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Information Diversity and Complementarities in Trading and Information Acquisition

ITAY GOLDSTEIN and LIYAN YANG*

ABSTRACT

We analyze a model in which different traders are informed of different fundamentals that affect the security value. We identify a source for strategic complementarities in trading and information acquisition: aggressive trading on information about one fundamental reduces uncertainty in trading on information about the other fundamental, encouraging more trading and information acquisition on that fundamental. This tends to amplify the effect of exogenous changes in the underlying information environment. Due to complementarities, greater diversity of information in the economy improves price informativeness. We discuss the relation between our model and recent financial phenomena and derive testable empirical implications.

SECURITY PRICES REFLECT INFORMATION available to traders about future payoffs. Uncertainty about these payoffs typically involves multiple dimensions. Obvious examples include multinational firms, for which there is uncertainty originating from the different countries where the firm operates, and conglomerates, for which there is uncertainty about the different industries the firm operates in. More generally, even focused firms are exposed to multiple dimensions of uncertainty: cash flows depend on the demand for firms' products and the technology they develop, on firms' idiosyncratic developments and the way they are affected by the macroeconomy or the industry, and on the success of firms' operations in traditional lines of business and in new speculative lines of

*Itay Goldstein is with Wharton School, University of Pennsylvania and Liyan Yang is with Joseph L. Rotman School of Management, University of Toronto. This paper was previously circulated under the title "Information diversity and market efficiency spirals." We thank the Editor (Bruno Biais) and three anonymous referees for constructive comments that have significantly improved the paper. We also thank Efstathios Avdis, Bradyn Breon-Drish, Henry Cao, Pierre Chaigneau, Alex Edmans, Vincent Glode, Richard Kihlstrom, Péter Kondor, Albert "Pete" Kyle, Doron Levit, Igor Makarov, Konstantin Milbradt, Tarun Ramadorai, Bart Taub, Dimitri Vayanos, Xinli Wang, Kathy Yuan, and audiences at the Georgia Tech Finance Seminar, the HEC Montreal Finance Seminar, the Maryland Smith School Finance Seminar, the Princeton University Finance Seminar, the Purdue Finance Seminar, the Rutgers Finance Seminar, the Virginia McIntire School Finance Seminar, the Wharton Finance Seminar, the 2012 Adam Smith Asset Pricing Conference (Oxford, the United Kingdom), the 2012 EFA Annual Meeting (Copenhagen, Denmark), the 2012 SED Annual Meeting (Limassol, Cyprus), the 2012 Tel Aviv Finance Conference (Tel Aviv, Israel), the 2013 NFA Annual Meeting (Quebec City, Canada), and the 2013 WFA Annual Meeting (Tahoe, the United States) for comments and suggestions. Yang thanks the Social Sciences and Humanities Research Council of Canada for financial support.

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business. In the modern world, information is so complex that traders tend to have a comparative advantage or specialize in different types of information. The price of the security is then based on the trading activities of the different types of informed traders, reflecting the overall expected value of the security given the information in the market.

A key question in understanding the workings of financial markets is what is the nature of interaction between the different types of informed traders. Suppose that people informed about one dimension of uncertainty trade more aggressively on their information or that there is more information produced on this dimension of uncertainty. Does this encourage people with expertise on other dimensions to trade more aggressively on their information and produce more information, or does it deter them from doing so? If the former holds, then there is *strategic complementarity* between the different groups in trading on their information and producing information. If the latter holds, then there is *strategic substitutability* between these differentially informed groups. There is long-standing interest in the literature in understanding mechanisms for strategic complementarity versus substitutability in financial markets, since complementarities are generally thought to amplify shocks, whereas substitutabilities are thought to attenuate them.

In this paper, we address these questions using a model in the spirit of the seminal work of Grossman and Stiglitz (1980). In particular, we extend their setting to include two dimensions of uncertainty about the payoff from the traded security, say, the technology of the firm and the demand for its products. We first consider an economy in which traders are endowed with different types of information; for example, some traders are informed about the technology and others are informed about the demand for the firm's products. As in Grossman and Stiglitz (1980), traders are risk averse, and trade in a market with uninformed traders and noisy supply. We then extend the model to endogenize the decision to produce information. Finally, we allow traders to become informed about the two dimensions simultaneously.

Consider the trading stage and suppose that the size of the group of technology-informed traders increases. Will this cause traders informed about product demand to trade more or less aggressively on their information? The presence of more technology-informed traders implies that more of their information will get into the price. When they trade, demand-informed traders condition on the price of the security (as is typically the case in the Grossman-Stiglitz framework), and hence the information in the price affects their trading decision. The additional information about technology has two opposite effects on how demand-informed traders trade on their own information. First, the increased technology information in the price reduces the uncertainty that demand-informed traders have to face concerning technology issues when they trade, which causes demand-informed traders to trade more aggressively on what they know without being exposed to risks they do not understand. This is the "uncertainty reduction effect," which favors strategic complementarities in trading on different types of information. Second, the additional information about technology in the price also makes demand-informed traders use the price more to infer the level of the fundamental of technology. This will make demand-informed traders trade less aggressively on their own information. More specifically, suppose demand-informed traders have a positive signal about the demand for the firm's product, which causes them to take a large long position. But, holding the price constant, when the price contains more information about technology, a strong demand fundamental implies a weak technology fundamental, and thus demand-informed traders will scale down their positions, trading less aggressively on their information. We call this effect the "inference augmentation effect," and we note that it favors strategic substitutability in trading. Overall, when the uncertainty reduction effect dominates the inference augmentation effect, trading on one type of information is a complement to trading on the other type of information.

It is important to note that strategic complementarities do not arise naturally in most models of financial markets.[1] In particular, the uncertainty reduction effect that we identify here as a source of strategic complementarities is muted in traditional models in the literature that have one dimension of uncertainty (these models go back to Grossman and Stiglitz (1980) and Hellwig (1980)). Indeed, it appears that such an effect is consistent with many financial phenomena in recent years. For example, as uncertainty about the macroeconomy and government policy was growing, commentators argued that traders were pulling out of the market in an attempt to limit their exposure to risks not within their range of expertise. Traders, who traditionally trade on information concerning traditional aspects of the activities of financial institutions and other firms, appear to have become concerned about other dimensions that might be driving these institutions' cash flows, such as their exposure to both more exotic assets and counterparty risks (as in the motivation for the recent paper by Caballero and Simsek (2013)). Since these traders do not know much about such dimensions, the uncertainty involved with trading the securities of these institutions induced traders to pull out of the market, not even trading on the information they had. We therefore find that, once it becomes more difficult to obtain information on one dimension of uncertainty from the price, traders reduce their trading on information they may have involving other dimensions of uncertainty. This is the complementarity that might lead changes in the underlying informational environment to have large effects.

The result of trading complementarities has important implications for market outcomes in our model. For example, in equilibrium, trading on the two types of information is complementary to each other when the trading intensity on one dimension of uncertainty is relatively close to the intensity on the other dimension of uncertainty. As a result, we find that the diversity of information in the economy improves the overall amount of information revealed by prices. That is, fixing the total mass of informed traders, an economy with a diverse information structure—that is, with a greater balance between the two

[1] There is a growing literature on sources for complementarities in financial markets. In Section V, we discuss a few papers that are closely related to ours in that they consider different types of information/fundamentals.

groups of informed traders—will exhibit greater price informativeness than an economy with a concentrated information structure—that is, with many more traders informed about one dimension than the other.

We next extend our analysis to study traders' information acquisition decisions. We start by allowing traders to become informed about one of the two dimensions of uncertainty. We again analyze the interaction between the two types of information. Suppose there are more technology-informed traders in the market. What will be the effect on agents' incentives to acquire information about demand? On the one hand, a variant of the traditional "Grossman-Stiglitz effect" exists in our model, reducing the incentive to produce information about demand when there are more technology-informed traders in the market. On the other hand, the uncertainty reduction effect mentioned above creates strategic complementarity in the information acquisition stage as well: knowing that more technology information will go into the price, traders know that they will face less uncertainty when trading on demand information, and hence have a stronger incentive to produce information about product demand. We identify conditions under which this effect dominates, so that acquiring two types of information is complementary.

Finally, we present an extension in which traders can also acquire the two types of information simultaneously. Assuming a convex cost structure in information acquisition, or that different agents have a comparative advantage in acquiring different kinds of information, we show that the much greater complexity in the analysis does not change our results qualitatively.

Our theory has provided many empirical implications, some of which we review in Sections II.D and III.D. These implications pertain to settings in which it is natural to think of the firm's activities as containing multiple dimensions of uncertainty. It would thus be relatively easy to test these implications in the context of multinational firms—analyzing the effect of changes in the information structure or investor base in one country on trading and information production in the other country—or firms operating in multiple industries. More generally, however, most firms are subject to multiple dimensions of uncertainty even if they are less easy to separate. As mentioned above, firms' cash flows depend on technology and product demand, as well as on their sensitivity to macroeconomic and idiosyncratic shocks. In such settings, it is natural to say that different types of investors specialize in different types of information, in which case one can test our hypotheses using measures of the size of different investor bases and their trading activities. Overall, our theory highlights the importance of looking at the interactions across types of investors and types of information to understand the overall efficiency of the financial market in generating and processing information.

The remainder of this paper is organized as follows. Section I presents the model and characterizes the equilibrium in the trading stage. In Section II, we analyze the interaction between trading on the two types of information, we provide a full characterization of when our model features complementarity versus substitutability in trading, and we highlight implications for the effect of information diversity. In Section III, we endogenize the information acquisition decision and examine when it will exhibit strategic substitutability versus strategic complementarity. Section IV presents an extension in which traders can become informed about the two dimensions simultaneously. In Section V, we discuss the relation of our paper to the literature. Finally, Section VI concludes.

I. The Model

A. Setup

Two assets are traded in the financial market: one riskless asset (bond) and one risky asset (stock). The bond is in unlimited supply; its payoff is one, and its price is normalized to one. The stock has a total supply of one unit; it has a price of $\tilde{p}$, which is determined endogenously in the financial market, and its payoff $\tilde{v}$ is given by

$$\tilde{v} = \tilde{v}_1 + \tilde{v}_2. \tag{1}$$

As we see in (1), the payoff of the stock consists of two ingredients, $\tilde{v}_1$ and $\tilde{v}_2$, sometimes referred to as fundamentals, which are independent and identically distributed (i.i.d.) according to a normal distribution function: $\tilde{v}_i \sim N(0, 1/\rho)$ $(i = 1, 2)$, where $\rho > 0$ represents the common prior precision of $\tilde{v}_1$ and $\tilde{v}_2$. The idea is that there are two dimensions of uncertainty about the payoff from the stock, captured by the variables $\tilde{v}_1$ and $\tilde{v}_2$, and, as we discuss below in more detail, they are potentially observable to different traders.[2]

In the basic setup, described in this section and analyzed in the next section, three types of rational traders trade the bond and the stock in the financial market: (1) $\tilde{v}_1$-informed traders (of mass $\lambda_1 > 0$), who observe the realization of the first component $\tilde{v}_1$ of the stock payoff; (2) $\tilde{v}_2$-informed traders (of mass $\lambda_2 > 0$), who observe the realization of the second component $\tilde{v}_2$ of the stock payoff; and (3) uninformed traders (of mass $\lambda_u > 0$), who do not observe any information. All three types of traders condition their trades on the stock price $\tilde{p}$, as is typical in rational expectations equilibrium models (e.g., Grossman and Stiglitz (1980)). Traders' utility from consumption C is given by the usual constant absolute risk aversion (CARA) function, $-e^{-\gamma C}$, where γ is the risk aversion parameter. Finally, to prevent fully revealing prices, we assume that there are noise traders who trade a random amount $\tilde{x} \sim N(0, 1/\chi)$ (with $\chi > 0$) of the stock, which is independent of the realizations of $\tilde{v}_1$ and $\tilde{v}_2$.[3]

While we assume here that the masses of informed traders, λ_1 and λ_2, are exogenous, we endogenize them later in Section III, where we analyze information acquisition decisions and the learning complementarities that arise in

[2] Several papers in the finance literature also specify that the value of the traded security is affected by more than one fundamental (e.g., Goldman (2005), Yuan (2005), and Kondor (2012), among others).

[3] It is well known that the behavior of such noise traders can be endogenized based on hedging needs or other noninformational motives to trade. For simplicity, we take their behavior here to be exogenous.

our setup. Also, to deliver our message most effectively, in our basic setup we assume that informed traders can only observe one ingredient of the stock payoff, that is, they can be informed about $\tilde{v}_1$ or $\tilde{v}_2$ but not both. As pointed out by Paul (1993, p. 1477), such an assumption "is in the spirit of Hayek's view that one of the most important functions of the price system is the decentralized aggregation of information and that no one person or institution can process all information relevant to pricing." However, to demonstrate the robustness of our results, we consider an extension in Section IV in which traders are allowed to acquire information about the two ingredients at the same time. We show that in the trading game our main results go through when there exist some speculators in the market (of mass $\lambda_{12} > 0$) who are informed about both ingredients at the same time, in addition to the masses λ_1 and λ_2 introduced above.[4] Interestingly, with respect to the information acquisition decision, we show that strictly positive endogenous values of λ_1, λ_2, and λ_{12} obtain at the same time only in knife-edge cases. Hence, our focus on the case in which traders are informed about either $\tilde{v}_1$ or $\tilde{v}_2$ is not only useful for simplifying the analysis and consistent with the spirit of Hayek, but also a natural outcome of endogenous equilibrium behavior.

B. Equilibrium Definition and Characterization

The equilibrium concept that we use is the rational expectations equilibrium (REE), as in Grossman and Stiglitz (1980). In equilibrium, traders trade to maximize their expected utility given their information set, where $\tilde{v}_i$-informed traders know $\tilde{v}_i$ and $\tilde{p}$ $(i = 1, 2)$, and uninformed traders know only $\tilde{p}$. The price $\tilde{p}$ is determined, in turn, by the market-clearing condition, whereby the sum of demands from the three types of rational traders and the noise traders is equal to the supply of the stock, which is equal to one. As in most of the literature, we consider a linear equilibrium, where the price $\tilde{p}$ linearly depends on the signals $\tilde{v}_1$ and $\tilde{v}_2$ and the noisy trading $\tilde{x}$:[5]

$$\tilde{p} = \alpha_0 + \alpha_1\tilde{v}_1 + \alpha_2\tilde{v}_2 + \alpha_x\tilde{x}. \tag{2}$$

The coefficients α_0, α_1, α_2, and α_x are endogenously determined.

As is well known, the CARA-normal setup assumed here implies that the demand function of a trader of type $t \in \{1, 2, u\}$ ($\tilde{v}_1$-informed, $\tilde{v}_2$-informed, and uninformed) is

$$D_t(\mathcal{F}_t) = \frac{E(\tilde{v}|\mathcal{F}_t) - \tilde{p}}{\gamma Var(\tilde{v}|\mathcal{F}_t)}, \tag{3}$$

where $\mathcal{F}_t$ is the trader's information set. Essentially, traders have a speculative motive to trade, which is reflected in the numerator of (3), according to which

[4] Obviously, to prevent the model from becoming trivial, we need to add an additional noise component to the payoff in this case.

[5] Recent work of Breon-Drish (2013) suggests that in the CARA-normal setup the linear equilibrium is unique among the class of continuous equilibria.

they buy (sell) the stock when its price is lower (higher) than the expected payoff. But, as we see in the denominator, speculators trade less aggressively when they are exposed to higher variance in the final payoff and when they are more risk-averse. We can now construct $E(\tilde{v}|\mathcal{F}_t)$ and $Var(\tilde{v}|\mathcal{F}_t)$ for the different traders and plug them into the demand functions to solve for the equilibrium.

The $\tilde{v}_i$-informed traders have information set $\mathcal{F}_i = \{\tilde{p}, \tilde{v}_i\}$. Since they know the ingredient $\tilde{v}_i$, they only need to forecast the other ingredient $\tilde{v}_j$. Suppose that the coefficients in the price function (2) are different from zero (this will be shown to be the case in equilibrium in the proof of Proposition 1). Then the information set $\mathcal{F}_i$ is equivalent to the signal

$$\tilde{s}_{j|i} \equiv \frac{\tilde{p} - \alpha_0 - \alpha_i \tilde{v}_i}{\alpha_j} = \tilde{v}_j + \frac{\alpha_x}{\alpha_j}\tilde{x}, \quad \text{for } i, j = 1, 2, j \neq i, \tag{4}$$

which is a signal about $\tilde{v}_j$ with normally distributed noise and precision $(\alpha_j/\alpha_x)^2\chi$. Applying Bayes's rule, we can compute the two conditional moments $E(\tilde{v}_j|\mathcal{F}_i)$ and $Var(\tilde{v}_j|\mathcal{F}_i)$ and determine the demand function $D_i(\tilde{p}, \tilde{v}_i)$ of $\tilde{v}_i$-informed traders.

The uninformed traders only observe the price $\tilde{p}$, that is, $\mathcal{F}_u = \{\tilde{p}\}$. The price $\tilde{p}$ is equivalent to the following signal in predicting the total payoff $\tilde{v}$:

$$\tilde{s}_u \equiv \frac{\tilde{p} - \alpha_0}{\alpha_x} = \frac{\alpha_1}{\alpha_x}\tilde{v}_1 + \frac{\alpha_2}{\alpha_x}\tilde{v}_2 + \tilde{x}. \tag{5}$$

We can then apply Bayes's rule to compute $E(\tilde{v}|\tilde{p})$ and $Var(\tilde{v}|\tilde{p})$ and obtain uninformed traders' demand function $D_u(\tilde{p})$.

