The Positive Effects of Biased Self-Perceptions in Firms*

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Abstract. We study a firm in which the marginal productivity of agents' effort increases with the effort of others. We show that the presence of an agent who overestimates his marginal productivity may make all agents better off, including the biased agent himself. This Pareto improvement is obtained even when compensation contracts are set endogenously to maximize firm value. We show that the presence of a leader improves coordination, but self-perception biases can never be Pareto-improving when they affect the leader. Self-perception biases are also shown to affect job assignments within firms and the likelihood and value of mergers.

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1. Introduction

Cooperation and coordination among agents are essential for the success of a firm.¹ According to Alchian and Demsetz (1972), firms form endogenously to combine and exploit complementary activities. Yet even when synergistic

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¹ For example, the success and viability of integrated firms (Grossman and Hart, 1986; Scharfstein and Stein, 2000), partnerships (Farrell and Scotchmer, 1988; Levin and Tadelis, 2005), strategic alliances (Jensen and Meckling, 1995; Holmström and Roberts, 1998), and joint

factors are brought together in a single firm, the realization of their potential value is not automatic. The classic team model developed by Holmström (1982) shows that moral-hazard and free-rider problems abound when the effort choices of the teams' agents are unobservable. Because agents make decisions that are in their self-interest, their unmonitored actions often fail to conform to their organization's objectives, unless their incentives are properly adjusted. These problems are exacerbated when there are complementarities between agents, as one agent may not fully internalize the impact of his decisions on those of others. In this paper, we use insights from psychology to study the impact that agents' biases will have on these problems and their solutions.

A large body of the psychology literature shows that individuals tend to overestimate their skills.² In business settings, Larwood and Whittaker (1977) find that managers tend to believe that they are better than the average manager, and Cooper et al. (1988) find that entrepreneurs see their own chances for success as higher than that of their peers. We incorporate such self-perception biases into the problem of a firm that must hire two agents for production. We assume that one agent (the *overconfident* agent) overestimates the degree to which his effort contributes to the firm's success (i.e., the marginal product of his effort). We show that not only can this bias make the firm more valuable by naturally overcoming the usual free-rider and effort coordination problems, but it can also make both agents, including the overconfident one, better off.

The idea is that agents who overestimate their marginal productivity work harder. This extra effort directly reduces free-riding; however, where there are complementarities between agents it does even more because the effort of one agent increases the marginal productivity of the other, and as a result he too finds that his effort is more valuable. In turn, this second agent also exerts more effort, thereby making the firm even more productive. When the production synergies between the two agents are substantial, even the biased agent ends up benefitting from his overinvestment in effort, as he shares the benefits of his colleague's extra effort (but still suffers the cost of his own overinvestment). Therefore, in our setting, overconfidence can generate a Pareto improvement. Importantly, this result holds even when compensation contracts are chosen

ventures (Alchian and Woodward, 1987; Kamien et al., 1992; Aghion and Tirole, 1994) are affected by the efficiency with which various entities and agents interact.

 $^{^2}$ Langer and Roth (1975), and Taylor and Brown (1988) document that individuals tend to perceive themselves as having more ability than they actually do. According to Kunda (1987), they also tend to believe in theories that imply that their own attributes cause desirable outcomes.

endogenously by the firm to maximize value. The firm then compensates the overconfident agent more than the other as the incentive for him to overinvest in effort is what makes synergistic production possible and valuable.

Our results add to the growing literature in corporate finance that studies the behavioral biases of managers and CEOs in firms. A number of recent papers provide evidence that the overconfidence of corporate managers affects the decisions they make for their firms. Malmendier and Tate (2005, 2007) use the tendency of CEOs to hold stock options too long as a proxy for overconfidence. Others, such as Ben-David et al. (2007) and Sautner and Weber (2006), use survey evidence to estimate the overconfidence of top executives. Finally, Puri and Robinson (2006, 2007) use the Survey of Consumer Finance to establish a link between optimism, work ethic, and the propensity to become an entrepreneur.

In this context, our paper makes a twofold contribution. First, we show that overconfidence can benefit all parties-the biased agent himself, his colleagues, and the firm-and thus generate Pareto improvements. This result is important for overconfidence to persist within the firm because suboptimal behavior is likely to be eliminated by economic forces over time. If the biases of the firm's employees cause losses, as studies of managerial biases often implicitly assume, the biased agents will be replaced. Moreover, if agents lose due to their own biases, they are likely to realize their mistakes and leave. Our result that self-perception biases can actually increase economic surplus and benefit all agents provides a justification for the presence and survival of biases in firms. Second, we show that it may be inappropriate to study the effects of managerial biases without considering the endogenous contractual incentives that managers face and the endogenous behavior of other agents. Our analysis demonstrates that the relation between biases and decisions does not depend solely on the direct effect of the bias; the overall effect on decisions also depends on changes in contracts and in the behavior of other agents.

In addition, we use the model to study the interaction between self-perception biases and organizational design. In particular, we incorporate leadership into our two-agent framework by letting the firm appoint one of the two agents as a leader, whose effort choice is partially revealed to the other agent (the follower) before the latter's own effort choice. This structure naturally increases cooperation as it creates an incentive for the leader to work harder in the knowledge that this choice will affect the follower's effort. Essentially, one agent leads the other by example, as in Hermalin (1998). However, here the leader's influence on the follower's action is due to complementarities between the two agents, and not due to superior information. Using this framework, we study how self-perception biases and leadership interact in contributing to agents' welfare and to firm value. We find that biases can generate Pareto improvements only when the rational agent is the leader. We also show that firm value is maximized with a rational leader when complementarities are strong, but with a biased leader otherwise. To our knowledge, this set of results on the optimal organizational design of a firm in the presence of behavioral biases is new in the literature. A related result is in Rotemberg and Saloner (2000) who show that CEOs with a vision can enhance the incentives of other workers to innovate.

Our analysis generates many novel empirical predictions. First, because overconfidence is expected to benefit all parties when complementarities are sufficiently strong, our analysis suggests that overconfidence is more likely to persist in firms that require more synergistic teamwork to be successful. Second, firms are more likely to appoint rational agents as leaders when complementarities are important. Third, overconfident agents are expected to receive more performance-based compensation as their bias magnifies their own and their colleagues' commitment to the firm, so that motivating them is particularly effective in increasing productivity. We also consider various extensions of the model that yield additional empirical predictions discussed throughout the paper. These extensions study the effect of self-perception biases on mergers and on task allocation within the firm, the interaction between self-perception biases and reputation motives, and the effect of learning about agents' skills and biases.

Our paper is related to others that examine the effects of behavioral traits in effort coordination problems. Rotemberg (1994) shows that when complementarities between a team's agents exist, the presence of some altruistic agents can generate Pareto improvements. Kandel and Lazear (1992) show that effort coordination problems can be overcome when peer pressure effectively imposes an extra cost on agents who do not make enough effort. In our model, however, it is not the concern for others or the concern of others that improve cooperation. Instead, biased agents simply think that their contribution is large enough to justify their costly effort. The externalities matter little to overconfident agents, but they do foster cooperation within the organization. In that sense, our model is closer to that of Kelsev and Spanjers (2004) who show how the ambiguity aversion of some agents leads them to use personal effort as insurance for the effort of others, thereby alleviating free-riding. Also related to our model is the work of Gaynor and Kleindorfer (1991) who show that misperceptions about the firm's production function can have beneficial effects.

Finally, the positive ex ante effects of ex post biased behavior have been studied in other contexts. A prime example is Fershtman and Judd (1987) who show that a firm in Cournot competition may choose to commit to an ex post inefficient strategy in order to affect the actions of the other firm. Similarly,

Kyle and Wang (1997) show that the presence of a biased manager creates an analogous commitment for a money management firm. Finally, Heifetz et al. (2007) argue that biased agents may be better equipped to survive in the long run because of the effects that their biases can have on the behavior of other agents. Our paper differs from these in context (that of a firm), scope (the interaction of self-perception biases with compensation contracts and organizational design), and results (the possibility that biases may lead not just to increases in value but actually to Pareto improvements).

The rest of the paper is organized as follows. Section 2 describes our main model in which a firm hires two agents, one rational and one biased. This model is solved and analyzed in Section 3. Section 4 analyzes the effects of making one of the agents the leader, comparing individual welfare and firm value in the rational-leader and biased-leader scenarios. Various extensions of the model are considered in Section 5. Finally, Section 6 offers some empirical implications of our model, discusses a number of applications, and concludes. All proofs are given in the appendix.

2. The Main Model

2.1 THE FIRM

We consider an all-equity firm, owned by risk-neutral shareholders (the *principal*), requiring the effort of two agents (indexed by i = 1, 2) for production. For simplicity, we assume that the firm's existing assets are worth zero (any non-negative constant would do) and that production derives from a single one-period project, which can either succeed or fail with probability π and $1 - \pi$, respectively. Without loss of generality, we assume that the appropriate discount rate for the project is zero, so that the project's value is simply its expected cash flow. The project generates $\sigma > 0$ dollars at the end of the period if it succeeds, and zero if it fails. Therefore, the firm's end-of-period cash flow is given by

$$\tilde{v} \equiv \begin{cases} \sigma & \text{prob. } \pi \\ 0 & \text{prob. } 1 - \pi. \end{cases}$$
(1)

The probability of success π is endogenous: it depends on the choice of effort $e_i \in [0, 1]$ by each agent *i*. The probability of success is given by

$$\pi = a_1 e_1 + a_2 e_2 + s e_1 e_2,\tag{2}$$

where $a_i \ge 0$, s > 0, and $a_1 + a_2 + s < 1$. Parameter a_i measures the direct effect of agent *i*'s effort. It can be interpreted as the agent's ability level.

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Parameter *s* captures the effect of the interaction between the two agents. In assuming that s > 0, we posit a situation in which the interaction is synergistic, that is, the two agents create positive externalities on each other. This is consistent with Alchian and Demsetz's (1972) view that firms are formed in order to exploit positive externalities or complementarities. For reasons that will become clearer later, we also assume that $\sigma \le 2$, which allows us to focus on interior solutions.

Agents are risk-neutral. They have zero wealth and are protected by limited liability, so that all contractual transfers from the firm to agents must be nonnegative.³ Their effort decisions are made simultaneously and are unobservable to the other agent and to the firm, rendering effort decisions non-contractible. As such, because only two project outcomes (i.e., end-of-period states of the world) are possible, compensation contracts must specify how much each agent receives for a successful project ($\tilde{v} = \sigma$) and for a failed project ($\tilde{v} = 0$), respectively. These contracts are chosen by the firm, knowing that agents choose their effort to maximize their expected utility and that they sustain a private utility cost of effort given by $c(e_i) = \frac{1}{2}e_i^2$. With risk-neutral principal and agents, compensation for failed projects reduces the motivation for effort and does not improve risk-sharing. Hence, it is never optimal for the firm to reward its agents for failed projects, and all compensation is paid only when $\tilde{v} = \sigma$. We denote agent *i*'s compensation in that state by w_i . Given a compensation contract w_i , we can therefore write the utility of agent *i* at the end of the period as

$$\tilde{U}_i \equiv w_i \mathbf{1}_{\{\tilde{v}=\sigma\}} - \frac{1}{2}e_i^2,\tag{3}$$

where $\mathbf{1}_{\{E\}}$ denotes an indicator function for event *E*. Finally, we assume that the reservation utility of the two agents is low enough that their participation constraint never binds.

2.2 SELF-PERCEPTION BIASES

We assume that agent 2 suffers from a self-perception bias, thinking that he is more skilled than he really is, and therefore overestimates the contribution of his effort to the project's chance of success. Specifically, he thinks his ability is $A_2 \ge a_2$, although it is actually only a_2 . We denote the agent's *self-perception bias* by $b \equiv A_2 - a_2 \in [0, 1 - a_1 - a_2 - s)$, and also refer to it as his *level of overconfidence*. We assume that agent 2's overconfidence amounts to a disagreement between the two agents about his skill. In particular, we

³ Our Pareto optimality results hold as long as the agents' wealth does not allow sufficiently large negative transfers.

assume that the principal and agent 1 know that agent 2 is biased, and that agent 2 knows that this is what they think but disagrees with them. In essence, therefore, the firm and agent 1 agree to disagree with agent 2 as in Morris (1996).⁴

Our characterization of agent 2's bias is consistent with a trait that has been extensively documented in behavioral psychology. Langer and Roth (1975), and Taylor and Brown (1988) show that people tend to overestimate their own skills, and Larwood and Whittaker (1977) show that business managers have the same bias. Similarly, Greenwald (1980) documents that people's self-evaluations tend to be unrealistically positive. Dunning et al. (1989) find that such biases are more pronounced when the definition of competence is ambiguous, which is likely to be the case in many business environments. In a group context, Caruso et al. (2006) provide evidence that individuals believe their contribution to a group's output to be greater than it really is. As we show in the following section, such self-perception biases may be useful when agents must cooperate within a firm.

3. Analysis of the Model

3.1 EQUILIBRIUM WITH EXOGENOUS COMPENSATION CONTRACTS

To study the role played by the bias of agent 2 in equilibrium, we start by fixing the compensation contracts of the two agents exogenously at w_1 and w_2 . Section 3.3 endogenizes these compensation contracts by letting the firm set w_1 and w_2 to maximize its value. Clearly, the firm will never choose contracts that promise its agents more total compensation than the firm's profits, as it would be more profitable to go out of business instead. We accordingly assume that $w_1 + w_2 \le \sigma$ even for this section.

