we endogenize the information-acquisition decision and analyze when it will exhibit strategic substitutability vs. strategic complementarity. Section IV. presents an extension where traders can become informed about the two dimensions simultaneously. In Section V., we discuss the relation of our paper to the literature. Finally, Section VI. concludes.

I. The Model

A. Setup

There are two assets traded in the financial market: one riskless asset (bond) and one risky asset (stock). The bond is in unlimited supply; its payoff is 1, and its price is normalized to 1. The stock has a total supply of 1 unit; it has a price of \( \tilde{p} \), which is determined endogenously in the financial market, and its payoff \( \tilde{v} \) is given by:

\[
\tilde{v} = \tilde{v}_1 + \tilde{v}_2. \tag{1}
\]

As we see in (1), the payoff of the stock is composed of two ingredients, \( \tilde{v}_1 \) and \( \tilde{v}_2 \), sometimes referred to as fundamentals, which are independent and identically distributed (i.i.d.) according to a normal distribution function: \( \tilde{v}_i \sim N(0, 1/\rho) \) \((i = 1, 2)\), where \( \rho > 0 \) represents the common prior precision of \( \tilde{v}_1 \) and \( \tilde{v}_2 \). The idea is that there are two dimensions of uncertainty about the payoff from the stock, captured by the variables \( \tilde{v}_1 \) and \( \tilde{v}_2 \), and, as we will discuss below in more detail, they are potentially observable to different traders.\(^2\)
In the basic setup, described in this section and analyzed in the next section, there are three types of rational traders trading the bond and the stock in the financial market: (1) \( \tilde{v}_1 \)-informed traders (of mass \( \lambda_1 > 0 \)), who observe the realization of the first component \( \tilde{v}_1 \) of the stock payoff; (2) \( \tilde{v}_2 \)-informed traders (of mass \( \lambda_2 > 0 \)), who observe the realization of the second component \( \tilde{v}_2 \) of the stock payoff; and (3) uninformed traders (of mass \( \lambda_u > 0 \)), who do not observe any information. All three types of traders condition their trades on the stock price \( \tilde{p} \), as is typical in rational-expectations-equilibrium models (e.g., Grossman and Stiglitz (1980)). Their utility from consumption \( C \) is given by the usual constant-absolute-risk-aversion (CARA) function, \( e^{-\gamma C} \), where \( \gamma \) is the risk-aversion parameter. Finally, to prevent fully revealing prices, we assume that there are noise traders who trade a random amount \( \tilde{x} \sim N(0, \chi) \) (with \( \chi > 0 \)) of the stock, which is independent of the realizations of \( \tilde{v}_1 \) and \( \tilde{v}_2 \).

While we currently assume that the masses of informed traders, \( \lambda_1 \) and \( \lambda_2 \), are exogenous, we endogenize them later in Section III., where we analyze information-acquisition decisions and the learning complementarities that arise in our setup. Also, to deliver our message most effectively, in our basic setup, we assume that informed traders can only observe one ingredient of the stock payoff, i.e., they can be informed about \( \tilde{v}_1 \) or \( \tilde{v}_2 \) but not about both of them. As is pointed out by Paul (1993, p. 1477), such an assumption “is in the spirit of Hayek’s view that one of the most important functions of the price system is the decentralized aggregation of information and that no one person or institution can process all information relevant to pricing.” However, to demonstrate the robustness of our results, we consider an extension in Section IV., where traders are allowed to acquire information simultaneously about the two ingredients. We show that in the trading game our main results go through when there are some speculators in the market (of mass \( \lambda_{12} > 0 \)) who are informed about...
the two ingredients simultaneously, in addition to the masses $\lambda_1$ and $\lambda_2$ introduced above. Interestingly, considering the information-acquisition decision, we show that only in knife-edge cases, we will have strictly positive endogenous values of $\lambda_1$, $\lambda_2$, and $\lambda_{12}$ at the same time. Hence, our focus on the case where traders are informed either about $\tilde{v}_1$ or about $\tilde{v}_2$ is not only useful for simplicity and consistent with the spirit of Hayek’s view, but it is also a natural outcome of endogenous equilibrium behavior.

### B. Equilibrium Definition and Characterization

The equilibrium concept that we use is the rational-expectations equilibrium (REE), as in Grossman and Stiglitz (1980). In equilibrium, traders trade to maximize their expected utility given their information set, where $\tilde{v}_i$-informed traders know $\tilde{v}_i$ and $\tilde{p}$ ($i = 1, 2$), and uninformed traders know only $\tilde{p}$. The price $\tilde{p}$ is determined, in turn, by the market-clearing condition, whereby the sum of demands from the three types of rational traders and the noise traders is equal to the supply of the stock, which is equal to one. As in most of the literature, we consider a linear equilibrium, where the price $\tilde{p}$ linearly depends on the signals $\tilde{v}_1$ and $\tilde{v}_2$ and the noisy trading $\tilde{x}$:

$$\tilde{p} = \alpha_0 + \alpha_1 \tilde{v}_1 + \alpha_2 \tilde{v}_2 + \alpha_x \tilde{x}.$$  

The coefficients $\alpha_0$, $\alpha_1$, $\alpha_2$, and $\alpha_x$ will be endogenously determined.

As is well known, the CARA-normal setup assumed here implies that the demand function of a trader of type $t \in \{1, 2, u\}$ ($\tilde{v}_1$-informed, $\tilde{v}_2$-informed, and uninformed) is:

$$D_t(\mathcal{F}_t) = \frac{E(\tilde{v}_|\mathcal{F}_t) - \tilde{p}}{\gamma Var(\tilde{v}_|\mathcal{F}_t)},$$  

where $\mathcal{F}_t$ is the trader’s information set. Essentially, traders have a speculative motive to trade, which is reflected in the numerator of (3), according to which they buy (sell) the stock.
when its price is lower (higher) than the expected payoff. But, as we see in the denominator, speculators will trade less aggressively when they are exposed to a higher variance in the final payoff and when they are more risk averse. We can now construct \( E(\tilde{v}|\mathcal{F}_i) \) and \( Var(\tilde{v}|\mathcal{F}_i) \) for the different traders and plug them in the demand functions to solve for the equilibrium.

The \( \tilde{v}_i \)-informed traders have information set \( \mathcal{F}_i = \{ \tilde{p}, \tilde{v}_i \} \). Since they know the ingredient \( \tilde{v}_i \), they only need to forecast the other ingredient \( \tilde{v}_j \). Suppose that the coefficients in the price function (2) are different from 0 (this will be verified to be the case in equilibrium in the proof of Proposition 1). Then, the information set \( \mathcal{F}_i \) is equivalent to the following signal:

\[
\tilde{s}_{ji} \equiv \frac{\tilde{p} - \alpha_0 - \alpha_i \tilde{v}_i}{\alpha_j} = \tilde{v}_j + \frac{\alpha_x}{\alpha_j} \tilde{x}, \quad \text{for } i, j = 1, 2, j \neq i,
\]

which is a signal about \( \tilde{v}_j \) with normally distributed noise with precision \((\alpha_j/\alpha_x)^2 \chi\). Applying Bayes’ rule, we can compute the two conditional moments \( E(\tilde{v}_j|\mathcal{F}_i) \) and \( Var(\tilde{v}_j|\mathcal{F}_i) \) and determine the demand function \( D_i(\tilde{p}, \tilde{v}_i) \) of \( \tilde{v}_i \)-informed traders.

The uninformed traders only observe the price \( \tilde{p} \); that is, \( \mathcal{F}_u = \{ \tilde{p} \} \). The price \( \tilde{p} \) is equivalent to the following signal in predicting the total payoff \( \tilde{v} \):

\[
\tilde{s}_u \equiv \frac{\tilde{p} - \alpha_0}{\alpha_x} = \frac{\alpha_1}{\alpha_x} \tilde{v}_1 + \frac{\alpha_2}{\alpha_x} \tilde{v}_2 + \tilde{x}.
\]

We can then apply Bayes’ rule to compute \( E(\tilde{v}|\tilde{p}) \) and \( Var(\tilde{v}|\tilde{p}) \) and obtain uninformed traders’ demand function \( D_u(\tilde{p}) \).

The equilibrium price is determined by the market-clearing condition for the risky asset:

\[
\lambda_1 D_1(\tilde{p}, \tilde{v}_1) + \lambda_2 D_2(\tilde{p}, \tilde{v}_2) + \lambda_u D_u(\tilde{p}) + \tilde{x} = 1.
\]

Plugging the expressions for \( D_i(\tilde{p}, \tilde{v}_i) \) and \( D_u(\tilde{p}) \) into the above market-clearing condition, we can solve for the price \( \tilde{p} \) as a function of the variables \( \tilde{v}_1, \tilde{v}_2 \) and \( \tilde{x} \). Then, comparing coefficients with those in the conjectured price function in (2), we get the following proposition that characterizes the linear rational-expectations equilibrium (with the proof included.
in the appendix).

**PROPOSITION 1:** For any $\lambda_1 > 0$ and $\lambda_2 > 0$, there exists a unique linear rational-expectations equilibrium, in which

$$\hat{p} = \alpha_0 + \alpha_1 \hat{v}_1 + \alpha_2 \hat{v}_2 + \alpha_x \hat{\epsilon}.$$  

The coefficients $\alpha_0 < 0$, $\alpha_1 > 0$, $\alpha_2 > 0$, and $\alpha_x > 0$ are given as a function of the exogenous parameters of the model in the proof in the appendix.

## II. Trading Intensities and Price Informativeness

### A. Definitions and Characterization

In our model, there are two types of fundamentals, $\hat{v}_1$ and $\hat{v}_2$. The focus of our analysis is on how aggressively traders trade on information about these fundamentals. This will eventually determine the level of price informativeness. A unit increase in $\hat{v}_i$ will cause a $\hat{v}_i$-informed trader to buy $\frac{\partial D_i(\hat{p}, \hat{v}_i)}{\partial \hat{v}_i}$ more stocks, and so as a group $\hat{v}_i$-informed traders will buy $\lambda_i \frac{\partial D_i(\hat{p}, \hat{v}_i)}{\partial \hat{v}_i}$ more stocks. We accordingly use this amount to represent the aggregate trading intensity $I_i$ on information $\hat{v}_i$, that is,

$$I_i \equiv \lambda_i \frac{\partial D_i(\hat{p}, \hat{v}_i)}{\partial \hat{v}_i}, \text{ for } i = 1, 2. \quad (7)$$

We use $\frac{1}{\text{Var}(\hat{v}|\hat{p})}$, the reciprocal of the variance of $\hat{v}$ conditional on the price, to measure the price informativeness about the payoff $\hat{v}$. That is, we define

$$\text{price informativeness} \equiv \frac{1}{\text{Var}(\hat{v}|\hat{p})}. \quad (8)$$

This also corresponds to how much residual uncertainty is faced by the uninformed traders after conditioning on the price.

In equilibrium, trading intensity $I_i$ affects price informativeness through its impact on
the price function. Specifically, following Bond and Goldstein (2011), we show that $I_i$ is equal to the ratio $\frac{\alpha_i}{\alpha_x}$ between the sensitivity of the price to fundamental shocks $\tilde{v}_i$ and the sensitivity of the price to noise $\tilde{x}$:

$$I_i = \lambda_i \frac{\partial D_i(\tilde{p}, \tilde{v}_i)}{\partial \tilde{v}_i} = \frac{\alpha_i}{\alpha_x}, \text{ for } i = 1, 2. \tag{9}$$

This expression is intuitive. Trading intensity on the fundamental $\tilde{v}_i$ is defined by the extent to which a change in this fundamental affects the overall demand for the stock. It is this intensity that brings the information about $\tilde{v}_i$ to the market and determines the extent to which the price reflects information about this fundamental vs. just noise.

We now link the trading intensities to overall price informativeness. From (5), we can compute $\text{Var}(\tilde{v}|\tilde{p})$. Then, using (9), we find that the price informativeness $\frac{1}{\text{Var}(\tilde{v}|\tilde{p})}$ depends on the trading intensities $I_1$ and $I_2$ as follows:

$$\frac{1}{\text{Var}(\tilde{v}|\tilde{p})} = \frac{I_1^2 \rho + I_2^2 \rho + \rho^2 \chi^{-1}}{(I_1 - I_2)^2 + 2 \rho \chi^{-1}}. \tag{10}$$

Note that $I_1$ and $I_2$ positively affect the price informativeness $\frac{1}{\text{Var}(\tilde{v}|\tilde{p})}$. That is, $\frac{\partial \text{Var}^{-1}(\tilde{v}|\tilde{p})}{\partial I_1} > 0$ and $\frac{\partial \text{Var}^{-1}(\tilde{v}|\tilde{p})}{\partial I_2} > 0$ (see equations (A13) and (A14) in the appendix). This result is intuitive, as one would expect that the price system will reveal more information about the total asset payoff ($\tilde{v}_1 + \tilde{v}_2$) when traders trade more aggressively on their information about the underlying fundamentals.

**B. Trading Complementarity vs. Substitutability**

We now examine how the two trading intensities interact in our model. Specifically, we will derive two best response functions $I_i = h_i(I_j; \lambda_i, \rho, \gamma, \chi)$ (for $i = 1, 2$), which will jointly determine the two trading intensities $I_1$ and $I_2$ in equilibrium. We are interested in the slope of the response functions. If $h_i(I_j; \lambda_i, \rho, \gamma, \chi)$ is increasing in $I_j$, then we say
that trading on information $\tilde{v}_i$ is a *complement* to trading on information $\tilde{v}_j$, because in this case, $\tilde{v}_i$-informed traders trade more aggressively on their information about $\tilde{v}_i$ when $\tilde{v}_j$-informed traders trade more aggressively on $\tilde{v}_j$. Similarly, if $h_i(I_j; \lambda_i, \rho, \gamma, \chi)$ is decreasing in $I_j$, then trading on information $\tilde{v}_i$ is a *substitute* to trading on information $\tilde{v}_j$. Studying these concepts of trading complementarity/substitutability will have important implications for the workings of financial markets, as we will discuss in the next subsection.

The demand function of $\tilde{v}_i$-informed traders is given by

$$D_i(\tilde{p}, \tilde{v}_i) = \frac{E(\tilde{v}_j|F_i) - \tilde{p}}{\gamma \text{Var}(\tilde{v}_j|F_i)}.$$

We can now use the definition of trading intensity $I_i$ in (9) to get:

$$I_i \equiv \frac{\partial D_i(\tilde{p}, \tilde{v}_i)}{\partial \tilde{v}_i} = \frac{\lambda_i}{\gamma} \frac{1}{\text{Var}(\tilde{v}_j|F_i)} + \frac{\partial}{\partial \tilde{v}_i} \frac{\lambda_i E(\tilde{v}_j|F_i)}{\gamma \text{Var}(\tilde{v}_j|F_i)}.$$

Using the signal observed by $\tilde{v}_i$-informed traders in (4) and applying Bayes’ rule, we can compute the conditional moments, $E(\tilde{v}_j|F_i)$ and $\text{Var}(\tilde{v}_j|F_i)$, and express $I_i$ as follows (recall that $I_i = \frac{\alpha_i}{\alpha_x}$):

$$I_i = \lambda_i \gamma^{-1}(\rho + \chi I_j^2) - \lambda_i \gamma^{-1} \chi I_i I_j.$$

We can see that an increase in the signal $\tilde{v}_i$ affects the $\tilde{v}_i$-informed traders’ demand in two ways. First, there is a direct effect, according to which an increase in $\tilde{v}_i$ implies a direct increase in the payoff from the stock, leading the $\tilde{v}_i$-informed traders to increase their demand for a given price. This positive effect is represented by the first term on the right-hand side of (11) and (12). Second, there is an indirect effect working through the inference that the $\tilde{v}_i$-informed traders can make about $\tilde{v}_j$: Holding the price constant, an increase in $\tilde{v}_i$ implies a lower expectation about $\tilde{v}_j$, and so reduces the traders’ demand for the stock. This negative effect is captured by the second term on the right-hand side of (11) and (12).

We can now rearrange terms in (12) and express $I_i$ as a best-response function to $I_j$:

$$I_i = h_i(I_j; \lambda_i, \rho, \gamma, \chi) \equiv \frac{\lambda_i (\rho + \chi I_j^2)}{\gamma + \lambda_i \chi I_j}, \text{ for } i, j = 1, 2 \text{ and } j \neq i.$$
To see more clearly the ways in which $I_j$ affects $I_i$ and their relation to the direct and indirect effects in (12), it is more useful to write this function as follows:

$$I_i = \lambda_i \gamma^{-1}(\rho + \chi I_j^2) - \lambda_i \gamma^{-1}(\rho + \chi I_i^2) \frac{\lambda_i \gamma^{-1}\chi I_j}{1 + \lambda_i \gamma^{-1}\chi I_i} \equiv I_{i,\text{Direct}}(I_j) - I_{i,\text{Indirect}}(I_j).$$  \hfill (14)

Now, we can study the ways that trading intensity $I_j$ influences trading intensity $I_i$.

First, an increase in $I_j$ strengthens the positive direct effect that the signal $\tilde{v}_i$ has on the demand of $\tilde{v}_i$-informed traders, i.e., it increases $I_{i,\text{Direct}}(I_j)$. This is because a higher level of $I_j$ implies that there is more information about $\tilde{v}_j$ in the price and this reduces the residual uncertainty $\text{Var}(\tilde{v}_j|\mathcal{F}_i)$ that $\tilde{v}_i$-informed traders are exposed to when they trade, and hence makes them trade more aggressively on their information about $\tilde{v}_i$. We label this effect the “uncertainty reduction effect,” that is, we define:

$$\text{uncertainty reduction effect} \equiv \frac{\partial I_{i,\text{Direct}}(I_j)}{\partial I_j} > 0.$$  \hfill (15)

Second, an increase in $I_j$ also strengthens the negative indirect effect in the expression for $I_i$, i.e., it increases $I_{i,\text{Indirect}}(I_j)$. Recall that the second term on the right-hand side of (12) (which is the origin of $I_{i,\text{Indirect}}(I_j)$) captures the fact that an increase in $\tilde{v}_i$, holding the price constant, provides indication that $\tilde{v}_j$ is lower, and this makes the traders demand less of the stock. This implies a reduction in trading intensity $I_i$. This force becomes stronger as trading intensity $I_j$ increases. This is because when the price is more sensitive to $\tilde{v}_j$, $\tilde{v}_i$-informed traders use the price more to infer information about $\tilde{v}_j$ and so an increase in $\tilde{v}_i$ with a fixed price provides a stronger negative indication about the realization of $\tilde{v}_j$. We label this effect the “inference augmentation effect” because it captures the effect of trading intensity $I_j$ on trading intensity $I_i$ via the effect of $I_j$ on the ability of $\tilde{v}_i$-informed traders to make an inference from the price about the signal they do not know. That is, we define:

$$\text{inference augmentation effect} \equiv \frac{\partial I_{i,\text{Indirect}}(I_j)}{\partial I_j} > 0.$$  \hfill (16)
Equations (14), (15) and (16) imply that the slope of the best response function $h_i$ is jointly determined by the uncertainty reduction and inference augmentation effects:

$$\frac{\partial h_i(I_j; \lambda_i, \rho, \gamma, \chi)}{\partial I_j} = \text{uncertainty reduction effect} - \text{inference augmentation effect.} \quad (17)$$

The uncertainty reduction effect generates strategic complementarity in trading: When $\tilde{v}_j$-informed traders trade more aggressively on $\tilde{v}_j$, there is more information about $\tilde{v}_j$ in the price, $\tilde{v}_i$-informed traders face lower uncertainty about what they do not know, and so trade more aggressively on their information. The inference augmentation effect generates strategic substitutability in trading: When $\tilde{v}_j$-informed traders trade more aggressively on $\tilde{v}_j$ and the price becomes more informative about $\tilde{v}_j$, $\tilde{v}_i$-informed traders use the price to update more about what they do not know, and because this inference is in opposite direction to their signal, they trade less aggressively on their information. Based on the relative strength of these two effects, the best response function may have either a positive or a negative slope: Either strategic complementarity or strategic substitutability dominates. Analyzing (14), (15), and (16), we can see that $h_i(I_j; \lambda_i, \rho, \gamma, \chi)$ is decreasing in $I_j$ when $I_j$ is low, and then increasing when $I_j$ is high. Formally, straightforward calculations yield the following proposition.

**PROPOSITION 2:** Trading on information $\tilde{v}_i$ is a complement (substitute) to trading on information $\tilde{v}_j$ if $I_j$ is sufficiently large (small). That is, $\frac{\partial h_i(I_j; \lambda_i, \rho, \gamma, \chi)}{\partial I_j} > 0$ if and only if $I_j > \frac{\sqrt{\gamma^2 + \lambda_i^2 \rho \chi - \gamma}}{\lambda_i^2 \chi \lambda_i}$. 

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C. Implications of the Interaction between the Two Trading Intensities

C.1. Trading Intensity Multipliers

So far, we discussed how trading intensities $I_1$ and $I_2$ are determined and how they interact with each other. In particular, we discussed when our model features strategic complementarity vs. substitutability, that is, when an increase in trading intensity on one type of information provides incentives for traders to trade more or less aggressively on the other type of information. In this subsection, we discuss the implications of the two trading intensities being complements or substitutes. We start by introducing the concept of “trading intensity multiplier,” which is generated by the interaction between $I_1$ and $I_2$.

The trading intensities $I_1$ and $I_2$ are determined by the system of equations in (13) as a function of the underlying parameters of the model. Changes in these parameters affect $I_1$ and $I_2$, and the magnitude of this effect depends on the multiplier. Formally, let $Q$ be one of the five exogenous parameters ($\lambda_1, \lambda_2, \rho, \gamma, \chi$) that determine $(I_1, I_2)$, then we have the following proposition (the proof is provided in the appendix).

**PROPOSITION 3:** (a) The effect of an exogenous parameter $Q$ on the trading intensity $I_i$ about $\tilde{v}_i$ is given by:

$$\frac{dI_i}{dQ} = M \left( \frac{\partial h_i}{\partial Q} + \frac{\partial h_i}{\partial I_j} \frac{\partial I_j}{\partial Q} \right),$$

where the term $\left( \frac{\partial h_i}{\partial Q} + \frac{\partial h_i}{\partial I_j} \frac{\partial I_j}{\partial Q} \right)$ captures the “direct effect” of changing $Q$ on $I_i$ and the coefficient $M$ is a “multiplier” given by:

$$M = \left( 1 - \frac{\partial h_1}{\partial I_2} \frac{\partial h_2}{\partial I_1} \right)^{-1} > 0.$$  

(b) Suppose that $\lambda_1 > 0$ and $\lambda_2 > 0$. Then, (i) when $\frac{1}{2} < \frac{I_1}{I_2} < 2$, $M > 1$, and so the effect of $Q$ on $I_i$ is amplified in equilibrium; (ii) when $\frac{I_1}{I_2} < \frac{1}{2}$ or when $\frac{I_1}{I_2} > 2$, $0 < M < 1$, and
so the effect of $Q$ on $I_i$ is attenuated in equilibrium; and (iii) when $\frac{I_1}{I_2} = \frac{1}{2}$ or when $\frac{I_1}{I_2} = 2$, $\mathcal{M} = 1$.

The direct effect of a change in a parameter $Q$ on trading intensity $I_i$ can be thought of as the initial impact before the interdependence between the two trading intensity measures is considered. It is given by $\left(\frac{\partial h_i}{\partial Q} + \frac{\partial h_i}{\partial I_j} \frac{\partial h_j}{\partial Q}\right)$. The interdependence then creates a multiplier that either amplifies or attenuates the direct effect in equilibrium. It is given by $\mathcal{M} = \left(1 - \frac{\partial h_1}{\partial I_2} \frac{\partial h_2}{\partial I_1}\right)^{-1}$, which, as the proof of the proposition shows, is strictly positive. We therefore label $\mathcal{M}$ the “trading intensity multiplier.”

Whether the direct effect is attenuated or amplified in equilibrium then depends on whether $\mathcal{M}$ is smaller than or bigger than 1, which in turn depends on the signs of the cross derivatives $\frac{\partial h_1}{\partial I_2}$ and $\frac{\partial h_2}{\partial I_1}$ (i.e., on whether trading on the two types of information is a complement or a substitute). Recall that, by Proposition 2, the two response functions in (13) start decreasing and then increasing. So, when $I_1$ and $I_2$ are very far from each other (specifically, when $\frac{I_1}{I_2} < \frac{1}{2}$ or when $\frac{I_1}{I_2} > 2$), one of the cross derivatives is positive, while the other is negative. As a result, the interaction between the two trading intensity measures tends to attenuate the initial effect; that is, $0 < \mathcal{M} < 1$. In contrast, when $I_1$ and $I_2$ are close to each other, both response functions are upward sloping so that trading on one type of information is a complement to the other. Hence, $\mathcal{M} > 1$, so the two trading intensity measures reinforce each other and the initial effect due to a change in exogenous parameters will get amplified in equilibrium.

C.2. The Effect of an Increase in $\lambda_i$

To illustrate the effects described above, let us consider the effect of an increase in one parameter of the model – the size $\lambda_i$ of the $\tilde{v}_i$-informed traders population – on the two
trading intensities in equilibrium. An increase in $\lambda_i$ can reflect a positive change in the number of traders, who are experts in one dimension of the firm’s activities and trade in the firm’s share. In the next subsection, we discuss possible real-world interpretations in more detail. Note that here we assume that the increase in $\lambda_i$ is exogenous, and study the trading implications for a fixed $\lambda_j$. In Section III., where we endogenize information acquisition, we will show that an increase in $\lambda_i$ can also lead to an increase in $\lambda_j$, resulting in an additional effect.

By Part (a) of Proposition 3, we can express the effect of the increase in $\lambda_i$ on the equilibrium levels of $I_i$ and $I_j$ as follows:

\[
\frac{dI_i}{d\lambda_i} = M \frac{\partial h_i}{\partial \lambda_i},
\]

\[
\frac{dI_j}{d\lambda_i} = M \frac{\partial h_j}{\partial I_i} \frac{\partial h_i}{\partial \lambda_i} = \frac{\partial h_j}{\partial I_i} dI_i.
\]

By the expression of $h_i$ in (13), we can easily see that $\frac{\partial h_i}{\partial \lambda_i}$ is positive.

Hence, we can reach the following conclusions regarding the effect of the increase in $\lambda_i$: First, an increase in $\lambda_i$ always increases the trading intensity on information $\tilde{v}_i$. This is intuitive since when more people are informed about $\tilde{v}_i$, the aggregate increase in demand following an improvement in $\tilde{v}_i$ will be larger.

Second, the effect of an increase in $\lambda_i$ on the trading intensity on information $\tilde{v}_i$ is greater when the multiplier $M$ is higher. In particular, the direct effect is attenuated when $M$ is below 1 and amplified when $M$ is above 1. This is where the role of the interaction between the two trading intensities comes into play. As we saw in Proposition 3, when $I_1$ and $I_2$ are relatively close to each other, the increase in one leads to an increase in the other and so on, so that $M$ is above 1. Hence, the direct effect of $\lambda_i$ on $I_i$ gets amplified, leading to a much stronger overall effect.

Third, an increase in $\lambda_i$ has an ambiguous effect on the trading intensity on information
\( \tilde{v}_j \). The sign of this effect is pinned down by the sign of \( \frac{\partial h_j}{\partial I_i} \), i.e., by whether trading on information \( \tilde{v}_j \) is a complement or a substitute to trading on information \( \tilde{v}_i \). From Proposition 2, we know that \( \frac{\partial h_j}{\partial I_i} > 0 \) if and only if \( I_i \) is sufficiently large, which will be true when \( \lambda_i \) is sufficiently large (this statement is formally proved in the appendix). Hence, when the mass of traders informed about \( \tilde{v}_i \) is sufficiently large, then an increase in this mass will lead to an increase in trading intensity about \( \tilde{v}_j \); otherwise the effect is opposite.

We summarize the above discussion in the following corollary.

**COROLLARY 1:** An increase in the size \( \lambda_i \) of the \( \tilde{v}_i \)-informed traders population,

(a) increases the trading intensity \( I_i \) on information \( \tilde{v}_i \) (i.e., \( \frac{dI_i}{d\lambda_i} > 0 \)); and the magnitude of this increase is higher when the multiplier \( M \) is larger;

(b) increases the trading intensity \( I_j \) on information \( \tilde{v}_j \) if and only if trading on \( \tilde{v}_j \) is a complement to trading on \( \tilde{v}_i \) (i.e., \( \frac{dI_j}{d\lambda_i} > 0 \) iff \( \frac{\partial h_i}{\partial I_i} > 0 \) or iff \( I_i > \frac{1}{2} I_j \)), which is true when \( \lambda_i \) is sufficiently large.

**C.3. The Effect of Information Diversity**

The interaction between the two types of informed traders and its effect on the multiplier have interesting implications for the effect of information diversity in our model. We now explore these implications.

Recall that the mass of traders informed about \( \tilde{v}_1 \) is \( \lambda_1 \) and the mass of traders informed about \( \tilde{v}_2 \) is \( \lambda_2 \). Information diversity is a function of the difference between \( \lambda_1 \) and \( \lambda_2 \) (for a fixed total size of the informed-traders population). Formally, we set \( \lambda_1 + \lambda_2 = \Lambda \) (where \( \Lambda \) is a constant), and define the following measure of information diversity:

\[
\Delta \equiv 1 - \frac{|\lambda_1 - \lambda_2|}{\Lambda} \in [0, 1].
\]  

A higher \( \Delta \) means that the two groups of informed traders are closer in size, and so the total
amount of information is more equally distributed between the two types of informed traders; hence, there is more diversity of information in the economy. By this logic, a situation with less diversity is one where most people know the same thing and so $\Delta$ is low.

To see the effect of diversity, note that when $\Delta$ is close to 1, $\lambda_1$ is close to $\lambda_2$, and so $I_1$ and $I_2$ are close to each other. Then, by Proposition 3, the trading intensity multiplier $M$ is greater than 1. On the other hand, when $\Delta$ is close to 0, either $\lambda_1$ or $\lambda_2$ is close to 0, and so $I_1$ and $I_2$ are far from each other, implying that the trading intensity multiplier $M$ is smaller than 1. This link between $\Delta$ and $M$ has important implications for price informativeness. Specifically, the following corollary shows that as information diversity increases, the price informativeness $\frac{1}{\text{Var}(\hat{v}|\hat{p})}$ goes up (the proof is included in the appendix).

**COROLLARY 2:** When the total size of informed traders population is fixed, information diversity, defined by $\Delta$ in (22), has a positive effect on the informativeness of the price regarding $\hat{v}$, that is, $\frac{d\text{Var}^{-1}(\hat{v}|\hat{p})}{d\Delta} > 0$.

To understand this result, compare the following two economies with the same total mass of informed traders $\lambda_1 + \lambda_2 = \Lambda$, but with levels of diversity at the two ends of the spectrum: (1) Economy I, where $\lambda_1 = \Lambda - \varepsilon \approx \Lambda$, $\lambda_2 = \varepsilon \approx 0$, and $\Delta \approx 0$ (a “concentrated” economy); and (2) Economy II, where $\lambda_1 = \lambda_2 = \frac{\Lambda}{2}$ and $\Delta = 1$ (a “diverse” economy). These two economies can be obtained by injecting a total mass $(\Lambda - 2\varepsilon)$ of informed traders to an initial economy where there is almost no information (i.e., $\lambda_1 = \lambda_2 = \varepsilon \approx 0$) along two different paths. To obtain Economy I, we only add traders who are informed about $\hat{v}_1$, while keeping the mass of agents informed about $\hat{v}_2$ close to 0. In contrast, to obtain Economy II, we simultaneously add traders informed about $\hat{v}_1$ and traders informed about $\hat{v}_2$.

Adding informed traders along both paths improves price informativeness, since $\frac{d\text{Var}^{-1}(\hat{v}|\hat{p})}{d\lambda_i} > 0$ for $i = 1, 2$ (see equation (A19) in the appendix). However, the impact of the new infor-
information is different on the two different paths leading to the two economies because of the trading-intensity-multiplier effect identified by Proposition 3. Along the path to obtain Economy I, the multiplier $M$ is smaller than 1, and thus the impact of the new added information is attenuated, while along the path to obtain Economy II, the multiplier $M$ is greater than 1, and the impact of the new added information is amplified. As a result, the total impact of the added mass $(\Lambda - 2\varepsilon)$ of informed traders on price informativeness $\frac{1}{\text{Var}(\hat{v} \mid \hat{p})}$ is larger in Economy II than in Economy I.

Overall, price informativeness is higher in our model when there is more diversity of information, or when there is more balance between the amount of information available on different dimensions. This is because the effect of adding more informed agents on price informativeness is greater when the two trading intensities are relatively close to each other, as then the uncertainty reduction effect dominates and trading on the two types of information is complementary, so that both types of traders trade more aggressively, impounding more information into the price and increasing price informativeness.

D. Empirical Implications

Corollaries 1 and 2 present hypotheses that can be tested empirically. There are many settings to which our model potentially applies, in which empirical researchers can test these hypotheses. In this subsection, we describe some of these settings and link them more directly to the results in the two corollaries in order to highlight how they can be potentially tested.

At the basis of our model is the idea that there are two (or more) dimensions of uncertainty that affect a firm’s cash flows. There are many settings in which this is naturally the case. One example is a multinational firm: The firm operates in several countries and its cash flow depends on developments of different countries. Then, investors operating in different
countries are more likely to be informed about the aspects involving their own country, and our model with heterogeneously informed traders follows directly. Another example is a conglomerate: This is a large firm that operates across different industries or business lines. There are different investors who specialize in information about the different industries, and again our model follows directly.