The equilibrium price is determined by the market-clearing condition for the risky asset:

$$\lambda_1 D_1(\tilde{p}, \tilde{v}_1) + \lambda_2 D_2(\tilde{p}, \tilde{v}_2) + \lambda_u D_u(\tilde{p}) + \tilde{x} = 1. \tag{6}$$

Plugging the expressions for $D_i(\tilde{p}, \tilde{v}_i)$ and $D_u(\tilde{p})$ into the above market-clearing condition, we can solve for the price $\tilde{p}$ as a function of the variables $\tilde{v}_1$, $\tilde{v}_2$, and $\tilde{x}$. Then, comparing coefficients with those in the conjectured price function in (2), we get the following proposition that characterizes the linear REE (with the proof included in the Appendix).

PROPOSITION 1: *For any $\lambda_1 > 0$ and $\lambda_2 > 0$, there exists a unique linear REE, in which*

$$\tilde{p} = \alpha_0 + \alpha_1\tilde{v}_1 + \alpha_2\tilde{v}_2 + \alpha_x\tilde{x}.$$

The coefficients $\alpha_0 < 0$, $\alpha_1 > 0$, $\alpha_2 > 0$, and $\alpha_x > 0$ are given as a function of the exogenous parameters of the model in the proof in the Appendix.

II. Trading Intensities and Price Informativeness

A. Definitions and Characterization

In our model, there are two types of fundamentals, $\tilde{v}_1$ and $\tilde{v}_2$. The focus of our analysis is on how aggressively traders trade on information about these fundamentals, which ultimately determines the level of price informativeness. A unit increase in $\tilde{v}_i$ will cause a $\tilde{v}_i$-informed trader to buy $\frac{\partial D_i(\tilde{p},\tilde{v}_i)}{\partial \tilde{v}_i}$ more stock, and so as a group $\tilde{v}_i$-informed traders will buy $\lambda_i \frac{\partial D_i(\tilde{p},\tilde{v}_i)}{\partial \tilde{v}_i}$ more stock. Accordingly, we use this amount to represent the aggregate trading intensity I_i on information $\tilde{v}_i$, that is,

$$I_i \equiv \lambda_i \frac{\partial D_i(\tilde{p}, \tilde{v}_i)}{\partial \tilde{v}_i}, \quad \text{for} \;\; i = 1, 2. \tag{7}$$

We use $\frac{1}{Var(\tilde{v}|\tilde{p})}$, the reciprocal of the variance of $\tilde{v}$ conditional on the price, to measure the price informativeness about the payoff $\tilde{v}$. That is, we define

$$\text{price informativeness} \equiv \frac{1}{Var(\tilde{v}|\tilde{p})}. \tag{8}$$

This also corresponds to how much residual uncertainty uninformed traders face after conditioning on the price.

In equilibrium, trading intensity I_i affects price informativeness through its impact on the price function. Specifically, following Bond and Goldstein (2015), we show that I_i is equal to the ratio $\frac{\alpha_i}{\alpha_x}$ between the sensitivity of the price to fundamental shocks $\tilde{v}_i$ and the sensitivity of the price to noise $\tilde{x}$: [6]

$$I_i \equiv \lambda_i \frac{\partial D_i(\tilde{p}, \tilde{v}_i)}{\partial \tilde{v}_i} = \frac{\alpha_i}{\alpha_x}, \quad \text{for} \;\; i = 1, 2. \tag{9}$$

This expression is intuitive. Trading intensity on the fundamental $\tilde{v}_i$ is defined by the extent to which a change in this fundamental affects the overall demand for the stock. It is this intensity that brings the information about $\tilde{v}_i$ to the market and determines the extent to which the price reflects information about this fundamental versus noise.

We now link the trading intensities to overall price informativeness. From (5), we can compute $Var(\tilde{v}|\tilde{p})$. Then, using (9), we find that the price informativeness $\frac{1}{Var(\tilde{v}|\tilde{p})}$ depends on the trading intensities I_1 and I_2 as follows:

$$\frac{1}{Var(\tilde{v}|\tilde{p})} = \frac{I_1^2\rho + I_2^2\rho + \rho^2\chi^{-1}}{(I_1 - I_2)^2 + 2\rho\chi^{-1}}. \tag{10}$$

[6] To see this formally, note that a unit increase in $\tilde{v}_i$ will cause the group of $\tilde{v}_i$-informed traders to buy $\lambda_i \frac{\partial D_i(\tilde{p},\tilde{v}_i)}{\partial \tilde{v}_i}$ more stock. If it happens that the noise traders supply the same number of extra shares, then, by the market-clearing condition, the price will not change. That is, changing $\tilde{v}_i$ by one unit has the same price impact as changing $\tilde{x}$ by $\lambda_i \frac{\partial D_i(\tilde{p},\tilde{v}_i)}{\partial \tilde{v}_i}$ units. In other words, $|\frac{\partial \tilde{p}}{\partial \tilde{v}_i}| = |\frac{\partial \tilde{p}}{\partial \tilde{x}} \lambda_i \frac{\partial D_i(\tilde{p},\tilde{v}_i)}{\partial \tilde{v}_i}|$ or $\alpha_i = \alpha_x \lambda_i \frac{\partial D_i(\tilde{p},\tilde{v}_i)}{\partial \tilde{v}_i}$ by the price function (2).

Note that I_1 and I_2 positively affect the price informativeness $\frac{1}{Var(\tilde{v}|\tilde{p})}$. That is, $\frac{\partial Var^{-1}(\tilde{v}|\tilde{p})}{\partial I_1} > 0$ and $\frac{\partial Var^{-1}(\tilde{v}|\tilde{p})}{\partial I_2} > 0$ (see equations (A13) and (A14) in the Appendix). This result is intuitive, as one would expect the price system to reveal more information about the total asset payoff $(\tilde{v}_1 + \tilde{v}_2)$ when traders trade more aggressively on their information about the underlying fundamentals.

B. Trading Complementarity versus Substitutability

We now examine how the two trading intensities interact in our model. Specifically, we derive two best response functions $I_i = h_i\left(I_j; \lambda_i, \rho, \gamma, \chi\right)$ (for $i = 1, 2$), which jointly determine the two trading intensities I_1 and I_2 in equilibrium. We are interested in the slope of the response functions. If $h_i\left(I_j; \lambda_i, \rho, \gamma, \chi\right)$ is increasing in I_j, then we say that trading on information $\tilde{v}_i$ is a *complement* to trading on information $\tilde{v}_j$ because, in this case, $\tilde{v}_i$-informed traders trade more aggressively on their information about $\tilde{v}_i$ when $\tilde{v}_j$-informed traders trade more aggressively on $\tilde{v}_j$. Similarly, if $h_i\left(I_j; \lambda_i, \rho, \gamma, \chi\right)$ is decreasing in I_j, then trading on information $\tilde{v}_i$ is a *substitute* to trading on information $\tilde{v}_j$.[7] These concepts of trading complementarity/substitutability have important implications for the workings of financial markets, as we discuss in the next subsection.

The demand function of $\tilde{v}_i$-informed traders is given by $D_i(\tilde{p}, \tilde{v}_i) = \frac{E(\tilde{v}|\mathcal{F}_i) - \tilde{p}}{\gamma Var(\tilde{v}|\mathcal{F}_i)} = \frac{\tilde{v}_i + E(\tilde{v}_j|\mathcal{F}_i) - \tilde{p}}{\gamma Var(\tilde{v}_j|\mathcal{F}_i)}$. We can now use the definition of trading intensity I_i in (9) to get

$$I_i \equiv \lambda_i \frac{\partial D_i(\tilde{p}, \tilde{v}_i)}{\partial \tilde{v}_i} = \frac{\lambda_i}{\gamma} \frac{1}{Var(\tilde{v}_j|\mathcal{F}_i)} + \frac{\partial}{\partial \tilde{v}_i} \frac{\lambda_i E(\tilde{v}_j|\mathcal{F}_i)}{\gamma Var(\tilde{v}_j|\mathcal{F}_i)}. \tag{11}$$

Using the signal observed by $\tilde{v}_i$-informed traders in (4) and applying Bayes's rule, we can compute the conditional moments, $E(\tilde{v}_j|\mathcal{F}_i)$ and $Var(\tilde{v}_j|\mathcal{F}_i)$, and express I_i as follows (recall that $I_i = \frac{\alpha_i}{\alpha_x}$):

$$I_i = \underbrace{\lambda_i \gamma^{-1}(\rho + \chi I_j^2)}_{\text{direct effect from observing } \tilde{v}_i} - \underbrace{\lambda_i \gamma^{-1} \chi I_i I_j}_{\text{indirect effect from inferring } \tilde{v}_j}. \tag{12}$$

We can see that an increase in the signal $\tilde{v}_i$ affects the $\tilde{v}_i$-informed traders' demand in two ways. First, there is a direct effect, according to which an increase in $\tilde{v}_i$ implies a direct increase in the payoff from the stock, leading the $\tilde{v}_i$-informed traders to increase their demand for a given price. This positive effect is represented by the first term on the right-hand side of (11) and (12).

[7] Our notion of strategic complement/substitute is standard in the literature. For example, Paul (1993, p. 1476) writes: "Two variables are strategic complements if the equilibrium response to an increase in one variable is for the other variable to increase and vice versa" and "(t)wo variables are strategic substitutes if the equilibrium response to an increase in one variable is for the other variable to decrease and vice versa."

Second, there is an indirect effect working through the inference that the $\tilde{v}_i$-informed traders make about $\tilde{v}_j$: holding the price constant, an increase in $\tilde{v}_i$ implies a lower expectation about $\tilde{v}_j$, and so reduces the traders' demand for the stock. This negative effect is captured by the second term on the right-hand side of (11) and (12).

We can now rearrange terms in (12) and express I_i as a best response function to I_j:

$$I_i = h_i(I_j; \lambda_i, \rho, \gamma, \chi) \equiv \frac{\lambda_i \left(\rho + \chi I_j^2\right)}{\gamma + \lambda_i \chi I_j}, \quad \text{for } i, j = 1, 2,\ j \neq i. \tag{13}$$

To see more clearly the ways in which I_j affects I_i and their relation to the direct and indirect effects in (12), it is more useful to write this function as follows:

$$\begin{aligned} I_i = & \underbrace{\lambda_i \gamma^{-1}\left(\rho + \chi I_j^2\right)}_{\text{direct effect from observing } \tilde{v}_i} - \underbrace{\lambda_i \gamma^{-1}\left(\rho + \chi I_j^2\right) \frac{\lambda_i \gamma^{-1} \chi I_j}{1 + \lambda_i \gamma^{-1} \chi I_j}}_{\text{indirect effect from inferring } \tilde{v}_j} \\ \equiv & I_{i,Direct}(I_j) - I_{i,Indirect}(I_j). \end{aligned} \tag{14}$$

Now we can study the ways that trading intensity I_j influences trading intensity I_i. First, an increase in I_j strengthens the positive direct effect that the signal $\tilde{v}_i$ has on the demand of $\tilde{v}_i$-informed traders, that is, it increases $I_{i,Direct}(I_j)$. This is because a higher level of I_j implies that there is more information about $\tilde{v}_j$ in the price, which reduces the residual uncertainty $Var(\tilde{v}_j|\mathcal{F}_i)$ that $\tilde{v}_i$-informed traders face when they trade and hence induces them to trade more aggressively on their information about $\tilde{v}_i$. We label this effect the "uncertainty reduction effect," that is,

$$\text{uncertainty reduction effect} \equiv \frac{\partial I_{i,Direct}(I_j)}{\partial I_j} > 0. \tag{15}$$

Second, an increase in I_j also strengthens the negative indirect effect in the expression for I_i, that is, it increases $I_{i,Indirect}(I_j)$. Recall that the second term on the right-hand side of (12) (which is the origin of $I_{i,Indirect}(I_j)$) captures the fact that an increase in $\tilde{v}_i$, holding the price constant, indicates that $\tilde{v}_j$ is lower, which induces the traders to demand less of the stock. This implies a reduction in trading intensity I_i. This force becomes stronger as trading intensity I_j increases. This is because, when the price is more sensitive to $\tilde{v}_j$, $\tilde{v}_i$-informed traders use the price more in inferring information about $\tilde{v}_j$ and so an increase in $\tilde{v}_i$ with a fixed price provides a stronger negative indication about the realization of $\tilde{v}_j$. We label this effect the inference augmentation effect because it captures the effect of trading intensity I_j on trading intensity

I_i via the effect of I_j on the ability of $\tilde{v}_i$-informed traders to make an inference from the price about the signal they do not know. That is,

$$\text{inference augmentation effect} \equiv \frac{\partial I_{i,Indirect}(I_j)}{\partial I_j} > 0. \tag{16}$$

Equations (14), (15), and (16) imply that the slope of the best response function h_i is jointly determined by the uncertainty reduction and inference augmentation effects:

$$\frac{\partial h_i\left(I_j;\lambda_i,\rho,\gamma,\chi\right)}{\partial I_j} = \text{uncertainty reduction effect} - \text{inference augmentation effect}. \tag{17}$$

The uncertainty reduction effect generates strategic complementarity in trading: when $\tilde{v}_j$-informed traders trade more aggressively on $\tilde{v}_j$, and hence there is more information about $\tilde{v}_j$ in the price, $\tilde{v}_i$-informed traders face lower uncertainty about what they do not know, and so trade more aggressively on their information. The inference augmentation effect generates strategic substitutability in trading: when $\tilde{v}_j$-informed traders trade more aggressively on $\tilde{v}_j$ and the price becomes more informative about $\tilde{v}_j$, $\tilde{v}_i$-informed traders use the price to update their expectation to a greater extent, and, because this inference is in opposite direction of their signal, they trade less aggressively on their information. Based on the relative strength of these two effects, the best response function may take a positive or a negative slope, that is, either strategic complementarity or strategic substitutability can dominate. Analyzing (14), (15), and (16), we can see that $h_i\left(I_j;\lambda_i,\rho,\gamma,\chi\right)$ is decreasing in I_j when I_j is low, and increasing when I_j is high. Formally, straightforward calculations yield the following proposition.

PROPOSITION 2: *Trading on information $\tilde{v}_i$ is a complement (substitute) to trading on information $\tilde{v}_j$ if I_j is sufficiently large (small). That is, $\frac{\partial h_i(I_j;\lambda_i,\rho,\gamma,\chi)}{\partial I_j} > 0$ if and only if $I_j > \frac{\sqrt{\gamma^2+\lambda_i^2\rho\chi}-\gamma}{\chi\lambda_i}$.*

C. Implications of the Interaction between the Two Trading Intensities

C.1. Trading Intensity Multipliers

So far, we discuss how trading intensities I_1 and I_2 are determined and how they interact with each other. In particular, we highlight when our model features strategic complementarity versus substitutability, that is, when an increase in trading intensity on one type of information provides incentives for traders to trade more or less aggressively on the other type of information. In this subsection, we discuss implications of the two trading intensities being complements or substitutes. We start by introducing the concept of trading intensity multiplier, which is generated by the interaction between I_1 and I_2.

The trading intensities I_1 and I_2 are determined by the system of equations in (13) as a function of the underlying parameters of the model. Changes in these parameters affect I_1 and I_2; the magnitude of this effect depends on the multiplier. Formally, let Q be one of the five exogenous parameters $(\lambda_1, \lambda_2, \rho, \gamma, \chi)$ that determine (I_1, I_2). Then we have the following proposition (the proof is provided in the Appendix):

PROPOSITION 3:

(a) *The effect of an exogenous parameter Q on the trading intensity I_i about $\tilde{v}_i$ is given by*

$$\underbrace{\frac{dI_i}{dQ}}_{\text{total effect}} = \mathcal{M}\underbrace{\left(\frac{\partial h_i}{\partial Q} + \frac{\partial h_i}{\partial I_j}\frac{\partial h_j}{\partial Q}\right)}_{\text{direct effect}}, \tag{18}$$

where the term $(\frac{\partial h_i}{\partial Q} + \frac{\partial h_i}{\partial I_j}\frac{\partial h_j}{\partial Q})$ captures the direct effect of changing Q on I_i and the coefficient $\mathcal{M}$ is a multiplier given by

$$\mathcal{M} = \left(1 - \frac{\partial h_1}{\partial I_2}\frac{\partial h_2}{\partial I_1}\right)^{-1} > 0. \tag{19}$$

(b) *Suppose that $\lambda_1 > 0$ and $\lambda_2 > 0$. Then (i) when $\frac{1}{2} < \frac{I_1}{I_2} < 2$, $\mathcal{M} > 1$ and so the effect of Q on I_i is amplified in equilibrium, (ii) when $\frac{I_1}{I_2} < \frac{1}{2}$ or $\frac{I_1}{I_2} > 2$, $0 < \mathcal{M} < 1$ and so the effect of Q on I_i is attenuated in equilibrium, and (iii) when $\frac{I_1}{I_2} = \frac{1}{2}$ or $\frac{I_1}{I_2} = 2$, $\mathcal{M} = 1$.*

The direct effect of a change in a parameter Q on trading intensity I_i, which is given by $(\frac{\partial h_i}{\partial Q} + \frac{\partial h_i}{\partial I_j}\frac{\partial h_j}{\partial Q})$, can be thought of as the initial impact before the interdependence between the two trading intensity measures is considered. The interdependence then creates a multiplier, which is given by $\mathcal{M} = (1 - \frac{\partial h_1}{\partial I_2}\frac{\partial h_2}{\partial I_1})^{-1}$, that either amplifies or attenuates the direct effect in equilibrium. As the proof of the proposition shows, the multiplier is strictly positive. We therefore label $\mathcal{M}$ the trading intensity multiplier.

