Government Guarantees and Financial Stability

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Abstract

Government guarantees to financial institutions are intended to reduce the likelihood of runs and bank failures, but are also usually associated with distortions in banks’ risk-taking decisions. We build a model to analyze these trade-offs based on the global-games literature and its application to bank runs. We derive several results, some of which against common wisdom. First, guarantees reduce the probability of a run, taking as given the amount of bank risk taking, but lead banks to take more risk, which in turn might lead to an increase in the probability of a run. Second, guarantees against fundamental-based failures and panic-based runs may lead to more efficiency than guarantees against panic-based runs alone. Finally, there are cases where following the introduction of guarantees banks take less risk than would be optimal.

Keywords: panic runs, fundamental runs, government guarantees, bank moral hazard

JEL classifications: G21, G28

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1 Introduction

Government guarantees to financial institutions are common all over the world. They come in different forms, such as deposit insurance provided to depositors who put their money in commercial banks, or implicit guarantees for a bailout provided ex post upon the bank’s failure. The recent financial crisis has led to renewed interest and debate about the role of government guarantees and their desirability. On the one hand, government guarantees are thought to have a positive role in preventing panic among investors, and hence they help to stabilize the financial system. They also help in mitigating the negative consequences once a panic has happened. On the other hand, they might create adverse incentives for banks to engage in excessive risk taking. This might even lead to an overall increase in financial fragility.

In this paper, we present a model to analyze the trade-off involved in the choice of the level of government guarantees. A model that captures the various effects of guarantees is notoriously rich and hard to solve. Such a model needs to endogenize the probability of a run on banks and how it is affected by banks’ risk choices and government guarantees. It also needs to endogenize banks’ risk choices and how they are affected by government guarantees, taking into account investors’ expected run behavior. We make technical progress in this paper by putting all these ingredients together in a tractable framework and derive some surprising results on the various effects of government guarantees that shed light on the optimal level and design of government guarantees.

Our starting point is the seminal Diamond and Dybvig (1983) economy, which has served researchers for years in studying runs and financial fragility. In this framework, banks offer deposit contracts to investors, who might face early liquidity needs, and by that provide risk sharing among them. While banks may improve investors’ welfare due to the risk sharing they provide, the deposit contracts also expose banks to the risk of a bank run, where many depositors panic and withdraw early out of the self-fulfilling belief that other depositors will do so and the bank will fail. In the original framework, Diamond and Dybvig (1983) propose a deposit insurance scheme that eliminates runs altogether and restores full efficiency. In their model, banking crises happen only due to a coordination failure. Then, by ensuring that depositors will receive the promised payment independently of the other depositors’ withdrawal decisions, deposit insurance prevents the bank-run equilibrium without entailing any disbursement for the government and so the first best allocation is achieved.

The situation in the real world, however, is more complex. Runs occur not only because of panics as in
Diamond and Dybvig (1983), but also because of a deterioration in the fundamentals of the banks’ assets (see evidence in Gorton (1988), Calomiris and Gorton (1991) and Calomiris and Mason (2003)). In this case, even with deposit insurance in place, bank runs might still occur and the design of the guarantee becomes of paramount importance. The government needs to decide whether deposit insurance covers depositors just in cases of illiquidity due to coordination failures or also in cases of insolvency where the banks’ assets fail to produce the required cash. This will have implications for the amount of disbursement that the guarantee scheme entails and for the choices made by the banks concerning the level of risk they end up taking. As mentioned above, analyzing the desirability of government guarantees and their design, in the light of these considerations, requires a model where the probability of a crisis and the deposit contract offered by the bank are determined endogenously and affected by the extent and form of government intervention.

To conduct this analysis, we build on the model developed in Goldstein and Pauzner (2005), in which depositors’ withdrawal decisions are uniquely determined using the global-game methodology, and so the probability of a run and how it is affected by the banking contract and by government policy can be determined. Importantly, Goldstein and Pauzner (2005) study the interaction between the demand deposit contract and the probability of a run. In their model the run probability depends on the banking contract, and the bank decides on the banking contract taking into account its effect on the probability of a run. We add a government to this model to study how the government’s guarantees policy interacts with the banking contract and the probability of a run.

In our model, there are two periods. Banks raise funds from risk-averse consumers in the form of deposits and invest them in risky projects whose return depends on the fundamentals of the economy. Depositors derive utility from consuming both a private and a public good. At the interim date, each depositor receives an imperfect signal regarding the fundamentals and decides when to withdraw based on the information received. In deciding whether to run or not, depositors compare the payoff they would get from going to the bank prematurely and waiting until maturity. These payoffs depend on the fundamentals and the proportion of depositors running.

In this setting, the equilibrium outcome is that runs occur when the fundamentals are below a unique threshold. But, within the range where they occur, they can be classified into panic-based runs or fundamental-based runs. The former type of run is one that is generated by the self-fulfilling belief of depositors that other depositors will run. The latter type of run happens at the lower part of the run region, where the fundamentals are low enough to make running a dominant strategy for depositors. Overall, the probability
of the occurrence of a run (and of both types of runs) is uniquely determined and depends on the amount of risk chosen by the bank as represented by the deposit contract offered to depositors (i.e., the amount offered to depositors demanding early withdrawal). Banks consider this effect when they design the demand-deposit contract. As in Diamond and Dybvig (1983), there is perfect competition with free entry in the banking sector, and so banks offer a contract that maximizes depositors’ expected utility. Here, they recognize the implications that the contract has for the possibility of a run and take them into account when deciding on the contract.

As in Goldstein and Pauzner (2005), we first show that the decentralized solution, i.e., without government intervention, is inefficient. There are two sources of inefficiency. First, in equilibrium, banks choose to offer deposit contracts that lead to inefficient fundamental-based and panic-based runs. While they internalize the cost of the runs, the benefit from risk sharing is large enough to lead banks to offer contracts that entail some inefficient runs. Second, still, banks reduce the amount of liquidity they offer to depositors demanding early withdrawal since they internalize the effect of this liquidity on the probability of inefficient runs. Hence, in equilibrium, the amount of risk sharing that is offered to depositors is lower than what depositors would have liked if there was no concern of a run.

Then, we enrich the model by adding the government, which attempts to reduce the probability of runs by guaranteeing depositors to receive a minimum repayment through the transfer of resources from the public good to the banking sector. We consider two different guarantee schemes. The first one is only meant to prevent the occurrence of panic runs. Depositors are guaranteed to receive at least a minimum repayment if the bank’s project is successful irrespective of what the other depositors do. By eliminating the negative externality that a run imposes, this scheme prevents the occurrence of panic runs with a mere announcement effect. In other words, as in Diamond and Dybvig (1983), this guarantee scheme does not entail any actual disbursement in equilibrium for the government and thus does not lead to distortions in the bank’s choice of the deposit contract. However, unlike in Diamond and Dybvig (1983), fundamental runs still occur in our framework, as depositors are not protected against the risk that the assets of their bank fail. Hence, there is still inefficiency under this guarantee scheme, as fundamental runs can also be inefficient: even though it is a dominant strategy to run, a run might still be collectively inefficient. When this guarantee scheme is in place, banks increase the amount they offer to depositors in case of early withdrawal. This leads to an increase in the probability of fundamental-based runs.

Interestingly, even though panic-based runs are eliminated under this guarantee scheme, overall runs
might become more likely due to the increased likelihood of fundamental-based runs. This is consistent with evidence presented by Demirguc-Kunt and Detragiache (1998) that crises might become more likely in the presence of deposit insurance. Importantly, however, overall welfare is always higher under this insurance scheme than in the decentralized solution. The fact that banks increase the amount they offer for early withdrawals and might increase the likelihood of runs overall does not imply they are acting against depositors' interests. Banks provide contracts that maximize depositors' expected welfare under the constraints. With these government guarantees in place, the implications of increasing the short-term rate for the probability of a run are less severe, and so banks choose to increase it more, reducing the extent of the second inefficiency of the decentralized solution mentioned above.

Still, other guarantee schemes may be more efficient. The facts that depositors are not protected against the failure of the banks’ projects and that inefficient fundamental-based runs might be triggered as a result limit the efficiency increase coming from this guarantee scheme. We then consider a second guarantee scheme, in which depositors receive at least a minimum guaranteed payment, irrespective of what the others do and irrespective of the bank’s available resources. That is, they get some protection against illiquidity and insolvency of the bank. For a given short-term rate set by the banking contract, this guarantee scheme reduces the probability of both panic-based and fundamental-based crises. But, crises still occur, leading to actual disbursements, and so leading to non-trivial costs of increasing the level of guarantees. Hence, the government is limited here in how much it helps the banking system.

An important result we observe under the second guarantee scheme is that, due to the fact that disbursement actually happens in equilibrium, there is a wedge between the optimal short-term rate (that the government would like to set) and the one that banks set in their contracts. Banks internalize the effect of the rate on the probability of a run among their depositors, but they do not internalize the effect it has on the amounts that the government needs to spend and so on the level of the public good. This is because overall government spending and the amount left for the public good are determined by the decisions of all banks combined and are very slightly affected by the decision of each particular bank. This is where the intuition of moral hazard often featured in the public debate – according to which banks take too much risk and set a too high deposit rate due to the deposit insurance (see, e.g., Calomiris (1990) and Acharya and Mora (2014)) – starts to show up in our model. Interestingly, however, unlike this conventional wisdom, we show that government guarantees do not always lead to this effect, but sometimes lead to the exactly opposite effect: banks set too low deposit rates once the government provides a guarantee against panic-
and fundamental-based runs. The important detail is whether the government ends up paying to depositors more in case there is no run and the bank ends up failing for fundamental reasons or in case there is a run and the bank faces a shortage of liquidity. If the former holds, then the cost of a run from the point of view of banks is higher than from the point of view of the government and the banks set too low of a deposit rate (generating the opposite of the common wisdom). If the latter holds, then the cost of a run from the point of view of banks is lower than from the point of view of the government and the banks set too high of a deposit rate (generating the common wisdom).

Finally, another interesting result of our analysis is that a broader guarantee scheme that protects depositors against both panic runs and the risk of bank failure can be better than one that only address panic runs (i.e., bank’s illiquidity). This occurs despite the fact that the latter completely removes the occurrence of panic runs and does not entail any disbursement for the government and any distortion in the choice of the deposit contract by the bank. The reason is that the former may be able to reduce the risk of inefficient fundamental-based runs.

Overall, the analysis in our paper provides a step towards understanding the implications and determination of government guarantees. The novelty of the paper is to analyze the effects of the introduction of guarantees in the banking sector in a context in which both fundamental and panic crises are possible and both banks’ and depositors’ decisions are endogenously determined. The paper is linked to other papers that analyze the distortions entailed by deposit insurance and other forms of guarantees. Boot and Greenbaum (1993) and Cooper and Ross (2002) highlight that public guarantees eliminate runs but at the cost of reducing the incentive of depositors to monitor banks, thus increasing the occurrence of crises and the disbursement for the government. More closely related is Keister (2014), who analyzes the desirability of bailouts in a setting with limited commitment in which banks anticipate that self-fulfilling runs can occur with a certain exogenous probability. The main difference in our paper is that we are able to endogenize the probability with which both fundamental and panic runs can occur, which allows a fuller analysis of the consequences of government guarantee policy. Indeed, the results of our analysis, as described above, have not been obtained in any of these and other related papers.

The ability to endogenize the probability of a run in our model is achieved by relying on the global-games literature that goes back to Carlsson and van Damme (1993). For an early review, see Morris and Shin (2003). Our paper builds more directly on Goldstein and Pauzner (2005), by deriving a bank-run model where the banking contract is determined endogenously, and the property of global strategic complementarities fails to

The paper proceeds as follows. Section 2 describes the model without government intervention. Section 3 derives the decentralized equilibrium. Section 4 analyzes the guarantee schemes. It first characterizes a scheme against panic runs and then one protecting depositors against both panic runs and bank failure. Section 5 uses a parametric example to illustrate the properties of the model. Section 6 concludes. All proofs are contained in the appendix.

2 The basic model

The basic model is based on Goldstein and Pauzner (2005). There are three dates \( t = 0, 1, 2 \) and a continuum \([0,1]\) of banks and consumers.

Banks raise one unit of funds from consumers in exchange for a deposit contract as specified below, and invest in a risky project. For each unit invested at date 0, the project returns 1 if liquidated at date 1 and a stochastic return \( \tilde{R} \) at date 2 given by

\[
\tilde{R} = \begin{cases} 
R > 1 & \text{w. p. } p(\theta) \\
0 & \text{w. p. } 1 - p(\theta).
\end{cases}
\]

The variable \( \theta \), which represents the state of the economy, is uniformly distributed over \([0,1]\). We assume that \( p(\theta) = \theta \) and that \( E_{\theta}[p(\theta)]R > 1 \), which implies that the expected long-run return of the project is superior to the short-run return.

Each consumer is endowed with 1 unit at date 0 and nothing thereafter. At date 0, each consumer deposits his endowment at the bank in exchange for a promised payment \( c_1 \) at date 1 or a risky payoff \( \tilde{c}_2 \) at date 2. Consumers are ex ante identical but can be of two types ex post: each of them has a probability \( \lambda \) of being early and consuming at date 1, and \( 1 - \lambda \) of being late and consuming at either date. Consumers privately learn their type at date 1.

In addition to the consumption of the good received by the bank, each consumer derives utility also from the provision of a public good \( g \) and thus his preferences are given by

\[
U(c, g) = u(c) + v(g)
\]
with $u'(c) > 0$, $v'(g) > 0$, $u''(c) < 0$, $v''(g) < 0$, $u(0) = v(0) = 0$ and relative risk aversion coefficient, $-cu''(c)/u'(c)$, greater than one for any $c \geq 1$.

The state of the economy $\theta$ is realized at the beginning of date 1, but is not publicly revealed until date 2. After $\theta$ is realized at date 1, each consumer receives a private signal $x_i$ of the form

$$x_i = \theta + \varepsilon_i,$$

where $\varepsilon_i$ are small error terms that are independently and uniformly distributed over $[-\varepsilon, +\varepsilon]$. After the signal is realized, consumers decide whether to withdraw at date 1 or wait until date 2.

The bank satisfies consumers’ withdrawal demands by liquidating the long term asset. If the liquidation proceeds are not enough to repay the promised $c_1$ to the depositors withdrawing at date 1, each of them receives a pro-rata share of the liquidation proceeds.

The banking sector is perfectly competitive. Banks make no profits and choose the deposit contract $(c_1, c_2)$ at date 0 that maximizes depositors’ expected utility. As a consequence, the payment $\tilde{c}_2$ equals the return of the nonliquidated units of the bank’s project divided by the number of remaining late depositors.

The timing of the model is as follows. At date 0, each bank chooses the promised payment $c_1$. At date 1, after receiving the private signal about the state of the fundamentals $\theta$, depositors decide whether to withdraw early or wait until date 2. At date 2, the bank’s project realizes and waiting late depositors receive a pro-rata share of the realized returns of the project units remaining at the bank.

3 The decentralized equilibrium

We start by analyzing depositors’ withdrawal decisions at date 1 for a given deposit contract promising a fixed payment $c_1$ to the depositors withdrawing at date 1. As in Diamond and Dybvig (1983), $c_1$ must be lower than the amount $\frac{1-\lambda c_1}{\lambda} R$ that each late depositor receives at date 2 when only the $\lambda$ early depositors withdraw at date 1 and the bank’s project returns $R$. Otherwise late depositors would always have an incentive to run, independently of the signal they receive at date 1. Moreover, $c_1$ must be at least 1 but less than $\min\{1/\lambda, R\}$ for the reasons explained below.