At the time each agent makes his effort decision, he does not know how much effort the other will exert, but anticipates the equilibrium level. Let us first take the position of the biased agent, agent 2. Anticipating e_1 by the first agent, he seeks to solve the following maximization problem (the "B"

⁴ Our results hold under the alternative assumption that agent 2 thinks that agent 1 agrees with his self-estimated skill. The derivation and proofs of these results are available from the authors upon request. The assumption that agent 1 realizes that agent 2 is biased is important for some, but not all, of our results. In particular, it does affect our welfare analysis as it pertains to agent 2. This is because our welfare results depend on whether other agents change their behavior when teamed with a biased agent. Still, as long as agent 1 assigns a positive probability to the possibility of agent 2 being biased, our welfare results will hold. We show this in Section 5.4.

subscript denotes that he is biased):

$$\max_{e_2 \in [0,1]} \mathbb{E}_{\scriptscriptstyle B} \big[\tilde{U}_2 \big] = w_2 \mathbb{E}_{\scriptscriptstyle B} \big[\pi \big] - c(e_2) = w_2 \big[a_1 e_1 + (a_2 + b) e_2 + s e_1 e_2 \big] - \frac{1}{2} e_2^2.$$

From this, it is easy to show that agent 2 chooses

$$e_2 = w_2(a_2 + b + se_1). \tag{4}$$

A similar maximization problem for the rational agent gives

$$e_1 = w_1(a_1 + se_2). (5)$$

Both agents work harder when they are paid more (large w_i), when they are more skilled (higher a_i), when the other agent works harder (large e_j), and when the synergies between them are greater (large s). Agent 2 also works harder as his opinion of his own skill is more inflated (as b gets larger). In other words, because the biased agent thinks that his effort is more productive than it really is, he is less reluctant to invest in effort.

The result that skilled (and biased) agents work harder derives directly from the assumption that the marginal productivity of effort is increasing in a_i . The same result obtains if we assume that the marginal disutility of effort is less at all effort levels for higher-ability agents. In fact, this assumption is made in a number of papers positing skill heterogeneity, whether the models concentrate on signaling (e.g., Spence, 1973), rank-order tournaments (e.g., Lazear and Rosen, 1981), screening (e.g., Garen, 1985), or multi-period contracting (Lewis and Sappington, 1997). Admittedly, there is no economic theory justifying the assumption of a positive relationship between skill and effort. Indeed, one can easily imagine contexts in which a highly skilled agent simply scales back on effort, nevertheless achieving the same result as the greater effort of less skilled agents while experiencing more leisure utility. Interestingly, however, Schor (1993) documents that, in practice, workers do exactly the opposite: they allocate the time that suddenly becomes available for leisure to extra work. Hence, it appears that individuals benefit from leisure up to a certain extent, but above this level derive more utility from work. This behavior prompts Simon (1991, p. 33) to ask "why do employees not substitute leisure for work more consistently than they do?" In this light, the agent's decision in our model should be interpreted as an allocation of effort to activities that he perceives as more productive; the overconfident agent just overestimates (by b) how productive he can make these activities. Consistent with the evidence of Felson (1984), these overconfident agents tend to work harder.

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The resulting equilibrium effort level of the two agents is derived in the following lemma.

Lemma 1. In equilibrium, agent 1 chooses

$$e_1 = \frac{\left[a_1 + (a_2 + b)sw_2\right]w_1}{1 - s^2w_1w_2},\tag{6}$$

and agent 2 chooses

$$e_2 = \frac{(a_2 + b + a_1 s w_1) w_2}{1 - s^2 w_1 w_2}.$$
(7)

It is easy to verify that the effort levels of the two agents in equilibrium are increasing in w_1 , w_2 , a_1 , a_2 , s, and b.⁵ The last two are crucial. As b increases, so does agent 2's perception of his own ability and productivity. From his perspective, this overestimated productivity warrants an extra effort; that is, his effort does not require as much effort from agent 1 as before. As b increases, agent 1 also works harder because, knowing that agent 2 works harder, he knows that the potential synergistic gains, through s, from their combined effort are greater than before. This makes his effort more valuable and increases his willingness to pay its cost. In other words, when the efforts of the agents are synergistic, the marginal productivity of one agent increases in the other's effort, hence greater effort by one increases the effort of the other.

3.2 FIRM VALUE AND AGENT WELFARE

Before we turn to the principal's problem of choosing w_1 and w_2 optimally (Section 3.3), let us examine how overconfidence affects the value of the firm and the welfare of the two agents (i.e., their expected utility from working at this firm) when contracts are exogenous. The intuition developed here will apply not only to our later results with endogenous contracts, but also to situations in which compensation cannot be contracted. For example, exogenous contracts can describe how the various parties will benefit from joint ventures between firms, from the reputation that their joint success creates, and from the provision of a public good.

Firm value is simply profit, that is, the portion of output that does not go to agents as compensation. With compensation contracts w_1 and w_2 , it is

⁵ Note that, as is shown in the proof of Lemma 1, our assumption that $\sigma \leq 2$ guarantees that $s^2 w_1 w_2 < 1$ and that (6) and (7) are positive.

given by

$$F \equiv (\sigma - w_1 - w_2) \mathbb{E}[\pi] = (\sigma - w_1 - w_2)(a_1e_1 + a_2e_2 + se_1e_2).$$
(8)

The welfare of agent 2 can be assessed from two perspectives. First, we could calculate expected utility as it is perceived ex ante, that is, assuming that his effort contributes an additional probability of success of b. This, we think, is uninteresting as agent 2 is then trivially better off as b increases because he thinks the firm's project is more likely to succeed. However, such welfare gains are illusions, as the agent will not actually experience this utility (on average) ex post. As Gervais et al. (2007) note, a more appropriate measure of welfare is the utility that this agent will get on average at the end of the period. This measure takes into account the fact that the agent overinvests in effort provision but is calculated using the true probability of each outcome given this behavior. This measure also represents how agent 2 will feel, on average, at the end of the period. As such, although we do not consider the issues per se in our model, it is more likely to determine whether in the long run the agent will be induced to stay with the firm or to move to other firms or activities that do furnish the reservation utility. For these reasons, we use this as our measure of welfare. We start with the following proposition.

Proposition 1. Given exogenous contracts, in equilibrium:

- (*i*) firm value is increasing in b;
- (*ii*) the welfare of agent 1 is increasing in b;
- (iii) the welfare of agent 2 is increasing in b if and only if

$$(1 - 2s^2 w_1 w_2)b < s(a_1 + a_2 s w_2)w_1.$$
⁽⁹⁾

Parts (i) and (ii) of the proposition show that an increase in the overconfidence of agent 2 increases firm value and always improves the welfare of agent 1. This is not surprising. When agent 2 overestimates his skill, he works harder and so makes the firm's project more likely to succeed, benefitting his co-worker and the firm. More interesting is part (iii), which shows that agent 2 sometimes gains from an increase in his own bias, that is, when (9) is satisfied. This condition can be satisfied for two reasons. It is satisfied when $s^2w_1w_2 > \frac{1}{2}$, regardless of the values of a_1 , a_2 and b. In this case, it is easy to verify that the agents gain so much from their synergy that both would be better off by committing to maximum effort, that is, to $e_1 = e_2 = 1$. Because increasing b raises the equilibrium level of effort towards this maximum effort, both are better off. When $s^2w_1w_2 < \frac{1}{2}$, synergistic payoffs are not large enough to warrant full effort even if the two agents could coordinate on e_1 and e_2 . Hence,

increasing the bias of agent 2 is not always beneficial to him. Instead, he is better off only if

$$b < \frac{s(a_1 + a_2 s w_2) w_1}{1 - 2s^2 w_1 w_2},\tag{10}$$

that is, if his overconfidence is not too extreme. Intuitively, this result derives from the tradeoff between agent 2's overinvestment in effort and the synergistic feedback effect of agent 1's increased effort. When b (and e_2) is small, a marginal increase in e_2 creates a synergistic gain that outweighs the extra cost of effort. As b (and e_2) gets larger, however, the marginal cost of effort increases⁶ and agent 2 hurts himself through his effort decisions.

Notice that the right-hand side of (10) is increasing in w_1 , w_2 , a_1 , a_2 and s. As the actual marginal productivity, individual or joint, of the two agents increases, the greater cost of effort associated with the bias of agent 2 becomes more worthwhile. Interestingly, while overconfidence can benefit agent 2 when w_2 is arbitrarily low, w_1 must be sufficiently high for this bias to pay off. Otherwise, the knowledge that agent 2 overinvests in effort does not alter agent 1's effort choice enough for the synergistic feedback effect to benefit agent 2. Taken together, the three parts of Proposition 1 imply that the overconfidence of agent 2 creates a Pareto improvement for the firm and the two agents when (9) holds.

Our result on the welfare of agent 2 relates to Bénabou and Tirole (2002) who show how some behavioral biases can enhance personal motivation and welfare. In their work, the individual is studied in isolation: self-deception improves welfare when the motivation gains from ignoring negative signals outweigh the losses from ignoring positive ones. In contrast, we treat the interactions of biased individuals with others. In particular, the gains from the biased decisions of some individuals (their mis-allocation of effort) are not the result of improved self-motivation, but of their effect on the motivation of others.

To conclude this section, we note a special case in which $w_1 + w_2 = \sigma$, that is, the entire profit is distributed as compensation to the two agents. Because there is no residual claimant, this could describe a partnership in which the two partners share the joint product of their efforts.⁷ In this context, synergies may include the central role played by firm reputation, as in Tirole (1996), especially in human-capital-intensive activities. In such firms, the partner's

⁶ To be precise, the marginal effect on the cost of effort from an increase in e_2 is $\frac{\partial}{\partial e_2} \left(\frac{e_2^2}{2}\right) = e_2$.

⁷ In fact, the case with $w_1 = w_2 = \frac{\sigma}{2}$ is the analogue to Holmström's (1982) team problem in which agents share the team's output equally. The only difference is that we allow for a synergistic term in the team's production function.

effort contributes to the firm's reputation, which increases the productivity of the other partners' effort. For example, a lawyer who expects his peers to shirk realizes that the reputation of the firm is likely to deteriorate, hence his effort will not significantly affect the overall value.⁸ In this light, condition (10) indicates that overconfidence in partnerships will benefit everyone only if there are synergies and if every partner has a large enough stake.⁹

3.3 THE FIRM'S CHOICE OF CONTRACTS

So far we have analyzed exogenously specified compensation contracts w_1 and w_2 . Clearly, however, knowing how the two agents react to their own and their colleague's contracts, the firm will seek to maximize its value by tailoring the contracts to the agents' characteristics and their interactions. The firm's problem is to choose the pair of contracts $\{w_1, w_2\}$ that maximizes its profits net of compensation as given in (8), knowing that its agents will choose the effort levels of e_1 and e_2 derived in Lemma 1.

To solve for the optimal contract, we make one additional assumption about the firm's production function: that $a_1 = a_2 = 0$, so that production derives exclusively from the cooperation of the two agents. This simplification is necessary to get closed-form solutions for the firm's choice of contracts. Since Proposition 1 shows that the benefits of overconfidence can only come from synergies, this added assumption does not affect the main point of our analysis. Moreover, because the firm's existing assets are assumed to be worth zero, the value of the firm corresponds to the value of this cooperation. That is, all other sources of firm value, which are irrelevant to our results, are simply left out. Still, our results with $a_1 = a_2 = 0$ are followed by some numerical analysis that considers the effect of making a_1 and a_2 positive.

Lemma 2. Suppose that production is generated solely by the cooperation of the two agents (i.e., $a_1 = a_2 = 0$). To maximize its value, the firm sets the contracts of the two agents to

$$w_1 = \frac{2\sigma}{8 - s^2 \sigma^2} \quad and \quad w_2 = \frac{\sigma}{2}.$$
 (11)

It is easy to verify that w_2 is greater than w_1 . The firm realizes that the overconfidence of agent 2 makes the cost of his effort seem cheap relative to

⁸ The importance of reputation in the context of partnerships is emphasized in papers by Morrison and Wilhelm (2004) and by Levin and Tadelis (2005).

⁹ For a given b > 0 in (10), agent 2 is worse off than if he were rational if w_2 is too close to σ , because in this case $w_1 = \sigma - w_2$ is near zero.

his expected gain from it. As such, an increase in w_2 has a greater impact on the overconfident agent's effort than the same increase in w_1 has on the rational agent's. Indeed, as long as $w_2 \le 1$ (which is the case in (11) since $\sigma \le 2$),

$$\frac{\partial e_2}{\partial w_2} = \frac{b}{\left(1 - 2s^2 w_1 w_2\right)^2} > \frac{bs w_2}{\left(1 - 2s^2 w_1 w_2\right)^2} = \frac{\partial e_1}{\partial w_1}.$$

Therefore, the firm always gains value by paying the overconfident agent more than the rational one. Of course, since value comes exclusively from cooperation, this does not mean that all compensation goes to the overconfident agent. To capture this value, the firm must increase the rational agent's compensation, and it does so more substantially when the synergies are large (i.e., w_1 is increasing in s). Moreover, the benefit of increasing the overconfident agent's compensation is limited, as the cost eventually outweighs the effort benefit.

The result that the contracts in (11) do not depend on b follows directly from our assumption that $a_1 = a_2 = 0$, which implies that the agents' effort levels in (6) and (7) are both proportional to b, making the probability of a successful project (se_1e_2) and the firm's value in (8) proportional to b^2 for any choice of state-contingent compensation w_1 and w_2 . Of course, the expected compensation of the two agents is increasing in b. Indeed, using the optimal contract from Lemma 2, it is easy to confirm that the equilibrium probability of a successful project (upon which w_1 and w_2 are paid to the agents) is

$$\Pr\{\tilde{v}=\sigma\} = se_1e_2 = \frac{b^2s^2\sigma^3(8-s^2\sigma^2)}{8(4-s^2\sigma^2)^2},$$

which is equal to zero at b = 0 and strictly increasing in b. Later, our numerical analysis will also show that the optimal contracts w_1 and w_2 depend on b when a_1 and a_2 are positive.

With endogenous contracts, the firm takes full advantage of the potential value of overconfidence. In what follows, we analyze the impact of this on agent welfare. Models that allow for overconfident agents (or, more generally, irrational agents), generally find that these agents systematically lose out to more rational parties once the latter optimize their behavior. Indeed, interactions of biased agents with rational agents or value-maximizing firms often result in a simple transfer of economic surplus from the irrational to the rational. In essence, in these models the irrational agents unknowingly leave money on the table, which others are more than happy to take. However, as we can now show, this need not be the case when the biased actions of some agents commit them to the firm and, through synergies, their colleagues with them.

Proposition 2. With the value-maximizing contracts of Lemma 2,

- (*i*) firm value is increasing in b;
- (*ii*) the welfare of agent 1 is increasing in b;
- (iii) the welfare of agent 2 is increasing in b if and only if $s^2\sigma^2 > \frac{8}{3}$.