More generally, however, there are many more cases where uncertainty about the firm comes from various sources: Firms’ cash flows depend on the demand for their products and the technology that they develop; firms’ cash flows depend on their own idiosyncratic developments and the way they are affected by the macroeconomy or the industry; firms’ cash flows depend on the success of their operations in traditional lines of business and new speculative lines of business. In these cases and others, it is very reasonable to assume that the population of investors investing in the firm is not homogeneous, but rather contains different subsets that specialize in different dimensions of uncertainty.

Testing the results in Corollaries 1 and 2 requires measures of the size of the investor base that specializes in each dimension of uncertainty. In the context of a multinational firm, these measures can be obtained based on data on the size of the investor base in different countries. While not all investors who are based in a given country will be informed about the developments of the firm pertaining to that country, the large literature on home bias suggests that investors are more likely to be informed of the elements closer to their geographical and/or cultural backgrounds, and so the (relative) sizes of investor bases in different countries may provide a good approximation for the (relative) sizes of groups informed about different fundamentals. Moreover, to get a more direct measure of the populations of informed traders, one may look at analyst coverage or institutional holdings in different countries, since the literature has suggested that analysts and institutions are indeed in-
formed (e.g., Dennis and Strickland (2002)). In the case of a conglomerate, tracking the
different populations of informed traders might be more challenging, since it is not immedi-
ately clear how many investors specialize in one industry vs. another. One clearer case is
following a merger between two firms. The uncertainty about the newly merged firm’s cash
flows comes from the lines of business of the two original firms, and one can proxy for the
size of different investor bases on the basis of data on investors (or, alternatively, analyst
coverage or institutional holdings) in the two original firms. More generally, for other cases
of multidimensional uncertainty, one can proxy for the differences in sizes of different groups
of informed agents based on the composition of the investors’ base. For example, to the
extent that institutional investors and individual investors are likely to be informed about
different aspects of the firm’s value, one can look at the relative size of these two groups of
investors and its implications for price informativeness. Alternatively, one could argue that
only institutional investors are likely to be informed, in which case the heterogeneity within
the base of institutional investors can help to gauge the relative sizes of different groups of
informed investors. Indeed, it is well known that different institutions have different styles
of investment (e.g., Fung and Hsieh (1997), Chan, Chen, and Lakonishok (2002)), and so
are likely to be informed of different aspects of the firm’s value.

With these proxies for the sizes of different investor bases, one can then test the hypothe-
ses presented in the previous subsection. Corollary 2 presents a very clear hypothesis: The
informativeness of the price about firm cash flows will increase when the investor base is
more balanced (information is more diverse). That is, typical measures of price informativeness,
such as the price nonsynchronicity measure in Morck, Yeung, and Yu (2000) or the
VAR-approach based measure in Hasbrouck (1991), will increase when the sizes of different
investor bases are closer to each other. Depending on the application, measures of the size of
investor bases will be based on investors in different countries (for the multinational firm), investors originating from different original firms (for the newly merged entity), or more generally different classes of investors (institutional investors employing different styles of investment).

In Corollary 1, the hypotheses are a bit more subtle. One would like to know how an increase in the size of one group affects the informativeness on different dimensions of information or the trading intensity of the two groups. In the case of multinational firms, the question is whether an increase in the investor base in one country affects the sensitivity of the price to innovations in that country and in another country. In other cases, a more indirect test would be to look at the effect of an increase in the size of one investor base on the overall trading activity of that base of investors (Part (a) of the corollary) and of the other base of investors (Part (b) of the corollary). As for Part (a), we would expect that an increase in the size $\lambda_i$ of one group of investors will increase the trading intensity $I_i$ of that group of investors, which can be gauged by looking at their trading activities, such as order flows or turnovers. Moreover, we expect that the overall effect will be stronger when the multiplier $M$, characterized by equation (19), is larger. For Part (b), we have a very sharp hypothesis: An increase in the size $\lambda_i$ of one investor base increases the trading intensity $I_j$ of the other base of investors if and only if $I_i > \frac{1}{2} I_j$, a condition that can be pinned down by comparing the trading activities (such as order flows or turnovers) of the two investor bases in the data.

For the above tests, it is important to note that our paper makes the simplifying assumption that the two fundamentals $\tilde{v}_1$ and $\tilde{v}_2$ are symmetric in the sense that they have the same unconditional variance. Clearly, this assumption will not hold perfectly in reality, and was made to deliver the theoretical insights in the most transparent way. Overall, the driving
forces of our results are not driven by this assumption, and so we expect our main predictions to be qualitatively similar even if the two fundamentals have different variances. But, for empirical testing, it is important to keep this feature of the model in mind and adjust the tests accordingly. Consider, for example, the variable $\Delta$ in Corollary 2: if $\text{Var}(\tilde{v}_1) > \text{Var}(\tilde{v}_2)$, then increasing $\lambda_1$ and decreasing $\lambda_2$ by the same amount will have direct effects on informativeness beyond the effects of diversity that we studied in the paper, since it implies that the two groups of informed traders in aggregate know more about the overall asset payoff. To address this issue in empirical testing, it is thus important to normalize the sizes of the informed traders populations by the unconditional variances of the different fundamentals.

Finally, an alternative avenue for empirical tests, instead of testing Corollaries 1 and 2, is to test the basic idea of the “uncertainty reduction effect” directly in the data. This effect generates complementarities in our model and its essential spirit is that informed traders would choose to trade more (less) aggressively when facing less (more) uncertainty. For example, one can test whether the arrival of policy-uncertainty or macroeconomic shocks cause traders, who do not specialize in this type of information, to face more uncertainty and hence scale down their trading. As mentioned in the introduction, casual observations suggest that this force has been at work in the crisis and its aftermath.

III. Endogenous Information Acquisition

So far, we assumed that the masses of agents who are informed about the two fundamentals $\tilde{v}_1$ and $\tilde{v}_2 - \lambda_1$ and $\lambda_2$, respectively – were exogenous. We now endogenize these parameters and examine how they are determined in light of the incentives to become informed in our model. The new interesting result that we get relative to the literature is that
sometimes there will be a dominant strategic complementarity in information acquisition, whereby the increase in the mass of agents acquiring information on one fundamental will lead more agents to acquire information on the other fundamental. This is because of the uncertainty reduction effect identified earlier.

A. Information Acquisition in Equilibrium

We assume that traders can acquire the signal $\tilde{v}_1$ at cost $c_1 > 0$, and the signal $\tilde{v}_2$ at cost $c_2 > 0$. Traders who choose to acquire $\tilde{v}_1$ ($\tilde{v}_2$) become part of the $\lambda_1$ ($\lambda_2$) group in the trading model described in previous sections, while those who choose not to acquire information become part of the $\lambda_u$ group. For now, we assume that any trader has an opportunity to become informed only about one of the two fundamentals. In Section IV., we present an extension that allows traders to become informed about both fundamentals at the same time, and show that our main results go through. Moreover, it turns out that only in knife-edge cases, we will have strictly positive masses of agents acquiring information about $\tilde{v}_1$, acquiring information about $\tilde{v}_2$, and acquiring information about $\tilde{v}_1$ and $\tilde{v}_2$, at the same time. Hence, our focus on the case where traders are informed either about $\tilde{v}_1$ or about $\tilde{v}_2$ is natural.

We assume that the overall mass of rational traders ($\lambda_1 + \lambda_2 + \lambda_u$) is very large. We make this assumption for simplicity to ensure that there will always be some traders who decide to stay uninformed in equilibrium (i.e., $\lambda_u > 0$). As a result, we do not have to consider the corner scenarios where all traders become informed of either $\tilde{v}_1$ or $\tilde{v}_2$; we only need to consider four possible cases of information market equilibrium: $(\lambda_1 = \lambda_2 = 0)$, $(\lambda_1 > 0, \lambda_2 = 0)$, $(\lambda_1 = 0, \lambda_2 > 0)$, and $(\lambda_1 > 0, \lambda_2 > 0)$. The case of $\lambda_u > 0$ is of course empirically relevant, since in reality, it is unlikely that every trader is informed.
By computing the unconditional expected utilities of different types of traders, we can obtain the value of acquiring information $\tilde{v}_i$ as:

$$\phi_i (I_1, I_2) \equiv \frac{1}{2\gamma} \log \left( \frac{Var(\tilde{v}_i | \bar{p})}{Var(\tilde{v}_j | F_i)} \right)$$

$$= \frac{1}{2\gamma} \log (\rho + I_j^2 \chi) - \frac{1}{2\gamma} \log \left( \frac{1}{Var(\tilde{v} | \bar{p})} \right),$$

(23)

for $i, j = 1, 2$ and $j \neq i$, where the second equality follows from using the signal observed by $\tilde{v}_i$-informed traders in (4) and applying Bayes’ rule. Intuitively, the value of acquiring information $\tilde{v}_i$ is increasing in the quality of information available to the trader after purchasing the signal $\tilde{v}_i$ (given by $Var^{-1}(\tilde{v}_j | F_i) = \rho + I_j^2 \chi$) and decreasing in the quality of information available to the trader by only observing the price (given by price informativeness $Var^{-1}(\tilde{v} | \bar{p})$). Note that we express the information value $\phi_i$ as a function of trading intensity measures $(I_1, I_2)$, but indirectly it of course depends on $(\lambda_1, \lambda_2)$ that affect $(I_1, I_2)$ through the system (13).

In the equilibrium of the information-acquisition stage, a trader will acquire a signal as long as the cost of doing so does not exceed the benefit. Given that there are always traders who decide not to acquire any signal, if some traders choose to acquire information on one fundamental, then the value of that information must be equal to its cost; that is, traders are indifferent between acquiring the signal and not acquiring the signal. Similarly, if all traders decide not to acquire information on one fundamental, then the value of that information must be smaller than its cost. Formally, suppose $(\lambda_1^*, \lambda_2^*) \in \mathbb{R}_+^2$ is an information market equilibrium, then: (a) if $\lambda_i^* > 0$ for some $i = 1, 2$, then $\phi_i (I_1, I_2) = c_i$; and (b) if $\lambda_i^* = 0$ for some $i = 1, 2$, then $\phi_i (I_1, I_2) \leq c_i$.

We can show that for any parameter configuration $(c_1, c_2, \rho, \chi, \gamma) \in \mathbb{R}_+^5$, there exists a unique information market equilibrium, as characterized by the following proposition (with the proof included in the appendix).
PROPOSITION 4: (a) For any exogenous parameters \((c_1, c_2, \rho, \chi, \gamma) \in \mathbb{R}_+^5\), there exists a unique information market equilibrium \((\lambda_1^*, \lambda_2^*)\).

(b) If \((e^{2\gamma c_1} - 1)(e^{2\gamma c_2} - 1) < 1\), then \(\lambda_1^* > 0\) and \(\lambda_2^* > 0\). Otherwise, (i) if \(0 < c_1 < \frac{\log 2}{2\gamma}\), then \(\lambda_1^* > 0\) and \(\lambda_2^* = 0\); (ii) if \(0 < c_2 < \frac{\log 2}{2\gamma}\), then \(\lambda_1^* = 0\) and \(\lambda_2^* > 0\); and (iii) if \(c_1 \geq \frac{\log 2}{2\gamma}\) and \(c_2 \geq \frac{\log 2}{2\gamma}\), then \(\lambda_1^* = \lambda_2^* = 0\).

Part (b) of this proposition states that if the information-acquisition costs \(c_1\) and \(c_2\) are relatively small – i.e., if \((e^{2\gamma c_1} - 1)(e^{2\gamma c_2} - 1) < 1\) – then there will be two types of informed traders active in equilibrium (i.e., \(\lambda_1^* > 0\) and \(\lambda_2^* > 0\)). If both \(c_1\) and \(c_2\) are large, or specifically, if both of them are greater than \(\frac{\log 2}{2\gamma}\), then no traders will find it optimal to acquire any information (i.e., \(\lambda_1^* = \lambda_2^* = 0\)). In the intermediate ranges, we will have only one type of informed traders acquiring the type of information that is relatively cheaper. Figure 1 graphically illustrates these results for \(\gamma = 3\). In the following subsections, we will focus on the case of \((e^{2\gamma c_1} - 1)(e^{2\gamma c_2} - 1) < 1\), so that there are two types of informed traders active in equilibrium.

[FIGURE 1 ABOUT HERE]

B. Learning Complementarity vs. Substitutability

We now analyze the strategic interactions among traders in the decision to produce information. In particular, we show that learning the two independent pieces of information \(\tilde{\nu}_i\) and \(\tilde{\nu}_j\) can be complementary, in the sense that an increase in the mass of agents acquiring information on one fundamental will increase the incentive of agents to acquire information about the other fundamental.

Formally, we will examine how the information value \(\phi_i\) changes with the sizes \((\lambda_1, \lambda_2)\) of the informed traders populations. If \(\phi_i\) is increasing in \(\lambda_j\) – i.e., if \(\frac{\partial \phi_i}{\partial \lambda_j} > 0\) – then we say
that acquiring information \( \tilde{v}_i \) exhibits a *strategic complementarity* to acquiring information \( \tilde{v}_j \), because in this case, more traders will acquire information \( \tilde{v}_i \) when more traders acquire information \( \tilde{v}_j \). If \( \phi_i \) is decreasing in \( \lambda_j \) – i.e., if \( \frac{\partial \phi_i}{\partial \lambda_j} < 0 \) – then acquiring information \( \tilde{v}_i \) exhibits a *strategic substitutability* to acquiring information \( \tilde{v}_j \). Similarly, if \( \frac{\partial \phi_i}{\partial \lambda_i} > 0 \) \((\frac{\partial \phi_i}{\partial \lambda_i} < 0)\), then there is a strategic complementarity (substitutability) among agents acquiring signal \( \tilde{v}_i \). These concepts of learning complementarity/substitutability are consistent with Grossman and Stiglitz (1980).

By equation (23), we have:

\[
\frac{\partial \phi_i}{\partial \lambda_j} = \frac{1}{2\gamma} \left( \frac{\partial \log (\rho + I_j^2 \chi)}{\partial I_j} \right) \frac{dI_j}{d\lambda_j} - \frac{1}{2\gamma} \frac{\partial \log \left( \frac{1}{\text{Var}(\tilde{v}_i|p)} \right)}{\partial \lambda_j}.
\]  

(24)

So, an increase in the population \( \lambda_j \) of \( \tilde{v}_j \)-informed traders has two opposite effects on the benefit \( \phi_i \) of acquiring signal \( \tilde{v}_i \). First, an increase in \( \lambda_j \) will increase the trading intensity \( I_j \) on information \( \tilde{v}_j \) (i.e., \( \frac{dI_j}{d\lambda_j} > 0 \) by Corollary 1). This increased \( I_j \) directly reduces the remaining uncertainty of a \( \tilde{v}_i \)-informed trader, as reflected by the term \( \frac{\partial \log (\rho + I_j^2 \chi)}{\partial I_j} > 0 \) in equation (24). This allows the \( \tilde{v}_i \)-informed trader to trade more aggressively and increases his expected utility, increasing the benefit from acquiring information about \( \tilde{v}_i \). This essentially builds on the uncertainty reduction effect we identified in earlier sections. Before, we have shown that this effect creates a positive link between the two trading intensities, and here we show that this effect extends to also imply an increase in the incentive to produce one kind of information when more people acquire the other kind of information.

Second, an increase in \( \lambda_j \) also causes the price to be more informative about the total cash flow \( \tilde{v} \). This reduces the incentive of uninformed traders to become informed about \( \tilde{v}_i \), which is part of \( \tilde{v} \), as they can now gain more information about \( \tilde{v} \) from the price. This effect is the standard Grossman-Stiglitz substitution effect, whereby having more informed
traders, reduces the incentive to become informed. This negative effect is reflected by the term \(-\frac{1}{2\gamma} \frac{\partial \log \left( \frac{\lambda_j}{\lambda_i} \right)}{\partial \lambda_j}\) in equation (24), which is indeed negative as shown by equation (A19) in the appendix.

When the positive uncertainty reduction effect dominates the negative Grossman-Stiglitz effect, an increase in \(\lambda_j\) will increase \(\phi_i\), leading to a complementarity. We can show that this is true when \(I_j > I_i\); that is, \(\frac{\partial \phi_i}{\partial \lambda_j} > 0\) if and only if \(I_j > I_i\). Interestingly, learning the same information is always a strategic substitute, i.e., \(\frac{\partial \phi_i}{\partial \lambda_i} < 0\). This is because the uncertainty reduction effect discussed above operates through the trading intensity \(I_j\) about the other component \(\tilde{v}_j\), while increasing \(\lambda_i\) mainly increases \(I_i\). Thus, the complementarity effect in acquiring different information is not present in the traditional unidimensional Grossman-Stiglitz framework, and can only be uncovered by considering the two-dimension framework in our paper. We summarize these results in the following proposition (with the proof included in the appendix).

**PROPOSITION 5:** Suppose \((e^{2\gamma_1} - 1)(e^{2\gamma_2} - 1) < 1\), so that \(\lambda_1^* > 0\) and \(\lambda_2^* > 0\). Acquiring information on the same fundamental is a strategic substitute: As more traders become informed of \(\tilde{v}_i\), the value \(\phi_i\) of acquiring \(\tilde{v}_i\) decreases; that is, \(\frac{\partial \phi_i}{\partial \lambda_i} < 0\). Acquiring information on different fundamentals can be a strategic substitute or a complement: As more traders become informed of \(\tilde{v}_j\), the value of acquiring \(\tilde{v}_i\) can decrease or increase, and \(\frac{\partial \phi_i}{\partial \lambda_j} > 0\) if and only if \(I_j > I_i\).

**C. The Impact of Information-Acquisition Cost**

To illustrate the implications of strategic interactions in information acquisition, in this subsection, we discuss results of comparative-statics analysis examining the impact of changing the exogenous cost \(c_i\) of acquiring information \(\tilde{v}_i\) on the equilibrium fractions \((\lambda_1^*, \lambda_2^*)\)
of informed traders and on the price informativeness $\frac{1}{\text{Var}(\tilde{v}|\tilde{p}^*)}$ in the overall equilibrium. The comparative-statics analysis is based on the equilibrium conditions $\phi_1 (I_1^*, I_2^*) = c_1$ and $\phi_2 (I_1^*, I_2^*) = c_2$ in the information-acquisition stage and on the system in (13) characterizing trading intensity measures in the trading stage. The cost of information $c_i$ represents a measure of the easiness of acquiring information on one fundamental $\tilde{v}_i$: A proliferation of sources of information about the firm (say, abundant disclosure, large analyst/media coverage, and advanced communication technologies) leads to easier access to information and corresponds to a low value of $c_i$ (e.g., Fishman and Hagerty (1989), Kim and Verrecchia (1994)).

As we show in Proposition 6, a decrease in the cost $c_i$ of acquiring signal $\tilde{v}_i$ always increases the equilibrium size $\lambda_i^*$ of the population of $\tilde{v}_i$-informed traders. This is intuitive since a lower $c_i$ implies a higher net benefit from knowing $\tilde{v}_i$. More interestingly, the complementarity effect emphasized in Proposition 5 implies that a decrease in $c_i$, the cost of acquiring information $\tilde{v}_i$, may also increase the equilibrium size $\lambda_j^*$ of the population of $\tilde{v}_j$-informed traders. Specifically, Proposition 6 shows that $\frac{d\lambda_j^*}{dc_i} < 0$ if and only if $c_i < c_j$. This is because $c_i < c_j$ implies that $I_i^* > I_j^*$, which, according to Proposition 5, generates $\frac{\partial \phi_j}{\partial \lambda_i} > 0$. That is, in this case, an increase in $\lambda_i$ increases the incentive to acquire information $\tilde{v}_j$, and so a decrease in $c_i$—directly increasing $\lambda_i$—will indirectly cause an increase in the population of traders informed about $\tilde{v}_j$. Finally, the proposition also shows that a decrease in the cost $c_i$ of acquiring signal $\tilde{v}_i$ always leads to an increase in the price informativeness measure $\frac{1}{\text{Var}(\tilde{v}|\tilde{p}^*)}$. The proposition is stated as follows (with its proof included in the appendix):

**PROPOSITION 6:** Suppose $(e^{2\gamma_1} - 1)(e^{2\gamma_2} - 1) < 1$, so that $\lambda_i^* > 0$ and $\lambda_j^* > 0$. A decrease in the cost $c_i$ of acquiring information $\tilde{v}_i$:

(a) increases the equilibrium size $\lambda_i^*$ of $\tilde{v}_i$-informed traders (i.e., $\frac{d\lambda_i^*}{dc_i} < 0$);
(b) increases the equilibrium size $\lambda_j^*$ of $\tilde{v}_j$-informed traders if and only if acquiring $\tilde{v}_j$ is a complement to acquiring $\tilde{v}_i$ (i.e., $\frac{d\lambda_j^*}{dc_i} < 0$ iff $\frac{\partial \phi_j}{\partial \lambda_i} > 0$ or iff $c_i < c_j$); and

(c) increases the price informativeness (i.e., $\frac{d\text{Var}^{-1}(\tilde{v}_j \tilde{p}^*)}{dc_i} < 0$).

D. Empirical Implications

The results in Proposition 6 can be tested empirically. Most interesting are of course the results in Part (b) of the proposition, which relate to the interaction between the two types of information in our model. These results characterize the effect of a decrease in the cost of acquiring one type of information on the number of traders acquiring the other type of information. Building on our discussion in Section II.D., one can test these results in the settings, where it is natural to think about multiple dimensions of uncertainty.

To test these results one would also need to get a proxy of the costs of information production. Ideally, the change in the cost of information production can be considered exogenous, so that its causal effect on the populations of informed traders can be examined. One potential source of exogenous variation in the cost of information production is regulation. For example, greater disclosure requirements will imply that traders have easier access to information. Consider then the case of a multinational firm: One can analyze the effect that changes in disclosure regulation in one country have on the amount of information produced by traders who trade the stock of the multinational firm and are based in another country.

Another source of exogenous variation in information costs was highlighted recently by Kelly and Ljungqvist (2012). They study changes in the number of sellside analysts who cover a stock due to exogenous reasons such as brokerage firms closing their research operations. To the extent that these analysts specialize in one dimension of firm uncertainty – a particular
country or industry – one can then check what this change does to the amount of information produced by the market on the other dimensions of uncertainty.

Finally, another way to view our model with two groups of traders is to think about technical traders, who process information about prices and order flows and trade on it, and fundamental traders, who trade on information about firm cash flows. One can recast the model to think about the interaction between these two groups. A natural experiment in this context is the introduction of automated quote dissemination on the New York Stock Exchange (NYSE) in 2003, which was studied by Hendershott, Jones, and Menkveld (2011). This change corresponds to an exogenous decrease in the cost \( c_i \) of acquiring and processing information for technical traders who are active in fast computerized trading. To test Part (b) of Proposition 6, one can examine the effect of this change on the behavior of more traditional fundamental traders. In addition, consistent with Part (c), Hendershott, Jones, and Menkveld (2011) find that introducing autoquoting indeed enhances the informativeness of quotes.\(^{12}\)

IV. An Extension with Some Traders Acquiring Both Signals

A. Setup

In this section, we analyze an extended economy where traders can potentially acquire two signals \( \tilde{v}_1 \) and \( \tilde{v}_2 \) simultaneously. To keep things interesting we also generalize the payoff of the risky asset as follows:

\[
\tilde{v} = \tilde{v}_1 + \tilde{w}_2 + \tilde{w},
\]  

(25)
where $\tilde{v}_i \sim N(0, 1/\rho)$ ($i = 1, 2$) is still a forecastable fundamental and $\tilde{w} \sim N(0, 1/\omega)$ (with $\omega > 0$) is the residual noise, which is introduced so that traders who observe both signals still face uncertainty when they trade. The random variables $\tilde{v}_1, \tilde{v}_2$, and $\tilde{w}$ are mutually independent.

Traders can still acquire the signal $\tilde{v}_1$ at cost $c_1 > 0$, and the signal $\tilde{v}_2$ at cost $c_2 > 0$. Now, we also allow traders to acquire both $\tilde{v}_1$ and $\tilde{v}_2$, but at a cost of $c_1 + c_2 + k$, where $k \geq 0$. Parameter $k$ represents an increasing marginal cost of information acquisition. It can also come as a result of a model with asymmetric expertise in information acquisition. We provide more detailed interpretation for this parameter in the next subsection. Our baseline model, presented in the previous subsections, corresponds to the case of $k = \infty$ (i.e., traders cannot observe two signals) and $\omega = \infty$ (i.e., there is no residual uncertainty in the asset payoff). In this section, we will show that our main results are robust to the general case of $0 < k < \infty$ and $0 < \omega < \infty$. For the ease of exposition, we denote

$$\tilde{v}_{12} \equiv \tilde{v}_1 + \tilde{v}_2 \text{ and } c_{12} \equiv c_1 + c_2 + k,$$

(26)

which respectively correspond to the signal and the information-acquisition cost for traders who obtain both signals $\tilde{v}_1$ and $\tilde{v}_2$.

Now, in the trading stage, there are potentially four types of rational traders: (1) $\tilde{v}_1$-informed traders observing $\tilde{v}_1$ (of mass $\lambda_1 \geq 0$), (2) $\tilde{v}_2$-informed traders observing $\tilde{v}_2$ (of mass $\lambda_2 \geq 0$), (3) $\tilde{v}_{12}$-informed traders observing $\tilde{v}_1$ and $\tilde{v}_2$ (of mass $\lambda_{12} \geq 0$), and (4) uninformed traders (of mass $\lambda_u > 0$).

All other features of the model are still the same. That is, traders have CARA utility functions with a risk-aversion parameter $\gamma > 0$. There are two tradable assets: the stock and the bond. All four types of rational traders condition their trades on the stock price $\tilde{p}$. Noise traders trade a random amount $\tilde{x} \sim N(0, 1/\chi)$ (with $\chi > 0$) of the stock, which is
independent of the realizations of $\tilde{v}_1$, $\tilde{v}_2$ and $\tilde{w}$.

B. Interpretation of the Parameter $k$

There are two possible interpretations for the parameter $k$. First, it can capture a convex cost structure of the information-acquisition technology, as in Verrecchia (1982) and others. That is, there is a large mass of ex-ante identical traders. They can obtain signal $\tilde{v}_i$ at a cost $c_i$, reducing the uncertainty they are exposed to in the payoff of the stock by $\rho^{-1}$. They can further reduce their uncertainty by an additional $\rho^{-1}$ through acquiring the other signal $\tilde{v}_j$. However, this extra reduction of uncertainty will not only cost them $c_j$ – which is the cost they would pay if they acquired this signal only – but also an additional cost of $k$. To the extent that traders (either individuals or institutions) have limited capacity for processing information, such a convex cost structure is quite natural.

Second, $k$ can come from a setting where different traders have different expertise in information acquisition. Suppose that there are two types of traders who are ex ante different: Each type has an advantage in gathering a particular type of information; and if they want to acquire the other type of information, they have to pay an extra cost of $k$. Traders with advantage in acquiring $\tilde{v}_1$ will decide between acquiring it at a cost of $c_1$, acquiring both signals at a cost of $c_{12}$, and staying uninformed. The choice faced by the other type of traders is analogous. This is then equivalent to the extension of the model we study here.\textsuperscript{13} This setting captures the idea that different traders have easier access to different sources of information. This can be the result of traders originating from different countries trading the stock of a multinational firm, or different institutions with expertise in different styles or industries, as we discussed in previous sections.

Under both interpretations, one may ask whether the additional cost $k$ can be avoided
by merging two traders, one informed about $\tilde{v}_1$ at cost $c_1$ and the other informed about $\tilde{v}_2$ at cost $c_2$. For example, an intermediary can hire two such traders and form one combined institution. In our view, however, it is unlikely that such combinations happen without any friction. There are agency costs, coordination costs, and organizational costs that imply that combining two traders with different pieces of information will be at an additional cost, and one can think of $k$ in our model as the lowest cost possible that is required to combine two types of trading expertise under one roof. Hence, we are agnostic about the source of this cost $k$, and study the equilibrium outcomes when traders can be informed about the two dimensions by paying this additional given cost (in addition to the costs of acquiring each type of information). Our model does not make an assumption about the size of $k$, and we can provide a characterization of equilibrium outcomes for all levels of $k$.

C. Results

As in the baseline model, we first take the masses $\lambda_1, \lambda_2$, and $\lambda_{12}$ as given to solve the financial market equilibrium, and then endogenize them in the information market equilibrium. We find that our main results are robust in this extended economy with $k \in (0, \infty)$ and $\omega \in (0, \infty)$. Since the analysis of this extension is quite complicated, we provide all the formal results together with their proofs in an online appendix. Here, we just summarize the main points.

In the trading stage, we can characterize the demand functions $D_t(\bar{p}, \tilde{v}_t)$ of the three types of informed traders (for $t \in \{1, 2, 12\}$). The trading intensity on information $\tilde{v}_i$ is defined as

$$I_i \equiv \lambda_i \frac{\partial D_i(\bar{p}, \tilde{v}_t)}{\partial \tilde{v}_i} + \lambda_{12} \frac{\partial D_{12}(\bar{p}, \tilde{v}_{12})}{\partial \tilde{v}_i},$$

which represents how all traders informed of the signal $\tilde{v}_i$ (either in addition to the other
signal or not) respond to a change in $\tilde{v}_i$. We then compute two best response functions $I_i = h_i (I_j; \lambda_i, \lambda_{12}, \rho, \gamma, \chi, \omega)$ (for $i, j = 1, 2, j \neq i$) that jointly determine the two trading intensities $I_1$ and $I_2$. As in Proposition 2 in the baseline model, we can still show that trading on information $\tilde{v}_i$ is a complement (substitute) to trading on information $\tilde{v}_j$ if $I_j$ is sufficiently large (small) in this extended economy.

We also show that there exists a linear rational-expectations equilibrium, with the price function given in the form of equation (2). Similar to Corollary 1, an increase in the size $\lambda_i$ of $\tilde{v}_i$-informed traders (a) increases the trading intensity $I_i$ on information $\tilde{v}_i$, and (b) increases the trading intensity $I_j$ on information $\tilde{v}_j$ if and only if trading on $\tilde{v}_j$ is complementary to trading on $\tilde{v}_i$, which is true when $\lambda_i$ is sufficiently large. We can also show that our Corollary 2 continues to hold. Specifically, for any given $\lambda_{12} \geq 0$, we still fix $\lambda_1 + \lambda_2$ at a constant $\Lambda > 0$, and define information diversity as $\Delta \equiv 1 - \frac{|\lambda_1 - \lambda_2|}{\Lambda}$, which is given by equation (22). Then, we can show that information diversity $\Delta$ increases the price informativeness $\frac{1}{\text{Var}(\tilde{v} | \tilde{p})}$.

In the information-acquisition stage, the benefit of acquiring information $\tilde{v}_t$ is given by $\phi_t (I_1, I_2) \equiv \frac{1}{2\theta} \log \left( \frac{\text{Var}(\tilde{v} | \tilde{p})}{\text{Var}(\tilde{v} | \mathcal{F}_t)} \right)$, where $\mathcal{F}_t = \{\tilde{v}_t, \tilde{p}\}$ for $t \in \{1, 2, 12\}$. As in the baseline model, the equilibrium in the information-acquisition stage is still defined by the usual no-deviation conditions. We show that for any exogenous parameter configuration $(k, c_1, c_2, \rho, \chi, \gamma, \omega) \in \mathbb{R}^{7+}$, there exists an information market equilibrium. In addition, except for a set of parameters with zero Lebesgue measure, the equilibrium is unique, and it has at most two types of active informed traders (recall that the three informed types of $t$ are $\{1, 2, 12\}$). For the parameter set with zero Lebesgue measure, each parameter configuration can produce a continuum of equilibria with $\lambda_1^* > 0, \lambda_2^* > 0$, and $\lambda_{12}^* > 0$, but the resulting trading intensities $I_1^*$ and $I_2^*$ are still unique. We further characterize more explicitly the information market equilibrium by two threshold values $\tilde{k}_1$ and $\tilde{k}_2$ (with $0 < \tilde{k}_1 < \tilde{k}_2$), which are in turn
determined by four primitive parameters \((\rho, \gamma, \chi, \omega)\) in explicit functional forms.

Figure 2 illustrates the results that are proved formally and generally in the online appendix. In the figure, we illustrate the types of equilibria in the space of \((c_1, c_2)\). Here, we choose \(\rho = 50, \chi = 50, \omega = 25,\) and \(\gamma = 3.\) Under those parameter values, \(\bar{k}_1 = 0.020\) and \(\bar{k}_2 = 0.028.\) Parameter \(k\) takes the value of 0.022, 0.016, and 0 in Panels (b), (c) and (d), respectively. Its choice in Panel (a) does not affect the result (it is above \(\bar{k}_2\)). We can see that if \(k > \bar{k}_2,\) the equilibrium distribution of trader types is very similar to our baseline model: No traders acquire the two signals simultaneously, and when \((c_1, c_2)\) are relatively small, traders decide to acquire signals \(\tilde{v}_1\) and \(\tilde{v}_2\) in isolation (i.e., \(\lambda_1^* > 0\) and \(\lambda_2^* > 0\)). For \(k \in (0, \bar{k}_2),\) the sets of \((c_1, c_2)\) generating an equilibrium with \(\lambda_1^* > 0\) and \(\lambda_2^* > 0\) still have a positive measure, and as we decrease \(k,\) their sizes decrease, and equilibria with \(\lambda_{12}^* > 0\) become more and more common.

