Trading Frenzies and Their Impact on Real Investment

Itay Goldstein, Emre Ozdenoren, and Kathy Yuan*

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Abstract

We study a model where a capital provider learns from the price of a firm's security in deciding how much capital to provide for new investment. This feedback effect from the financial market to the investment decision gives rise to trading frenzies, where speculators all wish to trade like others, generating large pressure on prices. Coordination among speculators is sometimes desirable for price informativeness and investment efficiency, but speculators' incentives push in the opposite direction, so that they coordinate exactly when it is undesirable. We analyze the effect of various market parameters on the likelihood of trading frenzies to arise.

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Trading frenzies in financial markets occur when many speculators rush to trade in the same direction leading to large pressure on prices. Financial economists have long been searching for the sources of trading frenzies, asking what causes strategic complementarities in speculators' behavior. This phenomenon is particularly puzzling given that the price mechanism in financial markets naturally leads to strategic substitutes, whereby the expected change in price caused by speculators' trades makes others want to trade in the opposite direction.

We argue in this paper that the potential effect that financial-market trading has on the real economy, i.e., on firms' cash flows, may provide the mechanism for trading frenzies to arise. Intuitively, suppose that speculators in the financial market short sell a stock, leading to a decrease in its price. Since the stock price provides information about the firm's profitability, it affects decisions by various agents, such as capital providers. Seeing the decrease in price, capital providers update downwards their expectation of the firm's profitability. This weakens the firm's access to capital and thus hurts its performance.[1] As a result, the firm's value decreases, and short sellers are able to make a profit. This creates a source for complementarities, whereby the expected change in value caused by speculators' trades makes others want to trade in the same direction, and generates a trading frenzy.

We develop a model to study and analyze this phenomenon. In particular, we study an environment where a capital provider decides how much capital to provide to a firm for the purpose of making new real investment. The decision of the capital provider depends on his assessment of the productivity of the proposed investment. In his decision, the capital provider uses two sources of information: his private information and the information aggregated by the price of the firm's security which is traded in the financial market. The reliance of capital provision on financial-market prices establishes the effect that the financial market has on the real economy. We refer to this effect as the 'feedback effect'.[2]

The financial market in our model contains many small speculators trading a security, whose payoff is correlated with the cash flow obtained from the firm's investment. Speculators trade on the basis of information they have about the productivity of the investment. They have access to two signals: the first signal is independent across speculators (conditional on the realization of the productivity), while the second one is correlated among them.[3] The

[1] Other agents that may be affected by the information in the price are managers, employees, customers, etc.

[2] In our model the financial market is a secondary market, and hence the only feedback from it to the firm's cash flow is informational; there is no transfer of cash from the market to the firm.

[3] In our model, the correlation is perfect, but this is not essential.

correlated signals introduce common noise in information into the model, which can be due to a rumor, for example. A trading frenzy occurs when speculators put large weight on the correlated signal relative to the idiosyncratic signal, and so they tend to trade similarly to each other.

To close the model, we introduce noisy price-elastic supply in the financial market. The market is cleared at a price for which the demand from speculators equals the exogenous supply. The endogenous price, in turn, reflects information about the productivity of the investment, as aggregated from speculators' trades. But, given the structure of information and trading, the information in the price contains noise from two sources – the noisy supply and the common noise in speculators' information. The information in the price is then used by the capital provider, together with his private information, when making the decision about capital provision and investment.

Analyzing the weight speculators put on the correlated signal relative to the idiosyncratic signal, we shed light on the determinants of trading frenzies. In a world with no strategic effects, this weight is naturally given by the ratio of precisions between the correlated and the idiosyncratic signals. But, in the equilibrium of our model, there are two strategic effects that shift the weight away from this ratio of precisions. The first effect is the usual outcome of a price mechanism. When speculators put weight on the correlated information, this information gets more strongly reflected in the price, and then the incentive of each individual speculator to put weight on the correlated information decreases. This generates strategic substitutes and pushes the weight that speculators put on the correlated information below the ratio of precisions.[4] The second effect arises due to the feedback effect from the price to the capital provision decision. When speculators put weight on the correlated information, this information gets to have a stronger effect on the capital provision to the firm and hence on the real value of its traded security. Then, the incentive of each speculator to put weight on this information increases. This leads to strategic complementarities that make speculators put a larger weight on the correlated signal.