It is not surprising to see in parts (i) and (ii) that rational agents and firms benefit from the presence of biased agents, who work harder than they should. What is more surprising is part (iii), which shows that agents sometimes gain from their overconfidence, *even when everyone else is optimizing*; that is, even when compensation is endogenous. For this to be the case, the feedback from the biased agent's overinvestment in effort must be sufficiently large. This occurs when the synergies are large and when the firm stands to gain greatly from agent cooperation, as it can then afford more incentive compensation for both agents.

As noted above, these results are obtained under the assumption that $a_1 = a_2 = 0$. This is because we cannot get closed-form solutions if a_1 or a_2 is positive. To confirm that the results in Proposition 2 do not depend on this assumption, we solve the model numerically with positive values for a_i . The effect is illustrated in Figures 1 and 2. To highlight the effect of agent 2's overconfidence, we assume that both agents are equally skilled (i.e., $a_1 = a_2 > 0$) so that the two agents are identical when b = 0. First, Figure 1 shows the equilibrium compensation contracts for the rational (solid line) and biased (dashed line) agents as a function of the bias b, and contrasts these with what they would get if both were rational (dotted line). Consistent with our earlier results, the biased agent is always paid more than the rational agent, regardless of the value of b. In fact, the biased agent always gets more (and the rational agent less) than he would if both were rational. Interestingly, with a_1 and a_2 positive, the compensation of the biased agent increases (and that of the rational agent decreases) in b, because the potential contribution of synergies to value becomes relatively more important (than that from a_1 and a_2) as the bias of agent 2 increases. Consequently, as b increases, the contracts slowly converge to the values derived in Lemma 2.

Figure 2 shows the welfare of the two agents and compares it with the welfare of a rational agent paired with another rational agent (dotted line). The rational agent is always better off with a biased, harder-working colleague; his utility is increasing in the degree of the bias. In other words, he gains more from his colleague's extra effort than he loses in lowered wages. As in Proposition 2, Pareto improvements are possible even when compensation contracts are chosen by the firm. Indeed, as Figure 2 shows, this result is not driven by our earlier assumption that $a_1 = a_2 = 0$. Moreover, as in Proposition 1, agent 2



Figure 1. Compensation and overconfidence.

The graph shows the equilibrium compensation of the rational agent (agent 1, continuous line) and the overconfident agent (agent 2, dashed line). The dotted line shows the equilibrium compensation that would result for the two agents if they were both rational (i.e., if b were zero). The following parameter values were used: $a_1 = a_2 = 0.05$, s = 0.6, and $\sigma = 2$.





is better off as a result of his bias as long as it is not too extreme. That is, although agent 2 is always paid more than agent 1, his overinvestment in effort eventually outweighs his gain from synergies.

4. Leadership

As we saw in Section 3, an overconfident agent can increase a firm's value by raising the equilibrium levels of effort. That is, some inherent behavioral traits of agents make them valuable teammates and make their teams valuable to firms. Naturally, firms not only control compensation contracts, but also affect the way agents interact. Here, we consider the case in which a firm calls upon one of the two agents to lead production. For our model, the question is: do rational or overconfident agents make better leaders? To answer, we modify the setup of Section 2 to accommodate the presence of a leader.

4.1 INTRODUCING A LEADER

In Section 3, the simultaneous choice of effort by the two agents is partially responsible for the lack of cooperation. That is, because neither agent observes the other's effort choice or its product, they fail to internalize some of the synergistic externalities that they create for one another. In this section, we let the firm organize in such a way that the effort choice e_L of one agent, the *leader*, is observed imperfectly by the other agent, the *follower*, who then makes his own effort decision e_F . To capture this and keep the analysis tractable, we modify (2) and assume that

$$\pi = a_{\rm L}\tilde{\epsilon}_{\rm L} + a_{\rm F}\tilde{\epsilon}_{\rm F} + s\tilde{\epsilon}_{\rm L}\tilde{\epsilon}_{\rm F},\tag{12}$$

where $\tilde{\epsilon}_{L}$ is equal to one with probability e_{L} and to zero with probability $1 - e_{L}$, and $\tilde{\epsilon}_{F}$ is similarly defined. Clearly, (2) and (12) are equivalent when the two agents choose their effort simultaneously, as their decisions are then based on $E[\pi]$, which is the same for both specifications. However, now we use (12) and assume that the follower observes $\tilde{\epsilon}_{L}$ before making his effort choice.¹⁰ Given this information, the follower knows that the conditional probability of a successful project is $E[\pi | \tilde{\epsilon}_{L}] = a_{L}\tilde{\epsilon}_{L} + a_{F}e_{F} + s\tilde{\epsilon}_{L}e_{F}$, so that the marginal contribution of his effort is $a_{F} + s$ when $\tilde{\epsilon}_{L} = 1$, and only a_{F} when $\tilde{\epsilon}_{L} = 0$. Making effort choices sequential, therefore, allows the follower to know more about potential synergies and to adjust his effort choice accordingly. Anticipating this, the leader can foster greater cooperation by choosing a higher effort level, which makes $\tilde{\epsilon}_{L} = 1$ more likely. As before, we assume that the firm cannot observe either agent's effort, and so it pays the two agents only when the project is successful. We now denote these payments by w_{L} for

 $^{^{10}}$ If instead we use (2) and assume that the follower observes the actual effort level of the leader, the analysis becomes intractable.

the leader and by $w_{\rm F}$ for the follower. Also, as in our main model, we start by assuming that $a_{\rm L} \ge 0$ and $a_{\rm F} \ge 0$.

The model is solved in essentially the same way as the no-leader model in Section 3, except that the follower chooses his effort based on the observation of $\tilde{\epsilon}_{\rm L}$. This, of course, means that the follower will choose a different effort level depending on whether the leader is successful ($\tilde{\epsilon}_{\rm L} = 1$) or not ($\tilde{\epsilon}_{\rm L} = 0$). We denote the follower's effort in these two scenarios by $e_{\rm FI}$ and $e_{\rm F0}$, respectively. As before, we first derive the equilibrium with exogenous contracts $w_{\rm L}$ and $w_{\rm F}$ before turning our attention to the firm's problem of choosing contracts and its leader.

Lemma 3. With a biased leader (and a rational follower), the equilibrium effort levels are given by $e_{L} = w_{L}[a_{L} + b + sw_{F}(2a_{F} + s)]$, $e_{FI} = w_{F}(a_{F} + s)$, and $e_{F0} = w_{F}a_{F}$. With a rational leader (and a biased follower), the equilibrium effort levels are given by $e_{L} = w_{L}[a_{L} + sw_{F}(2a_{F} + b + s)]$, $e_{FI} = w_{F}(a_{F} + b + s)$, and $e_{F0} = w_{F}(a_{F} + b)$.

As a follower, the biased agent makes an effort that exceeds that of an otherwise identical but rational agent by $bw_{\rm F}$. As before, this is the direct product of the additional probability of success that he thinks his effort generates. Interestingly, however, the biased leader does not always work harder than the rational leader. The difference between the effort level of the biased leader and that of the rational leader being equal to $bw_{\rm L}(1 - sw_{\rm F})$, the rational leader works harder when the synergies between the two agents (s) and the compensation incentives for the follower ($w_{\rm F}$) are large enough. When this is the case (i.e., when $sw_{\rm F} > 1$), firm value is clearly greater with a rational leader and a biased follower; switching roles unequivocally reduces effort.

It is important to see that the effort levels $(e_{FI} \text{ and } e_{F0})$ of the rational follower do not depend on *b*; only the frequency with which they are chosen does. This is because the rational follower knows exactly when the biased agent is successful and therefore does not need to work harder in anticipation of potential synergies. When the rational agent leads, however, he anticipates the greater effort that the biased follower will exert, and internalizes this in his own effort choice. In particular, the synergies between the two agents induce him to increase effort by sbw_Lw_F relative to the level that would be chosen if paired with a rational follower; that is, sw_L times the follower's extra effort generated by his bias, bw_F .

In our setting, the leader uses his choice of effort to influence the follower. In particular, the leader can internalize the externalities that his choice has on his colleague, as his actions affect the follower's. In that sense, he leads by example. This notion of leadership is similar to that developed by Hermalin (1998), where the leader has some information about the profitability of a project and uses his public effort choice to increase his credibility in signaling it to the other agents. Our model differs from Hermalin's in that our leader has no informational advantage. The leadership role consists solely in moving first and in the product of his effort being publicly observable. Another difference is that we allow a biased agent to lead a rational follower or to follow a rational leader, the aim being to study the effects of overconfidence in each of these two organizational structures.

It is easy to show that both agents work harder with a leader, regardless of whether the leader is the rational or the biased agent. The mechanism is simple: the fact that the product of his action is observed by the follower commits the leader to exert greater effort. Because of complementarities, this greater effort creates an incentive for the follower as well to work harder, on average. And this is another difference from Hermalin (1998): because our model's leader has no information about the project's fundamentals, he can only induce the follower to work harder when there are synergies (i.e., when s > 0).

4.2 FIRM VALUE AND AGENT WELFARE WITH EXOGENOUS CONTRACTS

In the rest of this section, we explore how the firm's organizational structure and the self-perception biases of its agents combine to affect individual welfare and firm value. As before, we start with exogenously specified contracts $w_{\rm L}$ and $w_{\rm F}$, where $w_{\rm L} + w_{\rm F} \leq \sigma$. Our first result is the analogue to Proposition 1 when either of the two agents is designated as leader.

Proposition 3. With a biased leader, the welfare of the leader is decreasing in *b*, while that of the rational follower is increasing in *b*. With a rational leader, the welfare of the leader is increasing in *b*, while that of the biased follower is increasing in *b* if and only if $b < sw_{L} \left[a_{L} + \frac{1}{2} (2a_{F} + s)sw_{F} \right]$. Firm value is increasing in *b* with either a biased leader or a rational leader.

As before, the rational agent always benefits from an increase in the bias of his teammate: he shares the product of the biased agent's overinvestment in effort but not the cost, and he can also optimally adjust his response to his teammate's behavior. A more interesting result is that an increase in the biased agent's misperception can only be Pareto-improving if the leader is rational. That is, the biased agent can benefit from his own misperception only when he is the follower, not when he is the leader. This is because the overconfident agent can potentially benefit from his bias only when it serves as a commitment device for him to exert more effort than would be rationally optimal. However, in our model, leadership itself is a commitment device, and it leaves no additional commitment value for the bias of the overconfident leader. Instead, only the costs of the overconfident agent's overinvestment in effort subsist, and therefore his bias always makes him worse off when he leads. However, overconfidence does have some commitment value when the biased agent is the follower, as is shown by the fact that the rational leader's effort level is increasing in *b*. Still, when the bias is too extreme, he overinvests in effort and suffers a loss. Only a sufficiently small bias can create a Pareto improvement.¹¹

4.3 THE FIRM'S DECISIONS

Now let us analyze the decisions of the firm when it can designate a leader. This means solving for the contracts that maximize firm value depending on which agent leads. To optimize its performance, the firm can choose to appoint either the rational or the overconfident agent as its leader. As in Section 3.3, in order to get closed-form solutions, we must assume that $a_L = a_F = 0$. Again, because it is the cooperation between the two agents that lies at the heart of our paper, this assumption does not affect our message. The following lemma derives the optimal compensation contracts under both leadership scenarios.¹²

Lemma 4. When the firm is led by a biased agent, the compensation contracts that maximize firm value are

$$w_{\rm L} = \frac{\sigma - w_{\rm F}}{2} \in \left(\frac{\sigma}{4}, \frac{\sigma}{3}\right) \quad and$$

$$w_{\rm F} = \frac{2s^2\sigma - 3b + \sqrt{4s^4\sigma^2 + 4s^2\sigma b + 9b^2}}{8s^2} \in \left(\frac{\sigma}{3}, \frac{\sigma}{2}\right). \tag{13}$$

¹¹ We would like to thank our referee for pointing out the intuition in this paragraph.

¹² With a menu of contracts, the firm could seek to elicit information from the follower about the outcome of the leader's effort (i.e., whether $\tilde{\epsilon}_{\rm L} = 1$ or $\tilde{\epsilon}_{\rm L} = 0$). This information could then be used to better motivate the leader's effort through incentives that depend directly on $\tilde{\epsilon}_{\rm L}$. However, it can be shown that, with risk-neutral agents and projects that can succeed only when both agents exert effort, the firm never gains from doing this. In particular, when the leader is biased, it is always cheaper for the firm to motivate him with compensation that is paid only for successful projects ($\tilde{v} = \sigma$), as the biased agent thinks that such projects are more likely than they really are. That is, the biased leader overvalues his contribution to the probability that $\tilde{v} = \sigma$ conditional on $\tilde{\epsilon}_{\rm L} = 1$. When the leader is rational, the firm's value is unaffected by whether it compensates him based on the outcome of his own effort ($\tilde{\epsilon}_{\rm L}$) or based on the final outcome of the project (\tilde{v}). Details of this analysis are available from the authors upon request.

When it is led by a rational agent, the compensation contracts that maximize firm value are

$$w_{\rm L} = \frac{\sigma}{4} \quad and \quad w_{\rm F} = \frac{\sigma}{2}.$$
 (14)

Interestingly, in both regimes the follower gets paid more than the leader, i.e., $w_{\rm F} > w_{\rm L}$. The intuition here can be seen if we analyze the effects of $w_{\rm L}$ and $w_{\rm F}$ on the probability of success. In both regimes, this is *s* times the product of the effort level of the leader, $e_{\rm L}$, and the effort level of the follower after the leader's effort leads to a successful outcome (i.e., after $\tilde{\epsilon}_{\rm L} = 1$), $e_{\rm Fl}$. While the wage of the leader $(w_{\rm L})$ affects only $e_{\rm L}$, the follower's wage $(w_{\rm F})$ affects both $e_{\rm Fl}$ and $e_{\rm L}$. This is because the leader anticipates the effort of the follower, whereas the follower already knows whether the leader was productive or not. More precisely, the effect of increasing $w_{\rm L}$ on the probability of success is $s \cdot \frac{\partial e_{\rm L}}{\partial w_{\rm L}} \cdot e_{\rm Fl}$, while the effect of increasing $w_{\rm F}$ is $s \left(\frac{\partial e_{\rm L}}{\partial w_{\rm F}} \cdot e_{\rm Fl} + \frac{\partial e_{\rm Fl}}{\partial w_{\rm F}} \cdot e_{\rm L}\right)$. Using Lemma 3, it is easy to verify that both direct effects are equal when $w_{\rm F} = w_{\rm L}$; that is, $s \cdot \frac{\partial e_{\rm L}}{\partial w_{\rm F}} \cdot e_{\rm Fl} = s \cdot \frac{\partial e_{\rm Fl}}{\partial w_{\rm F}} \cdot e_{\rm L}$. Because increasing $w_{\rm F}$ also generates an indirect effect, $s \cdot \frac{\partial e_{\rm L}}{\partial w_{\rm F}} \cdot e_{\rm Fl}$, it is more beneficial for the firm to increase the compensation of the follower than that of the leader.¹³

Having endogenized the compensation contracts under both firm structures, we turn our attention to the problem of choosing a leader.