Early depositors always withdraw at date 1 to satisfy their consumption needs. Late depositors compare the expected payoffs from going to the bank at date 1 or 2 and withdraw at the date when they expect to obtain the highest utility. Late depositors’ expected payoffs depend on the realization of the fundamentals $\theta$ as well as on the proportion $n$ of depositors withdrawing early. Since the signal $x_i$ provides information on
the expected date 2 repayment and the actions of the other depositors, each late depositor bases his decision on the signal he receives. When the signal is high, a late depositor attributes a high posterior probability to the event that the bank’s project yields the positive return $R$ at date 2. Also, upon observing a high signal, he infers that the others have also received a high signal. This lowers his belief about the likelihood of a run and thus his own incentive to withdraw at date 1. Conversely, when the signal is low, a late depositor has a high incentive to withdraw early as he attributes a high likelihood to the possibility that the project’s date 2 return will be zero and that the other depositors run.

We assume that there are two regions of extremely bad or extremely good fundamentals, where each late depositor’s action is based on the realization of the fundamentals irrespective of his beliefs about the others’ behavior. As shown in Goldstein and Pauzner (2005), the existence of these two extreme regions, no matter how small they are, guarantees the uniqueness of the equilibrium in depositors’ withdrawal decisions. We start with the lower region.

**Lower Dominance Region.** When the fundamentals are very bad ($\theta$ is very low), the expected utility from waiting until date 2, $\theta u \left( \frac{1-\lambda c_1}{1-\lambda} R \right)$, is lower than that from withdrawing at date 1, $u(c_1)$, even if only the early depositors were to withdraw ($n = \lambda$). If, given his signal, a late depositor is sure that this is the case, running is a dominant strategy. We then denote by $\theta(c_1)$ the value of $\theta$ that solves

$$u(c_1) = \theta u \left( \frac{1-\lambda c_1}{1-\lambda} R \right),$$

that is

$$\theta(c_1) = \frac{u(c_1)}{u \left( \frac{1-\lambda c_1}{1-\lambda} R \right)}$$

We refer to the interval $[0, \theta(c_1))$ as the lower dominance region, where runs are only driven by bad fundamentals. The threshold $\theta(c_1)$ is increasing in $c_1$ as we have

$$\frac{\partial \theta(c_1)}{\partial c_1} = \frac{u'(c_1) + \theta(c_1) \left( \frac{\lambda R}{1-\lambda c_1} \right) u' \left( \frac{1-\lambda c_1}{1-\lambda} R \right)}{u \left( \frac{1-\lambda c_1}{1-\lambda} R \right)} > 0$$

since $u'(c) > 0$. Thus, as $c_1$ increases, the lower dominance region becomes bigger and fundamental runs are more likely to occur.

For the lower dominance region to exist for any $c_1 \geq 1$, there must be feasible values of $\theta$ for which all late depositors receive signals that assure them to be in this region. Since the noise contained in the signal $x_i$ is at most $\varepsilon$, each late depositor withdraws at date 1 if he observes $x_i < \theta(c_1) - \varepsilon$. It follows that all
depositors receive a signal that assures them that \( \theta \) is in the lower dominance region when \( \theta < \theta(c_1) - 2\varepsilon \). Given that \( \theta \) is increasing in \( c_1 \), this condition is satisfied for any \( c_1 \geq 1 \) if \( \theta(1) > 2\varepsilon \).

**Upper Dominance Region.** The upper dominance region of \( \theta \) corresponds to the range \([\overline{\theta}, 1]\) in which fundamentals are so good that no late depositors withdraw at date 1. Hence, like in Goldstein and Pauzner (2005), we need to modify both the function \( p(\theta) \) and the investment technology available to the bank by assuming that in the range \([\overline{\theta}, 1]\) the project is safe, i.e., \( p(\theta) = 1 \), and yields the same return \( R > 1 \) at dates 1 and 2. Given \( c_1 < \min\{1/\lambda, R\} \), this ensures that the bank does not need to liquidate more units than the number \( n \) of depositors withdrawing at date 1. Then, when a late depositor observes a signal such that he believes that the fundamentals \( \theta \) are in the upper dominance region, he is certain to receive his payment \( \frac{1-c_1}{1-\theta} R \) at date 2, irrespective of his beliefs on other depositors’ action, and thus he has no incentives to run.

Similarly to before, the upper dominance region exists if there are feasible values of \( \theta \) for which all late depositors receive signals that assure them to be in this range. This is the case if \( \overline{\theta} < 1 - 2\varepsilon \).

**The Intermediate Region**

The two dominance regions are just extreme ranges of fundamentals in which late depositors have a dominant strategy that depends only on the fundamentals \( \theta \). When the signal indicates that \( \theta \) is in the intermediate range \([\theta(c_1), \overline{\theta}]\), a depositor’s decision to withdraw early depends on the realization of \( \theta \) as well as on his beliefs regarding other late depositors’ actions.

To determine late depositors’ withdrawal decisions in this region, we calculate their utility differential between withdrawing at date 2 and at date 1 as given by

\[
v(\theta, n) = \begin{cases} 
\theta u \left( \frac{1-c_1}{1-n} R \right) - u(c_1) & \text{if } \lambda \leq n \leq \hat{n} \\
0 - u \left( \frac{n}{\hat{n}} \right) & \text{if } \hat{n} \leq n \leq 1,
\end{cases}
\]

where \( n \) represents the proportion of depositors withdrawing at date 1 and

\[
\hat{n} = 1/c_1
\]

is the value of \( n \) at which the bank exhausts its resources if it pays \( c_1 \geq 1 \) to all withdrawing depositors. The expression for \( v(\theta, n) \) states that as long as the bank does not exhaust its resources at date 1, i.e., for \( n \leq \hat{n} \), late depositors waiting until date 2 obtain the residual \( \frac{1-c_1}{1-n} R \) with probability \( \theta \) while those withdrawing early obtain \( c_1 \). By contrast, for \( n \geq \hat{n} \) the bank liquidates its entire project at date 1. Each late depositor

\footnote{Except for the range \([\overline{\theta}, 1]\), the probability \( p(\theta) \) is still increasing in \( \theta \) and given by \( p(\theta) = \theta \).}
receives nothing if he waits until date 2 and the pro-rata share 1/n when withdrawing early.2

As Figure 1 illustrates, the function \( v(\theta, n) \) decreases in \( n \) for \( n \leq \hat{n} \) and increases with it afterwards. This implies that a late depositor’s incentive to withdraw early is highest when \( n = \hat{n} \) rather than when \( n = 1 \), as it is usually the case in standard global games where the equilibrium builds on the property of global strategic complementarity (e.g., Morris and Shin, 1998). However, since \( v(\theta, n) \) decreases in \( n \) whenever it is positive and crosses zero only once for \( n \leq \hat{n} \) remaining always below afterwards, the model exhibits the property of one-sided strategic complementarity as in Goldstein and Pauzner (2005). This implies that there still exists a unique equilibrium in which a late depositor runs if his signal is below the threshold \( \hat{\theta}^*(c_1) \). At this signal value, a late depositor is indifferent between withdrawing at date 1 and waiting at date 2. Formally, \( \hat{\theta}^*(c_1) \) is such that the expected utility

\[
\int_{\bar{\theta}}^{\hat{\theta}} u(c_1) dn + \int_{\hat{\theta}}^{1} u(\frac{1}{n}) dn \quad \text{from withdrawing at date 1 equates the expected utility}
\]

\[
\int_{\bar{\theta}}^{\hat{\theta}} \theta u \left( \frac{1-\lambda c_1}{1-n_R R} \right) dn + \int_{\hat{\theta}}^{1} u(0) dn \quad \text{from waiting until date 2.}
\]

**Proposition 1.** The model has a unique equilibrium in which late depositors run if they observe a signal below the threshold \( \hat{\theta}^*(c_1) \) and do not run above. At the limit, as \( \varepsilon \to 0 \), \( \hat{\theta}^*(c_1) \) simplifies to

\[
\hat{\theta}^*(c_1) = \frac{u(c_1) \left[ 1 - \lambda c_1 \right] + \lambda c_1 \int_{n=\bar{n}}^{1} u(\frac{1}{n})}{c_1 \int_{n=\bar{n}}^{\hat{\theta}} u \left( \frac{1-\lambda c_1}{1-n_R R} \right)}.
\]  

(7)

The threshold \( \hat{\theta}^*(c_1) \) is increasing in \( c_1 \), i.e., \( \frac{\partial \hat{\theta}^*(c_1)}{\partial c_1} > 0 \).

The proposition states that a late depositor’s action depends uniquely on the signal he receives as it provides information on other depositors’ actions. In the range \([\underline{\theta}(c_1), \bar{\theta}]\) late depositors do not have a dominant strategy and, due to strategic complementarity, each of them withdraws at date 1 in the interval \([\underline{\theta}(c_1), \hat{\theta}^*(c_1)]\) because he expects the others to do the same. Thus, in the intermediate region runs are panic-driven. These occur only if \( c_1 > 1 \) because this implies that the bank has to liquidate more than one unit for each depositor withdrawing at date 1 and, thus that the date 2 payoff of the late depositors is decreasing in the number \( n \) of withdrawing depositors. If \( c_1 \leq 1 \) there would be no coordination problem among depositors and runs would only be driven by fundamentals.

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2 Di

erently from Goldstein and Pauzner (2005), we assume that depositors receive a pro-rata share of the liquidation proceeds when the bank liquidates its entire project instead of being repaid according to a sequential service constraint. This assumption allows to maintain the properties of the function \( v(\theta, n) \) but it simplifies the analysis concerning the effect of \( c_1 \) on the probability of runs.
The threshold $\theta^*(c_1)$ increases with the promised payment $c_1$. The higher $c_1$ the lower is the payoff $\tilde{c}_2$ and thus the stronger is the incentive for each late depositors to withdraw early. This implies that the bank’s choice of the optimal deposit contract has a direct impact on the probability of occurrence of runs at date 1.

Now that we have characterized depositors’ withdrawal decisions at date 1, we turn to date 0 and compute the optimal deposit contract $c_1$. We focus on the limit case where $\varepsilon \to 0$ and only complete runs occur since all late depositors receive the same signal and take the same action.

Each bank chooses $c_1$ at date 0 to maximize the expected utility of a representative depositor, which is given by

$$\int_0^{\theta^*(c_1)} u(1) d\theta + \int_{\theta^*(c_1)}^1 \left[ \lambda u(c_1) + (1 - \lambda)\theta u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] d\theta + \int_0^1 v(g) d\theta.$$ (8)

The first term represents depositors’ expected utility for $\theta < \theta^*(c_1)$ when, given that a run occurs, the bank liquidates its entire project and each depositor obtains 1 instead of the promised payment $c_1$. The second term is depositors’ expected utility for $\theta \geq \theta^*(c_1)$ when the bank continues until date 2. The $\lambda$ early consumers withdraw early and obtain $c_1$, while the $(1 - \lambda)$ late depositors wait and receive the payment $\frac{1 - \lambda c_1}{1 - \lambda} R$ with probability $\theta$. The last term is the utility that depositors receive from the consumption of the public good $g$.

Before deriving the bank’s choice, we first note that the optimal $c_1$ must be an interior solution so that runs occur when $\theta < \theta^*(c_1) < \tilde{\theta}$. Also, it must be lower than $\min \{1/\lambda, R\}$. Otherwise runs would always occur and depositors’ expected utility would be lower than in the case $c_1 = 1$. We have the following result.

**Proposition 2** The optimal deposit contract $c_1^D > 1$ in the decentralized solution solves

$$\lambda \int_{\theta^*(c_1)}^1 \left[ u'(c_1) - \theta Ru' \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] d\theta + \frac{\partial \theta^*(c_1)}{\partial c_1} \left[ \lambda u(c_1) + (1 - \lambda)\theta^*(c_1)u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(1) \right] = 0$$ (9)

In choosing the promised payment to early depositors the bank trades off the marginal benefit of a higher $c_1$ with its marginal cost. The former, as represented by the first term in (9), is the better risk sharing obtained from the transfer of consumption from late to early depositors. The latter, which is captured by the second term in (9), is the loss in expected utility $\left[ \lambda u(c_1) + (1 - \lambda)\theta^*(c_1)u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(1) \right]$ due to the increased probability of panic runs as measured by $\frac{\partial \theta^*(c_1)}{\partial c_1}$.

At the optimum, the bank chooses $c_1^D > 1$ even if this entails panic runs. The reason is that when $c_1 = 1$, the difference between early and late depositors’ expected payment is maximal. A slight increase
of \( c_1 \) provides a large benefit in terms of risk sharing given that depositors have a relative risk aversion coefficient greater than 1. Furthermore, at \( c_1 = 1 \) the loss in terms of expected utility in the case of a run is zero. Thus, increasing \( c_1 \) slightly above 1 entails a small cost and is always optimal.

In what follows, we refer to the bank’s choice of \( c_1 \) as the risk taken by the bank. The level of \( c_1 \) influences the bank’s exposure to liquidity risk as it determines the threshold \( \theta^*(c_1) \) and thus the probability of a run occurring in the range \( \theta < \theta^*(c_1) \). Moreover, the choice of \( c_1 \) also indirectly affects the exposure of the bank to insolvency risk, as measured by \( E[1 - \theta \mid \theta \geq \theta^*(c_1)] \), since it influences the probability \( \theta \geq \theta^*(c_1) \) with which the bank survives until date 2. In particular, if \( c_1 \) increases, \( \theta^*(c_1) \) increases and the insolvency risk becomes smaller.

The decentralized solution entails two inefficiencies. First, the anticipation of the probability of runs induces the bank to choose a smaller value of \( c_1^D \) than the one that it would choose if runs were not anticipated. In the latter case, the bank would in fact choose \( c_1 \) to maximize

\[
\int_0^1 \left[ \lambda u(c_1) + (1 - \lambda) \theta u \left( \frac{1 - c_1}{1 - \theta} R \right) \right] d\theta + \int_0^1 v(g) d\theta.
\]

The solution to this problem, which we denote \( c_1^{NR} \), solves the following:

\[
u'(c_1) = \int_0^1 \theta R u' \left( \frac{1 - c_1}{1 - \lambda} R \right) d\theta. \tag{10}\]

The solution \( c_1^{NR} \) maximizes the gains from risk sharing and corresponds to the first best in Goldstein and Pauzner (2005). It also coincides with the promised repayment to depositors in Diamond and Dybvig (1983), with the only difference that the bank’s project is now risky and thus the term of the right hand side includes the probability \( \theta \) that the project succeeds. Comparing (9) and (10), it can be seen that \( c_1^{NR} > c_1^D \) for two reasons. As the first term in (9) shows, the occurrence of runs reduces the states in which the project succeeds at date 2 and early and late depositors can obtain the promised repayments \( c_1 \) and \( 1 - \frac{c_1}{1 - \lambda} R \). Moreover, as the second term in (9) shows, the repayment \( c_1 \) affects the probability with which a run occurs and depositors obtain 1 instead of the promised repayments. Both of these two effects reduce the optimal \( c_1 \) in the decentralized solution relative to the case in (10) where runs are not anticipated and \( c_1 \) maximizes risk sharing only.

Second, runs may lead to an inefficient liquidation of the bank project. Runs occur when the fundamental \( \theta \) falls below the threshold \( \theta^*(c_1) \). They are due to bad fundamentals in the range \( [0, \theta^*(c_1)] \) and to panics in

\[
\text{When } c_1 = 1, \text{ the term } \left[ \lambda u(c_1) + (1 - \lambda) \theta^*(c_1) u \left( \frac{1 - c_1}{1 - \theta} R \right) - u(1) \right] \text{ simplifies to } (1 - \lambda) [\theta^*(1) u(R) - u(1)] = 0 \text{ with } \theta^*(1) = \left[ \frac{u(1)}{u'(R)} \right].
\]
the range \([\theta(c_1), \theta^*(c_1)]\). The bank liquidates the project at date 1 to pay the withdrawing depositors. For each unit of project that the bank liquidates, the return \(R\) is foregone with probability \(\theta\). Liquidating the project is then inefficient for any \(\theta > \theta(1)\) since the utility \(u(1)\) that a depositor obtains from the liquidated unit is lower than the expected utility \(\theta u(R)\) he would obtain from the same unit if invested until date 2. Since \(c_1^D > 1\), it follows that \(\theta(1) < \theta(c_1) < \theta^*(c_1)\). Thus, in the decentralized solution panic runs are always inefficient while fundamental runs are inefficient only in the range \((\theta(1), \theta(c_1))\). The inefficiency in this range of fundamental runs arises from the fact that running is a dominant strategy for \(\theta < \theta(c_1)\). In other words, as it can be seen in (2), when taking their withdrawal decision, depositors anticipate to obtain \(c_1\) when running instead of 1 as each of them takes his own decision irrespective of the other depositors’ action.