This second effect is what causes a trading frenzy, leading speculators to put large weight on their correlated information, and to trade in a coordinated fashion. When this effect dominates, our model generates a pattern that looks like a 'run' on a stock by many speculators, who are driven by common noise in their correlated signals (e.g. rumor), leading to a price decline, lack of provision of new capital, and collapse of real value. This echoes some highly publicized events such as the bear raid on Overstock.com in 2005 or the bear

[4]Strategic substitutes due to the price mechanism appear in various forms in the literature on financial markets. See, for example, Grossman and Stiglitz (1980).

raids on Bear Stearns and Lehman Brothers in 2008. Our model also generates a similar pattern in the other direction: many speculators buy the stock, leading to a price increase, provision of more new capital, and increase in real value (in the case where the firm was financially constrained). This resembles events like the internet boom in the late 1990s or the real estate boom that preceded the recent crisis.

Using comparative-statics analysis, we investigate when trading frenzies are more likely to arise. First, we show that speculators are more likely to trade in a coordinated fashion when the supply in the financial market is more elastic with respect to the price. This can be interpreted as a more liquid market. In such a market, the strategic substitutes due to the price mechanism are weak, as informed demand is easily absorbed by the elastic supply without having much of a price impact. Hence, speculators tend to put more weight on correlated information and trade more similarly to each other. Second, we find that when there is small variance in the supply function, i.e., when there is small variance in noise/liquidity trading in the financial market, speculators tend to put large weights on their correlated signals and thus to act in a coordinated fashion. This is because in these situations, the capital provider relies more on the information in the price since the price is less noisy, and so the feedback effect from the market to the firm's cash flows strengthens, increasing the scope of strategic complementarities. Third, the precision of various sources of information also plays an important role in shaping the incentive to rely on correlated vs. uncorrelated information. Intuitively, there will be more coordination when speculators' correlated signals are sharper and when their uncorrelated signals are noisier. Interestingly, there will be more coordination when the capital provider has less precise information of his own, as then the feedback from the market to his decision is stronger.

Another question we ask is whether trading frenzies are good or bad for the efficiency of the capital provision decision. We find that they are sometimes good and sometimes bad, and that there is a conflict between the level of coordination in equilibrium and the one that maximizes the efficiency of the capital provision decision. The efficiency of the capital provision decision is maximized when the informativeness of the price is highest. It turns out that when there is high variance of noise/liquidity trading in the market, higher degree of coordination among speculators increases price informativeness. This is because, in noisy markets, coordination among speculators is beneficial in suppressing the noise in liquidity trading that reduces the informativeness of the price. In such markets, trading frenzies among speculators are actually desirable because they enable decision makers to detect some trace of informed trading in a market subject to large volume of liquidity trading and noise. On the other hand, when the market is less noisy, the importance of coordination among speculators declines, and the additional noise that coordination adds via the excess weight that speculators put on their correlated information (which translates into weight on common noise) makes coordination undesirable. Hence, the conflict arises because high levels of coordination are desirable in noisy markets, but in equilibrium, speculators coordinate more in less noisy markets.

Finally, our model assumes that speculators submit market orders, as in Kyle (1985), i.e., they do not condition on the price when they trade. This is consistent with many real-world situations where traders are prepared to take on price risk to achieve greater immediacy in trading. In the penultimate section of the paper, we extend the model to allow speculators to condition on the price. Strategic interactions disappear if speculators observe exactly the same message from the market that is observed by the capital provider. But, this assumption is not very realistic, as capital providers are exposed to various sources of market information –e.g., future prices (if the capital provider acts with a lag), prices from other markets, or even rumors – that are not all perfectly observable to speculators when they trade. Hence, we develop a version of the model where the speculators condition their trades on the price, but the capital provider is exposed to another piece of information correlated with the price. We show how strategic interactions reemerge and discuss when they lead to frenzies like in our main model.