Proposition 4. As the overconfidence of the biased agent (b) increases from zero, the firm is more valuable when the rational agent is appointed as the leader if $s\sigma > 1$, and it is more valuable when the biased agent is appointed as the leader otherwise.

The rational agent makes a better leader when synergies are large and when successful projects yield large cash flows. To understand this, recall that the value of appointing the rational agent as the leader comes from the fact that he works harder in anticipation of the follower's bias. For this value to be large, *s* must be large, as synergies are responsible for this portion of his effort. Indeed, Lemma 3 (with $a_L = a_F = 0$) shows that e_L is close to zero when *s* is close to zero. However, a large value of *s* is not sufficient for the gain to be

¹³ While our result that the leader is paid less than the follower seems counter-factual, it is important to note that it is obtained in a model with one leader and one follower. In reality, it is likely that the leader's actions influence those of many followers and generate multiple layers of synergies; hence, proper incentives at the top echelons of the firm will justify greater compensation. These issues, although interesting in their own right, are beyond the scope of our paper.

large; the two agents must also have sufficiently large incentives. That is, it is necessary that $w_{\rm L}$ and $w_{\rm F}$ be large enough, as these incentives serve to amplify the anticipatory component of production. Of course, the firm only has the power to create the necessary incentives when σ is large, as otherwise there is little economic surplus to be shared with the agents.

5. Extensions

We now consider some alternative specifications, which allow us to extend our predictions about the effects of overconfidence in firms. In all these extensions, the two agents choose their effort simultaneously, so we can use either (2) or (12), as they are then equivalent. Also, in order to emphasize the role played by effort complementarities between the agents, we retain the assumption that $a_1 = a_2 = 0$.

5.1 MERGERS AND ACQUISITIONS

Alchian and Demsetz (1972) argue that firms will take advantage of production synergies by acquiring the inputs to production that are more valuable when pooled. In our model, we have assumed so far that the firm has already attracted its synergistic labor force and looks for the optimal way to motivate it. Now we step back and ask how the potentially synergistic effort of an outside agent or firm can be acquired, making the merged firm more valuable than the sum of its parts.¹⁴ Again, our emphasis is on the role of overconfidence.

Suppose that firm 1 is owned by a principal who currently operates with a single rational agent. We still assume without loss of generality that the firm's assets in place are worth zero. With $a_1 = a_2 = 0$ and no synergies from a second agent, this firm is trivially worth zero.¹⁵ Firm 2 is privately-owned by its only agent, an *entrepreneur*. We assume for simplicity that firm 2's production function is the same as firm 1's and that its existing assets are also worth zero. The principal realizes that pooling the labor force of the two firms would create valuable synergies for his firm, in the form of \tilde{v} as specified in (1). In particular, the principal contemplates offering the entrepreneur w_2 if the joint project is successful (i.e., if $\tilde{v}_1 = \sigma$) in return for his participation in the project. In what follows, we consider the possibility that such an acquisition can be made and assess the value it creates.

¹⁴ For more on the role of synergies in mergers, see Hietala et al. (2003).

¹⁵ The value of zero is normalized. We could have assumed that the assets in place have a positive value and that the agent performs some (non-synergistic) tasks that also contribute value.

The entrepreneur who owns the second firm values it according to his own beliefs: if rational, he knows it is worth zero, but if overconfident (i.e., if b > 0), he thinks it is worth more. In that case, he thinks that his effort creates σbe_2 . Since it costs him $c(e_2) = \frac{1}{2}e_2^2$ in utility, he chooses $e_2 = \sigma b$. His (perceived) expected utility from owning this firm is therefore

$$\mathbf{E}_{\scriptscriptstyle B}[\tilde{U}_2] = \sigma \operatorname{Pr}_{\scriptscriptstyle B}\{\tilde{v}_2 = \sigma\} - c(e_2) = \frac{\sigma^2 b^2}{2}.$$
(15)

This quantity, which is increasing in the entrepreneur's bias, becomes his (endogenous) reservation utility when the first firm makes its bid. That is, in order to agree to join forces, he must expect at least this much in utility.

Given that firm 1's agent is rational, the principal realizes that the merger creates value only if the entrepreneur is overconfident, as otherwise Lemma 1 tells us that in equilibrium both agents will shirk once they work for the same merged firm. Hence, overconfidence now has both a positive and a negative effect. On the one hand, it makes cooperation between the two agents possible and valuable once the firms merge, but it also makes the entrepreneurial firm more expensive to acquire, as the owner overvalues it. That is, in the decision to accept firm 1's offer, he trades off a share of his own private (but overestimated) contribution to value for a share of the merged firm's synergies. The following proposition characterizes the terms of the deal when one is possible.

Proposition 5. A deal to merge the two firms is possible if and only if $s\sigma > 1$. If $s^2\sigma^2 \ge \frac{8}{3}$, then firm 1 offers $w_2 = \frac{\sigma}{2}$ to the entrepreneur and pays agent 1 $w_1 = \frac{2\sigma}{8-s^2\sigma^2}$. If instead $s^2\sigma^2 \in (1, \frac{8}{3})$, then the firm offers

$$w_2 = \frac{4}{s\left(s\sigma + \sqrt{s^2\sigma^2 + 8}\right)}\tag{16}$$

to the entrepreneur and pays agent 1

$$w_1 = \frac{s^2 \sigma^2 + s \sigma \sqrt{s^2 \sigma^2 + 8} - 4}{4s^2 \sigma}.$$
 (17)

When $s^2\sigma^2 \ge \frac{8}{3}$, firm 1 offers the agents the contracts derived in Lemma 2. In this case, the synergistic benefit of pooling the production of the two agents is so great that the entrepreneur expects more than (15) in utility from w_2 . That is, the firm benefits by strongly motivating agent 2 with extra compensation and, as a result, the latter's utility constraint for participating in the merger is not binding, as he gets more from his share of the production synergies than he loses from the private production forgone. When $s^2\sigma^2 \in (1, \frac{8}{3})$, however, the value-maximizing contract of Lemma 2 is not enough to convince the entrepreneur to merge. To conclude the deal, firm 1 must offer a larger w_2 (i.e., $w_2 > \frac{\sigma}{2}$). In effect, the entrepreneur's overconfidence makes the threat to reject the offer credible and increases his bargaining power. Hence, while the value of the merged firm is the same as in the proof of Proposition 2 when $s^2\sigma^2 \ge \frac{8}{3}$, it is less than that when $s^2\sigma^2 \in (1, \frac{8}{3})$, because the overconfident entrepreneur captures some of the surplus that his own bias creates in the first place. Finally, when $s\sigma \le 1$, the synergy from pooled resources is so small that the firm cannot offer a contract that the entrepreneur values more than his own private firm. From the entrepreneur's standpoint, sharing private production is not worth any share of what little synergy merging would create.

5.2 TASK DIFFICULTY

Suppose that the roles of the agents in the firm's production are not symmetrical; that is, the tasks they must perform have different degrees of difficulty. We capture this by assuming that the effort cost of agent *i* is given by $c(e_i) = \frac{k_i}{2}e_i^2$, where a large k_i corresponds to a difficult task.¹⁶

Proposition 6. When the task difficulties are given by k_1 and k_2 , the firm maximizes its value by setting the agents' contracts to

$$w_1 = \frac{2\sigma k_1 k_2}{8k_1 k_2 - s^2 \sigma^2} \quad and \quad w_2 = \frac{\sigma}{2}.$$
 (18)

The overconfidence of agent 2 makes the firm more valuable and both agents better off as long as $s^2\sigma^2 > \frac{8k_1k_2}{3}$. The value of the firm is greater when the more difficult task is assigned to the rational agent.

As in Proposition 2, the firm offers the higher compensation to the biased agent, and the Pareto optimality of overconfidence requires that *s* and σ be sufficiently large. When k_2 is large, however, agent 2's overinvestment in effort in response to any given effort level e_1 by agent 1 is more costly. Moreover, when k_1 is large, the overinvestment in effort by the biased agent does not prompt the rational agent to increase his own effort level by much. Both effects make overconfidence less likely to benefit everyone.

The last part of the proposition adds one more dimension to the firm's organizational choices. When tasks vary in difficulty, the firm gains by assigning the harder ones to rational agents. Since, in our model, production

¹⁶ We need to assume that $\sigma^2 < 4k_1k_2$ for interior solutions to obtain.

is generated by synergy between the two agents, the effort level of the rational agent (e_1) is proportional to that of the biased agent (e_2) . That is, the rational agent chooses his effort only based on the effort of the biased agent, so that increasing k_1 and k_2 have a similar stifling effect on the effort choice of the rational agent. For the biased agent, however, increasing k_2 has an additional stifling effect, reducing the part of effort motivated not by the other agent's effort but by his own bias. In other words, the effort of the biased agent can be viewed as the 'engine' that gets the team of agents going. As such, it is critical for the principal not to slow the biased agent down, which means assigning him the easier tasks. To our knowledge, this result that different jobs should be assigned to different employees based on these agents' self-perceptions is new to the job design literature (e.g., Holmström and Milgrom, 1991).

5.3 REPUTATION

In our model, the agents get their (positive) utility exclusively from compensation. If agents can take their team's performance and the associated reputation to other tasks or jobs, they get extra utility from a project success. This would be especially true for younger agents whose early performance will affect their lifetime prospects. In our one-period model, we can capture these reputation effects by assuming that agents get extra utility from project success, at no cost to the firm. Such an assumption is consistent with the idea that agents can get better jobs after performing well. We assume that, when the project is successful, agent *i* receives reputation utility of r_i in addition to his compensation.¹⁷ The following proposition shows how this affects the results of Lemma 2 and Proposition 2.

Proposition 7. When agents care about their reputation, the firm maximizes its value by setting their contracts to

$$w_1 = \frac{2(\sigma + r_1 + r_2)}{8 - s^2(\sigma + r_1 + r_2)^2} - r_1 \quad and \quad w_2 = \frac{r_1 - r_2 + \sigma}{2}.$$
 (19)

The overconfidence of agent 2 makes the firm more valuable and both agents better off as long as $s^2(\sigma + r_1 + r_2)^2 > \frac{8}{3}$.

Again, s must be large enough, and it suffices that both σ and s be large enough for the overconfidence of agent 2 to create a Pareto improvement.

¹⁷ For interior solutions to obtain, we need to impose the restrictions that $\sigma + r_1 + r_2 < 2$ and r_1 and r_2 are small enough (smaller than $\frac{\sigma}{3}$ is sufficient). Also, our notion of reputation is different from the collective reputation studied by Tirole (1996), as it is the individual agents, not the team, who carry the reputation that successful outcomes generate.

However, when agents also care about their reputation, σ does not need to be as large as in Proposition 2. Interestingly, it does not matter whether it is the reputation utility of agent 1 or agent 2 that is large. Through *s*, the greater incentive for one agent to work harder means the other agent also should work harder. That is, overconfidence is useful in any firm where some agents get large private benefits from the success of synergistic production. Finally, note that each agent's compensation decreases in his own reputation benefit and increases in that of the other. This is because inducing extra effort is cheaper when the agent cares about his own reputation, however, it pays to provide the agent with more incentive compensation when the other's concern for reputation increases the potential for synergy.

5.4 UNKNOWN TYPES AND LEARNING

Throughout this paper, we consider a situation in which the overconfidence of agent 2 and the rationality of agent 1 are known to each other and to the firm. We now show that the overconfidence of agents need not be known for our results to obtain. The mere possibility of agents being overconfident is enough to generate extra firm value and agent welfare through better cooperation.

Suppose that every agent *i* is either overconfident $(\tilde{b}_i = b)$ with probability $\phi \in (0, 1)$ or rational $(\tilde{b}_i = 0)$ with probability $1 - \phi$. Others, including the firm, do not know the agent's type. Of course, overconfident agents misinterpret their own type: they think that they are more skilled than they actually are. Not knowing types, the firm offers the same contract *w* to both agents.¹⁸ Although agents know their own type, they know that the other agent will choose a different effort level depending on his type. We denote the effort level of the rational type by e_{R} and that of the biased type by e_{B} .

A rational agent *i* knows that his effort will only contribute to the project's probability of success through synergy with the other agent. From his perspective, the expected effort level of this other agent is $\phi e_{\rm B} + (1 - \phi)e_{\rm R}$, and so his expected utility from effort level e_i is $ws[\phi e_{\rm B} + (1 - \phi)e_{\rm R}]e_i - \frac{1}{2}e_i^2$. This implies that his optimal choice of effort is

$$e_i = ws[\phi e_{\scriptscriptstyle \mathrm{B}} + (1 - \phi)e_{\scriptscriptstyle \mathrm{R}}] \equiv e_{\scriptscriptstyle \mathrm{R}}.$$
(20)

Similar reasoning leads a biased agent j to choose an effort level of

$$e_j = w \Big(b + s \big[\phi e_{\scriptscriptstyle B} + (1 - \phi) e_{\scriptscriptstyle R} \big] \Big) \equiv e_{\scriptscriptstyle B}.$$
⁽²¹⁾

¹⁸ Because compensation contracts are uni-dimensional, the firm cannot use a menu of contracts to screen agents either.