4 Introducing government guarantees

So far we have characterized the banks’ choice of the deposit contract in the presence of runs. We have shown that the solution in the decentralized economy is inefficient in terms of risk sharing and possible premature liquidation of worthy assets. In this section, we analyze whether the introduction of government guarantees can improve upon the decentralized allocation. We consider the case where the government can transfer part of the public good to the banking sector so to guarantee that depositors receive a certain minimum repayment when the bank is in trouble. The important feature of this analysis is that, differently from Diamond and Dybvig (1983), the design of the guarantees is of paramount importance.

In Diamond and Dybvig (1983), runs are only panic-driven and the inefficiency of the decentralized solution arises exclusively because of the multiplicity of equilibria. In the good equilibrium, banks implement the optimal level of risk sharing and only early depositors withdraw at date 1. In the bad equilibrium, all depositors withdraw at date 1, thus forcing the premature liquidation of the bank’s project. The latter emerges when there is a coordination problem among depositors. Although the bank is solvent if let operate until date 2, for some (unspecified) reason late depositors fear that other depositors will withdraw prematurely and precipitate a run. The possibility of the occurrence of the bad equilibrium is not anticipated by the bank and therefore it does not affect the banks’ promised repayments to depositors.

In such a context, the introduction of deposit insurance works as an equilibrium selection device. The guarantee to receive the promised repayment irrespective of the bank’s available resources removes the coordination problem among late depositors. This in turn eliminates the illiquidity problem that the bank faces when depositors withdraw prematurely. The bank always remains solvent and the first-best allocation
is achieved. The precise design of the guarantee scheme does not matter for its effectiveness. Specifically, it does not matter when the guarantee is paid, at date 1 or at date 2. What matters is only that each late depositor believes that the repayment he obtains does no longer depend on the action of the others. Moreover, as it has a mere announcement effect, the introduction of deposit insurance does not entail any disbursement for the government and it does not introduce any distortion in the choice of the deposit contract. This means that banks choose the same deposit contract that the government would choose. In sum, deposit insurance eliminates the bad equilibrium, thus removing the illiquidity problem that the bank faces when depositors withdraw prematurely.

Our framework differs from the one in Diamond and Dybvig (1983) in various respects. In our model the return of the bank’s asset is risky so that runs can be both fundamental and panic based. This implies that the bank can face situations of pure illiquidity, when depositors withdraw prematurely because of coordination problems (for \( \theta \in [\theta(c_1), \theta^*(c_1)] \)), but also of insolvency when the bank continues until date 2 and with probability \( 1 - \theta \) its assets return zero. Because of this distinction between illiquidity and insolvency, the precise design of the guarantee becomes important. The government can introduce guarantees that are only meant to eliminate the panic runs due to coordination failure among late depositors, as in Diamond and Dybvig (1983), or guarantees that can also address the risk of failure of the bank’s project at date 2. The two types of guarantees differ significantly in terms of their effectiveness in preventing/limiting runs, the disbursement that they entail for the government and the distortion that they may introduce in the bank’s choice of the deposit contract.

Below we characterize the two types of guarantees. In either case, the introduction of the guarantee modifies the timing of the model as follows. At date 0, the government chooses the amount to guarantee \( \overline{\tau} \) and then the bank chooses \( c_1 \). At date 1, after receiving the signal about the state of fundamentals \( \theta \), depositors decide whether to withdraw early or wait until date 2. As before, for each guarantee scheme considered, we solve the model backward. We first characterize depositors’ withdrawal decisions for given \( \overline{\tau} \) and \( c_1 \) and obtain the thresholds \( \hat{\theta}(c_1, \overline{\tau}) \) and \( \theta^*(c_1, \overline{\tau}) \) for the fundamental and panic runs, respectively. Then, we characterize the bank’s choice of \( c_1 \), for given \( \overline{\tau} \), and finally the government’s choice of \( \overline{\tau} \).

### 4.1 Guarantees against panic runs as in Diamond and Dybvig (1983)

We start by analyzing a guarantee scheme that eliminates the occurrence of panic runs as in Diamond and Dybvig (1983). As highlighted in the analysis of the decentralized economy, panic runs arise because of the
strategic complementarity between depositors’ actions. The greater the number of depositors withdrawing at date 1, the more units of the long term asset the bank needs to liquidate prematurely. This, in turn, increases the incentive for a late depositor to run since his repayment in the case he waits is reduced. The coordination failure among depositors leads to a panic-driven run for \( \theta \) in the interval \([\theta(c_1), \theta^*(c_1)]\).

As in Diamond and Dybvig (1983), panic runs can be prevented through a guarantee scheme that eliminates the strategic complementarity between depositors’ actions. The government can achieve this by promising depositors to receive a repayment \( \tau \) when the bank project succeeds equal to the amount they would obtain from the bank when no runs have occurred, i.e., \( \tau = \frac{1 - \lambda c_1}{1 - \lambda} R \), at date 2. This removes late depositors’ incentives to withdraw at date 1 for fears of a run (that is for \( \theta \geq \theta(c_1) \)) because the repayment that each late depositor expects to receive from waiting until date 2 when the bank’s project is successful is now independent of the other depositors’ withdrawal decisions. Importantly, setting the guarantee to the maximum level \( \tau = \frac{1 - \lambda c_1}{1 - \lambda} R \) that eliminates panic runs is optimal in this framework. The reason is that, as in Diamond and Dybvig (1983), the scheme against bank illiquidity has a pure announcement effect and it does not entail any disbursement, no matter how large the guarantee is. By increasing the minimum guaranteed repayment \( \tau \), the government reduces, ceteris paribus, the probability of a run, thus inducing the banks to choose a higher \( c_1 \). Since it can improve the amount of risk-sharing offered to depositors without suffering any cost, the government finds it optimal to fully prevent panics by setting the guarantee equal to the amount that late depositors would receive at date 2 from the bank in case of no runs at date 1 and of project success at date 2.4

Although it eliminates panic runs, this guarantee leaves fundamental runs in equilibrium for \( \theta \) in the interval \([0, \theta(c_1)]\). As the guarantee does not cover depositors in the case the bank turns out to be insolvent at date 2, late depositors still choose to run when they expect bad fundamentals, that is when \( \theta < \theta(c_1) \).

We can now turn to the bank’s choice of the deposit contract in the presence of a guarantee scheme that eliminates panic runs. Given that only fundamental runs are going to occur, each bank chooses \( c_1 \) to maximize

\[
Max_{c_1} \int_0^{\theta(c_1)} u(1) d\theta + \int_{0}^{1} \left[ \lambda u(c_1) + (1 - \lambda) \theta u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] d\theta + \int_0^1 v(g) d\theta
\]  

(11)

\footnote{We can prove this result formally by deriving depositors’ withdrawal decisions and the bank’s choice of the deposit contract for a generic amount of guarantee \( \tau \), and then by proving that the government finds it optimal to set \( \tau = \frac{1 - \lambda c_1}{1 - \lambda} R \) in equilibrium. These derivations follow the same steps as in the other sections of the paper and so, for brevity, we prefer to omit them here. Interested readers can find all derivations in Appendix B of Allen et al. (2014).}
The problem is similar to the one in the decentralized economy in that the bank sets \( c_1 \) to the level that maximizes depositors’ expected utility. The terms in (11) have the same meaning as in (8), with the difference that runs are now only fundamental-driven and thus the relevant threshold in (11) is \( \theta(c_1) \). Interestingly, the provision of the public good is the same as in the decentralized solution and depositors still obtain the same utility from it, as represented by the last term in (11). The reason is that, as mentioned above, the guarantee scheme against panic runs has only an announcement effect and does not entail any transfer of the public good to the banking sector. Depositors do not enjoy any guarantee either when for \( \theta < \theta(c_1) \) a fundamental run occurs at date 1 or when the bank’s asset returns zero at date 2.

The solution to the problem in (11) must be interior. That is, it must be that \( \theta(c_1) < \theta \) at the equilibrium \( c_1 \). If this was not the case, runs would always occur and depositors’ ex ante utility would be \( u(1) \). This is always lower than the expected utility that a depositor obtains when \( \theta(c_1) < \theta \) as given by \( u(1) \) for \( \theta < \theta(c_1) \) and by \( \lambda u(c_1) + (1 - \lambda) \theta u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \) for \( \theta \geq \theta(c_1) \). Since \( \int_{\theta(c_1)}^{1} \theta u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) > u(c_1) > u(1) \), the solution to the bank’s maximization problem must be an interior.

The first order condition that determines the deposit contract \( c_{1}^{DD} \) chosen by the banks solves

\[
\lambda \int_{\theta(c_1)}^{1} \left[ u'(c_1) - \theta Ru' \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] d\theta + \\
- \frac{\partial \theta(c_1)}{\partial c_1} \left[ \lambda u(c_1) + (1 - \lambda) \theta u(c_1) u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(1) \right] = 0,
\]

and we have the following result.

**Proposition 3** The deposit contract chosen by the banks in the case of a guarantee scheme against panic runs is higher than in the decentralized economy, i.e., \( c_{1}^{DD} > c_1^D \).

As in the decentralized economy, in choosing the deposit contract the bank trades off the marginal benefit of a higher \( c_1 \) with its marginal cost. The former, as represented by the first term in (12), is the better risk sharing obtained from the transfer of consumption from late to early depositors. The latter, as captured by the second term in (12), is the loss in expected utility \( \left[ \lambda u(c_1) + (1 - \lambda) \theta u(c_1) u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(1) \right] \) due to the increased probability of fundamental runs as measured by \( \frac{\partial \theta(c_1)}{\partial c_1} \). The solution \( c_{1}^{DD} \) is now larger than \( c_1^D \) in the decentralized economy. The reason is that both the run threshold \( \theta(c_1) \) and its sensitivity to changes in \( c_1 \), as represented by \( \frac{\partial \theta(c_1)}{\partial c_1} \), are lower than the respective ones in the decentralized solution. This means that the marginal benefit of an increase in \( c_1 \) in terms of better risk sharing is higher than the one in the decentralized economy, while its marginal cost is lower. Thus, the bank has an incentive to choose a higher
than in the case without government intervention.

An important implication of Proposition 3 concerns the effect that the introduction of this type of guarantee has on the stability of the banking system. Since $c_1^{DD} > c_1^P$ and $\theta(c_1)$ increases with $c_1$, the probability of fundamental runs is higher with the guarantee than without, i.e., $\theta(c_1^{DD}) > \theta(c_1^P)$. Moreover, in case the level of $c_1^{DD}$ is very high relative to $c_1^P$, it could even be that the probability of fundamental runs in the case of guarantees is higher than the overall probability of runs in the decentralized economy, that is $\theta(c_1^{DD}) > \theta^*(c_1^P)$. In this case, the introduction of government guarantees aiming at preventing panic runs has the perverse effect of increasing the instability of the banking system — albeit linked to the expectation of bad fundamentals and not to panics — through the increased repayment promised in the deposit contract.

We will show an example of when this happens below in Section 5. It is important to notice that, even when it leads to an increase in the likelihood of a banking crisis, this guarantee scheme always improves welfare relative to the decentralized solution. Banks choose to increase the repayment $c_1$ relative to the decentralized economy because, with the guarantee scheme in place, the negative consequences of an increase in the short-term repayment are less severe than in the decentralized economy. As a consequence, despite the fact that this increases the likelihood of a banking crisis, banks offer better risk-sharing to depositors, thus improving their expected utility and overall welfare.

A last point to notice concerning this guarantee scheme is that it does not introduce any distortion in the banks’ choice of the deposit contract. In other words, banks choose the same deposit contract that the government, which takes into account both the costs of providing the guarantee and depositors’ utility, would choose. The reason is that, as already mentioned, the guarantee does not entail any disbursement for the government. Thus, in choosing the deposit contract, both the banks and the government take into account only the effect that this choice has on the expected utility from the private good. In other words, banks internalize all the effects of a change in the repayment $c_1$ when making their choice. As we will show in details in the next section, this is no longer the case when the government guarantees depositors also against bank failure.

4.2 Guarantees against runs and bank failure

The guarantee scheme analyzed in the previous section removes the coordination problem among depositors and, consequently, the occurrence of panic runs. However, depositors are still exposed to the risk that their bank turns out to be insolvent and thus they run at the bank when they expect the bank to fail at date 2,
i.e., when they receive a signal $\theta < \theta(c_1)$. This implies that fundamental runs still occur when guarantees are designed to protect depositors only against panic runs. We now analyze a broader guarantee scheme, which protects depositors also against the failure of their bank. This scheme allows the government to reduce significantly both panic and fundamental runs. Interestingly, as we will see in Section 5, this scheme can be more effective in reducing runs than the one aimed at protecting depositors only against panic runs, although both types of runs still occur in equilibrium. Because of the lower probability of runs, this broader guarantee can also reach a higher level of social welfare than the scheme analyzed in the previous section, despite the fact that it entails a disbursement for the government and an inefficiency in the choice of the deposit contract by the bank as well as in the level of the guarantee $\tau$ chosen by the government. To study this, we first characterize the equilibrium when this guarantee is introduced, and we then analyze its efficiency by comparing the equilibrium to the allocation in which the government chooses both the deposit contract and the amount of guarantee.

### 4.2.1 The equilibrium when the bank chooses the deposit contract and the government sets the guarantee

Consider a guarantee scheme where the government promises depositors to receive a minimum amount $\tau$ whenever their bank is unable to repay them the promised repayments, either because of a run at date 1 or because of the bank turning out to be insolvent at date 2. This means that when there is a run at date 1 depositors obtain $\tau$ if $\tau > 1$ and 1 if $\tau \leq 1$, while in the absence of a run they obtain at date 2 the amount $\frac{1 - \lambda c_1}{1 - \lambda} R$ with probability $\theta$ and $\tau$ with probability $1 - \theta$. The government finances the provision of the guarantee by transferring resources from the public good $g$ to the banking sector and will therefore choose $\tau$ greater or smaller than 1 depending on the size of $g$. To make the analysis interesting, we restrict attention to the parameter space for $g$ such that in equilibrium the government always chooses $\tau < c_1$. This ensures that both fundamental and panic runs will still occur in equilibrium.

We start by characterizing depositors’ withdrawal decisions at date 1 in the two extreme regions of fundamentals. The upper dominance region is like in the decentralized economy. The upperbound of the lower dominance region $\theta(c_1, \tau)$ is the solution to

\[
u(c_1) = \theta \nu \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) + (1 - \theta) \nu(\tau).\]

The terms have the same meaning as in (2) with the difference that depositors have now utility $\nu(\tau)$ at date 2 when, with probability $1 - \theta$ the bank’s project fails and the guarantee $\tau$ is paid. The solution is then
equal to
\[ \hat{\theta}(c_1, \bar{v}) = \frac{u(c_1) - u(\bar{v})}{u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(\bar{v})}. \]  
(13)

The threshold \( \hat{\theta}(c_1, \bar{v}) \) is non-negative as \( c_1 \geq \bar{v} \), and it is smaller than the one in the decentralized economy and that in the allocation with the guarantee against panic runs as given by (3). As they anticipate to obtain \( \bar{v} \) when the project returns zero, late depositors have lower incentives to run. It follows that \( \hat{\theta}(c_1, \bar{v}) \) is decreasing in \( \bar{v} \), i.e., \( \frac{\partial \hat{\theta}(c_1, \bar{v})}{\partial \bar{v}} < 0 \).

To characterize depositors’ withdrawal decisions in the intermediate region, we need to distinguish the case when \( \bar{v} \leq \bar{v}_1 \) and the one when \( \bar{v} > \bar{v}_1 \) as this determines whether depositors obtain the guarantee only at date 2 or also when running at date 1.