Whether the direct effect is attenuated or amplified in equilibrium depends on whether $\mathcal{M}$ is less than or greater than one, which in turn depends on the signs of the cross-derivatives $\frac{\partial h_1}{\partial I_2}$ and $\frac{\partial h_2}{\partial I_1}$ (i.e., on whether trading on the two types of information is a complement or a substitute). Recall that, by Proposition 2, the two response functions in (13) are decreasing and then increasing. So, when I_1 and I_2 are very far from each other (specifically, when $\frac{I_1}{I_2} < \frac{1}{2}$ or $\frac{I_1}{I_2} > 2$), one of the cross-derivatives is positive while the other is negative. As a result, the interaction between the two trading intensity measures tends to attenuate the initial effect, that is, $0 < \mathcal{M} < 1$. In contrast, when I_1 and I_2 are close to each other, both response functions are upward sloping so that trading on one type of information is a complement to the other.[8] Hence, $\mathcal{M} > 1$, so the two trading intensity measures reinforce each other and the initial effect due to a change in exogenous parameters is amplified in equilibrium.

C.2. The Effect of an Increase in λ_i

To illustrate the effects described above, let us consider the effect of an increase in one parameter of the model—the size λ_i of the population of $\tilde{v}_i$-informed traders—on the two trading intensities in equilibrium. An increase in λ_i can reflect a positive change in the number of traders who are experts in one dimension of the firm's activities and trade in the firm's share. In the next subsection, we discuss possible real-world interpretations in more detail. Note that here we assume that the increase in λ_i is exogenous and study the trading implications for a fixed λ_j. In Section III, where we endogenize information acquisition, we show that an increase in λ_i can also lead to an increase in λ_j, resulting in an additional effect.

By part (a) of Proposition 3, we can express the effect of the increase in λ_i on the equilibrium levels of I_i and I_j as follows:[9]

$$\frac{dI_i}{d\lambda_i} = \mathcal{M}\frac{\partial h_i}{\partial \lambda_i}, \tag{20}$$

$$\frac{dI_j}{d\lambda_i} = \mathcal{M}\frac{\partial h_j}{\partial I_i}\frac{\partial h_i}{\partial \lambda_i} = \frac{\partial h_j}{\partial I_i}\frac{dI_i}{d\lambda_i}. \tag{21}$$

Further, by the expression of h_i in (13), we can easily see that $\frac{\partial h_i}{\partial \lambda_i}$ is positive. This leads to three conclusions regarding the effect of the increase in λ_i. First, an increase in λ_i always increases the trading intensity on information $\tilde{v}_i$. This is intuitive since, when more people are informed about $\tilde{v}_i$, the aggregate increase in demand following an improvement in $\tilde{v}_i$ will be larger.

Second, the effect of an increase in λ_i on the trading intensity on information $\tilde{v}_i$ is greater when the multiplier $\mathcal{M}$ is higher. In particular, the direct effect is attenuated when $\mathcal{M}$ is below one and amplified when $\mathcal{M}$ is above one. This is where the role of the interaction between the two trading intensities comes into play. As we saw in Proposition 3, when I_1 and I_2 are relatively close to each other, the increase in one leads to an increase in the other and so on, so that $\mathcal{M}$ is above one. Hence, the direct effect of λ_i on I_i gets amplified, leading to a much stronger overall effect.

Third, an increase in λ_i has an ambiguous effect on the trading intensity on information $\tilde{v}_j$. The sign of this effect is pinned down by the sign of $\frac{\partial h_j}{\partial I_i}$, that is, by whether trading on information $\tilde{v}_j$ is a complement or a substitute to trading on information $\tilde{v}_i$. From Proposition 2, we know that $\frac{\partial h_j}{\partial I_i} > 0$ if and only if I_i is

[8] We show that in equilibrium it is never the case that both response functions are simultaneously negatively sloped.

[9] Note that $\frac{\partial h_j}{\partial \lambda_i} = 0$.

sufficiently large, which is true when λ_i is sufficiently large (this statement is formally proved in the Appendix). Hence, when the mass of traders informed about $\tilde{v}_i$ is sufficiently large, then an increase in this mass will lead to an increase in trading intensity about $\tilde{v}_j$; otherwise, the effect is the opposite.

We summarize the above discussion in the following corollary.

COROLLARY 1: *An increase in the size λ_i of the population of $\tilde{v}_i$-informed traders*

(a) increases the trading intensity I_i on information $\tilde{v}_i$ (i.e., $\frac{dI_i}{d\lambda_i} > 0$), and the magnitude of this increase is higher when the multiplier $\mathcal{M}$ is larger, and
(b) increases the trading intensity I_j on information $\tilde{v}_j$ if and only if trading on $\tilde{v}_j$ is a complement to trading on $\tilde{v}_i$ (i.e., $\frac{dI_j}{d\lambda_i} > 0$ iff $\frac{\partial h_j}{\partial I_i} > 0$ or $I_i > \frac{1}{2}I_j$), which is true when λ_i is sufficiently large.

C.3. The Effect of Information Diversity

The interaction between the two types of informed traders and its effect on the multiplier have interesting implications for the effect of information diversity in our model. We now explore these implications.

Recall that the mass of traders informed about $\tilde{v}_1$ is λ_1 and the mass of traders informed about $\tilde{v}_2$ is λ_2. Information diversity is a function of the difference between λ_1 and λ_2 (for a fixed total size of the informed-traders population). Formally, let $\lambda_1 + \lambda_2 = \Lambda$ (where Λ is a constant). Then, we define the following measure of information diversity:

$$\Delta \equiv 1 - \frac{|\lambda_1 - \lambda_2|}{\Lambda} \in \left[0, 1\right]. \tag{22}$$

A higher Δ means that the two groups of informed traders are closer in size, and so the total amount of information is more equally distributed between the two types of informed traders, that is, there is more diversity of information in the economy. By this logic, a situation with less diversity is one in which most people know the same thing and so Δ is low.

To see the effect of diversity, note that, when Δ is close to one, λ_1 is close to λ_2, and so I_1 and I_2 are close to each other. Then, by Proposition 3, the trading intensity multiplier $\mathcal{M}$ is greater than one. On the other hand, when Δ is close to zero, either λ_1 or λ_2 is close to zero, and so I_1 and I_2 are far from each other, implying that the trading intensity multiplier $\mathcal{M}$ is smaller than one. This link between Δ and $\mathcal{M}$ has important implications for price informativeness. Specifically, the following corollary shows that, as information diversity increases, the price informativeness $\frac{1}{Var(\tilde{v}|\tilde{p})}$ goes up (the proof is included in the Appendix).

COROLLARY 2: *When the total size of the informed-traders population is fixed, information diversity, defined by Δ in (22), has a positive effect on the informativeness of the price of $\tilde{v}$, that is, $\frac{dVar^{-1}(\tilde{v}|\tilde{p})}{d\Delta} > 0$.*

To understand this result, compare the following two economies with the same total mass of informed traders $\lambda_1 + \lambda_2 = \Lambda$, but with levels of diversity at the two ends of the spectrum: (1) Economy I, where $\lambda_1 = \Lambda - \varepsilon \approx \Lambda$, $\lambda_2 = \varepsilon \approx 0$, and $\Delta \approx 0$ (a "concentrated" economy), and (2) Economy II, where $\lambda_1 = \lambda_2 = \frac{\Lambda}{2}$ and $\Delta = 1$ (a "diverse" economy). These two economies can be obtained by injecting a total mass $(\Lambda - 2\varepsilon)$ of informed traders into an initial economy where there is almost no information (i.e., $\lambda_1 = \lambda_2 = \varepsilon \approx 0$) along two different paths. To obtain Economy I, we only add traders who are informed about $\tilde{v}_1$, while keeping the mass of agents informed about $\tilde{v}_2$ close to zero. In contrast, to obtain Economy II, we simultaneously add traders informed about $\tilde{v}_1$ and traders informed about $\tilde{v}_2$.

Adding informed traders along both paths improves price informativeness, since $\frac{\partial Var^{-1}(\tilde{v}|\tilde{p})}{\partial \lambda_i} > 0$ for $i = 1, 2$ (see equation (A19) in the Appendix). However, the impact of the new information is different on the two different paths leading to the two economies because of the trading intensity multiplier effect identified by Proposition 3. Along the path to obtain Economy I, the multiplier $\mathcal{M}$ is smaller than one, and thus the impact of the new added information is attenuated, while, along the path to obtain Economy II, the multiplier $\mathcal{M}$ is greater than one, and the impact of the new added information is amplified. As a result, the total impact of the added mass $(\Lambda - 2\varepsilon)$ of informed traders on price informativeness $\frac{1}{Var(\tilde{v}|\tilde{p})}$ is larger in Economy II than in Economy I.

Overall, price informativeness is higher in our model when there is more diversity of information, or when there is more balance between the amount of information available on different dimensions. This is because the effect of adding more informed agents on price informativeness is greater when the two trading intensities are relatively close to each other, as in that case the uncertainty reduction effect dominates and trading on the two types of information is complementary, so that both types of traders trade more aggressively, impounding more information into the price and increasing price informativeness.

D. Empirical Implications

Corollaries 1 and 2 present hypotheses that can be tested empirically. There are many settings to which our model potentially applies, that is, in which empirical researchers can test these hypotheses. In this subsection, we describe some of these settings and link them more directly to the results in the two corollaries to highlight how they can be tested.

Our model is based on the idea that two (or more) dimensions of uncertainty affect a firm's cash flows. There are many settings in which this is naturally the case. One example is a multinational firm that operates in several countries. Such a firm's cash flow depends on developments in the different countries. Investors operating in different countries are more likely to be informed about developments involving their own country, and thus our model with heterogeneously informed traders follows directly. Another example is a conglomerate that operates across different industries or business lines. Given the presence of investors who specialize in information about particular industries, our model again follows directly.

Uncertainty about the firm may also come from various other sources. For instance, firms' cash flows depend on the demand for their products and the technology that they develop, on their own idiosyncratic developments and the way they are affected by the macroeconomy or the industry, or on the success of their operations in traditional lines of business and in new speculative lines of business. In such cases, it is reasonable to assume that the population of investors investing in the firm is not homogeneous, but rather contains subsets that specialize in different dimensions of uncertainty.

Testing the results in Corollaries 1 and 2 requires measures of the size of the investor base that specializes in each dimension of uncertainty. In the context of a multinational firm, these measures can be obtained from data on the size of the investor base in different countries. While not all investors in a given country will be informed about the developments of the firm pertaining to that country, a large literature on home bias suggests that investors are more likely to be informed about developments closer to their geographical and/or cultural backgrounds, and so the (relative) size of investor bases in different countries may provide a good approximation for the (relative) size of groups informed about different fundamentals. To get a more direct measure of the populations of informed traders, one may look at analyst coverage or institutional holdings in different countries, since the literature suggests that analysts and institutions are indeed informed (e.g., Dennis and Strickland (2002)). In the case of a conglomerate, tracking different populations of informed traders might be more challenging, since it is not immediately clear how many investors specialize in one industry versus another. A clearer case is that of a merger between two firms. Uncertainty about the newly merged firm's cash flows comes from the lines of business of the two original firms, and one can proxy for the size of different investor bases on the basis of data on investors (or, alternatively, analyst coverage or institutional holdings) in the two original firms. More generally, for other cases of multidimensional uncertainty, one can proxy for differences in the size of different groups of informed agents based on the composition of the investor base. For example, to the extent that institutional investors and individual investors are likely to be informed about different aspects of the firm's value, one can look at the relative size of these two groups of investors and its implications for price informativeness. Alternatively, one could argue that only institutional investors are likely to be informed, in which case heterogeneity among institutional investors can be used to gauge the relative size of different groups of informed investors. Indeed, it is well known that different institutions have different styles of investment (e.g., Fung and Hsieh (1997), Chan, Chen, and Lakonishok (2002)), and so are likely to be informed about different aspects of the firm's value.

Using the above proxies for the size of different investor bases, one can test the hypotheses presented in the previous subsection. Corollary 2 presents a clear hypothesis: the price is more informative about firm cash flows when the investor base is more balanced (information is more diverse). That is, typical measures of price informativeness, such as the price nonsynchronicity measure of Morck, Yeung, and Yu (2000) or the VAR-approach based measure of Hasbrouck (1991), will increase when different investor bases have more similar size. Depending on the application, measures of investor base size will be based on investors in different countries (for the multinational firm), investors associated with different original firms (for the newly merged entity), or different classes of investors (e.g., institutional investors employing different styles of investment).

In Corollary 1, the hypotheses are a bit more subtle. One would like to know how an increase in the size of one group affects informativeness on different dimensions of information or the trading intensity of the two groups. In the case of multinational firms, the question is whether an increase in the investor base in one country affects the sensitivity of the price to innovations in that country and in another country. In other cases, a more indirect test would be to look at the effect of an increase in the size of one investor base on the overall trading activity of that base of investors (part (a) of the corollary) and on that of the other base of investors (part (b) of the corollary). With respect to part (a), we would expect an increase in the size λ_i of one group of investors to increase the trading intensity I_i of that group of investors, which can be assessed by looking at their trading activities, such as order flows or turnover. Moreover, we expect the overall effect to be stronger when the multiplier $\mathcal{M}$, characterized by equation (19), is larger. Turning to part (b), we have a very sharp hypothesis: an increase in the size λ_i of one investor base increases the trading intensity I_j of the other investor base if and only if $I_i > \frac{1}{2}I_j$, a condition that can be pinned down by comparing the trading activities (such as order flows or turnover) of the two investor bases in the data.

In the above tests, it is important to note that our paper makes the simplifying assumption that the two fundamentals $\tilde{v}_1$ and $\tilde{v}_2$ are symmetric in the sense that they have the same unconditional variance. This assumption will not hold perfectly in reality, but was made to deliver the theoretical insights in the most transparent way. Because our results are not driven by this assumption, we expect our main predictions to be qualitatively similar even if the two fundamentals have different variances. For empirical testing, however, it is important to keep this feature of the model in mind and adjust the tests accordingly. Consider, for example, the variable Δ in Corollary 2: if $Var(\tilde{v}_1) > Var(\tilde{v}_2)$, then increasing λ_1 and decreasing λ_2 by the same amount will have direct effects on informativeness beyond the effects of diversity that we study in the paper, since this change in the composition of informed trader population implies that in aggregate the two groups of informed traders know more about the overall asset payoff. To address this issue in empirical testing, it is thus important to normalize the sizes of the informed trader populations by the unconditional variances of the different fundamentals.

Finally, an alternative to testing Corollaries 1 and 2 is to test the basic idea of the uncertainty reduction effect directly in the data. The essential spirit of this effect is that informed traders will choose to trade more (less) aggressively when facing less (more) uncertainty. For example, one can test this effect by checking whether the arrival of policy uncertainty or macroeconomic shocks cause traders who do not specialize in this type of information to face more uncertainty and hence scale down their trading. As mentioned in the introduction, casual observations suggest that such a force has been at work in the crisis and its aftermath.

III. Endogenous Information Acquisition

So far, we assume that the masses of agents who are informed about the fundamentals $\tilde{v}_1$ and $\tilde{v}_2$—λ_1 and λ_2, respectively—were exogenous. We now endogenize these parameters and examine how they are determined in light of the incentives to become informed in our model. The new interesting result that we get relative to the literature is that sometimes there will be a dominant strategic complementarity in information acquisition, whereby the increase in the mass of agents acquiring information on one fundamental leads more agents to acquire information on the other fundamental. This is because of the uncertainty reduction effect identified earlier.

A. Information Acquisition in Equilibrium

We assume that traders can acquire the signal $\tilde{v}_1$ at cost $c_1 > 0$, and the signal $\tilde{v}_2$ at cost $c_2 > 0$. Traders who choose to acquire $\tilde{v}_1$ ($\tilde{v}_2$) become part of the λ_1 (λ_2) group in the trading model described in previous sections, while those who choose not to acquire information become part of the λ_u group. For now, we assume that any trader has an opportunity to become informed about only one of the two fundamentals. In Section IV, we present an extension that allows traders to become informed about both fundamentals at the same time, and show that our main results go through. Moreover, it turns out that only in knife-edge cases do we have strictly positive masses of agents acquiring information about $\tilde{v}_1$, about $\tilde{v}_2$, and about $\tilde{v}_1$ and $\tilde{v}_2$ at the same time. Hence, our focus on the case in which traders are informed about either $\tilde{v}_1$ or $\tilde{v}_2$ is natural.

We assume that the overall mass of rational traders ($\lambda_1 + \lambda_2 + \lambda_u$) is very large. We make this assumption for simplicity to ensure that there will always be some traders who decide to stay uninformed in equilibrium (i.e., $\lambda_u > 0$). As a result, we do not have to consider the corner scenarios in which all traders become informed about either $\tilde{v}_1$ or $\tilde{v}_2$; we only need to consider four possible cases of information market equilibrium: ($\lambda_1 = \lambda_2 = 0$), ($\lambda_1 > 0, \lambda_2 = 0$), ($\lambda_1 = 0, \lambda_2 > 0$), and ($\lambda_1 > 0, \lambda_2 > 0$). The case of $\lambda_u > 0$ is of course empirically relevant, since in reality it is unlikely that every trader is informed.

By computing the unconditional expected utilities of different types of traders, we obtain the value of acquiring information $\tilde{v}_i$ as

$$\phi_i(I_1, I_2) \equiv \frac{1}{2\gamma} \log \left[\frac{Var(\tilde{v}|\tilde{p})}{Var(\tilde{v}_j|\mathcal{F}_i)} \right] = \frac{1}{2\gamma} \log \left(\rho + I_j^2 \chi \right) - \frac{1}{2\gamma} \log \left(\frac{1}{Var(\tilde{v}|\tilde{p})} \right), \tag{23}$$

for $i, j = 1, 2,\ j \neq i$, where the second equality follows from using the signal observed by $\tilde{v}_i$-informed traders in (4) and applying Bayes's rule. Intuitively, the value of acquiring information $\tilde{v}_i$ is increasing in the quality of information available to the trader after purchasing the signal $\tilde{v}_i$ (given by $Var^{-1}(\tilde{v}_j|\mathcal{F}_i) = \rho + I_j^2\chi$) and decreasing in the quality of information available to the trader by only observing the price (given by price informativeness $Var^{-1}(\tilde{v}|\tilde{p})$). Note that we express the information value ϕ_i as a function of trading intensity measures (I_1, I_2), but it of course depends indirectly on (λ_1, λ_2), which affect (I_1, I_2) through the system (13).

