Good disclosure, bad disclosure

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\textbf{Abstract}

We study real-efficiency implications of disclosing public information in a model with multiple dimensions of uncertainty where market prices convey information to a real decision maker. Paradoxically, when disclosure concerns a variable that the real decision maker cares to learn about, disclosure negatively affects price informativeness, and in markets that are effective in aggregating private information, this negative price-informativeness effect can dominate so that better disclosure negatively impacts real efficiency. When disclosure concerns a variable that the real decision maker already knows much about, disclosure always improves price informativeness and real efficiency. Our analysis has important empirical and policy implications for different contexts such as disclosure of stress test information and regulation of credit ratings.

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1. Introduction

One of the main conclusions often coming up following financial crises and failures is the need to provide more precise public information. Regulatory efforts of this kind go back at least to the Securities Exchange Act of 1934 and have been refined and reinforced many times over the years since then, such as in the Sarbanes-Oxley Act and more recently in the Dodd–Frank Act.\textsuperscript{1}

The idea behind these regulations is to create an environment with more abundant public information that will allow investors to make more informed capital allocation

\textsuperscript{1} For instance, Sarbanes-Oxley Act was passed as an “act to protect investors by improving the accuracy and reliability of corporate disclosures made pursuant to the securities laws, and for other purposes.” Greenstone et al. (2006,399) state: “Since the passage of the Securities Act of 1933 and the Securities Exchange Act of 1934, the federal government has actively regulated U.S. equity markets. The centerpiece of these efforts is the mandated disclosure of financial information.”
decisions. These attempts come in multiple channels: requiring firms to provide better disclosure to their investors, addressing problems in credit rating agencies to make credit ratings a more precise and reliable source of public information, disclosing publicly banks’ stress test results, releasing central bank projections about the aggregate economy, and others.

As is often the case with regulation, its intended consequences are very appealing and well understood: providing information seems quite desirable as we want decision makers to make more informed and efficient decisions. However, unintended consequences often appear, reducing the effectiveness of the act, and sometimes even creating overall undesirable outcomes. In the case of disclosing public information in financial markets, a very natural unintended consequence to consider is that the disclosure of such information might crowd out other types of information, possibly even making inferior the overall set of information available to decision makers.

In particular, going back at least to Hayek (1945), economists believe that market prices are an important source of information for real decision makers. They aggregate disperse pieces of information from many traders who trade in financial markets for their own profit motives. This aggregation process at the end generates a signal—the price—that can be very valuable in capital allocation decisions and would not be available without the trading process in the financial market. The trading process and the information aggregation are expected to be affected by disclosure of public information. The question then is whether the provision of more public information—via mandatory disclosure, credit ratings, or stress tests—encourages or discourages the processing of information via market prices. If the latter happens, then another question is whether the provision of public information increases or decreases the overall quality of information available to real decision makers.

In this paper, we propose a model to examine these questions. Our model shows that improving the precision of publicly disclosed information sometimes improves and sometimes reduces the quality of information generated by the financial market. In the latter case, the financial market attenuates the direct positive effect that disclosure has on real efficiency, and in fact this price-informativeness effect of disclosure can be so strong as to generate an overall negative effect on real efficiency. Whether this negative outcome arises depends on the type of information being disclosed and the efficiency of the financial markets in aggregating disperse information. We discuss how these subtle conclusions can inform policymakers in various contexts when designing disclosure policy and deciding which type of information to disclose publicly and how much.

In our model, speculators trade a risky asset in the financial market based on their disperse private signals and the available public information. As a result of the trading process, the price of the risky asset reveals some of the private information of speculators. The decision maker on the real side of the economy—who makes investment/production decisions affecting the cash flows of the risky asset—bases his decisions on the public information, his own private information, and the information in the price. His actions establish the effect that public information has on the real economy. Our mechanism works through the interactions between the exogenous public information and the endogenous price information, both of which affect the forecast quality of real decision makers.

Our model emphasizes that there are different dimensions for information disclosure, and the effect of disclosure depends on what kind of information is disclosed. The importance of studying different dimensions of information in the price formation process has been discussed in Goldstein and Yang (2015). In general, the performance and cash flows of firms depend on different fundamentals, and it is important to take account of what information gets reflected in the price to understand price informativeness and its real effects. Specifically, in our model, we assume that the profitability of the production technology determining the asset cash flow is affected by two independent factors, factor \( \tilde{a} \) and factor \( f \). The key difference between them in our model is based on the direction of information asymmetry between the real decision maker and speculators: relative to speculators, the real decision maker knows more about one factor \( \tilde{a} \) than the other \( f \), and hence he is more keen to learn about factor \( f \) of which he is relatively uninformed. Also, markets are incomplete in the sense that the security traded in the financial market is a claim on the overall cash flow, which is affected by both factors, and there are no separate securities tied to each one of the individual factors. This feature of our model is consistent with financial securities in the real world.

To fix ideas, one can consider our model to describe a financially constrained firm that needs funds from capital providers (such as banks) to make investments. The real decision maker in this example represents the capital providers. The speculators are hedge funds or mutual funds who trade the firm’s shares whose cash flows depend on the factors \( \tilde{a} \) and \( f \) and the capital provided by the capital providers. Public information can be disclosure mandated by the regulator from the firm or a credit rating agency. Thinking about the two dimensions of information, the capital providers can have fairly precise information about the quality of the firm’s products, which they get directly from the firm (factor \( \tilde{a} \)), but they have a harder time evaluating the competition that the firm faces and its interaction with other firms in the product market (factor \( f \)). Indeed, this kind of information requires aggregation from different sources to be precise.

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2 For example, the Financial Accounting Standards Board (FASB) states: “The benefits of financial reporting information include better investment, credit, and similar resource allocation decisions, which in turn result in more efficient functioning of the capital markets and lower costs of capital for the economy as a whole.” (FASB Financial Accounting Series, NO.1260–001 July 6, 2006, “Conceptual Framework for Financial Reporting: Objective of Financial Reporting and Qualitative Characteristics of Decision-Useful Financial Reporting Information,” Section QCS3, p. 35.)

3 It is also possible that the real decision maker has absolute informational advantage in both factors over speculators, and what matters for our results to hold is that the real decision maker has a relative informational advantage in one factor over speculators, similar to David Ricardo’s international trade theory of comparative advantage.
and so learning about it from the financial market is valuable. Both factors affect the firm’s cash flows, and so speculators trade on both types of information.

When public disclosure is about firm competition (factor $f$), it has two effects on the creditors’ forecast quality and hence on real efficiency in our model. The direct effect is to provide new information, which is positive: the creditors become better informed and make more efficient decisions after observing more information about $f$, no matter how noisy it is. The second effect is a negative indirect effect: a more accurate public signal about $f$ will lead speculators in the financial market to put a lower weight on their own private information about this factor, which in turn reduces the price informativeness about it. This negative indirect effect of disclosure on real efficiency will attenuate the positive direct effect, thereby making the overall effect modest. In addition, the indirect effect can be so strong that it even dominates the direct effect, leading to a negative overall effect of disclosure on real efficiency. This outcome will arise when the market aggregates speculators’ private information very effectively, and so the loss of efficiency due to the reduction in price information is pronounced.

When public disclosure is about product quality (factor $\tilde{d}$), the indirect effect becomes positive. To see the intuition, note that, in this case, speculators’ trading is determined by the public signal about $\tilde{d}$ and their private signals about $\tilde{d}$ and $\tilde{f}$. When the public signal provides more information about $\tilde{d}$, speculators will put a higher weight on this public information and less on their private information in forecasting factor $\tilde{d}$. As a result, their private information-based trading will reflect more of their private information about factor $\tilde{f}$ than their private information about factor $\tilde{d}$. Then, the price better aggregates information about $\tilde{f}$, which is what creditors try to learn, and real efficiency undoubtedly improves.

Summarizing the insights, we can see that disclosing more precise public information regarding the factor that the real decision maker already knows about much enables the market to better aggregate and reveal information about what the real decision maker tries to learn, thus making prices more informative and improving real efficiency. Paradoxically, disclosing more precise public information about the factor that the real decision maker wants to learn can backfire, as it interferes with the ability of the market to reveal this type of information, attenuating the positive direct effect of better disclosure. In cases in which the market is very effective in processing information, the indirect effect can be stronger than the direct effect, implying that better disclosure can reduce the overall quality of information available to decision makers and harm real efficiency. These results reveal that the overall contribution of disclosure should not be only measured by the value of the information it contains but rather also by the way it facilitates information generation by the market. In an environment with more than one dimension of uncertainty, which characterizes virtually all relevant real-world situations, prices might aggregate information on a dimension that is not very useful to the real decision makers. The role of public disclosure can then be to make prices aggregate information on the dimension that the real decision makers care to learn. This can be achieved by publicly disclosing information on what they already know.

An important question in evaluating the practical implications of these insights is how do we know, as policymakers or empiricists, which dimensions of information are known to the real decision makers and which ones are not? In general, markets have a comparative advantage in providing information that needs to be aggregated from many sources. In the context of the leading example above, information about product market competition and interactions requires such aggregation and so is more easily obtained from the market than can directly be available to the creditors. Hence, disclosure of this type of information might be harmful when the disclosure is not based on a very precise signal (e.g., the rating agency also does not have the advantage of obtaining aggregated information), and when the market does not have high levels of noise trading (which can be measured using common proxies in the market microstructure literature). Similar principles would guide the classification of information in other contexts. To provide another example, one could think of the context of the disclosure of stress test results for financial institutions, which has been under public debate recently (see Goldstein and Sapra, 2013 for a survey). The real decision maker in this case stands for the creditors who have to decide whether to extend more credit to the financial institution. Creditors have private information on some aspects of the bank’s situation, such as the quality of its loans (factor $\tilde{d}$), but may not have very good information about other aspects such as the network externalities between the bank and other counterparties (factor $\tilde{f}$). In this context, releasing information about loan quality will be undoubtedly beneficial, whereas the effect of releasing information about the network is more ambiguous. We discuss these issues in more detail in Section 4.3 and also offer some empirical implications.

The remainder of this section provides a review of related literature. Section 2 provides the description of the model. In Section 3, we characterize the equilibrium outcomes. Section 4 contains the main results on the effect of different types of disclosure on real efficiency and discusses empirical and policy implications of our analysis. In Section 5, we provide some discussion of the key ingredients of the model and robustness analysis. Section 6 concludes. All proofs and additional technical material are in the appendix.

1.1. Related literature

Our paper contributes to the literature on information disclosure in financial markets, which has been reviewed by Verrecchia (2001), Kanodia (2007), and Goldstein and Yang (2017). Previous studies use trading models to explore the implications of public information for the cost of capital (Diamond and Verrecchia, 1991; Hughes et al., 2007; Lambert et al., 2007), for private information acquisition and price informativeness (Lundholm, 1991; Demski and Feltham, 1994), and for disagreement and trading volume (Kim and Verrecchia, 1991; Kondor, 2012). Other papers analyze welfare implications of disclosure. Hirshleifer (1971) and Hakansson et al. (1982) point out
that public information destroys risk-sharing opportunities and thereby impairs social welfare. Kurlat and Veldkamp (2015) show that disclosure might harm welfare by eliminating access to high-risk, high-return assets. Another line of research explores the effect of disclosure in the presence of payoff externalities and coordination motives across agents, e.g., Morris and Shin (2002), Angeletos and Pavan (2004), Angeletos and Pavan (2007), Goldstein et al. (2011), Colombo et al. (2014), Vives (2017). None of these papers examined the different implications of disclosure about different dimensions of information, which is the focus of our paper.

Our paper is also related to the literature on the real effect of financial markets, where trading and prices in financial markets affect real investment/production decisions, which in turn affect firms’ cash flows. This is known as the “feedback effect.” Bond et al. (2012) provide a review of this literature. Several papers provide related empirical evidence; see, e.g., Luo (2005), Chen et al. (2007), Bakke and Whited (2010), Edmans et al. (2012), Foucault and Frésard (2014). Solving a model with a feedback loop between the market price and the firm’s cash flows is known to be challenging and requires nonstandard approaches to modeling the financial market. We adopt in this paper the basic framework of Goldstein et al. (2013). While they focus on coordination among speculators, we focus on patterns of trading on different types of information and how they are affected by public information and thus, the model presented here is different along these dimensions. Only a small number of other papers in this literature explore the interaction between disclosure of public information and the feedback effect; we discuss them further below.

The mechanism in our paper is related to that in Amador and Weill (2010). They construct a monetary model and show that releasing public information about monetary and/or productivity shocks can reduce welfare through reducing the informational efficiency of the good price system, which relates to the indirect effect of disclosure in our financial market model. One difference in our paper is that we build a model of a financial market, which is quite different, and so we analyze how these forces interact in the context of firms and security prices and the feedback loop between them. This creates differences in both the content and the techniques (see mention of the complications of the feedback-effect literature above). Perhaps more importantly, our paper highlights the different implications of different types of disclosure and so highlights that disclosure can be good or bad, depending on the type of information being disclosed and how effective the market is. This provides a rich set of implications for policy and empirical work. In contrast, in Amador and Weill (2010), there is only one dimension of information, and the indirect effect of disclosing information is always negative.