We start with the case \( \bar{v} \leq \bar{v}_1 \) when the guarantee is only paid at date 2. A late depositor’s utility differential between withdrawing at date 2 versus date 1, denoted \( v(\theta, n, \bar{v}) \), is then given by
\[ v(\theta, n, \bar{v}) = \begin{cases} 
\theta u \left( \frac{1 - n c_1}{1 - n} R \right) + (1 - \theta)u (\bar{v}) - u(c_1) & \text{if } \lambda \leq n \leq \bar{v} \\
u(\bar{v}) - u(c_1) & \text{if } \bar{v} \leq n \leq \hat{n} \\
u(\bar{v}) - u\left( \frac{1}{n} \right) & \text{if } \hat{n} \leq n \leq 1.
\end{cases} \]  
(14)

The expression for \( v(\theta, n, \bar{v}) \) has three intervals. In the first interval, for \( \lambda \leq n \leq \bar{v} = \frac{R - \bar{v}}{R c_1 - \bar{v}} \), depositors waiting until date 2 receive \( \frac{1 - n c_1}{1 - n} R > \bar{v} \) with probability \( \theta \) and \( \bar{v} \) with probability \( 1 - \theta \), while those withdrawing early obtain \( c_1 \). As \( n \) reaches \( \bar{v} \), the repayment offered by the bank to late depositors falls below \( \bar{v} \) and thus they always receive \( \bar{v} \) when waiting until date 2 for \( n \geq \bar{v} \). Depositors withdrawing at date 1 receive the promised repayment \( c_1 \) as long as \( n \leq \hat{n} = \frac{1}{c_1} \), that is as long as the bank has enough resources to pay \( c_1 \) from the liquidation of the project at date 1. As \( n \) grows further (i.e., for any \( \hat{n} \leq n \leq 1 \)), the bank liquidates its entire project for a value of 1 and each depositor receives the pro-rata share \( \frac{1}{n} \) when withdrawing at date 1. Since \( \frac{1}{n} \geq 1 \geq \bar{v} \), the guarantee is never paid to depositors withdrawing at date 1.

This is no longer the case when \( \bar{v} > \bar{v}_1 \) since the pro-rata share \( \frac{1}{n} \) can fall below \( \bar{v} \) when a large number of depositors withdraw at date 1. Then, in the case \( \bar{v} > 1 \), the expression for \( v(\theta, n, \bar{v}) \) becomes
\[ v(\theta, n, \bar{v}) = \begin{cases} 
\theta u \left( \frac{1 - n c_1}{1 - n} R \right) + (1 - \theta)u (\bar{v}) - u(c_1) & \text{if } \lambda \leq n \leq \bar{v} \\
u(\bar{v}) - u(c_1) & \text{if } \bar{v} \leq n \leq \hat{n} \\
u(\bar{v}) - u\left( \frac{1}{n} \right) & \text{if } \hat{n} \leq n \leq 1 \\
u(\bar{v}) - u(\bar{v}) & \text{if } \bar{v} \leq n \leq 1.
\end{cases} \]  
(15)
The function $v(\theta, c_1, \overline{\tau})$ has now four intervals. The first three are the same as in (14) and the terms have the same meaning as there. The difference is that now there is a fourth interval for $\hat{n} = \frac{1}{\theta} \leq n \leq 1$ where the pro rata share $\frac{1}{n}$ falls below $\overline{\tau}$ and depositors withdrawing at date 1 start receiving the guarantee $\overline{\tau}$.

Insert Figures 2a and 2b

The functions $v(\theta, n, \overline{\tau})$ are illustrated in Figures 2a and 2b for the case $\overline{\tau} \leq 1$ and $\overline{\tau} > 1$, respectively. When $\overline{\tau} \leq 1$, the function $v(\theta, n, \overline{\tau})$ crosses zero only once and exhibits the property of one-sided strategic complementarity, which guarantees that the model has a unique threshold equilibrium. By contrast, when $\overline{\tau} > 1$, it crosses zero for $n < \hat{n}$, it stays below zero for $\hat{n} \leq n \leq \tilde{n}$ and it is equal to zero for $\tilde{n} \leq n \leq 1$. Despite this, there still exists a unique threshold equilibrium, as we show below.

As in the decentralized economy, the threshold signal $x^*(c_1, \overline{\tau})$ can be found as the solution to the indifference condition that equates a depositor’s expected utility from withdrawing early with the one from waiting until date 2. We have the following result.

**Proposition 4** The model with a guarantee scheme against runs and bank failure has a unique threshold equilibrium in which late depositors run if they observe a signal below the threshold $x^*(c_1, \overline{\tau})$ and do not run above. At the limit as $\varepsilon \to 0$, the equilibrium threshold $x^*(c_1, \overline{\tau})$ simplifies to

$$
\theta^*(c_1, \overline{\tau}) = \frac{\int_{n=\lambda}^{\hat{n}} u(c_1) + \int_{n=\hat{n}}^{\min(\overline{n}, 1)} u(\frac{1}{n}) - \int_{n=\lambda}^{\min(\overline{n}, 1)} u(\overline{\tau})}{\int_{n=\lambda}^{\overline{\tau}} u \left( \frac{1-nc_1}{\overline{\tau}n} R \right) - u(\overline{\tau})}.
$$

(16)

The threshold $\theta^*(c_1, \overline{\tau})$ is increasing in $c_1$ and decreasing in $\overline{\tau}$, i.e., $\frac{\partial \theta^*(c_1, \overline{\tau})}{\partial c_1} > 0$ and $\frac{\partial \theta^*(c_1, \overline{\tau})}{\partial \overline{\tau}} < 0$.

The proposition characterizes the equilibrium threshold $\theta^*(c_1, \overline{\tau})$ as a function of the deposit contract $c_1$ chosen by the bank and the level of guarantees $\overline{\tau}$ set by the government. The expression for $\theta^*(c_1, \overline{\tau})$ depends on whether the level of guarantees $\overline{\tau}$ is above or below 1 as this determines the date at which depositors enjoy the guarantee. As in the decentralized economy, the threshold increases with the promised repayment $c_1$. For a given $\overline{\tau}$, a higher $c_1$ induces more runs as it increases depositors’ payoff at date 1, while lowering that at date 2. The introduction of the guarantee has instead a positive effect on the stability of the banking sector. For a given $c_1$, a higher $\overline{\tau}$ reduces the probability of a run as it increases the expected payment that late depositors receive if they wait until date 2. This is the case irrespective of whether $\overline{\tau}$ is above or below 1. This highlights the positive effect of government intervention on the stability in the banking sector.

Having characterized depositors’ withdrawal decisions, we can now turn to date 0 and analyze the bank’s choice of $c_1$ and the government’s choice of $\overline{\tau}$. We start with the former. Given $\overline{\tau}$ and anticipating depositors’
withdrawal decisions, each bank chooses \( c_1 \) to maximize
\[
\text{Max}_{c_1} \int_0^{\theta^*(c_1, \varpi)} u(\max(1, \varpi)) d\theta + \int_{\theta^*(c_1, \varpi)}^1 \left[ \lambda u(c_1) + (1 - \lambda) \left( \theta u \left( \frac{1 - \lambda c_1 R}{1 - \lambda} \right) + (1 - \theta) u(\varpi) \right) \right] d\theta +
E [v(g, c_1^*, \varpi)]
\]

where \( c_1^* \) denotes the equilibrium value of \( c_1 \) chosen by all banks, and \( E [v(g, c_1^*, \varpi)] \) is the expected utility from the public good as given by
\[
E [v(g, c_1^*, \varpi)] = \int_0^{\theta^*(c_1^*, \varpi)} v(g) d\theta + \int_{\theta^*(c_1^*, \varpi)}^1 [\theta v(g) + (1 - \theta) v(g - (1 - \lambda) \varpi)] d\theta \tag{17}
\]
when \( \varpi \leq 1 \), and by
\[
E [v(g, c_1^*, \varpi)] = \int_0^{\theta^*(c_1^*, \varpi)} v(g - \varpi + 1) d\theta + \int_{\theta^*(c_1^*, \varpi)}^1 [\theta v(g) + (1 - \theta) v(g - (1 - \lambda) \varpi)] d\theta \tag{18}
\]
when \( \varpi > 1 \).

For a given \( \varpi \), each bank chooses the deposit contract \( c_1 \) that maximizes depositors’ expected utility. The first term in (17) represents depositors’ expected utility when a run occurs for \( \theta < \theta^*(c_1, \varpi) \) and depositors obtain either \( u(1) \) if \( \varpi \leq 1 \) or \( u(\varpi) \) if \( \varpi > 1 \). The second term in (17) represents depositors’ expected utility for \( \theta \geq \theta^*(c_1, \varpi) \) when there is no run. In this case all depositors obtain the promised repayments depending on their type and late depositors receive the guarantee \( \varpi \) when the bank turns out to be insolvent, which occurs with probability \( 1 - \theta \).

The last term in (17) represents the utility from the public good. Differently from the scheme that guarantees depositors only against panic runs, the provision of the guarantee entails now a disbursement for the government, which depends on whether \( \varpi \) is greater or smaller than 1. The government finances such a disbursement by transferring the necessary amount of public good \( g \) to the banking sector. As they are atomistic, banks do not internalize the cost of the guarantee in their choice of \( c_1 \) and, consequently, the expressions for \( E [v(g, c_1^*, \varpi)] \) depend on the equilibrium choice \( c_1^* \) rather than \( c_1 \).

When \( \varpi \leq 1 \), the guarantee is paid only to the late depositors waiting until date 2 when the bank’s project fails. The first term in (18) represents the utility that depositors obtain from the public good \( g \) when, for \( \theta < \theta^*(c_1^*, \varpi) \), a run occurs; while the second term represents the utility from the public good when, for \( \theta \geq \theta^*(c_1^*, \varpi) \), there is no run. In this case, depositors obtain a level of utility \( v(g) \) with probability \( \theta \) and \( v(g - (1 - \lambda) \varpi) \) with probability \( 1 - \theta \) when the bank’s project fails and the government uses \( (1 - \lambda) \varpi \) units of the public good to repay \( \varpi \) to each of the \( (1 - \lambda) \) waiting late depositors.

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By contrast, when \( \tau > 1 \), the government pays the guarantee also to depositors running at date 1. The first term in (19) represents the utility to depositors when, for \( \theta < \theta^* (c_1, \tau) \), a run occurs and the government transfers \( \tau - 1 \) resources to the banking sector, thus providing only \( g - \tau + 1 \) units of the public good. The second term is the same as in (18).

As in the previous sections, the solution for \( c_1 \) must be interior, that is \( \theta^* (c_1, \tau) < \bar{\theta} \) at the equilibrium choice of \( c_1 \). If that was not the case runs would always occur and the ex ante utility would be \( u(\max(1, \tau)) \).

This is always lower than the expected utility a depositor would obtain when \( \theta^* (c_1, \tau) \leq \bar{\theta} \). In this case, a depositor obtains \( u(\max(1, \tau)) \) for \( \theta < \theta^* (c_1, \tau) \) and \( \lambda u(c_1) + (1 - \lambda) \left( \theta u \left( \frac{1 - \lambda c_1}{1 - \lambda R} \right) + (1 - \theta) u(\tau) \right) \) for \( \theta \geq \theta^* (c_1, \tau) \). Since \( \int_{\theta^* (c_1, \tau)}^{1} \left( \frac{1 - \lambda c_1}{1 - \lambda R} \right) d\theta > u(c_1) > u(\tau) \), the solution must be an interior.

The first order condition that determines the deposit contract \( c_1^{IN} \) in the case of a guarantee scheme that pays in the case of bank failure solves

\[
\lambda \int_{\theta^* (c_1, \tau)}^{1} \left[ u'(c_1) - \theta Ru' \left( \frac{1 - \lambda c_1}{1 - \lambda R} \right) \right] d\theta + \frac{\partial \theta^* (c_1, \tau)}{\partial c_1} \left[ \lambda u(c_1) + (1 - \lambda) \left( \theta^* (c_1, \tau) u \left( \frac{1 - \lambda c_1}{1 - \lambda R} \right) + (1 - \theta^* (c_1, \tau)) u(\tau) \right) - u(\max(1, \tau)) \right] = 0,
\]

and we have the following result.

**Proposition 5** The deposit contract \( c_1^{IN} \) in the case of a guarantee scheme against panic runs and bank failure is higher than in the decentralized economy and is increasing in \( \tau \), i.e., \( c_1^{IN} > c_1^D \) and \( \frac{dc_1^{IN}}{d\tau} > 0 \).

As usual, in choosing the promised payment to early depositors the bank trades off the marginal benefit of a higher \( c_1 \) with its marginal cost. The former, as represented by the first term in (20), is the better risk sharing obtained from the transfer of consumption from late to early depositors. The latter, as captured by the second term in (20), is the loss in expected utility due to the increased probability of panic runs as measured by \( \frac{\partial \theta^* (c_1, \tau)}{\partial c_1} \). The bank’s choice of \( c_1 \) crucially depends on the size of the guarantee \( \tau \) as this affects the probability of a run \( \theta^* (c_1, \tau) \) as well as the payments that depositors receive. As in the case of the guarantee only against panic runs, \( c_1^{IN} \) is larger than in the decentralized economy, which corresponds to the case \( \tau = 0 \) and the difference between \( c_1^{IN} \) and \( c_1^D \) increases with the size of \( \tau \).

Having characterized the bank’s choice of the deposit contract, we now turn to the government’s choice of \( \tau^{IN} \). We distinguish the case when \( \tau \leq 1 \) and when \( \tau > 1 \). We start with the former.

Given the bank’s \( c_1^{IN} \), at date 0 the government chooses \( \tau^{IN} \) to maximize depositors’ total expected utility, which is given by the expression in (17) evaluated at \( c_1 = c_1^{IN}(\tau) \).\(^5\) When \( \tau \leq 1 \), the level of guarantee \( \tau^{IN} \)

\(^5\)In order to keep the notation simple, we simply use \( c_1 \) instead of \( c_1^{IN}(\tau) \) in the formulas below.
is the solution to

\[
\int_{\theta^*(c_1, \bar{\tau})}^{1} (1 - \lambda)(1 - \theta) \left[ u'(\bar{\tau}) - v'(g - (1 - \lambda)\bar{\tau}) \right] d\theta +
\frac{\partial \theta^*(c_1, \bar{\tau})}{\partial c_1} \left[ \lambda u(c_1) + (1 - \lambda) \left( \theta^*(c_1, \bar{\tau}) u \left( \frac{1 - \lambda c_1}{1 - \lambda} \right) + (1 - \theta^*(c_1, \bar{\tau})) u(\bar{\tau}) \right) - u(1) \right] +
\frac{\partial \theta^*(c_1, \bar{\tau})}{\partial c_1} \frac{dc_1}{dc} \left[ \theta^*(c_1, \bar{\tau}) v(g) + (1 - \theta^*(c_1, \bar{\tau})) v(g - (1 - \lambda)\bar{\tau}) - v(g) \right] = 0.
\]

The solution to the first order conditions gives the choice of \( \bar{\tau} \) as a function of the amount of public resources available \( g \). The first term in (21) represents the net marginal benefit/cost of an increase in \( \bar{\tau} \).