In the equilibrium of the information acquisition stage, a trader will acquire a signal as long as the cost of doing so does not exceed the benefit. Given that there are always traders who decide not to acquire any signal, if some traders choose to acquire information on one fundamental, then the value of that information must be equal to its cost; that is, traders are indifferent between acquiring the signal and not acquiring the signal. Similarly, if all traders decide not to acquire information on one fundamental, then the value of that information must be smaller than its cost. Formally, suppose $(\lambda_1^*, \lambda_2^*) \in \mathbb{R}_+^2$ is an information market equilibrium. Then (a) if $\lambda_i^* > 0$ for some $i = 1, 2$, then $\phi_i(I_1, I_2) = c_i$, whereas (b) if $\lambda_i^* = 0$ for some $i = 1, 2$, then $\phi_i(I_1, I_2) \leq c_i$.

We can show that, for any parameter configuration $(c_1, c_2, \rho, \chi, \gamma) \in \mathbb{R}_{++}^5$, there exists a unique information market equilibrium, as characterized by the following proposition (the proof is included in the Appendix).

PROPOSITION 4:

- *(a) For any exogenous parameters* $(c_1, c_2, \rho, \chi, \gamma) \in \mathbb{R}_{++}^5$, *there exists a unique information market equilibrium* $(\lambda_1^*, \lambda_2^*)$.
- *(b) If* $(e^{2\gamma c_1} - 1)(e^{2\gamma c_2} - 1) < 1$, *then* $\lambda_1^* > 0$ *and* $\lambda_2^* > 0$. *Otherwise, (i) if* $0 < c_1 < \frac{\log 2}{2\gamma}$, *then* $\lambda_1^* > 0$ *and* $\lambda_2^* = 0$; *(ii) if* $0 < c_2 < \frac{\log 2}{2\gamma}$, *then* $\lambda_1^* = 0$ *and* $\lambda_2^* > 0$; *and (iii) if* $c_1 \geq \frac{\log 2}{2\gamma}$ *and* $c_2 \geq \frac{\log 2}{2\gamma}$, *then* $\lambda_1^* = \lambda_2^* = 0$.

Part (b) of this proposition states that, if the information acquisition costs c_1 and c_2 are relatively small, that is, if $(e^{2\gamma c_1} - 1)(e^{2\gamma c_2} - 1) < 1$, then there will be two types of informed traders active in equilibrium (i.e., $\lambda_1^* > 0$ and $\lambda_2^* > 0$). If both c_1 and c_2 are large, or, more specifically, if both are greater than $\frac{\log 2}{2\gamma}$, then no traders will find it optimal to acquire any information (i.e., $\lambda_1^* = \lambda_2^* = 0$). In the intermediate ranges, we will have only one type of informed trader acquiring the type of information that is relatively cheaper. Figure 1 illustrates these results for $\gamma = 3$. In the following subsections, we focus on the case $(e^{2\gamma c_1} - 1)(e^{2\gamma c_2} - 1) < 1$, so that two types of informed traders are active in equilibrium.

B. Learning Complementarity versus Substitutability

We now analyze the strategic interactions among traders in the decision to produce information. In particular, we show that learning the two independent

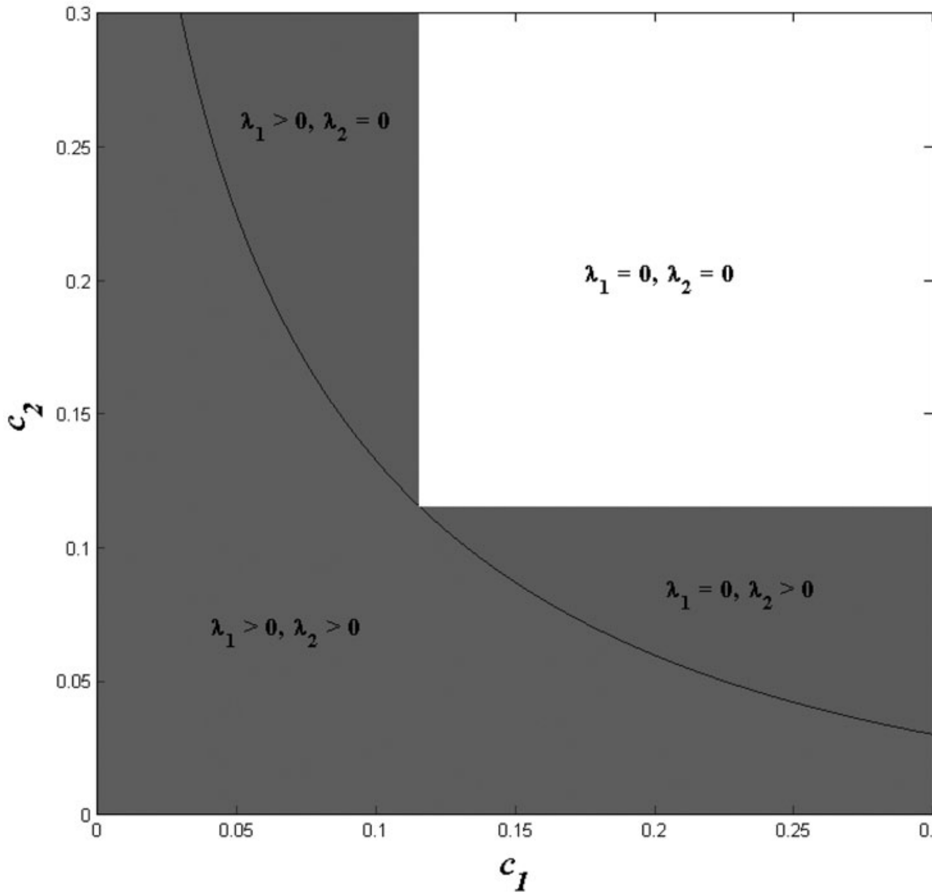

Figure 1. Trader-type distribution in the baseline model. This figure plots the regimes of trader types in equilibrium in the space of (c_1, c_2) for the baseline model. Parameter c_1 is the cost of acquiring information $\tilde{v}_1$, while parameter c_2 is the cost of acquiring information $\tilde{v}_2$. The absolute risk aversion coefficient is $\gamma = 3$.

pieces of information $\tilde{v}_i$ and $\tilde{v}_j$ can be complementary, in the sense that an increase in the mass of agents acquiring information on one fundamental will increase the incentive of agents to acquire information about the other fundamental.

Formally, we examine how the information value ϕ_i changes with the sizes (λ_1, λ_2) of the informed-trader populations. If ϕ_i is increasing in λ_j, that is, if $\frac{\partial \phi_i}{\partial \lambda_j} > 0$, then we say that acquiring information $\tilde{v}_i$ exhibits *strategic complementarity* to acquiring information $\tilde{v}_j$, because in this case more traders will acquire information $\tilde{v}_i$ when more traders acquire information $\tilde{v}_j$. If ϕ_i is decreasing in λ_j, that is, if $\frac{\partial \phi_i}{\partial \lambda_j} < 0$, then acquiring information $\tilde{v}_i$ exhibits *strategic* *substitutability* to acquiring information $\tilde{v}_j$.[10] Similarly, if $\frac{\partial \phi_i}{\partial \lambda_i} > 0$ ($\frac{\partial \phi_i}{\partial \lambda_i} < 0$), then there is strategic complementarity (substitutability) among agents acquiring signal $\tilde{v}_i$. These concepts of learning complementarity/substitutability are consistent with Grossman and Stiglitz (1980).

By equation (23), we have:

$$\frac{\partial \phi_i}{\partial \lambda_j} = \frac{1}{2\gamma} \underbrace{\frac{\partial \log\left(\rho + I_j^2 \chi\right)}{\partial I_j}}_{\text{uncertainty reduction effect}} \frac{dI_j}{d\lambda_j} - \frac{1}{2\gamma} \underbrace{\frac{\partial \log\left(\frac{1}{Var(\tilde{v}|\tilde{p})}\right)}{\partial \lambda_j}}_{\text{Grossman-Stiglitz effect}} . \tag{24}$$

So, an increase in the population λ_j of $\tilde{v}_j$-informed traders has two opposite effects on the benefit ϕ_i of acquiring signal $\tilde{v}_i$. First, an increase in λ_j increases the trading intensity I_j on information $\tilde{v}_j$ (i.e., $\frac{dI_j}{d\lambda_j} > 0$ by Corollary 1). The increased I_j directly reduces the remaining uncertainty of a $\tilde{v}_i$-informed trader, as reflected by the term $\frac{\partial \log(\rho + I_j^2 \chi)}{\partial I_j} > 0$ in equation (24). This allows the $\tilde{v}_i$-informed trader to trade more aggressively and increases his expected utility, increasing the benefit from acquiring information about $\tilde{v}_i$. This result essentially builds on the uncertainty reduction effect we identify in earlier sections. Before, we show that this effect creates a positive link between the two trading intensities; here, we show that this effect also implies an increase in the incentive to produce one kind of information when more people acquire the other kind of information.

Second, an increase in λ_j causes the price to be more informative about the total cash flow $\tilde{v}$, which reduces the incentive of uninformed traders to become informed about $\tilde{v}_i$, which is part of $\tilde{v}$, as they can now gain more information about $\tilde{v}$ from the price. This effect is the standard Grossman-Stiglitz substitution effect, whereby having more informed traders reduces the incentive to become informed. This negative effect is reflected by the term $-\frac{1}{2\gamma}\frac{\partial \log(\frac{1}{Var(\tilde{v}|\tilde{p})})}{\partial \lambda_j}$ in equation (24), which is indeed negative as shown by equation (A19) in the Appendix.

When the positive uncertainty reduction effect dominates the negative Grossman-Stiglitz effect, an increase in λ_j will increase ϕ_i, leading to a complementarity. We can show that this is true when $I_j > I_i$, that is, $\frac{\partial \phi_i}{\partial \lambda_j} > 0$ if and only if $I_j > I_i$. Interestingly, learning the same information is always a strategic substitute, that is, $\frac{\partial \phi_i}{\partial \lambda_i} < 0$. This is because the uncertainty reduction effect discussed above operates through the trading intensity I_j about the other

[10] As before, we use the word "strategic" to capture the notion that traders interact in information-acquisition behaviors, because their decisions on acquiring information affect each other's incentives to do so. See related discussions in Section V. However, we note that our traders are "small" in the sense that they do not account for any effect that their behavior has on the cross-sectional distributions of information or on the equilibrium asset price. Rather, their optimal strategies are affected by the strategies of others in a way that generates strategic substitutabilities or complementarities.

component $\tilde{v}_j$, while increasing λ_i mainly increases I_i.[11] Thus, the complementarity effect in acquiring different information is not present in the traditional unidimensional Grossman-Stiglitz framework, and can only be uncovered by considering the two-dimensional framework in our paper. We summarize these results in the following proposition (with the proof included in the Appendix).

PROPOSITION 5: *Suppose* $(e^{2\gamma c_1} - 1)(e^{2\gamma c_2} - 1) < 1$*, so that* $\lambda_1^* > 0$ *and* $\lambda_2^* > 0$. *Acquiring information on the same fundamental is a strategic substitute: as more traders become informed about* $\tilde{v}_i$*, the value* ϕ_i *of acquiring* $\tilde{v}_i$ *decreases, that is,* $\frac{\partial \phi_i}{\partial \lambda_i} < 0$. *Acquiring information on different fundamentals can be a strategic substitute or a complement: as more traders become informed about* $\tilde{v}_j$*, the value of acquiring* $\tilde{v}_i$ *can decrease or increase, and* $\frac{\partial \phi_i}{\partial \lambda_j} > 0$ *if and only if* $I_j > I_i$.

C. The Impact of Information Acquisition Cost

To illustrate the implications of strategic interactions in information acquisition, in this subsection we discuss results of comparative-statics analysis examining the impact of changing the exogenous cost c_i of acquiring information $\tilde{v}_i$ on the equilibrium fractions $(\lambda_1^*, \lambda_2^*)$ of informed traders and on price informativeness $\frac{1}{Var(\tilde{v}|\tilde{p}^*)}$ in the overall equilibrium. The comparative-statics analysis is based on the equilibrium conditions $\phi_1(I_1^*, I_2^*) = c_1$ and $\phi_2(I_1^*, I_2^*) = c_2$ in the information acquisition stage and on the system in (13) characterizing trading intensity measures in the trading stage. The cost of information c_i represents a measure of the ease of acquiring information on one fundamental $\tilde{v}_i$: an increase in the number of information sources about the firm (say, abundant disclosure, large analyst and media coverage, and advanced communication technologies) increases access to information and corresponds to a low value of c_i (e.g., Fishman and Hagerty (1989), Kim and Verrecchia (1994)).

As we show in Proposition 6, a decrease in the cost c_i of acquiring signal $\tilde{v}_i$ always increases the equilibrium size λ_i^* of the population of $\tilde{v}_i$-informed traders. This is intuitive since a lower c_i implies a higher net benefit from knowing $\tilde{v}_i$. More interestingly, the complementarity effect emphasized in Proposition 5 implies that a decrease in c_i, the cost of acquiring information $\tilde{v}_i$, may also increase the equilibrium size λ_j^* of the population of $\tilde{v}_j$-informed traders. Specifically, Proposition 6 shows that $\frac{d\lambda_j^*}{dc_i} < 0$ if and only if $c_i < c_j$. This is because $c_i < c_j$ implies that $I_i^* > I_j^*$, which, according to Proposition 5, generates $\frac{\partial \phi_j}{\partial \lambda_i} > 0$. That is, in this case, an increase in λ_i increases the incentive to acquire information

[11] Formally, we have $\frac{\partial \phi_i}{\partial \lambda_i} = \frac{1}{2\gamma} \frac{\partial \log(\rho + I_j^2 \chi)}{\partial I_j} \frac{dI_j}{d\lambda_i} - \frac{1}{2\gamma} \frac{\partial \log(\frac{1}{Var(\tilde{v}|\tilde{p})})}{\partial \lambda_i}$. So, the strength of the uncertainty reduction effect $\frac{\partial \log(\rho + I_j^2 \chi)}{\partial I_j}$ is related to $\frac{dI_j}{d\lambda_i}$. By Corollary 1, we have $\frac{dI_j}{d\lambda_i} = \frac{\partial h_j}{\partial I_i} \frac{dI_i}{d\lambda_i}$, which can be positive or negative depending on the sign of $\frac{\partial h_j}{\partial I_i}$. If $\frac{\partial h_j}{\partial I_i} < 0$, then $\frac{dI_j}{d\lambda_i} < 0$ and the uncertainty reduction effect works in the same direction as the Grossman-Stiglitz effect. If $\frac{\partial h_j}{\partial I_i} > 0$, we have $\frac{\partial h_j}{\partial I_i} < 1$ by equation (A8), and hence the effect of λ_i on I_j is smaller that its effect on I_i, and thus the uncertainty reduction effect is limited.

$\tilde{v}_j$, and so a decrease in c_i—directly increasing λ_i—will indirectly cause an increase in the population of traders informed about $\tilde{v}_j$. Finally, the proposition also shows that a decrease in the cost c_i of acquiring signal $\tilde{v}_i$ always leads to an increase in the price informativeness measure $\frac{1}{Var(\tilde{v}|\tilde{p}^*)}$. The proposition is stated as follows (the proof is included in the Appendix):

PROPOSITION 6: *Suppose $(e^{2\gamma c_1} - 1)(e^{2\gamma c_2} - 1) < 1$, so that $\lambda_1^* > 0$ and $\lambda_2^* > 0$. A decrease in the cost c_i of acquiring information $\tilde{v}_i$*

(a) increases the equilibrium size λ_i^* of $\tilde{v}_i$-informed traders (i.e., $\frac{d\lambda_i^*}{dc_i} < 0$),

(b) increases the equilibrium size λ_j^* of $\tilde{v}_j$-informed traders if and only if acquiring $\tilde{v}_j$ is a complement to acquiring $\tilde{v}_i$ (i.e., $\frac{d\lambda_j^*}{dc_i} < 0$ iff $\frac{\partial \phi_j}{\partial \lambda_i} > 0$ or $c_i < c_j$), and

(c) increases the price informativeness (i.e., $\frac{dVar^{-1}(\tilde{v}|\tilde{p}^*)}{dc_i} < 0$).

D. Empirical Implications

The results in Proposition 6 can be tested empirically. Most interesting of course are the results in part (b) of the proposition, which relate to the interaction between the two types of information in our model. These results characterize the effect of a decrease in the cost of acquiring one type of information on the number of traders acquiring the other type of information. Building on our discussion in Section II.D, one can test these results in settings in which it is natural to think about multiple dimensions of uncertainty.

To test these results one would also need a proxy for the costs of information production. Ideally, the change in the cost of information production can be considered exogenous, so that its causal effect on the populations of informed traders can be examined. One potential source of exogenous variation in the cost of information production is regulation. For example, greater disclosure requirements will imply that traders have easier access to information. Consider then the case of a multinational firm: one can analyze the effect that changes in disclosure regulation in one country have on the amount of information produced by traders who trade the stock of the multinational firm and are based in another country.

Another source of exogenous variation in information costs is highlighted in recent work by Kelly and Ljungqvist (2012). The authors study changes in the number of sell-side analysts who cover a stock due to exogenous reasons such as brokerage firms closing their research operations. To the extent that these analysts specialize in one dimension of firm uncertainty—a particular country or industry—one can check how this change affects the amount of information produced by the market on the other dimensions of uncertainty.