[FIGURE 2 ABOUT HERE]

Importantly, except for a set of parameters with zero Lebesgue measure, there is never an equilibrium with \(\lambda_1^* > 0, \lambda_2^* > 0,\) and \(\lambda_{12}^* > 0\) at the same time. There are equilibria where none of them is positive, one of them is positive or two of them are positive. This suggests that our baseline model, not allowing for the possibility of acquiring the two signals simultaneously, captures the most interesting case. This is because if we study a case with \(\lambda_{12}^* > 0,\) it implies that we would not have two groups of traders heterogeneously informed without one group’s information being dominated by the other group’s information. This would miss the basic spirit of our model. Our baseline model can be viewed as a special case of the extended model described in this section (and solved in the online appendix) where \(k\) is sufficiently large.

Finally, in the online appendix we also show that our Proposition 5 continues to be valid
in the extended economy. That is, when the economy operates such that \( \lambda_1^* > 0, \lambda_2^* > 0, \) and \( \lambda_{12}^* = 0, \) acquiring information on the same fundamental is a strategic substitute (i.e., \( \frac{\partial \phi_i}{\partial \lambda_i} < 0 \)), and acquiring information on different fundamentals can be a strategic substitute or a complement (i.e., \( \frac{\partial \phi_j}{\partial \lambda_i} > 0 \) if and only if \( I_i \) is sufficiently large relative to \( I_j \)).

V. Relation to the Literature

Our paper is related to other papers in the literature that analyze complementarity/substitutability between different types of information in financial markets. Admati and Pfleiderer (1987) study a different notion of complementarity/substitutability, as they look at it from the perspective of one investor. Specifically, they define two signals to be complements (substitutes) if the benefit of one investor observing two signals simultaneously is greater (smaller) than the sum of the benefits of the same investor observing these two signals in isolation. Based on whether signals tend to be complements or substitutes, Admati and Pfleiderer (1987) then analyze under what circumstances information will be concentrated at the hands of a few investors rather than spread out across many investors. They propose two effects – the “unlocking effect” and the “aggregation effect” – which, respectively, favor substitutability and complementarity according to their definition. The “unlocking effect” occurs when a trader having one signal can use it to unlock information about the other signal from market prices in a multi-asset setting. The “aggregation effect” occurs when the price is almost a sufficient statistic for the underlying information, and thus each individual signal has relatively little incremental value, but the bundle of all signals is still superior to the price. Lundholm (1991) builds on this framework to analyze the effect of public disclosure on the information-acquisition activities of traders in financial markets.
In contrast, our focus is on strategic complementarities/substitutabilities, which are about the interactions among different investors and study how an investor’s benefit of trading or acquiring a signal is affected by other investors trading or acquiring signals. Our paper takes the view that different investors are informed about different aspects of the value of the security, and so we naturally stay away from the focus of Admati and Pfleiderer (1987) on whether information will be concentrated at the hands of a few investors or not. Only in Section IV., we allow investors to acquire two types of signals, but even there we stick to our basic premise that information is naturally dispersed across investors by assuming that different investors have an advantage in acquiring different types of information or that there is a convex cost structure in information acquisition. Our focus is thus on the interactions across groups of investors and on the effect of diversity of information – whether the two groups of informed investors are close in size or not – on the overall informativeness of the price. Not surprisingly, given the difference in the notion of complementarity/substitutability and the underlying setting, the effects leading to complementarity and substitutability in our paper, which are highlighted in Sections II. and III., are quite different from the effects highlighted in Admati and Pfleiderer (1987).

The papers by Paul (1993) and Lee (2010) share our notion of strategic complementarity/substitutability, in looking at the interaction across traders in trading on and producing different information. Specifically, Paul (1993) emphasizes a substitution effect coming from the competition among traders trading on the same type of information. Lee (2010), which builds on Subrahmanyam and Titman (1999), emphasizes a complementarity effect across different types of traders coming from the fact that trades based on different types of information provide noise for each other in the market-order based model. However, the models in both papers are based on market orders, as in Kyle (1985), where traders do not observe
prices or condition on prices when they trade. As a result, the effects highlighted by these papers are quite different from the effects in our model, given that the complementarity and substitutability in trading intensity in our model originate from the updating that traders make based on the information they glean from market prices.

Another related literature analyzes models of trading in multiple securities, e.g., Admati (1985) and Bernhardt and Taub (2008). Perhaps most closely related to our paper is a recent paper by Cespa and Foucault (2012), who identify a channel that shares the same spirit to our “uncertainty reduction effect” in the trading stage. They use a multi-asset model to study liquidity spillovers between different securities when dealers trading in one asset observe the price of the other traded asset and learn information concerning the asset they trade. The cross-asset learning in their model generates a self-reinforcing feedback loop between price informativeness and liquidity, which drives liquidity comovement across markets and can lead to multiple equilibria with different levels of illiquidity. In contrast, we use a one-asset setting to examine the interaction between trading intensities on information about different aspects of the same asset and characterize its impact on price informativeness and private information production. Unlike Cespa and Foucault (2012) where the direct effect of a change in exogenous parameters always gets amplified when all informed traders observe prices, in our model, the direct effect can be either amplified or attenuated, depending on the competition between the “uncertainty reduction effect” and the “inference augmentation effect.” This is because traders informed of different fundamentals trade against each other in our model, while in Cespa and Foucault (2012) they specialize in trading different assets. As a result of this difference, our model does not have the multiplicity of equilibria that Cespa and Foucault (2012) have.

Considering the empirical implications of our model, some of which are highlighted in
Sections II.D. and III.D., recall that they involve interactions among differentially informed groups of traders trading in the same security. Hence, among the above mentioned papers, they may pertain only to Paul (1993) and Lee (2010). However, given the different mechanisms, our model also generates distinct empirical implications. First, complementarities in our setting, unlike in the other settings, are generated by changes in the overall uncertainty traders are exposed to. This can be directly tested empirically. Second, our analysis goes beyond the analyses in the above mentioned papers by analyzing the effects of information diversity on price informativeness, which, as we highlighted in Sections II.D. and III.D., can be tested empirically. Finally, the results in Paul (1993) and Lee (2010) are specific to markets based on market orders, whereas our results are based on limit-order markets, and this can again be considered in empirical analysis.

VI. Conclusion

Does information in financial markets attract or deter the transmission and production of more information? We extend the seminal Grossman and Stiglitz (1980) model to include two dimensions of uncertainty in the value of the traded asset, and provide new insights into this question by uncovering a rich set of interactions. We identify an uncertainty reduction effect whereby traders trading more aggressively on information about one fundamental will reduce the uncertainty faced by those traders informed about the other fundamental and thus encourage them to trade more aggressively and produce more information. We show that when this effect dominates, producing and trading on two types of information can be complementary. This effect also implies that greater diversity of information in the economy enhances price informativeness, which highlights that it is not only the total size of
informed population but also the composition that matters in determining traders’ behavior and market outcomes. Finally, our analysis sheds lights on a variety of financial phenomena and makes empirically testable predictions.

Appendix. Proofs

Proof of Proposition 1:

Using (4), we can compute

\[ D_i(\tilde{p}, \tilde{v}_i) = \frac{(\rho + (\alpha_j/\alpha_x)^2 \chi) \tilde{p}_i + (\alpha_j/\alpha_x)^2 \chi \frac{\tilde{a}_j - \alpha_i \tilde{v}_i}{\alpha_j} - (\rho + (\alpha_j/\alpha_x)^2 \chi) \tilde{p}}{\gamma} . \]

Plugging this expression and \( D_u(\tilde{p}) = \frac{\tilde{p} - E(\tilde{v}|\tilde{p})}{\gamma Var(\tilde{v}|\tilde{p})} \) into the market clearing condition, and rearranging terms, we have:

\[
\lambda_1 \left[ \rho + \left( \alpha_2/\alpha_x \right)^2 \chi - \left( \alpha_2/\alpha_x \right)^2 \chi/\alpha_2 \right] \tilde{p} \\
+ \lambda_2 \left[ \rho + \left( \alpha_1/\alpha_x \right)^2 \chi - \left( \alpha_1/\alpha_x \right)^2 \chi/\alpha_1 \right] \tilde{p} + \lambda_u \frac{\tilde{p} - E(\tilde{v}|\tilde{p})}{\gamma Var(\tilde{v}|\tilde{p})} \\
= \lambda_1 \left( \alpha_2/\alpha_x \right)^2 \chi \frac{-\alpha_0}{\alpha_2} + \lambda_2 \left( \alpha_1/\alpha_x \right)^2 \chi \frac{-\alpha_0}{\alpha_1} - \gamma \\
+ \lambda_1 \left[ \rho + \left( \alpha_2/\alpha_x \right)^2 \chi - \left( \alpha_1/\alpha_x \right) \left( \alpha_2/\alpha_x \right) \chi \right] \tilde{v}_1 \\
+ \lambda_2 \left[ \rho + \left( \alpha_1/\alpha_x \right)^2 \chi - \left( \alpha_1/\alpha_x \right) \left( \alpha_2/\alpha_x \right) \chi \right] \tilde{v}_2 + \gamma \tilde{x}. \quad (A1)
\]

Note that the left-hand side of the above equation is only related to \( \tilde{p} \), while the right-hand side is only related to \( \tilde{v}_1, \tilde{v}_2, \) and \( \tilde{x} \). Hence, based on (2) and (A1), we form the following system of two equations in terms of two unknowns, \( \left( \alpha_1/\alpha_x \right) \) and \( \left( \alpha_2/\alpha_x \right) \):

\[
\left( \alpha_1/\alpha_x \right) = \lambda_1 \left[ \rho + \left( \alpha_2/\alpha_x \right)^2 \chi - \left( \alpha_1/\alpha_x \right) \left( \alpha_2/\alpha_x \right) \chi \right] \gamma^{-1}; \quad (A2)
\]

\[
\left( \alpha_2/\alpha_x \right) = \lambda_2 \left[ \rho + \left( \alpha_1/\alpha_x \right)^2 \chi - \left( \alpha_1/\alpha_x \right) \left( \alpha_2/\alpha_x \right) \chi \right] \gamma^{-1}. \quad (A3)
\]

By equation (A3), we can express \( \left( \alpha_2/\alpha_x \right) \) in terms of \( \left( \alpha_1/\alpha_x \right) \) as follows (this is equation
(13) for \( i = 2 \) in the main text):

\[
\frac{\alpha_2}{\alpha_x} = \frac{\lambda_2 \left[ \rho + \chi (\alpha_1/\alpha_x)^2 \right]}{\gamma + \lambda_2 \chi (\alpha_1/\alpha_x)}. \tag{A4}
\]

Then, plugging the above expression into equation (A2), we have the following cubic polynomial in \((\alpha_1/\alpha_x)\):

\[
\gamma \chi^2 \lambda_2 (\lambda_1 + \lambda_2) (\alpha_1/\alpha_x)^3 + 2 \chi \lambda_2 (\gamma^2 - \chi \rho \lambda_1 \lambda_2) (\alpha_1/\alpha_x)^2 \\
+ \gamma (\gamma^2 - \chi \rho \lambda_1 \lambda_2) (\alpha_1/\alpha_x) - \rho \lambda_1 (\gamma^2 + \chi \rho \lambda_2^2) = 0. \tag{A5}
\]

There exists a positive solution to the above polynomial, because when \( \lambda_1 > 0 \) and \( \lambda_2 > 0 \), the left-hand side of equation (A5) is negative at \((\alpha_1/\alpha_x) = 0\) and becomes positive as \((\alpha_1/\alpha_x)\) is sufficiently large. In addition, the coefficients of the cubic polynomial have one sign change regardless of the sign of \((\gamma^2 - \chi \rho \lambda_1 \lambda_2)\). Hence, by Descartes’ “rule of signs,” the cubic polynomial has one (unique) positive real root, that is, the equilibrium coefficient ratio \( \frac{\alpha_1}{\alpha_x} \) is unique and positive. By equation (A4), we also know that the equilibrium coefficient ratio \( \frac{\alpha_2}{\alpha_x} \) is positive.

By (5) and applying Bayes’ rule, we have:

\[
E(\tilde{v}|\tilde{p}) = \beta_{\tilde{v},\tilde{p}} \tilde{s}_u, \tag{A6}
\]

\[
Var(\tilde{v}|\tilde{p}) = \frac{(\alpha_1/\alpha_x - \alpha_2/\alpha_x)^2 + 2 \rho \chi^{-1}}{(\alpha_1/\alpha_x)^2 + (\alpha_2/\alpha_x)^2 + \rho \chi^{-1}}, \tag{A7}
\]

where \( \beta_{\tilde{v},\tilde{p}} = \frac{(\alpha_1/\alpha_x + (\alpha_2/\alpha_x)}{(\alpha_1/\alpha_x)^2 + (\alpha_2/\alpha_x)^2 + \rho \chi^{-1}} \). Using the above equations to express out \( \frac{\tilde{p} - E(\tilde{v}|\tilde{p})}{\sqrt{Var(\tilde{v}|\tilde{p})}} \) in equation (A1) delivers

\[
\left( A_{\rho 0} - A_{\rho x} \frac{1}{\alpha_x} \right) \tilde{p} = A_1 \tilde{v} + A_2 \tilde{v} + \gamma \tilde{x} + \left( -A_0 \frac{\alpha_0}{\alpha_1} - A_2 \frac{\alpha_0}{\alpha_2} - A_{\rho x} \frac{\alpha_0}{\alpha_x} - \gamma \right),
\]

where the coefficients of \( A \)'s are known positive values – which are determined by \((\alpha_1/\alpha_x)\).
and \((\alpha_2/\alpha_x)\) – and defined as follows:

\[
A_{p0} = \frac{\lambda_1}{\text{Var}(\tilde{v}|F_1)} + \frac{\lambda_2}{\text{Var}(\tilde{v}|F_2)} + \frac{\lambda_u}{\text{Var}(\tilde{v}|\hat{p})},
\]

\[
A_{px} = \lambda_1 \left( \frac{\alpha_2}{\alpha_x} \right) \chi + \lambda_2 \left( \frac{\alpha_1}{\alpha_x} \right) \chi + \frac{\lambda_u \beta_{\tilde{v},\hat{p}}}{\text{Var}(\tilde{v}|\hat{p})},
\]

\[
A_1 = \gamma \left( \frac{\alpha_1}{\alpha_x} \right), \quad A_2 = \gamma \left( \frac{\alpha_2}{\alpha_x} \right),
\]

\[
A_{01} = \lambda_2 \left( \frac{\alpha_1}{\alpha_x} \right)^2 \chi, \quad A_{02} = \lambda_1 \left( \frac{\alpha_2}{\alpha_x} \right)^2 \chi, \quad A_{0x} = \lambda_u \frac{\beta_{\tilde{v},\hat{p}}}{\text{Var}(\tilde{v}|\hat{p})}.
\]

Thus, we can solve for \(\alpha_x\):

\[
\alpha_x = \frac{\gamma}{A_{p0} - A_{px} \frac{1}{\alpha_x}} \Rightarrow \alpha_x = \frac{\gamma + A_{px}}{A_{p0}} > 0.
\]

Combining the known ratios \((\alpha_1/\alpha_x)\) and \((\alpha_2/\alpha_x)\) with the value of \(\alpha_x\) gives the values of \(\alpha_1\) and \(\alpha_2\), which are positive. Once we know \(\alpha_1\), \(\alpha_2\) and \(\alpha_x\), then we can solve \(\alpha_0\) using

\[
\alpha_0 = \frac{-A_{01} \frac{\alpha_1}{\alpha_x} - A_{02} \frac{\alpha_1}{\alpha_x} - A_{0x} \frac{\alpha_1}{\alpha_x} - \gamma}{A_{p0} - A_{px} \frac{1}{\alpha_x}} \Rightarrow \alpha_0 = -\frac{\gamma}{\left( A_{p0} - A_{px} \frac{1}{\alpha_x} \right) + A_{01} \frac{1}{\alpha_x} + A_{02} \frac{1}{\alpha_x} + A_{0x} \frac{1}{\alpha_x}}.
\]

We can further use the solved expressions of \(A_{01}\), \(A_{02}\), \(A_{0x}\), \(A_{px}\) and \(\alpha_x\) to simplify the denominator of the above expression of \(\alpha_0\) and show that \(\alpha_0 = -\frac{\gamma}{A_{p0}} < 0\). QED.

**Proof of Proposition 3:**

Taking total differentiation of equation (13) (for \(i = 1, 2\)) with respect to \(Q\) implies:

\[
\frac{dI_1}{dQ} = \frac{\partial h_1}{\partial Q} + \frac{\partial h_1}{\partial I_2} \frac{dI_2}{dQ} \quad \text{and} \quad \frac{dI_2}{dQ} = \frac{\partial h_2}{\partial Q} + \frac{\partial h_2}{\partial I_1} \frac{dI_1}{dQ}.
\]

Solving for \(\frac{dI_1}{dQ}\) and \(\frac{dI_2}{dQ}\) delivers

\[
\frac{dI_1}{dQ} = \frac{\partial h_3}{\partial Q} + \frac{\partial h_3}{\partial I_2} \frac{dI_2}{dQ} \quad \text{and} \quad \frac{dI_2}{dQ} = \frac{\partial h_3}{\partial Q} + \frac{\partial h_3}{\partial I_1} \frac{dI_1}{dQ},
\]

which is equation (18).

Next, we examine the sign and magnitude of \(\mathcal{M} = \left( 1 - \frac{\partial h_1}{\partial I_2} \frac{\partial h_2}{\partial I_1} \right)^{-1}\). By equation (13),
direct computation shows
\[
\frac{\partial h_i}{\partial I_j} = 1 - \frac{\gamma^2 + \chi \rho \lambda^2_i}{(\gamma + \chi \lambda_i I_j)^2}, \text{ for } i, j = 1, 2 \text{ and } j \neq i. \tag{A8}
\]
Using equations (A2) and (A3), we can express \(\lambda_i\) in terms of \(I_1\) and \(I_2\) as follows:
\[
\lambda_i = \frac{\gamma I_i}{\rho + I^2_j \chi - I_1 I_2 \chi}, \text{ for } i, j = 1, 2 \text{ and } j \neq i. \tag{A9}
\]
Plugging the above expression into equation (A8) yields:
\[
\frac{\partial h_i}{\partial I_j} = \frac{(2I_j - I_i) \chi I_i}{\rho + \chi I^2_j}, \text{ for } i, j = 1, 2 \text{ and } j \neq i. \tag{A10}
\]
Thus, we have
\[
\mathcal{M}^{-1} = 1 - \frac{\partial h_1}{\partial I_2} \frac{\partial h_2}{\partial I_1} = 1 - \frac{(2I_2 - I_1) \chi I_1 (2I_1 - I_2) \chi I_2}{\rho + \chi I^2_1} = \frac{(\rho + 2\chi I_1 I_2) (\rho + \chi (I_1 - I_2)^2)}{(\rho + \chi I^2_1) (\rho + \chi I^2_2)} > 0.
\]
That is, \(\mathcal{M} > 0\).

Whether \(M > 1\) depends on whether \(\frac{\partial h_1}{\partial I_2}\) and \(\frac{\partial h_2}{\partial I_1}\) have the same sign. Specifically, we have three cases.

Case 1. If \(\frac{\partial h_1}{\partial I_2} = 0\) for some \(i\), then \(\mathcal{M} = 1\). By equation (A10), this will be true if and only if
\[
\frac{\partial h_i}{\partial I_j} = \frac{(2I_j - I_i) \chi I_i}{\rho + \chi I^2_j} = 0 \Rightarrow I_i = I_j = 2.
\]
Case 2. If \(\frac{\partial h_1}{\partial I_2} > 0\) for \(i = 1, 2\), then \(\mathcal{M} > 1\). By equation (A10), this will be true if and only if
\[
\frac{\partial h_i}{\partial I_j} = \frac{(2I_j - I_i) \chi I_i}{\rho + \chi I^2_j} > 0 \Rightarrow \frac{I_i}{I_j} < 2, \forall i \Rightarrow \frac{1}{2} < \frac{I_1}{I_2} < 2.
\]
Case 3. If \(\frac{\partial h_1}{\partial I_2} > 0\) and \(\frac{\partial h_2}{\partial I_1} < 0\), then \(0 < \mathcal{M} < 1\). This will be true if and only if \(\frac{I_j}{I_i} > 2\), i.e., \(\frac{I_1}{I_2} > 2\) or \(\frac{I_1}{I_2} < \frac{1}{2}\).

Note that it is not possible to have both \(\frac{\partial h_1}{\partial I_2} < 0\) and \(\frac{\partial h_2}{\partial I_1} < 0\), because these two inequalities combine to imply \(2 < \frac{I_1}{I_2} < \frac{1}{2}\), which is impossible. QED.
Proof of Corollary 1:

In the main text, we have proven Part (a) and the first part of Part (b). We only need to show that $\frac{\partial h_i}{\partial I_j} > 0$ if $\lambda_i$ is sufficiently large. By (A10), $\frac{\partial h_i}{\partial I_j} > 0$ if and only if $I_i > \frac{1}{2} I_j$. For a fixed $\lambda_j > 0$, if $\lambda_i \to \infty$, then by equation (13), $I_i = \frac{\lambda_i (\rho + \chi I_j^2)}{\gamma + \lambda_i \chi I_j} = \frac{\rho + \chi I_j^2}{\lambda_i + \chi I_j} \to \frac{\rho + \chi I_j^2}{\chi I_j} = \frac{\rho}{\chi I_j} + I_j > I_j$, and so $\frac{\partial h_i}{\partial I_j} < 0$. QED.

Proof of Corollary 2:

Without loss of generality, we assume that $\lambda_1 > \lambda_2$, and as a result, increasing diversity $\Delta$ while fixing $\Lambda$ is equivalent to decreasing $\lambda_1$ and increasing $\lambda_2$.

Formally, we have

$$\lambda_1 + \lambda_2 = \Lambda \text{ and } \Delta = 1 - \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} = \frac{2\lambda_2}{\lambda_1 + \lambda_2}.$$  

Then, taking total differentiation of the above system, yields:

$$d\lambda_1 + d\lambda_2 = d\Lambda = 0,$$

$$d\left(\frac{2\lambda_2}{\lambda_1 + \lambda_2}\right) = 2 \left( -\frac{\lambda_2}{(\lambda_1 + \lambda_2)^2} d\lambda_1 + \frac{(\lambda_1 + \lambda_2) - \lambda_2}{(\lambda_1 + \lambda_2)^2} d\lambda_2 \right) = d\Delta,$$

which implies:

$$\frac{d\lambda_1}{d\Delta} = -\frac{\Lambda}{2} \text{ and } \frac{d\lambda_2}{d\Delta} = \frac{\Lambda}{2}.$$  

So, by equation (13) and the chain rule, we have

$$\frac{\partial h_1}{\partial \Delta} = \frac{\partial h_1}{\partial \lambda_1} \frac{d\lambda_1}{d\Delta} = -\frac{\partial h_1}{\partial \lambda_1} \frac{\Lambda}{2} \text{ and } \frac{\partial h_2}{\partial \Delta} = \frac{\partial h_2}{\partial \lambda_2} \frac{d\lambda_2}{d\Delta} = \frac{\partial h_2}{\partial \lambda_2} \frac{\Lambda}{2}.$$  

Setting $Q = \Delta$ in equation (18) and using the above expressions of $\frac{\partial h_1}{\partial \Delta}$ and $\frac{\partial h_2}{\partial \Delta}$, we obtain

$$\frac{dI_1}{d\Delta} = \mathcal{M} \left( \frac{\partial h_1}{\partial \Delta} + \frac{\partial h_1}{\partial I_2} \frac{\partial h_2}{\partial \Delta} \right) = \mathcal{M} \left( -\frac{\partial h_1}{\partial \lambda_1} + \frac{\partial h_1}{\partial I_2} \frac{\partial h_2}{\partial \lambda_2} \right) \frac{\Lambda}{2}.$$
Then, using equations (A8) and (A9), we have

\[
\frac{dI_1}{d\Delta} = M \frac{\Lambda}{2} \gamma \left[ -\frac{\rho + \chi I_2^2}{(\gamma + \chi \frac{\gamma I_1}{\rho + \chi I_2^2} I_2)} + \frac{(2I_2 - I_1) \chi I_1}{(\gamma + \chi \frac{\gamma I_1}{\rho + \chi I_2^2} I_2)} \right] .
\]  

(A11)

Similarly, we can compute:

\[
\frac{dI_2}{d\Delta} = M \frac{\Lambda}{2} \gamma \left[ -\frac{\rho + \chi I_1^2}{(\gamma + \chi \frac{\gamma I_2}{\rho + \chi I_1^2} I_1)} + \frac{(2I_1 - I_2) \chi I_2}{(\gamma + \chi \frac{\gamma I_1}{\rho + \chi I_2^2} I_2)} \right] .
\]  

(A12)

Direct computation from the expression of \( \text{Var}_1(\tilde{v}|\tilde{p}) \) in (10) shows:

\[
\frac{\partial \text{Var}_1(\tilde{v}|\tilde{p})}{\partial I_1} = 2\chi \rho (I_1 + I_2) \frac{(\rho + \chi I_2^2 - \chi I_1 I_2)}{[2\rho + \chi (I_1 - I_2)^2]^2} > 0,
\]  

(A13)

\[
\frac{\partial \text{Var}_1(\tilde{v}|\tilde{p})}{\partial I_2} = 2\chi \rho (I_1 + I_2) \frac{(\rho + \chi I_1^2 - \chi I_1 I_2)}{[2\rho + \chi (I_1 - I_2)^2]^2} > 0,
\]  

(A14)

where the inequalities follow from equations (A2) and (A3), namely, \( (\rho + I_j^2 \chi - I_1 I_2 \chi) = \frac{\gamma I_1}{\chi} > 0 \).

By chain rule:

\[
\frac{d\text{Var}_1(\tilde{v}|\tilde{p})}{d\Delta} = \frac{d\text{Var}_1(\tilde{v}|\tilde{p})}{dI_1} \frac{dI_1}{d\Delta} + \frac{d\text{Var}_1(\tilde{v}|\tilde{p})}{dI_2} \frac{dI_2}{d\Delta}.
\]

Plugging equations (A11)-(A14) into the above equation delivers:

\[
\frac{d\text{Var}_1(\tilde{v}|\tilde{p})}{d\Delta} = M \gamma \lambda \chi^2 \rho (I_1^2 - I_2^2) (I_1 + I_2) \left[ \frac{\rho^3 + \chi \rho^2 (I_1 + I_2)^2}{[2\rho + \chi (I_1 - I_2)^2]^2 \gamma^2 (\rho + \chi I_2^2) (\rho + \chi I_2^2)^2} \right] > 0,
\]

because \( I_1 > I_2 \) by \( \lambda_1 > \lambda_2 \). QED.

**Proof of Proposition 4:**

The idea of proving Proposition 4 goes as follows. As we mentioned in the main text, there are four possible types of information market equilibria, depending on whether \( \lambda_1 \) and \( \lambda_2 \) are zero or positive. For any given parameter configuration \( (\rho, \chi, \gamma) \in \mathbb{R}^3_{++} \), we
identify all those values of \((c_1, c_2) \in \mathbb{R}^2_{++}\) that support an information market equilibrium of each type. We then show that the union of those identified values of \((c_1, c_2)\) forms the whole space of \(\mathbb{R}^2_{++}\), which implies the existence of an information market equilibrium. In addition, we show that each parameter configuration can only support a unique information market equilibrium.

**Case 1.** \(\lambda_1 = \lambda_2 = 0\):

By equation (13), we have \(I_1 = I_2 = 0\) in this case. Since no trader finds acquiring any information to be beneficial in equilibrium, we have \(\phi_i (0, 0) \leq c_i\) for \(i = 1, 2\). By equations (10) and (23), we can express \(\phi_i (I_1, I_2)\) as follows:

\[
\phi_i (I_1, I_2) = \frac{1}{2\gamma} \log \left( \frac{\rho + I_j^2 \chi_j}{I_1^2 \rho + I^2 \rho^2 \chi^{-1}} \right), \quad \text{for } i, j = 1, 2 \text{ and } j \neq i. \tag{A15}
\]

So, \(\phi_i (0, 0) = \frac{\log 2}{2\gamma}\). The set of \((c_1, c_2)\) supporting an equilibrium of \((\lambda_1 = \lambda_2 = 0)\) is

\[
S_0 \equiv \left\{ (c_1, c_2) \in \mathbb{R}^2_{++} : c_1 \geq \frac{\log 2}{2\gamma} \text{ and } c_2 \geq \frac{\log 2}{2\gamma} \right\}.
\]

**Case 2.** \(\lambda_1 > 0\) and \(\lambda_2 = 0\):

By equation (13) and \(\lambda_1 > 0\) and \(\lambda_2 = 0\), we have:

\[
I_1 = \lambda_1 \rho \gamma^{-1} \text{ and } I_2 = 0
\]

and hence,

\[
\lambda_1 = \gamma \rho^{-1} I_1. \tag{A16}
\]

So, the condition of \(\lambda_1 > 0\) implies \(I_1 > 0\).

Since in this case, traders only acquire signal \(\tilde{v}_1\), the value \(\phi_1\) of signal \(\tilde{v}_1\) must be equal to its cost \(c_1\) and the value \(\phi_2\) of signal \(\tilde{v}_2\) must be no larger than its cost \(c_2\); that is,

\[
\phi_1 (I_1, 0) = c_1 \text{ and } \phi_2 (I_1, 0) \leq c_2.
\]
Thus, the set of \((c_1, c_2)\) supporting an equilibrium of \((\lambda_1 > 0, \lambda_2 = 0)\) is

\[
S_1 \equiv \{(c_1, c_2) \in \mathbb{R}^2_+ : c_1 = \phi_1(I_1, 0), c_2 \geq \phi_2(I_1, 0) \text{ for all } I_1 > 0\}.
\]

We next analytically characterize the set \(S_1\). By (A15), we have

\[
\phi_1(I_1, 0) = \frac{1}{2\gamma} \log \left( \frac{I_1^2 + 2\rho \chi^{-1}}{I_1^2 + \rho \chi^{-1}} \right),
\]

which is decreasing in \(I_1\). Since \(c_1 = \phi_1(I_1, 0)\), the range of \(c_1\) in \(S_1\) is \((\lim_{I_1 \to \infty} \phi_1(I_1, 0), \phi_1(0, 0)) = \left(0, \frac{\log 2}{2\gamma}\right)\).

The lower bound of \(c_2\) is \(\phi_2(I_1, 0)\). By (A15), we have

\[
\phi_2(I_1, 0) = \frac{1}{2\gamma} \log \left( \frac{(\rho + I_1^2 \chi)(I_1^2 + 2\rho \chi^{-1})}{I_1^2 \rho + \rho^2 \chi^{-1}} \right),
\]

which is increasing in \(I_1\). Combining with \(c_1 = \phi_1(I_1, 0)\), we can cancel \(I_1\) and express the lower bound of \(c_2\) as a decreasing function in \(c_1\): \(\frac{1}{2\gamma} \log \left(\frac{e^{2\gamma c_1}}{e^{2\gamma c_1} - 1}\right)\).

So, we can analytically characterize \(S_1\) as follows:

\[
S_1 = \left\{(c_1, c_2) \in \mathbb{R}^2_+ : c_1 \in \left(0, \frac{\log 2}{2\gamma}\right) \text{ and } c_2 \geq \frac{1}{2\gamma} \log \left(\frac{e^{2\gamma c_1}}{e^{2\gamma c_1} - 1}\right)\right\}.
\]

In addition, for any \((c_1, c_2) \in S_1\), there exists a unique information market equilibrium with \(\lambda_1 > 0\) and \(\lambda_2 = 0\). Specifically, from \(\phi_1(I_1, 0) = \frac{1}{2\gamma} \log \left(\frac{I_1^2 + 2\rho \chi^{-1}}{I_1^2 + \rho \chi^{-1}}\right) = c_1\), we can determine a unique \(I_1 > 0\). Then, using \(\lambda_1 = \gamma \rho^{-1} I_1\) in (A16), we can compute a unique \(\lambda_1\).

**Case 3.** \(\lambda_1 = 0\) and \(\lambda_2 > 0\):

This is symmetric to the above case, and the set of \((c_1, c_2)\) supporting an equilibrium of \((\lambda_1 = 0, \lambda_2 > 0)\) is

\[
S_2 \equiv \left\{(c_1, c_2) \in \mathbb{R}^2_+ : c_2 \in \left(0, \frac{\log 2}{2\gamma}\right) \text{ and } c_1 \geq \frac{1}{2\gamma} \log \left(\frac{e^{2\gamma c_2}}{e^{2\gamma c_2} - 1}\right)\right\}.
\]

Also, for any \((c_1, c_2) \in S_2\), there exists a unique information market equilibrium with \(\lambda_1 = 0\) and \(\lambda_2 > 0\).

**Case 4.** \(\lambda_1 > 0\) and \(\lambda_2 > 0\):
To establish this result, we first characterize the two constant boundaries of the two functions bounding
\( I_1 > 0, \ I_2 > 0, \ \rho + I_2^2 \chi - I_1 I_2 \chi > 0 \) and \( \rho + I_1^2 \chi - I_1 I_2 \chi > 0 \).