Another closely related paper is by Bond and Goldstein (2015) who analyze a feedback model and discuss different implications of disclosure. In particular, their analysis also suggests that disclosure can either reduce or raise price informativeness, depending on the type of information being disclosed. Aside from the fact that their model uses different techniques and is set up in a different context, both their mechanism and results are different from ours. At a general level, their paper does not explore the way speculators trade on multiple dimensions of information and shift between them as a result of disclosure. This is the key behind our results on the different effects of different types of disclosure. To see this, consider their two results on disclosure: Propositions 4 and 5. In Proposition 4, they show that disclosure by the government of its information about the fundamental will lead speculators to completely stop trading on this information. This corner solution, highlighting the negative effect of disclosure, can be viewed as a special case of the negative effect of disclosure in our model and is due to the particular information and payoff structure in their paper. In Proposition 5, they show that disclosure by the government of its objective function will lead speculators to trade more on the fundamental information, highlighting the positive effect of disclosure. This positive effect of disclosure, however, originates from a very different economic force than the one in our model. Specifically, Bond and Goldstein (2015) consider risk-averse speculators, and the positive effect of disclosure is due to the reduction in risk when more information is available. In our model, on the other hand, all agents are risk neutral, and so the results are driven by our key force according to which traders shift weights across different signals in their trading behavior in response to different types of disclosure. Finally, it is important to note that in Bond and Goldstein (2015), there is no direct positive effect of disclosing information on real efficiency, since the information being disclosed is already known to the decision maker (the government in their model). Hence, they do not study when the indirect effect is strong enough to overcome the direct effect as we do here.

A few other recent papers also present models in which disclosure harms price efficiency or investment efficiency, albeit through different channels. In Gao and Liang (2013), disclosure crowds out private information production, reduces price informativeness, and so harms managers’ learning and investments. In Banerjee et al. (2018), public information can lower price efficiency because traders who face more fundamental information choose to acquire nonfundamental information exclusively. In Edmans et al. (2016), only hard information (such as earnings) can be disclosed, and disclosing hard information distorts the manager’s investment incentives by changing the relative weight between hard and soft information. In Han et al. (2016), disclosure attracts noise trading that reduces price informativeness and harms managers’ learning quality. Our results highlight the importance of disclosing different types of fundamental information, and they are not driven by anything related to information production, manager incentives, or noise trading.

2. The model

There are three dates, $t = 0, 1$, and 2. At date 0, a continuum $[0, 1]$ of “speculators” trade one risky asset based on private and public information about factors related to the asset’s future cash flows. The equilibrium asset price aggregates speculators’ private information through their trading. At date 1, a representative “real decision maker,” who sees the public information and the equilibrium asset
price, makes inference from the price to guide his actions, which in turn determine the cash flow of the risky asset that was traded in the previous period. At date 2, the cash flow is realized, and all agents get paid and consume.

The risky asset can be interpreted as a stock of a financially constrained firm that needs capital from outside capital providers to make investments. The real decision maker represents the capital providers—such as banks, equity investors, and venture capital firms—that decide how much capital to provide to the firm for the purpose of making new real investment based on their assessment of the productivity of the proposed investment. The public information can be thought of as announcements made by the firm about its future prospects or as economic statistics published by government agencies or central banks. Speculators can be thought of as mutual funds or hedge funds who trade the firm’s stock based on their private information. In Section 4.3, we discuss more empirical settings to which our setup potentially applies.

2.1. Investment

The firm in our economy has access to the following production technology:

\[ Q(K) = \tilde{A}fK, \]

where \( K \) is the amount of investment made by the real decision maker at date 1, \( Q(K) \) is the date 2 output that is generated by the investment \( K \), and \( \tilde{A} \geq 0 \) and \( f \geq 0 \) are two productivity factors. Let \( \tilde{a} \) and \( f \) be the natural logs of \( \tilde{A} \) and \( f \), i.e., \( \tilde{a} \equiv \log \tilde{A} \) and \( f \equiv \log f \). We assume that \( \tilde{a} \) and \( f \) are normally distributed as follows:

\[ \tilde{a} \sim N(0, \tau_\tilde{a}^{-1}) \quad \text{and} \quad f \sim N(0, \tau_f^{-1}), \]

where \( \tilde{a} \) and \( f \) are mutually independent, and \( \tau_\tilde{a} > 0 \) and \( \tau_f > 0 \), respectively, are their precision (inverse of variance).

Factors \( \tilde{a} \) and \( f \) represent two dimensions of uncertainty that affect the cash flow of the traded firm. For example, one dimension can be a factor related to the aggregate economy, and the other one can be firm-specific (e.g., Greenwood et al., 1996; Veldkamp and Wolfers, 2007). More generally, cash flows depend on the demand for firms’ products and the technology they develop and on the success of firms’ operations in traditional lines of business and in new speculative lines of business; thus, the feature of multiple dimensions of uncertainty follows directly.\(^4\)

The real decision maker can be more informed about some particular aspect of the firm, as he has comparative advantage in processing some types of information. We assume that, relative to financial market speculators, the real decision maker has better information about factor \( \tilde{a} \) than factor \( f \). We consider an extreme version of this asymmetric knowledge by assuming that the real decision maker knows perfectly factor \( \tilde{a} \) but nothing about factor \( f \) beyond the prior distribution. We have analyzed an extension in which the real decision maker has noisy signals about both factors, and our results go through as long as the signal quality about one factor is sufficiently different from the signal quality about the other factor.

At date 1, the real decision maker chooses the level of investment \( K \). As in Goldstein et al. (2013), making investment incurs a private cost of \( C(K) = \frac{1}{2}cK^2 \), where \( c > 0 \). The cost can be the monetary cost of raising the capital or the private effort incurred in monitoring the investment. We also follow Goldstein et al. (2013) and assume that the real decision maker captures proportion \( \beta \in (0, 1) \) of the full output \( Q(K) \), and thus his payoff from the investment is \( \beta Q(K) \). So, conditional on his information set \( I_R \), the real decision maker chooses \( K \) as follows:

\[ K^* = \arg \max_k \mathbb{E} \left( \beta \tilde{A}fK - \frac{1}{2}cK^2 \middle| I_R \right) = \frac{\beta}{c} \tilde{A}E(f | I_R). \tag{1} \]

The real decision maker’s information set \( I_R \) consists of factor \( \tilde{a} \), the public information as specified below, and the endogenous equilibrium asset price.

2.2. Private and public information

Each speculator \( i \) observes two private noisy signals about \( \tilde{a} \) and \( f \), respectively:

\[ \tilde{x}_i = \tilde{a} + \tilde{\epsilon}_{x,i} \quad \text{and} \quad \tilde{y}_i = f + \tilde{\epsilon}_{y,i}, \]

where \( \tilde{\epsilon}_{x,i} \sim N(0, \tau_{\tilde{a}}^{-1}) \) (with \( \tau_{\tilde{a}} > 0 \)); \( \tilde{\epsilon}_{y,i} \sim N(0, \tau_f^{-1}) \) (with \( \tau_f > 0 \)); and they are mutually independent and independent of \( \{\tilde{a}, f\} \). The endogenous market price \( \tilde{P} \) aggregates speculators’ private signals \( \{\tilde{x}_i, \tilde{y}_i\} \) through their trading in the financial market. Hence, \( \tilde{P} \) contains information about \( \tilde{a} \) and \( f \), which is useful for the real decision maker to make investment decisions.

All agents, including speculators and the real decision maker, observe two noisy public signals, \( \tilde{\omega} \) and \( \tilde{\eta} \) about the two productivity factors:

\[ \tilde{\omega} = \tilde{a} + \tilde{\epsilon}_{\omega} \quad \text{and} \quad \tilde{\eta} = f + \tilde{\epsilon}_{\eta}, \tag{2} \]

where \( \tilde{\epsilon}_{\omega} \sim N(0, \tau_{\omega}^{-1}) \) (with \( \tau_{\omega} > 0 \)); \( \tilde{\epsilon}_{\eta} \sim N(0, \tau_{\eta}^{-1}) \) (with \( \tau_{\eta} > 0 \)); and they are mutually independent and independent of \( \{\tilde{a}, f\} \). Parameters \( \tau_{\omega} \) and \( \tau_{\eta} \) control the precision of the two public signals. We will follow the literature (e.g., Morris and Shin, 2002; Amador and Weill, 2010) and conduct comparative statics exercises with respect to \( \tau_{\omega} \) and \( \tau_{\eta} \) to examine the efficiency implications of releasing different types of public information.

2.3. Trading and price formation

At \( t = 0 \), speculators submit market orders, as in Kyle (1985), to trade the risky asset in the financial market. They can buy or sell up to one unit of the risky asset, and thus speculator \( i \)’s demand for the asset is \( d(i) \in [-1, 1] \). This position limit can be justified by borrowing/short sales constraints faced by speculators.\(^5\) Speculators are risk

\(^4\) Several papers in the finance literature have also specified that the value of the traded security is affected by more than one fundamental, e.g., Froot et al. (1992), Goldman (2005), Kondor (2012), Goldstein and Yang (2015), among others.

\(^5\) The specific size of this position limit is not crucial; what is crucial is that speculators cannot take unlimited positions.
neutral, and therefore they choose their positions to maximize the expected trading profits conditional on their information sets \( I_i = \{ \xi_i, \tilde{y}_i, \tilde{\varnothing}_i, \tilde{\eta}_i \} \).

The traded asset is a claim on the portion of the aggregate output that remains after removing the real decision maker’s share. Specifically, the aggregate output is \( Q = \tilde{A}F(K) \). So, after removing the \( \beta \) fraction of \( Q \), the remaining \((1 - \beta)\) fraction constitutes the cash flow on the risky asset:

\[
\tilde{V} = (1 - \beta)Q = (1 - \beta)\tilde{A}F(K).
\] (3)

A speculator’s profit from buying one unit of the asset is given by \( \tilde{V} - \tilde{P} \), and similarly, his profit from shorting one unit is \( \tilde{P} - \tilde{V} \). So, speculator \( i \) chooses demand \( d(i) \) to solve:

\[
\max_{d(\tilde{\xi}) \in [-1,1]} \left[ d(i)E(\tilde{V} - \tilde{P}|I_i) \right].
\] (4)

Since each speculator is atomistic and is risk neutral, he will optimally choose to either buy up to the one-unit position limit or short up to the one-unit position limit. We denote the aggregate demand from speculators as \( D = \sum_{i=1}^{i} d(i)di \), which is the fraction of speculators who buy the asset minus the fraction of those who short the asset.

As in Goldstein et al. (2013), we assume the following noisy supply curve provided by (unmodeled) liquidity traders:

\[
L(\tilde{\xi}, \tilde{P}) = 1 - 2\Phi(\tilde{\xi} - \lambda \log \tilde{P}),
\] (5)

where \( \tilde{\xi} \sim N(0, \tau_{\xi}^{-1}) \) (with \( \tau_{\xi} > 0 \)) is an exogenous demand shock independent of other shocks in the economy. Function \( \Phi(\cdot) \) denotes the cumulative standard normal distribution function. Thus, the supply curve \( L(\tilde{\xi}, \tilde{P}) \) is strictly increasing in the price \( \tilde{P} \) and decreasing in the demand shock \( \tilde{\xi} \). The parameter \( \lambda > 0 \) captures the elasticity of the supply curve with respect to the price, and it can be interpreted as the liquidity of the market. That is, when \( \lambda \) is high, the supply is very elastic with respect to the price and thus the demand from informed speculators can be easily absorbed by noise trading without moving the price very much. An assumption that the noise trading depends on the price is needed here to determine an equilibrium price. An alternative formulation would allow speculators to condition their trades on the price, and then noise trading does not need to depend on the price to close the model. We consider this formulation in Section 5.3 and show that it does not lend itself to analytical tractability. But with numerical analysis, we show that our main qualitative results are robust to this variation of the model.

The market clears by equating the aggregate demand \( D \) from speculators with the noisy supply \( L(\tilde{\xi}, \tilde{P}) \):

\[
D = L(\tilde{\xi}, \tilde{P}).
\] (6)

This market-clearing condition will determine the equilibrium price \( \tilde{P} \).

2.4. Equilibrium definitions

Definition 1. An equilibrium consists of a price function, \( P(\tilde{a}, \tilde{f}, \tilde{\varnothing}, \tilde{\eta}, \tilde{\xi}) : \mathbb{R}^3 \rightarrow \mathbb{R} \), an investment policy for the real decision maker, \( K(\tilde{a}, \tilde{P}, \tilde{\varnothing}, \tilde{\eta}) : \mathbb{R}^4 \rightarrow \mathbb{R} \), a trading strategy of speculators, \( d(\tilde{x}, \tilde{y}, \tilde{\varnothing}, \tilde{\eta}) : \mathbb{R}^4 \rightarrow [-1,1] \), and the corresponding aggregate demand function for the asset \( D(\tilde{a}, \tilde{f}, \tilde{\varnothing}, \tilde{\eta}) \) such that: (a) for the real decision maker, \( K(\tilde{a}, \tilde{P}, \tilde{\varnothing}, \tilde{\eta}) = \tilde{F}(\tilde{a}, \tilde{P}, \tilde{\varnothing}, \tilde{\eta}) \); (b) for speculator \( i \), \( d(\tilde{x}_i, \tilde{y}_i, \tilde{\varnothing}_i, \tilde{\eta}_i) \) solves \( (4) \); (c) the market-clearing condition \( (6) \) is satisfied; and (d) the aggregate asset demand is given by

\[
D(\tilde{a}, \tilde{f}, \tilde{\varnothing}, \tilde{\eta}) = \int_0^1 d(\tilde{x}, \tilde{y}, \tilde{\varnothing}, \tilde{\eta})di = E\left[ d(\tilde{x}, \tilde{y}, \tilde{\varnothing}, \tilde{\eta})|\tilde{a}, \tilde{f}, \tilde{\varnothing}, \tilde{\eta} \right],
\] (7)

where the expectation is taken over \( (\tilde{\xi}, \tilde{\eta}) \).