Finally, another way to view our model with two groups of traders is to think about technical traders, who process and trade on information about prices and order flows, and fundamental traders, who trade on information about firm cash flows. One can recast the model to think about the interaction between these two groups. A natural experiment in this context is the introduction of automated quote dissemination on the New York Stock Exchange in 2003, which Hendershott, Jones, and Menkveld (2011) study. This change corresponds to an exogenous decrease in the cost c_i of acquiring and processing information for technical traders who are active in fast computerized trading. To test part (b) of Proposition 6, one can examine the effect of this change on the behavior of more traditional fundamental traders. In addition, consistent with part (c), Hendershott, Jones, and Menkveld (2011) find that the introduction of autoquoting indeed enhances the informativeness of quotes.[12]

IV. An Extension with Some Traders Acquiring Both Signals

A. Setup

In this section, we analyze an extended economy where traders can potentially acquire two signals $\tilde{v}_1$ and $\tilde{v}_2$ simultaneously. To keep things interesting we also generalize the payoff of the risky asset as follows:

$$\tilde{v} = \tilde{v}_1 + \tilde{v}_2 + \tilde{w}, \tag{25}$$

where $\tilde{v}_i \sim N(0, 1/\rho)$ $(i = 1, 2)$ is still a forecastable fundamental and $\tilde{w} \sim N(0, 1/\omega)$ (with $\omega > 0$) is the residual noise, which is introduced so that traders who observe both signals still face uncertainty when they trade. The random variables $\tilde{v}_1$, $\tilde{v}_2$, and $\tilde{w}$ are mutually independent.

Traders can still acquire the signal $\tilde{v}_1$ at cost $c_1 > 0$, and the signal $\tilde{v}_2$ at cost $c_2 > 0$. Now, we also allow traders to acquire both $\tilde{v}_1$ and $\tilde{v}_2$, but at a cost of $c_1 + c_2 + k$, where $k \geq 0$ represents an increasing marginal cost of information acquisition. Parameter k can also arise as a result of a model with asymmetric expertise in information acquisition. We provide more detailed interpretation of this parameter in the next subsection. Our baseline model, presented in the previous subsections, corresponds to the case in which $k = \infty$ (i.e., traders cannot observe two signals) and $\omega = \infty$ (i.e., there is no residual uncertainty in the asset payoff). In this section, we show that our main results are robust to the general case of $0 < k < \infty$ and $0 < \omega < \infty$. For ease of exposition, we denote

$$\tilde{v}_{12} \equiv \tilde{v}_1 + \tilde{v}_2 \quad \text{and} \quad c_{12} \equiv c_1 + c_2 + k, \tag{26}$$

which correspond respectively to the signal and the information acquisition cost for traders who obtain both signals $\tilde{v}_1$ and $\tilde{v}_2$.

Now, in the trading stage, there are potentially four types of rational traders: (1) $\tilde{v}_1$-informed traders observing $\tilde{v}_1$ (of mass $\lambda_1 \geq 0$), (2) $\tilde{v}_2$-informed traders observing $\tilde{v}_2$ (of mass $\lambda_2 \geq 0$), (3) $\tilde{v}_{12}$-informed traders observing $\tilde{v}_1$ and $\tilde{v}_2$ (of mass $\lambda_{12} \geq 0$), and (4) uninformed traders (of mass $\lambda_u > 0$).

[12] Of course, this finding is also consistent with a standard "unidimensional" model where the single fundamental pertains to technical information. However, the standard model does not speak to the implications for traders informed about other types of information, which is the key prediction of our mechanism.

All other features of the model are the same. In particular, traders have CARA utility functions with risk-aversion parameter $\gamma > 0$. There are two tradable assets: the stock and the bond. All four types of rational traders condition their trades on the stock price $\tilde{p}$. And noise traders trade a random amount $\tilde{x} \sim N(0, 1/\chi)$ (with $\chi > 0$) of the stock, which is independent of the realizations of $\tilde{v}_1$, $\tilde{v}_2$, and $\tilde{w}$.

B. Interpretation of the Parameter k

There are two possible interpretations for the parameter k. First, it can capture a convex cost structure of the information acquisition technology, as in Verrecchia (1982) and others. Suppose there is a large mass of ex ante identical traders. They can obtain signal $\tilde{v}_i$ at a cost c_i, reducing the payoff uncertainty they face by ρ^{-1}. They can further reduce their uncertainty by an additional ρ^{-1} by acquiring the other signal $\tilde{v}_j$. However, this extra reduction in uncertainty will cost them not only c_j—which is the cost they would pay if they acquired this signal only—but also k. To the extent that traders (either individuals or institutions) have limited capacity to process information, such a convex cost structure is quite natural.

Second, k can come from a setting in which different traders have different expertise in information acquisition. Suppose that there are two types of traders who are ex ante different. Each type has an advantage in gathering a particular type of information; if they want to acquire the other type of information, they have to pay an extra cost of k. Traders with advantage in acquiring $\tilde{v}_1$ will decide between acquiring $\tilde{v}_1$ at a cost of c_1, acquiring both signals at a cost of c_{12}, and staying uninformed. The choice the other type of traders face is analogous. This setting is therefore equivalent to the extension of the model we study here,[13] and it captures the idea that different traders have varying access to different sources of information. As we discussed before, this can be the result of, for instance, traders from different countries trading the stock of a multinational firm, or different institutions with expertise in different styles or industries.

Under both interpretations, one may ask whether the additional cost k can be avoided by merging two traders, one informed about $\tilde{v}_1$ at cost c_1 and the other informed about $\tilde{v}_2$ at cost c_2. For example, an intermediary can hire two such traders and form one combined institution. In our view, however, it is unlikely that such combinations happen without any friction, as agency costs, coordination costs, and organizational costs imply that combining two traders with different pieces of information will be at an additional cost. One can think of k in our model as the lowest cost required to combine two types of trading expertise under one roof. We are agnostic about the source of this cost k. Rather, we study the equilibrium outcomes that obtain when traders can be informed about the two dimensions by paying this cost (in addition to

[13] Note that, as in the baseline model, we still assume that there are a large number of traders in the economy so that, in equilibrium, there exist traders who optimally choose to stay uninformed.

the costs of acquiring each type of information). Note that our model does not make an assumption about the size of k and we can provide a characterization of equilibrium outcomes for all levels of k.

C. Results

As in the baseline model, we first take the masses $\lambda_1, \lambda_2,$ and λ_{12} as given to solve the financial market equilibrium, and then we endogenize them in the information market equilibrium. We find that our main results are robust in this extended economy with $k \in (0, \infty)$ and $\omega \in (0, \infty)$. Since the analysis of this extension is quite complicated, we provide the formal results together with their proofs in an Internet Appendix.[14] Here, we summarize the main points.

In the trading stage, we can characterize the demand functions $D_t(\tilde{p}, \tilde{v}_t)$ of the three types of informed traders (for $t \in \{1, 2, 12\}$). We define the trading intensity on information $\tilde{v}_i$ as

$$I_i \equiv \lambda_i \frac{\partial D_i(\tilde{p}, \tilde{v}_i)}{\partial \tilde{v}_i} + \lambda_{12} \frac{\partial D_{12}(\tilde{p}, \tilde{v}_{12})}{\partial \tilde{v}_i},$$

which represents how all traders informed about the signal $\tilde{v}_i$ (either in addition to the other signal or not) respond to a change in $\tilde{v}_i$. We then compute two best response functions $I_i = h_i\left(I_j; \lambda_i, \lambda_{12}, \rho, \gamma, \chi, \omega\right)$ (for $i, j = 1, 2, j \neq i$) that jointly determine the two trading intensities I_1 and I_2. As in Proposition 2 in the baseline model, we can still show that trading on information $\tilde{v}_i$ is a complement (substitute) to trading on information $\tilde{v}_j$ if I_j is sufficiently large (small) in this extended economy.

We also show that there exists a linear REE, with the price function given in the form of equation (2). Similar to Corollary 1, an increase in the size λ_i of $\tilde{v}_i$-informed traders (a) increases the trading intensity I_i on information $\tilde{v}_i$, and (b) increases the trading intensity I_j on information $\tilde{v}_j$ if and only if trading on $\tilde{v}_j$ is complementary to trading on $\tilde{v}_i$, which is true when λ_i is sufficiently large. We further show that our Corollary 2 continues to hold. Specifically, for any given $\lambda_{12} \geq 0$, we still fix $\lambda_1 + \lambda_2$ at a constant $\Lambda > 0$, and define information diversity as $\Delta \equiv 1 - \frac{|\lambda_1 - \lambda_2|}{\Lambda}$, which is given by equation (22). We then show that information diversity Δ increases the price informativeness $\frac{1}{Var(\tilde{v}|\tilde{p})}$.

In the information acquisition stage, the benefit of acquiring information $\tilde{v}_t$ is given by $\phi_t(I_1, I_2) \equiv \frac{1}{2\gamma} \log[\frac{Var(\tilde{v}|\tilde{p})}{Var(\tilde{v}|\mathcal{F}_t)}]$, where $\mathcal{F}_t = \{\tilde{v}_t, \tilde{p}\}$ for $t \in \{1, 2, 12\}$. As in the baseline model, the equilibrium in the information acquisition stage is still defined by the usual no-deviation conditions. We show that, for any exogenous parameter configuration $(k, c_1, c_2, \rho, \chi, \gamma, \omega) \in \mathbb{R}^7_{++}$, there exists an information market equilibrium. In addition, except for a set of parameters with zero Lebesgue measure, the equilibrium is unique, and has at most two types of active informed traders (recall that the three informed types of t are

[14] The Internet Appendix may be found in the online version of this article on the *Journal of Finance* website.

$\{1, 2, 12\}$). For the parameter set with zero Lebesgue measure, each parameter configuration can produce a continuum of equilibria with $\lambda_1^* > 0$, $\lambda_2^* > 0$, and $\lambda_{12}^* > 0$, but the resulting trading intensities I_1^* and I_2^* are still unique.[15] We further characterize the information market equilibrium by two threshold values $\bar{k}_1$ and $\bar{k}_2$ (with $0 < \bar{k}_1 < \bar{k}_2$), which are in turn determined by four primitive parameters $(\rho, \gamma, \chi, \omega)$ in explicit functional forms.

Figure 2 illustrates the results that are proved formally and generally in the Internet Appendix. In the figure, we illustrate the types of equilibria in the space of (c_1, c_2). Here, we choose $\rho = 50$, $\chi = 50$, $\omega = 25$, and $\gamma = 3$. Under these parameter values, $\bar{k}_1 = 0.020$ and $\bar{k}_2 = 0.028$. Parameter k takes the value of 0.022, 0.016, and zero in Panels B, C, and D, respectively. Its choice in Panel A does not affect the result (it is above $\bar{k}_2$). We can see that, if $k > \bar{k}_2$, the equilibrium distribution of trader types is very similar to our baseline model: no trader acquires the two signals simultaneously, and when (c_1, c_2) are relatively small, traders acquire signals $\tilde{v}_1$ and $\tilde{v}_2$ in isolation (i.e., $\lambda_1^* > 0$ and $\lambda_2^* > 0$). For $k \in (0, \bar{k}_2)$, the sets of (c_1, c_2) generating an equilibrium with $\lambda_1^* > 0$ and $\lambda_2^* > 0$ still have a positive measure, and, as we decrease k, their sizes decrease and equilibria with $\lambda_{12}^* > 0$ become more common.

Importantly, except for a set of parameters with zero Lebesgue measure, there is never an equilibrium with $\lambda_1^* > 0$, $\lambda_2^* > 0$, and $\lambda_{12}^* > 0$ at the same time. There are equilibria in which none of these parameters are positive, one of them is positive, or two of them are positive. This suggests that our baseline model, which does not allow for the possibility of acquiring the two signals simultaneously, captures the most interesting case. This is because, if we study a case in which $\lambda_{12}^* > 0$, we would not have two groups of traders heterogeneously informed without one group's information being dominated by the other group's information. This would miss the basic spirit of our model. Our baseline model can thus be viewed as a special case of the extended model described in this section (and solved in the Internet Appendix) where k is sufficiently large.

Finally, in the Internet Appendix we also show that our Proposition 5 continues to be valid in the extended economy. That is, when the economy is such that $\lambda_1^* > 0$, $\lambda_2^* > 0$, and $\lambda_{12}^* = 0$, acquiring information on the same fundamental is a strategic substitute (i.e., $\frac{\partial \phi_i}{\partial \lambda_i} < 0$), and acquiring information on different fundamentals can be a strategic substitute or complement (i.e., $\frac{\partial \phi_j}{\partial \lambda_i} > 0$ if and only if I_i is sufficiently large relative to I_j).

V. Relation to the Literature

Our paper is related to other papers in the literature that analyze complementarity versus substitutability between different types of information in

[15] The proofs for these results are still similar to, although much more complicated than, those in the baseline model. That is, we fix $(k, \rho, \chi, \gamma, \omega) \in \mathbb{R}^5_{++}$, and characterize the sets of (c_1, c_2) that support a particular type of equilibrium, and then show that these sets are mutually exclusive and their union forms the whole space $\mathbb{R}^2_{++}$ of (c_1, c_2).

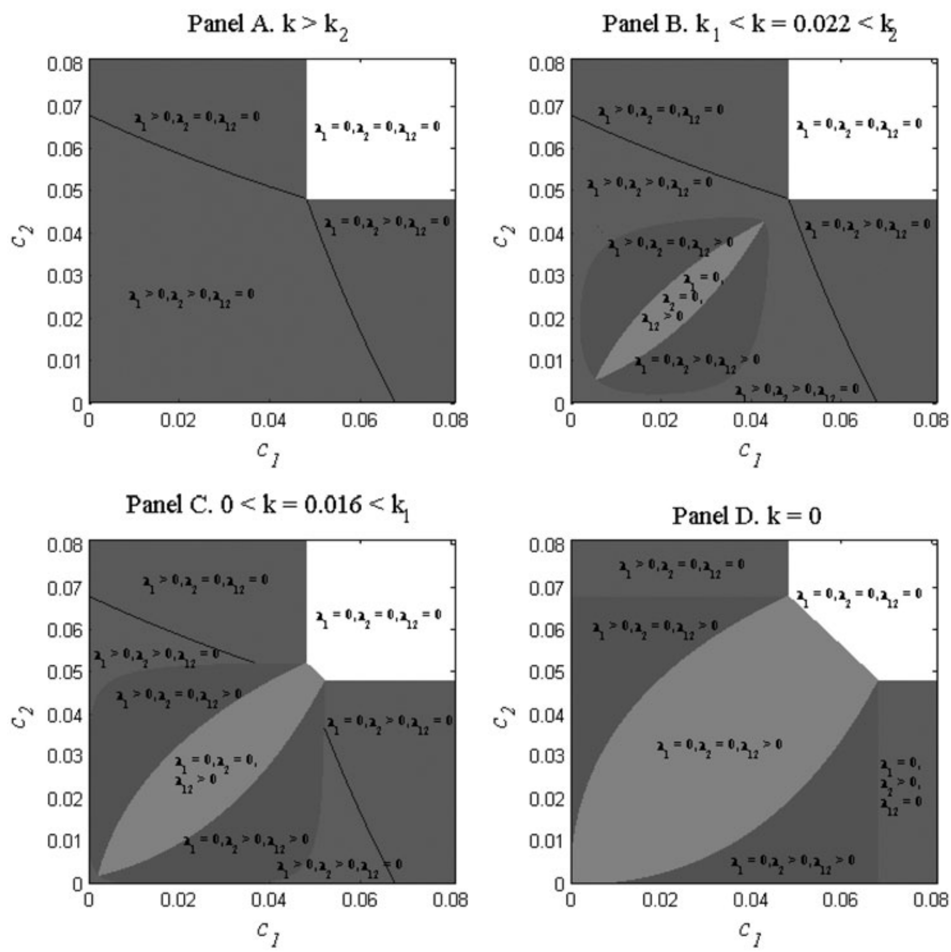

Figure 2. Trader-type distribution in the extended model. This figure plots the regimes of trader types in equilibrium in the space of (c_1, c_2) for the extended model in Section IV. Parameter c_1 is the cost of acquiring information $\tilde{v}_1$, while parameter c_2 is the cost of acquiring information $\tilde{v}_2$. Parameter k is the extra cost that traders have to pay in order to acquire both signals at the same time. It takes the value of 0.022, 0.016, and zero in Panels B, C, and D, respectively. The common prior precision of $\tilde{v}_1$ and $\tilde{v}_2$ is $\rho = 50$, the precision of the unforecastable residual in the stock payoff is $\omega = 25$, the precision of noise trading is $\chi = 50$, and the absolute risk aversion coefficient is $\gamma = 3$. These parameter values jointly imply $\bar{k}_1 = 0.020$ and $\bar{k}_2 = 0.028$, the two threshold values of parameter k.

financial markets. Admati and Pfleiderer (1987) study a different notion of complementarity/substitutability, focusing on the perspective of one investor. Specifically, they define two signals to be complements (substitutes) if the benefit of one investor observing two signals simultaneously is greater (smaller) than the sum of the benefits of the *same* investor observing these two signals in isolation. Based on whether the signals tend to be complements or substitutes, they then analyze under what circumstances information will concentrate in the hands of a few investors rather than be spread out across many investors. The authors propose two effects—the "unlocking effect" and the "aggregation effect"—which respectively favor substitutability and complementarity according to their definition. The unlocking effect occurs when a trader having one signal can use it to unlock information about the other signal from market prices in a multi-asset setting. The aggregation effect occurs when the price is almost a sufficient statistic for the underlying information, and thus each individual signal has relatively little incremental value but the bundle of all signals is still superior to the price. Lundholm (1991) builds on this framework to analyze the effect of public disclosure on the information acquisition activities of traders in financial markets.