Solving for e_{R} and e_{B} in (20) and (21) yields the equilibrium effort level of the two types of agents:

$$e_{\rm R} = \frac{\phi b s w^2}{1 - s w}$$
 and $e_{\rm B} = \frac{b w \left[1 - (1 - \phi) s w \right]}{1 - s w}$. (22)

Note that it is no longer the presence but the possible presence of a biased agent that makes rational agents exert effort. Indeed, $e_{R} > 0$ if and only if $\phi > 0$. In fact, it is possible for both agents to be rational and exert effort, even though both would shirk if they knew each other's type. The following proposition derives the contract that the firm will offer to both agents in equilibrium.

Proposition 8. With unknown agent types, the firm maximizes its value by setting both agents' contracts to

$$w = \frac{3 - \sqrt{9 - 4s\sigma}}{2s}.$$
(23)

The potential presence of overconfidence (i.e., the fact that $\phi > 0$) makes the firm more valuable and both types of agents better off as long as

$$s\sigma > \frac{5}{4}$$
 and $\phi > \frac{3 + \sqrt{9 - 4s\sigma}}{2s\sigma} - 1.$ (24)

It is again critical that s and σ be large enough for overconfidence to have a Pareto-improving effect. When the overconfidence of agents is unknown, however, this is not sufficient. In addition, ϕ must be large; that is, it must be likely that agents are biased. Otherwise, the commitment value of overconfidence is low since agents cannot count on effort overinvestment by their colleagues. As a result, the agents who happen to be overconfident end up overworking and have excessive effort costs.

Using (20) and (21), it is easy to verify that $e_{\rm B} = e_{\rm R} + bw$ so that, as before, overconfident agents work harder than they should, given their colleague's effort level. The implications here, when types are unknown, are important. Because overconfident agents work harder, project success will tend to be associated with agent overconfidence. This is stated more formally in the following proposition.

Proposition 9. After a successful project ($\tilde{v} = \sigma$), the firm's agents assign a higher posterior probability to the other agent being overconfident. Similarly, the firm assigns a higher posterior probability to at least one agent being overconfident and also to both agents being overconfident.

Interestingly, even when both agents happen to be rational (probability $(1-\phi)^2$), the success of the project makes them put more weight on the possibility that their colleague is overconfident. Hence, if they were to interact in a second period, the first-period success would further enhance the impact of the potential presence of overconfidence. More generally, although our one-period model does not allow us to specify the time-series dynamics, it is clear that cooperation and project success should lead to more cooperation and more success, regardless of agent types. In other words, overconfidence breeds success, and success engenders overconfidence. An extension of the model in which agents also learn about their ability shows that biased agents mis-attribute any increase in output to their own skill, confirming their ex ante beliefs and slowing down the learning process. Depending on their higher-order beliefs, sometimes these agents may never learn their true ability and remain biased even asymptotically.¹⁹

6. Conclusion

Teamwork synergy and cooperation have long been identified as important factors in firms' production. How firms extract the full value from potential synergy is less clear. As is shown by Holmström (1982), when agents share their team's output but their contribution to that output is unobservable, a free-riding problem emerges. Because an agent sustains the full burden of his effort but gets only a fraction of the benefit, he scales back on effort and counts on that of others. In equilibrium, the team does not achieve its full first-best production. This problem is exacerbated by the presence of complementarities: because agents do not fully account for the positive externalities that their effort creates, cooperation is suboptimal within the team, and more value is lost. With both problems, mechanisms that increase the effort exerted by the team's agents recover some of the lost surplus.

This paper explores the role of biased self-perceptions in firms that face effort coordination problems. When agents overestimate their skills and thus the marginal product of their effort, they naturally tend to work harder as, for them, the extra cost of effort is more than compensated for by the extra reward they mistakenly expect. Naturally, this reduces free-riding. Such agents also care less about potential complementarities: their own marginal product warrants the extra cost of effort regardless of synergy. Interestingly, this can make the firm and all the agents, including the biased ones, better off. On the one hand, the overinvestment in effort by a biased agent costs some utility, however, it also has a beneficial feedback effect, as the other

¹⁹ These results and proofs are available from the authors upon request.

agents react to the synergistic increase in their marginal product by working harder, thereby increasing the firm's output and the biased agent's share of that payoff.

Our model also generates a set of predictions that can be used to guide future empirical work. First, we find that, when complementarities within the firm are sufficiently strong, agent overconfidence leads to Pareto improvements. In fact, when one agent's effort is a substitute for another agent (when s < 0in the model), overconfidence can only make one worse off, as the colleague then scales back on his own effort. This suggests that overconfidence is more likely to persist in firms that benefit more from synergistic teamwork. Second, rational agents make better leaders when complementarities are large. This has implications for the relation between the firm's organization, the bias of its workers, and the nature of production. Third, we find that overconfident agents will receive greater incentive compensation because their bias makes such compensation particularly effective in increasing their firm's productivity. Fourth, for similar reasons, the firm will choose to allocate easier tasks to overconfident rather than rational agents. Fifth, the effect of overconfidence in synergistic teamwork is enhanced when any of the team's agents care about reputation. Sixth, overconfidence makes mergers valuable but can also deter them: mergers are expected to take place when overconfidence can prompt the realization of sufficiently strong synergy, but when the target firm is run by an overconfident agent, it is harder to convince him to join forces.

We believe that our model can also apply to other settings in corporate finance. One is the multi-division firm. Division managers typically face effort coordination problems. As in our model, each division manager bears the full costs of his effort but shares the gains with others. This point has been discussed and demonstrated in a number of articles, including Boot and Schmeits (2000), and Scharfstein and Stein (2000). Moreover, complementarities, which are important in our model, may arise in this context as a simple result of production synergy. They may also be a product of financing spillovers made possible by efficient internal capital markets. Indeed, Stein (1997) shows that the success of one division provides more resources to the firm and thus enables other divisions to get more financing for their own investments. This may heighten the incentive of other division managers to be productive (although we should also point out that the competition for resources may have an opposite effect on endogenous incentives, as shown by Brusco and Panunzi (2005)).

Another setting to which our model naturally applies is venture capital. The idea that the venture capital function is plagued by a two-sided moral hazard problem between the venture capitalist (VC) and the entrepreneur can be found in Sahlman (1990), Lerner (1995), Hellmann and Puri (2002), and

Kaplan and Strömberg (2004). These authors argue that, in addition to the contribution that the entrepreneur's effort is bound to have on success, the VC's effort in monitoring, advising, and organizing the company can also affect its fate. Therefore, it is reasonable to think of the relationship between the entrepreneur and the VC as an effort coordination problem in which the effort of one benefits both, as in the model of venture capital proposed by Casamatta (2003). In this context, complementarities between the entrepreneur and the VC are also likely to exist because the dedication of one to the company could potentially increase the dedication of the other. For example, a VC with limited human capital may choose to allocate more of it to a company in which the entrepreneur appears to be fully engaged. Likewise, the entrepreneur is less likely to turn his attention to alternative outside opportunities if he senses the committed support of the VC.

Appendix

Proof of Lemma 1. In equilibrium, it must be the case that e_1 is agent 1's optimal response to an effort level of e_2 by agent 2, and that e_2 is agent 2's optimal response to an effort level of e_1 by agent 1. That is, e_1 and e_2 must solve (4) and (5). It is easy to verify that (6) and (7) are the unique solution to this problem. To finish, we need to verify that e_1 and e_2 are both positive as, otherwise, the second-order conditions are not satisfied and corner solutions are optimal for e_1 and e_2 . Because s < 1 and $\sigma \le 2$, we have $s^2w_1w_2 < w_1w_2 \le \frac{\sigma}{2} \cdot \frac{\sigma}{2} \le 1$, which implies that the denominators in (6) and (7) are both positive.

Proof of Proposition 1. The firm's value is given by (8) using the equilibrium effort levels e_1 and e_2 derived in Lemma 1. From (6) and (7), we have

$$\frac{\partial e_1}{\partial b} = \frac{sw_1w_2}{1 - s^2w_1w_2} \quad \text{and} \quad \frac{\partial e_2}{\partial b} = \frac{w_2}{1 - s^2w_1w_2}.$$
 (A1)

Since s < 1 and $w_1 w_2 \le \frac{\sigma}{2} \cdot \frac{\sigma}{2} \le 1$, both of these quantities are positive, and so

$$\frac{\partial F}{\partial b} = (\sigma - w_1 - w_2) \left(a_1 \frac{\partial e_1}{\partial b} + a_2 \frac{\partial e_2}{\partial b} + se_2 \frac{\partial e_1}{\partial b} + se_1 \frac{\partial e_2}{\partial b} \right) > 0.$$

For any given effort levels e_1 and e_2 , we have

$$\mathbf{E}[\tilde{U}_1] = w_1 \mathbf{E}[\pi] - c(e_1) = w_1 \left(a_1 e_1 + a_2 e_2 + s e_1 e_2 \right) - \frac{e_1^2}{2}, \tag{A2}$$

so that, using (A1),

$$\begin{aligned} \frac{\partial \mathbf{E}[\tilde{U}_1]}{\partial b} &= w_1 \left(a_1 \frac{\partial e_1}{\partial b} + a_2 \frac{\partial e_2}{\partial b} + se_2 \frac{\partial e_2}{\partial b} + se_1 \frac{\partial e_1}{\partial b} \right) - e_1 \frac{\partial e_1}{\partial b} \\ &= \frac{w_1 (a_1 s w_1 w_2 + a_2 w_2 + s^2 e_2 w_1 w_2 + se_1 w_2) - se_1 w_1 w_2}{1 - s^2 w_1 w_2} \\ &= \frac{w_1 w_2 (a_1 s w_1 + a_2 + s^2 e_2 w_1)}{1 - s^2 w_1 w_2}, \end{aligned}$$

which is clearly positive. Similarly,

$$\mathbf{E}[\tilde{U}_2] = w_2 \mathbf{E}[\pi] - c(e_2) = w_2 (a_1 e_1 + a_2 e_2 + s e_1 e_2) - \frac{e_2^2}{2},$$
(A3)

so that, using (A1),

$$\frac{\partial \mathbf{E}[\tilde{U}_2]}{\partial b} = w_2 \left(a_1 \frac{\partial e_1}{\partial b} + a_2 \frac{\partial e_2}{\partial b} + se_2 \frac{\partial e_2}{\partial b} + se_1 \frac{\partial e_1}{\partial b} \right) - e_2 \frac{\partial e_2}{\partial b}$$
$$= \frac{w_2 (a_1 s w_1 w_2 + a_2 w_2 + s^2 e_2 w_1 w_2 + se_1 w_2) - e_2 w_2}{1 - s^2 w_1 w_2}$$
$$= \frac{w_2 [a_1 s w_1 w_2 + a_2 w_2 + se_1 w_2 - (1 - s^2 w_1 w_2)e_2]}{1 - s^2 w_1 w_2}.$$

This quantity is positive if and only if the expression in brackets is positive. Using the fact that $(1 - s^2 w_1 w_2)e_2 = (a_2 + b + a_1 s w_1)w_2$ from (7), this condition can be rewritten as

$$a_1sw_1w_2 + a_2w_2 + se_1w_2 - (a_2 + b + a_1sw_1)w_2 > 0,$$

which simplifies to $se_1 - b > 0$. Using (6), this inequality can be rewritten as

$$\frac{\left[a_1 + (a_2 + b)sw_2\right]sw_1}{1 - s^2w_1w_2} > b,$$

which further simplifies to (9).

Proof of Lemma 2. With $a_1 = a_2 = 0$, the value of the firm is

$$F = (\sigma - w_1 - w_2) \mathbf{E}[\pi] = (\sigma - w_1 - w_2) s e_1 e_2,$$

where e_1 and e_2 are given by (6) and (7) with $a_1 = a_2 = 0$. That is,

$$F = \frac{(\sigma - w_1 - w_2) b^2 s^2 w_1 w_2^2}{(1 - s^2 w_1 w_2)^2}.$$

The firm chooses w_1 and w_2 to maximize this quantity. It is straightforward to verify that the first-order conditions for this maximization problem are equivalent to

$$(\sigma - w_2)(1 + s^2 w_1 w_2) - 2w_1 = 0$$
 and
 $2(\sigma - w_1) - w_2(3 - s^2 w_1 w_2) = 0.$

The only real values for w_1 and w_2 that solve these equations are given by (11), and the second-order conditions can be verified easily.

Proof of Proposition 2. Because the optimal compensation contract in Lemma 2 does not vary with b, we can use the results of Proposition 1 to prove this proposition. Parts (i) and (ii) follow directly from parts (i) and (ii) of Proposition 1. When $a_1 = a_2 = 0$, condition (9) in part (iii) of Proposition 1 reduces to $s^2w_1w_2 > \frac{1}{2}$. After we replace w_1 and w_2 by their optimal values in Lemma 2, this inequality reduces to $s^2\sigma^2 > \frac{8}{3}$.

Proof of Lemma 3. Biased leader scenario. If the biased agent is successful (i.e., if $\tilde{\epsilon}_{\rm L} = 1$), the rational agent's expected utility is $w_{\rm F}(a_{\rm L} + a_{\rm F}e_{\rm F} + se_{\rm F}) - \frac{1}{2}e_{\rm F}^2$. His effort choice is therefore $e_{\rm FI} = w_{\rm F}(a_{\rm F} + s)$. If the biased agent is not successful (i.e., if $\tilde{\epsilon}_{\rm L} = 0$), the rational agent's expected utility is $w_{\rm F}a_{\rm F}e_{\rm F} - \frac{1}{2}e_{\rm F}^2$. Therefore, his effort choice is $e_{\rm F0} = w_{\rm F}a_{\rm F}$. Taking these subsequent effort choices into account, the biased agent's expected utility is

$$\mathbf{E}_{\mathrm{B}}\left[\tilde{U}_{\mathrm{L}}\right] = w_{\mathrm{L}}\left[(a_{\mathrm{L}}+b)e_{\mathrm{L}}+a_{\mathrm{F}}\overline{e}_{\mathrm{F}}+se_{\mathrm{L}}e_{\mathrm{FI}}\right] - \frac{1}{2}e_{\mathrm{L}}^{2},$$

where $\overline{e}_{\rm F} \equiv e_{\rm L}e_{\rm FI} + (1 - e_{\rm L})e_{\rm F0} = (a_{\rm F} + se_{\rm L})w_{\rm F}$. His effort choice is therefore $e_{\rm L} = w_{\rm L} [a_{\rm L} + b + sw_{\rm F}(2a_{\rm F} + s)]$.