Given \( \lambda_1 > 0 \) and \( \lambda_2 > 0 \), the information market equilibrium implies:
\[
\phi_1 (I_1, I_2) = c_1 \text{ and } \phi_2 (I_1, I_2) = c_2.
\]

Thus, the set of \((c_1, c_2)\) supporting an equilibrium of \((\lambda_1 > 0, \lambda_2 > 0)\) is:
\[
S_{1,2} \equiv \left\{ (c_1, c_2) \in \mathbb{R}_++^2 : c_1 = \phi_1 (I_1, I_2), c_2 = \phi_2 (I_1, I_2), \right. \\
\text{for all } I_1 > 0, \ I_2 > 0, \text{ such that} \\
\left. \rho + I_2^2 \chi - I_1 I_2 \chi > 0 \text{ and } \rho + I_1^2 \chi - I_1 I_2 \chi > 0 \right\}.
\]

We next characterize \( S_{1,2} \). We first decompose the set \( S_{1,2} \) into two symmetric sets:
\[
S_{I_1 \geq I_2} \equiv \left\{ (c_1, c_2) \in \mathbb{R}_++^2 : c_1 = \phi_1 (I_1, I_2), c_2 = \phi_2 (I_1, I_2), \right. \\
\text{for all } (I_1, I_2) \in \mathbb{R}_++^2 \text{ such that } I_2 \leq I_1 < I_2 + \frac{\rho}{I_1 \chi} \right\}
\]
and
\[
S_{I_1 \leq I_2} \equiv \left\{ (c_1, c_2) \in \mathbb{R}_++^2 : c_1 = \phi_1 (I_1, I_2), c_2 = \phi_2 (I_1, I_2), \right. \\
\text{for all } (I_1, I_2) \in \mathbb{R}_++^2 \text{ such that } I_1 \leq I_2 < I_1 + \frac{\rho}{I_1 \chi} \right\}.
\]

Apparently, \( S_{1,2} = S_{I_1 \geq I_2} \cup S_{I_1 \leq I_2} \).

Given the symmetry of the two sets \( S_{I_1 \geq I_2} \) and \( S_{I_1 \leq I_2} \), we will analyze \( S_{I_1 \geq I_2} \) only. Basically, we will show that
\[
S_{I_1 \geq I_2} = \left\{ (c_1, c_2) \in \mathbb{R}_+^2 : c_1 \in \left( 0, \frac{\log 2}{2\gamma} \right) \text{ and } c_1 \leq c_2 < \frac{1}{2\gamma} \log \left( \frac{e^{2\gamma c_1}}{e^{2\gamma c_1} - 1} \right) \right\}. \tag{A17}
\]
To establish this result, we first characterize the two constant boundaries of \( c_1 \), and then characterize the two functions bounding \( c_2 \) for a given \( c_1 \).

In allocations supported by parameters in \( S_{I_1 \geq I_2} \), the cost \( c_1 \) is given by \( c_1 = \phi_1 (I_1, I_2) = \frac{1}{2\gamma} \log \left( \frac{\rho + I_2^2 \chi}{I_1^2 \rho + I_1^2 \rho + \rho^2 \chi - 1} \right) \) by \( \text{(A15)} \), which decreases with \( I_1 \). Thus, for a given \( I_2 > 0 \), \( c_1 \) achieves its infimum at \( I_1 = I_2 + \frac{\rho}{I_2 \chi} \) and its maximum at \( I_1 = I_2 \). That is, \( \phi_1 \left( I_2 + \frac{\rho}{I_2 \chi}, I_2 \right) < \)
c_1 \leq \phi_1 (I_2, I_2). Direct computation shows \( \phi_1 \left( I_2 + \frac{\rho}{I_2 \chi}, I_2 \right) = 0. \) Thus, for any given \( I_2 > 0, \) we have \( c_1 \in (0, \phi_1 (I_2, I_2)] \). Given \( I_2 \) can take any positive value in \( S_{I_1 \geq I_2}, \) the range of \( c_1 \) is given by

\[
\bigcup_{I_2 > 0} (0, \phi_1 (I_2, I_2)] = \left( 0, \max_{I_2 > 0} \phi_1 (I_2, I_2) \right] = (0, \phi_1 (0, 0)) = \left( 0, \frac{\log 2}{2 \gamma} \right).
\]

We now fix any \( c_1 \in \left( 0, \frac{\log 2}{2 \gamma} \right) \) and find all the corresponding values of \( c_2 \) in \( S_{I_1 \geq I_2} \) as follows. Given that \( \phi_1 (I_2, I_2) \) decreases with \( I_2, \) there exists a unique \( \overline{I}_{2,c_1}, \) which is determined by \( \phi_1 (\overline{I}_{2,c_1}, \overline{I}_{2,c_1}) = c_1. \) The pairs of \((I_1, I_2)\) that can be supported by the value of \( c_1 \) in \( S_{I_1 \geq I_2} \) must satisfy \( I_2 \leq \overline{I}_{2,c_1}: \) Otherwise, for any \( I_2 > \overline{I}_{2,c_1} \) and \( I_1 \in [I_2, I_2 + \frac{\rho}{I_2 \chi}], \)
we have \( \phi_1 (I_1, I_2) < \phi_1 (I_2, I_2) < \phi_1 (I_{2,c_1}, I_{2,c_1}) = c_1. \) For any \( I_2 \in (0, I_{2,c_1}], \) there exists a unique \( I_{1,I_2,c_1} \) that generates \( c_1 \) through \( \phi_1 (I_{1,I_2,c_1}, I_2) = c_1 \) and then determines the value of \( c_2 \) through \( c_2 = \phi_2 (I_{1,I_2,c_1}, I_2). \) By doing so, for the given \( c_1, \) all the corresponding values of \( c_2 \) can be generated through varying \( I_2 \in (0, \overline{I}_{2,c_1}]; \) that is, for the given \( c_1, \) we determine the constant \( \overline{I}_{2,c_1}, \) and then for any \( I_2 \in (0, \overline{I}_{2,c_1}], \) we have \( c_2 = \phi_2 (I_{1,I_2,c_1}, I_2), \) where \( I_{1,I_2,c_1} \)
is determined by \( \phi_1 (I_{1,I_2,c_1}, I_2) = c_1. \)

By the chain rule, we have:

\[
\frac{dc_2}{dI_2} = \frac{\partial \phi_2 (I_{1,I_2,c_1}, I_2)}{\partial I_{1,I_2,c_1}} \frac{dI_{1,I_2,c_1}}{dI_2} + \frac{\partial \phi_2 (I_{1,I_2,c_1}, I_2)}{\partial I_2},
\]

and by applying the implicitly function theorem to \( \phi_1 (I_{1,I_2,c_1}, I_2) = c_1, \) we can find

\[
\frac{dI_{1,I_2,c_1}}{dI_2} = -\frac{\partial \phi_1 (I_{1,I_2,c_1}, I_2) / \partial I_2}{\partial \phi_1 (I_{1,I_2,c_1}, I_2) / \partial I_{1,I_2,c_1}}.
\]

Plugging this equation into \( \frac{dc_2}{dI_2}, \) and using the expression forms of \( \phi_1 (I_1, I_2), \) we can show that \( \frac{dc_2}{dI_2} < 0. \) That is, \( c_2 = \phi_2 (I_{1,I_2,c_1}, I_2) \) decreases with \( I_2 \) for \( I_2 \in (0, \overline{I}_{2,c_1}]. \) Thus, for a given \( c_1, \) the lower bound for \( c_2 \) is \( \phi_2 \left( I_{1,I_2,c_1}, \overline{I}_{2,c_1} \right) \) and the upper bound is \( \phi_2 (I_{1,0,c_1}, 0). \) Using the fact of \( \phi_1 (\overline{I}_{2,c_1}, \overline{I}_{2,c_1}) = c_1 \) and \( \phi_1 (I_{1,I_2,c_1}, I_2) = c_1, \) we can show that \( \phi_2 \left( I_{1,I_2,c_1}, \overline{I}_{2,c_1} \right) = c_1 \) and \( \phi_2 (I_{1,0,c_1}, 0) = \frac{1}{2} \left( \frac{e^{2\gamma c_1}}{e^{2\gamma c_1} - 1} \right). \) This completes the proof of expression (A17).
We can obtain a similar expression for \( S_{I_i \leq I_2} \), and thus \( S_{1,2} \) is given by:

\[
S_{1,2} = \left\{ (c_1, c_2) \in \mathbb{R}_{++}^2 : c_1 \in \left( 0, \frac{\log 2}{2\gamma} \right) \text{ and } c_1 \leq c_2 < \frac{1}{2\gamma} \log \left( \frac{e^{2\gamma c_1}}{e^{2\gamma c_1} - 1} \right) \right\}
\]

\[
\cup \left\{ (c_1, c_2) \in \mathbb{R}_{++}^2 : c_2 \in \left( 0, \frac{\log 2}{2\gamma} \right) \text{ and } c_2 \leq c_1 < \frac{1}{2\gamma} \log \left( \frac{e^{2\gamma c_2}}{e^{2\gamma c_2} - 1} \right) \right\}
\]

\[
= \left\{ (c_1, c_2) \in \mathbb{R}_{++}^2 : (e^{2\gamma c_1} - 1)(e^{2\gamma c_2} - 1) < 1 \right\}
\]

The above proof also shows that for any \((c_1, c_2) \in S_{1,2}\), there exists a unique pair of \((I_1, I_2)\) that is supported by \((c_1, c_2)\) in this case. Take \(c_1 \leq c_2\) as an example. For a given \(c_1\), we know that the supported pair \((I_1, I_2)\) takes the form of \((I_{1,2,c_1}, I_2)\), where \(\phi_1(I_{1,2,c_1}, I_2) = c_1, \phi_2(I_{1,2,c_1}, I_2) = c_2\), and \(I_2 \in (0, 0, I_{2,c_1}]\). However, given \(\frac{dc_2}{dI_2} < 0\), different values of \(I_2 \in (0, I_{2,c_1}]\) will be supported by different values of \(c_2\). Thus, each \((c_1, c_2) \in S_{1,2}\) can only support a unique pair \((I_1, I_2)\). Then, using equation (A9), we can determine a unique pair \((\lambda_1, \lambda_2)\) through the determined \((I_1, I_2)\).

Finally, by carefully checking the boundaries of the \(S\) sets, we see that their union forms the whole parameter space \(\mathbb{R}_{++}^5\). This means that for any exogenous parameter configuration \((c_1, c_2, \rho, \chi, \gamma) \in \mathbb{R}_{++}^5\), there exists an information market equilibrium. In addition, all of the \(S\) sets are mutually exclusive. Given that we have established that any parameter configuration in each \(S\) set supports only one information market equilibrium, the information market equilibrium is unique. QED.

**Proof of Proposition 5:**

By equations (21) and (A10), we have:

\[
\frac{dI_i}{d\lambda_i} = \mathcal{M} \frac{\gamma (\rho + \chi I_j^2)}{(\gamma + \lambda_i \chi I_j)^2} > 0 \quad \text{and} \quad \frac{dI_j}{d\lambda_i} = \frac{(2I_i - I_j) \chi I_j}{\rho + \chi I_i^2} \frac{dI_i}{d\lambda_i}, \text{ for } i, j = 1, 2 \text{ and } j \neq i. \quad (A18)
\]
By the chain rule, the absolute magnitude of the Grossman-Stiglitz effect is:
\[ \frac{1}{2\gamma} \frac{\partial \log (\text{Var}^{-1}(\tilde{v}|\tilde{p}))}{\partial \lambda_i} = \frac{\text{Var}(\tilde{v}|\tilde{p})}{2\gamma} \left( \frac{\partial \text{Var}^{-1}(\tilde{v}|\tilde{p})}{\partial I_i} \frac{dI_i}{d\lambda_i} + \frac{\partial \text{Var}^{-1}(\tilde{v}|\tilde{p})}{\partial I_j} \frac{dI_j}{d\lambda_i} \right). \]

Then, plugging the expressions of \( \text{Var}(\tilde{v}|\tilde{p}) \) (equation (10)), \( \frac{\partial \text{Var}^{-1}(\tilde{v}|\tilde{p})}{\partial I_i} \) and \( \frac{\partial \text{Var}^{-1}(\tilde{v}|\tilde{p})}{\partial I_j} \) (equations (A13) and (A14)), and \( \frac{dI_j}{d\lambda_i} \) (equation (A18)) into the above expression, we have:
\[ \frac{1}{2\gamma} \frac{\partial \log (\text{Var}^{-1}(\tilde{v}|\tilde{p}))}{\partial \lambda_i} = \frac{\chi (I_1 + I_2) (\rho^2 + \chi \rho I_i (I_1 + I_2) + \chi^2 I_1 I_2 (I_1 - I_2)^2) dI_i}{\gamma (\rho + \chi I_i^2)(\rho + \chi I_i^2 + \chi I_j^2)(2\rho + \chi (I_1 - I_2)^2) d\lambda_i} > 0, \]
(A19)

for \( i = 1, 2 \).

Now we prove \( \frac{\partial \phi_i}{\partial \lambda_i} < 0 \). By equation (23) and the expressions of \( \frac{dI_j}{d\lambda_i} \) in (A18) and \( \frac{1}{2\gamma} \frac{\partial \log (\text{Var}^{-1}(\tilde{v}|\tilde{p}))}{\partial \lambda_i} \) in (A19), we have:
\[ \frac{\partial \phi_i}{\partial \lambda_i} = \frac{1}{2\gamma} \frac{2I_j \chi}{\rho + I_j^2 \chi} \frac{dI_j}{d\lambda_i} - \frac{1}{2\gamma} \frac{\partial \log (\text{Var}^{-1}(\tilde{v}|\tilde{p}))}{\partial \lambda_i} = \left[ \frac{I_j \chi}{\gamma (\rho + I_j^2 \chi)} \frac{(2I_i - I_j) \chi I_j}{\rho + \chi I_j^2} - \chi (I_1 + I_2) \frac{(\rho^2 + \chi \rho I_i (I_1 + I_2) + \chi^2 I_1 I_2 (I_1 - I_2)^2) dI_i}{\gamma (\rho + \chi I_i^2)(\rho + \chi I_i^2 + \chi I_j^2)(2\rho + \chi (I_1 - I_2)^2) d\lambda_i} \right] \]
for \( i, j = 1, 2 \) and \( j \neq i \). Direct computation shows that bracket term in the above equation is negative:
\[ -\chi I_i \left[ \rho^2 I_1 + \rho^2 I_2 + \chi \rho I_i^3 + \chi \rho I_j^3 + \chi I_i (I_1 - I_2)^2 + 2\chi \rho I_i^2 I_j + \chi^2 I_j (2I_j^2 + I_j^2) (I_1 - I_j^2) \right] < 0, \]
\[ \lambda_i (\rho + \chi I_i^2)(\rho + \chi I_j^2)(\rho + \chi I_i^2 + \chi I_j^2)(2\rho + \chi (I_1 - I_2)^2) \]
where we have used the fact of \( (\rho + \chi I_i^2 - \chi I_i I_2) = \frac{dI_i}{d\lambda_i} \). Thus, \( \frac{\partial \phi_i}{\partial \lambda_i} < 0 \).

Next, we prove \( \frac{\partial \phi_i}{\partial \lambda_j} > 0 \) if and only if \( I_j > I_i \). Following the similar argument as above, we can show:
\[ \frac{\partial \phi_i}{\partial \lambda_j} = \frac{1}{2\gamma} \frac{2I_j \chi}{\rho + I_j^2 \chi} \frac{dI_j}{d\lambda_j} - \frac{1}{2\gamma} \frac{\partial \log (\text{Var}^{-1}(\tilde{v}|\tilde{p}))}{\partial \lambda_j} = \frac{\chi (I_j - I_i) (\rho + \chi I_j^2 - \chi I_i I_2)^2}{\gamma (\rho + \chi I_j^2)(2\rho + \chi I_j^2 - 2\chi I_i I_2)(\rho + \chi I_j^2 + \chi I_j^2) d\lambda_j}, \]
for \( i, j = 1, 2 \) and \( j \neq i \). Since \( \frac{dI_i}{d\lambda_j} > 0 \), we have \( \frac{\partial \phi_i}{\partial \lambda_j} > 0 \) if and only if \( I_j > I_i \). QED.
Proof of Proposition 6:

Setting $\phi_i (I_1, I_2) = c_i$ for $i = 1, 2$ delivers the following system:

\[
\begin{align*}
\log (\rho + I_2^2 \chi) - \log (Var^{-1} (\tilde{v} | \tilde{p}^*)) = 2\gamma c_1, \\
\log (\rho + I_1^2 \chi) - \log (Var^{-1} (\tilde{v} | \tilde{p}^*)) = 2\gamma c_2.
\end{align*}
\]  

(A20)

Suppose we decrease $c_1$. Applying the implicit function theorem to the above system delivers:

\[
\begin{align*}
\left\{\begin{array}{l}
-V ar (\tilde{v} | \tilde{p}^*) \frac{\partial Var^{-1} (\tilde{v} | \tilde{p}^*)}{\partial I_1^*} \frac{dI_1^*}{dc_1} + \left(-V ar (\tilde{v} | \tilde{p}^*) \frac{\partial Var^{-1} (\tilde{v} | \tilde{p}^*)}{\partial I_2^*} + \frac{2I_1^* \chi}{\rho + I_1^2 \chi}\right) \frac{dI_2^*}{dc_1} = 2\gamma, \\
-V ar (\tilde{v} | \tilde{p}^*) \frac{\partial Var^{-1} (\tilde{v} | \tilde{p}^*)}{\partial I_1^*} + \frac{2I_1^* \chi}{\rho + I_1^2 \chi} = 0.
\end{array}\right.
\end{align*}
\]  

Thus,

\[
\frac{dI_1^*}{dc_1} = -\frac{2\gamma V ar (\tilde{v} | \tilde{p}^*)}{D} \frac{\partial Var^{-1} (\tilde{v} | \tilde{p}^*)}{\partial I_1^*} \quad \text{and} \quad \frac{dI_2^*}{dc_1} = -\frac{2\gamma}{D} \left( \frac{V ar (\tilde{v} | \tilde{p}^*)}{\partial I_2^*} + \frac{2I_1^* \chi}{\rho + I_1^2 \chi} \right),
\]  

(A21)

where

\[
D = \frac{4\chi^2 (I_1^* + I_2^*) (\rho + \chi I_1^2 - \chi I_1^* I_2^*) ( \rho + \chi I_1^2 - \chi I_1^* I_2^*)}{(\rho + \chi I_1^2) (\rho + \chi I_1^2) (\rho + \chi I_1^2 + \chi I_2^2) (2\rho + \chi (I_1^* - I_2^*)^2)} > 0.
\]

We now derive the effect on $\lambda_1^*$ and $\lambda_2^*$ of decreasing $c_1$. By equation (A9),

\[
\frac{d\lambda_1^*}{dc_1} = \gamma \frac{(\rho + I_2^2 \chi) \frac{dI_1^*}{dc_1} + (I_1^* - 2I_2^*) I_1^* \chi \frac{dI_2^*}{dc_1}}{(\rho + I_2^2 \chi - I_1^* I_2^*)^2}.
\]

Then, plugging equation (10), (A13), (A14) and (A21) into the above expression yields:

\[
\frac{d\lambda_1^*}{dc_1} = \frac{-4\gamma^2 (\rho + \chi I_1^2 - \chi I_1^* I_2^*)}{D (\rho + I_2^2 \chi - I_1^* I_2^*)^2} \left(\frac{\rho^2 I_1^* + \rho^2 I_2^* + \chi \rho I_1^* I_2^* + \chi \rho I_1^3 + \chi \rho I_1^2 I_2^* + 2\chi \rho I_1^* I_2^*}{\rho + \chi I_1^2 - \chi I_1^* I_2^*} + \chi I_1^* (I_1^* - I_2^*)^2 \right) < 0.
\]

Similarly, we can compute:

\[
\frac{d\lambda_2^*}{dc_1} = \gamma \frac{I_2^* (I_2^* - 2I_1^*) \chi \frac{dI_1^*}{dc_1} + (\rho + I_1^2 \chi) \frac{dI_1^*}{dc_1}}{(\rho + \chi I_2^2 - \chi I_1^* I_2^*)^2} = \frac{-4\gamma^2 (I_1^* - I_2^*)}{D (\rho + \chi I_1^2 + \chi I_2^2) (2\rho + \chi (I_1^* - I_2^*)^2)}.
\]

Thus, \(\frac{d\lambda_1^*}{dc_1} < 0\) if and only if $I_1^* > I_2^*$. 

55
Note that we have
\[ I_1^* > I_2^* \Leftrightarrow c_1 < c_2, \]
because the system of determining \( I_1^* \) and \( I_2^* \) in (A20) implies:
\[
\log \left( \rho + I_2^{*2} \chi \right) - \log \left( \rho + I_1^{*2} \chi \right) = 2 \gamma (c_1 - c_2).
\]
Therefore, \( \frac{d \Delta \chi}{dc_1} < 0 \) if and only if \( c_1 < c_2 \).

Finally, we show \( \frac{d \text{Var}_{\chi}(\vec{v} || \hat{p}^*)}{dc_1} < 0 \). By the chain rule,
\[
\frac{d \text{Var}_{\chi}(\vec{v} || \hat{p}^*)}{dc_1} = \frac{\partial \text{Var}_{\chi}(\vec{v} || \hat{p}^*)}{\partial I_1^*} \frac{d I_1^*}{dc_1} + \frac{\partial \text{Var}_{\chi}(\vec{v} || \hat{p}^*)}{\partial I_2^*} \frac{d I_2^*}{dc_1}.
\]
Plugging equation (A21) into the above equation delivers:
\[
\frac{d \text{Var}_{\chi}(\vec{v} || \hat{p}^*)}{dc_1} = -\frac{2 \gamma}{D} \frac{\partial \text{Var}_{\chi}(\vec{v} || \hat{p}^*)}{\partial I_2^*} \frac{2 I_1^* \chi}{\rho + I_1^{*2} \chi} < 0.
\]
QED.
References


Kondor, Péter, 2012, The more we know about the fundamental, the less we agree on the price, *Review of Economic Studies* 79, 1175-1207.


Footnotes

1There is a growing literature on sources for complementarities in financial markets. In Section V., we discuss a few papers that are most closely related to ours in that they consider different types of information/fundamentals.

2Several papers in the finance literature have also specified that the value of the traded security is affected by more than one fundamental; e.g., Goldman (2005), Yuan (2005), and Kondor (2012), among others.

3It is well known that the behavior of such noise traders can be endogenized based on hedging needs or other non-informational motives to trade. For simplicity, we take their behavior here to be exogenous.

4Obviously, to prevent the model from becoming trivial, we need to add an additional noise component to the payoff in this case.

5The recent work of Breon-Drish (2013) suggests that in the CARA-normal setup the linear equilibrium is unique among the class of continuous equilibria.

6To see this formally, note that a unit increase in $\tilde{v}_i$ will cause the group of $\tilde{v}_i$-informed traders to buy $\lambda_i \frac{\partial D_i(\tilde{p}, \tilde{v}_i)}{\partial \tilde{v}_i}$ more stocks. If it happens that the noise traders supply the same number of extra shares, then, by the market clearing condition, the price will not change. That is, changing $\tilde{v}_i$ by one unit has the same price impact as changing $\tilde{x}$ by $\lambda_i \frac{\partial D_i(\tilde{p}, \tilde{v}_i)}{\partial \tilde{v}_i}$ units – i.e., $\left| \frac{\partial \tilde{p}}{\partial \tilde{v}_i} \right| = \left| \frac{\partial \tilde{p}}{\partial \tilde{x}} \lambda_i \frac{\partial D_i(\tilde{p}, \tilde{v}_i)}{\partial \tilde{v}_i} \right|$ or $\alpha_i = \alpha_x \lambda_i \frac{\partial D_i(\tilde{p}, \tilde{v}_i)}{\partial \tilde{v}_i}$ by the price function (2).

7Our notion of strategic complement/substitute is standard in the literature. For example, Paul (1993, p. 1476) wrote: “Two variables are strategic complements if the equilibrium response to an increase in one variable is for the other variable to increase and vice versa” and “(t)wo variables are strategic substitutes if the equilibrium response to an increase in one variable is for the other variable to decrease and vice versa.”

8We show that in equilibrium it is never the case that both response functions are simultaneously
negatively sloped.

9Note that \( \frac{\partial h_j}{\partial I_i} = 0 \).

10As before, we use the word “strategic” to capture the notion that traders interact in information-acquisition behaviors, because their decisions on acquiring information affect each other’s incentives to do so. See related discussions in Section V. However, we note that our traders are “small” in the sense that they do not account for any effect that their behavior has on the cross-sectional distributions of information or on the equilibrium asset price. Their optimal strategies are affected by the strategies of others in a way that generates strategic substitutabilities or complementarities.

11Formally, we have \( \frac{\partial \phi_i}{\partial I_j} = \frac{1}{2} \frac{\partial \log (\rho + I_j^2)}{\partial I_j} \frac{dI_j}{d\lambda_i} - \frac{1}{2} \frac{\partial \log \left( \frac{1}{\text{Var}(v_l)} \right)}{\partial \lambda_i} \). So, the strength of the uncertainty reduction effect \( \frac{\partial \log (\rho + I_j^2)}{\partial I_j} \) is related to \( \frac{dI_j}{d\lambda_i} \). By Corollary 1, we have \( \frac{dI_j}{d\lambda_i} = \frac{\partial h_j}{\partial I_i} \frac{dI_i}{d\lambda_i} \), which can be positive or negative, depending on the sign of \( \frac{\partial h_j}{\partial I_i} \). If \( \frac{\partial h_j}{\partial I_i} < 0 \), then \( \frac{dI_j}{d\lambda_i} < 0 \) and the uncertainty reduction effect works in the same direction as the Grossman-Stiglitz effect. If \( \frac{\partial h_j}{\partial I_i} > 0 \), we have \( \frac{\partial h_j}{\partial I_i} < 1 \) by equation (A8), and hence the effect of \( \lambda_i \) on \( I_j \) is smaller than its effect on \( I_i \); therefore, the uncertainty reduction effect is limited.

12Of course, this finding is also consistent with a standard “unidimensional” model where the single fundamental pertains to technical information. However, the standard model does not speak to the implications for traders informed of other type of information, which is the key prediction of our mechanism.

13Note that, as in the baseline model, we still assume that there are a large number of traders in the economy, so that in equilibrium, there exist traders who optimally choose to stay uninformed.

14The proofs for these results are still similar to, although much more complicated than, those in the baseline model. That is, we fix \((k, \rho, \chi, \gamma, \omega) \in \mathbb{R}^5_++\), and characterize the sets of \((c_1, c_2)\) that support a particular type of equilibrium, and then show these sets are mutually exclusive and their union forms the whole space \( \mathbb{R}^2_+ \) of \((c_1, c_2)\).
Figure 1. **Trader-type distribution in the baseline model.** This figure plots the regimes of trader types in equilibrium in the space of \((c_1, c_2)\) for the baseline model. The absolute risk aversion coefficient is \(\gamma = 3\).
This figure plots the regimes of trader types in equilibrium in the space of \((c_1, c_2)\) for the extended model in Section IV. The parameter values are: \(\rho = 50\), \(\chi = 50\), \(\omega = 25\) and \(\gamma = 3\), which implies \(\bar{k}_1 = 0.020\) and \(\bar{k}_2 = 0.028\). Parameter \(k\) takes the value of \(0.022\), \(0.016\), and \(0\) in Panels (b), (c), and (d), respectively.

**Figure 2: Trader-type distribution in the extended model.** This figure plots the regimes of trader types in equilibrium in the space of \((c_1, c_2)\) for the extended model in Section IV. The parameter values are: \(\rho = 50\), \(\chi = 50\), \(\omega = 25\) and \(\gamma = 3\), which implies \(\bar{k}_1 = 0.020\) and \(\bar{k}_2 = 0.028\). Parameter \(k\) takes the value of \(0.022\), \(0.016\), and \(0\) in Panels (b), (c), and (d), respectively.
Internet Appendix to “Information Diversity and Complementarities in Trading and Information Acquisition”

Itay Goldstein and Liyan Yang

This appendix provides formal results and proofs for the extension in Section IV, where traders are allowed to acquire the two signals simultaneously. We show that our main results hold in this extended economy. That is, (1) at the trading stage, there is strategic complementarity in trading, and information diversity increases the price informativeness; and (2) at the learning stage, there is strategic complementarity in information acquisition.

1 Setup

We briefly recap the setup of the extension. The payoff of the risky asset as follows:

\[ \tilde{v} = \tilde{v}_1 + \tilde{v}_2 + \tilde{w}, \]  

(IA1)

where \( \tilde{v}_i \sim N(0, 1/\rho) (i = 1, 2) \) is still the forecastable fundamental and \( \tilde{w} \sim N(0, 1/\omega) \) (with \( \omega > 0 \)) is the residual noise. The random variables \( \tilde{v}_1, \tilde{v}_2, \) and \( \tilde{w} \) are mutually independent.

Traders can acquire the signal \( \tilde{v}_i \) separately at cost \( c_i > 0 \) for \( i = 1, 2 \). They can also acquire both \( \tilde{v}_1 \) and \( \tilde{v}_2 \), but at a cost of \( c_1 + c_2 + k \) where \( k \geq 0 \). For the ease of expression, we denote

\[ \tilde{v}_{12} \equiv \tilde{v}_1 + \tilde{v}_2 \text{ and } c_{12} \equiv c_1 + c_2 + k, \]  

(IA2)

which respectively correspond to the signal and the information-acquisition cost associated with those traders who observe both signals \( \tilde{v}_1 \) and \( \tilde{v}_2 \).

In the trading stage, there are potentially four types of traders: (1) \( \tilde{v}_1 \)-informed traders observing \( \tilde{v}_1 \) (of mass \( \lambda_1 \geq 0 \)), (2) \( \tilde{v}_2 \)-informed traders observing \( \tilde{v}_2 \) (of mass \( \lambda_2 \geq 0 \)), (3) \( \tilde{v}_{12} \)-informed traders observing \( \tilde{v}_1 \) and \( \tilde{v}_2 \) (of mass \( \lambda_{12} \geq 0 \)), and (4) uninformed traders (of
mass $\lambda_n > 0$).

All other features of the model are still the same as the baseline model. That is, traders have CARA utility function with a risk-aversion parameter $\gamma > 0$. There are two tradable assets – the stock and the bond. All four types of traders condition their trades on the stock price $\tilde{p}$. Noise traders trade a random amount $\tilde{x} \sim N(0, 1/\chi)$ (with $\chi > 0$) of the stock, which is independent of the realizations of $\tilde{v}_1, \tilde{v}_2$ and $\tilde{w}$.

## 2 Financial Market Equilibrium Characterization

We still consider linear REE with the price function given by

$$\tilde{p} = \alpha_0 + \alpha_1 \tilde{v}_1 + \alpha_2 \tilde{v}_2 + \alpha_x \tilde{x}, \quad (IA3)$$

where the coefficients $\alpha$’s are endogenous.

Given the CARA-normal setup, the demand function takes the following universal expression:

$$D_t(\tilde{p}, \tilde{v}_i) = \frac{E(\tilde{v}_i | F_t) - \tilde{p}}{\gamma \text{Var} (\tilde{v}_i | F_t)}$$

for $t \in \{1, 2, 12, u\}$.