In contrast, our focus is on *strategic* complementarities/substitutabilities, which result from interactions among different investors, and we study how one investor's benefit of trading or acquiring a signal is affected by *other* investors trading or acquiring signals. Our paper takes the view that different investors are informed about different aspects of the value of the security, and so we naturally stay away from the focus of Admati and Pfleiderer (1987) on whether information concentrates in the hands of a few investors or not. Only in Section IV do we allow investors to acquire two types of signals, but even there we stick to our basic premise that information is naturally dispersed across investors by assuming that different investors have an advantage in acquiring different types of information or that there is a convex cost structure in information acquisition. Our focus is thus on the interactions across groups of investors and on the effect of diversity of information—whether the two groups of informed investors are close in size or not—on the overall informativeness of the price. Not surprisingly, given the difference in the notion of complementarity/substitutability and the underlying setting, the effects leading to complementarity and substitutability in our paper, which are highlighted in Sections II and III, are quite different from those highlighted in Admati and Pfleiderer (1987).

The papers by Paul (1993) and Lee (2010) share our notion of strategic complementarity/substitutability, in that they look at the interaction across traders in trading on and producing different information. Specifically, Paul (1993) emphasizes a substitution effect arising from competition among traders trading on the same type of information. Lee (2010), who builds on Subrahmanyam and Titman (1999), emphasizes a complementarity effect across different types of traders arising from the fact that trades based on different types of information provide noise for each other in the market-order-based model. However, the models in both papers are based on market orders, as in Kyle (1985), where traders do not observe prices or condition on prices when they trade. As a result, the effects highlighted by these papers are quite different from those in our model, given that the complementarity and substitutability in trading intensity in our model originate from traders updating their expectations based on the information they obtain from market prices.

Another related literature analyzes models of trading in multiple securities (e.g., Admati (1985) and Bernhardt and Taub (2008)). Perhaps most closely related to our paper is a recent paper by Cespa and Foucault (2014), who identify a channel that shares the same spirit to our uncertainty reduction effect in the trading stage. They use a multi-asset model to study liquidity spillovers between different securities when dealers trading in one asset observe the price of the other traded asset and learn information concerning the asset they trade. The cross-asset learning in their model generates a self-reinforcing feedback loop between price informativeness and liquidity, which drives liquidity comovement across markets and can lead to multiple equilibria with different levels of illiquidity. In contrast, we use a one-asset setting to examine the interaction between trading intensities on information about different aspects of the same asset and characterize its impact on price informativeness and private information production. Unlike Cespa and Foucault (2014), where the direct effect of a change in exogenous parameters always gets amplified when all informed traders observe prices, in our model the direct effect can be either amplified or attenuated depending on the competition between the uncertainty reduction effect and the inference augmentation effect. This is because traders informed about different fundamentals trade against each other in our model, while in Cespa and Foucault (2014) they specialize in trading different assets. As a result of this difference, our model does not have the multiplicity of equilibria that Cespa and Foucault (2014) have.

Considering the empirical implications of our model, some of which are highlighted in Sections II.D and III.D, recall that they involve interactions among differentially informed groups of traders trading in the same security. Hence, among the above-mentioned papers, they may relate only to Paul (1993) and Lee (2010). However, given the different mechanisms, our model also generates distinct empirical implications. First, complementarities in our setting, unlike in the other settings, are generated by changes in the overall uncertainty traders face. This can be directly tested empirically. Second, our analysis goes beyond the above papers by analyzing the effects of information diversity on price informativeness, which, as we highlight in Sections II.D and III.D, can be tested empirically. Finally, the results in Paul (1993) and Lee (2010) are specific to markets based on market orders, whereas our results are based on limit-order markets, which again can be considered in empirical analysis.

VI. Conclusion

Does information in financial markets attract or deter the transmission and production of more information? Extending the seminal Grossman and Stiglitz (1980) model to include two dimensions of uncertainty in the value of the traded asset, we provide new insights into this question by uncovering a rich set of interactions. We identify an uncertainty reduction effect whereby traders trading more aggressively on information about one fundamental reduce the uncertainty faced by those traders informed about the other fundamental and thus encourage them to trade more aggressively and produce more information.

We show that, when this effect dominates, producing and trading on two types of information can be complementary. This effect also implies that greater diversity of information in the economy enhances price informativeness, which highlights that it is not only the size of the informed population but also the composition that matters in determining traders' behavior and market outcomes. Finally, our analysis sheds lights on a variety of financial phenomena and makes empirically testable predictions.

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Appendix: Proofs

Proof of Proposition 1

Using (4), we can compute $D_i(\tilde{p}, \tilde{v}_i) = \frac{(\rho + (\alpha_j/\alpha_x)^2 \chi)\tilde{v}_i + (\alpha_j/\alpha_x)^2 \chi \frac{\tilde{p} - \alpha_0 - \alpha_i \tilde{v}_i}{\alpha_j} - (\rho + (\alpha_j/\alpha_x)^2 \chi)\tilde{p}}{\gamma}$. Plugging this expression and $D_u(\tilde{p}) = \frac{\tilde{p} - E(\tilde{v}|\tilde{p})}{\gamma Var(\tilde{v}|\tilde{p})}$ into the market-clearing condition and rearranging terms, we have

$$\begin{aligned} &\lambda_1 \left[\rho + (\alpha_2/\alpha_x)^2 \chi - (\alpha_2/\alpha_x)^2 \chi/\alpha_2\right] \tilde{p} \\ &\quad + \lambda_2 \left[\rho + (\alpha_1/\alpha_x)^2 \chi - (\alpha_1/\alpha_x)^2 \chi/\alpha_1\right] \tilde{p} + \lambda_u \frac{\tilde{p} - E(\tilde{v}|\tilde{p})}{Var(\tilde{v}|\tilde{p})} \\ &= \lambda_1 (\alpha_2/\alpha_x)^2 \chi \frac{-\alpha_0}{\alpha_2} + \lambda_2 (\alpha_1/\alpha_x)^2 \chi \frac{-\alpha_0}{\alpha_1} - \gamma \\ &\quad + \lambda_1 \left[\rho + (\alpha_2/\alpha_x)^2 \chi - (\alpha_1/\alpha_x)(\alpha_2/\alpha_x)\chi\right] \tilde{v}_1 \\ &\quad + \lambda_2 \left[\rho + (\alpha_1/\alpha_x)^2 \chi - (\alpha_1/\alpha_x)(\alpha_2/\alpha_x)\chi\right] \tilde{v}_2 + \gamma \tilde{x}. \end{aligned} \tag{A1}$$

Note that the left-hand side of the above equation is only related to $\tilde{p}$, while the right-hand side is only related to $\tilde{v}_1$, $\tilde{v}_2$, and $\tilde{x}$. Hence, based on (2) and (A1), we form the following system of two equations in terms of two unknowns, (α_1/α_x) and (α_2/α_x):

$$(\alpha_1/\alpha_x) = \lambda_1 \left[\rho + (\alpha_2/\alpha_x)^2 \chi - (\alpha_1/\alpha_x)(\alpha_2/\alpha_x)\chi\right] \gamma^{-1}, \tag{A2}$$

$$(\alpha_2/\alpha_x) = \lambda_2 \left[\rho + (\alpha_1/\alpha_x)^2 \chi - (\alpha_1/\alpha_x)(\alpha_2/\alpha_x)\chi\right] \gamma^{-1}. \tag{A3}$$

By equation (A3), we can express (α_2/α_x) in terms of (α_1/α_x) as follows (this is equation (13) for $i = 2$ in the main text):

$$\frac{\alpha_2}{\alpha_x} = \frac{\lambda_2 \left[\rho + \chi (\alpha_1/\alpha_x)^2\right]}{\gamma + \lambda_2 \chi (\alpha_1/\alpha_x)}. \tag{A4}$$

Then, plugging the above expression into equation (A2), we have the following cubic polynomial in (α_1/α_x):

$$\gamma\chi^2\lambda_2(\lambda_1+\lambda_2)(\alpha_1/\alpha_x)^3+2\chi\lambda_2\left(\gamma^2-\chi\rho\lambda_1\lambda_2\right)(\alpha_1/\alpha_x)^2 \\ +\gamma\left(\gamma^2-\chi\rho\lambda_1\lambda_2\right)(\alpha_1/\alpha_x)-\rho\lambda_1\left(\gamma^2+\chi\rho\lambda_2^2\right)=0. \tag{A5}$$

There exists a positive solution to the above polynomial, because, when $\lambda_1>0$ and $\lambda_2>0$, the left-hand side of equation (A5) is negative at $(\alpha_1/\alpha_x)=0$ and becomes positive when (α_1/α_x) is sufficiently large. In addition, the coefficients of the cubic polynomial have one sign change regardless of the sign of $\left(\gamma^2-\chi\rho\lambda_1\lambda_2\right)$. Hence, by Descarte's "rule of signs," the cubic polynomial has one (unique) positive real root, that is, the equilibrium coefficient ratio $\frac{\alpha_1}{\alpha_x}$ is unique and positive. By equation (A4), we also know that the equilibrium coefficient ratio $\frac{\alpha_2}{\alpha_x}$ is positive.

By (5) and applying Bayes's rule, we have

$$E(\tilde{v}|\tilde{p})=\beta_{\tilde{v},\tilde{p}}\tilde{s}_u, \tag{A6}$$

$$Var(\tilde{v}|\tilde{p})=\frac{(\alpha_1/\alpha_x-\alpha_2/\alpha_x)^2+2\rho\chi^{-1}}{(\alpha_1/\alpha_x)^2\rho+(\alpha_2/\alpha_x)^2\rho+\rho^2\chi^{-1}}, \tag{A7}$$

where $\beta_{\tilde{v},\tilde{p}}=\frac{(\alpha_1/\alpha_x)+(\alpha_2/\alpha_x)}{(\alpha_1/\alpha_x)^2+(\alpha_2/\alpha_x)^2+\rho\chi^{-1}}$. Using the above equations in equation (A1) delivers

$$\left(A_{p0}-A_{px}\frac{1}{\alpha_x}\right)\tilde{p}=A_1\tilde{v}_1+A_2\tilde{v}_2+\gamma\tilde{x}+\left(-A_{01}\frac{\alpha_0}{\alpha_1}-A_{02}\frac{\alpha_0}{\alpha_2}-A_{0x}\frac{\alpha_0}{\alpha_x}-\gamma\right),$$

where the A's are known positive values that are determined by (α_1/α_x) and (α_2/α_x) and defined as follows:

$$\begin{aligned} A_{p0} &= \frac{\lambda_1}{Var(\tilde{v}|\mathcal{F}_1)}+\frac{\lambda_2}{Var(\tilde{v}|\mathcal{F}_2)}+\frac{\lambda_u}{Var(\tilde{v}|\tilde{p})}, \\ A_{px} &= \lambda_1(\alpha_2/\alpha_x)\chi+\lambda_2(\alpha_1/\alpha_x)\chi+\frac{\lambda_u\beta_{\tilde{v},\tilde{p}}}{Var(\tilde{v}|\tilde{p})}, \\ A_1 &= \gamma(\alpha_1/\alpha_x),\ A_2=\gamma(\alpha_2/\alpha_x), \\ A_{01} &= \lambda_2(\alpha_1/\alpha_x)^2\chi,\ A_{02}=\lambda_1(\alpha_2/\alpha_x)^2\chi,\ \text{and } A_{0x}=\lambda_u\frac{\beta_{\tilde{v},\tilde{p}}}{Var(\tilde{v}|\tilde{p})}. \end{aligned}$$

Thus, we can solve for α_x:

$$\alpha_x=\frac{\gamma}{A_{p0}-A_{px}\frac{1}{\alpha_x}}\Rightarrow\alpha_x=\frac{\gamma+A_{px}}{A_{p0}}>0.$$

Combining the known ratios (α_1/α_x) and (α_2/α_x) with the value of α_x gives the values of α_1 and α_2, which are positive. Once we know α_1, α_2, and α_x, we can solve α_0 using

$$\alpha_0 = \frac{-A_{01}\frac{\alpha_0}{\alpha_1} - A_{02}\frac{\alpha_0}{\alpha_2} - A_{0x}\frac{\alpha_0}{\alpha_x} - \gamma}{A_{p0} - A_{px}\frac{1}{\alpha_x}} \Rightarrow \alpha_0 = -\frac{\gamma}{\left(A_{p0} - A_{px}\frac{1}{\alpha_x}\right) + A_{01}\frac{1}{\alpha_1} + A_{02}\frac{1}{\alpha_2} + A_{0x}\frac{1}{\alpha_x}}.$$

We can further use the solved expressions of A_{01}, A_{02}, A_{0x}, A_{px}, and α_x to simplify the denominator of the above expression of α_0 and show that $\alpha_0 = -\frac{\gamma}{A_{p0}} < 0$. □

Proof of Proposition 3

Total differentiation of equation (13) (for $i = 1, 2$) with respect to Q implies

$$\frac{dI_1}{dQ} = \frac{\partial h_1}{\partial Q} + \frac{\partial h_1}{\partial I_2}\frac{dI_2}{dQ} \quad \text{and} \quad \frac{dI_2}{dQ} = \frac{\partial h_2}{\partial Q} + \frac{\partial h_2}{\partial I_1}\frac{dI_1}{dQ}.$$

Solving for $\frac{dI_1}{dQ}$ and $\frac{dI_2}{dQ}$ delivers

$$\frac{dI_1}{dQ} = \frac{\frac{\partial h_1}{\partial Q} + \frac{\partial h_1}{\partial I_2}\frac{\partial h_2}{\partial Q}}{1 - \frac{\partial h_1}{\partial I_2}\frac{\partial h_2}{\partial I_1}} \text{ and } \frac{dI_2}{dQ} = \frac{\frac{\partial h_2}{\partial Q} + \frac{\partial h_2}{\partial I_1}\frac{\partial h_1}{\partial Q}}{1 - \frac{\partial h_1}{\partial I_2}\frac{\partial h_2}{\partial I_1}},$$

which is equation (18).

Next, we examine the sign and magnitude of $\mathcal{M} = \left(1 - \frac{\partial h_1}{\partial I_2}\frac{\partial h_2}{\partial I_1}\right)^{-1}$. By equation (13), direct computation shows

$$\frac{\partial h_i}{\partial I_j} = 1 - \frac{\gamma^2 + \chi\rho\lambda_i^2}{\left(\gamma + \chi\lambda_i I_j\right)^2}, \text{ for } i, j = 1, 2, \ j \neq i. \tag{A8}$$

Using equations (A2) and (A3), we can express λ_i in terms of I_1 and I_2 as follows:

$$\lambda_i = \frac{\gamma I_i}{\rho + I_j^2\chi - I_1 I_2\chi}, \text{ for } i, j = 1, 2, \ j \neq i. \tag{A9}$$

Plugging the above expression into equation (A8) yields

$$\frac{\partial h_i}{\partial I_j} = \frac{\left(2I_j - I_i\right)\chi I_i}{\rho + \chi I_j^2}, \text{ for } i, j = 1, 2, \ j \neq i. \tag{A10}$$

Thus, we have

$$\mathcal{M}^{-1} = 1 - \frac{\partial h_1}{\partial I_2}\frac{\partial h_2}{\partial I_1} = 1 - \frac{(2I_2 - I_1)\chi I_1}{\rho + \chi I_2^2}\frac{(2I_1 - I_2)\chi I_2}{\rho + \chi I_1^2}$$
$$= \frac{(\rho + 2\chi I_1 I_2)\left(\rho + \chi(I_1 - I_2)^2\right)}{(\rho + \chi I_2^2)(\rho + \chi I_1^2)} > 0.$$

That is, $\mathcal{M} > 0$.

Whether $M > 1$ depends on whether $\frac{\partial h_1}{\partial I_2}$ and $\frac{\partial h_2}{\partial I_1}$ have the same sign. We thus have three cases.

Case 1. If $\frac{\partial h_i}{\partial I_j} = 0$ for some i, then $\mathcal{M} = 1$. By equation (A10), this will be true if and only if

$$\frac{\partial h_i}{\partial I_j} = \frac{\left(2I_j - I_i\right)\chi I_i}{\rho + \chi I_j^2} = 0 \Rightarrow \frac{I_i}{I_j} = 2.$$

Case 2. If $\frac{\partial h_i}{\partial I_j} > 0$ for $i = 1, 2$, then $\mathcal{M} > 1$. By equation (A10), this will be true if and only if

$$\frac{\partial h_i}{\partial I_j} = \frac{\left(2I_j - I_i\right)\chi I_i}{\rho + \chi I_j^2} > 0 \Rightarrow \frac{I_i}{I_j} < 2, \forall i \Rightarrow \frac{1}{2} < \frac{I_1}{I_2} < 2.$$

Case 3. If $\frac{\partial h_i}{\partial I_j} > 0$ and $\frac{\partial h_j}{\partial I_i} < 0$, then $0 < \mathcal{M} < 1$. This will be true if and only if $\frac{I_j}{I_i} > 2$, that is, $\frac{I_1}{I_2} > 2$ or $\frac{I_1}{I_2} < \frac{1}{2}$.

Note that it is not possible to have both $\frac{\partial h_1}{\partial I_2} < 0$ and $\frac{\partial h_2}{\partial I_1} < 0$, because these two inequalities combine to imply $2 < \frac{I_j}{I_i} < \frac{1}{2}$, which is impossible. □

Proof of Corollary 1

In the main text, we have proven part (a) and the first part of part (b). We only need to show that $\frac{\partial h_i}{\partial I_j} > 0$ if λ_i is sufficiently large. By (A10), $\frac{\partial h_i}{\partial I_j} > 0$ if and only if $I_i > \frac{1}{2} I_j$. For a fixed $\lambda_j > 0$, if $\lambda_i \to \infty$, then by equation (13) we have $I_i = \frac{\lambda_i(\rho + \chi I_j^2)}{\gamma + \lambda_i \chi I_j} = \frac{\rho + \chi I_j^2}{\frac{\gamma}{\lambda_i} + \chi I_j} \to \frac{\rho + \chi I_j^2}{\chi I_j} = \frac{\rho}{\chi I_j} + I_j > I_j$, and so $\frac{\partial h_i}{\partial I_j} < 0$. □

Proof of Corollary 2

Without loss of generality, we assume that $\lambda_1 > \lambda_2$, and as a result increasing diversity Δ while fixing Λ is equivalent to decreasing λ_1 and increasing λ_2.