As in Goldstein et al. (2013), we will focus on linear monotone equilibria in which a speculator buys the asset if and only if a linear combination of his signals is above a cutoff threshold and sells it otherwise.

Definition 2. A linear monotone equilibrium is an equilibrium in which \( d(\tilde{x}, \tilde{y}, \tilde{\varnothing}, \tilde{\eta}) = 1 \) if \( \tilde{x} + \Phi_y \tilde{y} + \Phi_\varnothing \tilde{\varnothing} + \Phi_\eta \tilde{\eta} > g \) for constants \( \Phi_y, \Phi_\varnothing, \Phi_\eta \), and \( g \) and \( d(\tilde{x}, \tilde{y}, \tilde{\varnothing}, \tilde{\eta}) = -1 \) otherwise.

3. Equilibrium characterizations

In this section, we illustrate the steps for constructing a linear monotone equilibrium. The equilibrium characterization boils down to a fixed-point problem of solving for the weight that speculators put on the signal \( \tilde{y} \) about factor \( \tilde{f} \) when they trade the risky asset. Specifically, we first conjecture a trading strategy of speculators and use the market-clearing condition to determine the asset price and hence the information that the real decision maker can learn from the price. We then update the real decision maker’s belief and characterize his investment rule, which in turn determines the cash flow of the traded asset. Finally, given the implied price and cash flow in the first two steps, we solve for speculators’ optimal trading strategy and compare it with the initial conjectured trading strategy to solve for its underlying parameters.

3.1. Price informativeness

In a linear monotone equilibrium, speculators buy the asset whenever \( \tilde{x}_i + \Phi_y \tilde{y}_i + \Phi_\varnothing \tilde{\varnothing}_i + \Phi_\eta \tilde{\eta}_i > g \), where \( \Phi_y, \Phi_\varnothing, \Phi_\eta \), and \( g \) are endogenous parameters that will be determined in equilibrium. This condition is equivalent to

\[
\tilde{y}_i > \frac{g - (\tilde{x}_i + \Phi_y \tilde{y}_i + \Phi_\varnothing \tilde{\varnothing}_i + \Phi_\eta \tilde{\eta}_i)}{\Phi_y + \Phi_\varnothing + \Phi_\eta}.
\]

Using our normal distribution functions, speculators’ aggregate purchase can be
characterized by \(1 - \Phi\left(\frac{g - (\bar{a} + \phi_f \bar{f}) - \phi_w \bar{w} - \phi_\eta \bar{\eta}}{\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}}\right)\) and their aggregate selling is \(\Phi\left(\frac{g - (\bar{a} + \phi_f \bar{f}) - \phi_w \bar{w} - \phi_\eta \bar{\eta}}{\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}}\right)\). Thus, the net aggregate demand from speculators is

\[
D(\bar{a}, \bar{f}, \bar{\omega}, \bar{\eta}) = 1 - 2\Phi\left(\frac{g - (\bar{a} + \phi_f \bar{f}) - \phi_w \bar{w} - \phi_\eta \bar{\eta}}{\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}}\right) - \Phi\left(\frac{g - (\bar{a} + \phi_f \bar{f}) - \phi_w \bar{w} - \phi_\eta \bar{\eta}}{\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}}\right).
\]

(8)

The market-clearing condition (6) together with Eqs. (5) and (8) indicates that

\[
1 - 2\Phi\left(\frac{g - (\bar{a} + \phi_f \bar{f}) - \phi_w \bar{w} - \phi_\eta \bar{\eta}}{\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}}\right) = 1 - 2\Phi(\xi - \lambda \log \bar{P}),
\]

which implies that the equilibrium price is given by

\[
\bar{P} = \exp\left(\frac{\bar{a} + \phi_f \bar{f} + \phi_w \bar{w} + \phi_\eta \bar{\eta} - g \lambda}{\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}} + \frac{\xi}{\lambda}\right).
\]

(9)

Recall that the real decision maker has the information set \(\{\bar{a}, \bar{P}, \bar{\omega}, \bar{\eta}\}\) and thus, he knows the realizations of \(\bar{a}, \bar{\omega},\) and \(\bar{\eta}\). As a result, the price \(\bar{P}\) is equivalent to the following signal in predicting factor \(\bar{f}\):

\[
\tilde{s}_p = \frac{\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}} \log \bar{P} - \bar{a} - \phi_w \bar{w} - \phi_\eta \bar{\eta} + g}{\phi_y},
\]

(10)

where the normally distributed noise is

\[
\tilde{\epsilon}_p = \frac{\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}} \xi}{\phi_y},
\]

(11)

which has a precision of

\[
\tau_p = \frac{1}{\text{Var}(\tilde{\epsilon}_p)} = \frac{\phi_y^2 \tau_x \tau_y \tau \xi}{\tau_y + \phi_y^2 \tau_x}.
\]

(12)

The endogenous precision \(\tau_p\) captures how much information the real decision maker can learn from the price about factor \(\bar{f}\), which he does not know. As we will see, \(\tau_p\) will affect real efficiency through guiding the real decision maker’s investment decisions. We will be interested in studying how the public signals’ precision levels \(\tau_w\) and \(\tau_\eta\) affect \(\tau_p\) and then real efficiency. We will show that \(\tau_w\) and \(\tau_\eta\) affect \(\tau_p\) only through their effects on \(\phi_y\), the weight that speculators put on their signals about factor \(\bar{f}\) when they trade. Specifically, if speculators trade more aggressively on their information about \(\bar{f}\) (i.e., when \(\phi_y\) increases), the price will be more informative about factor \(\bar{f}\), all other things being equal. As a result, the real decision maker can glean more information from the price, which increases real efficiency.

3.2. Optimal investment policy

The real decision maker has information set \(I_R = \{\bar{a}, \bar{P}, \bar{\omega}, \bar{\eta}\}\). By Eq. (1), in forming the optimal investment, the real decision maker needs to forecast factor \(\bar{f}\). The public signal \(\bar{\eta}\) directly provides information about \(\bar{f}\). We have already characterized how the real decision maker uses price \(\bar{P}\) to form a signal \(\tilde{s}_p\) in predicting factor \(\bar{f}\). That is, the real decision maker’s information set equips him with two signals in forecasting \(\bar{f}\): \(\bar{\eta}\) and \(\tilde{s}_p\). By Bayes’ rule and Eq. (11), we compute the real decision maker’s optimal investment as follows:

\[
K^* = \exp\left[\left(\frac{-\beta}{e} + \frac{1}{2} \frac{\tau_\eta}{\tau_f + \tau_\eta + \tau_p}\right) + \bar{a}\right]
\]

\[
+ \left(\frac{\tau_\eta}{\tau_f + \tau_\eta + \tau_p}\right) \bar{\eta} + \left(\frac{\tau_p}{\tau_f + \tau_\eta + \tau_p}\right) \tilde{s}_p.
\]

(13)

3.3. Optimal trading strategy

Using the expression of \(\bar{P}\) in Eq. (9), the cash flow expression \(V = (1 - \beta) \hat{A} F K^*\), and the investment rule in Eq. (13), we can compute the expected price and cash flow conditional on speculator \(i\)’s information set \(\{\bar{x}_i, \bar{y}_i, \bar{\omega}_i, \bar{\eta}\}\) as follows:

\[
E(\tilde{P}|\bar{x}_i, \bar{y}_i, \bar{\omega}_i, \bar{\eta}) = \exp\left(b_{x_i}^0 + b_{y_i}^0 \bar{y}_i + b_{\omega_i}^0 \bar{\omega}_i + b_{\eta_i}^0 \bar{\eta}\right).
\]

(14)

\[
E(\tilde{V}|\bar{x}_i, \bar{y}_i, \bar{\omega}_i, \bar{\eta}) = \exp\left(b_{x_i}^0 + b_{y_i}^0 \bar{y}_i + b_{\omega_i}^0 \bar{\omega}_i + b_{\eta_i}^0 \bar{\eta}\right).
\]

(15)

where the endogenous coefficients \(b\)'s are given in Appendix A.

Speculator \(i\) will choose to buy the asset if and only if his expectation for the value of the asset is higher than his expectation for the price, that is, \(E(\tilde{V}|\bar{x}_i, \bar{y}_i, \bar{\omega}_i, \bar{\eta}) > E(\tilde{P}|\bar{x}_i, \bar{y}_i, \bar{\omega}_i, \bar{\eta})\). Thus, we have

\[
E(\tilde{V}|\bar{x}_i, \bar{y}_i, \bar{\omega}_i, \bar{\eta}) > E(\tilde{P}|\bar{x}_i, \bar{y}_i, \bar{\omega}_i, \bar{\eta}) \iff \left(\frac{b_{x_i}^0}{b_{y_i}^0} \bar{y}_i + \left(\frac{b_{\omega_i}^0}{b_{\eta_i}^0}\right) \bar{\omega}_iight) \bar{\eta} + \left(\frac{b_{\omega_i}^0}{b_{\eta_i}^0} - 1\right) \bar{\eta} > b_{x_i}^0 - b_{y_i}^0.
\]

Recall that we conjecture the speculators’ trading strategy as buying the asset whenever \(\bar{x}_i + \phi_y \bar{y}_i + \phi_\omega \bar{\omega}_i + \phi_\eta \bar{\eta} > g\). Hence, we require that in equilibrium,

\[
\phi_y = \frac{b_{x_i}^0 - b_{y_i}^0}{b_{x_i}^0 - b_{y_i}^0},
\]

(16)

\[
\phi_\omega = \frac{b_{\omega_i}^0 - b_{\eta_i}^0}{b_{x_i}^0 - b_{y_i}^0},
\]

(17)

and

\[
\phi_\eta = \frac{b_{\omega_i}^0 - b_{\eta_i}^0}{b_{x_i}^0 - b_{y_i}^0}.
\]

(18)

provided that \(b_{x_i}^0 - b_{y_i}^0 > 0\). The right-hand side \(b_{x_i}^0 - b_{y_i}^0\) of Eq. (16) depends only on \(\phi_y\) (through the term of \(\phi_y\) in \(b_{y_i}^0\) and \(b_{\omega_i}^0\) and the term of \(\tau_\eta \) in \(b_{\eta_i}^0\)). Therefore, we use Eq. (16) to compute \(\phi_y\) and then plug this solved \(\phi_y\) into Eqs. (17) and (18) to compute \(\phi_\omega\) and \(\phi_\eta\).

Proposition 1. (a) A linear monotone equilibrium is characterized by the following two conditions in terms of the weight
that speculators put on private signal $\tilde{y}$ about factor $\tilde{f}$:

$$
\phi_y = \left(1 + \frac{\partial^2 \nu/\partial \tau_x^2}{\tau_y + \phi_y \tau_x} - \frac{\phi_y}{\lambda \sqrt{\tau_x^{-1} + \phi_y \tau_x^{-1}}}\right) \frac{\tau_y}{\tau_y + \phi_y \tau_x},
$$

and

$$
\phi_y^2 > \tau_y \left(\frac{1}{4X^2} - \frac{1}{\tau_y}\right).
$$

(b) When $\lambda > \frac{\phi_y}{2\tau_y}$, there exists a linear monotone equilibrium with $\phi_y > 0$. The equilibrium is unique when $\lambda$ is sufficiently large.

4. The effect of disclosure

In this section, we study the implications of disclosure in the model, focusing on real efficiency—the surplus generated by real investment decisions. Ideally, we should conduct a full welfare analysis by examining how public disclosure affects the expected utility levels of all agents in the economy. However, for tractability, we assume that noise traders trade the risky asset according to Eq. (5), which precludes welfare analysis on them.

Our specific measure of real efficiency follows Goldstein et al. (2013), reflecting the expected net benefit of investment evaluated in equilibrium:

$$
RE = E\left(\tilde{A} \tilde{K}^* - \frac{\xi}{2} K^{**}\right).
$$

In Appendix A, we use Eq. (1) and the law of iterated expectation to compute

$$
RE = \beta \left(1 - \frac{\beta}{2}\right) \exp\left[\frac{2}{\tau_y} \frac{\tau_y}{\tau_f} - \operatorname{Var}(\tilde{f}|\tilde{y}, \tilde{s}_p)\right],
$$

where

$$
\operatorname{Var}(\tilde{f}|\tilde{y}, \tilde{s}_p) = \frac{1}{\tau_y + \tau_n + \tau_p}.
$$

In our model, disclosure affects real efficiency through changing the real decision maker’s information set. The more precise information that the real decision maker has, the more efficient his investment decisions. This fact is clearly captured by expression (21): recall that the real decision maker knows factor $\tilde{a}$, and so he only needs to forecast the other factor $\tilde{f}$; therefore, the term $\operatorname{Var}(\tilde{f}|\tilde{y}, \tilde{s}_p)$ captures the efficiency loss due to remaining uncertainty relative to a full information economy. We now examine the real efficiency implications of releasing public information about the two different factors.

4.1. The effect of disclosure about factor $\tilde{f}$

Eq. (22) demonstrates that the quality of public disclosure $\tilde{y}$ about factor $\tilde{f}$, measured by $\tau_y$, has two effects on the overall quality of the real decision maker’s information (and hence real efficiency). The first is a positive direct effect of providing new information, which is related to the term $\tau_y$ in Eq. (22). The second effect is an endogenous indirect effect: public information affects the trading of speculators (more specifically, the loading $\phi_y$ on private information about $\tilde{f}$) and hence the price informativeness about factor $\tilde{f}$, which in turn affects the amount of information that the real decision maker can learn from the price, i.e., the term $\tau_p$ in Eq. (22). Formally, by Eqs. (21) and (22), we have

$$
\frac{\partial \tau_p}{\partial \tau_n} \propto \frac{\partial (\tau_f + \tau_n + \tau_p)}{\partial \tau_n} = \frac{1}{\text{direct effect}} + \frac{\partial \tau_p}{\partial \tau_n},
$$

where

$$
\frac{\partial \tau_p}{\partial \tau_n} = \frac{2\tau_p \tau_y}{\phi_y (\tau_y + \phi_y \tau_x)} \frac{\partial \phi_y}{\partial \tau_n}.
$$

which follows from applying the chain rule to Eq. (12).