For $\tilde{v}_i$-informed traders, their information set is $F_i = \{\tilde{p}, \tilde{v}_i\}$. The price $\tilde{p}$ is still equivalent to the signal in predicting $\tilde{v}_j$:

$$\tilde{s}_{ji} \equiv \frac{\tilde{p} - \alpha_0 - \alpha_i \tilde{v}_i}{\alpha_j} = \tilde{v}_j + (\alpha_x/\alpha_j) \tilde{x}, \text{ for } i, j = 1, 2 \text{ and } i \neq j,$$

and hence the relevant moments are:

$$E(\tilde{v}_i | F_i) = \tilde{v}_i + E(\tilde{v}_j | \tilde{s}_{ji}) = \tilde{v}_i + \frac{(\alpha_j/\alpha_x)^2 \chi \tilde{s}_{ji}}{\rho + (\alpha_j/\alpha_x)^2 \chi}, \quad (IA4)$$

$$\text{Var}(\tilde{v}_i | F_i) = \text{Var}(\tilde{v}_j | \tilde{p}, \tilde{v}_i) + \text{Var}(\tilde{w} | \tilde{p}, \tilde{v}_i) = [\rho + (\alpha_j/\alpha_x)^2 \chi]^{-1} + \omega^{-1}. \quad (IA5)$$

So, their demand function is:

$$D_i(\tilde{p}, \tilde{v}_i) = \frac{E(\tilde{v}_i | F_i) - \tilde{p}}{\gamma \text{Var} (\tilde{v}_i | F_i)} = \tilde{v}_i + \frac{(\alpha_j/\alpha_x)^2 \chi \tilde{s}_{ji}}{\rho + (\alpha_j/\alpha_x)^2 \chi} - \tilde{p}. \quad (IA6)$$

$\tilde{v}_{12}$-informed traders have an information set $F_{12} = \{\tilde{p}, \tilde{v}_1, \tilde{v}_2\}$. The conditional moments in their demand are:

$$E(\tilde{v} | F_{12}) = E(\tilde{v}_1 + \tilde{v}_2 + \tilde{w} | \tilde{p}, \tilde{v}_{12}) = \tilde{v}_1 + \tilde{v}_2, \quad (IA7)$$

$$\text{Var}(\tilde{v} | F_{12}) = \text{Var}(\tilde{v}_1 + \tilde{v}_2 + \tilde{w} | \tilde{p}, \tilde{v}_{12}) = \text{Var}(\tilde{w} | \tilde{p}, \tilde{v}_{12}) = \omega^{-1}, \quad (IA8)$$

and hence their demand function is:

$$D_{12}(\tilde{p}, \tilde{v}_1, \tilde{v}_2) = \frac{E(\tilde{v} | F_{12}) - \tilde{p}}{\gamma \text{Var} (\tilde{v} | F_{12})} = \tilde{v}_1 + \tilde{v}_2 - \tilde{p}. \quad (IA9)$$
The trading intensity on information $\tilde{v}_i$ is defined as:

$$I_i \equiv \lambda_i \frac{\partial D_i(\tilde{p}_i, \tilde{v}_i)}{\partial \tilde{v}_i} + \lambda_{12} \frac{\partial D_{12}(\tilde{p}, \tilde{v}_{12})}{\partial \tilde{v}_i}$$

for $i = 1, 2$. (IA10)

As we did for the main model, we can still show $I_i = \frac{\alpha_i}{\alpha_x}$. Using this result, the definition of $I_i$ in (IA10) and the expressions of $D_i(\tilde{p}_i, \tilde{v}_i)$ and $D_{12}(\tilde{p}, \tilde{v}_{12})$ in (IA6) and (IA9), we can obtain the best response functions:

$$I_i = h_i (I_j; \lambda_i, \lambda_{12}, \rho, \gamma, \chi, \omega) \equiv \frac{(\rho + I_j^2 \chi) (\lambda_i + \lambda_{12}) + \lambda_{12} \omega}{\gamma [1 + (\rho + I_j^2 \chi) \omega^{-1}] + \lambda_i I_j \chi},$$

for $i, j = 1, 2$ and $i \neq j$. (IA11)

In order to compute the equilibrium, we use the two best response functions to form one equation in terms of $I_1$:

$$I_1 = \frac{\left(\rho + \left(\frac{(\rho + I_j^2 \chi)(\lambda_2 + \lambda_{12}) + \lambda_{12} \omega}{\gamma [1 + (\rho + I_j^2 \chi) \omega^{-1}] + \lambda_2 I_j \chi}\right)^2 \chi \right) (\lambda_1 + \lambda_{12}) + \lambda_{12} \omega}{\gamma [1 + \left(\rho + \left(\frac{(\rho + I_j^2 \chi)(\lambda_2 + \lambda_{12}) + \lambda_{12} \omega}{\gamma [1 + (\rho + I_j^2 \chi) \omega^{-1}] + \lambda_2 I_j \chi}\right)^2 \chi \right) \omega^{-1}] + \lambda_1 \left(\frac{(\rho + I_j^2 \chi)(\lambda_2 + \lambda_{12}) + \lambda_{12} \omega}{\gamma [1 + (\rho + I_j^2 \chi) \omega^{-1}] + \lambda_2 I_j \chi}\right) \chi}. $$

The above equation always admits a positive solution by the intermediate value theorem. However, it is difficult to establish the uniqueness, as the above equation is a fifth order polynomial. Our simulation analysis suggests that the equilibrium is always unique. Once we have computed $I_1 = \frac{\alpha_i}{\alpha_x}$, the derivations for $\alpha$’s are standard. To summarize, we have the following proposition.

**Proposition A1** For any $\lambda_1 > 0$, $\lambda_2 > 0$ and $\lambda_{12} \geq 0$, there exists a linear rational-expectations equilibrium with the price function given by equation (IA3).

Still, when $\frac{\partial h_i}{\partial I_j} > 0$, we say that trading on $\tilde{v}_i$ is a complement to trading on $\tilde{v}_j$, and when $\frac{\partial h_i}{\partial I_j} < 0$, we say that trading on $\tilde{v}_i$ is a substitute to trading on $\tilde{v}_j$. Through direct computation, we can get the following proposition which extends Proposition 2 in the paper.

**Proposition A2** Trading on information $\tilde{v}_i$ is a complement (substitute) to trading on information $\tilde{v}_j$ if and only if trading intensity $I_j$ is sufficiently high (low). That is, $\frac{\partial h_i}{\partial I_j} > 0$ if and only if $I_j > \frac{-\gamma + \sqrt{\gamma^2 + \chi (\lambda_{12} + \lambda_i) (\rho \lambda_i + (\omega + \rho) \lambda_{12})}}{\chi (\lambda_{12} + \lambda_i)}$. 
3 Implications of Trading Intensity Interactions

3.1 A Useful Lemma

The following lemma puts some restrictions on the admissible values of \((I_1, I_2)\) in equilibrium and it is very useful for the subsequent proofs.

**Lemma R** For any exogenously given \(\lambda_1 > 0, \lambda_2 > 0\) and \(\lambda_{12} \geq 0\), we have: (a) \((\gamma I_i - \lambda_{12} \omega) > 0\) and (b) \(\rho + I_2^2 \chi - I_i I_j \chi > 0\), for \(i, j = 1, 2\), and \(i \neq j\).

**Proof.** Using the best response function (IA11), we can express \(\lambda_i\) as functions of \((I_1, I_2)\) and other exogenous parameters as follows:

\[
\lambda_i = \frac{(\gamma I_i - \lambda_{12} \omega) (1 + (\rho + I_i^2 \chi) \omega^{-1})}{\rho + I_i^2 \chi - I_i I_j \chi} \quad \text{for} \quad i = 1, 2. \tag{IA12}
\]

By the assumption of \(\lambda_i > 0\), we must have \(\frac{\gamma I_i - \lambda_{12} \omega}{\rho + I_i^2 \chi - I_i I_j \chi} > 0\). Now we show that both its numerator and its denominator are positive. Given that \((\gamma I_i - \lambda_{12} \omega)\) and \((\rho + I_i^2 \chi - I_i I_j \chi)\) are positive, or one is positive and the other is negative. We discuss case by case. If \(\rho + I_2^2 \chi - I_1 I_2 \chi > 0\) and \(\rho + I_1^2 \chi - I_i I_j \chi > 0\), then \(\lambda_1 > 0\) and \(\lambda_2 > 0\) requires \(\lambda_{12} < \gamma \omega^{-1} \min \{I_1, I_2\}\). In contrast, if one expression is positive and the other is negative, for example, if \(\rho + I_2^2 \chi - I_1 I_2 \chi > 0\) and \(\rho + I_1^2 \chi - I_i I_j \chi < 0\), then we must have \(\rho + I_2^2 \chi - I_1 I_2 \chi > 0 > \rho + I_1^2 \chi - I_i I_j \chi \Rightarrow I_2 > I_1\). However, if \(\rho + I_2^2 \chi - I_1 I_2 \chi > 0\), we will have \((I_1 \gamma \omega^{-1} - \lambda_{12}) > 0\) by \(\lambda_1 > 0\), and at the same time, if \(\rho + I_1^2 \chi - I_1 I_2 \chi < 0\), we will have \((I_2 \gamma \omega^{-1} - \lambda_{12}) < 0\), which implies \((I_1 \gamma \omega^{-1} - \lambda_{12}) > 0 > (I_2 \gamma \omega^{-1} - \lambda_{12}) \Rightarrow I_1 > I_2\). A contradiction. ■

3.2 Trading Intensity Multiplier

We can follow similar steps as in the main text and obtain a proposition on trading intensity multiplier as follows.

**Proposition A3** For any exogenously given \(\lambda_1 > 0, \lambda_2 > 0\) and \(\lambda_{12} \geq 0\), the effect of an exogenous parameter \(Q \in \{\lambda_1, \lambda_2, \lambda_{12}, \rho, \gamma, \chi, \omega\}\) on the trading intensity \(I_i\) about \(\bar{v}_i\) is given
by:

\[
\frac{dI_i}{dQ} = M \left( \frac{\partial h_i}{\partial Q} + \frac{\partial h_i}{\partial I_j} \frac{\partial I_j}{\partial Q} \right),
\]

(IA13)

where the term \( \left( \frac{\partial h_i}{\partial Q} + \frac{\partial h_i}{\partial I_j} \frac{\partial I_j}{\partial Q} \right) \) captures the “direct effect” of changing \( Q \) on \( I_i \) and the coefficient \( M \) is a “multiplier” given by:

\[
M = \left( 1 - \frac{\partial h_1}{\partial I_2} \frac{\partial h_2}{\partial I_1} \right)^{-1} > 0.
\]

(IA14)

**Proof.** Similar to the proof of Proposition 3 in the paper, we take total differentiation of the best response functions (for \( i = 1, 2 \)) with respect to \( Q \) and obtain:

\[
\frac{dI_1}{dQ} = \frac{\partial h_1}{\partial Q} + \frac{\partial h_1}{\partial I_2} \frac{dI_2}{dQ} \quad \text{and} \quad \frac{dI_2}{dQ} = \frac{\partial h_2}{\partial Q} + \frac{\partial h_2}{\partial I_1} \frac{dI_1}{dQ}.
\]

Solving for \( \frac{dI_1}{dQ} \) and \( \frac{dI_2}{dQ} \) delivers

\[
\frac{dI_1}{dQ} = \frac{\frac{\partial h_1}{\partial Q} + \frac{\partial h_1}{\partial I_2} \frac{dI_2}{dQ}}{1 - \frac{\partial h_1}{\partial I_2} \frac{\partial h_2}{\partial I_1}} \quad \text{and} \quad \frac{dI_2}{dQ} = \frac{\frac{\partial h_2}{\partial Q} + \frac{\partial h_2}{\partial I_1} \frac{dI_1}{dQ}}{1 - \frac{\partial h_1}{\partial I_2} \frac{\partial h_2}{\partial I_1}},
\]

which is equations (IA13) and (IA14). So, we are only left to show \( M > 0 \).

Computing \( \frac{\partial h_i}{\partial I_j} \) from the best response function (IA11) and then plugging in equation (IA12), we can express \( \frac{\partial h_i}{\partial I_j} \) as follows:

\[
\frac{\partial h_i}{\partial I_j} = \frac{\chi (\gamma I_i - \omega \lambda_{12}) - \omega I_i + 2\omega I_j - \rho I_i + \gamma I_i I_j^2}{\omega + \rho + \gamma I_j^2} - \frac{\gamma \rho + \gamma \chi I_j^2 - \chi \omega \lambda_{12} I_j}{\gamma \rho + \gamma \chi I_j^2 - \chi \omega \lambda_{12} I_j}, \quad \text{for} \ i, j = 1, 2 \ \text{and} \ j \neq i.
\]

(IA15)

So,

\[
\frac{1}{M} = 1 - \frac{\partial h_1}{\partial I_2} \frac{\partial h_2}{\partial I_1} = \frac{(\gamma I_1 - \omega \lambda_{12})(\gamma I_2 - \omega \lambda_{12}) Z_M}{(\omega + \rho + \gamma I_2^2)(\omega + \rho + \gamma I_1^2)(\gamma \rho + \gamma \chi I_2^2 - \chi \omega \lambda_{12} I_2)(\gamma \rho + \gamma \chi I_1^2 - \chi \omega \lambda_{12} I_1)},
\]

where

\[
Z_M = (\omega + \rho + \gamma I_2^2) \left( \frac{\gamma \rho}{\gamma I_2 - \omega \lambda_{12}} + \chi I_2 \right) (\omega + \rho + \gamma I_1^2) \left( \frac{\gamma \rho}{\gamma I_1 - \omega \lambda_{12}} + \chi I_1 \right) - \chi (-\omega I_1 + 2\omega I_2 - \rho I_1 + \gamma I_1 I_2^2) \chi (-\omega I_2 + 2\omega I_1 - \rho I_2 + \gamma I_2 I_1^2).
\]

Note that by Lemma R, we have \((\gamma I_1 - \omega \lambda_{12}) > 0\) and \((\gamma I_2 - \omega \lambda_{12}) > 0\), and hence \((\gamma \rho + \gamma \chi I_2^2 - \chi \omega \lambda_{12} I_2) > 0\), and \((\gamma \rho + \gamma \chi I_1^2 - \chi \omega \lambda_{12} I_1) > 0\). Thus, \(M\) has the same sign as \(Z_M\).
By \((\gamma_1 - \omega \lambda_{12}) > 0\) and \((\gamma_2 - \omega \lambda_{12}) > 0\), we have:
\[
Z_M > (\omega + \rho + \chi I_2^2) (\chi I_2) (\omega + \rho + \chi I_1^2) (\chi I_1)
- \chi (-\omega I_1 + 2\omega I_2 - \rho I_1 + \chi I_1 I_2^2) \chi (-\omega I_2 + 2\omega I_1 - \rho I_2 + \chi I_2 I_1^2)
= 2\chi^2 (\omega (I_1 - I_2)^2 + \rho (I_1^2 + I_2^2)) (\omega + \chi I_1 I_2) > 0.
\]
Therefore, \(M > 0\). \(\square\)

### 3.3 The Effect of the Size \(\lambda_i\)

The following proposition shows that Corollary 1 in our main text is robust in this extended economy.

**Corollary A1** For any exogenously given \(\lambda_1 > 0\), \(\lambda_2 > 0\) and \(\lambda_{12} \geq 0\), an increase in the size \(\lambda_i\) of \(\bar{v}_i\)-informed traders population

(a) increases the trading intensity \(I_i\) on information \(\bar{v}_i\) (i.e., \(\frac{dI_i}{d\lambda_i} > 0\));

(b) increases the trading intensity \(I_j\) on information \(\bar{v}_j\) if and only if trading on \(\bar{v}_j\) is complementary to trading on \(\bar{v}_i\) (i.e., \(\frac{dI_j}{d\lambda_i} > 0\) if and only if \(\frac{\partial h_i}{\partial \lambda_i} > 0\)); and

(c) increases the trading intensity \(I_j\) on information \(\bar{v}_j\) if \(\lambda_i\) is sufficiently large (i.e., \(\frac{dI_j}{d\lambda_i} > 0\) if \(\lambda_i\) is sufficiently large).

**Proof.** Suppose we increase \(\lambda_1\). By Proposition A3, we can compute
\[
\frac{dI_1}{d\lambda_1} = M \frac{\partial h_1}{\partial \lambda_1} \quad \text{and} \quad \frac{dI_2}{d\lambda_1} = M \frac{\partial h_2}{\partial \lambda_1} \cdot \frac{\partial h_1}{\partial I_1} \frac{dI_1}{d\lambda_1}.
\] (IA16)
In order to sign \(\frac{dI_1}{d\lambda_1}\) and \(\frac{dI_2}{d\lambda_1}\), we need to compute \(\frac{\partial h_1}{\partial \lambda_i}\) and \(\frac{\partial h_2}{\partial \lambda_i}\).

By (IA11), direct computation shows
\[
\frac{\partial h_i}{\partial \lambda_i} = \frac{(\rho + I_j^2 \chi) (\gamma (1 + (\rho + I_i^2 \chi) \omega^{-1}) + \lambda_i I_j \chi) - ((\rho + I_j^2 \chi) (\lambda_i + \lambda_{12}) + \lambda_{12} \omega) I_j \chi}{(\gamma (1 + (\rho + I_j^2 \chi) \omega^{-1}) + \lambda_i I_j \chi)^2},
\]
for \(i, j = 1, 2\) and \(i \neq j\). Then, plugging in \(\lambda_i = \frac{(\gamma \lambda_i - \lambda_{12} \omega)(1 + (\rho + I_j^2 \chi) \omega^{-1})}{\rho + I_j^2 \chi - \lambda_i I_j \chi}\) (i.e., equation (IA12)) into the above expression, we have:
\[
\frac{\partial h_i}{\partial \lambda_i} = \frac{\omega (\rho + \chi I_j^2 - \chi I_1 I_2)}{(\omega + \rho + \chi I_2^2) (\gamma \rho + \gamma I_j^2 - \chi \omega \lambda_{12} I_j)}, \text{for } i, j = 1, 2 \text{ and } j \neq i.
\] (IA17)
So, we have \(\frac{\partial h_i}{\partial \lambda_i} > 0\), since \(\gamma \chi I_j^2 - \chi \omega \lambda_{12} I_j > 0\) by Lemma R. Also, by (IA16), we have
\[
\frac{dI_1}{d\lambda_1} = M \frac{\partial h_1}{\partial \lambda_i} > 0,
\]
which establishes parts (a) and (b) of Proposition A1.
Now let us prove part (c). By (IA15) and the fact of \((\gamma I_2 - \omega \lambda_{12}) > 0\) in Lemma R, we have
\[
\frac{\partial h_2}{\partial I_1} = \chi (\gamma I_2 - \omega \lambda_{12}) + \omega I_2 + 2\omega I_1 - \rho I_2 + \chi I_2 I_1^2 > 0 \iff (-\omega I_2 + 2\omega I_1 - \rho I_2 + \chi I_2 I_1^2) > 0.
\]
Now suppose \(\lambda_1 \to \infty\). First, we show that as \(\lambda_1\) increases, \(I_2\) is always bounded for a given \(\lambda_2 > 0\) and \(\lambda_{12} \geq 0\). To see this, note that \((\gamma I_2 - \omega \lambda_{12}) > 0\) in Lemma R, and by (IA11),
\[
I_2 = \frac{(\rho + I_2^2 \chi)(\lambda_2 + \lambda_{12}) + \lambda_{12} \omega}{\gamma (1 + (\rho + I_2^2 \chi) \omega^{-1}) + \lambda_2 I_1 \chi} < \frac{(\rho + I_2^2 \chi)(\lambda_2 + \lambda_{12}) + (\lambda_2 + \lambda_{12}) \omega}{\gamma (1 + (\rho + I_2^2 \chi) \omega^{-1})} = \frac{(\lambda_2 + \lambda_{12}) \omega}{\gamma}
\]
and as a result, \(I_2 \in \left(\frac{\lambda_{12} \omega}{\gamma}, \frac{(\lambda_2 + \lambda_{12}) \omega}{\gamma}\right)\) is bounded for a given \(\lambda_2 > 0\) and \(\lambda_{12} \geq 0\).

Second, by (IA11), as \(\lambda_1 \to \infty\), we have:
\[
I_1 = \frac{(\rho + I_2^2 \chi)(\lambda_1 + \lambda_{12}) + \lambda_{12} \omega}{\gamma (1 + (\rho + I_2^2 \chi) \omega^{-1}) + \lambda_1 I_2 \chi} \to \frac{\rho + I_2^2 \chi}{I_2 \chi} = I_2 + \frac{\rho}{I_2 \chi}.
\]
This implies as \(\lambda_1 \to \infty\),
\[
-\omega I_2 + 2\omega I_1 - \rho I_2 + \chi I_2 I_1^2 \\
\rightarrow -\omega I_2 + 2\omega \left(I_2 + \frac{\rho}{I_2 \chi}\right) - \rho I_2 + \chi I_2 \left(I_2 + \frac{\rho}{I_2 \chi}\right)^2 \\
= \frac{\rho^2 + \chi^2 I_2^4 + 2\omega \rho + \chi \omega I_2^2 + \chi \rho I_2^2}{\chi I_2} > 0.
\]
Thus, \(\frac{\partial h_2}{\partial I_1} > 0\) for sufficiently large \(\lambda_1\). ■

### 3.4 The Effect of Information Diversity

For any exogenous \(\lambda_{12} \geq 0\), we fix \(\lambda_1 + \lambda_2 = \Lambda > 0\) and then define information diversity as \(\Delta \equiv 1 - \frac{\lambda_1 - \lambda_{12}}{\lambda_1 + \lambda_{12}}\). We can still show that information diversity improves price informativeness, which extends Corollary 2 in the paper.

**Corollary A2** When total amount of information is fixed (i.e., when \(\lambda_1 + \lambda_2\) and \(\lambda_{12}\) are fixed), information diversity increases the price informativeness regarding \(\tilde{v}\), that is, \(\frac{d\text{Var}^{-1}(\tilde{v}|\tilde{p})}{d\Delta} > 0\).

**Proof.** Without loss of generality, we assume that \(\lambda_1 > \lambda_2\), and as a result, increasing diversity \(\Delta\) while fixing \(\Lambda\) is equivalent to decreasing \(\lambda_1\) and increasing \(\lambda_2\).

We prove this proposition in two steps. First, we will use Proposition A3 to compute \(\frac{dI_1}{d\Delta}\) and \(\frac{dI_2}{d\Delta}\) and then use the chain rule to get the expression of \(\frac{d\text{Var}^{-1}(\tilde{v}_1 + \tilde{v}_2|\tilde{p})}{d\Delta}\). Second, we will
sign this expression of \( \frac{d\text{Var}(\tilde{v}_1 + \tilde{v}_2|\tilde{p})}{d\Delta} \), and this step is a bit complicated.

\[
\text{Step 1. Compute the Expression of } \frac{d\text{Var}(\tilde{v}_1 + \tilde{v}_2|\tilde{p})}{d\Delta}.
\]

As in the proof for Corollary 2 in the paper, we can still show

\[
\frac{d\lambda_1}{d\Delta} = -\frac{\Lambda}{2} \quad \text{and} \quad \frac{d\lambda_2}{d\Delta} = \frac{\Lambda}{2}.
\]

Note that \( \lambda_i \) only directly affects \( h_i \) but not \( h_j \), and hence:

\[
\frac{\partial h_1}{\partial \Delta} = \frac{\partial h_1}{\partial \lambda_1} \frac{d\lambda_1}{d\Delta} = -\frac{\partial h_1}{\partial \lambda_1} \frac{\Lambda}{2} \quad \text{and} \quad \frac{\partial h_2}{\partial \Delta} = \frac{\partial h_2}{\partial \lambda_2} \frac{d\lambda_2}{d\Delta} = \frac{\partial h_2}{\partial \lambda_2} \frac{\Lambda}{2}.
\]

Thus, by Proposition A3, we have:

\[
\frac{dI_1}{d\Delta} = \mathcal{M} \left( \frac{\partial h_1}{\partial \Delta} + \frac{\partial h_1}{\partial \lambda_1} \frac{dI_1}{d\Delta} \right) = \mathcal{M} \left( \frac{\partial h_1}{\partial \lambda_1} \frac{\Lambda}{2} \right), \tag{IA18}
\]

\[
\frac{dI_2}{d\Delta} = \mathcal{M} \left( \frac{\partial h_2}{\partial \Delta} + \frac{\partial h_2}{\partial \lambda_1} \frac{dI_1}{d\Delta} \right) = \mathcal{M} \left( \frac{\partial h_2}{\partial \lambda_1} \frac{\Lambda}{2} \right). \tag{IA19}
\]

Given

\[
\text{Var}(\tilde{v}|\tilde{p}) = \text{Var}(\tilde{v}_1 + \tilde{v}_2|\tilde{p}) + \omega^{-1},
\]

we know \( \frac{d\text{Var}^{-1}(\tilde{v}|\tilde{p})}{d\Delta} \) and \( \frac{d\text{Var}(\tilde{v}_1 + \tilde{v}_2|\tilde{p})}{d\Delta} \) have opposite signs. So, we compute \( \frac{d\text{Var}(\tilde{v}_1 + \tilde{v}_2|\tilde{p})}{d\Delta} \).

By the chain rule and equations (IA18) and (IA19),

\[
\frac{d\text{Var}(\tilde{v}_1 + \tilde{v}_2|\tilde{p})}{d\Delta} = \frac{\partial \text{Var}(\tilde{v}_1 + \tilde{v}_2|\tilde{p})}{\partial I_1} \frac{dI_1}{d\Delta} + \frac{\partial \text{Var}(\tilde{v}_1 + \tilde{v}_2|\tilde{p})}{\partial I_2} \frac{dI_2}{d\Delta} = \mathcal{M} \frac{\Lambda}{2} Z_V, \tag{IA20}
\]

where

\[
Z_V = -\left( \frac{\partial \text{Var}(\tilde{v}_1 + \tilde{v}_2|\tilde{p})}{\partial I_1} \frac{dI_1}{d\Delta} + \frac{\partial \text{Var}(\tilde{v}_1 + \tilde{v}_2|\tilde{p})}{\partial I_2} \frac{dI_2}{d\Delta} \right) \frac{\partial h_1}{\partial \lambda_1} + \left( \frac{\partial \text{Var}(\tilde{v}_1 + \tilde{v}_2|\tilde{p})}{\partial I_1} \frac{dI_1}{d\Delta} + \frac{\partial \text{Var}(\tilde{v}_1 + \tilde{v}_2|\tilde{p})}{\partial I_2} \frac{dI_2}{d\Delta} \right) \frac{\partial h_2}{\partial \lambda_2}. \tag{IA21}
\]

Thus, \( \frac{d\text{Var}(\tilde{v}_1 + \tilde{v}_2|\tilde{p})}{d\Delta} \) have the same sign as \( Z_V \).

We can show that the expression

\[
\text{Var}(\tilde{v}_1 + \tilde{v}_2|\tilde{p}) = \frac{(I_1 - I_2)^2 + 2\rho \chi^{-1}}{I_1^2 \rho + I_2^2 \rho + \rho^2 \chi^{-1}} \tag{IA22}
\]

still holds. So, we can directly compute:

\[
\frac{\partial \text{Var}(\tilde{v}_1 + \tilde{v}_2|\tilde{p})}{\partial I_i} = -2\chi (I_1 + I_2) \left( \rho + \chi I_j^2 - \chi I_i I_j \right) \quad \text{for } i, j = 1, 2 \text{ and } j \neq i. \tag{IA23}
\]

Plugging in the expressions of \( \frac{\partial \text{Var}(\tilde{v}_1 + \tilde{v}_2|\tilde{p})}{\partial I_i} \) and \( \frac{\partial h_i}{\partial \lambda_i} \) (i.e., (IA23), (IA15) and (IA17)) into equation (IA21), and simplifying, we can show that \( \frac{d\text{Var}(\tilde{v}_1 + \tilde{v}_2|\tilde{p})}{d\Delta} \) have the same sign as the
As a result, configuration, we can have a pair following expression:

\[ H(I_1, I_2; \lambda_{12}, \rho, \omega, \chi, \gamma) \]

\[ \equiv (\omega + \rho + \chi I_1^2) (\gamma \rho + \gamma \chi I_1^2 - \chi \omega \lambda_{12} I_1) (\rho + \chi I_2^2 - \chi I_1 I_2)^3 \]

\[ - (\rho + \chi I_2^2 - \chi I_1 I_2) (\rho + \chi I_1^2 - \chi I_1 I_2)^2 \chi (\gamma I_1 - \omega \lambda_{12}) (2\omega I_2 - \omega I_1 - \rho I_1 + \chi I_1 I_2) \]

\[ - (\omega + \rho + \chi I_2^2) (\gamma \rho + \gamma \chi I_2^2 - \chi \omega \lambda_{12} I_2) (\rho + \chi I_1^2 - \chi I_1 I_2)^3 \]

\[ + (\rho + \chi I_1^2 - \chi I_1 I_2) (\rho + \chi I_2^2 - \chi I_1 I_2)^2 \chi (\gamma I_2 - \omega \lambda_{12}) (2\omega I_1 - \omega I_2 - \rho I_2 + \chi I_1 I_2) \]

Step 2. Prove \( H(I_1, I_2; \lambda_{12}, \rho, \omega, \chi, \gamma) < 0 \):

We show that the above expression \( H(I_1, I_2; \lambda_{12}, \rho, \omega, \chi, \gamma) \) is always negative (and hence \( \frac{d\text{Var}(\hat{\alpha}_1 + \hat{\alpha}_2 | \hat{\beta})}{d\Delta} < 0 \) or \( \frac{d\text{Var}^{-1}(\hat{\alpha}_1 + \hat{\alpha}_2 | \hat{\beta})}{d\Delta} > 0 \)) as follows. For any given exogenous parameter configuration, we can have a pair \((I_1, I_2)\) defining the financial market equilibrium. Then, we fix \((I_1, I_2)\) and parameters other than \(\gamma\), and view \( H(I_1, I_2; \lambda_{12}, \rho, \omega, \chi, \gamma) \) as a function of \(\gamma\). We know that \( \gamma I_1 - \lambda_{12} \omega > 0 \) by Lemma R, that is, \( \gamma > \frac{\lambda_{12} \omega}{I_1} \) and \( \gamma > \frac{\lambda_{12} \omega}{I_2} \), which are two fixed values (once \((I_1, I_2)\) is fixed), and we will show that \( H(I_1, I_2; \lambda_{12}, \rho, \omega, \chi, \gamma) \) is decreasing and that \( H(I_1, I_2; \lambda_{12}, \rho, \omega, \chi, \gamma) \) is negative at the larger value of \( \frac{\lambda_{12} \omega}{I_1} \), \( \frac{\lambda_{12} \omega}{I_2} \). As a result, \( H(I_1, I_2; \lambda_{12}, \rho, \omega, \chi, \gamma) < 0 \).

First, we prove \( \frac{\partial H(I_1, I_2; \lambda_{12}, \rho, \omega, \chi, \gamma)}{\partial \gamma} < 0 \). Direct computation shows:

\[
\frac{\partial H(I_1, I_2; \gamma, \omega, \rho, \chi, \lambda_{12})}{\partial \gamma} = -\chi (I_1 - I_2)(I_1 + I_2)
\]

\[
\times \left[ I_1 I_2 (I_1 - I_2)^2 ((I_1 - I_2)^2 (\omega + \rho) + \rho I_1 I_2) \chi^3 \\
+ \rho I_1 I_2 (I_1 - I_2)^2 (3\omega + 4\rho) + 4\rho I_1 I_2) \chi^2 \\
+ \rho^2 (\omega I_1^2 + \omega I_2^2 + 2\omega I_1 I_2 + 3\rho I_1 I_2) \chi + \omega \rho^3 \right].
\]

Given \( \lambda_1 > \lambda_2 \), we know \( I_1 > I_2 \). To see this, note that by equation (IA12):

\[
\lambda_1 = \frac{(\gamma I_1 - \lambda_{12} \omega) (1 + (\rho + \bar{T}_2 \chi) \omega^{-1})}{\rho + \bar{T}_2 \chi - I_1 I_2 \chi} > \lambda_2 = \frac{(\gamma I_2 - \lambda_{12} \omega) (1 + (\rho + \bar{T}_2 \chi) \omega^{-1})}{\rho + \bar{T}_2 \chi - I_1 I_2 \chi}
\]
Given that decreasing. is decreasing, we have

Thus, at least one of\( T_1 = (I_1 - I_2) (\gamma I_1 - \lambda I I) \) \( (\rho + I_2^2 \gamma - I_1 I_2) \gamma (I_1 - \lambda I I) (1 + (\rho + I_2^2 \gamma) \omega^{-1}) \)

\[-(\rho + I_2^2 \gamma - I_1 I_2) \gamma (I_2 - \lambda I I) (1 + (\rho + I_2^2) \omega^{-1}) \]

\[= \frac{(I_1 - I_2)}{\omega} \left[ \frac{I_1 I_2 (-\omega \lambda I_1 - \omega \lambda I_2 + 2 \gamma I_1 I_2) \gamma^2}{\omega} + \left( \omega (\gamma I_1^2 + \gamma I_2^2 - \omega \lambda I_1 - \omega \lambda I_2) \gamma \right) + \gamma \rho I_1 + I_2^2 - I_1 I_2 \right] \]

\[> 0 \]

\[\Rightarrow I_1 > I_2.\]

This in turn implies \( \frac{\partial H(I_1, I_2, \lambda, \rho, \omega, \chi, \gamma)}{\partial \gamma} < 0 \) and hence \( H(I_1, I_2; \lambda, \rho, \omega, \chi, \gamma) \) is monotonically decreasing.

Second, we show \( H\left(I_1, I_2; \lambda, \rho, \omega, \chi, \frac{\lambda I_1 \omega}{I_2}\right) < 0 \). Direct computation shows:

\[H\left(I_1, I_2; \lambda, \rho, \omega, \chi, \frac{\lambda I_1 \omega}{I_2}\right) = -\chi \omega \lambda I_1 (I_1 - I_2) (\rho + \chi I_1^2 - \chi I_1 I_2) I_1^{-1} T_1(I_1, I_2, \omega, \rho, \chi),\]

where

\[T_1(I_1, I_2, \rho, \omega, \chi) \equiv I_1 (I_1 - I_2) \left(\omega I_1^3 - \omega I_2^3 + \rho I_1^3 - \rho I_2^3 + 3 \omega I_1 I_2^2 - 3 \omega I_1^2 I_2 + 2 \rho I_1 I_2^2\right) \gamma^2 + \rho I_1 \left(5 \omega I_1^2 + 3 \omega I_2^2 + 3 \rho I_1^2 + 4 \rho I_2^2 - 8 \omega I_1 I_2 - \rho I_1 I_2\right) \gamma + \left(2 \rho^3 I_1 + 5 \omega \rho^2 I_1 + \omega \rho^2 I_2\right),\]

and

\[H\left(I_1, I_2; \lambda, \rho, \omega, \chi, \frac{\lambda I_1 \omega}{I_2}\right) = -\chi \omega \lambda I_1 (I_1 - I_2) (\rho + \chi I_1^2 - \chi I_1 I_2) I_2^{-1} T_2(I_1, I_2, \omega, \rho, \chi),\]

where

\[T_2(I_1, I_2, \rho, \omega, \chi) \equiv I_2 (I_1 - I_2) \left(\omega I_1^3 - \omega I_2^3 + \rho I_1^3 - \rho I_2^3 + 3 \omega I_1 I_2^2 - 3 \omega I_1^2 I_2 - 2 \rho I_1 I_2^2\right) \gamma^2 + \rho I_2 \left(3 \omega I_1^2 + 5 \omega I_2^2 + 4 \rho I_1^2 + 3 \rho I_2^2 - 8 \omega I_1 I_2 - \rho I_1 I_2\right) \gamma + \left(2 \rho^3 I_2 + \omega \rho^2 I_1 + 5 \omega \rho^2 I_2\right).\]

Note that

\[T_1(I_1, I_2, \rho, \omega, \chi) + T_2(I_1, I_2, \rho, \omega, \chi) = (I_1 + I_2) (2 \rho + \chi (I_1 - I_2)^2) \left(\rho^2 + 3 \omega \rho + \chi \omega (I_1 - I_2)^2 + \chi \rho (I_1^2 + I_2^2 + I_1 I_2)\right) > 0.\]

Thus, at least one of \( T_1(I_1, I_2, \rho, \omega, \chi) \) and \( T_2(I_1, I_2, \rho, \omega, \chi) \) is positive, which in turn implies that at least one of \( H\left(I_1, I_2; \lambda, \rho, \omega, \chi, \frac{\lambda I_1 \omega}{I_1}\right) \) and \( H\left(I_1, I_2; \lambda, \rho, \omega, \chi, \frac{\lambda I_1 \omega}{I_2}\right) \) is negative.