Formally, we have

$$\lambda_1 + \lambda_2 = \Lambda \text{ and } \Delta = 1 - \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} = \frac{2\lambda_2}{\lambda_1 + \lambda_2}.$$

Then, total differentiation of the above system yields

$$d\lambda_1 + d\lambda_2 = d\Lambda = 0,$$
$$d\left(\frac{2\lambda_2}{\lambda_1 + \lambda_2}\right) = 2\left(-\frac{\lambda_2}{(\lambda_1 + \lambda_2)^2} d\lambda_1 + \frac{(\lambda_1 + \lambda_2) - \lambda_2}{(\lambda_1 + \lambda_2)^2} d\lambda_2\right) = d\Delta,$$

which implies

$$\frac{d\lambda_1}{d\Delta} = -\frac{\Lambda}{2} \text{ and } \frac{d\lambda_2}{d\Delta} = \frac{\Lambda}{2}.$$

So, by equation (13) and the chain rule, we have

$$\frac{\partial h_1}{\partial \Delta} = \frac{\partial h_1}{\partial \lambda_1}\frac{d\lambda_1}{d\Delta} = -\frac{\partial h_1}{\partial \lambda_1}\frac{\Lambda}{2} \text{ and } \frac{\partial h_2}{\partial \Delta} = \frac{\partial h_2}{\partial \lambda_2}\frac{d\lambda_2}{d\Delta} = \frac{\partial h_2}{\partial \lambda_2}\frac{\Lambda}{2}.$$

Setting $Q = \Delta$ in equation (18) and using the above expressions of $\frac{\partial h_1}{\partial \Delta}$ and $\frac{\partial h_2}{\partial \Delta}$, we obtain

$$\frac{dI_1}{d\Delta} = \mathcal{M}\left(\frac{\partial h_1}{\partial \Delta} + \frac{\partial h_1}{\partial I_2}\frac{\partial h_2}{\partial \Delta}\right) = \mathcal{M}\left(-\frac{\partial h_1}{\partial \lambda_1} + \frac{\partial h_1}{\partial I_2}\frac{\partial h_2}{\partial \lambda_2}\right)\frac{\Lambda}{2}.$$

Then, using equations (A8) and (A9), we have

$$\frac{dI_1}{d\Delta} = \mathcal{M}\frac{\Lambda}{2}\gamma\left[-\frac{\rho+\chi I_2^2}{\left(\gamma+\chi\frac{\gamma I_1}{\rho+I_2^2\chi-I_1I_2\chi}I_2\right)^2} + \frac{(2I_2-I_1)\,\chi I_1}{\rho+\chi I_2^2}\frac{\rho+\chi I_1^2}{\left(\gamma+\chi\frac{\gamma I_2}{\rho+I_1^2\chi-I_1I_2\chi}I_1\right)^2}\right]. \tag{A11}$$

Similarly, we can compute

$$\frac{dI_2}{d\Delta} = \mathcal{M}\frac{\Lambda}{2}\gamma\left[\frac{\rho+\chi I_1^2}{\left(\gamma+\chi\frac{\gamma I_2}{\rho+I_1^2\chi-I_1I_2\chi}I_1\right)^2} - \frac{(2I_1-I_2)\,\chi I_2}{\rho+\chi I_1^2}\frac{\rho+\chi I_2^2}{\left(\gamma+\chi\frac{\gamma I_1}{\rho+I_2^2\chi-I_1I_2\chi}I_2\right)^2}\right]. \tag{A12}$$

Direct computation from the expression of $Var^{-1}(\tilde{v}|\tilde{p})$ in (10) shows that

$$\frac{\partial Var^{-1}(\tilde{v}|\tilde{p})}{\partial I_1} = \frac{2\chi\rho\,(I_1+I_2)\left(\rho+\chi I_2^2-\chi I_1I_2\right)}{\left[2\rho+\chi(I_1-I_2)^2\right]^2} > 0, \tag{A13}$$

$$\frac{\partial Var^{-1}(\tilde{v}|\tilde{p})}{\partial I_2} = \frac{2\chi\rho\,(I_1+I_2)\left(\rho+\chi I_1^2-\chi I_1I_2\right)}{\left[2\rho+\chi(I_1-I_2)^2\right]^2} > 0, \tag{A14}$$

where the inequalities follow from equations (A2) and (A3), namely, $(\rho + I_j^2\chi - I_1I_2\chi) = \frac{\gamma I_i}{\lambda_i} > 0$.

By the chain rule, we have

$$\frac{dVar^{-1}(\tilde{v}|\tilde{p})}{d\Delta} = \frac{\partial Var^{-1}(\tilde{v}|\tilde{p})}{\partial I_1}\frac{dI_1}{d\Delta} + \frac{\partial Var^{-1}(\tilde{v}|\tilde{p})}{\partial I_2}\frac{dI_2}{d\Delta}.$$

Plugging equations (A11) to (A14) into the above equation delivers

$$\frac{dVar^{-1}(\tilde{v}|\tilde{p})}{d\Delta} = \frac{\mathcal{M}\gamma\Lambda\chi^2\rho\left(I_1^2-I_2^2\right)(I_1+I_2)\begin{bmatrix} c\rho^3+\chi\rho^2(I_1+I_2)^2 \\ +3\chi^2\rho I_1 I_2 (I_1-I_2)^2+\chi^3 I_1 I_2 (I_1-I_2)^4 \end{bmatrix}}{[2\rho+\chi(I_1-I_2)^2]^2\gamma^2(\rho+\chi I_2^2)(\rho+\chi I_1^2)} > 0,$$

because $I_1 > I_2$ by $\lambda_1 > \lambda_2$. □

Proof of Proposition 4

The proof of Proposition 4 goes as follows. As we mentioned in the main text, there are four possible types of information market equilibria, depending on whether λ_1 and λ_2 are zero or positive. For any given parameter configuration $(\rho, \chi, \gamma) \in \mathbb{R}_{++}^3$, we identify all those values of $(c_1, c_2) \in \mathbb{R}_{++}^2$ that support an information market equilibrium of each type. We then show that the union of the identified values of (c_1, c_2) forms the whole space of $\mathbb{R}_{++}^2$, which implies the existence of an information market equilibrium. In addition, we show that each parameter configuration can only support a unique information market equilibrium.

Case 1. $\lambda_1 = \lambda_2 = 0$

By equation (13), we have $I_1 = I_2 = 0$ in this case. Since no trader finds acquiring information to be beneficial in equilibrium, we have $\phi_i(0, 0) \leq c_i$ for $i = 1, 2$. By equations (10) and (23), we can express $\phi_i(I_1, I_2)$ as follows:

$$\phi_i(I_1, I_2) = \frac{1}{2\gamma}\log\left(\frac{\rho + I_j^2\chi}{\frac{I_1^2\rho+I_2^2\rho+\rho^2\chi^{-1}}{(I_1-I_2)^2+2\rho\chi^{-1}}}\right), \text{ for } i, j = 1, 2,\ j \neq i. \quad \text{(A15)}$$

So, $\phi_i(0, 0) = \frac{\log 2}{2\gamma}$. The set of (c_1, c_2) supporting an equilibrium of $(\lambda_1 = \lambda_2 = 0)$ is

$$\mathcal{S}_0 \equiv \left\{(c_1, c_2) \in \mathbb{R}_{++}^2 : c_1 \geq \frac{\log 2}{2\gamma} \text{ and } c_2 \geq \frac{\log 2}{2\gamma}\right\}.$$

Case 2. $\lambda_1 > 0$ *and* $\lambda_2 = 0$

By equation (13) and $\lambda_1 > 0$ and $\lambda_2 = 0$, we have

$$I_1 = \lambda_1\rho\gamma^{-1} \text{ and } I_2 = 0,$$

and hence

$$\lambda_1 = \gamma\rho^{-1}I_1. \quad \text{(A16)}$$

So, the condition $\lambda_1 > 0$ implies $I_1 > 0$.

Since in this case traders only acquire signal $\tilde{v}_1$, the value ϕ_1 of signal $\tilde{v}_1$ must be equal to its cost c_1 and the value ϕ_2 of signal $\tilde{v}_2$ must be no larger than its cost c_2, that is,

$$\phi_1(I_1, 0) = c_1 \text{ and } \phi_2(I_1, 0) \le c_2.$$

Thus, the set of (c_1, c_2) supporting an equilibrium of $(\lambda_1 > 0, \lambda_2 = 0)$ is

$$\mathcal{S}_1 \equiv \left\{(c_1, c_2) \in \mathbb{R}^2_{++} : c_1 = \phi_1(I_1, 0), c_2 \ge \phi_2(I_1, 0) \text{ for all } I_1 > 0\right\}.$$

We next analytically characterize the set $\mathcal{S}_1$. By (A15), we have

$$\phi_1(I_1, 0) = \frac{1}{2\gamma}\log\left(\frac{I_1^2 + 2\rho\chi^{-1}}{I_1^2 + \rho\chi^{-1}}\right),$$

which is decreasing in I_1. Since $c_1 = \phi_1(I_1, 0)$, the range of c_1 in $\mathcal{S}_1$ is $(\lim_{I_1\to\infty}\phi_1(I_1, 0), \phi_1(0, 0)) = (0, \frac{\log 2}{2\gamma})$.
The lower bound of c_2 is $\phi_2(I_1, 0)$. By (A15), we have

$$\phi_2(I_1, 0) = \frac{1}{2\gamma}\log\left(\frac{\left(\rho + I_1^2\chi\right)\left(I_1^2 + 2\rho\chi^{-1}\right)}{I_1^2\rho + \rho^2\chi^{-1}}\right),$$

which is increasing in I_1. Combining with $c_1 = \phi_1(I_1, 0)$, we can cancel I_1 and express the lower bound of c_2 as a decreasing function in c_1: $\frac{1}{2\gamma}\log(\frac{e^{2\gamma c_1}}{e^{2\gamma c_1}-1})$.

So, we can analytically characterize $\mathcal{S}_1$ as follows:

$$\mathcal{S}_1 = \left\{(c_1, c_2) \in \mathbb{R}^2_{++} : c_1 \in \left(0, \frac{\log 2}{2\gamma}\right) \text{ and } c_2 \ge \frac{1}{2\gamma}\log\left(\frac{e^{2\gamma c_1}}{e^{2\gamma c_1} - 1}\right)\right\}.$$

In addition, for any $(c_1, c_2) \in \mathcal{S}_1$, there exists a unique information market equilibrium with $\lambda_1 > 0$ and $\lambda_2 = 0$. Specifically, from $\phi_1(I_1, 0) = \frac{1}{2\gamma}\log(\frac{I_1^2+2\rho\chi^{-1}}{I_1^2+\rho\chi^{-1}}) = c_1$, we can determine a unique $I_1 > 0$. Then, using $\lambda_1 = \gamma\rho^{-1}I_1$ in (A16), we can compute a unique λ_1.

Case 3. $\lambda_1 = 0$ *and* $\lambda_2 > 0$
This is symmetric to the above case, and the set of (c_1, c_2) supporting an equilibrium of $(\lambda_1 = 0, \lambda_2 > 0)$ is

$$\mathcal{S}_2 \equiv \left\{(c_1, c_2) \in \mathbb{R}^2_{++} : c_2 \in \left(0, \frac{\log 2}{2\gamma}\right) \text{ and } c_1 \ge \frac{1}{2\gamma}\log\left(\frac{e^{2\gamma c_2}}{e^{2\gamma c_2} - 1}\right)\right\}.$$

Also, for any $(c_1, c_2) \in \mathcal{S}_2$, there exists a unique information market equilibrium with $\lambda_1 = 0$ and $\lambda_2 > 0$.

Case 4. $\lambda_1 > 0$ *and* $\lambda_2 > 0$
Equation (A9) and the conditions $\lambda_1 > 0$ and $\lambda_2 > 0$ imply the following restrictions on (I_1, I_2):

$$I_1 > 0,\ I_2 > 0, \rho + I_2^2\chi - I_1I_2\chi > 0,\ \text{and}\ \rho + I_1^2\chi - I_1I_2\chi > 0.$$

Given $\lambda_1 > 0$ and $\lambda_2 > 0$, the information market equilibrium implies

$$\phi_1(I_1, I_2) = c_1 \text{ and } \phi_2(I_1, I_2) = c_2.$$

Thus, the set of (c_1, c_2) supporting an equilibrium of $(\lambda_1 > 0, \lambda_2 > 0)$ is

$$\mathcal{S}_{1,2} \equiv \left\{ \begin{array}{c} c\,(c_1, c_2) \in \mathbb{R}^2_{++} : c_1 = \phi_1(I_1, I_2), c_2 = \phi_2(I_1, I_2), \\ \text{for all } I_1 > 0, I_2 > 0, \text{ such that} \\ \rho + I_1^2\chi - I_1I_2\chi > 0 \text{ and } \rho + I_2^2\chi - I_1I_2\chi > 0 \end{array} \right\}.$$

We next characterize $\mathcal{S}_{1,2}$. We first decompose the set $\mathcal{S}_{1,2}$ into two symmetric sets:

$$\mathcal{S}_{I_1\geq I_2} \equiv \left\{ \begin{array}{l} (c_1, c_2) \in \mathbb{R}^2_{++} : c_1 = \phi_1(I_1, I_2), c_2 = \phi_2(I_1, I_2), \\ \text{for all } (I_1, I_2) \in \mathbb{R}^2_{++} \text{ such that } I_2 \leq I_1 < I_2 + \frac{\rho}{I_2\chi} \end{array} \right\}$$

and

$$\mathcal{S}_{I_1\leq I_2} \equiv \left\{ \begin{array}{l} (c_1, c_2) \in \mathbb{R}^2_{++} : c_1 = \phi_1(I_1, I_2), c_2 = \phi_2(I_1, I_2), \\ \text{for all } (I_1, I_2) \in \mathbb{R}^2_{++} \text{ such that } I_1 \leq I_2 < I_1 + \frac{\rho}{I_1\chi} \end{array} \right\}.$$

Apparently, $\mathcal{S}_{1,2} = \mathcal{S}_{I_1\geq I_2} \cup \mathcal{S}_{I_1\leq I_2}$.

Given the symmetry of the two sets $\mathcal{S}_{I_1\geq I_2}$and $\mathcal{S}_{I_1\leq I_2}$, we analyze $\mathcal{S}_{I_1\geq I_2}$ only. Basically, we show that

$$\mathcal{S}_{I_1\geq I_2} = \left\{(c_1, c_2) \in \mathbb{R}^2_{++} : c_1 \in \left(0, \frac{\log 2}{2\gamma}\right) \text{ and } c_1 \leq c_2 < \frac{1}{2\gamma}\log\left(\frac{e^{2\gamma c_1}}{e^{2\gamma c_1} - 1}\right)\right\}. \tag{A17}$$

To establish this result, we first characterize the two constant boundaries of c_1, and then characterize the two functions bounding c_2 for a given c_1.

In allocations supported by parameters in $\mathcal{S}_{I_1\geq I_2}$, the cost c_1 is given by $c_1 = \phi_1(I_1, I_2) = \frac{1}{2\gamma}\log\left(\frac{\rho + I_2^2\chi}{\frac{I_1^2\rho + I_2^2\rho + \rho^2\chi^{-1}}{(I_1 - I_2)^2 + 2\rho\chi^{-1}}}\right)$ by (A15), which decreases with I_1. Thus, for a given $I_2 > 0$, c_1 achieves its infimum at $I_1 = I_2 + \frac{\rho}{I_2\chi}$ and its maximum at $I_1 = I_2$. That is, $\phi_1(I_2 + \frac{\rho}{I_2\chi}, I_2) < c_1 \leq \phi_1(I_2, I_2)$. Direct computation shows $\phi_1(I_2 + \frac{\rho}{I_2\chi}, I_2) = 0$. Thus, for any given $I_2 > 0$, we have $c_1 \in (0, \phi_1(I_2, I_2)]$. Given that I_2 can take any positive value in $\mathcal{S}_{I_1\geq I_2}$, the range of c_1 is given by

$$\cup_{I_2>0}(0, \phi_1(I_2, I_2)] = \left(0, \max_{I_2>0}\phi_1(I_2, I_2)\right] = (0, \phi_1(0, 0)) = \left(0, \frac{\log 2}{2\gamma}\right).$$

We now fix any $c_1 \in (0, \frac{\log 2}{2\gamma})$ and find all the corresponding values of c_2 in $\mathcal{S}_{I_1 \geq I_2}$ as follows. Given that $\phi_1(I_2, I_2)$ decreases with I_2, there exists a unique $\bar{I}_{2,c_1}$, which is determined by $\phi_1(\bar{I}_{2,c_1}, \bar{I}_{2,c_1}) = c_1$. The pairs of (I_1, I_2) that can be supported by the value of c_1 in $\mathcal{S}_{I_1 \geq I_2}$ must satisfy $I_2 \leq \bar{I}_{2,c_1}$; otherwise, for any $I_2 > \bar{I}_{2,c_1}$ and $I_1 \in [I_2, I_2 + \frac{\rho}{I_2 \chi})$, we have $\phi_1(I_1, I_2) < \phi_1(I_2, I_2) < \phi_1(\bar{I}_{2,c_1}, \bar{I}_{2,c_1}) = c_1$. For any $I_2 \in (0, \bar{I}_{2,c_1}]$, there exists a unique I_{1,I_2,c_1} that generates c_1 through $\phi_1(I_{1,I_2,c_1}, I_2) = c_1$ and then determines the value of c_2 through $c_2 = \phi_2(I_{1,I_2,c_1}, I_2)$. For the given c_1, all the corresponding values of c_2 can be generated by varying $I_2 \in (0, \bar{I}_{2,c_1}]$; that is, for the given c_1, we determine the constant $\bar{I}_{2,c_1}$ and then for any $I_2 \in (0, \bar{I}_{2,c_1}]$, we have $c_2 = \phi_2(I_{1,I_2,c_1}, I_2)$, where I_{1,I_2,c_1} is determined by $\phi_1(I_{1,I_2,c_1}, I_2) = c_1$.