Rational leader scenario. If the rational agent is successful (i.e., if $\tilde{\epsilon}_{\rm L} = 1$), the biased agent's expected utility is $w_{\rm F}[a_{\rm L} + (a_{\rm F} + b)e_{\rm F} + se_{\rm F}] - \frac{1}{2}e_{\rm F}^2$. His effort choice is therefore $e_{\rm F1} = w_{\rm F}(a_{\rm F} + b + s)$. If the rational agent is not successful (i.e., if $\tilde{\epsilon}_{\rm L} = 0$), the biased agent's expected utility is $w_{\rm F}(a_{\rm F} + b)e_{\rm F} - \frac{1}{2}e_{\rm F}^2$. Therefore, his effort choice is $e_{\rm F0} = w_{\rm F}(a_{\rm F} + b)$. Taking these subsequent effort choices into account, the rational agent's expected utility is

$$\mathbf{E}\big[\tilde{U}_{\rm L}\big] = w_{\rm L}\big(a_{\rm L}e_{\rm L} + a_{\rm F}\overline{e}_{\rm F} + se_{\rm L}e_{\rm Fl}\big) - \frac{1}{2}e_{\rm L}^2,$$

where $\overline{e}_{\rm F} \equiv e_{\rm L}e_{\rm F1} + (1 - e_{\rm L})e_{\rm F0} = (a_{\rm F} + b + se_{\rm L})w_{\rm F}$. His effort choice is therefore $e_{\rm L} = w_{\rm L}[a_{\rm L} + sw_{\rm F}(2a_{\rm F} + b + s)]$.

Proof of Proposition 3. With a leader, the probability that the project is successful is

$$E[\pi] = a_{L}e_{L} + a_{F}[e_{L}e_{F1} + (1 - e_{L})e_{F0}] + se_{L}e_{F1},$$
(A4)

and so the value of the firm is given by

$$F = (\sigma - w_{\rm L} - w_{\rm F}) \mathbf{E}[\pi]$$

= $(\sigma - w_{\rm L} - w_{\rm F}) \Big(a_{\rm L} e_{\rm L} + a_{\rm F} \Big[e_{\rm L} e_{\rm F1} + (1 - e_{\rm L}) e_{\rm F0} \Big] + s e_{\rm L} e_{\rm F1} \Big).$ (A5)

The effort cost of the leader is $c(e_L) = \frac{e_L^2}{2}$, whereas that of the follower is

$$\overline{C}_{\rm F} \equiv \Pr\{\tilde{\epsilon}_{\rm L} = 1\}c(e_{\rm F1}) + \Pr\{\tilde{\epsilon}_{\rm L} = 0\}c(e_{\rm F0}) = e_{\rm L}\frac{e_{\rm F1}^2}{2} + (1 - e_{\rm L})\frac{e_{\rm F0}^2}{2}.$$
 (A6)

Biased leader scenario. We can use (A4) and (A6) to calculate the expected utility of the two agents. The biased leader's expected utility is given by

$$\begin{split} \mathbf{E}\big[\tilde{U}_{L}\big] &= w_{L}\mathbf{E}[\pi] - c(e_{L}) = w_{L}\Big(a_{L}e_{L} + a_{F}\big[e_{L}e_{FI} + (1 - e_{L})e_{F0}\big] + se_{L}e_{FI}\Big) - \frac{e_{L}^{2}}{2} \\ &= w_{L}\Big(a_{L}e_{L} + a_{F}\big[e_{L}w_{F}(a_{F} + s) + (1 - e_{L})w_{F}a_{F}\big] + se_{L}w_{F}(a_{F} + s)\Big) - \frac{e_{L}^{2}}{2} \\ &= w_{L}e_{L}\big[a_{L} + sw_{F}(2a_{F} + s)\big] + w_{L}w_{F}a_{F}^{2} - \frac{e_{L}^{2}}{2}, \end{split}$$

where we have used e_{F1} and e_{F0} derived in Lemma 3 for the second equality. From the same lemma, we know that $e_{\text{L}} = w_{\text{L}} [a_{\text{L}} + b + sw_{\text{F}}(2a_{\text{F}} + s)]$, so that $\frac{\partial e_{\text{L}}}{\partial b} = w_{\text{L}}$. Therefore

$$\frac{\partial \mathbf{E}[\tilde{U}_{\scriptscriptstyle \mathrm{L}}]}{\partial b} = w_{\scriptscriptstyle \mathrm{L}}^2 \big[a_{\scriptscriptstyle \mathrm{L}} + s w_{\scriptscriptstyle \mathrm{F}} (2a_{\scriptscriptstyle \mathrm{F}} + s) \big] - e_{\scriptscriptstyle \mathrm{L}} w_{\scriptscriptstyle \mathrm{L}} = -b w_{\scriptscriptstyle \mathrm{L}}^2 < 0.$$

The rational follower's expected utility is given by

$$\begin{split} \mathbf{E}\big[\tilde{U}_{\rm F}\big] &= w_{\rm F}\mathbf{E}[\pi] - \overline{C}_{\rm F} = w_{\rm F}\Big(a_{\rm L}e_{\rm L} + a_{\rm F}\big[e_{\rm L}e_{\rm FI} + (1-e_{\rm L})e_{\rm F0}\big] + se_{\rm L}e_{\rm FI}\Big) \\ &- e_{\rm L}\frac{e_{\rm FI}^2}{2} - (1-e_{\rm L})\frac{e_{\rm F0}^2}{2} \\ &= w_{\rm F}\Big(a_{\rm L}e_{\rm L} + a_{\rm F}\big[e_{\rm L}w_{\rm F}(a_{\rm F}+s) + (1-e_{\rm L})w_{\rm F}a_{\rm F}\big] + se_{\rm L}w_{\rm F}(a_{\rm F}+s)\Big) \\ &- e_{\rm L}\frac{w_{\rm F}^2(a_{\rm F}+s)^2}{2} - (1-e_{\rm L})\frac{w_{\rm F}^2a_{\rm F}^2}{2} \\ &= w_{\rm F}e_{\rm L}\left[a_{\rm L} + \frac{1}{2}w_{\rm F}(a_{\rm F}+s)^2 - \frac{1}{2}w_{\rm F}a_{\rm F}^2\right] + \frac{w_{\rm F}^2a_{\rm F}^2}{2}, \end{split}$$

and therefore

$$\frac{\partial \mathrm{E}[\tilde{U}_{\mathrm{F}}]}{\partial b} = w_{\mathrm{F}}w_{\mathrm{L}}\left[a_{\mathrm{L}} + \frac{1}{2}w_{\mathrm{F}}(a_{\mathrm{F}} + s)^2 - \frac{1}{2}w_{\mathrm{F}}a_{\mathrm{F}}^2\right] > 0.$$

Using e_{F1} and e_{F0} from Lemma 3 in (A5), we have

$$F = (\sigma - w_{\rm L} - w_{\rm F}) \Big(a_{\rm L} e_{\rm L} + a_{\rm F} \Big[e_{\rm L} w_{\rm F} (a_{\rm F} + s) + (1 - e_{\rm L}) w_{\rm F} a_{\rm F} \Big] + s e_{\rm L} w_{\rm F} (a_{\rm F} + s) \Big),$$

and so

$$\frac{\partial F}{\partial b} = (\sigma - w_{\mathrm{L}} - w_{\mathrm{F}}) \Big(a_{\mathrm{L}} w_{\mathrm{L}} + a_{\mathrm{F}} \Big[w_{\mathrm{L}} w_{\mathrm{F}}(a_{\mathrm{F}} + s) - w_{\mathrm{L}} w_{\mathrm{F}} a_{\mathrm{F}} \Big] + s w_{\mathrm{L}} w_{\mathrm{F}}(a_{\mathrm{F}} + s) \Big) > 0.$$

Rational leader scenario. In this case, the rational leader's expected utility is given by

$$\begin{split} \mathbf{E}\big[\tilde{U}_{L}\big] &= w_{L}\mathbf{E}[\pi] - c(e_{L}) = w_{L}\Big(a_{L}e_{L} + a_{F}\big[e_{L}e_{F1} + (1 - e_{L})e_{F0}\big] + se_{L}e_{F1}\Big) - \frac{e_{L}^{2}}{2} \\ &= w_{L}\Big(a_{L}e_{L} + a_{F}\big[e_{L}w_{F}(a_{F} + b + s) + (1 - e_{L})w_{F}(a_{F} + b)\big] \\ &+ se_{L}w_{F}(a_{F} + b + s)\Big) - \frac{e_{L}^{2}}{2} \\ &= w_{L}e_{L}\big[a_{L} + sw_{F}(2a_{F} + b + s)\big] + w_{L}w_{F}a_{F}(a_{F} + b) - \frac{e_{L}^{2}}{2} \\ &= \frac{e_{L}^{2}}{2} + w_{L}w_{F}a_{F}(a_{F} + b), \end{split}$$

where we have used $e_{\rm L} = w_{\rm L} [a_{\rm L} + sw_{\rm F}(2a_{\rm F} + b + s)]$ from Lemma 3 for the last equality. Since $\frac{\partial e_{\rm L}}{\partial b} = sw_{\rm L}w_{\rm F}$, we have

$$\frac{\partial \mathrm{E}[\tilde{U}_{\scriptscriptstyle \mathrm{L}}]}{\partial b} = e_{\scriptscriptstyle \mathrm{L}} s w_{\scriptscriptstyle \mathrm{L}} w_{\scriptscriptstyle \mathrm{F}} + w_{\scriptscriptstyle \mathrm{L}} w_{\scriptscriptstyle \mathrm{F}} a_{\scriptscriptstyle \mathrm{F}} > 0.$$

The biased follower's expected utility is given by

$$\begin{split} \mathbf{E}[\tilde{U}_{\rm F}] &= w_{\rm F} \mathbf{E}[\pi] - \overline{C}_{\rm F} = w_{\rm F} \Big(a_{\rm L} e_{\rm L} + a_{\rm F} \big[e_{\rm L} e_{\rm F1} + (1 - e_{\rm L}) e_{\rm F0} \big] + s e_{\rm L} e_{\rm F1} \Big) \\ &- e_{\rm L} \frac{e_{\rm F1}^2}{2} - (1 - e_{\rm L}) \frac{e_{\rm F0}^2}{2} \\ &= w_{\rm F} \Big(a_{\rm L} e_{\rm L} + a_{\rm F} \big[e_{\rm L} w_{\rm F} (a_{\rm F} + b + s) + (1 - e_{\rm L}) w_{\rm F} (a_{\rm F} + b) \big] \end{split}$$

$$+ se_{L}w_{F}(a_{F} + b + s) - e_{L}\frac{w_{F}^{2}(a_{F} + b + s)^{2}}{2} - (1 - e_{L})\frac{w_{F}^{2}(a_{F} + b)^{2}}{2}$$
$$= w_{F}e_{L}\left[a_{L} + \frac{1}{2}w_{F}s(2a_{F} + s)\right] + \frac{1}{2}w_{F}^{2}(a_{F}^{2} - b^{2}),$$

and therefore

$$\frac{\partial \mathbf{E}[\tilde{U}_{\mathrm{F}}]}{\partial b} = w_{\mathrm{F}} s w_{\mathrm{L}} w_{\mathrm{F}} \left[a_{\mathrm{L}} + \frac{1}{2} w_{\mathrm{F}} s (2a_{\mathrm{F}} + s) \right] - b w_{\mathrm{F}}^2,$$

which is increasing if and only if $b < sw_{L} \left[a_{L} + \frac{1}{2}(2a_{F} + s)sw_{F} \right]$. Using e_{FI} and e_{F0} from Lemma 3 in (A5), we have

$$F = (\sigma - w_{\rm L} - w_{\rm F}) \Big(a_{\rm L} e_{\rm L} + a_{\rm F} \Big[e_{\rm L} w_{\rm F} (a_{\rm F} + b + s) + (1 - e_{\rm L}) w_{\rm F} (a_{\rm F} + b) \Big] + s e_{\rm L} w_{\rm F} (a_{\rm F} + b + s) \Big) = (\sigma - w_{\rm L} - w_{\rm F}) \Big(e_{\rm L} \Big[a_{\rm L} + s w_{\rm F} (2a_{\rm F} + b + s) \Big] + w_{\rm F} a_{\rm F} (a_{\rm F} + b) \Big) = (\sigma - w_{\rm L} - w_{\rm F}) \Big[e_{\rm L}^{2} + w_{\rm F} a_{\rm F} (a_{\rm F} + b) \Big],$$

and so

$$\frac{\partial F}{\partial b} = (\sigma - w_{\rm L} - w_{\rm F})(2e_{\rm L}sw_{\rm L}w_{\rm F} + w_{\rm F}a_{\rm F}) > 0.$$

Proof of Lemma 4. The firm's problem is to choose $w_{\rm L}$ and $w_{\rm F}$ in order to maximize

$$F = (\sigma - w_{\text{L}} - w_{\text{F}})E[\pi] = (\sigma - w_{\text{L}} - w_{\text{F}})se_{\text{L}}e_{\text{FI}},$$

as value only gets created when $\tilde{\epsilon}_{\rm L} = 1$. When the leader is biased, we have $e_{\rm L} = w_{\rm L}(b + s^2 w_{\rm F})$ and $e_{\rm Fl} = s w_{\rm F}$ from Lemma 3 (with $a_{\rm L} = a_{\rm F} = 0$). After simplifications, the first-order conditions (with respect to $w_{\rm L}$ and to $w_{\rm F}$ respectively) for this maximization problem are

$$0 = \sigma - 2w_{\rm L} - w_{\rm F}, \quad \text{and} \tag{A7}$$

$$0 = (\sigma - w_{\rm L} - w_{\rm F})(b + 2s^2 w_{\rm F}) - w_{\rm F}(b + s^2 w_{\rm F}).$$
(A8)

The first of these conditions implies that $w_{L} = \frac{\sigma - w_{F}}{2}$. Using this in (A8), the second condition reduces to

$$-4s^{2}w_{\rm F}^{2}-(3b-2s^{2}\sigma)w_{\rm F}+\sigma b=0.$$

It is easy to verify that this quadratic equation has a unique positive root, and that this root is greater than $\frac{\sigma}{3}$ and less than $\frac{\sigma}{2}$, implying that $w_{\text{L}} \in (\frac{\sigma}{4}, \frac{\sigma}{3})$.