Given that \( H(I_1, I_2; \lambda_1, \rho, \omega, \chi, \cdot) \) is monotonically decreasing and that \( \frac{\lambda I_1 \omega}{I_1} < \frac{\lambda I_1 \omega}{I_2} \), we know that \( H\left(I_1, I_2; \lambda, \rho, \omega, \chi, \frac{\lambda I_1 \omega}{I_2}\right) \) must be negative. Since \( \gamma > \frac{\lambda I_1 \omega}{I_2} \) and \( H(I_1, I_2; \lambda, \rho, \omega, \chi, \gamma) \) is decreasing, we have \( H(I_1, I_2; \lambda, \rho, \omega, \chi, \gamma) < 0. \)
4 Endogenous Information Acquisition

The value of information $\tilde{v}_i$ is

$$\phi_i (I_1, I_2) = \frac{1}{2\gamma} \log \left[ \frac{\text{Var} (\tilde{v} | \tilde{p})}{\text{Var} (\tilde{v} | \tilde{v}_i, \tilde{p})} \right], \quad \text{for } i = 1, 2$$

(IA25)

and the value of seeing two signals $\tilde{v}_1$ and $\tilde{v}_2$ together is:

$$\phi_{12} (I_1, I_2) = \frac{1}{2\gamma} \log \left[ \frac{\text{Var} (\tilde{v} | \tilde{p})}{\text{Var} (\tilde{v} | \tilde{v}_1, \tilde{v}_2, \tilde{p})} \right],$$

(IA26)

where the conditional variances are given by equations (IA5), (IA8) and (IA22), and are reproduced in terms of $(I_1, I_2)$ as follows:

$$\text{Var} (\tilde{v} | \mathcal{F}_1) = \left( \rho + I_2^2 \chi \right)^{-1} + \omega^{-1},$$

(IA27)

$$\text{Var} (\tilde{v} | \mathcal{F}_2) = \left( \rho + I_1^2 \chi \right)^{-1} + \omega^{-1},$$

(IA28)

$$\text{Var} (\tilde{v} | \tilde{v}_{12}, \tilde{p}) = \omega^{-1},$$

(IA29)

$$\text{Var} (\tilde{v} | \tilde{p}) = \frac{(I_1 - I_2)^2 + 2 \rho \chi^{-1}}{I_1^2 \rho + I_2^2 \rho + \rho^2 \chi^{-1}} + \omega^{-1}.$$

(IA30)

Both $\phi_i$ and $\phi_{12}$ are functions of $I_1$ and $I_2$. So we can work directly with these two variables, and after we determine them, we can use the financial market equilibrium condition (IA11) to figure out $\lambda_1, \lambda_2$, and $\lambda_{12}$.

We still assume that there are a large number of traders existing in the market, so that some traders always choose to stay uninformed in equilibrium (i.e., $\lambda_u^* > 0$). The information market equilibrium is still defined by the usual non-deviation conditions. That is, suppose $(\lambda_1^*, \lambda_2^*, \lambda_{12}^*) \in \mathbb{R}_+^3$ is an information market equilibrium, then (i) if $\lambda_t^* > 0$ for some $t \in \{1, 2, 12\}$, then $\phi_t (I_1, I_2) = c_t$; and (ii) if $\lambda_t^* = 0$ for some $t \in \{1, 2, 12\}$, then $\phi_t (I_1, I_2) \leq c_t$ (Note that $c_{12} \equiv c_1 + c_2 + k$).

We define the following function:

$$F (I_1, I_2) \equiv \phi_{12} (I_1, I_2) - \phi_1 (I_1, I_2) - \phi_2 (I_1, I_2)$$

$$= \frac{1}{2\gamma} \log \left[ \frac{\text{Var} (\tilde{v} | \tilde{v}_1, \tilde{p}) \text{Var} (\tilde{v} | \tilde{v}_2, \tilde{p})}{\text{Var} (\tilde{v} | \tilde{v}_{12}, \tilde{p}) \text{Var} (\tilde{v} | \tilde{p})} \right].$$

(IA31)

This function is related to Admati and Pfleiderer’s (1987) notion of complements/substitutes: If $F (I_1, I_2) \geq 0$ (\leq 0) at an equilibrium allocation, then $\tilde{v}_1$ and $\tilde{v}_2$ are a complement (substitute) under the definitions of Admati and Pfleiderer (1987).
4.1 Existence and Uniqueness of Information Market Equilibrium

We can show that for any \((c_1, c_2, k, \rho, \gamma, \chi, \omega) \in \mathbb{R}^7_+\), there exists an information market equilibrium. In addition, the equilibrium is unique except for a set of \((c_1, c_2, k, \rho, \gamma, \chi, \omega)\) with Lebesgue measure zero in the parameter space of \(\mathbb{R}^7_+\). These results extend Proposition 4 in the paper.

The idea of proving this existence/uniqueness result goes as follows. We first characterize eight cases depending on whether \(\lambda_1, \lambda_2\) and \(\lambda_{12}\) are positive or zero, and in each case, the equilibrium conditions put certain restrictions on the trading intensities \(I_1\) and \(I_2\). By doing so, for any given parameter configuration \((k, \rho, \gamma, \chi, \omega) \in \mathbb{R}^5_+\), we can form problems to identify those values of \((c_1, c_2) \in \mathbb{R}^2_+\) that support an information market equilibrium. Second, we then analytically characterize the sets of \((c_1, c_2)\) identified in the first step, and show that their union forms the whole space of \(\mathbb{R}^2_+\), which implies the existence of information market equilibrium: For any \((c_1, c_2, k, \rho, \gamma, \chi, \omega) \in \mathbb{R}^7_+\), we can find a pair \((I_1, I_2)\) that is supported by this parameter configuration, and then we use the financial market equilibrium condition (IA11) to back out the underlying \(\lambda\)'s.

In addition, we show that the information market equilibrium is unique within each characterized set of \((c_1, c_2)\) and that those sets of \((c_1, c_2)\) are mutually exclusive, which in turn jointly imply the uniqueness of information market equilibrium. Note that this approach also analytically characterizes the equilibrium along the way.

The proof is quite complicated and so we put it at the end of this file as an independent section (Section 5). Here, we just state the following proposition (as an extension of Proposition 4 in the paper).

Proposition A4 (a) For any exogenous parameter \((c_1, c_2, k, \rho, \gamma, \chi, \omega) \in \mathbb{R}^7_+\), there exists an information market equilibrium. Except for a set of parameters with zero Lebesgue measure, the equilibrium is unique, and it has at most two types of informed traders.

(b) Let \(\bar{k}_2 \equiv k_2(\rho, \gamma, \chi, \omega) = \frac{1}{2\gamma} \log \left[ \frac{\left( \frac{\rho + \omega + \sqrt{\omega^2 + 2\rho \chi}}{2\chi} \right)^{-1} + \omega^{-1}}{\omega^{-1} \left( \frac{\rho + \omega + \sqrt{\omega^2 + 2\rho \chi}}{2\chi} \right)^{-1} + \omega^{-1}} \right] \). Then,

(i) if \(k > \bar{k}_2\), then \(\lambda_{12} = 0\) at any information market equilibrium, and there are four possible types of trader distributions supported by parameter sets with positive Lebesgue
measures: \((\lambda_1 = 0, \lambda_2 = 0, \lambda_{12} = 0)\), \((\lambda_1 > 0, \lambda_2 = 0, \lambda_{12} = 0)\), \((\lambda_1 = 0, \lambda_2 > 0, \lambda_{12} = 0)\), and \((\lambda_1 > 0, \lambda_2 > 0, \lambda_{12} = 0)\);

(ii) if \(0 < k \leq k_2\), then there are seven possible types of trader distributions supported by parameter sets with positive Lebesgue measures: 

\[(\lambda_1 = 0, \lambda_2 = 0, \lambda_{12} = 0), (\lambda_1 > 0, \lambda_2 = 0, \lambda_{12} = 0), (\lambda_1 = 0, \lambda_2 > 0, \lambda_{12} = 0), (\lambda_1 > 0, \lambda_2 > 0, \lambda_{12} = 0), (\lambda_1 > 0, \lambda_2 = 0, \lambda_{12} = 0), (\lambda_1 > 0, \lambda_2 > 0, \lambda_{12} = 0), \text{ and } (\lambda_1 = 0, \lambda_2 > 0, \lambda_{12} > 0);\]

(iii) if \(k = 0\), then there are six possible types of trader distributions supported by parameter sets with positive Lebesgue measures, and they are same as (ii) excluding the type \((\lambda_1 > 0, \lambda_2 > 0, \lambda_{12} = 0)\).

**Proof.** See Section 5. ■

### 4.2 Learning Complementarities vs. Substitutes

The following proposition extends Proposition 5 in the paper to this extended economy with residual uncertainty \(\tilde{w}\).

**Proposition A5** Suppose the economy is operating in the region of \((\lambda_1 > 0, \lambda_2 > 0, \lambda_{12} = 0)\). Acquiring information on the same fundamental is a strategic substitute: As more traders become informed of \(\tilde{v}_i\), the value \(\phi_i\) of acquiring \(\tilde{v}_i\) decreases; that is, \(\frac{\partial \phi_i}{\partial \lambda_i} < 0\). Acquiring information on different fundamentals can be a strategic substitute or a complement: As more traders become informed of \(\tilde{v}_i\), the value of acquiring \(\tilde{v}_j\) can decrease or increase, and \(\frac{\partial \phi_i}{\partial \lambda_i} > 0\) if and only if \(I_i > \frac{2(\omega + \rho)I_j}{(\chi_I^2 + \omega) + \sqrt{(\chi_I^2 + \omega)^2 + 4\chi_I^2(\omega + \rho)}}\).

**Proof.** We take \(\lambda_1\) as an example and check the sign of \(\frac{\partial \phi_1}{\partial \lambda_1}\) and \(\frac{\partial \phi_2}{\partial \lambda_1}\). We first compute \(\frac{dI_1}{d\lambda_1}\) and \(\frac{dI_2}{d\lambda_1}\) and then use the chain rule to obtain expressions of \(\frac{\partial \phi_1}{\partial \lambda_1}\) and \(\frac{\partial \phi_2}{\partial \lambda_1}\) and sign them.

By equation (IA16), we know

\[
\frac{dI_1}{d\lambda_1} = M \frac{\partial h_1}{\partial \lambda_1} \quad \text{and} \quad \frac{dI_2}{d\lambda_1} = \frac{\partial h_2}{\partial I_1}.
\]

Setting \(\lambda_{12} = 0\) in equations (IA15) and (IA17) delivers:

\[
\frac{\partial h_1}{\partial \lambda_1} = -\frac{\omega (\rho + \chi I_2^2 - \chi I_1 I_2)^2}{\gamma (\rho + \chi I_2^2)(\omega + \rho + \chi I_2^2)} \quad \text{and} \quad \frac{\partial h_2}{\partial I_1} = \chi I_2 \frac{2\omega I_1 - \omega I_2 - \rho I_2 + \chi I_1^2 I_2}{(\rho + \chi I_1^2)(\omega + \rho + \chi I_1^2)}.
\]

Thus,

\[
\frac{dI_1}{d\lambda_1} > 0 \quad \text{and} \quad \frac{dI_2}{d\lambda_1} = \chi I_2 \frac{2\omega I_1 - \omega I_2 - \rho I_2 + \chi I_1^2 I_2}{(\rho + \chi I_1^2)(\omega + \rho + \chi I_1^2)}.
\]
Next, we prove $\frac{\partial \phi_1}{\partial \lambda_1} < 0$. By the expression of $\phi_1 (I_1, I_2)$ and applying the chain rule, we have:

$$\frac{\partial \phi_1}{\partial \lambda_1} \propto \frac{1}{\text{Var} (\tilde{v} | \tilde{p})} \left[ \frac{\partial \text{Var} (\tilde{v}_1 + \tilde{v}_2 | \tilde{p})}{\partial I_1} \frac{dI_1}{d\lambda_1} + \frac{\partial \text{Var} (\tilde{v}_1 + \tilde{v}_2 | \tilde{p})}{\partial I_2} \frac{dI_2}{d\lambda_1} \right] - \frac{(\rho + I_2^2 \chi)^{-2}}{(\rho + I_2^2 \chi)^{-1} + \omega^{-1}} \frac{dI_2}{d\lambda_1} \frac{dI_2}{d\lambda_1}$$

$$= \frac{1}{\text{Var} (\tilde{v} | \tilde{p})} \left[ \frac{\partial \text{Var} (\tilde{v}_1 + \tilde{v}_2 | \tilde{p})}{\partial I_1} \frac{dI_1}{d\lambda_1} + \frac{\partial \text{Var} (\tilde{v}_1 + \tilde{v}_2 | \tilde{p})}{\partial I_2} \frac{dI_2}{d\lambda_1} \right] - \frac{(\rho + I_2^2 \chi)^{-2}}{(\rho + I_2^2 \chi)^{-1} + \omega^{-1}} \frac{dI_2}{d\lambda_1} \frac{dI_2}{d\lambda_1}$$

$$\propto \left( \frac{1}{\text{Var} (\tilde{v} | \tilde{p})} \left[ \frac{\partial \text{Var} (\tilde{v}_1 + \tilde{v}_2 | \tilde{p})}{\partial I_1} \frac{dI_1}{d\lambda_1} + \frac{\partial \text{Var} (\tilde{v}_1 + \tilde{v}_2 | \tilde{p})}{\partial I_2} \frac{dI_2}{d\lambda_1} \right] - \frac{(\rho + I_2^2 \chi)^{-2}}{(\rho + I_2^2 \chi)^{-1} + \omega^{-1}} \frac{dI_2}{d\lambda_1} \frac{dI_2}{d\lambda_1} \right) - \frac{(\rho + I_2^2 \chi)^{-2}}{(\rho + I_2^2 \chi)^{-1} + \omega^{-1}} \frac{dI_2}{d\lambda_1} \frac{dI_2}{d\lambda_1}$$

where the last equality follows from the expression of $\text{Var} (\tilde{v} | \tilde{p})$ in (IA30), $\frac{\partial \text{Var}(\tilde{v}_1 + \tilde{v}_2 | \tilde{p})}{\partial I_1}$ in (IA23), and where $N_1 (I_1, I_2; \rho, \chi, \omega)$ is a positive polynomial function of $(I_1, I_2; \rho, \chi, \omega)$.

Finally, we sign $\frac{\partial \phi_2}{\partial \lambda_1}$. By the expression of $\phi_2 (I_1, I_2)$ and applying the chain rule, we have:

$$\frac{\partial \phi_2}{\partial \lambda_1} \propto \frac{1}{\text{Var} (\tilde{v} | \tilde{p})} \left[ \frac{\partial \text{Var} (\tilde{v}_1 + \tilde{v}_2 | \tilde{p})}{\partial I_1} \frac{dI_1}{d\lambda_1} + \frac{\partial \text{Var} (\tilde{v}_1 + \tilde{v}_2 | \tilde{p})}{\partial I_2} \frac{dI_2}{d\lambda_1} \right] - \frac{(\rho + I_2^2 \chi)^{-2}}{(\rho + I_2^2 \chi)^{-1} + \omega^{-1}} \frac{dI_1}{d\lambda_1} \frac{dI_1}{d\lambda_1}$$

$$= \left( \frac{1}{\text{Var} (\tilde{v} | \tilde{p})} \left[ \frac{\partial \text{Var} (\tilde{v}_1 + \tilde{v}_2 | \tilde{p})}{\partial I_1} \frac{dI_1}{d\lambda_1} + \frac{\partial \text{Var} (\tilde{v}_1 + \tilde{v}_2 | \tilde{p})}{\partial I_2} \frac{dI_2}{d\lambda_1} \right] - \frac{(\rho + I_2^2 \chi)^{-2}}{(\rho + I_2^2 \chi)^{-1} + \omega^{-1}} \frac{dI_1}{d\lambda_1} \frac{dI_1}{d\lambda_1} \right) - \frac{(\rho + I_2^2 \chi)^{-2}}{(\rho + I_2^2 \chi)^{-1} + \omega^{-1}} \frac{dI_1}{d\lambda_1} \frac{dI_1}{d\lambda_1}$$

where the last equality follows from the expression of $\text{Var} (\tilde{v} | \tilde{p})$ in (IA30), $\frac{\partial \text{Var}(\tilde{v}_1 + \tilde{v}_2 | \tilde{p})}{\partial I_1}$ in (IA23), and $\frac{\partial \phi_2}{\partial \lambda_1}$ in (IA33).

Thus,

$$\frac{\partial \phi_2}{\partial \lambda_1} > 0 \iff \omega I_1 - \omega I_2 - \rho I_2 + \chi I_1 I_2^2 + \chi I_2^2 I_2 > 0$$

$$\iff I_1 > \frac{2I_2 (\omega + \rho)}{(\chi I_2^2 + \omega) + \sqrt{(\chi I_2^2 + \omega)^2 + 4\chi I_2^2 (\omega + \rho)}}.$$
5 Proof of Proposition A4

5.1 Defining the Sets of \((c_1, c_2)\) That Support an Equilibrium

Depending on whether the three \(\lambda\)'s are positive or zero, we consider eight cases of trader type distributions.

Case 1. \(\lambda_{12} = 0, \lambda_1 = 0\) and \(\lambda_2 = 0\)

By equation (IA11), we have \(I_1 = I_2 = 0\) in this case. No trader finds acquiring any information to be optimal, and thus, the equilibrium conditions imply:

\[
\phi_i (0, 0) \leq c_i \text{ for } i = 1, 2, \quad (IA34)
\]
\[
\phi_{12} (0, 0) \leq c_1 + c_2 + k. \quad (IA35)
\]

Thus, the values of \((c_1, c_2)\) that support the allocations in this case are given by the following set:

\[
C^1 \equiv \{(c_1, c_2) \in \mathbb{R}^2_+ : c_1 \geq \phi_1 (0, 0), c_2 \geq \phi_2 (0, 0) \text{ and } c_1 + c_2 + k \geq \phi_{12} (0, 0)\}.
\]

This set corresponds to \(S_0\) in the proof of Proposition 4 in the paper.

Case 2. \(\lambda_{12} > 0, \lambda_1 = 0\) and \(\lambda_2 = 0\)

By equation (IA11), we have:

\[
I_1 = I_2 \text{ and } \lambda_{12} = \frac{\gamma}{\omega} I_1. \quad (IA36)
\]

So, the condition of \(\lambda_{12} > 0\) implies the following restriction on \((I_1, I_2)\):

\[
I_1 = I_2 > 0. \quad (IA37)
\]

Given \(\lambda_1 = 0\) and \(\lambda_2 = 0\), no trader finds it optimal to acquire one signal separately in equilibrium, which means that the values of acquiring these signals are no larger than their respective costs, i.e.,

\[
\phi_i (I_1, I_1) \leq c_i \text{ for } i = 1, 2, \quad (IA38)
\]

where we have used the restriction of \(I_1 = I_2\) in the above condition. The fact of \(\lambda_{12} > 0\) implies that the value \(\phi_{12} (I_1, I_2)\) of observing two signals simultaneously is exactly equal to the total acquisition cost \(c_1 + c_2 + k\); that is,

\[
\phi_{12} (I_1, I_1) = c_1 + c_2 + k. \quad (IA39)
\]

Combining equations (IA38) and (IA39) delivers another restriction on \((I_1, I_2)\):

\[
F (I_1, I_1) \equiv \phi_{12} (I_1, I_1) - \phi_1 (I_1, I_1) - \phi_1 (I_1, I_1) \geq k. \quad (IA40)
\]

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Conditions (IA38) and (IA39), subject to the constraints (IA37) and (IA40) on \((I_1, I_2)\), characterize the following set of \((c_1, c_2)\) that supports the allocations in Case 1:

\[
C^2 \equiv \left\{ (c_1, c_2) \in \mathbb{R}^2_{+} : c_1 \geq \phi_1 (I, I), c_2 \geq \phi_2 (I, I), c_1 + c_2 + k = \phi_{12} (I, I) \right\}.
\]

for all \(I > 0\) such that \(F (I, I) \geq k\)

**Case 3.** \(\lambda_{12} = 0, \lambda_1 > 0\) and \(\lambda_2 = 0\)

By equation (IA11) and \(\lambda_1 > 0, \lambda_2 = 0\) and \(\lambda_{12} = 0\), we have:

\[
I_2 = 0 \text{ and } I_1 = \frac{\rho \lambda_1}{\gamma (1 + \rho \omega^{-1})} \tag{IA41}
\]

and hence

\[
\lambda_1 = \frac{\gamma (1 + \rho \omega^{-1})}{\rho} I_1. \tag{IA42}
\]

So, the condition of \(\lambda_1 > 0\) implies

\[
I_1 > 0. \tag{IA43}
\]

In this case, traders only acquire signal \(\tilde{\nu}_1\), and those actively informed traders must come from the type-1 traders. Thus, the value \(\phi_1\) of signal \(\tilde{\nu}_1\) must be equal to its cost \(c_1\):

\[
\phi_1 (I_1, 0) = c_1, \tag{IA44}
\]

where we have used the fact of \(I_2 = 0\). (If \(\lambda_1\) comes from the type-2 traders, then \(\phi_1 (I_1, 0) = c_1 + k\), which implies that all type-1 traders would acquire signal \(\tilde{\nu}_1\). A contradiction.)

The fact of \(\lambda_2 = 0\) and \(\lambda_{12} = 0\) implies

\[
\phi_2 (I_1, 0) \leq c_2, \tag{IA45}
\]

\[
\phi_{12} (I_1, 0) \leq c_1 + c_2 + k \Leftrightarrow \phi_{12} (I_1, 0) - \phi_1 (I_1, 0) - k \leq c_2, \tag{IA46}
\]

where the last inequality in (IA46) has used condition (IA44).

The two inequalities (IA45) and (IA46) define the lower bound for \(c_2\) that can support the allocations in Case 3. Depending on the sizes of the left-hand-side of these two inequalities, we have two subcases:

**Subcase 3.1.** If \(\phi_2 (I_1, 0) \geq \phi_{12} (I_1, 0) - \phi_1 (I_1, 0) - k\), or equivalently, if

\[
F (I_1, 0) \equiv \phi_{12} (I_1, 0) - \phi_1 (I_1, 0) - \phi_2 (I_1, 0) \leq k, \tag{IA47}
\]

then, the lower bound for \(c_2\) is determined by (IA45). That is, (IA44) and (IA45) (subject to the restrictions \(I_1 > 0, I_2 = 0\) and (IA47)) jointly characterize the following set of \((c_1, c_2)\) that supports equilibrium allocations in this case:

\[
C^3.1 \equiv \left\{ (c_1, c_2) \in \mathbb{R}^2_{+} : c_1 = \phi_1 (I_1, 0), c_2 \geq \phi_2 (I_1, 0), \right\}
\]

for all \(I_1 > 0\) such that \(F (I_1, 0) \leq k\).
This set $C^{3.1}$ corresponds to $S_1$ in the proof of Proposition 4 in the paper.

Subcase 3.2. If $\phi_2 (I_1, 0) < \phi_{12} (I_1, 0) - \phi_1 (I_1, 0) - k$, or equivalently, if

$$F (I_1, 0) \equiv \phi_{12} (I_1, 0) - \phi_1 (I_1, 0) - \phi_2 (I_1, 0) > k,$$

then the lower bound for $c_2$ is determined by (IA48). So, (IA44) and (IA46) (subject to the restrictions $I_1 > 0$, $I_2 = 0$ and (IA48)) jointly characterize the following set of $(c_1, c_2)$ that support equilibrium allocations in this case:

$$C^{3.2} \equiv \left\{ (c_1, c_2) \in \mathbb{R}_+^2 : c_1 = \phi_1 (I_1, 0), c_2 \geq \phi_{12} (I_1, 0) - \phi_1 (I_1, 0) - k, \right. $$

for all $I_1 > 0$ such that $F (I_1, 0) > k$

Case 4. $\lambda_{12} = 0$, $\lambda_1 = 0$ and $\lambda_2 > 0$

This case is symmetric to Case 3, and we can switch subscripts 1 and 2 and define sets $C^{4.1}$ and $C^{4.2}$ accordingly.

Case 5. $\lambda_{12} > 0$, $\lambda_1 > 0$ and $\lambda_2 = 0$

By equation (IA11) and $\lambda_{12} > 0$, $\lambda_1 > 0$ and $\lambda_2 = 0$, we have:

$$\lambda_{12} = \frac{\gamma}{\omega} I_2 > 0 \Rightarrow I_2 > 0$$

and

$$\lambda_1 = \frac{\gamma (\rho + I_2^2 \chi + \omega) (I_1 - I_2)}{\omega (\rho + I_2^2 \chi - I_1 I_2 \chi)}. $$

Given $\lambda_1 > 0$ and $\lambda_2 = 0$, we can show $I_1 > I_2$. Specifically, by (IA11) and $I_2 = \frac{\omega \lambda_{12}}{\gamma}$, we know $I_1 > I_2 \Leftrightarrow \gamma \chi I_2^2 - \chi \omega \lambda_{12} I_2 + \gamma \rho > 0 \Leftrightarrow \gamma \rho > 0$, which is clearly true. Thus, by $\lambda_1 > 0$ and equation (IA50), we have:

$$I_1 > I_2 \text{ and } \rho + I_2^2 \chi - I_1 I_2 \chi > 0. $$

The information market equilibrium conditions defining $\lambda_1 > 0$ and $\lambda_{12} > 0$ are:

$$\phi_1 (I_1, I_2) = c_1,$$

$$\phi_{12} (I_1, I_2) = c_1 + c_2 + k \Rightarrow c_2 = \phi_{12} (I_1, I_2) - \phi_1 (I_1, I_2) - k.$$  

The fact of $\lambda_2 = 0$ further implies

$$\phi_2 (I_1, I_2) \leq c_2.$$  

So, conditions (IA53) and (IA54) jointly imply:

$$F (I_1, I_2) \equiv \phi_{12} (I_1, I_2) - \phi_1 (I_1, I_2) - \phi_2 (I_1, I_2) \geq k.$$  

Thus, conditions (IA52) and (IA53), subject to restrictions (IA49), (IA51) and (IA55), jointly
determine the following set of \((c_1, c_2)\) that supports the allocations in Case 5:

\[
C^5 \equiv \left\{ (c_1, c_2) \in \mathbb{R}^2_+ : c_1 = \phi_1 (I_1, I_2), c_2 = \phi_{12} (I_1, I_2) - \phi_1 (I_1, I_2) - k \right\}.
\]

for all \(I_1 > I_2 > 0\) such that \(\rho + I_2^2 \chi - I_1 I_2 \chi > 0\) and \(F (I_1, I_2) \geq k\).

**Case 6.** \(\lambda_{12} > 0, \lambda_1 = 0\) and \(\lambda_2 > 0\)

This case is symmetric to Case 5, and we can switch subscripts 1 and 2 and define a set \(C^6\) accordingly.

**Case 7.** \(\lambda_{12} = 0, \lambda_1 > 0\) and \(\lambda_2 > 0\)

By equation (IA11) and \(\lambda_{12} = 0\), we can express \(\lambda_1\) and \(\lambda_2\) as functions of \(I_1\) and \(I_2\):

\[
\lambda_1 = \frac{\gamma (1 + (\rho + I_2^2 \chi) \omega^{-1}) I_1}{\rho + I_2^2 \chi - I_1 I_2 \chi} \quad \text{and} \quad \lambda_2 = \frac{\gamma (1 + (\rho + I_1^2 \chi) \omega^{-1}) I_2}{\rho + I_1^2 \chi - I_1 I_2 \chi},
\]

The condition of \(\lambda_1 > 0\) and \(\lambda_2 > 0\) implies the following restriction on \((I_1, I_2)\):

\[
I_1 > 0, I_2 > 0, \rho + I_2^2 \chi - I_1 I_2 \chi > 0 \quad \text{and} \quad \rho + I_1^2 \chi - I_1 I_2 \chi > 0.
\]

Given \(\lambda_1 > 0\) and \(\lambda_2 > 0\), the information market equilibrium implies:

\[
\phi_1 (I_1, I_2) = c_1 \quad \text{and} \quad \phi_2 (I_1, I_2) = c_2.
\]

The condition of \(\lambda_{12} = 0\) requires:

\[
\phi_{12} (I_1, I_2) \leq c_1 + c_2 + k,
\]

which together with (IA58) implies the following restriction on \((I_1, I_2)\):

\[
F (I_1, I_2) \equiv \phi_{12} (I_1, I_2) - \phi_1 (I_1, I_2) - \phi_2 (I_1, I_2) \leq k.
\]

Thus, condition (IA58), subject to restrictions (IA57) and (IA59), determines the following set of \((c_1, c_2)\) that supports the allocations in this case:

\[
C^7 \equiv \left\{ (c_1, c_2) \in \mathbb{R}^2_+ : c_1 = \phi_1 (I_1, I_2), c_2 = \phi_2 (I_1, I_2), \right. \]

\[
\left\{ \begin{array}{l}
\text{for all } I_1 > 0, I_2 > 0, \text{ such that } \\
\rho + I_1^2 \chi - I_1 I_2 \chi > 0, \rho + I_2^2 \chi - I_1 I_2 \chi > 0 \text{ and } F (I_1, I_2) \leq k
\end{array} \right\}.
\]

This set \(C^7\) corresponds to \(S_{1,2}\) in the proof of Proposition 4 in the paper.

**Case 8.** \(\lambda_{12} > 0, \lambda_1 > 0\) and \(\lambda_2 > 0\) (Multiplicty)

By equation (IA11), we know that

\[
I_1 > 0 \text{ and } I_2 > 0.
\]

The information market equilibrium conditions consistent with \(\lambda_{12} > 0, \lambda_1 > 0\) and \(\lambda_2 > 0\) are:

\[
\phi_1 (I_1, I_2) = c_1, \phi_2 (I_1, I_2) = c_2 \quad \text{and} \quad \phi_{12} (I_1, I_2) = c_1 + c_2 + k.
\]
which in turn implies the following restriction on \((I_1, I_2)\):

\[
F(I_1, I_2) \equiv \phi_{12}(I_1, I_2) - \phi_1(I_1, I_2) - \phi_2(I_1, I_2) = k. \tag{IA62}
\]

But given a pair \((I_1, I_2)\), we need to pin down three possible values \(\lambda_1\), \(\lambda_2\) and \(\lambda_{12}\) from two equations characterized by the two best response functions (IA11) for \(i = 1, 2\). The idea is to take \(\lambda_{12}\) as given, and solve for \(\lambda_1\) and \(\lambda_2\) as functions of \((I_1, I_2, \lambda_{12})\) from (IA11):

\[
\lambda_1 = \frac{(\rho + I_2^2\chi + \omega)(I_1\gamma\omega^{-1} - \lambda_{12})}{\rho + I_2^2\chi - I_1I_2\chi} \quad \text{and} \quad \lambda_2 = \frac{(\rho + I_1^2\chi + \omega)(I_2\gamma\omega^{-1} - \lambda_{12})}{\rho + I_1^2\chi - I_1I_2\chi}, \tag{IA63}
\]

which is equation (IA12). Then, conditions \(\lambda_1 > 0\) and \(\lambda_2 > 0\) together with (IA63) would put more restrictions on \((I_1, I_2, \lambda_{12})\) that can support an equilibrium in Case 8. Specifically, by Lemma R, the implied restriction on \((I_1, I_2)\) is

\[
\rho + I_2^2\chi - I_1I_2\chi > 0 \quad \text{and} \quad \rho + I_1^2\chi - I_1I_2\chi > 0. \tag{IA64}
\]

Thus, the complete set of conditions for pairs \((I_1, I_2)\) that are consistent with Case 8 are (IA60), (IA62) and (IA64). Given any pair \((I_1, I_2)\) that satisfies these conditions, we can compute a *continuum* of trader distributions as follows:

\[
0 < \lambda_{12} < \gamma\omega^{-1} \min \{I_1, I_2\}, \lambda_1 = \frac{(\rho + I_2^2\chi + \omega)(I_1\gamma\omega^{-1} - \lambda_{12})}{\rho + I_2^2\chi - I_1I_2\chi} \quad \text{and} \quad \lambda_2 = \frac{(\rho + I_1^2\chi + \omega)(I_2\gamma\omega^{-1} - \lambda_{12})}{\rho + I_1^2\chi - I_1I_2\chi}. \tag{IA65}
\]

Condition (IA61), subject to (IA60), (IA62) and (IA64), will fully characterize the following set of \((c_1, c_2)\) that is consistent with allocations in Case 8:

\[
C^8 \equiv \left\{ (c_1, c_2) \in \mathbb{R}_{++}^2 : c_1 = \phi_1(I_1, I_2), c_2 = \phi_2(I_1, I_2), \right. \\
\left. \text{for all } I_1 > 0, I_2 > 0, \text{ such that} \right. \\
\left. \rho + I_1^2\chi - I_1I_2\chi > 0, \rho + I_2^2\chi - I_1I_2\chi > 0 \text{ and } F(I_1, I_2) = k \right\},
\]

which is a subset of \(C^7\).