By the chain rule, we have

$$\frac{dc_2}{dI_2} = \frac{\partial \phi_2\left(I_{1,I_2,c_1}, I_2\right)}{\partial I_{1,I_2,c_1}} \frac{\partial I_{1,I_2,c_1}}{\partial I_2} + \frac{\partial \phi_2\left(I_{1,I_2,c_1}, I_2\right)}{\partial I_2},$$

and by applying the implicit function theorem to $\phi_1(I_{1,I_2,c_1}, I_2) = c_1$, we can find

$$\frac{\partial I_{1,I_2,c_1}}{\partial I_2} = -\frac{\partial \phi_1\left(I_{1,I_2,c_1}, I_2\right)/\partial I_2}{\partial \phi_1\left(I_{1,I_2,c_1}, I_2\right)/\partial I_{1,I_2,c_1}}.$$

Plugging this equation into $\frac{dc_2}{dI_2}$, and using the expression forms of $\phi_i(I_1, I_2)$, we can show that $\frac{dc_2}{dI_2} < 0$. That is, $c_2 = \phi_2(I_{1,I_2,c_1}, I_2)$ decreases with I_2 for $I_2 \in (0, \bar{I}_{2,c_1}]$. Thus, for a given c_1, the lower bound for c_2 is $\phi_2(I_{1,\bar{I}_{2,c_1},c_1}, \bar{I}_{2,c_1})$ and the upper bound is $\phi_2(I_{1,0,c_1}, 0)$. Using the fact that $\phi_1(\bar{I}_{2,c_1}, \bar{I}_{2,c_1}) = c_1$ and $\phi_1(I_{1,I_2,c_1}, I_2) = c_1$, we can show that $\phi_2(I_{1,\bar{I}_{2,c_1},c_1}, \bar{I}_{2,c_1}) = c_1$ and $\phi_2(I_{1,0,c_1}, 0) = \frac{1}{2\gamma}\log\left(\frac{e^{2\gamma c_1}}{e^{2\gamma c_1}-1}\right)$. This completes the proof of expression (A17).

We can obtain a similar expression for $\mathcal{S}_{I_1 \leq I_2}$, and thus $\mathcal{S}_{1,2}$ is given by

$$\begin{aligned}
\mathcal{S}_{1,2} &= \left\{(c_1, c_2) \in \mathbb{R}^2_{++} : c_1 \in \left(0, \frac{\log 2}{2\gamma}\right) \text{ and } c_1 \leq c_2 < \frac{1}{2\gamma}\log\left(\frac{e^{2\gamma c_1}}{e^{2\gamma c_1}-1}\right)\right\} \\
&\quad \cup \left\{(c_1, c_2) \in \mathbb{R}^2_{++} : c_2 \in \left(0, \frac{\log 2}{2\gamma}\right) \text{ and } c_2 \leq c_1 < \frac{1}{2\gamma}\log\left(\frac{e^{2\gamma c_2}}{e^{2\gamma c_2}-1}\right)\right\} \\
&= \left\{(c_1, c_2) \in \mathbb{R}^2_{++} : (e^{2\gamma c_1}-1)(e^{2\gamma c_2}-1) < 1\right\}.
\end{aligned}$$

The above proof also shows that, for any $(c_1, c_2) \in \mathcal{S}_{1,2}$, there exists a unique pair of (I_1, I_2) that is supported by (c_1, c_2) in this case. Take $c_1 \leq c_2$ as an example. For a given c_1, we know that the supported pair (I_1, I_2) takes the form of (I_{1,I_2,c_1}, I_2), where $\phi_1(I_{1,I_2,c_1}, I_2) = c_1$, $\phi_2(I_{1,I_2,c_1}, I_2) = c_2$, and $I_2 \in (0, \bar{I}_{2,c_1}]$. However, given $\frac{dc_2}{dI_2} < 0$, different values of $I_2 \in (0, \bar{I}_{2,c_1}]$ will be supported by different values of c_2. Thus, each $(c_1, c_2) \in \mathcal{S}_{I_1 \geq I_2}$ can only support a unique pair (I_1, I_2). Then, using equation (A9), we can determine a unique pair (λ_1, λ_2) through the determined (I_1, I_2).

Finally, by carefully checking the boundaries of the $\mathcal{S}$ sets, we see that their union forms the whole parameter space $\mathbb{R}^5_{++}$. This means that, for any exogenous parameter configuration $(c_1, c_2, \rho, \chi, \gamma) \in \mathbb{R}^5_{++}$, there exists an information market equilibrium. In addition, all of the $\mathcal{S}$ sets are mutually exclusive. Given that we have established that any parameter configuration in each $\mathcal{S}$ set supports only one information market equilibrium, the information market equilibrium is unique. □

Proof of Proposition 5

By equations (21) and (A10), we have

$$\frac{dI_i}{d\lambda_i} = \mathcal{M}\frac{\gamma\left(\rho+\chi I_j^2\right)}{\left(\gamma+\lambda_i\chi I_j\right)^2} > 0 \text{ and } \frac{dI_j}{d\lambda_i} = \frac{\left(2I_i - I_j\right)\chi I_j}{\rho+\chi I_i^2}\frac{dI_i}{d\lambda_i}, \quad \text{for } i, j = 1, 2,\ j \neq i. \tag{A18}$$

By the chain rule, the absolute magnitude of the Grossman-Stiglitz effect is

$$\frac{1}{2\gamma}\frac{\partial \log\left(Var^{-1}(\tilde{v}|\tilde{p})\right)}{\partial \lambda_i} = \frac{Var(\tilde{v}|\tilde{p})}{2\gamma}\left(\frac{\partial Var^{-1}(\tilde{v}|\tilde{p})}{\partial I_i}\frac{dI_i}{d\lambda_i} + \frac{\partial Var^{-1}(\tilde{v}|\tilde{p})}{\partial I_j}\frac{dI_j}{d\lambda_i}\right).$$

Then, plugging the expressions of $Var(\tilde{v}|\tilde{p})$ (equation (10)), $\frac{\partial Var^{-1}(\tilde{v}|\tilde{p})}{\partial I_i}$ and $\frac{\partial Var^{-1}(\tilde{v}|\tilde{p})}{\partial I_j}$ (equations (A13) and (A14)), and $\frac{dI_j}{d\lambda_i}$ (equation (A18)) into the above expression, we have

$$\frac{1}{2\gamma}\frac{\partial \log\left(Var^{-1}(\tilde{v}|\tilde{p})\right)}{\partial \lambda_i} = \frac{\chi\left(I_1+I_2\right)\left(\rho^2+\chi\rho I_i\left(I_1+I_2\right)+\chi^2 I_1 I_2\left(I_1-I_2\right)^2\right)}{\gamma\left(\rho+\chi I_i^2\right)\left(\rho+\chi I_1^2+\chi I_2^2\right)\left(2\rho+\chi\left(I_1-I_2\right)^2\right)}\frac{dI_i}{d\lambda_i} > 0, \tag{A19}$$

for $i = 1, 2$.

Now we prove $\frac{\partial \phi_i}{\partial \lambda_i} < 0$. By equation (23) and the expressions of $\frac{dI_j}{d\lambda_i}$ in (A18) and $\frac{1}{2\gamma}\frac{\partial \log\left(Var^{-1}(\tilde{v}|\tilde{p})\right)}{\partial \lambda_i}$ in (A19), we have:

$$\begin{aligned}
\frac{\partial \phi_i}{\partial \lambda_i} &= \frac{1}{2\gamma}\frac{2I_j\chi}{\rho+I_j^2\chi}\frac{dI_j}{d\lambda_i} - \frac{1}{2\gamma}\frac{\partial \log[Var^{-1}(\tilde{v}|\tilde{p})]}{\partial \lambda_i} \\
&= \left[\frac{I_j\chi}{\gamma\left(\rho+I_j^2\chi\right)}\frac{\left(2I_i-I_j\right)\chi I_j}{\rho+\chi I_i^2} - \frac{\chi\left(I_1+I_2\right)\left(\rho^2+\chi\rho I_i\left(I_1+I_2\right)+\chi^2 I_1 I_2\left(I_1-I_2\right)^2\right)}{\gamma\left(\rho+\chi I_i^2\right)\left(\rho+\chi I_1^2+\chi I_2^2\right)\left(2\rho+\chi\left(I_1-I_2\right)^2\right)}\right]\frac{dI_i}{d\lambda_i},
\end{aligned}$$

for $i, j = 1, 2,\ j \neq i$. Direct computation shows that the bracketed term in the above equation is negative:

$$-\frac{\chi I_i\left[\rho^2 I_1+\rho^2 I_2+\chi\rho I_1^3+\chi\rho I_2^3+\chi\rho I_j\left(I_1-I_2\right)^2+2\chi\rho I_i^2 I_j+\chi^2 I_j\left(2I_i^2+I_j^2\right)\left(I_1-I_2\right)^2\right]}{\lambda_i\left(\rho+\chi I_1^2\right)\left(\rho+\chi I_2^2\right)\left(\rho+\chi I_1^2+\chi I_2^2\right)\left(2\rho+\chi\left(I_1-I_2\right)^2\right)}<0,$$

where we have used the fact that $(\rho + \chi I_j^2 - \chi I_1 I_2) = \frac{\gamma I_i}{\lambda_i}$. Thus, $\frac{\partial\phi_i}{\partial\lambda_i} < 0$.

Next, we prove $\frac{\partial\phi_i}{\partial\lambda_j} > 0$ if and only if $I_j > I_i$. Following a similar argument as above, we can show that

$$\begin{aligned}\frac{\partial\phi_i}{\partial\lambda_j} &= \frac{1}{2\gamma}\frac{2I_j\chi}{\rho+I_j^2\chi}\frac{dI_j}{d\lambda_j}-\frac{1}{2\gamma}\frac{\partial\log\left[Var^{-1}\left(\tilde{v}|\tilde{p}\right)\right]}{\partial\lambda_j}\\ &= \frac{\chi\left(I_j-I_i\right)\left(\rho+\chi I_j^2-\chi I_1 I_2\right)^2}{\gamma\left(\rho+\chi I_j^2\right)\left(2\rho+\chi I_1^2+\chi I_2^2-2\chi I_1 I_2\right)\left(\rho+\chi I_1^2+\chi I_2^2\right)}\frac{dI_j}{d\lambda_j},\end{aligned}$$

for $i, j = 1, 2,\ j \neq i$. Since $\frac{dI_j}{d\lambda_j} > 0$, we have $\frac{\partial\phi_i}{\partial\lambda_j} > 0$ if and only if $I_j > I_i$. □

Proof of Proposition 6

Setting $\phi_i(I_1, I_2) = c_i$ for $i = 1, 2$ delivers the following system:

$$\begin{cases}\log\left(\rho+I_2^{*2}\chi\right)-\log\left(Var^{-1}\left(\tilde{v}|\tilde{p}^*\right)\right)=2\gamma c_1,\\ \log\left(\rho+I_1^{*2}\chi\right)-\log\left(Var^{-1}\left(\tilde{v}|\tilde{p}^*\right)\right)=2\gamma c_2.\end{cases} \tag{A20}$$

Suppose we decrease c_1. Applying the implicit function theorem to the above system delivers

$$\begin{cases}-Var\left(\tilde{v}|\tilde{p}^*\right)\frac{\partial Var^{-1}\left(\tilde{v}|\tilde{p}^*\right)}{\partial I_1^*}\frac{dI_1^*}{dc_1}+\left(-Var\left(\tilde{v}|\tilde{p}^*\right)\frac{\partial Var^{-1}\left(\tilde{v}|\tilde{p}^*\right)}{\partial I_2^*}+\frac{2I_2^*\chi}{\rho+I_2^{*2}\chi}\right)\frac{dI_2^*}{dc_1}=2\gamma,\\ \left(-Var\left(\tilde{v}|\tilde{p}^*\right)\frac{\partial Var^{-1}\left(\tilde{v}|\tilde{p}^*\right)}{\partial I_1^*}+\frac{2I_1^*\chi}{\rho+I_1^{*2}\chi}\right)\frac{dI_1^*}{dc_1}-Var\left(\tilde{v}|\tilde{p}^*\right)\frac{\partial Var^{-1}\left(\tilde{v}|\tilde{p}^*\right)}{\partial I_2^*}\frac{dI_2^*}{dc_1}=0.\end{cases}$$

Thus,

$$\frac{dI_1^*}{dc_1}=-\frac{2\gamma}{D}Var\left(\tilde{v}|\tilde{p}^*\right)\frac{\partial Var^{-1}\left(\tilde{v}|\tilde{p}^*\right)}{\partial I_2^*}\text{ and }\frac{dI_2^*}{dc_1}=-\frac{2\gamma}{D}\left(-Var\left(\tilde{v}|\tilde{p}^*\right)\frac{\partial Var^{-1}\left(\tilde{v}|\tilde{p}^*\right)}{\partial I_1^*}+\frac{2I_1^*\chi}{\rho+I_1^{*2}\chi}\right), \tag{A21}$$

where

$$D=\frac{4\chi^2\left(I_1^{*2}+I_2^{*2}\right)\left(\rho+\chi I_2^{*2}-\chi I_1^* I_2^*\right)\left(\rho+\chi I_1^{*2}-\chi I_1^* I_2^*\right)}{\left(\rho+\chi I_2^{*2}\right)\left(\rho+\chi I_1^{*2}\right)\left(\rho+\chi I_1^{*2}+\chi I_2^{*2}\right)\left(2\rho+\chi\left(I_1^*-I_2^*\right)^2\right)}>0.$$

We now derive the effect on λ_1^* and λ_2^* of decreasing c_1. By equation (A9),

$$\frac{d\lambda_1^*}{dc_1} = \gamma \frac{\left(\rho + I_2^{*2}\chi\right)\frac{dI_1^*}{dc_1} + \left(I_1^* - 2I_2^*\right) I_1^* \chi \frac{dI_2^*}{dc_1}}{\left(\rho + I_2^{*2}\chi - I_1^* I_2^* \chi\right)^2}.$$

Then, plugging equations (10), (A13), (A14), and (A21) into the above expression yields

$$\frac{d\lambda_1^*}{dc_1} = -\frac{4\gamma^2\chi\left(\rho + \chi I_1^{*2} - \chi I_1^* I_2^*\right)}{D\left(\rho + I_2^{*2}\chi - I_1^* I_2^*\chi\right)^2} \frac{\begin{bmatrix}\rho^2 I_1^* + \rho^2 I_2^* + \chi\rho I_1^{*3} + \chi\rho I_2^{*3} + 2\chi\rho I_1^* I_2^{*2} \\ +\chi I_1^*\left(I_1^* - I_2^*\right)^2\left(\rho + \chi I_1^{*2} + 2\chi I_2^{*2}\right)\end{bmatrix}}{\left(\rho + \chi I_1^{*2}\right)\left(\rho + \chi I_1^{*2} + \chi I_2^{*2}\right)\left(2\rho + \chi\left(I_1^* - I_2^*\right)^2\right)} < 0.$$

Similarly, we can compute

$$\frac{d\lambda_2^*}{dc_1} = \gamma \frac{I_2^*\left(I_2^* - 2I_1^*\right)\chi \frac{dI_1^*}{dc_1} + \left(\rho + I_1^{*2}\chi\right)\frac{dI_2^*}{dc_1}}{\left(\rho + I_1^{*2}\chi - I_1^* I_2^*\chi\right)^2}$$

$$= -\frac{4\gamma^2\chi\left(I_1^* - I_2^*\right)}{D\left(\rho + \chi I_1^{*2} + \chi I_2^{*2}\right)\left(2\rho + \chi\left(I_1^* - I_2^*\right)^2\right)}.$$

Thus, $\frac{d\lambda_2^*}{dc_1} < 0$ if and only if $I_1^* > I_2^*$.

Note that we have

$$I_1^* > I_2^* \Leftrightarrow c_1 < c_2,$$

because the system of determining I_1^* and I_2^* in (A20) implies

$$\log\left(\rho + I_2^{*2}\chi\right) - \log\left(\rho + I_1^{*2}\chi\right) = 2\gamma\left(c_1 - c_2\right).$$

Therefore, $\frac{d\lambda_2^*}{dc_1} < 0$ if and only if $c_1 < c_2$.

Finally, we show that $\frac{dVar^{-1}(\tilde{v}|\tilde{p}^*)}{dc_1} < 0$. By the chain rule,

$$\frac{dVar^{-1}\left(\tilde{v}|\tilde{p}^*\right)}{dc_1} = \frac{\partial Var^{-1}\left(\tilde{v}|\tilde{p}^*\right)}{\partial I_1^*}\frac{dI_1^*}{dc_1} + \frac{\partial Var^{-1}\left(\tilde{v}|\tilde{p}^*\right)}{\partial I_2^*}\frac{dI_2^*}{dc_1}.$$

Plugging equation (A21) into the above equation delivers

$$\frac{dVar^{-1}\left(\tilde{v}|\tilde{p}^*\right)}{dc_1} = -\frac{2\gamma}{D}\frac{\partial Var^{-1}\left(\tilde{v}|\tilde{p}^*\right)}{\partial I_2^*}\frac{2I_1^*\chi}{\rho + I_1^{*2}\chi} < 0.$$

□

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Supporting Information

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Appendix S1: Internet Appendix.