When the leader is rational, we have $e_{\rm L} = s w_{\rm L} w_{\rm F} (b+s)$ and $e_{\rm FI} = w_{\rm F} (b+s)$ from Lemma 3 (with $a_{\rm L} = a_{\rm F} = 0$). After simplifications, the first-order conditions (with respect to $w_{\rm L}$ and to $w_{\rm F}$ respectively) for this maximization problem are

$$0 = \sigma - 2w_{\rm L} - w_{\rm F}, \quad \text{and} \tag{A9}$$

$$0 = (\sigma - w_{\rm L} - w_{\rm F})2s(b+s) - s(b+s).$$
(A10)

It is easy to verify that $w_{\rm L} = \frac{\sigma}{4}$ and $w_{\rm F} = \frac{\sigma}{2}$ uniquely solve these equations.

Proof of Proposition 4. We can use the effort levels and contracts derived in Lemma 3 and Lemma 4 to calculate the value of the firm. With a biased leader, this value is given by

$$F = (\sigma - w_{\scriptscriptstyle \rm L} - w_{\scriptscriptstyle \rm F}) s e_{\scriptscriptstyle \rm L} e_{\scriptscriptstyle \rm Fl} = (\sigma - w_{\scriptscriptstyle \rm L} - w_{\scriptscriptstyle \rm F}) s^2 w_{\scriptscriptstyle \rm L} w_{\scriptscriptstyle \rm F} (b + s^2 w_{\scriptscriptstyle \rm F})$$

with $w_{\rm L}$ and $w_{\rm F}$ as given in (13). After replacing $w_{\rm L}$ and $w_{\rm F}$ and simplifying, this becomes

$$F = \frac{(6s^{2}\sigma + 3b - \Lambda)^{2} (2s^{2}\sigma - 3b + \Lambda) (2s^{2}\sigma + 5b + \Lambda)}{16,384 s^{4}},$$

where $\Lambda \equiv \sqrt{4s^4\sigma^2 + 4s^2\sigma b + 9b^2}$. Tedious but straightforward manipulations yield

$$\left(\frac{\partial F}{\partial b}\right)_{b=0} = \frac{s^2 \sigma^3}{32}.$$
(A11)

With a rational leader, the firm's value is given by

$$F = (\sigma - w_{\rm L} - w_{\rm F})se_{\rm L}e_{\rm F1} = (\sigma - w_{\rm L} - w_{\rm F})s^2w_{\rm L}w_{\rm F}^2(b+s)^2$$

with $w_{\rm L}$ and $w_{\rm F}$ as given in (14). After replacing $w_{\rm L}$ and $w_{\rm F}$ and simplifying, this becomes

$$F = \frac{s^2 \sigma^4 (b+s)^2}{64}.$$

It is then straightforward to show that

$$\left(\frac{\partial F}{\partial b}\right)_{b=0} = \frac{s^3 \sigma^4}{32}.$$
(A12)

The firm will appoint the rational agent as its leader when (A12) is greater than (A11) or, equivalently, when $s\sigma > 1$.

Proof of Proposition 5. We know from Lemma 2 that, once the two agents work for the same firm, firm value is maximized with $w_1 = \frac{2\sigma}{8-s^2\sigma^2}$ and $w_2 = \frac{\sigma}{2}$. With this contract, we know from Lemma 1 that the equilibrium effort levels of the two agents are given by

$$e_1 = \frac{bs\sigma^2}{2(4-s^2\sigma^2)}$$
 and $e_2 = \frac{b\sigma(8-s^2\sigma^2)}{4(4-s^2\sigma^2)}$.

The (biased) expected utility of agent 2 is then given by

$$\mathbf{E}_{\mathbf{B}}[\tilde{U}_{2}] = w_{2}(be_{2} + se_{1}e_{2}) - \frac{e_{2}^{2}}{2} = \frac{b^{2}\sigma^{2}(8 - s^{2}\sigma^{2})^{2}}{32(4 - s^{2}\sigma^{2})^{2}}.$$

This contract meets the entrepreneur's (i.e., agent 2's) reservation utility if this quantity is at least $\frac{b^2\sigma^2}{2}$, that is, if $s^2\sigma^2 \ge \frac{8}{3}$. Otherwise (i.e., if $s^2\sigma^2 < \frac{8}{3}$), the entrepreneur must be offered more than $\frac{\sigma}{2}$ for him to give up his own firm. More precisely, given the equilibrium effort levels of the two agents in Lemma 1, the compensation contracts must satisfy

$$\frac{b^2 \sigma^2}{2} \le \mathbf{E}_{\scriptscriptstyle \mathrm{B}} \big[\tilde{U}_2 \big] = w_2 \big(be_2 + se_1 e_2 \big) - \frac{e_2^2}{2} = \frac{b^2 w_2^2}{2(1 - s^2 w_1 w_2)^2}$$

or equivalently, $w_2 \ge \frac{\sigma}{1+s^2\sigma w_1}$. Given that the firm gains nothing from offering the entrepreneur more than his reservation wage, its problem is to choose w_1 and w_2 to maximize

$$F = (\sigma - w_1 - w_2)E[\pi] = (\sigma - w_1 - w_2)se_1e_2 = \frac{(\sigma - w_1 - w_2)sb^2w_1w_2^2}{(1 - s^2w_1w_2)^2}$$
(A13)

subject to

$$w_2 = \frac{\sigma}{1 + s^2 \sigma w_1}.\tag{A14}$$

Using (A14) in (A13) and simplifying, the firm's problem reduces to choosing w_1 to maximize

$$F = \left(\sigma - w_1 - \frac{\sigma}{1 + s^2 \sigma w_1}\right) b^2 s^2 \sigma^2 w_1.$$

The first-order condition for this maximization problem is

$$\sigma - 2w_1 - \frac{\sigma}{(1+s^2\sigma w_1)^2} = 0,$$

which can be shown to be equivalent to

$$-2s^4\sigma^2w_1^2 - s^2\sigma(4 - s^2\sigma^2)w_1 + 2(s^2\sigma^2 - 1) = 0.$$

Because the first two terms of this quadratic expression are negative for positive w_1 , there is a unique positive root if and only if $s^2\sigma^2 > 1$ (and otherwise, there is no pair of compensation contracts that can attract the entrepreneur to the first firm). This root is given by (17) and, since $s^2\sigma^2 < \frac{8}{3}$, it can be shown to be smaller than $\frac{1}{s^2\sigma}$. Using (17) in (A14), we get (16) which, given that $w_1 < \frac{1}{s^2\sigma}$, is greater than $\frac{\sigma}{2}$ and smaller than σ .

Proof of Proposition 6. Agent 1's expected utility is given by $E[\tilde{U}_1] = w_1 s e_1 e_2 - \frac{k_1}{2} e_1^2$, which is maximized by choosing

$$e_1 = \frac{w_1 s e_2}{k_1}.$$
 (A15)

Similarly, agent 2's (biased) expected utility is given by $E[\tilde{U}_2] = w_2(be_2 + se_1e_2) - \frac{k_2}{2}e_2^2$, which is maximized by choosing

$$e_2 = \frac{w_2(b + se_1)}{k_2}.$$
 (A16)

Solving for e_1 and e_2 in (A15) and (A16), we get

$$e_1 = \frac{bsw_1w_2}{k_1k_2 - s^2w_1w_2}$$
 and $e_2 = \frac{k_1bw_2}{k_1k_2 - s^2w_1w_2}$.

The firm's problem is to choose w_1 and w_2 to maximize

$$F = (\sigma - w_1 - w_2) \mathbb{E}[\pi] = (\sigma - w_1 - w_2) se_1 e_2$$
$$= \frac{(\sigma - w_1 - w_2) k_1 b^2 s w_1 w_2^2}{(k_1 k_2 - s^2 w_1 w_2)^2}.$$

It is easy to verify that the contracts in (18) solve this maximization problem. With these contracts, it is straightforward to verify that

$$F = \frac{b^2 s^2 \sigma^4}{16k_2 (4k_1 k_2 - s^2 \sigma^2)},$$
(A17)

$$E[\tilde{U}_1] = \frac{k_1 b^2 s^2 \sigma^4}{8(4k_1 k_2 - s^2 \sigma^2)^2}, \quad \text{and}$$
(A18)

$$E[\tilde{U}_2] = \frac{b^2 \sigma^2 (8k_1k_2 - s^2 \sigma^2) (3s^2 \sigma^2 - 8k_1k_2)}{32k_2 (4k_1k_2 - s^2 \sigma^2)^2}.$$
 (A19)

Since $s^2\sigma^2 < 4k_1k_2$ (see footnote 16), it is clear that *F* and $\mathbb{E}[\tilde{U}_1]$ are increasing in *b*, whereas $\mathbb{E}[\tilde{U}_2]$ is increasing in *b* as long as $s^2\sigma^2 > \frac{8k_1k_2}{3}$. From (A17), since k_1k_2 is unaffected when the costly task is assigned to one or the other agent, it is clear that firm value will be lower if the costly task is assigned to agent 2 (i.e., $k_2 > k_1$).

Proof of Proposition 7. With an effort level e_1 , agent 1's expected utility is $(w_1 + r_1)se_1e_2 - \frac{1}{2}e_1^2$, which is maximized at

$$e_1 = (w_1 + r_1)se_2. (A20)$$

Similarly, agent 2's (biased) expected utility is $(w_2 + r_2)(b + se_1)e_2 - \frac{1}{2}e_2^2$, which is maximized at

$$e_2 = (w_2 + r_2)(b + se_1). \tag{A21}$$

Solving for e_1 and e_2 in (A20) and (A21), we get

$$e_1 = \frac{bs(w_1 + r_1)(w_2 + r_2)}{1 - s^2(w_1 + r_1)(w_2 + r_2)}$$
 and $e_2 = \frac{b(w_2 + r_2)}{1 - s^2(w_1 + r_1)(w_2 + r_2)}$

The firm's problem is to choose w_1 and w_2 to maximize

$$F = (\sigma - w_1 - w_2) \mathbf{E}[\pi] = (\sigma - w_1 - w_2) se_1 e_2$$
$$= \frac{(\sigma - w_1 - w_2) b^2 s^2 (w_1 + r_1) (w_2 + r_2)^2}{\left[1 - s^2 (w_1 + r_1) (w_2 + r_2)\right]^2}.$$

It is easy to verify that the contracts in (19) solve this maximization problem. The rest of the proof is similar to that of Proposition 6.

Proof of Proposition 8. From the firm's perspective, the effort level of the two agents is a pair of random variables (independent of each other) with a mean of

$$\overline{e} \equiv \phi e_{\scriptscriptstyle B} + (1-\phi)e_{\scriptscriptstyle R} = \frac{\phi bw}{1-sw},$$

where the last equality is obtained by replacing e_{R} and e_{B} by their equilibrium values in (22) and by simplifying. As such, the firm's problem is to choose w

in order to maximize

$$F = (\sigma - 2w)E[\pi] = (\sigma - 2w)s\overline{e}^2 = \frac{(\sigma - 2w)s\phi^2 b^2 w^2}{(1 - sw)^2}.$$
 (A22)

The first-order condition for this maximization problem is

$$0 = \frac{dF}{dw} = s\phi^2 b^2 \frac{(2\sigma - 6w)w(1 - sw)^2 + 2s(\sigma - 2w)w^2(1 - sw)}{(1 - sw)^4}$$
$$= \frac{2s\phi^2 b^2 w}{(1 - sw)^3} (\sigma - 3w + sw^2).$$

It is easy to verify that (23) is the unique $w \in (0, \frac{\sigma}{2})$ that satisfies this condition and that the second-order condition is satisfied for this w. The fact that Fis proportional to ϕ^2 in (A22) and that the optimal w does not depend on ϕ implies that firm value is increasing in ϕ . The expected utility of rational agents is given by

$$E[\tilde{U}_i] = w \operatorname{Pr} \{\tilde{v} = \sigma\} - c(e_{R}) = wse_{R}\overline{e} - \frac{e_{R}^2}{2}$$
$$= wse_{R}[\phi e_{B} + (1 - \phi)e_{R}] - \frac{e_{R}^2}{2}$$

which, after replacing e_{R} and e_{B} by their equilibrium values in (22) and simplifying, reduces to

$$\mathrm{E}[\tilde{U}_i] = \frac{\phi^2 b^2 s^2 w^4}{2(1-sw)^2}.$$

Since the optimal w does not depend on ϕ , this quantity is clearly increasing in ϕ . The expected utility of biased agents is given by

$$E[\tilde{U}_j] = w \operatorname{Pr} \{\tilde{v} = \sigma\} - c(e_{\scriptscriptstyle B}) = wse_{\scriptscriptstyle B}\overline{e} - \frac{e_{\scriptscriptstyle B}^2}{2}$$
$$= wse_{\scriptscriptstyle B}[\phi e_{\scriptscriptstyle B} + (1-\phi)e_{\scriptscriptstyle R}] - \frac{e_{\scriptscriptstyle B}^2}{2}$$

which, after replacing e_{R} and e_{B} by their equilibrium values in (22) and simplifying, reduces to

$$\mathbf{E}[\tilde{U}_j] = \frac{b^2 w^2 [\phi^2 s^2 w^2 - (1 - sw)^2]}{2(1 - sw)^2}$$

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$$=\frac{b^2w^2[1-(1-\phi)sw][(1+\phi)sw-1]}{2(1-sw)^2}.$$

It is easy to verify that w in (23) is greater than $\frac{1}{s}$ so that the first expression in brackets is positive. Therefore, the welfare of biased agents is increasing in ϕ if and only if $(1 + \phi)sw - 1 > 0$, which is equivalent to

$$1 + \phi > \frac{1}{sw} = \frac{2}{3 - \sqrt{9 - 4s\sigma}} = \frac{2}{3 - \sqrt{9 - 4s\sigma}} \frac{3 + \sqrt{9 - 4s\sigma}}{3 + \sqrt{9 - 4s\sigma}}$$
$$= \frac{3 + \sqrt{9 - 4s\sigma}}{2s\sigma},$$

and in turn equivalent to the second inequality in (24). The first inequality in (24) ensures that $\frac{3+\sqrt{9-4s\sigma}}{2s\sigma} - 1 < 1$ since $\phi \in (0, 1)$.