### 5.2 Characterizing the Sets of \((c_1, c_2)\) That Support an Equilibrium

We now analytically characterize the sets of \(C\)'s and show that their union forms the whole space of \(\mathbb{R}_{++}^2\). So, for any given parameter configuration \((c_1, c_2, k, \rho, \omega, \chi, \gamma) \in \mathbb{R}_{++}^7\), there is always an equilibrium supported by it, which establishes the existence of an information market equilibrium. In addition, we will show that within each set of \((c_1, c_2)\), there exists a unique information market equilibrium supported by a given parameter configuration, and that all the \(C\) sets are mutually exclusive (except for the boundaries with a zero Lebesgue
measure). As a result, each parameter configuration can support only one pair of trading intensities \((I_1, I_2)\), which establishes the uniqueness of an information market equilibrium. We show these results by providing a series of lemmas.

### 5.2.1 The Properties of \(F(I_1, I_2)\)

The definitions of \(C\)'s are involved with the comparison between function \(F(I_1, I_2)\) and parameter \(k\), and thus it is useful to characterize the range of \(F(I_1, I_2)\). Examining Cases 1-8, we find that in equilibrium, it is always true that \(\rho+I_1^2\chi-I_1I_2\chi>0\) and \(\rho+I_2^2\chi-I_1I_2\chi>0\). So, we define the following set of \((I_1, I_2)\):

\[
\mathcal{I}_F \equiv \{(I_1, I_2) \in \mathbb{R}_+^2 : \rho+I_1^2\chi-I_1I_2\chi>0 \text{ and } \rho+I_2^2\chi-I_1I_2\chi>0\} \quad \text{(IA66)}
\]

and characterize the infimum and maximum values of \(F(I_1, I_2)\) over this set.

We can compute:

\[
Var(\tilde{v}|\tilde{v}_1, \tilde{p}) Var(\tilde{v}|\tilde{v}_2, \tilde{p}) - Var(\tilde{v}|\tilde{v}_12, \tilde{p}) Var(\tilde{v}|\tilde{p}) = \left( (\rho + I_1^2\chi)^{-1} + \omega^{-1} \right) \left( (\rho + I_2^2\chi)^{-1} + \omega^{-1} \right) - \omega^{-1} \left( I_1^2\rho + I_2^2\rho + \rho^2\chi^{-1} + \omega^{-1} \right)
\]

\[
= \frac{\omega \rho (\rho + \chi I_1^2 + \chi I_2^2) + \chi^2 I_1 I_2 (\rho + \chi I_1^2 - \chi I_1 I_2) + \chi^2 I_1 I_2 (\rho + \chi I_2^2 - \chi I_1 I_2) + \chi \rho I_1 I_2 (2\rho + \chi (I_1 - I_2)^2)}{(\rho + \chi I_1^2) (\rho + \chi I_2^2) (\rho + \chi I_1^2 + \chi I_2^2)} > 0
\]

for all \((I_1, I_2) \in \mathcal{I}_F\), which implies

\[
\frac{Var(\tilde{v}|\tilde{v}_1, \tilde{p}) Var(\tilde{v}|\tilde{v}_2, \tilde{p})}{Var(\tilde{v}|\tilde{v}_12, \tilde{p}) Var(\tilde{v}|\tilde{p})} > 1 \Rightarrow F(I_1, I_2) > 0 \text{ for all } (I_1, I_2) \in \mathcal{I}_F.
\]

In addition, we have \(\lim_{I_1=I_2 \to \infty} F(I_1, I_2) = 0\), and thus,

\[
\inf_{(I_1, I_2) \in \mathcal{I}_F} F(I_1, I_2) = 0.
\]

Direct computation shows that \(\frac{\partial F(I_1, I_2)}{\partial \rho}\) admits the same sign as the following function:

\[
f_i(I_1, I_2) \equiv -\chi^2 I_j I_i^3 - \chi (\omega + \chi I_j^2) I_i^3 - (\omega \rho + 2\chi \omega I_j^2 + \chi \rho I_j^2) I_i + I_j (\omega + \rho) (\rho + \chi I_j^2),
\]

(IA67)

for \(i = 1, 2, j = 1, 2\) and \(j \neq i\). Therefore, for any given \(I_j > 0\), we have \(f_i(I_1, I_2)\) is first positive and then becomes negative, meaning that \(F(I_1, I_2)\) first increases with \(I_i\) and then decreases with \(I_i\). Therefore, \(F(I_1, I_2)\) achieves a maximum value in an open subset of \(\mathcal{I}_F\).
Let \((I_{1,\text{max}}, I_{2,\text{max}})\) denote the maximum point. Then:

\[
f_1(I_{1,\text{max}}, I_{2,\text{max}}) = 0 \quad \text{and} \quad f_2(I_{1,\text{max}}, I_{2,\text{max}}) = 0 \Rightarrow
f_1(I_{1,\text{max}}, I_{2,\text{max}}) - f_2(I_{1,\text{max}}, I_{2,\text{max}})
= -(I_{1,\text{max}} - I_{2,\text{max}})
\left(\begin{array}{l}
\rho^2 + 2\omega \rho + 2\chi \omega I_{1,\text{max}}^2 \max F + 2\chi \omega I_{2,\text{max}}^2 \max F \\
+\chi \rho I_{1,\text{max}}^2 \max F + \chi \rho I_{2,\text{max}}^2 \max F + \chi^2 I_{1,\text{max}} \max F I_{2,\text{max}} \max F \\
+\chi^2 I_{2,\text{max}}^2 \max F I_{1,\text{max}} \max F + 2\chi^2 I_{1,\text{max}}^2 \max F I_{2,\text{max}} \max F
\end{array}\right)
= 0 \Rightarrow I_{1,\text{max}} = I_{2,\text{max}}.
\]

That is, \(F(I_1, I_2)\) achieves its maximum on the line of \(I_1 = I_2\).

Define

\[
F_{I_1=I_2}(I) \equiv F(I, I) = \frac{1}{2\gamma} \log \left[ \frac{\left(\frac{\rho}{\rho + I^2 \chi} - \omega^{-1}\right)^2}{\omega^{-1} \left(\frac{\rho}{2 + I^2 \chi} - \omega^{-1}\right)^2} \right]. \tag{IA68}
\]

Direct computation shows that the derivative of function \(F_{I_1=I_2}(\cdot)\) is:

\[
F_{I_1=I_2}'(I) \propto \rho^2 - 2\chi I^2 (\omega + \chi I^2), \tag{IA69}
\]

which is negative for \(I^2 < \frac{-\omega + \sqrt{\omega^2 + 2\rho^2}}{2\chi}\) and positive for \(I^2 > \frac{-\omega + \sqrt{\omega^2 + 2\rho^2}}{2\chi}\). Thus, \(F_{I_1=I_2}(I)\) first increases and then decreases with \(I\), and it achieves its maximum at the value of

\[
I_{\text{max}} \equiv \sqrt{\frac{-\omega + \sqrt{\omega^2 + 2\rho^2}}{2\chi}}. \tag{IA70}
\]

The resulting maximum value of \(F(I_1, I_2)\) is:

\[
\bar{k}_2 \equiv \max_{(I_1, I_2) \in \mathcal{I}_F} F(I_1, I_2) = \frac{1}{2\gamma} \log \left[ \frac{\left(\frac{\rho}{\rho + \frac{-\omega + \sqrt{\omega^2 + 2\rho^2}}{2\chi}} - \omega^{-1}\right)^2}{\omega^{-1} \left(\frac{\rho/2 + \frac{-\omega + \sqrt{\omega^2 + 2\rho^2}}{2\chi}}{2\chi} + \omega^{-1}\right)^2} \right]. \tag{IA71}
\]

We summarize the above discussion into the following lemma.

**Lemma F** For any \((I_1, I_2) \in \mathcal{I}_F\), \(F(I_1, I_2)\) is strictly positive, and \(\inf_{(I_1, I_2) \in \mathcal{I}_F} F(I_1, I_2) = 0\). \(F(I_1, I_2)\) achieves its maximum at \(I_1 = I_2 = \sqrt{\frac{-\omega + \sqrt{\omega^2 + 2\rho^2}}{2\chi}}\), and the maximum value is given by \(\bar{k}_2\) in (IA71).
5.2.2 Characterization of $C^1$

Direct computation shows
\[
\phi_1 (0, 0) = \phi_2 (0, 0) = \frac{1}{2\gamma} \log \left( \frac{2\rho^{-1} + \omega^{-1}}{\rho^{-1} + \omega^{-1}} \right), \tag{IA72}
\]
\[
\phi_{12} (0, 0) = \frac{1}{2\gamma} \log \left( \frac{2\rho^{-1} + \omega^{-1}}{\omega^{-1}} \right). \tag{IA73}
\]
Define
\[
\bar{k}_1 \equiv F (0, 0) = \frac{1}{2\gamma} \log \left[ \frac{(\rho^{-1} + \omega^{-1})^2}{\omega^{-1} (2\rho^{-1} + \omega^{-1})} \right]. \tag{IA74}
\]

Thus, if $k > \bar{k}_1$,
\[
c_1 \geq \phi_1 (0, 0), c_2 \geq \phi_2 (0, 0) \Rightarrow c_1 + c_2 + k \geq \phi_{12} (0, 0)
\]
and so, the condition of $c_1 + c_2 + k \geq \phi_{12} (0, 0)$ in set $C^1$ is redundant. If $0 \leq k \leq \bar{k}_1$, for some values of $(c_1, c_2)$, this condition is binding.

As a result, the set of $C^1$ can be characterized by the following lemma.

Lemma 1 If $k > \bar{k}_1$, then:
\[
C^1 = \left\{ (c_1, c_2) \in \mathbb{R}_++^2 : c_1 \geq \frac{1}{2\gamma} \log \left( \frac{2\rho^{-1} + \omega^{-1}}{\rho^{-1} + \omega^{-1}} \right) \text{ and } c_2 \geq \frac{1}{2\gamma} \log \left( \frac{2\rho^{-1} + \omega^{-1}}{\rho^{-1} + \omega^{-1}} \right) \right\},
\]
and if $0 \leq k \leq \bar{k}_1$, then:
\[
C^1 = \left\{ (c_1, c_2) \in \mathbb{R}_+^2 : c_1 \geq \frac{1}{2\gamma} \log \left( \frac{2\rho^{-1} + \omega^{-1}}{\rho^{-1} + \omega^{-1}} \right) \leq c_1 \leq \frac{1}{2\gamma} \log \left( \frac{\rho^{-1} + \omega^{-1}}{\omega^{-1}} \right) - k \right\}
\]
\[
\quad \quad \quad \quad \quad \quad \quad \text{and } c_2 \geq \frac{1}{2\gamma} \log \left( \frac{\rho^{-1} + \omega^{-1}}{\omega^{-1}} \right) - c_1 - k \right\}
\]
\[
\quad \quad \quad \quad \quad \quad \quad \cup \left\{ (c_1, c_2) \in \mathbb{R}_+^2 : c_1 > \frac{1}{2\gamma} \log \left( \frac{\rho^{-1} + \omega^{-1}}{\omega^{-1}} \right) - k \text{ and } c_2 \geq \frac{1}{2\gamma} \log \left( \frac{2\rho^{-1} + \omega^{-1}}{\rho^{-1} + \omega^{-1}} \right) \right\}.
\]

5.2.3 Characterization of $C^2$

By Lemma F, if $k > \bar{k}_2$, then $\max_{(I_1, I_2) \in \mathcal{I}_F} F (I_1, I_2) < k$, which implies that the set $C^2$ is empty. Now suppose $k \leq \bar{k}_2$. We will characterize $C^2$ explicitly in the following form:
\[
C^2 \equiv \left\{ (c_1, c_2) \in \mathbb{R}_+^2 : c_1 \in \left[ c_{I_1=I_2}, c_{I_1=I_2} \right] \text{ and } c_{2I_1=I_2} (c_1) \leq c_2 \leq c_{2I_1=I_2} (c_1) \right\},
\]
where $c_{I_1=I_2}$ and $c_{I_1=I_2}$ are two constants and $c_{2I_1=I_2} (\cdot)$ and $c_{2I_1=I_2} (\cdot)$ are two functions. To do so, we will follow three steps. We first characterize the possible values of $I$ that satisfy the condition of $F (I, I) \geq k$ in the definition of $C^2$. We next characterize the range $\left[ c_{I_1=I_2}, c_{I_1=I_2} \right]$ of $c_1$ in the set $C^2$. We finally characterize the boundaries $c_{2I_1=I_2} (\cdot)$ and $c_{2I_1=I_2} (\cdot)$, which in turn completely characterize the set $C^2$. 

The range of $I$.

According to the proof of Lemma F, $F_{I_1=I_2} (I) \equiv F (I, I)$ first increases and then decreases with $I$, and its maximum $\bar{k}_2$ is greater than $k$. So, the set of $I$ which satisfies the condition $F (I, I) \geq k$ is an interval.

By the expression of $F_{I_1=I_2} (I)$ in (IA68), we have $\lim_{I \to \infty} F_{I_1=I_2} (I) = 0$. Thus, there exists an $\bar{I} \in [I_{\text{max} F}, \infty]$ (i.e., $\bar{I}$ lies on the right branch of function $F_{I_1=I_2} (\cdot)$), which is determined by $F_{I_1=I_2} (\bar{I}) = k$, such that $F_{I_1=I_2} (I) < k$ for $I > \bar{I}$. This $\bar{I}$ determines the right end of the interval of $I$ satisfying $F (I, I) \geq k$.

The left end of the interval can take two possible values depending on the value of $F_{I_1=I_2} (0) = F (0, 0)$. If $F (0, 0) < k$, or equivalently, if $k > \bar{k}_1$ (since $\bar{k}_1 = F (0, 0)$ by (IA74)), then we have $\underline{I} \in (0, I_{\text{max} F}]$ and it is determined by $F_{I_1=I_2} (\underline{I}) = k$. If $k \leq \bar{k}_1$, then the left end $\underline{I}$ takes the value 0.

So, the possible values of $I$ in $C^2$ form an interval of $[\underline{I}, \bar{I}]$, and its two ends are determined as follows:

\begin{align*}
\text{If } k \leq \bar{k}_2, \text{ then } & \bar{I} \in [I_{\text{max} F}, \infty] \text{ and } F_{I_1=I_2} (\bar{I}) = k; \quad (\text{IA75}) \\
\text{If } \bar{k}_1 < k \leq \bar{k}_2, \text{ then } & \underline{I} \in (0, I_{\text{max} F}] \text{ and } F_{I_1=I_2} (\underline{I}) = k; \quad (\text{IA76}) \\
\text{If } 0 \leq k < \bar{k}_1, \text{ then } & \underline{I} = 0. \quad (\text{IA77})
\end{align*}

The range of $c_1$.

We define

\begin{align*}
\phi_{I_1=I_2} (I) & \equiv \phi_1 (I, I) = \phi_2 (I, I) = \frac{1}{2 \gamma} \log \left( \frac{(\rho/2 + I^2 \chi)^{-1} + \omega^{-1}}{(\rho + I^2 \chi)^{-1} + \omega^{-1}} \right), \quad (\text{IA78}) \\
\phi_{I_1=I_2,12} (I) & \equiv \phi_{12} (I, I) = \frac{1}{2 \gamma} \log \left( \frac{(\rho/2 + I^2 \chi)^{-1} + \omega^{-1}}{\omega^{-1}} \right). \quad (\text{IA79})
\end{align*}

For any $I \in [\underline{I}, \bar{I}]$, the possible range of $c_1$ in $C^2$ is $[\phi_{I_1=I_2} (I), \phi_{I_1=I_2,12} (I) - \phi_{I_1=I_2} (I) - k]$.

Thus, the minimum and maximum admissible values of $c_1$ are, respectively,

\begin{align*}
\underline{c}_{I_1=I_2} & \equiv \min_{I \in [\underline{I}, \bar{I}]} \phi_{I_1=I_2} (I) \quad \text{and} \quad \overline{c}_{I_1=I_2} \equiv \max_{I \in [\underline{I}, \bar{I}]} \left[ \phi_{I_1=I_2,12} (I) - \phi_{I_1=I_2} (I) - k \right]. \quad (\text{IA80})
\end{align*}

Direct computation shows $\phi'_{I_1=I_2} (I) < 0$, that is, $\phi_{I_1=I_2} (I)$ decreases with $I$. This makes sense, since the allocations supported by parameters in $C^2$ correspond to Grossman-Stiglitz economies, and thus, when a higher trading intensity $I$ causes the price to become more
informative, the information value \( \phi_{I_1=I_2} (I) \) becomes lower. As a result,

\[
\phi_{I_1=I_2} \equiv \min_{I \in [L, \bar{I}]} \phi_{I_1=I_2} (I) = \phi_{I_1=I_2} (\bar{I}) = \frac{1}{2\gamma} \log \left( \frac{(\rho/2 + \bar{I}^2 \chi)^{-1} + \omega^{-1}}{(\rho + \bar{I}^2 \chi)^{-1} + \omega^{-1}} \right). \quad (I A 81)
\]

Similarly, we can show that \( \phi_{I_1=I_2,12} (I) - \phi_{I_1=I_2} (I) \) also decreases with \( I \), and hence \( \phi_{I_1=I_2,12} \equiv \max_{I \in [L, \bar{I}]} [\phi_{I_1=I_2,12} (I) - \phi_{I_1=I_2} (I) - k] = \phi_{I_1=I_2,12} (I) - \phi_{I_1=I_2} (I) - k \). If \( k > \bar{k}_1 \), we have \( F_{I_1=I_2} (I) = \phi_{I_1=I_2,12} (I) - 2\phi_{I_1=I_2} (I) = k \), and hence \( \phi_{I_1=I_2,12} = \phi_{I_1=I_2} (I) \). If \( k \leq \bar{k}_1 \), we have \( \bar{I} = 0 \), and hence \( \phi_{I_1=I_2,12} = \frac{1}{2\gamma} \log \left( \frac{(\rho/2 + \bar{I}^2 \chi)^{-1} + \omega^{-1}}{(\rho + \bar{I}^2 \chi)^{-1} + \omega^{-1}} \right) - k \).

That is,

\[
\phi_{I_1=I_2,12} = \begin{cases} 
\frac{1}{2\gamma} \log \left( \frac{(\rho/2 + \bar{I}^2 \chi)^{-1} + \omega^{-1}}{(\rho + \bar{I}^2 \chi)^{-1} + \omega^{-1}} \right), & \text{if } k > \bar{k}_1, \\
\frac{1}{2\gamma} \log \left( \frac{\rho^{-1} + \omega^{-1}}{\omega^{-1}} \right) - k, & \text{if } k \leq \bar{k}_1.
\end{cases} \quad (I A 82)
\]

**The boundaries of \( C^2 \).**

The set \( C^2 \) is bounded by two curves in the \((c_1, c_2)\) space. These two curves can be characterized as two functions which take \( c_1 \) as an argument over the domain of \([c_{I_1=I_2}, \phi_{I_1=I_2}]\).

On the lower boundary, for any \( I \in [L, \bar{I}] \), we have \( c_1 = \phi_{I_1=I_2,12} - \phi_{I_1=I_2} - k = \frac{1}{2\gamma} \log \left( \frac{(\rho/2 + \bar{I}^2 \chi)^{-1} + \omega^{-1}}{\omega^{-1}} \right) \). From these two equations, we can cancel \( I \) and get the function defining the lower boundary as follows:

\[
c_{I_1=I_2} (c_1) \equiv \frac{1}{2\gamma} \log \left( \frac{1}{e^{2\gamma(c_1+k)} - 1} - \frac{\rho}{2\omega} \right)^{-1} + 1) - (c_1 + k). \quad (I A 83)
\]

The upper boundary \( \phi_{I_1=I_2} (c_1) \) is a function symmetric to \( c_{I_1=I_2} (c_1) \). After some computation, we can further express it more explicitly as a function of \( c_1 \) as follows. If \( k > \bar{k}_1 \), then for all \( c_1 \in [c_{I_1=I_2}, \phi_{I_1=I_2}] \):

\[
\phi_{I_1=I_2} (c_1) = \frac{1}{2\gamma} \log \left( 1 + \left( e^{2\gamma c_1} - 1 \right) \left[ \frac{\omega^2}{\rho^2} + \frac{e^{2\gamma c_1} + 1}{e^{2\gamma c_1} - 1} \frac{\omega}{\rho} + \frac{1}{4} + \frac{\omega + 1}{\rho + 1} \right] \right) - (c_1 + k). \quad (I A 84)
\]

If \( k \leq \bar{k}_1 \), then \( \phi_{I_1=I_2} (c_1) \) is given by \((I A 84)\) for \( c_1 \in [c_{I_1=I_2}, \phi_1 (0,0)] \), and by \( c_2 = \)}
In particular, at which is negative for all $c_1 \in [\phi_1 (0, 0), \bar{c}_1_{I_1=I_2}]$; that is:

$$
\phi_{12} (0, 0) - (c_1 + k) \quad \text{for} \quad c_1 \in [\phi_1 (0, 0), \bar{c}_1_{I_1=I_2}]; \quad \text{that is:}
$$

$$
\overline{c}_{2_{I_1=I_2}} (c_1) = \begin{cases} \\
\frac{1}{2\gamma} \log \left( 1 + \frac{1}{e} \left( \frac{(c_1 + 1)}{\varepsilon} \right) \frac{1}{4} \right) + \left( \frac{c_1}{\mu} + \frac{1}{2} \right) - (c_1 + k), \\
\frac{1}{2\gamma} \log \left( \frac{2\xi - 1}{\omega - 1} \right) - (c_1 + k),
\end{cases}
$$

for $\frac{1}{2\gamma} \log \left( \frac{2\xi - 1}{\omega - 1} \right) < c_1 \leq \frac{1}{2\gamma} \log \left( \frac{(\rho + P^2 \chi)^{-1} + \omega^{-1}}{-\omega - 1} \right) - k$.

(IA85)

In particular, at $c_1 = \bar{c}_{I_1=I_2}$, the lower and upper boundaries coincide, that is, $c_{2_{I_1=I_2}} (c_1_{I_1=I_2}) = c_{2_{I_1=I_2}} (c_1_{I_1=I_2})$ (by $F (\bar{I}, \bar{I}) = k \Rightarrow \frac{1}{2\gamma} \log \left( \frac{(\rho + P^2 \chi)^{-1} + \omega^{-1}}{-\omega - 1} \right) = \frac{1}{2\gamma} \log \left( \frac{(\rho + P^2 \chi)^{-1} + \omega^{-1}}{-\omega - 1} \right) - k$).

Finally, we know that for given $(c_1, c_2) \in C^2$, there exists a unique information market equilibrium with $\lambda_1 = \lambda_2 = 0$ and $\lambda_{12} > 0$. Specifically, for any given $(c_1, c_2) \in C^2$, we can determined a unique $I$ from equation (IA78): $\phi_{12} (I, I) = \frac{1}{2\gamma} \log \left( \frac{(\rho + P^2 \chi)^{-1} + \omega^{-1}}{-\omega - 1} \right) = c_1 + c_2 + k$. Then, with this determined $I$ in hand, by (IA36), we can determine a unique $\lambda_{12}$: $\lambda_{12} = \frac{\phi_{12} (I, I)}{\omega}$.

We summarize the above discussion into the following lemma.

**Lemma 2** (a) If $k > \bar{k}_2$, then the set $C^2$ is empty. If $k \leq \bar{k}_2$, then $C^2$ is given as follows:

$$
C^2 = \left\{ (c_1, c_2) \in \mathbb{R}^2_{+} : c_1 \in \left[ c_{1_{I_1=I_2}}, \bar{c}_{1_{I_1=I_2}} \right], \quad \text{and} \quad c_{2_{I_1=I_2}} (c_1) \leq c_2 \leq \overline{c}_{2_{I_1=I_2}} (c_1) \right\},
$$

where $c_{1_{I_1=I_2}}, \bar{c}_{1_{I_1=I_2}}$ and $\overline{c}_{2_{I_1=I_2}} (c_1)$ are given by equations (IA81), (IA82) and (IA83), respectively. The function $\overline{c}_{2_{I_1=I_2}} (c_1)$ is given by (IA84) if $\bar{k}_1 < k \leq \bar{k}_2$, and by (IA85) if $k \leq \bar{k}_1$.

(b) For any given $(c_1, c_2) \in C^2$, there exists a unique information market equilibrium with $\lambda_1 = \lambda_2 = 0$ and $\lambda_{12} > 0$.

**5.2.4 Characterization of $C^3$**

By (IA67), $\frac{\partial F (I_1, 0)}{\partial I_1}$ has the same sign as

$$
f_1 (I_1, 0) = -\chi \omega I_1^3 - \omega \rho I_1,
$$

which is negative for all $I_1 > 0$. Thus, $F (I_1, 0)$ is decreasing in $I_1$. So, if $k > F (0, 0) = \bar{k}_1$, then $F (I_1, 0) < k$ for all $I_1 > 0$ and hence $C^{3-2}$ is empty (i.e., only Subcase 3.1 is relevant).
In addition, we can show \( \lim_{I_1 \to -\infty} F(I_1, 0) = 0 \), and thus \( C^{3.1} \) is non-empty as long as \( k > 0 \).

Suppose \( k > \bar{k}_1 \). Then, \( F(I_1, 0) < k \) for all \( I_1 > 0 \) in the set \( C^{3.1} \). We can show that \( \phi_1(I_1, 0) = \frac{1}{2\gamma} \log \left( \frac{I_1^2 + 2\rho \gamma^{-1} + \omega^{-1}}{\rho^{-1} + \omega^{-1}} \right) \) decreases with \( I_1 \). Thus, the range of \( c_1 \) is

\[
\left( \lim_{I_1 \to -\infty} \phi_1(I_1, 0), \phi_1(0, 0) \right) = \left( 0, \frac{1}{2\gamma} \log \left( \frac{2\rho^{-1} + \omega^{-1}}{\rho^{-1} + \omega^{-1}} \right) \right),
\]

and any \( I_1 \in (0, \infty) \) corresponds to a \( c_1 \in \left( 0, \frac{1}{2\gamma} \log \left( \frac{2\rho^{-1} + \omega^{-1}}{\rho^{-1} + \omega^{-1}} \right) \right) \). The lower bound of \( c_2 \) is

\[
\phi_2(I_1, 0) = \frac{1}{2\gamma} \log \left( \frac{I_1^2 + 2\rho \gamma^{-1} + \omega^{-1}}{(\rho + I_1^2 \gamma)} + \omega^{-1} \right),
\]

which is increasing in \( I_1 \). Using \( c_1 = \frac{1}{2\gamma} \log \left( \frac{I_1^2 + 2\rho \gamma^{-1} + \omega^{-1}}{(\rho + I_1^2 \gamma)} + \omega^{-1} \right) \),

we can cancel \( I_1 \) and express the lower bound of \( c_2 \) as a function in \( c_1 \):

\[
\frac{1}{2\gamma} \log \left( \frac{(\rho^{-1} + \omega^{-1}) e^{2\gamma c_1}}{(e^{2\gamma c_1} - 1) \rho^{-1} + \omega^{-1} e^{2\gamma c_1}} \right).
\]

Suppose \( k \leq \bar{k}_1 \). Then, there exists an \( I_{1,CRIT} \in [0, \infty) \) with \( F(I_{1,CRIT}, 0) = k \), such that \( F(I_1, 0) \leq k \) for \( I_1 \geq I_{1,CRIT} \) and \( F(I_1, 0) > k \) for \( 0 < I_1 < I_{1,CRIT} \). Given that \( \phi_1(I_1, 0) \) decreases with \( I_1 \), the range of \( c_1 \) in set \( C^{3.1} \) is \( (\phi_1(\infty, 0), \phi_1(I_{1,CRIT}, 0)] = (0, \phi_1(I_{1,CRIT}, 0)] \),

and the range of \( c_1 \) in set \( C^{3.2} \) is \( (\phi_1(I_{1,CRIT}, 0), \phi_1(0, 0)) = (\phi_1(I_{1,CRIT}, 0), \frac{1}{2\gamma} \log \left( \frac{2\rho^{-1} + \omega^{-1}}{\rho^{-1} + \omega^{-1}} \right) \). The lower bound of \( c_2 \) in \( C^{3.1} \) is still given by function \( \phi_2(I_1, 0) = \frac{1}{2\gamma} \log \left( \frac{(\rho^{-1} + \omega^{-1}) e^{2\gamma c_1}}{(e^{2\gamma c_1} - 1) \rho^{-1} + \omega^{-1} e^{2\gamma c_1}} \right) \). The lower bound of \( c_2 \) in \( C^{3.1} \) is given by \( \phi_{12}(I, 0) - \phi_1(I, 0) - k = \frac{1}{2\gamma} \log \left( \frac{\rho^{-1} + \omega^{-1}}{\omega^{-1}} \right) - k \),

which is a constant.

Finally, we show that for any given \((c_1, c_2) \in C^3 \), there exists a unique information market equilibrium with \( \lambda_1 > 0 \) and \( \lambda_2 = \lambda_{12} = 0 \). Specifically, by (IA44), we have \( \phi_1(I_1, 0) = \frac{1}{2\gamma} \log \left( \frac{I_1^2 + 2\rho \gamma^{-1} + \omega^{-1}}{(\rho + I_1^2 \gamma)} + \omega^{-1} \right) = c_1 \), from which we can determine a unique \( I_1 > 0 \). Then, from (IA42), we can compute a unique \( \lambda_1 = \frac{\gamma(1 + \rho \omega^{-1})}{\rho} I_1 \).

To summarize, we have the following lemma.

**Lemma 3** (a) If \( k > \bar{k}_1 \), then:

\[
C^3 = \left\{ (c_1, c_2) \in \mathbb{R}^2_+ : c_1 \in \left( 0, \frac{1}{2\gamma} \log \left( \frac{2\rho^{-1} + \omega^{-1}}{\rho^{-1} + \omega^{-1}} \right) \right) \right\},
\]

and \( c_2 \geq \frac{1}{2\gamma} \log \left( \frac{(\rho^{-1} + \omega^{-1}) e^{2\gamma c_1}}{(e^{2\gamma c_1} - 1) \rho^{-1} + \omega^{-1} e^{2\gamma c_1}} \right) \),
and if \( k \leq \bar{k}_1 \), then:

\[
C^3 = \begin{cases} 
(c_1, c_2) \in \mathbb{R}^2_+ : & c_1 \in (0, \phi_1 (I_1, CRIT, 0)] \\
& \text{and } c_2 \geq \frac{1}{2\gamma} \log \left( \frac{(\rho^{-1} + \omega^{-1}) e^{2\gamma c_1}}{(e^{2\gamma c_1 - 1})^{\rho^{-1} + \omega^{-1} e^{2\gamma c_1}}} \right) \\
\cup & \left( c_1, c_2 \right) \in \mathbb{R}^2_+ : c_1 \in \left( \phi_1 (I_1, CRIT, 0), \frac{1}{2\gamma} \log \left( \frac{2\rho^{-1} + \omega^{-1}}{\rho^{-1} + \omega^{-1}} \right) \right) \\
& \text{and } c_2 \geq \frac{1}{2\gamma} \log \left( \frac{\rho^{-1} + \omega^{-1}}{\omega^{-1}} \right) - k
\end{cases}
\]

where \( I_1, CRIT \) is determined by \( F(I_1, CRIT, 0) = k \).

(b) For any given \((c_1, c_2) \in C^3\), there exists a unique information market equilibrium with \( \lambda_1 > 0 \) and \( \lambda_2 = \lambda_{12} = 0 \).

5.2.5 Characterization of \( C^4 \)

This is a case symmetric to \( C^3 \), and we can switch the subscripts 1 and 2 to obtain the set of \((c_1, c_2)\) supporting allocations in this case. The result is summarized in the following lemma.

Lemma 4 (a) If \( k > \bar{k}_1 \), then:

\[
C^4 = \begin{cases} 
(c_1, c_2) \in \mathbb{R}^2_+ : & c_2 \in (0, \frac{1}{2\gamma} \log \left( \frac{2\rho^{-1} + \omega^{-1}}{\rho^{-1} + \omega^{-1}} \right)) \\
& \text{and } c_1 \geq \frac{1}{2\gamma} \log \left( \frac{(\rho^{-1} + \omega^{-1}) e^{2\gamma c_2}}{(e^{2\gamma c_2 - 1})^{\rho^{-1} + \omega^{-1} e^{2\gamma c_2}}} \right)
\end{cases}
\]

and if \( k \leq \bar{k}_1 \), then:

\[
C^4 = \begin{cases} 
(c_1, c_2) \in \mathbb{R}^2_+ : & c_2 \in (0, \phi_2 (0, I_2, CRIT)] \\
& \text{and } c_1 \geq \frac{1}{2\gamma} \log \left( \frac{(\rho^{-1} + \omega^{-1}) e^{2\gamma c_2}}{(e^{2\gamma c_2 - 1})^{\rho^{-1} + \omega^{-1} e^{2\gamma c_2}}} \right) \\
\cup & \left( c_1, c_2 \right) \in \mathbb{R}^2_+ : c_2 \in \left( \phi_2 (0, I_2, CRIT), \frac{1}{2\gamma} \log \left( \frac{2\rho^{-1} + \omega^{-1}}{\rho^{-1} + \omega^{-1}} \right) \right) \\
& \text{and } c_1 \geq \frac{1}{2\gamma} \log \left( \frac{\rho^{-1} + \omega^{-1}}{\omega^{-1}} \right) - k
\end{cases}
\]

where \( I_2, CRIT \) is determined by \( F(0, I_2, CRIT) = k \).