Computing the different $b$’s in Eq. (16) and assuming that the supply elasticity $\lambda$ is very large, we get that $b^p_y$ and $b^2_y$ approach zero, and thus the expression in Eq. (16) determining $\phi_y$ reduces to

$$
\phi_y \approx \frac{b^p_y}{b^2_y} = \left(1 + \frac{\tau_y}{\tau_f + \tau_n + \tau_p} \frac{\tau_y}{\tau_y + \phi_y \tau_x} \right) \frac{1}{2 \tau_y + \tau_n + \tau_p}.
$$

Intuitively, when the supply elasticity $\lambda \rightarrow \infty$, the market is very liquid and so prices do not move that much; see Eq. (9). Hence, traders mainly use their information to update cash flows and not so much about prices. Then, the relative weight $\phi_y$ they put on their signal $\tilde{y}_l$ (about factor $\tilde{f}$) in their trading rule is determined by the extent to which they use signal $\tilde{y}_l$ to forecast cash flow relative to the extent they use signal $\tilde{s}$ (about factor $\tilde{a}$) to forecast cash flow. This is the ratio $b^p_y / b^2_y$.

Using the expression of $\tau_p$ in Eq. (12) and applying the implicit function theorem to Eq. (25), we can show

$$
\frac{\partial \phi_y}{\partial \tau_n} = \frac{\phi_y}{\tau_f + \tau_n + \tau_p} \left[\frac{\tau_y}{\tau_y + \phi_y \tau_x} \right] + \frac{1}{1 - \frac{2\tau_p}{\tau_y + \phi_y \tau_x}} < 0.
$$

That is, more precise public disclosure about factor $\tilde{f}$ causes speculators to trade less aggressively on their own private information about $\tilde{f}$.

To see the intuition note that in the expression of $E(V|\tilde{x}_l, \tilde{y}_l, \tilde{a}, \tilde{h})$ in Eq. (15), the public signal $\tilde{y}_l$ and the private signal $\tilde{y}_l$ are useful for predicting $\tilde{f}$, while the public signal $\tilde{a}$ and the private signal $\tilde{h}$ are useful for predicting $\tilde{a}$. When $\tau_n$ increases so that the public signal $\tilde{y}_l$ becomes a more informative signal about $\tilde{f}$, speculators put a higher weight $b^p_y$ on the signal $\tilde{y}_l$ and a lower weight $b^2_y$ on their own private signals $\tilde{h}_l$ in predicting $\tilde{f}$. This directly decreases $\phi_y$ given that $\phi_y = b^p_y / b^2_y$, as we see in Eq. (25).

This effect is captured by the numerator of Eq. (26). Moreover, there is a further “multiplier effect” captured by the denominator in Eq. (26): the decrease in $\phi_y$ reduces $\tau_p$, which causes the real decision maker to glean less information about $\tilde{f}$, making the asset value $V$ less sensitive to $\tilde{f}$. Thus, in anticipation of this outcome, speculators trade more aggressively on their private information $\tilde{x}_l$ about the other factor $\tilde{a}$, which increases $b^2_y$ in Eq. (15), and less aggressively on information $\tilde{y}_l$ about $\tilde{f}$, which decreases $b^p_y$. 


As a result, \( \phi_y \) decreases further given that \( \phi_y = \frac{\partial y}{\partial \tau} \). This amplification chain continues on and on until it converges to a much lower level of \( \phi_y \). Note that this second multiplier effect depends on the fact that the cash flows from the traded security are endogenous and affected by market prices, whereas the first basic effect would exist even in a model where the cash flows from the traded security do not depend on market prices (as in Subrahmanyan and Titman, 1999 and Foucault and Gehrig, 2008).

Since \( \frac{\partial \phi_y}{\partial \tau_\eta} < 0 \), we have \( \frac{\partial \tau_\eta}{\partial \tau_\eta} < 0 \) as well by Eq. (24). That is, the real decision maker learns less information from the price as a result of more disclosure about factor \( \tilde{f} \) so that the indirect effect of disclosing information about factor \( \tilde{f} \) is negative in Eq. (23). This negative indirect effect attenuates the positive direct effect, causing the overall effect of disclosure on real efficiency to be modest or even negative.

This result presents a paradox: recall that factor \( \tilde{f} \) is the variable that the real decision maker cares to learn about; still, disclosing more information about it publicly gives rise to a counter productive indirect effect through affecting the price informativeness, and this indirect effect can overturn the positive direct effect, reducing real efficiency overall.

We show that the negative indirect effect is stronger than the positive direct effect when public information is relatively imprecise (\( \tau_\eta \) is small) and the precision \( \tau_\xi \) of noise trading is large. The intuition is as follows. First, when the disclosure level is sufficiently high, the positive direct effect always dominates. For instance, if \( \tau_\eta \rightarrow \infty \), the real decision maker would know factor \( \tilde{f} \), and the allocation would be the first best, which achieves the maximum real efficiency. Thus, only when the disclosure level \( \tau_\eta \) is low is it possible for the negative indirect effect to dominate. Second, suppose \( \tau_\eta \) is low. When there is little noise trading (\( \tau_\xi \) is large), the market aggregates speculators’ private information effectively. Then, since the indirect effect operates through price informativeness, it is particularly strong in this case. By contrast, when \( \tau_\xi \) is small, the market has a lot of noise trading and its information aggregation role is limited, thereby weakening the indirect effect of disclosure via price informativeness.

Overall, greater disclosure about factor \( \tilde{f} \) interferes with the ability of the market to aggregate information about this factor. This effect tends to reduce real efficiency. The effect might be so strong as to outweigh the positive direct effect that precise disclosure about \( \tilde{f} \) has on real efficiency. To summarize, we have the following proposition.

**Proposition 2.** For a high enough level of supply elasticity \( \lambda \), increasing the precision \( \tau_\eta \) of public disclosure \( \tilde{y}_1 \) about factor \( \tilde{f} \) (a) decreases the relative weight \( \phi_y \) that speculators put on private signals \( \tilde{y}_1 \) (i.e., \( \frac{\partial \phi_y}{\partial \tau_\eta} < 0 \)); (b) decreases the precision \( \tau_p \) with which the real decision maker learns from the price regarding factor \( \tilde{f} \) (i.e., \( \frac{\partial \tau_p}{\partial \tau_\eta} < 0 \)), and so the indirect effect is negative; (c) increases real efficiency \( RE \) at high levels of disclosure (i.e., \( \frac{\partial RE}{\partial \tau_\eta} > 0 \) for large \( \tau_\eta \)); and (d) decreases (increases) real efficiency \( RE \) at low levels of disclosure if the precision \( \tau_\xi \) of noise trading is large (small) (i.e., for small \( \tau_\eta \), \( \frac{\partial RE}{\partial \tau_\eta} < 0 \) if \( \tau_\xi \) is large, and \( \frac{\partial RE}{\partial \tau_\eta} > 0 \) if \( \tau_\xi \) is small).

Fig. 1 graphically illustrates Proposition 2. We set \( \tau_\eta = \tau_f = \tau_y = \tau_{\omega} = \lambda = 1 \) in all these four panels. In Panels A1 and A2, we choose \( \tau_\xi = 0.5 \) so that the level \( \tau_\xi \) of noise trading is relatively high and the market does not aggregate private information that much. In Panels B1 and B2, we choose \( \tau_\xi = 10 \). And thus the level of noise trading is low and the market aggregates private information effectively. In Panels A1 and B1, we plot the weight \( \phi_y \) that speculators put on the private signal \( \tilde{y}_1 \) against the precision \( \tau_\eta \) of the public signal \( \tilde{y}_1 \). In Panels A2 and B2, we plot three variables against \( \tau_\eta \): (i) \( \tau_\eta \), the direct effect of public disclosure on the real decision maker’s forecast precision by providing new information about \( \tilde{f} \); (ii) \( \tau_p \), the indirect effect of public disclosure on the real decision maker’s forecast precision by affecting the informational content of the price; and (iii) \( \tau_\eta + \tau_p \), which is a proxy for real efficiency, since by Eqs. (21) and (22), real efficiency \( RE \) is a monotonic transformation of \( \tau_\eta + \tau_p \).

In Panels A1 and B1, we see that, consistent with Proposition 2, the relative weight \( \phi_y \) that speculators put on private information \( \tilde{y}_1 \) decreases with the precision \( \tau_\eta \) of public disclosure \( \tilde{y}_1 \). This pattern translates to a decreasing \( \tau_p \) as a function of \( \tau_\eta \) in Panels A2 and B2, which corresponds to the negative indirect effect of disclosure. As a result, the direct effect of increasing \( \tau_\eta \), as manifested by the increasing \( \tau_\eta \), is attenuated by the negative indirect effect in both panels.

In addition, in Panel A2 where \( \tau_\xi \) is relatively small, the direct effect dominates and real efficiency \( (\tau_\eta + \tau_p) \) increases with \( \tau_\eta \). By contrast, in Panel B2 where \( \tau_\xi \) is relatively large, the indirect effect dominates for low levels of disclosure while the direct effect dominates for high levels of disclosure, so that there exists a U-shape between real efficiency and disclosure. Hence, improving disclosure might backfire and reduce real efficiency. To understand the implications of this result, suppose that there are some technical constraints in achieving precision beyond some upper bound so that a social planner might be restricted to choosing an optimal disclosure level \( \tau_\eta^* \) from some given interval \([0, \tau_\eta]\). Then, depending on where this interval exactly is, we will see a “bang-bang” solution to the optimal choice \( \tau_\eta^* \). It will be optimal to either provide no public information at all (i.e., setting \( \tau_\eta^* = 0 \)) or provide the maximum feasible amount of public information (i.e., setting \( \tau_\eta^* = \tau_\eta \)).

### 4.2. The effect of disclosure about factor \( \tilde{a} \)

Since the real decision maker knows factor \( \tilde{a} \) perfectly, the public signal \( \tilde{\omega} \) about factor \( \tilde{a} \) does not directly provide information about the other factor \( \tilde{f} \). Therefore, the only channel for public disclosure to affect real efficiency is through its indirect effect on the endogenous precision of the information that the real decision maker can learn.

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3 Of course, when the real decision maker only observes a noisy signal about \( \tilde{a} \), the direct effect of the public signal \( \tilde{\omega} \) is still active. We have analyzed an extension which allows the real decision maker to see noisy signals about both factors and found that our results are robust in that extension.
from the asset price. Formally, by Eqs. (12), (21), and (22), we have
\[
\frac{\partial RE}{\partial \tau_{\omega}} \propto \frac{\partial \tau_p}{\partial \tau_{\omega}} = \frac{2\tau_p \tau_y}{\phi_y (\tau_y + \phi_y^2 \tau_x)} \frac{\partial \phi_y}{\partial \tau_{\omega}}.
\] (27)

As discussed in the previous section, when the supply elasticity \( \lambda \) is very large, \( \phi_y \) is determined by \( \phi_y = \frac{\phi_y}{\phi_x} \). By applying the implicit function theorem, we can show:
\[
\frac{\partial \phi_y}{\partial \tau_{\omega}} = \frac{\phi_y}{1 - \frac{2\tau_p \tau_y}{\tau_y + \phi_y^2 \tau_x}} > 0.
\] (28)

That is, more precise public disclosure about factor \( \bar{a} \) causes speculators to trade more aggressively on their private information about the other factor \( \bar{f} \) and increases the informativeness of the price about this factor.

The intuition for this result goes as follows: when \( \tau_{\omega} \) increases so that the public signal \( \bar{\omega} \) becomes a more informative signal about \( \bar{a} \), speculators put a higher weight \( b_{\omega}^y \) on the signal \( \bar{\omega} \) and a lower weight \( b_x^y \) on the signal \( \bar{x}_i \) in predicting \( \bar{a} \). Other things equal, this increases \( \phi_y \) given that \( \phi_y = \frac{\phi_y}{\phi_x} \). In addition, there is a multiplier effect, as captured by the denominator in Eq. (28): the increase in \( \phi_y \) improves \( \tau_p \) in Eq. (12), and so the real decision maker gleans more information on \( \bar{f} \) from the price, making the asset cash flow \( \bar{V} \) more responsive to \( \bar{f} \) through the real decision maker’s investments. This, in turn, causes speculators to rely more on their private signal \( \bar{y}_i \)—which is a signal about \( \bar{f} \)—in making their forecasts, which increases \( b_y^y \) in Eq. (15). Thus, \( \phi_y \) increases further given that \( \phi_y = \frac{\phi_y}{\phi_x} \), until the equilibrium value of \( \phi_y \) reaches a much higher level.

Overall, since \( \frac{\partial \phi_y}{\partial \tau_{\omega}} > 0 \) by Eq. (28), we have \( \frac{\partial \tau_p}{\partial \tau_{\omega}} > 0 \) as well in Eq. (27). That is, the real decision maker learns more information about factor \( \bar{f} \) from the price. By Eq. (27), real efficiency improves with better disclosure: greater disclosure about factor \( \bar{a} \) allows the market to do a better job of aggregating information about factor \( \bar{f} \), and the improved price informativeness increases real efficiency. Summarizing the above discussions, we have the following proposition.