By increasing \( \bar{\tau} \), the government transfers resources from the public good to the banking sector. When \( u'(\bar{\tau}) > v'(g - (1 - \lambda)\bar{\tau}) \), the marginal benefit from doing this is larger than the marginal cost. The second term in (21) represents the benefit in terms of expected utility from the private good deriving from the lower probability of panic runs, as measured by \( \frac{\partial \theta^*(c_1, \bar{\tau})}{\partial c_1} \). The last two terms in (21) capture the effects of a higher guarantee on the utility from the public good. The first of these terms, \( \frac{\partial \theta^*(c_1, \bar{\tau})}{\partial g} \left[ \theta^*(c_1, \bar{\tau}) v(g) + (1 - \theta^*(c_1, \bar{\tau})) v(g - (1 - \lambda)\bar{\tau}) - v(g) \right] \), is the direct effect that a higher \( \bar{\tau} \) has on the utility from the public good. As \( \bar{\tau} \) increases, the probability of runs decreases, thus leading to a lower utility from the public good as the government needs to transfer \( (1 - \lambda)\bar{\tau} \) resources to depositors with probability \( 1 - \theta^*(c_1, \bar{\tau}) \). The second term, \( \frac{\partial \theta^*(c_1, \bar{\tau})}{\partial c_1} \frac{dc_1}{dc} \left[ \theta^*(c_1, \bar{\tau}) v(g) + (1 - \theta^*(c_1, \bar{\tau})) v(g - (1 - \lambda)\bar{\tau}) - v(g) \right] \), is the indirect positive effect of a higher \( \bar{\tau} \) on the utility from the public good through the change of \( c_1 \). A higher \( \bar{\tau} \) increases \( c_1 \) and thus the probability of runs. This in turn increases the utility from the public good since, when \( \bar{\tau} \leq 1 \), the government does not provide any guarantee when runs occur.\(^6\)

For the case when \( \bar{\tau} > 1 \), the level of guarantee \( \bar{\tau}^N \) maximizes depositors’ total expected utility, which is as again in (17) evaluated at \( c_1 = c^N_1(\bar{\tau}) \). Thus, the \( \bar{\tau}^N \) chosen by the government is now given by the solution to

\[
\int_{0}^{\theta^*(c_1, \bar{\tau})} \left[ u'(\bar{\tau}) - v'(\bar{\tau} - 1) \right] d\theta +
\frac{\partial \theta^*(c_1, \bar{\tau})}{\partial c_1} \left[ \lambda u(c_1) + (1 - \lambda) \left( \theta^*(c_1, \bar{\tau}) u \left( \frac{1 - \lambda c_1}{1 - \lambda} \right) + (1 - \theta^*(c_1, \bar{\tau})) u(\bar{\tau}) \right) - u(1) \right] +
\frac{\partial \theta^*(c_1, \bar{\tau})}{\partial c_1} \frac{dc_1}{dc} \left[ \theta^*(c_1, \bar{\tau}) v(g) + (1 - \theta^*(c_1, \bar{\tau})) v(g - (1 - \lambda)\bar{\tau}) - v(g) \right] = 0.
\]

\( ^6 \)In the expression (21) there is no term capturing the indirect effect of \( \bar{\tau} \) on the utility from the private good. This is a result of the Envelope theorem since \( c_1 \) is chosen by each bank so as to maximize depositors’ expected utility from the private good. The same applies to the expression (22).
Expression (22) differs from (21) in several important respects. The reason is that when τ > 1 the guarantee is also paid to depositors running at date 1. This leads to an increase in the utility from the private good and to a corresponding reduction in the utility from the public good when a run occurs. The first term in (22), \( R \tilde{\theta}(c, \tau) \left[ u(\tau) - v'(g - \tau + 1) \right] \), represents the marginal net benefit (cost) of an increase in τ on the difference in depositors’ utilities from the private and public good. The second term in (22) is the same as the first term in (21). The last three terms in (22) are as those in (21) except that now running depositors obtain \( u(\tau) \) from the private good and \( v(g - \tau + 1) \) from the public good instead of \( a(1) \) and \( v(g) \), respectively. The implication is that depositors obtain now more in terms of private good from running but at the cost of a lower provision of public good.

To sum up, the amount of the guarantee is crucial in determining when the guarantee is paid to depositors and, consequently, when there is a reduction in the provision of public good. When \( \tau < 1 \), the guarantee is paid only at date 2 when the bank fails. When \( \tau > 1 \), the guarantee is also paid at date 1 when a run occurs so that the government faces a disbursement both at date 1 and at date 2. As a consequence, the sign of the third term in (22), \( \left[ \tilde{\theta}^*(c_1, \tau) v(g) + (1 - \tilde{\theta}^*(c_1, \tau)) v(g - (1 - \lambda)\tau) - v(g - \tau + 1) \right] \), can be either positive or negative. As we will see below, this has very important implications for the distortions that the guarantee scheme against panic runs and bank failure introduces in the choice of the deposit contract and the amount of public guarantees.

### 4.2.2 Distortions of the equilibrium allocation

Having characterized the choice of the bank and that of the government, we now turn to analyze whether the levels of \( c_1 \) and \( \tau \) obtained above are efficient. To do so, we compare them with the levels set by the government in the case it could choose both \( c_1 \) and \( \tau \). This case represents the (constrained) efficient allocation since, in choosing the deposit contract and the level of guarantee, the government takes explicit account of the disbursements needed to provide the guarantee when doing so. We start comparing the level of \( c_1^N \) set by the bank as given by the solution to (20) with the one chosen by the government, for a given, equal, level of guarantee \( \tau \) in the two cases. Then, we analyze how the distortion in the deposit contract \( c_1 \) affects the choice of the level of guarantee \( \tau \) by the government.

When the government chooses both \( c_1 \) and \( \tau \), it does it to maximize depositors’ total expected utility from the private and public goods as given in (17). The expressions for the utility of the public good \( E[v(g, c_1, \tau)] \) are still as in (18) and (19) with the difference that, since the government is not atomistic, they depend on
c_1 rather than on c_1^* . Thus, the optimal deposit contract c_1^G chosen by the government solves

\[
\lambda \int_{\theta^*(c_1, \tau)}^{1} \left[ u'(c_1) - \thetaRu' \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] d\theta + \\
- \frac{\partial \theta^*(c_1, \tau)}{\partial c_1} \left[ \lambda u(c_1) + (1 - \lambda) \left( \theta^*(c_1, \tau) u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) + (1 - \theta^*(c_1, \tau)) u(\tau) \right) - u(1) \right] + \\
- \frac{\partial \theta^*(c_1, \tau)}{\partial c_1} [\theta^*(c_1, \tau) v(g) + (1 - \theta^*(c_1, \tau)) v(g - (1 - \lambda)\tau) - v(g)] = 0
\]  

(23)

when \( \tau \leq 1 \), and

\[
\lambda \int_{\theta^*(c_1, \tau)}^{1} \left[ u'(c_1) - \thetaRu' \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] d\theta + \\
- \frac{\partial \theta^*(c_1, \tau)}{\partial c_1} \left[ \lambda u(c_1) + (1 - \lambda) \left( \theta^*(c_1, \tau) u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) + (1 - \theta^*(c_1, \tau)) u(\tau) \right) - u(\tau) \right] + \\
- \frac{\partial \theta^*(c_1, \tau)}{\partial c_1} [\theta^*(c_1, \tau) v(g) + (1 - \theta^*(c_1, \tau)) v(g - (1 - \lambda)\tau) - v(g - \tau + 1)] = 0
\]  

(24)

when \( \tau > 1 \). Once again, the difference between the two expressions above depends on whether the guarantee is paid only at date 2 as when \( \tau < 1 \), or also at date 1 as when \( \tau > 1 \), since this influences depositors’ utility and the public good provision in the case of a run.

We can now compare the choice of \( c_1 \) of the government \( c_1^G \) with that of the banks \( c_1^N \). We have the following result.

**Proposition 6** For a given \( \tau \), the guarantee scheme against runs and bank failure entails a distortion in the choice of the deposit contract \( c_1^N \) by the bank relative to the efficient level \( c_1^G \). Specifically,

i) For \( \tau \leq 1 \), \( c_1^N < c_1^G \);

ii) For \( \tau > 1 \), \( c_1^N < c_1^G \) if \( [\theta^*(c_1, \tau) v(g) + (1 - \theta^*(c_1, \tau)) v(g - (1 - \lambda)\tau) - v(g - \tau + 1)] < 0 \) and \( c_1^N > c_1^G \) otherwise.

The proposition highlights that the choice of \( c_1 \) by the bank differs from the one that the government would choose as the bank does not internalize the costs of providing the guarantee in terms of lower provision of the public good. This is a typical result in models where private agents, here the banks and their depositors, enjoy the benefits of a public form of insurance without fully paying for its costs. Interestingly, however, the proposition highlights that the distortion in our model can go either way, meaning that the bank can choose both a lower and a higher level of \( c_1 \) than the efficient level. Given that \( c_1 \) represents the risk taken by the bank (as it determines its exposure to runs at date 1 and failure at date 2), the result suggests that the introduction of a guarantee scheme in our model does not necessarily lead to the choice of an excessive
level of risk by the banks (i.e., \( c_1^N > c_1^F \)), but it can instead lead to cases where the bank chooses a lower level of risk that the efficient one.

The reason for this result lies in the fact that, depending on the value of \( \tau \), the guarantee is paid at different dates and thus entails different benefits or costs in terms of the provision of the public good. When \( \tau \leq 1 \), the guarantee is paid only at date 2 so that depositors obtain a higher level of public good when running than when waiting until date 2. As the bank does not take this into account, it chooses a lower level of \( c_1 \) than the one that the government would choose. By contrast, when \( \tau > 1 \), the guarantee is paid both at date 1 and date 2 and whether depositors benefit from a greater public good when there is no run or when a run occurs at date 1 depends on whether the bracket \( \theta^*(c_1, \tau)v(g) + (1 - \theta^*(c_1, \tau))v(g - (1 - \lambda)\tau) - v(g - \tau + 1) \) is negative or positive. In the former case, the expected utility from the public good when there is no run (i.e., \( \theta^*(c_1, \tau)v(g) + (1 - \theta^*(c_1, \tau))v(g - (1 - \lambda)\tau) \)) is lower than the one in the case of a run (i.e., \( v(g - \tau + 1) \)). Similarly to the case when \( \tau \leq 1 \), the bank chooses a lower level of \( c_1 \) than the one chosen by the government as it does not take account of the extra benefit in terms of higher public good when a run occurs. By contrast, when the bracket \( \theta^*(c_1, \tau)v(g) + (1 - \theta^*(c_1, \tau))v(g - (1 - \lambda)\tau) - v(g - \tau + 1) \) is positive, the provision of the public good is greater when there is no run than when there is a run at date 1. This implies that, since it does not internalize this extra benefit coming from the higher provision of public good when there is no run, the bank chooses a level of \( c_1 \), which is inefficiently high. Only in this case the equilibrium features the typical moral hazard problem as the bank takes "excessive" risk in terms of being too exposed to runs at date 1.

To sum up, the introduction of a guarantee against runs and bank failure introduces a distortion in the bank’s deposit contract \( c_1^N \) because it entails an actual disbursement for the government that is not internalized in the bank’s choice. Interestingly, the sign of the distortion is not univocal but depends on the level of the guarantee (i.e., on whether \( \tau \leq 1 \) or \( \tau > 1 \)) as this determines whether runs are costly—or rather beneficial— in terms of public good. When \( \tau \leq 1 \), runs are always beneficial in terms of a higher provision of public good and consequently, for any given \( \tau \), banks choose a lower level of \( c_1^N \) than the efficient one. When \( \tau > 1 \) this effect is ambiguous and the distortion can go either way.

These distortions have also implications for the government’s choice of the level of guarantee \( \tau \). When the government cannot directly choose \( c_1 \), it can only try to curb the moral hazard problem in the banks’ choice of \( c_1 \) when making its own choice of the level of guarantee \( \tau \). Specifically, the bank chooses an inefficiently low level of \( c_1 \) when runs are beneficial in terms of a provision of the public good (i.e., when \( \tau \leq 1 \) or \( \tau > 1 \)).
if \[ \theta^*(c_1, \pi) v(g) + (1 - \theta^*(c_1, \pi)) v(g - (1 - \lambda) \pi) - v(g - \pi + 1) < 0 \]. In these instances, the government chooses a level of guarantee \( \pi \) that differs from the efficient one (i.e., the one chosen in the case the government sets both \( c_1 \) and \( \pi \)) so as to increase the probability of a run. By contrast, when runs are costly in terms of lower provision of public good (i.e., when \( \pi > 1 \) if \[ \theta^*(c_1, \pi) v(g) + (1 - \theta^*(c_1, \pi)) v(g - (1 - \lambda) \pi) - v(g - \pi + 1) > 0 \)) and thus the bank chooses an excessively high \( c_1 \), the government sets \( \pi \) so as to "correct" the risk-taking choice of the bank and lead to a reduction in the likelihood of a run.

Despite the distortions associated to the introduction of the guarantee scheme and the fact that panic runs still occur in equilibrium, the intervention improves depositors' welfare. By reducing the likelihood of a run and its negative consequences, the guarantee induces banks to increase the repayment offered to the early withdrawing depositors, thus improving risk-sharing. Unlike the guarantee scheme against panic runs only, the intervention in this case entails an actual cost in terms of a lower provision of the public good. However, whenever the government chooses to provide the guarantee (i.e., \( \pi > 0 \)), the increase in the utility from the private good more than compensates the loss in terms of a lower utility from the public good. Hence, the intervention leads to an increase in depositors' welfare. We will show this in the next section.

5 A numerical example

In the previous section, we have characterized banks' choice of the deposit contract and depositors' withdrawal decisions in the presence of different guarantee schemes. In response to the government intervention, banks increase the repayment offered to early withdrawal depositors and each scheme improves depositors' expected utility relative to the allocation without guarantees. Despite these common features, we have shown that the precise design of the intervention is of paramount importance and the guarantee schemes analyzed above differ significantly in terms of their effectiveness in preventing the occurrence of runs, the costs and the potential distortions associated to the intervention. A guarantee scheme meant to protect depositors only against the illiquidity of the bank (like the one analyzed in Section 4.1) is fully effective in preventing the occurrence of panic crises and does not entail any disbursement for the government. This implies that this scheme does not lead to any distortion in the banks' risk choice. The potential drawback of this guarantee scheme is that fundamental runs still occur and that, as a consequence, the instability in the banking sector could be higher than in the case without guarantees. A more generous guarantee that protect depositors also against the risk that the bank is insolvent (like the one analyzed in Section 4.2) allows to reduce both the probability of panic and fundamental crises but, since it entails an actual disbursement for
the government, it leads to a distortion in the banks’ risk-taking decision. As banks do not internalize the costs of the intervention, there is a moral hazard problem associated to the guarantee. However, counter to the common wisdom, we have shown that banks do not always choose to take excessive risk in response to the introduction of the guarantees. What matters to determine whether the banks take too much or too little risk relative to the efficient level is whether the government ends up paying to depositors more in the case there is no run and the bank fails for fundamental reasons or in the case there is a run and the bank faces a shortage of liquidity. If the former is true, then the cost of a run from the point of view of the bank is higher than from the point of view of the government and banks’ risk-taking is too low. The opposite is true if the latter holds.

In this section we illustrate the properties of the model with the use of a numerical example. In particular, the goal of this section is to demonstrate that, in a reasonable parameter space, the following results hold: i) the guarantee scheme protecting depositors only against illiquidity of the bank can increase the overall instability of the banking sector relative to the case without guarantees; ii) in response to the introduction of the guarantees against both illiquidity and insolvency, banks choose a lower level of risk than the efficient one (in contrast with the common wisdom); iii) a more generous guarantee scheme protecting depositors against both illiquidity and insolvency of the bank can be better than one protecting them only against illiquidity in that it generates higher social welfare.

We assume that depositors’ utility functions from the private good \( u(c) \) and from the public good \( v(g) \) are given by

\[
u(c) = \frac{(c + f)^{1-\sigma}}{1-\sigma} - \frac{(f)^{1-\sigma}}{1-\sigma},
\]

and

\[
v(g) = \frac{(g + f)^{1-\sigma}}{1-\sigma} - \frac{(f)^{1-\sigma}}{1-\sigma},
\]

respectively. The two functions are normalization of a standard CRRA function with \( \sigma \) being the relative risk aversion coefficient. The parameter \( f \) prevents depositors’ utility to get to \(-\infty\), thus guaranteeing that the assumption \( u(0) = v(0) = 0 \) is satisfied. This parameter can be interpreted as a guaranteed level of consumption resulting, for example, from an additional endowment of resources that depositors receive after date 0, which then can be consumed but not invested in the bank. We also maintain \( p(\theta) = \theta \), with the upper dominance region corresponding to \( \theta = 1 \). We consider different amount of the public good \( g \) to show how the equilibrium of the model changes with the amount of available public resources in the economy.
Specifically, we report the results for the following cases: \( g = 0.7 \) and \( g = 1.5 \). The remaining parameters are set as follows \( \sigma = 3; \ R = 5; \ \lambda = 0.3 \) and \( f = 4 \).