(b) For any given \((c_1, c_2) \in C^4\), there exists a unique information market equilibrium with \( \lambda_2 > 0 \) and \( \lambda_1 = \lambda_{12} = 0 \).

5.2.6 Characterization of \( C^5 \)

By Lemma F, if \( k > \bar{k}_2 \), then \( \max_{(I_1, I_2) \in \mathcal{I}_F} F(I_1, I_2) < k \), which implies that the set \( C^5 \) is empty. Now suppose \( k \leq \bar{k}_2 \). We will follow the same logic as the characterization of the set
and characterize the set $C^5$ as follows:

$$C^5 \equiv \left\{(c_1, c_2) \in \mathbb{R}^2_{++} : c_2 \in \left[c_{21}, c_{22}\right] \text{ and } c_{11} > c_2 < c_{12} \right\},$$

where $c_{21}$ and $c_{22}$ are two constants and $c_{11}$ and $c_{12}$ are two functions. To do so, we will also follow three steps. We first characterize the possible values of $(I_1, I_2)$ that satisfy the conditions in the above expression of $C^5$. We next characterize the range $[c_{21}, c_{22}]$ of $c_2$. We finally characterize the boundaries $c_{11}$ and $c_{12}$, which in turn completely characterize the set $C^5$.

The range of $(I_1, I_2)$

The possible values of $(I_1, I_2)$ in $C^5$ form the following set:

$$I_{C^5} \equiv \left\{(I_1, I_2) \in \mathbb{R}^2_{++} : I_1 > I_2, \rho + I_2^2 \chi - I_1 I_2 \chi > 0 \text{ and } F(I_1, I_2) \geq k \right\}.$$

(i) If $k \leq \bar{k}_1$, then $I_{C^5}$ is equivalent to the following set:

$$I_{C^5, k \leq \bar{k}_1} \equiv \left\{(I_1, I_2) \in \mathbb{R}^2_{++} : 0 < I_2 \leq I \text{ and } I_2 < I_1 < \min \left\{I_2 + \frac{\rho}{I_2 \chi}, I_{1, \text{Right}}(I_2) \right\} \right\},$$

where $\bar{I}$ is given by equation (1A75) and where $I_{1, \text{Right}}(I_2)$ is the value at the right branch of $F(\cdot, I_2)$ such that $F(I_{1, \text{Right}}(I_2), I_2) = k$. Note that $I_{1, \text{Right}}(I_2)$ is always well-defined for any $0 < I_2 \leq \bar{I}$; for any given $I_2 > 0$, $F(\cdot, I_2)$ first increases and then decreases with $I_1$, and $\lim_{I_1 \to \infty} F(I_1, I_2) = 0$; since $F(I_2, I_2) \geq k$ for any $0 < I_2 \leq \bar{I}$, then there is a unique $I_{1, \text{Right}}(I_2)$ such that $F(I_{1, \text{Right}}(I_2), I_2) = k$.

To establish this equivalence, we need to show that $I_{C^5, k \leq \bar{k}_1} \subseteq I_{C^5}$ and $I_{C^5} \subseteq I_{C^5, k \leq \bar{k}_1}$. It is clear that $I_{C^5, k \leq \bar{k}_1} \subseteq I_{C^5}$. To show $I_{C^5} \subseteq I_{C^5, k \leq \bar{k}_1}$, it is sufficient to show that any pair $(I_1, I_2) \in I_{C^5}$ satisfies $0 < I_2 \leq \bar{I}$. We prove this by contradiction. Suppose that there is some pair $(I_1^0, I_2^0) \in I_{C^5}$ with $I_2^0 > \bar{I}$. Then $F(I_2^0, I_2^0) < k$ (by $I_2^0 > \bar{I}$) and $F(I_1^0, I_2^0) \geq k$ (by $(I_1^0, I_2^0) \in I_{C^5}$). So, $F(I_1^0, I_2^0) \leq F(I_1, I_2)$. Given $I_1^0 > I_2^0$, this implies that $I_2^0$ lies on the increasing left branch of $F(\cdot, I_2^0)$. Hence, $F(I_1^0, I_2^0) > 0$, which in turn implies $I_2^0 \leq I_{\max F}$, contradicting with the initial assumption of $I_2^0 > \bar{I} \geq I_{\max F}$.

(ii) If $\bar{k}_1 < k \leq \bar{k}_2$, then $I_{C^5}$ is equivalent to the following set:

$$I_{C^5, \bar{k}_1 < k \leq \bar{k}_2} \equiv \left\{(I_1, I_2) \in \mathbb{R}^2_{++} : I \leq I_2 \leq \bar{I} \text{ and } I_2 < I_1 < \min \left\{I_2 + \frac{\rho}{I_2 \chi}, I_{1, \text{Right}}(I_2) \right\} \right\} \cup \left\{(I_1, I_2) \in \mathbb{R}^2_{++} : I_2 \leq I_2 < I \text{ and } I_{1, \text{Left}}(I_2) I_2 < I_1 < \min \left\{I_2 + \frac{\rho}{I_2 \chi}, I_{1, \text{Right}}(I_2) \right\} \right\},$$

where $I_2 > 0$ is a constant and where $I_{1, \text{Right}}(I_2)$ is the value on the left increasing branch of $F(\cdot, I_2)$ such that $F(I_{1, \text{Left}}(I_2), I_2) = k$. 

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We next show that $I_2, I_{1,Left} (\cdot)$ and $I_{1,Right} (\cdot)$ are well-defined, and that $I_{C^5} \subseteq I_{C^5, k_1 < k \leq k_2}$. For any pair $(I_1, I_2) \in I_{C^5}$ with $I_2 \in [L, \bar{I}]$, $I_{1,Right} (I_2)$ is uniquely defined, since $F (\cdot, I_2)$ first increases and then decreases with $I_1$, $\lim_{I_1 \to \infty} F (I_1, I_2) = 0$, and $F (I_2, I_2) \geq k$. It is straightforward to check that those pairs belong to $I_{C^5, k_1 < k \leq k_2}$.

For any pair $(I_1, I_2) \in I_{C^5}$ with $I_2 \notin [L, \bar{I}]$, we must have $F (I_2, I_2) < k$ and $F (I_1, I_2) \geq k$. Given the assumption of $I_2 < I_1$, $I_2$ must lie on the increasing left branch of $F (\cdot, I_2)$, which implies $I_2 \leq I_{\text{max}} F$ and hence $I_2 < I$ (by $I_2 \notin [L, \bar{I}]$ and $\bar{I} \geq I_{\text{max}} F$). For any $I_2$ slightly smaller than $I$, the function $F (\cdot, I_2)$ will intersect with the line $k$ twice, on its left and right branches, respectively, which accordingly define the values of $I_{1,Left} (I_2)$ and $I_{1,Right} (I_2)$, respectively. As $I_2$ gradually decreases, $F (\cdot, I_2)$ will eventually intersect with $k$ only once when it achieves its maximum, and then any further decrease in $I_2$ will cause $F (\cdot, I_2)$ to be below the line $k$.

To prove the last statement, note the following. Denote $I_{1,\text{max}} F (I_2) \equiv \arg \max_{I_1} F (I_{1,\text{max}} F (I_2), I_2)$ and $M (I_2) \equiv F (I_{1,\text{max}} F (I_2), I_2)$. By the envelope theorem, we have $M' (I_2) = \frac{\partial F (I_{1,\text{max}} F (I_2), I_2)}{\partial I_2} \propto f_2 (I_{1,\text{max}} F (I_2), I_2)$. Direct computation shows that $f_2 (I_{1,\text{max}} F (I_2), I_2) - f_1 (I_{1,\text{max}} F (I_2), I_2) > 0$ for $I_1 > I_2$. Given the first-order-condition $f_1 (I_{1,\text{max}} F (I_2), I_2) = 0$ for any $I_2 > 0$, we thus have $f_2 (I_{1,\text{max}} F (I_2), I_2) > 0$ and $M (I_2) \equiv F (I_{1,\text{max}} F (I_2), I_2)$ increases with $I_2$ for allocations in $C^5$, and indeed, as $I_2$ decreases, the maximum value of $F (\cdot, I_2)$ decreases as well.

In addition, we also know that at $I_2 = 0$, $F (\cdot, I_2)$ achieves its maximum at $I_1 = 0$, that is, $I_{1,\text{max}} F (0) = 0$ and $M (0) = F (0, 0) = \bar{k}_1$. Given the condition of $\bar{k}_1 < k$, we know that as $I_2$ decreases toward 0, the curve $F (\cdot, I_2)$ will have no intersections with $k$.

The above discussion also implies that $I_2 > 0$ is given by

$$F (I_{1,\text{max}} F (I_2), I_2) = k.$$  \hfill (IA86)

So, when $I_2 \leq I_2 < L$, the curve $F (\cdot, I_2)$ intersects with $k$ twice. The intersection point on its left branch is $I_{1,Left} (I_2)$. (We also know that $I_2$ lies on the left branch of $F (\cdot, I_2)$ and that $F (\cdot, I_2) < k$, and thus $I_{1,Left} (I_2) > I_2$; that is, for $(I_1, I_2) \in I_{C^5}$ with $I_2 \notin [L, \bar{I}]$, the condition of $F (I_1, I_2) \geq k$ implies $I_1 \geq I_{1,Left} (I_2)$ and makes the condition $I_1 > I_2$ redundant.) The intersection point of $F (\cdot, I_2)$ with $k$ on the right branch determines $I_{1,Right} (I_2)$. Therefore, both $I_{1,Left} (I_2)$ and $I_{1,Right} (I_2)$ are well-defined, and any $(I_1, I_2) \in I_{C^5}$ with $I_2 \notin [L, \bar{I}]$ also belongs to $I_{C^5, k_1 < k \leq k_2}$, which completes the proof of $I_{C^5} \subseteq I_{C^5, k_1 < k \leq k_2}$.
It is also straightforward to check that $I_{c^5,k_1<k<k_2} \subseteq I_{c^5}$. Thus, $I_{c^5} = I_{c^5,k_1<k<k_2}$.

The range of $c_2$.

By

$$c_2 = \phi_{12} (I_1, I_2) - \phi_1 (I_1, I_2) - k = \frac{1}{2\gamma} \log \left( \frac{\rho + I_2^2 \chi}{\omega - 1} \right) - k,$$

(IA87)

we have $c_2$ is decreasing in $I_2$. So, the lower bound of $c_2$ is

$$c_{2,k_1 > I_2} = \frac{1}{2\gamma} \log \left( \frac{\rho - 1 + \omega^{-1}}{\omega - 1} \right) - k,$$

(IA88)

where the second equality follows from the definition of $I$ (i.e., $F (I, I) = k$).

For $k \leq k_1$, the upper bound of $c_2$ is

$$\overline{c}_{2,k_1 > I_2} = \frac{1}{2\gamma} \log \left( \frac{\rho^{-1} + \omega^{-1}}{\omega - 1} \right) - k,$$

(IA89)

and for $k_1 < k < k_2$, the upper bound of $c_2$ is

$$\overline{c}_{2,k_1 > I_2} = \frac{1}{2\gamma} \log \left( \frac{\rho + I_2^2 \chi^{-1} + \omega^{-1}}{\omega - 1} \right) - k.$$

(IA90)

The boundaries of $C^5$.

Equation (IA87) implies that $I_2$ and $c_2$ are related as follows:

$$I_2 (c_2) = \sqrt{\frac{\omega}{(e^{2\gamma (c_2 + k)} - 1) \chi} - \frac{\rho}{\chi}}.$$

(IA91)

By

$$c_1 = \phi_1 (I_1, I_2) = \frac{1}{2\gamma} \log \left( \frac{(I_1 - I_2)^2 + 2\rho \chi^{-1}}{(\rho + I_2^2 \chi^{-1} + \omega^{-1})} \right),$$

(IA92)

c_1 decreases with $I_1$. So, for any given $I_2$, or equivalently, any $c_2 \in [c_{2,k_1 > I_2}, \overline{c}_{2,k_1 > I_2}]$ by (IA91),

c_1 is bounded by $\phi_1 \left( \min \{ I_2 + \frac{\rho}{I_2 \chi}, I_{1,Right} (I_2) \} , I_2 \right)$ and $\phi_1 \left( \max \{ I_2, I_{1,Left} (I_2) \} , I_2 \right)$. That is,

$$c_{\min I_1 > I_2} (c_2) = \phi_1 \left( \min \{ I_2 (c_2) + \frac{\rho}{I_2 (c_2) \chi}, I_{1,Right} (I_2 (c_2)) \} , I_2 (c_2) \right).$$

(IA93)

In particular, when $I_2 (c_2) + \frac{\rho}{I_2 (c_2) \chi} \leq I_{1,Right} (I_2 (c_2))$, we have $c_{\min I_1 > I_2} (c_2) = \phi_1 \left( I_2 + \frac{\rho}{I_2 \chi}, I_2 \right) = 0$, where the second equality follows the expression of $\phi_1 (I_1, I_2)$. If $k \leq k_1$,

$$\overline{c}_{1,k_1 > I_2} (c_2) = \phi_1 (I_2 (c_2), I_2 (c_2))$$

$$= \frac{1}{2\gamma} \log \left( \frac{1}{(e^{2\gamma (c_2 + k)} - 1 - \frac{\rho}{2\omega})^{-1} + 1} \right) - (c_2 + k),$$

for $c_2 \in \left( \frac{1}{2\gamma} \log \left( \frac{\rho + I_2^2 \chi^{-1} + \omega^{-1}}{\omega - 1}\right) - k, \frac{1}{2\gamma} \log \left( \frac{\rho^{-1} + \omega^{-1}}{\omega - 1}\right) - k \right)$ (IA94)

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and if $\bar{k}_1 < k \leq \bar{k}_2$,

$$\bar{c}_{I_1 > I_2} (c_2) = \begin{cases} 
\frac{1}{2\gamma} \log \left( \frac{1}{e^{\gamma(c_2+k)} - \frac{\rho}{2\omega}^{-1}} + 1 \right), \\
- (c_2 + k), \\
\phi_1 (I_{1, Left} (I_2 (c_2)), I_2 (c_2)), \\
\frac{1}{2\gamma} \log \left( \frac{(\rho+I_2^2\chi)^{-1} + \omega^{-1}}{\omega} \right) - k 
\end{cases}$$

for $c_2 \in [\frac{1}{2\gamma} \log \left( \frac{(\rho+I_2^2\chi)^{-1} + \omega^{-1}}{\omega} \right) - k, 0]$.

It is also clear that for any $(c_1, c_2) \in C^5$, there exists a unique information market equilibrium with $\lambda_1 > 0$, $\lambda_2 = 0$, and $\lambda_{12} > 0$. Specifically, for any given $(c_1, c_2) \in C^5$, we can compute a unique $I_2$ from (IA87). This implies a unique $\lambda_{12}$ by (IA49): $\lambda_{12} = \frac{2}{\omega} I_2$. Then, given that $\phi_1 (I_1, I_2)$ decreases with $I_1$ in (IA92), the value of $I_1$ is uniquely determined by $c_1$ and $I_2$. Finally, using $I_1$ and $I_2$, we can determine $\lambda_1$ from equation (IA50).

We summarize the above results into the following lemma.

**Lemma 5** (a) If $k > \bar{k}_2$, then the set $C^5$ is empty. Otherwise, it is given by

$$C^5 = \left\{ (c_1, c_2) \in \mathbb{R}_+^2 : c_2 \in [c_{2I_1 > I_2}, \bar{c}_{2I_1 > I_2}] \text{ and } c_{1I_1 > I_2} (c_2) \leq c_1 < \bar{c}_{1I_1 > I_2} (c_2) \right\},$$

where the constant $c_{2I_1 > I_2}$ is given by (IA88), the function $c_{1I_1 > I_2} (\cdot)$ is given by (IA93), the constant is given by (IA90) for $\bar{k}_1 < k \leq \bar{k}_2$ and by (IA89) for $k \leq \bar{k}_1$, and the function $\bar{c}_{1I_1 > I_2} (\cdot)$ is given by (IA95) for $k_1 < k \leq \bar{k}_2$ and by (IA94) for $k \leq \bar{k}_1$. (b) For any $(c_1, c_2) \in C^5$, there exists a unique information market equilibrium with $\lambda_1 > 0$, $\lambda_2 = 0$, and $\lambda_{12} > 0$.

### 5.2.7 Characterization of $C^6$

This is a symmetric case to $C^5$, and the set $C^6$ can be characterized by the following lemma.

**Lemma 6** (a) If $k > \bar{k}_2$, then the set $C^6$ is empty. Otherwise, it is given by

$$C^6 = \left\{ (c_1, c_2) \in \mathbb{R}_+^2 : c_1 \in [c_{1I_1 < I_2}, \bar{c}_{1I_1 < I_2}] \text{ and } c_{2I_1 < I_2} (c_1) < c_2 < \bar{c}_{2I_1 < I_2} (c_1) \right\},$$

where the constants $c_{1I_1 < I_2}$ and $\bar{c}_{1I_1 < I_2}$ and the functions $c_{2I_1 < I_2} (\cdot)$ and $\bar{c}_{2I_1 < I_2} (\cdot)$ are defined similarly as in Lemma 5. (b) For any $(c_1, c_2) \in C^6$, there exists a unique information market equilibrium with $\lambda_1 = 0$, $\lambda_2 > 0$, and $\lambda_{12} > 0$. 

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5.2.8 Characterization of $C^7$

If $k > \bar{k}_2$, the condition of $F(I_1, I_2) \leq k$ is redundant in $C^7$, and $C^7$ becomes:

$$C^7_{k > \bar{k}_2} \equiv \left\{ (c_1, c_2) \in \mathbb{R}^{2_+} : c_1 = \phi_1(I_1, I_2), c_2 = \phi_2(I_1, I_2), \text{ for all } I_1 > 0, I_2 > 0, \text{ such that } \rho + I_1^\chi - I_1I_2^\chi > 0, \rho + I_2^\chi - I_1I_2^\chi > 0 \right\}.$$ 

We will focus on this case to characterize $C^7$, and the way of characterizing this case will also make straightforward the characterization of the other cases. The proof below is largely the same as the proof for Proposition 4 in the paper.

We first decompose the set $C^7_{k > \bar{k}_2}$ into two symmetric sets:

$$C^7_{k > \bar{k}_2, I_1 \geq I_2} \equiv \left\{ (c_1, c_2) \in \mathbb{R}^{2_+} : c_1 = \phi_1(I_1, I_2) \text{ and } c_2 = \phi_2(I_1, I_2), \text{ for all } (I_1, I_2) \in \mathbb{R}^2_+ \text{ such that } I_2 \leq I_1 < I_2 + \frac{\rho}{I_1^\chi} \right\}$$

and

$$C^7_{k > \bar{k}_2, I_1 \leq I_2} \equiv \left\{ (c_1, c_2) \in \mathbb{R}^{2_+} : c_1 = \phi_1(I_1, I_2) \text{ and } c_2 = \phi_2(I_1, I_2), \text{ for all } (I_1, I_2) \in \mathbb{R}^2_+ \text{ such that } I_1 \leq I_2 < I_1 + \frac{\rho}{I_1^\chi} \right\}.$$ 

Apparently, $C^7_{k > k_2} = C^7_{k > \bar{k}_2, I_1 \geq I_2} \cup C^7_{k > \bar{k}_2, I_1 \leq I_2}$.

Given the symmetry of the two sets $C^7_{k > \bar{k}_2, I_1 \geq I_2}$ and $C^7_{k > \bar{k}_2, I_1 \leq I_2}$, we will analyze $C^7_{k > \bar{k}_2, I_1 \geq I_2}$ only. Basically, we will show that

$$C^7_{k > \bar{k}_2, I_1 \geq I_2} = \left\{ (c_1, c_2) \in \mathbb{R}^{2_+} : c_1 \in \left(0, \frac{1}{2\gamma} \log \left(\frac{2\rho - 1 + \omega^{-1}}{\rho - 1 + \omega^{-1}}\right)\right), \text{ and } c_1 \leq c_2 < \frac{1}{2\gamma} \log \left(\frac{e^{-\gamma c_1}}{e^{-\gamma c_1} - 1}\right) \right\}. \quad \text{(IA96)}$$

To establish this result, we first characterize the two constant boundaries of $c_1$, and then characterize the two functions bounding $c_2$ for a given $c_1$.

In allocations supported by parameters in $C^7$, the cost $c_1$ is given by $c_1 = \phi_1(I_1, I_2) = \frac{1}{2\gamma} \log \left(\frac{(I_2 - I_1)^2 + 2\rho^{-1} \omega^{-1} + \omega^{-1}}{\rho + I_2^\chi} \right)$, which decreases with $I_1$. Thus, for a given $I_2 > 0$, $c_1$ achieves its infimum at $I_1 = I_2 + \frac{\rho}{I_2^\chi}$ and its maximum at $I_1 = I_2$. That is, $\phi_1 \left(I_2 + \frac{\rho}{I_2^\chi}, I_2\right) < c_1 \leq \phi_1 \left(I_2, I_2\right)$. Direct computation shows $\phi_1 \left(I_2 + \frac{\rho}{I_2^\chi}, I_2\right) = 0$. Thus, for any given $I_2 > 0$, we have $c_1 \in \left(0, \phi_1 \left(I_2, I_2\right)\right]$. Given $I_2$ can take any positive value in $C^7_{k > \bar{k}_2, I_1 \geq I_2}$, we have the range of $c_1$ is given by

$$\cup_{I_2 > 0} \left(0, \phi_1 \left(I_2, I_2\right)\right] = \left(0, \max_{I_2 > 0} \phi_1 \left(I_2, I_2\right)\right] = \left(0, \phi_1 \left(0, 0\right)\right] = \left(0, \frac{1}{2\gamma} \log \left(\frac{2\rho - 1 + \omega^{-1}}{\rho - 1 + \omega^{-1}}\right)\right].$$

We now fix any $c_1 \in \left(0, \frac{1}{2\gamma} \log \left(\frac{2\rho - 1 + \omega^{-1}}{\rho - 1 + \omega^{-1}}\right)\right]$ and find all the corresponding values of $c_2$ in $C^7_{k > \bar{k}_2, I_1 \geq I_2}$ as follows. Given that $\phi_1 \left(I_2, I_2\right)$ decreases with $I_2$, there exists a unique $I_2_{c_1}, I_1$, which
is determined by \( \phi_1 \left( \bar{I}_{2,c_1}, \bar{I}_{2,c_1} \right) = c_1 \). The pairs of \((I_1, I_2)\) that can be supported by the value of \(c_1\) in \( C^7_{k > \bar{k}_2, I_1 \geq I_2} \) must satisfy \( I_2 \leq \bar{I}_{2,c_1} \): Otherwise, for any \( I_2 > \bar{I}_{2,c_1} \) and \( I_1 \in [I_2, I_2 + \frac{\rho}{I_2^{\gamma_1}}) \), we have \( \phi_1 \left( I_1, I_2 \right) < \phi_1 \left( I_2, I_2 \right) < \phi_1 \left( \bar{I}_{2,c_1}, \bar{I}_{2,c_1} \right) = c_1 \). For any \( I_2 \in (0, \bar{I}_{2,c_1}] \), there exists a unique \( I_1, I_2, c_1 \) that generates \( c_1 \) through \( \phi_1 \left( I_1, I_2, c_1 \right) \) and then determines the value of \( c_2 \) through \( c_2 = \phi_2 \left( I_1, I_2, c_1, I_2 \right) \). By doing so, for the given \( c_1 \), all the corresponding values of \( c_2 \) can be generated through varying \( I_2 \in (0, \bar{I}_{2,c_1}] \); that is, for the given \( c_1 \), we determine the constant \( \bar{I}_{2,c_1} \), and then for any \( I_2 \in (0, \bar{I}_{2,c_1}] \), we have \( c_2 = \phi_2 \left( I_1, I_2, c_1, I_2 \right) \), where \( I_1, I_2, c_1 \) is determined by \( \phi_1 \left( I_1, I_2, c_1, I_2 \right) = c_1 \).

By the chain rule, we have:

\[
\frac{dc_2}{dI_2} = \frac{\partial \phi_2 \left( I_1, I_2, c_1, I_2 \right)}{\partial I_1} \frac{\partial I_1}{\partial I_2} + \frac{\partial \phi_2 \left( I_1, I_2, c_1, I_2 \right)}{\partial I_2},
\]

and by applying the implicitly function theorem to \( \phi_1 \left( I_1, I_2, c_1, I_2 \right) = c_1 \), we can find

\[
\frac{\partial I_1, I_2, c_1}{\partial I_2} = -\frac{\partial \phi_1 \left( I_1, I_2, c_1, I_2 \right)}{\partial I_2} \bigg/ \phi_2 \left( I_1, I_2, c_1, I_2 \right) = c_1.
\]

Plugging this equation into \( \frac{dc_2}{dI_2} \), and using the expression forms of \( \phi_1 \left( I_1, I_2 \right) \), we can show that

\[
\frac{dc_2}{dI_2} < 0.
\]

That is, \( c_2 = \phi_2 \left( I_1, I_2, c_1, I_2 \right) \) decreases with \( I_2 \) for \( I_2 \in (0, \bar{I}_{2,c_1}] \). Thus, for a given \( c_1 \), the lower bound for \( c_2 \) is \( \phi_2 \left( I_1, \bar{I}_{2,c_1}, c_1, \bar{I}_{2,c_1} \right) \) and the upper bound is \( \phi_2 \left( I_1, 0, c_1, 0 \right) \). Using the fact of \( \phi_1 \left( \bar{I}_{2,c_1}, \bar{I}_{2,c_1} \right) = c_1 \) and \( \phi_1 \left( I_1, I_2, c_1, I_2 \right) = c_1 \), we can show that \( \phi_2 \left( I_1, I_2, c_1, \bar{I}_{2,c_1} \right) = c_1 \) and \( \phi_2 \left( I_1, 0, c_1, 0 \right) = \frac{1}{2\gamma} \log \left( \frac{\left( \rho^{-1+\omega^{-1}} \right) e^{2\gamma_1}}{(e^{2\gamma_1-1})\rho^{-1+\omega^{-1}}e^{2\gamma_1}} \right) \). This completes the proof of expression (IA96).

We can obtain a similar expression for \( C^7_{k > \bar{k}_2, I_1 \leq I_2} \), and thus \( C^7_{k > \bar{k}_2} \) is given by:

\[
C^7_{k > \bar{k}_2} = \left\{ (c_1, c_2) \in \mathbb{R}^2_+ : c_1 \in \left( 0, \frac{1}{2\gamma} \log \left( \frac{2\rho^{-1+\omega^{-1}}}{\rho^{-1+\omega^{-1}}e^{2\gamma_1}} \right) \right) \right\} \\
\cup \left\{ (c_1, c_2) \in \mathbb{R}^2_+ : c_2 \in \left( 0, \frac{1}{2\gamma} \log \left( \frac{\left( \rho^{-1+\omega^{-1}} \right) e^{2\gamma_2}}{(e^{2\gamma_2-1})\rho^{-1+\omega^{-1}}e^{2\gamma_2}} \right) \right) \right\}. \tag{IA97}
\]

When \( k \leq \bar{k}_2 \), the condition of \( F \left( I_1, I_2 \right) \leq k \) in \( C^7 \) becomes relevant. However, the above proof for (IA96) has nothing to do with parameter \( k \), and in particular, we see that, for a given \( c_1 \), the function \( c_2 = \phi_2 \left( I_1, I_2, c_1, I_2 \right) \) still decreases with \( I_2 \). What the condition \( F \left( I_1, I_2 \right) \leq k \) does is to simply change the range of \( I_2 \) for a given \( c_1 \) (that is the corresponding part of \( (0, \bar{I}_{2,c_1}] \) in the proof of (IA96)). As a result, when \( k \leq \bar{k}_2 \), the set of \( C^7 \) can be expressed as \( C^7_{k \leq \bar{k}_2} = C^7_{k > \bar{k}_2} \cap (C^2 \cup C^5 \cup C^6) \). Also, given \( F \left( I_1, I_2 \right) > 0 \) for all \( (I_1, I_2) \) by Lemma F, we
know that $C^7$ is empty for $k = 0$.

The above proof also shows that for any $(c_1, c_2) \in C^7$, there exists a unique pair of $(I_1, I_2)$ that is supported by $(c_1, c_2)$ in this case. Take $c_1 \leq c_2$ and $k > \bar{k}_2$ as an example. For a given $c_1$, we know that the supported pair $(I_1, I_2)$ takes the form of $(I_{1,2,c_1}, I_2)$, where $\phi_1(I_{1,2,c_1}, I_2) = c_1$, $\phi_2(I_{1,2,c_1}, I_2) = c_2$ and $I_2 \in (0, \bar{I}_{2,c_1}]$. However, given $\frac{\partial \phi_2}{\partial I_2} < 0$, different values of $I_2 \in (0, \bar{I}_{2,c_1}]$ will be supported by different values of $c_2$. Thus, each $(c_1, c_2) \in C^7_{k>\bar{k}_2, I_1 \geq I_2}$ can only support a unique pair $(I_1, I_2)$. Then, using equation (IA56) we can determine a unique pair $(\lambda_1, \lambda_2)$ through the determined $(I_1, I_2)$.

We summarize all the above discussions in the following lemma.

**Lemma 7** (a) If $k > \bar{k}_2$, then $C^7$ is given by $C^7_{k>\bar{k}_2}$ in (IA97). If $0 < k \leq \bar{k}_2$, then $C^7$ is non-empty and is given by $C^7_{k>\bar{k}_2} \setminus (C^2 \cup C^5 \cup C^6)$. If $k = 0$, then $C^7$ is empty. (b) For any given $(c_1, c_2) \in C^7$, there exists a unique information market equilibrium with $\lambda_1 > 0, \lambda_2 > 0$ and $\lambda_{12} = 0$.

### 5.2.9 Characterization of $C^8$

In the proof of the above cases, we see that $C^8$ is given by the intersection boundaries of $C^5 \cup C^6$ and $C^7$. In addition, the above analysis on cases $C^5 \cup C^6$ and $C^7$ show that a given parameter pair $(c_1, c_2)$ can support a unique pair $(I_1, I_2)$ of equilibrium trading intensities. However, we also know that the uniquely determined pair $(I_1, I_2)$ can be supported by a continuum of trading type distributions $(\lambda_1, \lambda_2, \lambda_{12})$ in Case 8. Intuitively, the two parameters $(c_1, c_2)$ uniquely determine the two variables $(I_1, I_2)$, but there are three $\lambda$’s to be determined, which can not be uniquely pinned down. We formally summarize these results in the following lemma.

**Lemma 8** (a) If $k > \bar{k}_2$, then the set $C^8$ is empty. Otherwise, it is non-empty and has a zero Lebesgue measure in the parameter space $\mathbb{R}_{++}^7$. Specifically, if $\bar{k}_1 < k \leq \bar{k}_2$, then $C^8$ is given by the lower bound $c_1 = c_{1I_1 > I_2}(c_2)$ and part of the upper bound $c_1 = c_{1I_1 > I_2}(c_2)$ for $c_2 \in \left\{ \frac{1}{2\gamma} \log \left( \left( \frac{r+p^2\gamma}{\omega+1} \right)^{-1} + \omega^{-1} \right) - k, \frac{1}{2\gamma} \log \left( \left( \frac{r+p^2\gamma}{\omega+1} \right)^{-1} + \omega^{-1} \right) - k \right\}$ in $C^5$, and the counterparts in $C^6$; and if $k \leq \bar{k}_1$, then $C^8$ is given by the lower bounds $c_1 = c_{1I_1 > I_2}(c_2)$ in $C^5$ and
$c_2 = c_{I_1 < I_2} (c_1)$ in $\mathcal{C}^6$. (b) For any given $(c_1, c_2) \in \mathcal{C}^8$, there exists a unique pair $(I_1, I_2)$ in equilibrium and a continuum of trader distributions with $\lambda_1 > 0, \lambda_2 > 0$ and $\lambda_{12} > 0$.

By carefully checking the boundaries of the $\mathcal{C}$ sets in Part (a) of Lemmas 1-8, for any given $(k, \rho, \omega, \chi, \gamma) \in \mathbb{R}_{++}^5$, we see that their union forms the whole $(c_1, c_2)$-space $\mathbb{R}_{++}^2$. This means that for any exogenous parameter configuration $(c_1, c_2, k, \rho, \omega, \chi, \gamma) \in \mathbb{R}_{++}^7$, there exists an information market equilibrium. In addition, except that sets $\mathcal{C}_5, \mathcal{C}_6, \mathcal{C}_7$ and $\mathcal{C}_8$ can potentially intersect at their boundaries (which have a zero Lebesgue measure in the parameter space), all the $\mathcal{C}$ sets are mutually exclusive. Given that Part (b) of Lemmas 1-8 shows that there exists a unique information market equilibrium within each $\mathcal{C}$ set, we know that the information market equilibrium is unique except for a parameter set of zero Lebesgue measure.