**Proposition 3.** For a high enough level of supply elasticity \( \lambda \), increasing the precision \( \tau_{\omega} \) of public disclosure \( \bar{\omega} \) about factor \( \bar{a} \) (a) increases the relative weight \( \phi_y \) that speculators put on private signals \( \bar{y}_i \) (i.e., \( \frac{\partial \phi_y}{\partial \tau_{\omega}} > 0 \)); (b) increases the precision \( \tau_p \) with which the real decision maker learns from the price regarding factor \( \bar{f} \) (i.e., \( \frac{\partial \tau_p}{\partial \tau_{\omega}} > 0 \)), and so the indirect effect is positive; and (c) increases real efficiency \( RE \) (i.e., \( \frac{\partial \text{RE}}{\partial \tau_{\omega}} > 0 \)).
4.3. Empirical and policy implications

We now discuss empirical and policy implications coming out of Propositions 2 and 3. Our model setup captures the interactions among three types of agents: the speculators who trade the financial asset, the real decision maker who makes decisions that affect the real value of the firm, and the agent who discloses public information. Hence, our setup is one where the real decision maker differs from the agent that releases the public information. As a result, disclosure has a direct effect on real efficiency by revealing new information to the decision maker and an indirect effect through affecting the informativeness of market prices. In Section 5.3, we will consider a variation where the real decision maker is also the one making the disclosure, and so disclosure has only an indirect effect on real efficiency.

There are various empirical settings that naturally fit our main setup. As we mentioned at the beginning of Section 2, our leading example is a financially constrained firm that raises capital from outside capital providers to finance investments. The risky asset corresponds to the firm’s traded financial assets (stock or bond), and speculators are financial institutions who trade the firm’s asset. The agent who discloses the information can be the firm that releases public information about the investment’s profitability. It can also be the government or a rating agency that discloses information in the course of their evaluation of the firm’s prospects. The real decision maker can be thought of as the capital providers who determine capital provision and investment based on their private information, the public information released, and the asset price. There are two prime current examples at the center of policy debate, where the issue of quality of public information has been discussed recently. One is the disclosure of stress test results for banks, and one is the quality of information in credit ratings. We will now discuss them in more detail and explore their connection to our model. We will then conclude this section by discussing empirical implications of our model.

4.3.1. Disclosure of stress test results for financial institutions

In the new era of financial regulation following the subprime crisis of 2008, an important component of the supervisory toolkit is the stress tests for financial institutions to assess their ability to withstand future shocks. For instance, the Dodd–Frank Act requires the Federal Reserve to conduct supervisory stress tests of large bank holding companies and to publicly disclose the results of the stress tests. The Dodd–Frank Act also requires all federally regulated financial companies with $10 billion or more in total consolidated assets to conduct their own internal stress tests and to publicly disclose the results of these internal stress tests under the severely adverse scenario.

A key question that occupies policymakers and bankers is what level of disclosure of the stress test results is desirable. The debate over this question is described in a Wall Street Journal article.8 In this article, Fed Governor Daniel Tarullo expresses support for wide disclosure, saying, “(t)he disclosure of stress-test results allows investors and other counterparties to better understand the profiles of each institution.” However, the Clearing House Association expresses the concern that making the additional information public “could have unanticipated and potentially unwarranted and negative consequences to covered companies and U.S. financial markets.”

Our analysis in Propositions 2 and 3 provides a framework to think about some costs and benefits of stress test disclosure. One goal of stress tests, as indicated by the quote from Tarullo above, is to improve the quality of information that is available to bank creditors when they decide how much to lend to the bank. As in our framework, creditors can have private information on some aspects of the bank’s situation, say about the quality of its loans, but they may not have very good information about other aspects, say about the network externalities between the bank and other counterparties. Without stress test disclosure, creditors can use their information and glean information from market prices when making their decisions. Our analysis suggests that when considering the real efficiency implications of disclosure, it is important to think about the specific structure of information possessed by creditors and how it compares to the information being disclosed. Following our example, by Proposition 3, disclosure about the quality of banks’ loans will be undoubtedly beneficial, even though this is information that creditors already have. This is because it allows the market to process information about network externalities more efficiently and convey this information to creditors who can make more efficient decisions. However, by Proposition 2, disclosing information about network externalities might backfire. On the one hand, it directly provides useful information to creditors. But on the other hand, it makes the market less useful in providing this information. As a result, as Panel B2 of Fig. 1 shows, when the financial market is very effective in aggregating private information—which is the case when there is little noise trading—the information should be disclosed only when it is sufficiently precise, i.e., when the quality of stress tests is very high.

Thinking more about the practical implications of our results, it is important to clarify that public disclosure of stress test information is predicted to backfire and reduce efficiency in our framework when three conditions hold: First, this disclosure is in a dimension on which bank creditors do not have very precise direct information. Second, the information provided by stress tests is noisy. Third, absent disclosure, the market can do a fairly good job of aggregating this information. We think that our narrative in the previous paragraph provides a reasonable case in which these conditions hold. Information about the effect of network externalities on the risk of an individual bank, by its nature, needs to be aggregated from many sources. This is thus exactly the type of information that the market has a comparative advantage in providing as long as noise trading is not too prominent (which can be measured empirically using common proxies from the market microstructure literature). For the same reasons, regulators such as the Federal Reserve, when conducting and disclosing this type of information, might end up with only a

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8 “Lenders Stress over Test Results,” Wall Street Journal, March 5, 2012. Also see Goldstein and Sapra (2013) for a survey on the costs and benefits of disclosing stress test results.
noisy proxy, given that stress test techniques might not be as effective as markets in aggregating many pieces of information. Overall, in deciding what types of information to disclose, regulators should assess how likely it is that the three conditions mentioned above will hold, based on similar reasoning.

4.3.2. Precision of credit ratings

Many observers identify inaccurate credit ratings as one of the main contributors to the recent financial crisis, which has prompted an examination of the role of credit rating agencies (see Skreta and Veldkamp, 2009 and White, 2010 for related discussions). The existing studies have proposed that conflicts of interest and rating shopping have led to biased ratings (e.g., Skreta and Veldkamp, 2009; Opp et al., 2013). As Skreta and Veldkamp (2009) recognize, in theory, one obvious policy recommendation is to improve the accuracy of credit ratings by mandating disclosure of all shadow ratings. For instance, in China, the issuers of asset-backed securities are required to disclose at least two credit ratings.

Our analysis suggests that even if credit ratings become a more precise and reliable source of public information, real efficiency is not guaranteed. Suppose that creditors rely on their own information, on information in credit ratings, and on information in market prices of the firm’s securities when deciding how much capital to extend to the firm. Based on information provided by the firm, creditors can have high-quality information about the quality of the firm’s products, but they have a harder time evaluating the competition that the firm faces and its interaction with other firms. Both these factors affect the prospects of the firm. Creditors can gain some information from financial markets, which aggregate the signals of many different traders. Just like before, according to Proposition 3, if credit rating agencies base the ratings on information they get from the firm concerning product quality, then increasing the precision of ratings would increase price informativeness and have an overall positive effect on the efficiency of creditors’ decisions. But according to Proposition 2, if credit rating agencies base the ratings more on independent research they conduct concerning competition and market interactions, then a more precise rating will reduce price informativeness and might reduce the overall efficiency of the creditors’ decisions. This depends on the overall precision of the ratings and the quality of market information. Thus, our model generates implications for when greater precision of credit ratings is desirable and perhaps also what kind of information credit rating agencies should focus on in different circumstances.

As in the previous application, it is important to emphasize that greater precision of ratings will harm real efficiency when it focuses on information that, absent disclosure, the market does a good job of aggregating. Hence, it is natural to think about the firm’s competition with other firms in this context, as this is not “hard” information and is probably best revealed when aggregated across different sources. In this case, as long as there is not much noise trading, the disclosure of not so precise information from the rating agency (which is not the best way to aggregate information from different sources) might interfere with the aggregation function of the market and lead to an overall decrease in real efficiency.

4.3.3. Empirical implications

The discussion so far revolved around normative implications of our model concerning the optimal design of disclosure in settings like stress tests and credit ratings. We emphasized that when the information is in dimensions that require aggregation from many sources, then disclosure might backfire and reduce real efficiency, as it interferes with the natural ability of the market to provide this information. On the other hand, providing public information on issues that the market does not have comparative advantage in, because they do not require much aggregation, is always beneficial. Another important question revolves around the positive implications of our model. Can one come up with testable implications of our model providing ground for future empirical work? We now provide a couple of examples with this goal in mind.

First, one common type of disclosure involves macroeconomic projections. Central banks provide forecasts about important macroeconomic variables such as gross domestic product (GDP), inflation, and unemployment. There is no doubt that forecasting these variables can benefit a lot from aggregation of opinions from many market participants, and so this is the kind of information that the market has comparative advantage in processing. Thinking about individual firms, the projected effect of macroeconomic variables on their profits is also incorporated into their stock prices by the trading of speculators who specialize in this type of information, such as hedge funds and mutual funds. This information can then be used by the firm when making its investment decisions. Our model predicts that an increase in the precision of macroeconomic forecasts will decrease the efficiency of firms’ investment decisions when these forecasts are overall not very precise and when the market for the firm’s stocks does not have a high level of noise trading. In other cases, the efficiency of firms’ investment decisions will increase as a result of an increase in the precision of macroeconomic forecasts. These predictions can be tested fairly easily with existing proxies for the precision of macroeconomic forecasts, the efficiency of investment decisions, and the amount of noise trading in the price.

Second, thinking about different types of information disclosed by firms, one can classify them into variables that look more like our ̅f factor and variables that look more like our ̅d factor. For example, disclosing to the market hard facts about revenues, profits, or corporate events seems like disclosing information about factor ̅d in our model. This is because creditors can directly get this information from the firm, and these are hard facts on which information aggregation from many market participants is not likely to help much. On the other hand, disclosing information to the market about forecasts for future performance or for future synergies in case of an acquisition corresponds to disclosing information about factor ̅f in our model. This is because the information available to the management in this case is probably noisy, as it requires assessment of future developments that are not known to anyone for sure. This is exactly the kind of information
that the market has comparative advantage in processing given its ability to aggregate different opinions from many different market participants. With such classifications in mind, one can then test the results of our model concerning the effect of different types of disclosure.

5. Extensions and variations

In this section, we provide analysis and discussion of several extensions and variations of the model. In Section 5.1 we consider a model where the speculators observe only one private signal; we demonstrate that our mechanism crucially relies on speculators shifting weights between different private signals. In Section 5.2 we extend our model to endogenize the information acquisition decision by speculators and show that our results are robust to this extension. In Section 5.3 we consider a variation in which public information is disclosed by the real decision maker, which allows us to speak to a broader set of empirical settings. In this section, we also explore robustness on another dimension and allow speculators to condition their trades on the price so that the market liquidity parameter \( \lambda \) becomes endogenous.

5.1. The role of two private signals for speculators

The crux of the mechanism in our paper is that the quality of public information causes speculators to shift weights between their different private signals, causing changes in the informativeness of the price. To see this, consider an alternative model where each speculator is endowed with only a private signal \( \tilde{y}_i \) about factor \( \tilde{f} \). This is equivalent to assuming that \( \tau_x = 0 \) and \( \tau_y > 0 \) in our baseline model in Section 2.

As before, we conjecture that in this alternative setting, speculators buy the asset whenever \( \tilde{y}_i + \tilde{\phi}_\omega \tilde{\omega} + \tilde{\phi}_\eta \tilde{\eta} > \tilde{g} \), where \( \tilde{\phi}_\omega \), \( \tilde{\phi}_\eta \), and \( \tilde{\eta} \) are endogenous parameters. We can follow similar steps as in Section 3 and show that speculators’ aggregate net demand for the risky asset is

\[
\hat{D}(\tilde{f}, \tilde{\omega}, \tilde{\eta}) = 1 - 2\Phi\left(\frac{\tilde{g} - \tilde{\phi}_\omega \tilde{\omega} - \tilde{\phi}_\eta \tilde{\eta}}{\sqrt{\tau_y}}\right).
\]

So, using market-clearing condition (6), we can find that the equilibrium price would change to

\[
\tilde{p} = \exp\left(\frac{\tilde{f}}{\sqrt{\tau_y}} + \frac{\tilde{\eta}}{\lambda} - \frac{\tilde{g}}{\lambda \sqrt{\tau_y}} + \tilde{\phi}_\omega \tilde{\omega} + \tilde{\phi}_\eta \tilde{\eta}\right).
\]

Given that the real decision maker knows public information \( \tilde{\omega} \) and \( \tilde{\eta} \), the price \( \tilde{p} \) is equivalent to the following signal in predicting \( \tilde{f} \):

\[
\tilde{s}_p = \tilde{f} + \sqrt{\tau_y} \tilde{\eta},
\]

which has a precision of

\[
\tilde{\tau}_p = \frac{1}{\text{Var}(\sqrt{\tau_y} \tilde{\xi})} = \tau_y \tilde{\tau}_\xi.
\]

Clearly, the amount \( \tilde{\tau}_p \) of information that the real decision maker learns from the price is not affected by the public information precision \( \tau_\omega \) and \( \tau_\eta \). This shuts down the mechanism emphasized in our analysis. So, our main results in Propositions 2 and 3 disappear in this alternative economy with only one private signal \( \tilde{y}_i \).

Proposition 4. Suppose that each speculator observes only one private signal \( \tilde{x}_i \) or \( \tilde{y}_i \). Disclosure does not affect the amount of information that the real decision maker learns from prices, and so the indirect effect of disclosure is inactive.