In each table, we compare the decentralized allocation with the solution with the two guarantee schemes analyzed in the previous section and the efficient allocation corresponding to the case where the government chooses both the deposit contract \( c_1 \) and the level of guarantee \( \bar{\tau} \). In the tables, the intervention labelled "Guarantees against panic runs" corresponds to the one illustrated in Section 4.1, where the government guarantees depositors waiting until date 2 to receive a repayment as if no runs have occurred when the bank project succeeds. The intervention labelled "Guarantees against runs and bank failures", instead, corresponds to the one analyzed in Section 4.2, which protects depositors both against the risk of a panic run and bank failure since depositors receive a minimum repayment \( \bar{\tau} \) irrespective of the other depositors’ withdrawal decisions and banks’ available resources. Finally, the last row labelled "Government choosing both \( c_1 \) and \( \bar{\tau} \)" describes the choices of the government in the guarantee scheme against runs and bank failure in the case if the government chose directly both \( c_1 \) and \( \bar{\tau} \).

In each table, we report, in order, the probability of panic and fundamental crises (\( \theta^*, \ \theta \)), the equilibrium values for the deposit contract \( (c_1, c_2) \), the equilibrium level of guarantee \( \bar{\tau} \), the expected utility from the private and public good \( E[u(c_1, c_2, \bar{\tau})] \) and \( E[v(g, \bar{\tau})] \) and, in the last column, the percentage change in the social welfare relative to the decentralized economy due to the introduction of the guarantee.

We present two tables, that differ only for the value of the public good \( g \). This allows us to consider both cases where the amount guaranteed \( \bar{\tau}^N \) is below or above 1 in the scheme against runs and bank failure.

We start by reporting the example in the case when \( g = 0.7 \).

<table>
<thead>
<tr>
<th>( \theta/\theta^* )</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( \bar{\tau} )</th>
<th>( \Delta SW )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{c}_1 )</td>
<td>0.451436</td>
<td>1.0076</td>
<td>4.98372</td>
<td>0</td>
</tr>
<tr>
<td>( \bar{c}_2 )</td>
<td>0.463204</td>
<td>1.0762</td>
<td>4.7694</td>
<td>0</td>
</tr>
<tr>
<td>( \bar{\tau} )</td>
<td>0.488273</td>
<td>1.10762</td>
<td>4.7694</td>
<td>0.27</td>
</tr>
<tr>
<td>( \bar{\tau} )</td>
<td>0.355442</td>
<td>1.01445</td>
<td>4.96905</td>
<td>0.27</td>
</tr>
<tr>
<td>( \bar{\tau} )</td>
<td>0.373496</td>
<td>1.01445</td>
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<td>0.373496</td>
<td>1.01445</td>
<td>4.96905</td>
<td>0.27</td>
</tr>
</tbody>
</table>

The table shows that both types of guarantee schemes improve upon the decentralized economy, where the social welfare, as given by \( E[u(c_1, c_2, \bar{\tau})] + E[v(g, \bar{\tau})] \), in equilibrium is equal to 0.0225355. The scheme against panic runs allows to remove the panic runs completely, but it leads to a higher overall instability in
the banking sector, since the probability of fundamental runs ($\bar{\theta} = 0.488273$) is higher than the probability of panic runs in the decentralized economy ($\theta^* = 0.463162$). This happens because the removal of panics induces banks to choose a much higher repayment to early depositors than in the decentralized economy ($c_1^{DD} = 1.10762 > c_1^D = 1.0076$). This pushes the threshold $\theta$ with the guarantee against panic above $\theta^*$ in the decentralized economy, as all thresholds increase with $c_1$.

In the case of the guarantee scheme against runs and bank failure both panic and fundamental runs occur in equilibrium. Yet, the banking sector is more stable since the probability of runs with guarantees is significantly lower than in the decentralized economy (0.373496 against 0.463162). Also, importantly, this broader guarantee scheme achieves a higher level of social welfare than the other, more restricted, guarantee scheme against panic runs (the percentage increase in social welfare relative to the decentralized economy is $\Delta SW^IN = 0.45 > \Delta SW^{DD} = 0.11$). This happens precisely because, while it also entails a higher level of $c_1$ than in the decentralized economy ($c_1^{IN} = 1.01445 > c_1^D = 1.0076$), the scheme against runs and bank failure is more effective in reducing the probability of runs than the scheme against panics only.

Interestingly, regarding the distortions introduced by the guarantee scheme against runs and bank insolvency as captured in the last row of the table, it emerges that the guarantee does not introduce the standard moral hazard problem usually associated with government intervention (i.e., guarantees induce banks to take excessive risk). In fact, despite the fact that the guarantee induces the banks to choose a higher $c_1$ than in the decentralized economy, the repayment offered to early depositors is still low compared to the efficient one chosen by the government (1.01445 rather than 1.12229). This is due to the fact that, in this example, runs entail a benefit in terms of a higher utility from the public good relative to the case where the run does not occur.

The case with $g = 1.5$ is reported below. The only difference is that now the government chooses to provide a guarantee $\pi^{IN} = 1.055 > 1$ with the more generous scheme. Notwithstanding this, all the results described above for the case $g = 0.7$ remain valid. Both types of guarantees improve social welfare relative to the decentralized economy, where the social welfare in equilibrium is equal to 0.0286413. However, the broader guarantee scheme turns out to be better.

*Table 2: $g = 1.5*$

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To sum up, the numerical examples confirm the two main insights of the analysis. First, the introduction of guarantees does not always decrease the instability of the banking system. Specifically, the guarantee scheme against panic runs can lead to a higher probability of runs than in the decentralized economy, despite eliminating panics. Second, the broader guarantee scheme against runs and bank failure can lead to higher social welfare than the more limited guarantee scheme against panic runs, although it leads to inefficiencies in the choices of the deposit contract by the bank and the amount of guarantee by the government.

6 Concluding remarks

In this paper we develop a model where both panic and fundamental runs are possible and both banks’ and depositors’ decisions are endogenously determined. We highlight the effects of different guarantee schemes on depositors’ withdrawal decisions, on the bank’s risk-taking incentives and on the likelihood of runs and show that government intervention is always desirable as it reduces the inefficiency of the decentralized economy arising from the coordination problem among depositors. However, different guarantee schemes significantly differ in terms of their effectiveness in preventing runs, the disbursement that they entail for the government and the type of distortion that they may introduce in the bank’s choice of the deposit contract.

We show that a guarantee scheme against panic runs as in Diamond and Dybvig (1983) allows to completely eliminate panic runs and does not entail any disbursement for the government, thus leading to an efficient choice of risk by the bank. However, such a scheme may have the perverse effect of increasing the instability of the banking sector. Despite being only fundamental-driven, runs can be more likely than in the decentralized economy if the deposit contract chosen by the banks is significantly larger than the one chosen in the decentralized economy. This implies that, by being more effective in reducing the occurrence of both panic and fundamental runs, a scheme that also protects depositors against bank failure can lead to higher social welfare than the one only against panic runs.

In our numerical examples, we show that indeed a guarantee scheme against both runs and bank failure
is the most effective in reducing the instability in the banking sector than a less generous guarantee scheme even though it does not eliminates panic runs. Moreover, this scheme entails an actual disbursement for the government in terms of a lower provision of the public good and is inefficient in terms of the deposit contract offered by the bank and the amount of guarantee chosen by the government.

Finally, we show that the distortion in the choice of the deposit contract chosen by the bank can be different from the standard moral hazard problem usually associated with government intervention. Unlike the conventional wisdom, the introduction of the guarantees does not always induce banks to take excessive risk. We show that, when only the late depositors receive the guarantee, banks choose a lower repayment to the depositors than the efficient one. This is due to the fact that, in this case, the government does not transfer any resources to the depositors when a run occurs and, thus the provision of the public good is higher when a run occurs than when it does not. As a consequence, since banks do not internalize the effect that their choice has on the expected utility from the public good, they choose a deposit contract that is lower than the efficient one. When depositors receive the guarantee also in the case of a run, depending on whether the provision of the public good is larger in the case of a run or when it does not occur, we show that the standard moral hazard problem in the choice of the deposit contract can arise or we have, as before, that the banks choose a repayment to early depositors below the efficient one.

The paper offers a convenient framework to evaluate the implications of government guarantees, as the likelihood and types of runs as well as the risk taking implicit in the choice of the deposit contract are fully endogenous and depend on the presence and level of the guarantee. The analysis sheds light on the importance of the design of the guarantee scheme and offers insights for future research. One potentially interesting extension would be the analysis of the feedback effect between government solvency and financial stability. When the introduction of a guarantee scheme entails an actual disbursement for the government, it can threaten the solvency of the country and thus undermine the credibility of the guarantee. The threat of sovereign default represents a new source of risk that has been greatly overlooked in the literature on government intervention so far, but, as the recent financial and sovereign crises have shown, it is a very relevant form of risk.

Another possible extension would be to consider a situation of no commitment where the government only intervenes if it is ex post optimal. The credibility of the intervention will then be conditional on its ex post optimality, which in turn depends on the features of the guarantee introduced. There is a growing literature analyzing different forms of interventions in a context of limited commitment (e.g., Ennis and
Keister, 2009 and 2010 and Cooper and Kempf, 2011), but in frameworks where the probability of runs is taken as exogenously given. Addressing limited commitment in our framework seems an interesting future research avenue.

References


Rochet, J-C., and Vives, X., (2004), "Coordination failures and the lender of last resort: was Bagehot right after all?", *Journal of the European Economic Association*, 2(6), 1116-1147.


**Appendix**

**Proof of Proposition 1:** The proof follows Goldstein and Pauzner (2005). The arguments in their proof establish that there is a unique equilibrium in which depositors run if and only if the signal they receive is below a common signal $x^*(c_1)$. The number $n$ of depositors withdrawing at date 1 is equal to the probability of receiving a signal $x_i$ below $x^*(c_1)$ and, given that depositors’ signals are independent and uniformly
distributed over the interval $[\theta - \varepsilon, \theta + \varepsilon]$, it is given by:

$$n(\theta, x^*(c_1)) = \begin{cases} 
\lambda + (1 - \lambda) \frac{1}{\lambda} \left( \frac{x^*(c_1) - \theta - \varepsilon}{2\varepsilon} \right) & \text{if } \theta \leq x^*(c_1) - \varepsilon \\
\frac{x^*(c_1) - \theta - \varepsilon}{2\varepsilon} & \text{if } x^*(c_1) - \varepsilon \leq \theta \leq x^*(c_1) + \varepsilon \\
\lambda & \text{if } \theta \geq x^*(c_1) + \varepsilon 
\end{cases} \quad (25)$$

When $\theta$ is below $x^*(c_1) - \varepsilon$, all patient depositors receive a signal below $x^*(c_1)$ and run. When $\theta$ is above $x^*(c_1) + \varepsilon$, all late depositors wait until date 2 and only the $\lambda$ early consumers withdraw early. In the intermediate interval, when $\theta$ is between $x^*(c_1) - \varepsilon$ and $x^*(c_1) + \varepsilon$, there is a partial run as some of the late depositors run. The proportion of late consumers withdrawing early decreases linearly with $\theta$ as fewer agents observe a signal below the threshold.

Having characterized the proportion of agents withdrawing for any possible value of the fundamentals $\theta$, we can now compute the threshold signal $x^*(c_1)$. A patient depositor who receives the signal $x^*(c_1)$ must be indifferent between withdrawing at date 1 and at date 2. The threshold $x^*(c_1)$ can be then found as the solution to

$$f(\theta, c_1) = \int_{n=1}^{\frac{1}{\lambda}} \left[ \theta(n) u \left( \frac{1 - nc_1}{n} - u(c_1) \right) \right] + \int_{n=1}^{\frac{1}{\lambda}} \left[ u(0) - u \left( \frac{1}{n} \right) \right] = 0, \quad (26)$$

where, from (25), $\theta(n) = x^*(c_1) + \varepsilon - 2\varepsilon \frac{n-\lambda}{n}$. Equation (26) follows from (5) and requires that a late depositor’s expected utility when he withdrawing at date 1 is equal to that when he waits until date 2. Note that at the limit, when $\varepsilon \rightarrow 0$, $\theta(n) \rightarrow x^*(c_1)$, and we denote it as $\theta^*(c_1)$. Solving (26) with respect to $\theta^*(c_1)$ gives the threshold as in the proposition.

To prove that $\theta^*(c_1)$ is increasing in $c_1$, we use the implicit function theorem and obtain

$$\frac{\partial \theta^*(c_1)}{\partial c_1} = -\frac{\frac{\partial f(\theta^*(c_1), c_1)}{\partial \theta}}{\frac{\partial f(\theta^*(c_1), c_1)}{\partial c_1}}.$$ It is easy to see that $\frac{\partial f(\theta^*(c_1), c_1)}{\partial c_1} > 0$. Thus, the sign of $\frac{\partial \theta^*(c_1)}{\partial c_1}$ is given by the opposite sign of $\frac{\partial f(\theta^*(c_1), c_1)}{\partial \theta}$, where

$$\frac{\partial f(\theta^*(c_1), c_1)}{\partial c_1} = -\frac{1}{c_1} \left[ x^*(c_1) u \left( \frac{1 - x^*(c_1)}{1 - c_1} \right) - u(c_1) \right] + \frac{1}{c_1} \left[ u(0) - u(c_1) \right] - \int_{n=1}^{\frac{1}{\lambda}} u'(c_1) + \theta^*(c_1) \left( \frac{nR}{1 - n} \right) u' \left( \frac{1 - nc_1}{1 - n} R \right) \right] = 0.$$

Thus,

$$\frac{\partial \theta^*(c_1)}{\partial c_1} = \int_{n=1}^{\frac{1}{\lambda}} u'(c_1) + \theta^*(c_1) \left( \frac{nR}{1 - n} \right) u' \left( \frac{1 - nc_1}{1 - n} R \right) > 0, \quad (27)$$

and the proposition follows. \(\square\)

**Proof of Proposition 2:** Differentiating (8) with respect to $c_1$ gives the deposit contract $c_1^D$ as the solution to (9).

To show that $c_1^D > 1$, we evaluate (9) at $c_1 = 1$. From (7), at $c_1 = 1$ the threshold $\theta^*(c_1)$ simplifies to

$$\theta^*(1) = \frac{(1 - \lambda)u(1)}{(1 - \lambda)u(R)}.$$
and, from (3), it is then
\[ \theta^*(1) = \bar{\theta}(1). \]
Thus, when \( c_1 = 1 \), (9) can be rewritten as follows:
\[ \lambda \int_{\underline{\theta}(1)}^{1} [u'(1) - \theta Ru'(R)] d\theta - \frac{\partial \bar{\theta}(c_1)}{\partial c_1} |_{c_1 = 1} (1 - \lambda) [\bar{\theta}(1)u(R) - u(1)] \]
The second term is equal to zero because of the definition of \( \bar{\theta}(c_1) \) in (3), and thus the expression simplifies to
\[ \lambda \int_{\underline{\theta}(1)}^{1} [u'(1) - \theta Ru'(R)] d\theta. \]
Since the relative risk aversion coefficient is bigger than 1, it holds
\[ 1 \cdot u'(1) > Ru'(R), \]
so that \( \lambda \int_{\underline{\theta}(1)}^{1} [u'(1) - \theta Ru'(R)] d\theta > 0 \) and thus \( c_1^D > 1 \).