Proof of Proposition 9. Let \tilde{n} denote the number of overconfident agents working for the firm. Suppose that agent 1 is rational. From his perspective,

 $\Pr\{\tilde{v}=\sigma \mid \tilde{b}_2=b\}=se_{\scriptscriptstyle B} \quad \text{and} \quad \Pr\{\tilde{v}=\sigma \mid \tilde{b}_2=0\}=se_{\scriptscriptstyle R},$

so that

$$\begin{aligned} \Pr\{\tilde{n} = 1 \mid \tilde{v} = \sigma\} &= \Pr\{\tilde{b}_2 = b \mid \tilde{v} = \sigma\} \\ &= \frac{\Pr\{\tilde{v} = \sigma \mid \tilde{b}_2 = b\}\Pr\{\tilde{b}_2 = b\}}{\sum_{\beta \in \{0, b\}} \Pr\{\tilde{v} = \sigma \mid \tilde{b}_2 = \beta\}\Pr\{\tilde{b}_2 = \beta\}} \\ &= \frac{se_{\scriptscriptstyle B}\phi}{se_{\scriptscriptstyle B}\phi + se_{\scriptscriptstyle R}(1 - \phi)} = \frac{e_{\scriptscriptstyle B}\phi}{e_{\scriptscriptstyle B}\phi + e_{\scriptscriptstyle R}(1 - \phi)}.\end{aligned}$$

This quantity is greater than ϕ , the prior probability that a rational agent assigns to \tilde{n} being equal to one, if $e_{\text{B}} > e_{\text{R}}$, which can be shown to hold using (22). Now suppose that agent 1 is biased. From his perspective,

 $\Pr_{\mathsf{B}}\left\{\tilde{v}=\sigma \mid \tilde{b}_{2}=b\right\} = b + se_{\mathsf{B}} \text{ and } \Pr_{\mathsf{B}}\left\{\tilde{v}=\sigma \mid \tilde{b}_{2}=0\right\} = b + se_{\mathsf{R}},$

so that²⁰

$$\Pr_{\mathsf{B}}\{\tilde{n} = 1 \mid \tilde{v} = \sigma\} = \Pr_{\mathsf{B}}\{\tilde{b}_{2} = b \mid \tilde{v} = \sigma\}$$
$$= \frac{\Pr_{\mathsf{B}}\{\tilde{v} = \sigma \mid \tilde{b}_{2} = b\}\Pr\{\tilde{b}_{2} = b\}}{\sum_{\beta \in \{0, b\}}\Pr_{\mathsf{B}}\{\tilde{v} = \sigma \mid \tilde{b}_{2} = \beta\}\Pr\{\tilde{b}_{2} = \beta\}}$$

 $[\]frac{1}{20}$ Note that the overconfident agent thinks that he is skilled, not overconfident.

$$= \frac{(b+se_{\rm B})\phi}{(b+se_{\rm B})\phi+(b+se_{\rm R})(1-\phi)}$$
$$= \frac{(b+se_{\rm B})\phi}{b+s[e_{\rm B}\phi+e_{\rm R}(1-\phi)]}.$$

This quantity is greater than ϕ , the prior probability that an overconfident agent assigns to \tilde{n} being equal to one, if $e_{\rm B} > e_{\rm R}$, which is the case. Finally, from the firm's perspective,

$$\Pr\{\tilde{v} = \sigma \mid \tilde{n} = 2\} = \Pr\{\tilde{v} = \sigma \mid \tilde{b}_1 = b, \tilde{b}_2 = b\} = se_{\scriptscriptstyle B}^2,$$

$$\Pr\{\tilde{v} = \sigma \mid \tilde{n} = 1\} = \Pr\{\tilde{v} = \sigma \mid \tilde{b}_1 = b, \tilde{b}_2 = 0\}$$

$$= \Pr\{\tilde{v} = \sigma \mid \tilde{b}_1 = 0, \tilde{b}_2 = b\} = se_{\scriptscriptstyle B}e_{\scriptscriptstyle R},$$

$$\Pr\{\tilde{v} = \sigma \mid \tilde{n} = 0\} = \Pr\{\tilde{v} = \sigma \mid \tilde{b}_1 = 0, \tilde{b}_2 = 0\} = se_{\scriptscriptstyle R}^2.$$

Therefore,

$$\Pr\{\tilde{n} = 2 \mid \tilde{v} = \sigma\} = \frac{\Pr\{\tilde{v} = \sigma \mid \tilde{n} = 2\}\Pr\{\tilde{n} = 2\}}{\sum_{n=0}^{2} \Pr\{\tilde{v} = \sigma \mid \tilde{n} = n\}\Pr\{\tilde{n} = n\}}$$
$$= \frac{se_{\scriptscriptstyle B}^{2}\phi^{2}}{se_{\scriptscriptstyle B}^{2}\phi^{2} + 2se_{\scriptscriptstyle B}e_{\scriptscriptstyle R}\phi(1-\phi) + se_{\scriptscriptstyle R}^{2}(1-\phi)^{2}}$$
$$= \left(\frac{e_{\scriptscriptstyle B}\phi}{e_{\scriptscriptstyle B}\phi + e_{\scriptscriptstyle R}(1-\phi)}\right)^{2},$$

which is greater than the firm's prior, $\Pr{\{\tilde{n}=2\}} = \phi^2$, if $e_{\text{B}} > e_{\text{R}}$, which is the case. We can show that $\Pr{\{\tilde{n}=0 \mid \tilde{v}=\sigma\}} < \Pr{\{\tilde{n}=0\}}$ in a similar way.

References

Aghion, P. and Tirole, J. (1994) The management of innovation, *Quarterly Journal of Economics* **109**, 1185–1209.

Alchian, A. A. and Demsetz, H. (1972) Production, information costs, and economic organization, *American Economic Review* **62**, 777–795.

Alchian, A. and Woodward, S. E. (1987) Reflections on the theory of the firm, *Journal of Institutional and Theoretical Economics* **143**, 110–136.

Bénabou, R. and Tirole, J. (2002) Self-confidence and personal motivation, *Quarterly Journal* of *Economics* **117**, 871–915.

Ben-David, I., Graham, J. R. and Harvey, C. R. (2007) Managerial overconfidence and corporate policies, working paper, Duke University.

Boot, A. W. and Schmeits, A. (2000) Market discipline and incentive problems in conglomerate firms with applications to banking, *Journal of Financial Intermediation* **9**, 240–273.

Brusco, S. and Panunzi, F. (2005) Reallocation of corporate resources and managerial incentives in internal capital markets, *European Economic Review* **49**, 659–681.

Caruso, E. M., Epley, N. and Bazerman, M. H. (2006) The costs and benefits of undoing egocentric responsibility assessments in groups, *Journal of Personality and Social Psychology* **91**, 857–871.

Casamatta, C. (2003) Financing and advising: optimal financial contracts with venture capitalists, *Journal of Finance* 58, 2059–2085.

Cooper, A. C., Woo, C. Y. and Dunkelberg, W. C. (1988) Entrepreneurs' perceived chances for success, *Journal of Business Venturing* **3**, 97–108.

Dunning, D., Meyerowitz, J. A. and Holzberg, A. D. (1989) Ambiguity and self-evaluation: the role of idiosyncratic trait definitions in self-serving assessments of ability, *Journal of Personality and Social Psychology* **57**, 1082–1090.

Farrell, J. and Scotchmer, S. (1988) Partnerships, *Quarterly Journal of Economics* **103**, 279–297. Felson, R. B. (1984) The effects of self-appraisals of ability on academic performance, *Journal of Personality and Social Psychology* **47**, 944–952.

Fershtman, C. and Judd, K. (1987) Equilibrium incentives in oligopoly, *American Economic Review* 77, 927–940.

Garen, J. E. (1985) Worker heterogeneity, job screening, and firm size, *Journal of Political Economy* **93**, 715–739.

Gaynor, M. and Kleindorfer, P. (1991) Equilibrium misperceptions, *Economics Letters* 35, 27-30.

Gervais, S., Heaton, J. B. and Odean, T. (2007) Overconfidence, investment policy, and manager welfare, working paper, Duke University.

Greenwald, A. G. (1980) The totalitarian ego: fabrication and revision of personal history, *American Psychologist* **35**, 603–618.

Grossman, S. J. and Hart, O. D. (1986) The costs and benefits of ownership: a theory of vertical and lateral integration, *Journal of Political Economy* **94**, 691–719.

Heifetz, A., Shannon, C. M. and Spiegel, Y. (2007) What to maximize if you must, *Journal of Economic Theory* **133**, 31–57.

Hellmann, T. and Puri, M. (2002) Venture capital and the professionalization of start-up firms: empirical evidence, *Journal of Finance* **57**, 169–197.

Hermalin, B. E. (1998) Toward an economic theory of leadership: leading by example, *American Economic Review* **88**, 1188–1206.

Hietala, P., Kaplan, S. N. and Robinson, D. T. (2003) What is the price of hubris? Using takeover battles to infer overpayments and synergies, *Financial Management* **32**, 5–31.

Holmström, B. (1982) Moral hazard in teams, Bell Journal of Economics 13, 324-340.

Holmström, B. and Milgrom, P. (1991) Multitask principal-agent analyses: incentive contracts, asset ownership, and job design, *Journal of Law, Economics, and Organization* **7**, 25–52.

Holmström, B. and Roberts, J. (1998) The boundaries of the firm revisited, *Journal of Economic Perspectives* **12**, 73–94.

Jensen, M. C. and Meckling, W. H. (1995) Specific and general knowledge and organizational structure, *Journal of Applied Corporate Finance* **8**, 4–18.

Kamien, M. I., Muller, E. and Zang, I. (1992) Research joint ventures and R&D cartels, *American Economic Review* 82, 1293–1306.

Kandel, E. and Lazear, E. P. (1992) Peer pressure and partnerships, *Journal of Political Economy* **100**, 801–817.

Kaplan, S. N. and Strömberg, P. (2004) Characteristics, contracts, and actions: evidence from venture capitalist analyses, *Journal of Finance* **59**, 2177–2210.

Kelsey, D. and Spanjers, W. (2004) Ambiguity in partnerships, *Economic Journal* **114**, 528–546. Kunda, Z. (1987) Motivated inference: self-serving generation and evaluation of causal theories, *Journal of Personality and Social Psychology* **53**, 636–647.

Kyle, A. S. and Wang, F. A. (1997) Speculation duopoly with agreement to disagree: can overconfidence survive the market test? *Journal of Finance* **52**, 2073–2090.

Langer, E. and Roth, J. (1975) Heads I win, tails it's chance: the illusion of control as a function of the sequence of outcomes in a purely chance task, *Journal of Personality and Social Psychology* **32**, 951–955.

Larwood, L. and Whittaker, W. (1977) Managerial myopia: self-serving biases in organizational planning, *Journal of Applied Psychology* **62**, 194–198.

Lazear, E. P. and Rosen, S. (1981) Rank-order tournaments as optimum labor contracts, *Journal of Political Economy* **89**, 841–864.

Lerner, J. (1995) Venture capitalists and the oversight of private firms, *Journal of Finance* **50**, 301–318.

Levin, J. and Tadelis, S. (2005) Profit sharing and the role of professional partnerships, *Quarterly Journal of Economics* **120**, 131–171.

Lewis, T. R. and Sappington, D. E. M. (1997) Penalizing success in dynamic incentive contracts: no good deed goes unpunished? *RAND Journal of Economics* **28**, 346–358.

Malmendier, U. and Tate, G. (2005) CEO overconfidence and corporate investment, *Journal of Finance* **60**, 2661–2700.

Malmendier, U., and Tate, G. (2007) Who makes acquisitions? CEO overconfidence and the market's reaction, *Journal of Financial Economics*, forthcoming.

Morris, S. (1996) Speculative investor behavior and learning, *Quarterly Journal of Economics* **111**, 1111–1133.

Morrison, A. D. and Wilhelm, W. J. (2004) Partnership firms, reputation, and human capital, *American Economic Review* 94, 1682–1692.

Puri, M. and Robinson, D. T. (2006) Who are entrepreneurs and why do they behave that way? working paper, Duke University.

Puri, M. and Robinson, D. T. (2007) Optimism and economic choice, *Journal of Financial Economics*, forthcoming.

Rotemberg, J. J. (1994) Human relations in the workplace, *Journal of Political Economy* **102**, 684–717.

Rotemberg, J. J. and Saloner, G. (2000) Visionaries, managers, and strategic direction, *RAND Journal of Economics* **31**, 693–716.

Sahlman, W. A. (1990) The structure and governance of venture-capital organizations, *Journal of Financial Economics* **27**, 473–521.

Sautner, Z. and Weber, M. (2006) How do managers behave in stock option plans? evidence from exercise and survey data, working paper, University of Amsterdam.

Scharfstein, D. S. and Stein, J. C. (2000) The dark side of internal capital markets: divisional rent-seeking and inefficient investment, *Journal of Finance* **55**, 2537–2564.

Schor, J. B. (1993) *The Overworked American: The Unexpected Decline of Leisure*, Basic Books, New York.

Simon, H. A. (1991) Organizations and markets, *Journal of Economic Perspectives* **5**, 25–44. Spence, M. (1973) Job market signaling, *Quarterly Journal of Economics* **87**, 355–374.

Stein, J. C. (1997) Internal capital markets and the competition for corporate resources, *Journal of Finance* **52**, 111–133.

Taylor, S. and Brown, J. D. (1988) Illusion and well-being: a social psychological perspective on mental health, *Psychological Bulletin* **103**, 193–210.

Tirole, J. (1996) A theory of collective reputations (with applications to the persistence of corruption and to firm quality), *Review of Economic Studies* **63**, 1–22.