Proposition 4 demonstrates that the feature of speculators observing two private signals is crucial for establishing an effect of public signals on price informativeness. In our baseline model, the two private signals are about different factors. In the online Internet Appendix, we have also analyzed a setting in which each speculator receives two private signals about the same factor \( \tilde{f} \) that is not known to the real decision maker: one signal \( \tilde{y}_i \) is speculator specific, while the other signal \( \tilde{\xi} \) is common across speculators but not observed by the real decision maker. We find that this alternative setting generates results in the spirit of our main results due to public disclosure making speculators shift weights across their private signals. This contributes to our main point that disclosure has an effect on price informativeness by causing traders to change weights across different signals in their trading decisions.

5.2. Endogenous information acquisition

In our baseline model of Section 2 we take as exogenous the signals received by speculators. We now show that our results are robust to endogenous information acquisition of speculators. Our analysis of information acquisition closely follows Verrecchia (1982). Specifically, at the beginning of date 0, speculator \( i \) can acquire private signals \( \tilde{x}_i \) and \( \tilde{y}_i \) with precision levels \( \tau_{x,i} \) and \( \tau_{y,i} \) according to an increasing, convex, and smooth cost function, \( C(\tau_{x,i}, \tau_{y,i}) \). Following the literature (e.g., Gao and Liang, 2013), we assume that speculators acquire information before the public information is released, although they know the disclosure policy. That is, when acquiring information, speculators do not observe \( \tilde{\omega} \) and \( \tilde{\eta} \) but know parameters \( \tau_\omega \) and \( \tau_\eta \). At date 0, after speculators acquire information, public information is disclosed, and then the financial market opens. The order of events at dates 1 and 2 is the same as in Section 2.

Speculator \( i \)'s ex-ante expected trading gain net of information acquisition cost is

\[
\pi(\tau_{x,i}, \tau_{y,i}; \tau_x, \tau_y) = E[d(\tilde{x}_i, \tilde{y}_i, \tilde{\omega}, \tilde{\eta})E(\tilde{V} - \tilde{P} \tilde{x}_i, \tilde{y}_i, \tilde{\omega}, \tilde{\eta})]
\]

\[-C(\tau_{x,i}, \tau_{y,i}).
\]

where \( \tau_x \) and \( \tau_y \) are the precision levels acquired by a representative speculator in equilibrium. In the online Internet Appendix, we compute the expression of \( \pi(\tau_{x,i}, \tau_{y,i}; \tau_x, \tau_y) \). The optimal precision levels \( \tau_{x,i}^* \) and \( \tau_{y,i}^* \) are determined by the first-order conditions of maximizing \( \pi(\tau_{x,i}^*, \tau_{y,i}^*; \tau_x, \tau_y) \). In a symmetric equilibrium, we have \( \tau_{x,i}^* = \tau_x \) and \( \tau_{y,i}^* = \tau_y \) for \( i \in [0, 1] \). Thus, assuming interior solutions,
in equilibrium, we have
\[ \frac{\partial \pi(\tau_{x,i}, \tau_{y,j}; \xi, \eta)}{\partial \tau_{x,i}} \bigg|_{\tau_{x,i} = \tau_x} = 0 \quad \text{and} \quad \frac{\partial \pi(\tau_{x,i}, \tau_{y,j}; \xi, \eta)}{\partial \tau_{y,j}} \bigg|_{\tau_{y,j} = \tau_y} = 0. \]

These two first-order conditions, together with the two conditions that characterize the financial market equilibrium in Proposition 1, form a system of three equations and one inequality in terms of three unknowns (\( \tau_x \), \( \tau_y \), and \( \phi_y \)), which pins down the overall equilibrium with endogenous information acquisition.

There is no closed-form expression for \( \pi(\tau_{x,i}, \tau_{y,j}; \xi, \eta) \). Thus, we use Fig. 2 to numerically examine the implication of disclosure in this extended economy. We assume that the information acquisition cost function takes a quadratic form, i.e., \( C(\tau_{x,i}, \tau_{y,j}) = \frac{\gamma_x}{2} \tau_{x,i}^2 + \frac{\gamma_y}{2} \tau_{y,j}^2 \), where \( \gamma_x > 0 \) and \( \gamma_y > 0 \). We plot five variables against the precision of public information \( (\tau_{\xi}, \text{ or } \tau_{\omega}) \): \( \tau_x \), \( \tau_y \), \( \phi_y \), \( \tau_p \), and \( \text{RE} \). In the top and middle panels, we are interested in the implications of disclosing information about factor \( \tilde{f} \), and so we vary the precision \( \tau_{\xi} \) of public information about factor \( \tilde{f} \) and fix the precision \( \tau_{\omega} \) of public information about factor \( \tilde{a} \). In the bottom panels, we vary the precision \( \tau_{\omega} \) and fix the precision \( \tau_{\xi} \). The parameter values in Fig. 2 are similar to those in Fig. 1. Specifically, in the top panels, we set \( \tau_{\omega} = 1 \) and \( \tau_{\xi} = 0.5 \), while in the middle panels, we set \( \tau_{\omega} = 1 \) and \( \tau_{\xi} = 10 \). In the bottom panels, we set \( \tau_{\xi} = \tau_{\omega} = 1 \). In all panels, the other parameters are \( \tau_0 = \tau_f = \lambda = \gamma_x = \gamma_y = \xi = 1 \) and \( \beta = \frac{1}{2} \). While the figures show results for these particular parameter values, we conducted analysis with many different parameter values, and the main results we highlight in the discussion below are robust across them.

We focus our discussion on the nine right panels of Fig. 2, which conduct analysis in the spirit of Fig. 1, only allowing for the precision levels of private information to be endogenously determined. To facilitate comparison, we plot two curves in each one of the nine panels. The solid curves correspond to the equilibrium outcomes with endogenous private information of speculators. The dashed curves correspond to the equilibrium outcomes in economies where the speculators’ private information is exogenous. For instance, in Panel A3, the solid curve plots \( \phi_y \) against \( \tau_{\xi} \) when the values of \( \tau_x \) and \( \tau_y \) vary with \( \tau_{\xi} \), while the dashed curve plots \( \phi_y \) against \( \tau_{\xi} \) when \( \tau_x \) and \( \tau_y \) are fixed at their equilibrium values for \( \tau_{\xi} = 0 \).

The patterns of the solid curves in these nine right panels, which have been confirmed more generally for various parameter values, suggest that our results in the baseline model are robust to endogeneous information acquisition of speculators. Consistent with Proposition 2, we observe (i) that in Panels A3 and B3, disclosing information about factor \( \tilde{f} \) reduces the weight \( \phi_y \) that speculators put on their private signals \( \tilde{y}_i \); (ii) that in Panels A4 and B4, the
real decision maker learns less information from the price with greater disclosure about $\tilde{f}$; (iii) that in Panel A5 where the precision $\tau_{\xi}$ of noise trading is relatively low, disclosing information about $\tilde{f}$ improves real efficiency; and (iv) that in Panel B5 where the precision $\tau_{\xi}$ of noise trading is relatively high, real efficiency first decreases and then increases with greater disclosure about $\tilde{f}$. In addition, consistent with Proposition 3, we find that disclosing information about factor $\tilde{\alpha}$ increases $\phi_y$ in Panel C3, the precision $\tau_p$ of the price information in Panel C4, and real efficiency $RE$ in Panel C5.

Moreover, as these nine panels show, compared to the economies with exogenous private information, endogenous information acquisition strengthens our results. That is, the solid curves are steeper than the dashed curves. For instance, in Panel B3, an increase in $\tau_{\eta}$ reduces $\phi_y$ by a larger amount along the solid curve than along the dashed curve. This translates to a deeper drop in real efficiency in Panel B5 along the solid curve than along the dashed curve. This result reflects that the information acquisition decisions reinforce the trading decisions to create an overall bigger effect. An increase in the quality of public information about $\tilde{f}$ not only makes traders rely less on their private information about this factor when they trade but also makes them produce less information about it to begin with (see Panel B2), and so the effect is amplified. The crowding out of private information acquisition is similar to results in other papers in the literature (e.g., Diamond, 1985; Gao and Liang, 2013).

Interestingly, in Panel C3, an increase in the precision $\tau_{\omega}$ of public information about factor $\tilde{\alpha}$ also increases $\phi_y$ by a larger amount on the solid curve than on the dashed curve, leading to a greater increase in real efficiency when information acquisition is endogenous (Panel C5). This is because disclosing information about factor $\tilde{\alpha}$ motivates speculators to acquire more information about factor $\tilde{f}$ (Panel C2), and so increases the weight they put on this private information even more. This leads to an amplified positive effect on the efficiency of real decisions. Hence, in the spirit of the other results in our paper, when studying the effect of disclosure, it is important to distinguish between the disclosure of information about different factors. These points are missing in the traditional literature studying the effects of disclosure on private information acquisition (e.g., Diamond, 1985; Gao and Liang, 2013).

In thinking about how to empirically distinguish our model from previous models in the literature dealing with crowding out of private information by public disclosure, one can consider two avenues. First, as discussed in the previous paragraph, our model does not always generate “crowding out,” but rather sometimes generates “crowding in” of private information. It all depends on the type of information being disclosed and the type of information being privately produced and traded on. Hence, more detailed empirical tests are needed to differentiate between dimensions of information and how they respond to disclosure. Second, the crowding-out literature emphasized the channel through the production of information, which is discussed in this section, but the new element of our model is the channel through the intensity of trading for a given amount of information. Hence, to observe this in the data, one would need to obtain more detailed information about the actual trading by speculators and how it responds to disclosure of information, as opposed to looking at their information acquisition that is captured by analysts’ coverage, for example.

### 5.3. Public information disclosed by the real decision maker

This section presents a different framework combining two variations of our basic model that address important points related to empirical implications and robustness. First, in the framework described below, we consider a situation where the real decision maker is the one disclosing the information. As we mentioned in Section 4.3, this situation is relevant in many real-world settings. For example, a regulator discloses stress test results for banks and has to make a decision about intervention in the operations of banks. Or, a firm makes announcements about its future prospects and has to make an investment decision based on all the available information and without raising new capital. In our basic model, the real decision maker learns directly from the disclosed information, and the empirical implications of that structure have been discussed in Section 4.3, whereas in the version described here, the real decision maker does not learn from the disclosed information, simplifying the analysis and allowing us to address a range of other empirical implications.

Second, in the framework described below, speculators submit price-contingent demand schedules, and noise trading is independent of the price. Our basic model features a different set of assumptions where speculators submit market orders, and the price is pinned down by a price-dependent term in the noise-trading function. As we explain below, this is done for tractability, as the alternative considered in this section does not lend itself to analytical solutions. The downside of the basic model is that market liquidity is exogenously given by $\lambda$. The variation considered here allows us to endogenize market liquidity and explore how the endogenous liquidity affects our results. We combine this variation with the version where the decision maker is also disclosing the information, as this is a simpler framework and thus can help us focus on the implications of endogenous liquidity in the most transparent way. Still, as we discuss below, this variation of the model is not solvable in closed form, and we resort to numerical analysis in this section.

We now describe the setting. As before, we assume that the real decision maker knows perfectly factor $\tilde{\alpha}$. In order for the real decision maker to be able to disclose some information about factor $\tilde{f}$, we now also endow him with a private signal about $\tilde{f}$: $\tilde{s}_f = \tilde{f} + \tilde{e}_s$, where $\tilde{e}_s \sim N(0, \tau_{s}^{-1})$ (with $\tau_{s} > 0$) is independent of other shocks. This information structure still parsimoniously captures the idea that the real decision maker knows relatively more about factor $\tilde{\alpha}$ than factor $\tilde{f}$. The real decision maker releases two public signals, $\tilde{\omega}$ and $\tilde{\eta}$, which are respectively noisy versions of his two private signals as follows:

$$\tilde{\omega} = \tilde{\alpha} + \tilde{e}_\omega$$

and

$$\tilde{\eta} = \tilde{s}_f + \tilde{\epsilon}.$$
where $\tilde{e}_{\omega} \sim N(0, \tau_{\omega}^{-1})$ (with $\tau_{\omega} \geq 0$), $\tilde{e} \sim N(0, \tau^{-1})$ (with $\tau > 0$), and they are mutually independent and independent of $\{\tilde{a}, \tilde{f}, \tilde{\eta}\}$. Relating back to Eq. (2) in the basic model, we can define $\tilde{e}_{\omega} = \tilde{e} + \tilde{e}$ and rewrite $\tilde{\eta} = \tilde{f} + \tilde{e}_{\omega}$. In this setting, there is a natural upper bound for the precision $\tau_{\eta}$ of $\tilde{e}_{\omega}$, i.e., $\tau_{\eta} = \frac{\tau_{\omega}}{\sqrt{\tau_{\omega}} \tau_{\eta} \in [0, \tau_{\eta}]$. Essentially, the real decision maker can disclose to the public a signal that is not more precise than the signal that he observes.

We still consider linear monotone equilibria in which speculators buy the asset whenever a combination of their signals is above a threshold. As before, they have two private signals and two public signals. In addition, now they also condition on the price that serves two roles: it determines how much they need to pay for the asset and conveys information about its fundamentals. So, the conjectured trading rule is that a speculator will buy if and only if $\tilde{s}_i + \phi_y \tilde{y}_i + \phi_{\omega} \tilde{\omega} + \phi_\eta \tilde{\eta} - \phi_p \tilde{p} > g$, where $\tilde{p} \equiv \log \tilde{p}$ and $\phi$‘s and $g$ are endogenous parameters.

Following similar steps as in the baseline model, we can show that speculators’ aggregate net demand for the risky asset is $D(\tilde{a}, \tilde{f}, \tilde{\eta}, \tilde{p}) = 1 - 2 \Phi \left( \frac{g - (\phi_y \tilde{y}_i + \phi_{\omega} \tilde{\omega} + \phi_\eta \tilde{\eta} + \sqrt{\tau_{\omega}^{-1} + \phi_\eta^2 \tau_{\eta}^{-1}})}{\phi_{\tilde{p}}} \right)$.