**Proof of Proposition 3:** Denote \( FOC_{c_1}^{c_1^D}(c_1) \) as the first order condition in (12) which implicitly determines the deposit contract \( c_1^D \) chosen by the banks. To show that \( c_1^DD > c_1^D \), we need to compare (9) with (12) and show that \( FOC_{c_1}^{c_1^D}(c_1) \) evaluated at \( c_1 = c_1^D \) is greater than (9) evaluated at \( c_1 = c_1^D \), which is equal to zero. The first term in each expression only differ in the lower extreme of the integrals and it is easy to see that the first term in (9) is smaller than that in (12) since \( \theta^*(c_1) > \underline{\theta}(c_1) \). Thus, we only need to compare \( \frac{\partial \theta^*(c_1^D)}{\partial c_1} \left[ \lambda u(c_1^D) + (1 - \lambda) \theta^*(c_1^D)u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(1) \right] \) with \( \frac{\partial \underline{\theta}(c_1^D)}{\partial c_1} \left[ \lambda u(c_1^D) + (1 - \lambda) \underline{\theta}(c_1^D)u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(1) \right] \) and show that the former is larger than the latter. It is easy to see that
\[ \left[ \lambda u(c_1^D) + (1 - \lambda) \theta^*(c_1^D)u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(1) \right] > \left[ \lambda u(c_1^D) + (1 - \lambda) \underline{\theta}(c_1^D)u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(1) \right], \]
thus \( \theta^*(c_1^D) > \underline{\theta}(c_1^D) \). Then, we have to show that \( \frac{\partial \theta^*(c_1^D)}{\partial c_1} > \frac{\partial \underline{\theta}(c_1^D)}{\partial c_1} \). Substituting the expression for each derivative from (4) and (27), after a few manipulations, we can rewrite the condition above as follows
\[ u'(c_1) \int_{n=\lambda}^{n=\lambda} u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) + \int_{n=\lambda}^{n=\lambda} \theta^*(c_1)u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \left( \frac{nR}{1 - n} \right) u' \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) + \frac{\partial \theta^*(c_1)}{\partial c_1} u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) u' \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \left( \frac{\lambda R}{1 - \lambda} \right), \]
which holds since \( \frac{1 - \lambda c_1}{1 - \lambda} R > \frac{1 - n c_1}{1 - n} R \) and \( \frac{nR}{1 - n} > \frac{\lambda R}{1 - \lambda} \) for any \( n > \lambda \) and \( \theta^*(c_1) > \underline{\theta}(c_1) \). Thus, the proposition follows.

**Proof of Proposition 4:** We need to distinguish two cases depending on whether \( \bar{\tau} \) is larger or smaller than 1. Consider first the case where \( \bar{\tau} \leq 1 \). The proof is analogous to the one of Proposition 2. A patient depositor who receives the signal \( x^*(c_1, \bar{\tau}) \) must be indifferent between withdrawing at date 1 and date 2. The threshold \( x^*(c_1, \bar{\tau}) \) can be then found as the solution to
\[
 f(\theta, c_1, \bar{\tau}) = \int_{n=\lambda}^{n=\lambda} \theta(n)u \left( \frac{1 - n c_1}{1 - n} R \right) + (1 - \theta(n))u \left( \bar{\tau} - u(c_1) \right) + \int_{n=\lambda}^{n=\lambda} u \left( \bar{\tau} - u \left( \frac{1}{n} \right) \right) = 0 \quad (28) 
\]

\[ 37 \]
where, still from (25), \( \theta(n) = x^*(c_1, \tau) + \varepsilon - \frac{2\varepsilon(\alpha - \lambda)}{\lambda} \). Equation (28) follows from (14) and requires that a late depositor’s expected utility when he withdraws at date 1 is equal to that when he waits until date 2. At the limit, when \( \varepsilon \to 0 \), \( \theta(n) \to x^*(c_1, \tau) \), and the threshold \( \theta^*(c_1, \tau) \) solves (28).

The case where \( \tau > 1 \) is more involved since we need first to show that, despite the fact that the function \( v(\theta, n, \tau) \) is zero in the range \( n \leq n \leq 1 \), a unique threshold equilibrium exists. We then split the proof in two parts. First, we prove that a unique threshold equilibrium exists and then, we compute the equilibrium threshold.

**Existence of a unique threshold equilibrium**

The proof follows Goldstein and Pauzner (2005). Recall that the proportion of depositors running \( n(\theta, x^*) \) when they behave according to the same threshold strategy \( x^* \) is given by (25). Denote as \( \Delta(x_i, \hat{n}(\theta)) \) an agent’s expected difference in utility between withdrawing at date 2 rather than at date 1 when he holds beliefs \( \hat{n}(\theta) \) regarding the number of depositors running. The function \( \Delta(x_i, \hat{n}(\theta)) \) is given by

\[
\Delta(x_i, \hat{n}(\theta)) = \frac{1}{2\varepsilon} \int_{x_i - \varepsilon}^{x_i + \varepsilon} E_n [v(\theta, \hat{n}(\theta))] \, d\theta.
\]

Since for any realization of \( \theta \), the proportion of depositors running is deterministic, we can write \( n(\theta) \) instead of \( \hat{n}(\theta) \) and the function \( \Delta(x_i, n(\theta)) \) simplifies to

\[
\Delta(x_i, n(\theta)) = \frac{1}{2\varepsilon} \int_{x_i - \varepsilon}^{x_i + \varepsilon} v(\theta, n(\theta)) \, d\theta.
\]

Notice that when all depositors behave according to the same threshold strategy \( x^* \), \( \hat{n}(\theta) = n(\theta, x^*) \) defined in (25). The following lemma states a few properties of the function \( \Delta(x_i, n(\theta)) \).

**Lemma 1** i) The function \( \Delta(x_i, n(\theta)) \) is continuous in \( x_i \); ii) for any \( a > 0 \), \( \Delta(x_i + a, \hat{n}(\theta) + a) \) is nondecreasing in \( a \); iii) \( \Delta(x_i + a, \hat{n}(\theta) + a) \) is strictly increasing in \( a \) if there is a positive probability that \( n < \bar{n} \) and \( \theta < \bar{\theta} \).

**Proof of Lemma 1.** The proof follows Goldstein and Pauzner (2005). The function \( \Delta(\cdot) \) is continuous in \( x_i \) as a change in \( x_i \) only changes the limits of integration in the computation of \( \Delta(\cdot) \). The function \( \Delta(x_i + a, \hat{n}(\theta) + a) \) is nondecreasing in \( a \) since, as \( a \) increases, depositors see the same distribution of \( n \) but expect \( \theta \) to be higher. Since \( v(\theta, n) \) is nondecreasing in \( \theta \), \( \Delta(\cdot) \) is nondecreasing in \( a \). In order for \( \Delta(x_i + a, \hat{n}(\theta) + a) \) to be strictly increasing in \( a \), we need that \( \theta < \bar{\theta} \) and that there is a positive probability that \( n < \bar{n} \). This is the case because, when \( n < \bar{n} \) and \( \theta < \bar{\theta} \), \( v(\theta, n) \) is strictly increasing in \( \theta \), and, thus, \( \Delta(x_i + a, \hat{n}(\theta) + a) \) is strictly increasing in \( a \).

A threshold equilibrium with the threshold signal \( x^* \) exists, if and only if no depositor finds it optimal to run if he receives a signal higher than \( x^* \) and to wait if he receives a signal below \( x^* \):

\[
\Delta(x_i, n(\theta, x^*)) < 0 \quad \forall x_i < x^*; \tag{29}
\]
\[
\Delta(x_i, n(\theta, x^*)) > 0 \quad \forall x_i > x^*. \tag{30}
\]

By continuity, a depositor must be indifferent between withdrawing at date 1 rather than date 2 when he receives the signal \( x_i = x^* \)

\[
\Delta(x^*, n(\theta, x^*)) = 0. \tag{31}
\]
In the lower and upper dominance regions, \( \Delta(x^*, n(\theta, x^*)) < 0 \) and \( \Delta(x^*, n(\theta, x^*)) > 0 \), respectively. Thus, by continuity of \( \Delta(x^*, n(\theta, x^*)) \) in \( x_i \), there exists some \( x^* \) at which it equals to zero. To prove that the \( x^* \) is unique, we use the property stated in Lemma 1 that \( \Delta(x^*, n(\theta, x^*)) \) is strictly increasing in \( x_i \) in the range \( \theta \in [x^* - \varepsilon, x^* + \varepsilon] \) since from (25), there is always a positive probability that \( n < \pi \) in that range. Thus, there is only one value of \( x^* \), which is a candidate to be a threshold equilibrium. To show that it is indeed an equilibrium we have to show that no depositor has an incentive to deviate. This means that we have to show that, given that (31) holds, (29) and (30) also hold.

Let’s start from (29). Decompose the intervals \([x_i - \varepsilon, x_i + \varepsilon]\) and \([x^* - \varepsilon, x^* + \varepsilon]\) over which the integrals \( \Delta(x_i, n(\theta, x^*)) \) and \( \Delta(x^*, n(\theta, x^*)) \) into a common part \( c = [x_i - \varepsilon, x_i + \varepsilon] \cap [x^* - \varepsilon, x^* + \varepsilon] \) and two disjoint parts \( d_i = \frac{|x_i - x^*|}{\varepsilon} \) and \( d^* = \frac{|x^* - x^* + \varepsilon|}{\varepsilon} \). We can then rewrite the integrals \( \Delta(x_i, n(\theta, x^*)) \) and \( \Delta(x^*, n(\theta, x^*)) \) as follows:

\[
\Delta(x_i, n(\theta, x^*)) = \frac{1}{2\varepsilon} \int_{\theta \in c} v(\theta, n(\theta, x^*)) + \frac{1}{2\varepsilon} \int_{\theta \in d_i} v(\theta, n(\theta, x^*))
\]

\[
\Delta(x^*, n(\theta, x^*)) = \frac{1}{2\varepsilon} \int_{\theta \in c} v(\theta, n(\theta, x^*)) + \frac{1}{2\varepsilon} \int_{\theta \in d^*} v(\theta, n(\theta, x^*))
\]

For any \( \theta \in d_i, n = 1 \) since \( \theta \leq x^* - \varepsilon \). Thus, \( v(\theta, n(\theta, x^*)) = 0 \) and, in turn \( \Delta(x_i, n(\theta, x^*)) = 0 \) in that interval. In order to show that \( \Delta(x_i, n(\theta, x^*)) < 0 \), we need to show that \( \frac{1}{2\varepsilon} \int_{\theta \in c} v(\theta, n(\theta, x^*)) < 0 \). This is the case because (31) holds and the fundamentals in the range \( d^* \) are better than those in the range \( d_i \), which implies that \( \frac{1}{2\varepsilon} \int_{\theta \in d^*} v(\theta, n(\theta, x^*)) > \frac{1}{2\varepsilon} \int_{\theta \in d_i} v(\theta, n(\theta, x^*)) = 0 \). The proof for (30) is analogous.

**The equilibrium threshold**

Having proved the existence of a unique threshold equilibrium, we can now compute \( x^*(c_1, \bar{\pi}) \). Consider first the case where \( \bar{\pi} > 1 \). A patient depositor who receives the signal \( x^*(c_1, \bar{\pi}) \) must be indifferent between withdrawing at date 1 and at date 2. The threshold \( x^*(c_1, \bar{\pi}) \) can be then found as the solution to

\[
f(\theta, c_1, \bar{\pi}) = \int_{n=\lambda}^{\bar{\pi}} [\theta(n)u\left(\frac{1 - n c_1}{1 - n} R\right) + (1 - \theta(n))u(\bar{\pi} - u(c_1))] + \int_{n=\bar{\pi}}^{\bar{\pi}} [u(\bar{\pi}) - u(c_1)] \]

\[
+ \int_{n=\lambda}^{\bar{\pi}} [u(\bar{\pi}) - u\left(\frac{1}{n}\right)] + \int_{n=\lambda}^{\bar{\pi}} [u(\bar{\pi}) - u(c_1)] = 0.
\]  

(32)

As before, \( \theta(n) = x^*(c_1, \bar{\pi}) + \varepsilon - 2\varepsilon \frac{n - \lambda}{\bar{\pi} - \lambda} \) and, at the limit, when \( \varepsilon \to 0 \), \( \theta(n) \to x^*(c_1, \bar{\pi}) \), the threshold \( \theta^*(c_1, \bar{\pi}) \) solves (32).

To complete the proof, we need to compute the direct effect of \( c_1 \) and \( \bar{\pi} \) on the threshold \( \theta^*(c_1, \bar{\pi}) \). To prove that \( \theta^*(c_1, \bar{\pi}) \) is increasing in \( c_1 \), we use the implicit function theorem and obtain

\[
\frac{\partial \theta^*(c_1, \bar{\pi})}{\partial c_1} = -\frac{\frac{\partial f(\theta^*(c_1, \bar{\pi}), c_1, \bar{\pi})}{\partial c_1}}{\frac{\partial f(\theta^*(c_1, \bar{\pi}), c_1, \bar{\pi})}{\partial \theta^*(c_1, \bar{\pi})}}.
\]

We start with the case \( \bar{\pi} \leq 1 \). It is easy to see that the denominator is positive since

\[
\frac{\partial f(\theta^*(c_1, \bar{\pi}), c_1, \bar{\pi})}{\partial \theta^*} = \int_{n=\lambda}^{\bar{\pi}} \left[u\left(\frac{1 - n c_1}{1 - n} R\right) - u(\bar{\pi})\right] > 0.
\]

Thus, the sign of \( \frac{\partial \theta^*(c_1, \bar{\pi})}{\partial c_1} \) is given by the opposite sign of \( \frac{\partial f(\theta^*(c_1, \bar{\pi}), c_1, \bar{\pi})}{\partial c_1} \). After some manipulations, we obtain:

\[
\frac{\partial f(\theta^*(c_1, \bar{\pi}), c_1, \bar{\pi})}{\partial c_1} = -\int_{n=\lambda}^{\bar{\pi}} u'(c_1) - \int_{n=\lambda}^{\bar{\pi}} \theta^*(c_1, \bar{\pi})u'\left(\frac{1 - n c_1}{1 - n} R\right)\left(\frac{nR}{1 - n}\right) < 0.
\]
This implies
\[ \frac{\partial \theta^*(c_1, \tau)}{\partial c_1} = \int_{\tau}^{n} u'(c_1) + \int_{n-\lambda}^{c_1} \theta^*(c_1, \tau) u' \left( \frac{1-n_c}{1-n} R \right) \left( \frac{nR}{1-n} \right) > 0. \] (33)

We now turn to the effect of \( \tau \) on the threshold. To prove that \( \theta^*(c_1, \tau) \) is decreasing in \( \tau \), we use again the implicit function theorem and obtain
\[ \frac{\partial \theta^*(c_1, \tau)}{\partial \tau} = -\frac{\partial f(\theta^*, c_1, \tau)}{\partial \tau}. \]

The denominator is as before and it is positive. Thus, the sign of \( \frac{\partial \theta^*(c_1, \tau)}{\partial \tau} \) is given by the opposite sign of \( \frac{\partial f(\theta^*, c_1, \tau)}{\partial \tau} \). After some manipulations, we obtain
\[ \frac{\partial f(\theta^*, c_1, \tau)}{\partial \tau} = \int_{\tau}^{1} u'(\tau) - \int_{n-\lambda}^{c_1} \theta^*(c_1, \tau) u' \left( \tau \right) > 0, \]
which implies that
\[ \frac{\partial \theta^*(c_1, \tau)}{\partial \tau} = \frac{\int_{\tau}^{1} u'(\tau) - \int_{n-\lambda}^{c_1} \theta^*(c_1, \tau) u' \left( \tau \right)}{\int_{\tau}^{n} u \left( \frac{1-n_c}{1-n} R \right) - u \left( \tau \right)} < 0. \] (34)