We assume that the noise-trading function is similar to the one in Eq. (5), but we set $\lambda = 0$. This is because, once informed speculators condition their trading on the price, noise trading does no longer need to be dependent on the price for the price to be pinned down by the market-clearing condition. Then, the market-clearing condition generates the following equilibrium price function:

$$\tilde{p} = \exp \left( -g + \tilde{a} + \phi_y \tilde{y}_i + \phi_{\omega} \tilde{\omega} + \phi_\eta \tilde{\eta} + \sqrt{\tau_{\omega}^{-1} + \phi_\eta^2 \tau_{\eta}^{-1}} \right).$$

Importantly, now the price impact of noise trading is endogenous, i.e., $\frac{\partial \log \tilde{p}}{\partial \tilde{p}} = \frac{\sqrt{\tau_{\omega}^{-1} + \phi_\eta^2 \tau_{\eta}^{-1}}}{\phi_{\tilde{p}}}$. Where in the baseline model it was the exogenous $\frac{1}{\sqrt{\tau}}$. We follow the literature (e.g., Kyle, 1985) and use the inverse of price impact to measure market liquidity, i.e., Liquidity $\equiv \frac{\phi_{\tilde{p}}}{\sqrt{\tau_{\omega}^{-1} + \phi_\eta^2 \tau_{\eta}^{-1}}}$.

The real decision maker’s information set is $\{\tilde{a}, \tilde{s}_f, \tilde{\omega}, \tilde{\eta}, \tilde{p}\}$. To him, the price $\tilde{p}$ is still a signal $\tilde{p}$ in predicting $\tilde{f}$, as specified by Eq. (10)-(12). Also, the real decision maker does not learn directly from the public signals $\tilde{\omega}$ and $\tilde{\eta}$, given that they are noisy versions of his own information. Thus, the real decision maker’s information set is effectively $\{\tilde{a}, \tilde{s}_f, \tilde{p}\}$, and his date 1 optimal investment is:

$$K^* = \arg \max_K E \left( \beta \tilde{A} f K - \frac{c}{2} K^2 \right) \left( \tilde{a}, \tilde{s}_f, \tilde{p} \right)$$

$$= \exp \left[ \left( \frac{\log \beta}{c} + \frac{1}{2} \frac{1}{\tau_f + \tau_s + \tau_p} \right) + \tilde{a} \right]$$

$$+ \frac{\tau_s}{\tau_f + \tau_s + \tau_p} \tilde{s}_f + \frac{\tau_p}{\tau_f + \tau_s + \tau_p} \tilde{p}.$$ 

Accordingly, we can compute real efficiency as $RE = \frac{\tilde{p}}{c + \frac{c}{\tau_f}} \exp \left( \frac{\frac{1}{\tau_a} + \frac{1}{\tau_f}}{\frac{1}{\tau_f + \tau_a + \tau_p}} \right)$.

At date 0, speculators now can condition on the information in prices, and they make forecast about future cash flow as follows:

$$E(\tilde{V} | \tilde{s}_i, \tilde{y}_i, \tilde{\omega}, \tilde{p}) = b_{i0}^\prime + b_{i1}^\prime \tilde{s}_i + b_{i2}^\prime \tilde{y}_i + b_{i3}^\prime \tilde{\omega} + b_{i4}^\prime \tilde{p} + b_{i5}^\prime \tilde{p},$$

where $b_i$’s are given in the proof of Proposition 5 in Appendix B. Since speculators condition their trade on the price $\tilde{p}$, they do not need to forecast it. As a result, speculator $i$ will buy the asset if and only if

$$b_{i0}^\prime + b_{i1}^\prime \tilde{s}_i + b_{i2}^\prime \tilde{y}_i + b_{i3}^\prime \tilde{\omega} + b_{i4}^\prime \tilde{\eta} + b_{i5}^\prime \tilde{p} > \tilde{p}$$

and

$$b_{i6}^\prime + b_{i7}^\prime \tilde{s}_i + b_{i8}^\prime \tilde{y}_i + b_{i9}^\prime \tilde{\omega} + b_{i10}^\prime \tilde{\eta} - (1 - b_{i5}^\prime) \tilde{p} > 0,$$

which compares with the conjectured trading strategy, yielding the following equations that determine the equilibrium:

$$\phi_y = \frac{b_{i1}^\prime}{b_{i2}^\prime}, \phi_{\omega} = \frac{b_{i3}^\prime}{b_{i4}^\prime}, \phi_\eta = \frac{b_{i5}^\prime}{b_{i6}^\prime}, \text{ and } \phi_p = \frac{b_{i7}^\prime}{b_{i8}^\prime},$$

provided that $b_{i6}^\prime > 0$.

Proposition 5. In the economy where public information is disclosed by the real decision maker and speculators submit price-contingent demand schedules, a linear monotone equilibrium is characterized jointly by the following two conditions in terms of polynomials of $\phi_y$:

$$A_3 \phi_y^3 + A_2 \phi_y^2 + A_1 \phi_y + A_0 = 0,$$

$$B_4 \phi_y^4 + B_3 \phi_y^3 + B_2 \phi_y^2 + B_1 \phi_y + B_0 > 0,$$

where the coefficients of $A$‘s and $B$‘s are given in Appendix B.

Given the high-degree polynomials that determine the solution for the equilibrium outcome in this setting, analytical characterization of the effect of disclosure precision on trading and real efficiency is not attainable. The problem gets complicated by the fact that speculators also update based on prices in a model that features a feedback loop with real investment decisions. To gain insight into the results in this setting, we thus conduct extensive numerical analyses for different sets of parameters. In Fig. 3, we summarize the results for particular parameter values. Specifically, we have set $\tau_{s} = \tau_{f} = \tau_{b} = \tau_{s} = \tau_{f} = \tau_{b} = t = 1$, $\tau_{c} = 5$, and $\beta = \frac{1}{2}$. By setting $\tau_{s} = 5$ and $\tau_{f} = 1$, we try to capture the idea that the real decision maker, as the source of public information, can be more informed about factor $f$ than each individual speculator. Still, he can gain from the aggregation of information across many different speculators in the market. In the top three panels, we set $\tau_{s} = 1$ and check the implications of changing $\tau_{c}$. In the bottom three panels, we set $\tau_{c} = 1$ and examine the implications of changing $\tau_{c}$. As in the baseline model, we are still interested in the effect on variables $\phi_y$ and $RE$. In addition, we have also plotted the measure for market liquidity, $\text{Liquidity} \equiv \frac{\phi_{\tilde{p}}}{\sqrt{\tau_{\omega}^{-1} + \phi_\eta^2 \tau_{\eta}^{-1}}}$, to gain better understanding of how liquidity changes in disclosure when it is allowed to adjust and how this can affect the results concerning the key variables of interest $\phi_y$ and $RE$. While the results in the figure are for specific parameters, we have conducted analysis for many different sets of parameters and the results are consistent with what is shown here.
Inspecting Fig. 3, we can see the intuitive result in Panels A3 and B3 that disclosing public information about either factor improves market liquidity. This is a result of the fact that more precise public information implies that prices incorporate overall more information about the value of the asset, and so they respond less to noise trading. Still, despite the endogenous adjustment in liquidity, the other panels show that our main results about the effect of disclosure on trading and real efficiency from the baseline model still hold in this version. The only difference, of course, is that disclosure affects real efficiency only through the indirect effect of the informativeness of the market signal to the decision maker, since the decision maker does not learn directly from the disclosure. As in Proposition 2, disclosing information about \( \bar{f} \) causes speculators to trade less aggressively on private information about \( \bar{f} \), which harms the real decision maker’s learning from the price and decreases real efficiency. As in Proposition 3, disclosing information about \( \bar{d} \) makes speculators trade more aggressively on their private information about \( \bar{f} \), which improves the real decision maker’s learning from the price and promotes real efficiency. So, overall, public information can either increase or decrease real efficiency, depending on the type of information being disclosed, and this is true despite the fact that liquidity is endogenously determined in the model. We now discuss some empirical implications for the setting described here.

Stress tests and regulatory intervention: As mentioned before, disclosure has been hotly debated in the context of stress tests. We have emphasized the implications of this for the efficiency of decisions taken by banks’ counterparties, but the version of the model described here would have implications for the efficiency of decisions taken by the regulator himself who both discloses the information and potentially takes an action with regard to the bank. For instance, the regulator conducts stress tests for financial institutions and makes intervention decisions, such as whether to bail out some financial institutions, based on the stress test results as well as market information. Our results show that if the regulator discloses information about issues for which the regulator has relative information advantage over the financial market, then disclosure is desirable because the disclosed information encourages speculators to trade more on information that the regulator cares to learn, which in turn improves the regulator’s ability to learn from prices. If instead the regulator discloses information about issues that the regulator knows relatively less and wants to learn more from the financial market, then disclosure is unambiguously undesirable because the disclosed information reduces the incentives of speculators to trade on the information that the regulator cares to learn, thereby reducing the informativeness of price signals.

Disclosure by firms: A wide interest in disclosure surrounds the release of information by a firm about its future prospects. The analysis in this section has implications for mandatory and voluntary disclosure by firms. In the case of mandatory disclosure, the firm is required by regulators to disclose its information to the general public. Our analysis suggests that when the disclosure is about
issues that the firm has an informational advantage relative to the financial market, greater disclosure improves efficiency by allowing the market to do a better job of aggregating information about issues that the firm tries to learn from the market. In contrast, when the disclosure is about issues that the firm knows relatively less than market participants, greater disclosure interferes with the ability of the market to aggregate information useful for the firm and therefore is not warranted. In terms of voluntary disclosure, since the firm always discloses information that benefits the efficiency of its investment decisions, an empirical prediction is that firms tend to publicly disclose information about matters over which they have a relative higher precision and they do not want to learn more from the market about.

6. Conclusion

Public disclosure of information has been an important component of financial regulation for many years. One key question is whether the provision of more public information—via mandatory disclosure, credit ratings, stress tests, or macro statistics—improves real efficiency. In a world with other channels for learning, providing more public disclosure can crowd out other types of information. This is particularly relevant in the context of financial markets where prices are thought to provide useful information to decision makers. In this paper, we propose a framework to study these issues. Paradoxically, when disclosure is about a variable that the real decision maker wants to learn, there exists a negative indirect effect of disclosure on real efficiency through influencing the information aggregation function of financial markets. Moreover, when there is little noise trading in financial markets, the negative indirect effect can dominate the positive direct effect of providing new information so that better disclosure can harm real efficiency. Thus, although it appears attractive to disclose information concerning some variable that relevant decision makers care to learn about the most, the overall impact of such disclosure can be counter productive. On the other hand, disclosing public information about variables that real decision makers know quite well is always beneficial, since it leads the financial market to focus on other dimensions that the real decision makers want to learn. These insights can be quite useful for policy purposes by guiding policymakers in deciding which information would be more valuable to disclose publicly and when.

Appendix A. Additional materials

A.1. The expressions of the coefficients b’s in Eq. (14) and (15)

The b coefficients in $E(\tilde{y}_i, \tilde{y}_j, \tilde{o}, \tilde{\eta})$ are

$$
b_0^{\tilde{y}_i} = \frac{1}{\lambda^{\sqrt{\tau_x^{-1} + \phi_y^2} \tau_y^{-1}}} + \frac{1}{2\lambda^{2} \tau_x^{-1}} + \frac{1}{2\lambda^{2} \tau_y^{-1}} \right),
$$

$$
b_0^{\tilde{y}_j} = \frac{1}{\lambda^{\sqrt{\tau_x^{-1} + \phi_y^2} \tau_y^{-1}}} + \frac{1}{2\lambda^{2} \tau_x^{-1}} + \frac{1}{2\lambda^{2} \tau_y^{-1}} \right),
$$

$$
b_0^{\tilde{y}_j} = \frac{1}{\lambda^{\sqrt{\tau_x^{-1} + \phi_y^2} \tau_y^{-1}}} + \frac{1}{2\lambda^{2} \tau_x^{-1}} + \frac{1}{2\lambda^{2} \tau_y^{-1}} \right),
$$

$$
and \ b_0^{\tilde{y}_i} = \frac{1}{\lambda^{\sqrt{\tau_x^{-1} + \phi_y^2} \tau_y^{-1}}} + \frac{1}{2\lambda^{2} \tau_x^{-1}} + \frac{1}{2\lambda^{2} \tau_y^{-1}} \right),
$$

The $b$ coefficients in $E(\tilde{y}_i, \tilde{y}_j, \tilde{o}, \tilde{\eta})$ are

$$
\beta \left( \begin{array}{c}
\frac{\beta (1 - \beta)}{c} + \frac{\tau_f + \tau_y}{2(\tau_f + \tau_y)^2} \\
+ \frac{2}{\tau_o + \tau_x + \tau_o} \\
+ \frac{1}{2} \left( 1 + \frac{\tau_p}{\tau_f + \tau_o + \tau_p} \right)^2 \frac{1}{\tau_f + \tau_y + \tau_o}
\end{array} \right),
$$

$$
\frac{\beta}{c} \left( \begin{array}{c}
\frac{\beta (1 - \beta)}{c} + \frac{\tau_f + \tau_y}{2(\tau_f + \tau_y)^2} \\
+ \frac{2}{\tau_o + \tau_x + \tau_o} \\
+ \frac{1}{2} \left( 1 + \frac{\tau_p}{\tau_f + \tau_o + \tau_p} \right)^2 \frac{1}{\tau_f + \tau_y + \tau_o}
\end{array} \right),
$$

and $b_i^{\tilde{y}_j} = \frac{\tau_y}{\tau_f + \tau_o + \tau_y} + \frac{1}{2} \left( 1 + \frac{\tau_p}{\tau_f + \tau_o + \tau_p} \right)^2 \frac{1}{\tau_f + \tau_y + \tau_o}$.