The case \( \tau > 1 \) is analogous. In this case, the effect of \( c_1 \) and \( \tau \) on the threshold are given by
\[ \frac{\partial \theta^*(c_1, \tau)}{\partial c_1} = \frac{\int_{\tau}^{n} u'(c_1) + \theta^*(c_1, \tau) \int_{n-\lambda}^{c_1} u' \left( \frac{1-n_c}{1-n} R \right) \left( \frac{nR}{1-n} \right) > 0}{\int_{\tau}^{n} u \left( \frac{1-n_c}{1-n} R \right) - u \left( \tau \right)} \]
and
\[ \frac{\partial \theta^*(c_1, \tau)}{\partial \tau} = \frac{\int_{\tau}^{n} u'(c_1) - \theta^*(c_1, \tau) \int_{n-\lambda}^{c_1} u' \left( \tau \right)}{\int_{\tau}^{n} u \left( \frac{1-n_c}{1-n} R \right) - u \left( \tau \right)} < 0, \]
respectively. This completes the proof of the proposition. \( \square \)

**Proof of Proposition 5:** We consider first the case \( \tau \leq 1 \). Denote \( FOC_{c_1}^N(c_1, \tau) \) the first order condition that implicitly determines \( c_1^N \). This is given by (20) evaluated at \( \tau \leq 1 \) and, thus equal to
\[ \lambda \int_{\tau}^{1} u'(c_1) - \theta R u' \left( \frac{1 - \lambda c_1}{1 - \lambda R} \right) d\theta + \frac{\partial \theta^*(c_1, \tau)}{\partial \tau} \left[ \lambda u(c_1) + (1 - \lambda) \theta^*(c_1, \tau) u \left( \frac{1 - \lambda c_1}{1 - \lambda R} \right) - u \left( \tau \right) \right] + (1 - \lambda) u(\tau) - u(1) \right) = 0. \] (35)

To compute \( \frac{d c_1}{d \tau} \) we use the implicit function theorem. Thus, \( \frac{d c_1}{d \tau} = -\frac{FOC_{c_1}^N(c_1, \tau)}{\frac{\partial FOC_{c_1}^N(c_1, \tau)}{\partial \tau}} \). Since \( c_1^N \) is an interior solution, \( \frac{d c_1^N}{d \tau} > 0 \) if and only if \( \frac{\partial FOC_{c_1}^N(c_1, \tau)}{\partial \tau} > 0 \). We have
\[ \frac{\partial FOC_{c_1}^N(c_1, \tau)}{\partial \tau} = -\lambda \frac{\partial \theta^*(c_1, \tau)}{\partial \tau} \left[ u'(c_1) - \theta^*(c_1, \tau) R u' \left( \frac{1 - \lambda c_1}{1 - \lambda R} \right) \right] + \frac{\partial \theta^*(c_1, \tau)}{\partial \tau} \left[ \lambda u(c_1) + (1 - \lambda) \theta^*(c_1, \tau) u \left( \frac{1 - \lambda c_1}{1 - \lambda R} \right) - u \left( \tau \right) \right] + (1 - \lambda) u(\tau) - u(1) \right] \]
\[ + \frac{\partial \theta^*(c_1, \tau)}{\partial \tau} \left[ \lambda u(c_1) + (1 - \lambda) \theta^*(c_1, \tau) u \left( \frac{1 - \lambda c_1}{1 - \lambda R} \right) - u \left( \tau \right) \right] \]
\[ + \frac{\partial \theta^*(c_1, \tau)}{\partial \tau} \left[ \lambda u(c_1) + (1 - \lambda) \theta^*(c_1, \tau) u \left( \frac{1 - \lambda c_1}{1 - \lambda R} \right) - u \left( \tau \right) \right]. \]
Recall that \( \frac{\partial \theta^*(c_1, \tau)}{\partial c_1} > 0 \) and \( \frac{\partial \theta^*(c_1, \tau)}{\partial \tau} < 0 \). Deriving \( \frac{\partial \theta^*(c_1, \tau)}{\partial c_1} \), as given in (33), with respect \( \tau \), after a few manipulations, the cross derivative \( \frac{\partial \theta^*(c_1, \tau)}{\partial c_1 \partial \tau} \) becomes

\[
\frac{\partial \theta^*(c_1, \tau)}{\partial c_1 \partial \tau} = \frac{1}{\int_{n=\lambda}^{\tau} u \left( \frac{1-n c_1}{1-n} R \right) - u (\tau)} \left\{ -R(c_1 - 1) \left( \frac{n R}{1-n} \right) u' \left( \frac{1-n c_1}{1-n} R \right) + \frac{\partial \theta^*(c_1, \tau)}{\partial c_1} \int_{n=\lambda}^{\tau} u' (\tau) \right\} + \frac{\partial \theta^*(c_1, \tau)}{\partial \tau} \int_{n=\lambda}^{\tau} u' \left( \frac{1-n c_1}{1-n} R \right) \left( \frac{n R}{1-n} \right) \right\}
\]

Substituting the expression for \( \frac{\partial \theta^*(c_1, \tau)}{\partial c_1 \partial \tau} \) into that for \( \frac{\partial FOC_{c_1}(c_1^N, \tau)}{\partial \tau} \), after a few manipulations, we obtain:

\[
\frac{\partial FOC_{c_1}(c_1, \tau)}{\partial \tau} = -\lambda \frac{\partial \theta^*(c_1, \tau)}{\partial c_1} \left[ \left( \frac{R(c_1 - 1)}{n c_1 R - \lambda} \right) u'(c_1) - \theta^*(c_1, \tau) R u' \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] + \frac{\partial \theta^*(c_1, \tau)}{\partial c_1} \int_{n=\lambda}^{\tau} u' (\tau) \left[ \lambda u(c_1) + (1 - \lambda) \theta^*(c_1, \tau) \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(\tau) \right] + (1 - \lambda)(1 - \theta^*(c_1, \tau)) u'(\tau) + \frac{\partial \theta^*(c_1, \tau)}{\partial \tau} \int_{n=\lambda}^{\tau} u \left( \frac{1-n c_1}{1-n} R \right) - u (\tau) \left[ \lambda u(c_1) + (1 - \lambda) \theta^*(c_1, \tau) \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(\tau) \right] + (1 - \lambda)(1 - \theta^*(c_1, \tau)) u'(\tau) + (1 - \lambda)u(\tau) - u(1) \right].
\]

All the terms in the expression above are positive beside the bracket

\[
- \frac{\partial \theta^*(c_1, \tau)}{\partial c_1} \left[ \left( \frac{R(c_1 - 1)}{n c_1 R - \lambda} \right) u'(c_1) - \theta^*(c_1, \tau) R u' \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] + (1 - \lambda)(1 - \theta^*(c_1, \tau)) u'(\tau)
\]

and

\[
\frac{\partial \theta^*(c_1, \tau)}{\partial \tau} \int_{n=\lambda}^{\tau} u \left( \frac{1-n c_1}{1-n} R \right) - u (\tau) \left[ \lambda u(c_1) + (1 - \lambda) \theta^*(c_1, \tau) \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(\tau) \right] + (1 - \lambda)(1 - \theta^*(c_1, \tau)) u'(\tau) + (1 - \lambda)u(\tau) - u(1) \right].
\]

Let’s start to show that (37) is positive. In order for this to be true, we need to show that the term in the square bracket is negative. Substituting the expression for \( \frac{\partial \theta^*(c_1^N, \tau)}{\partial \tau} \) from (34), after a few manipulations, the term in the square bracket simplifies to

\[
- \int_{n=\lambda}^{1} u'(\tau) + \theta^*(c_1, \tau) \int_{n=\lambda}^{\tau} u'(\tau) \int_{n=\lambda}^{1} u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(\tau) \right] + \int_{n=\lambda}^{1} (1 - \theta^*(c_1, \tau)) u'(\tau),
\]

which can be rearranged as

\[
- \int_{n=\lambda}^{1} u'(\tau) \int_{n=\lambda}^{\tau} u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(\tau) \right] + \theta^*(c_1, \tau) \int_{n=\lambda}^{\tau} u'(\tau) \int_{n=\lambda}^{\tau} u \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(\tau) \right] + \int_{n=\lambda}^{1} (1 - \theta^*(c_1, \tau)) u'(\tau).
\]
After a few manipulations, the expression above can be rewritten as follows

\[
\begin{align*}
\quad u'(\tau) & \left[ -(\pi - \lambda)(1 - \theta^*(c_1, \tau)) \right] \\
& \quad \cdot \left[ \frac{\int_{n=\lambda}^{\tau} u \left( \frac{1 - \lambda c_1 z_1}{1 - n} R \right) - u(z)}{\int_{n=\lambda}^{\tau} u \left( \frac{1 - \lambda c_1 z_1}{1 - n} R \right) - u(z)} - (1 - \pi) \right] \\
& \quad \cdot \left[ \frac{\int_{n=\lambda}^{\tau} u \left( \frac{1 - \lambda c_1 z_1}{1 - n} R \right) - u(z)}{\int_{n=\lambda}^{\tau} u \left( \frac{1 - \lambda c_1 z_1}{1 - n} R \right) - u(z)} \right] \\
& \quad + \left[ (1 - \lambda)(1 - \theta^*(c_1, \tau)) \right].
\end{align*}
\]

To show that (37) is positive it suffices to show that

\[
(\pi - \lambda)(1 - \theta^*(c_1, \tau)) \cdot \left[ \frac{\int_{n=\lambda}^{\tau} u \left( \frac{1 - \lambda c_1 z_1}{1 - n} R \right) - u(z)}{\int_{n=\lambda}^{\tau} u \left( \frac{1 - \lambda c_1 z_1}{1 - n} R \right) - u(z)} > (1 - \lambda)(1 - \theta^*(c_1, \tau)).
\]

Rewriting the condition above as follows

\[
(1 - \lambda)(1 - \theta^*(c_1, \tau)) \cdot \left[ \frac{\int_{n=\lambda}^{\tau} u \left( \frac{1 - \lambda c_1 z_1}{1 - n} R \right) - u(z)}{\int_{n=\lambda}^{\tau} u \left( \frac{1 - \lambda c_1 z_1}{1 - n} R \right) - u(z)} > (1 - \lambda)(1 - \theta^*(c_1, \tau)),
\]

and it is easy to see that it always holds since \( \frac{1 - \lambda c_1 R}{1 - n} > \frac{1 - \lambda c_1 R}{1 - n} \) for any \( n > \lambda \).

In order to prove that \( \frac{\partial N}{\partial \theta > 0} \), we are left to show that (38) is dominated by some other term in (36). Thus, we show that (38) is smaller than the positive term \( -\lambda \frac{\partial \theta^*(c_1, \tau)}{\partial \bar{\tau}} \left[ u'(c_1) - \theta^*(c_1, \tau)R u' \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] \). To do this, first, recall that from (35) it holds

\[
\frac{\partial \theta^*(c_1, \tau)}{\partial \bar{\tau}} \left[ \lambda u(c_1) + (1 - \lambda)\theta^*(c_1, \tau) \right] u' \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(\tau) = (1 - \lambda)u(\tau) - u(1)
\]

Thus, a sufficient condition for \( \frac{\partial N}{\partial \theta > 0} \) is that

\[
-\lambda \frac{\partial \theta^*(c_1, \tau)}{\partial \bar{\tau}} \left[ u'(c_1) - \theta^*(c_1, \tau)R u' \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] >
\]

\[
\lambda \int_{\theta^*(c_1, \tau)}^{1} u'(c_1) - \theta R u' \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \, d\theta \int_{\theta^*(c_1, \tau)}^{\tau} u'(\tau) \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(\tau).
\]

Substituting the expression for \( \frac{\partial \theta^*(c_1, \tau)}{\partial \bar{\tau}} \), after a few manipulations, the condition above becomes

\[
\lambda \left[ u'(c_1) - \theta^*(c_1, \tau)R u' \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] \left[ \int_{\lambda}^{1} u'(\tau) - \theta^*(c_1, \tau) \int_{\theta^*(c_1, \tau)}^{\tau} u'(\tau) \right] >
\]

\[
\lambda \int_{\theta^*(c_1, \tau)}^{1} u'(c_1) - \theta R u' \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \, d\theta \int_{\theta^*(c_1, \tau)}^{\tau} u'(\tau),
\]

which can be simplified to

\[
\left[ u'(c_1) - \theta^*(c_1, \tau)R u' \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] u'(\tau) \left[ (1 - \lambda) - (\tau - \lambda)\theta^*(c_1, \tau) \right] >
\]

\[
(1 - \theta^*(c_1, \tau)) \left[ u'(c_1) - E[\theta \mid \theta > \theta^*(c_1, \tau)]R u' \left( \frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] (\tau - \lambda)u'(\tau).
\]

Since \( \theta^*(c_1, \tau) < E[\theta \mid \theta > \theta^*(c_1, \tau)] \) and \( [(1 - \lambda) - (\tau - \lambda)\theta^*(c_1, \tau)] > (1 - \theta^*(c_1, \tau))(\tau - \lambda) \), the condition above holds and the proposition follows. The proof for the case \( \tau \geq 1 \) is analogous. □
Proof of Proposition 6: Evaluating (20) taking \( \tau \leq 1 \) and comparing it with (23), it is easy to see that they only differ in the last term of (23) that is

\[
- \frac{\partial \theta^*(c_1, \tau)}{\partial c_1} \left[ \theta^*(c_1, \tau) v(g) + (1 - \theta^*(c_1, \tau)) v(g - (1 - \lambda)\tau) - v(g) \right]
\]

Since \([\theta^*(c_1, \tau) v(g) + (1 - \theta^*(c_1, \tau)) v(g - (1 - \lambda)\tau) - v(g)] < 0\), for given \( c_1 \) and \( \tau \), the expression in (20) is smaller than that in (23), thus implying that \( c_1^N < c_1^G \).

Evaluate now (20) taking \( \tau > 1 \) and compare it with (24). They only differ in the last term in (24), which is equal to

\[
- \frac{\partial \theta^*(c_1, \tau)}{\partial c_1} \left[ \theta^*(c_1, \tau) v(g) + (1 - \theta^*(c_1, \tau)) v(g - (1 - \lambda)\tau) - v(g - \tau + 1) \right]
\]

The bracket \([\theta^*(c_1, \tau) v(g) + (1 - \theta^*(c_1, \tau)) v(g - (1 - \lambda)\tau) - v(g - \tau + 1)]\) can be either positive or negative. Using the same argument as in the case with \( \tau \leq 1 \), it follows that \( c_1^N < c_1^G \) if the bracket is negative, and \( c_1^N > c_1^G \) if it is positive. The proposition follows. \( \square \)
Figure 1: Depositor’s utility differential in the decentralized economy. The figure shows how the utility differential for a late depositor between withdrawing at date 2 versus date 1 changes with the number $n$ of depositors withdrawing at date 1. The function is decreasing in $n$ for $\lambda \leq n < \frac{1}{c_1}$ and increasing for $\frac{1}{c_1} \leq n \leq 1$. It crosses zero only once for $n < \frac{1}{c_1}$ and remains below zero afterwards.
Figure 2a: Depositor’s utility differential with a guarantee against runs and bank failure when $\bar{c} \leq 1$. The figure shows how the utility differential for a late depositor between withdrawing at date 2 versus date 1 changes with the number $n$ of depositors withdrawing at date 1 for a given guarantee $\bar{c}$ chosen by the government. The function is decreasing in $n$ for $\lambda \leq n < \bar{n}$, constant in the range $\bar{n} \leq n < \hat{n}$ and increasing for $\hat{n} \leq n \leq 1$. It crosses zero only once for $n < \bar{n}$ and remains below afterwards.
Figure 2b: Depositor’s utility differential with a guarantee against runs and bank failure when \( \bar{\sigma} > 1 \). The figure shows how the utility differential for a late depositor between withdrawing at date 2 versus date 1 changes with the number \( n \) of depositors withdrawing at date 1 for a given guarantee \( \bar{\sigma} \) chosen by the government. The function is decreasing in \( n \) for \( \lambda \leq n < \bar{n} \), constant in the range \( \bar{n} \leq n < \hat{n} \), increasing for \( \hat{n} \leq n < \bar{n} \) and again constant in the range \( \bar{n} \leq n \leq 1 \). It crosses for \( n < \bar{n} \) and it takes value zero in the interval \( \bar{n} \leq n \leq 1 \).