A.2. Deriving the expression of real efficiency in Eq. (21)

By the law of iterated expectations, we have

$$
RE = E \left( \tilde{A} \tilde{K}^* - \frac{c}{2} K^* \right) 
= E \left[ E \left( \tilde{A} \tilde{K}^* - \frac{c}{2} K^* | \tilde{Y}_i, \tilde{Y}_j, \tilde{O} \right) \left| \tilde{Y}_i, \tilde{Y}_j, \tilde{O} \right. \right] 
= E \left[ \tilde{A} \tilde{K}^* E(\tilde{Y}_i, \tilde{Y}_j, \tilde{O}) \right] - \frac{c}{2} K^*.
$$

Replacing $K^*$ with Eq. (1) in the above equation, we can compute

$$
E \left[ \tilde{A} \tilde{K}^* E(\tilde{Y}_i, \tilde{Y}_j, \tilde{O}) \right] = \frac{\beta}{c} \left( 1 - \frac{\beta}{2} \right) E \left[ \tilde{A} \tilde{E}(\tilde{Y}_i, \tilde{Y}_j, \tilde{O}) \right] 
= \frac{\beta}{c} \left( 1 - \frac{\beta}{2} \right) E \left[ (\tilde{A} \tilde{E}(\tilde{Y}_i, \tilde{Y}_j, \tilde{O}) \right] 
= \frac{\beta}{c} \left( 1 - \frac{\beta}{2} \right) e^{2 \text{Var}(\tilde{Y}_i, \tilde{Y}_j)} E \left[ e^{2\text{Var}(\tilde{Y}_i, \tilde{Y}_j)} \right].
$$

Direct computation shows

$$
E \left[ e^{2\text{Var}(\tilde{Y}_i, \tilde{Y}_j)} \right] = e^{2 \text{Var}(\tilde{Y}_i, \tilde{Y}_j)} E \left[ e^{\text{Var}(\tilde{Y}_i, \tilde{Y}_j)} \right] = e^{2 \text{Var}(\tilde{Y}_i, \tilde{Y}_j)}.\]

Inserting the above expression into Eq. (A1), we obtain Eq. (21).
Appendix B. Proof of propositions

Proof of Proposition 1. Part (a). Inserting the expressions of $b_p^e, b_p^h, b_p^b$, and $\tau_p$ into Eq. (16) yields Eq. (19). Inserting the expressions of $b_t^e$ and $b_t^h$ into condition $b_t^e - b_t^h > 0$, we obtain condition (20).

Part (b). When $\lambda > \sqrt{\pi}$, the right-hand side (RHS) of condition (20) is negative so that it is always satisfied. Thus, the existence of a linear monotone equilibrium boils down to the existence of a solution to Eq. (19). Define the RHS of Eq. (19) as $B(\phi_y)$.

When $\lambda > \sqrt{\pi}$, $B(0) = \frac{\lambda^2}{\lambda^2 - \pi} > 0$. In addition, $\lim_{\phi_y \to \infty} B(\phi_y) = \frac{1}{\lambda^2 - \pi} < 0$. So, by the intermediate value theorem, there exists $\phi_y > 0$ satisfying Eq. (19).

We next prove the uniqueness for a sufficiently large $\lambda$. If we can prove that at the equilibrium level of $\phi_y$, the RHS in Eq. (19) always crosses the 45 degree line from above, then the equilibrium is unique. That is, we need to show $\frac{\partial B(\phi_y)}{\partial \phi_y} < 1$ for those values of $\phi_y$ satisfying Eq. (19). Suppose $\lambda \to \infty$. The RHS of Eq. (19) degenerates to

$$B^{\lambda \to \infty}(\phi_y) = \left( 1 + \frac{\tau_p}{\sqrt{\tau_p + \tau_y + \tau_0}} \right)^\frac{\tau_p}{\sqrt{\tau_p + \tau_y + \tau_0}}. \tag{B1}$$

Direct computation shows

$$\frac{\partial B^{\lambda \to \infty}(\phi_y)}{\partial \phi_y} = \frac{2\phi_y \tau_y \tau_x}{(\tau_y + \phi_y^2 \tau_x)^2}. \tag{B2}$$

By the expression of $\tau_p$ in Eq. (12), we can compute

$$\frac{\partial \tau_p}{\partial \phi_y} = \frac{2\phi_y \tau_y \tau_x}{(\tau_y + \phi_y^2 \tau_x)^2}. \tag{B3}$$

which is plugged in Eq. (B2), yielding

$$\frac{\partial B^{\lambda \to \infty}(\phi_y)}{\partial \phi_y} = \frac{2\phi_y \tau_y \tau_x}{(\tau_y + \phi_y^2 \tau_x)^2} \left( \frac{\tau_f + \tau_y + \tau_p}{\tau_f + \tau_y + \tau_p} \right)^2 \left( \frac{\tau_f + \tau_y + \tau_p}{\tau_f + \tau_y + \tau_p} \right)^2 \left( \frac{\tau_y + \phi_y^2 \tau_x}{\tau_y + \phi_y^2 \tau_x} \right)^2. \tag{B4}$$

By Eq. (B1), we have

$$\frac{\tau_y + \phi_y^2 \tau_x}{\tau_y + \phi_y^2 \tau_x + \tau_f + \tau_y + \tau_p} = 1 + \frac{\phi_y}{\tau_y + \tau_f + \tau_y + \tau_p} \frac{\tau_f + \tau_y + \tau_p}{\tau_f + \tau_y + \tau_p}$$

which is plugged into Eq. (B4), yielding

$$\frac{\partial B^{\lambda \to \infty}(\phi_y)}{\partial \phi_y} = \frac{1}{1 + \frac{\phi_y}{\tau_y + \tau_f + \tau_y + \tau_p}} \left( \frac{\tau_f + \tau_y + \tau_p}{\tau_f + \tau_y + \tau_p} \right)^2 \left( \frac{\tau_y + \phi_y^2 \tau_x}{\tau_y + \phi_y^2 \tau_x} \right)^2. \tag{B5}$$

By the expression of $\tau_p$ in Eq. (12), we have

$$\phi_y^2 \tau_x = \frac{\tau_p \tau_y}{\tau_y + \tau_x - \tau_p}. \tag{B6}$$

which is plugged into Eq. (B5),

$$\frac{\partial B^{\lambda \to \infty}(\phi_y)}{\partial \phi_y} = \frac{2\tau_y}{\tau_y + \tau_f + \tau_y + \tau_p} \frac{\tau_f + \tau_y + \tau_p}{\tau_f + \tau_y + \tau_p} \frac{\tau_y + \phi_y^2 \tau_x}{\tau_y + \phi_y^2 \tau_x} < 1,$$

since

$$\frac{2\tau_y}{\tau_y + \tau_f + \tau_y + \tau_p} < 1. \quad \frac{\tau_f + \tau_y + \tau_p}{\tau_f + \tau_y + \tau_p} < 1. \quad 0 < \tau_y \tau_x - \tau_p < \tau_y \tau_x. \quad \square$$

Proof of Proposition 2. Since parts (a) and (b) have been proved in the text, we only need to examine the real-efficiency implications in parts (c) and (d).

Part (c). By Eq. (26) and Eq. (B6), we can compute the indirect effect of disclosure as

$$\frac{\partial \tau_p}{\partial \eta} = -2\tau_p \tau_y \tau_x - \tau_p \left( \frac{\tau_f + \tau_y + \tau_p}{\tau_f + \tau_y + \tau_p} \right)^2 \left( \frac{\tau_f + \tau_y + \tau_p}{\tau_f + \tau_y + \tau_p} \right)^2 \left( \frac{\tau_y + \phi_y^2 \tau_x}{\tau_y + \phi_y^2 \tau_x} \right)^2. \tag{B6}$$

Clearly, as $\eta \to \infty$, we have $\frac{\partial \tau_p}{\partial \eta} \to 0$ because $\tau_p$ is bounded above by $\tau_y \tau_x$ by Eq. (12). So disclosure about factor $\bar{f}$ only has the positive direct effect for a high enough level of $\tau_p$.

Part (d). To show part (d), we examine the behavior of the indirect effect $\frac{\partial \tau_p}{\partial \eta}$ at $\eta = 0$. Consider the process of $\tau_x \to \infty$ or $\tau_x \to 0$. If $\lim_{\eta \to 0} \frac{f_1(\eta)}{f_2(\eta)} = 0$, then we denote $f_1 = o(f_2)$, meaning that $f_1$ converges at a faster rate than $f_2$. If $\lim_{\eta \to 0} \frac{f_2(\eta)}{f_2(\eta)}$ is bounded (but different from 0), then we denote $f_1 = O(f_2)$, meaning that $f_1$ and $f_2$ converge at the same rate. By Eqs. (25) and (12), we have $\phi_y = O(1)$ and $\tau_p = O(\tau_x)$. By Eq. (26) and the orders of $\phi_y$ and $\tau_p$, we have

$$\frac{\partial \phi_y}{\partial \tau_p} = - \frac{\phi_y}{\tau_f + \tau_y} + o(1).$$

So, by Eq. (24), we have

$$\frac{\partial \tau_p}{\partial \eta} \bigg|_{\eta = 0} = -\frac{2\phi_y^2 \tau_x \tau_y^2}{(\tau_y + \phi_y^2 \tau_x)^2 (\tau_f + \tau_y)} \tau_x + o(\tau_x). \tag{B7}$$

Thus, by Eq. (23), we have

$$\frac{\partial RE}{\partial \eta} \bigg|_{\eta = 0} \propto 1 + \frac{\partial \tau_p}{\partial \eta} \bigg|_{\eta = 0} = 0. \tag{B7}$$

As a result, $\frac{\partial RE}{\partial \eta} \bigg|_{\eta = 0} < 0$ for sufficiently large $\tau_x$, and $\frac{\partial RE}{\partial \eta} \bigg|_{\eta = 0} > 0$ for sufficiently small $\tau_x$. \quad \square$

Proof of Proposition 5. Speculator’s information set is $\{\xi, \tilde{y}, \tilde{\omega}, \tilde{\eta}, \tilde{p}\}$. To speculators, the price is equivalent to the following signal:

$$\tilde{p} = \phi_p \tilde{p} + g - \phi_p \tilde{\omega} = \tilde{b} + \phi_y \tilde{f} + \sqrt{\tau_x^{-1} + \phi_y^2 \tau_x^{-1}} \tilde{\xi}. \tag{B8}$$
We define
\[ \theta_0 = 2 - \frac{\tau_p}{\tau_f + \tau_s + \tau_p \phi_y}, \quad \theta_f = 1 + \frac{\tau_s}{\tau_f + \tau_s + \tau_p}, \quad \text{and} \]
\[ \theta_s = \frac{\tau_s}{\tau_f + \tau_s + \tau_p}. \]

Let \( \Delta \equiv \Var(\theta_0 \delta \theta_f \delta \theta_s \xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6) \) and let \( \delta_x, \delta_y, \delta_o, \delta_p, \delta_\tau \) be the loadings of \( \xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6 \) in the expectations \( E(\theta_0 \delta + \theta_f \delta + \theta_s \delta) \), respectively.

Then, we can compute
\[ b'_0 = \delta_0 - \delta_\tau + \delta_o \delta_p + \frac{\tau_p}{\tau_f + \tau_s + \tau_p \phi_y}, \]
\[ b'_1 = -\delta_\tau + \delta_o \delta_p + \frac{\tau_p}{\tau_f + \tau_s + \tau_p \phi_y}, \]
\[ b'_2 = \delta_0 + \delta_o \delta_p + \frac{\tau_p}{\tau_f + \tau_s + \tau_p \phi_y}, \]
\[ b'_3 = -\delta_\tau + \delta_o \delta_p + \frac{\tau_p}{\tau_f + \tau_s + \tau_p \phi_y}. \]

A linear monotone equilibrium is characterized jointly by two conditions: \( \phi_y = \frac{\tau'_0}{\tau'_f} \) and \( b'_0 > 0 \). Inserting the expressions of \( b' \)s into these two conditions and simplifying, we obtain the two polynomial conditions in Proposition 5, where the coefficients of \( A \)'s and \( B \)'s in the polynomials are given as follows:
\[ A_3 = -2 \tau'_0 \left( \tau_f + \tau_s + \tau_p + \tau_f \xi \right), \]
\[ A_2 = \tau_s \tau_f \left( \tau_f + \tau_s + \tau_p + \tau_f \xi \right), \]
\[ A_1 = -2 \tau_s \tau_f \left( \tau_f + \tau_s + \tau_p + \tau_f \xi \right), \]
\[ A_0 = \tau'_0 \left( \tau_f + \tau_s + \tau_p + \tau_f \xi \right), \]
\[ B_4 = 2 \tau'_0 \left( \tau_f + \tau_s + \tau_p + \tau_f \xi \right), \]
\[ B_3 = -\tau'_0 \tau_f \xi, \]
\[ B_2 = \tau'_0 \left( 2 \tau_f + 2 \tau_s + \tau_p + \tau_f \xi \right), \]
\[ B_1 = -\tau'_0 \tau_f \xi \left( 2 \tau_f + 2 \tau_s + \tau_f \xi \right), \]
\[ B_0 = 2 \tau'_0 \left( \tau_f + \tau_s + \tau_f \xi \right). \]

References


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