Our paper can provide a basis for policy discussions regarding the role of financial markets in the economy and potential ways to regulate them. For example, in the recent crisis, regulatory agencies around the world became concerned about the damaging effect of short sales and decided to put restrictions on them. Our model can provide some justification for such restrictions on short sales, or speculative trading in general, since they generate damaging real effects. In fact, only a model where the financial market has a feedback effect on the real economy can potentially explain the concerns expressed by regulators about financial-market trading and guide their policy decisions.

There is a small, but growing, branch of models in financial economics that consider the feedback effect from trading in financial markets to corporate decisions. The basic motivation for this literature goes back to Hayek (1945), who posited that market prices provide an important source of information for various decision makers. Empirical evidence for this link is provided by Luo (2005) and Chen, Goldstein, and Jiang (2007). On the theoretical side, earlier contributions to this literature include Fishman and Hagerty (1992), Leland (1992), Khanna, Slezak, and Bradley (1994), Boot and Thakor (1997), Dow and Gorton (1997), Subrahmanyam and Titman (1999), and Fulghieri and Lukin (2001).

Several recent papers in this literature are more closely related to the mechanism in our paper. Ozdenoren and Yuan (2008) show that the feedback effect from asset prices to the real value of a firm generates strategic complementarities. In their paper, however, the feedback effect is modeled exogenously and is not based on learning. As a result, their paper does not deliver the implications that our paper delivers on the effect of liquidity and various information variables on coordination and efficiency. Khanna and Sonti (2004) also model feedback exogenously and show how a single trader can increase the value of his existing inventory in the stock by trading to affect the value of the firm. Goldstein and Guembel (2008) do analyze learning by a decision maker, and show that this might lead to manipulation of the price by a single potentially informed trader. Hence, the manipulation equilibrium in their paper is not a result of strategic complementarities among heterogeneously informed traders. Hirshleifer et al. (2006) also analyze learning by a decision maker, and show that the feedback effect enables irrational traders who trade on common noise to make a profit. However, in their model, the decision of these traders to trade on noise is exogenous (they act irrationally) and is not endogenized as a result of a coordination problem. Dow, Goldstein, and Guembel (2007) show that the feedback effect generates complementarities in the decision to produce information, but not in the trading decision.

More generally, our paper is related to the literature on informational externalities in financial markets. In particular, Vives (1993), Amador and Weil (2011) and others show how the reliance of agents on public information imposes negative externalities on others, as it reduces the efficiency of learning. Our paper shows how the weight on common information increases due to strategic complementarities that emerge as a result of the informational feedback from the market to real investment. Other papers explore other sources of complementarities in financial markets. For example see, Froot, Scharfstein, and Stein (1992), Hirshleifer, Subrahmanyam, and Titman (1994), Bru and Vives (2002), Veldkamp (2006a and 2006b), Ganguli and Yang (2009), Amador and Weil (2011), and Garcia and Strobl (2011).

Our paper is most closely related to Goldstein, Ozdenoren, and Yuan (2011) and Angeletos, Lorenzoni, and Pavan (2010). Both of these papers derive endogenous complementarities as a result of learning from the aggregate action of agents. To analyze trading frenzies and their impact on real investments, we embed this mechanism in a model of financial markets where a capital provider learns from the price to make an investment decision. Modeling the financial market explicitly enriches the problem in various ways. For example, having a price mechanism introduces strategic substitutes that coexist with the strategic complementarities in the model. Also, the ability of speculators to learn from the aggregate action in the rational-expectations-equilibrium extension gives rise to other effects mentioned above.[5] Hence, our model is substantially different from the above mentioned models. In terms of results, our model generates new insights in the context of our study, such as the effect of supply elasticity and noise trading on coordination in financial markets. We also derive new results on the difference between the equilibrium level of coordination and the efficient level of coordination.

The remainder of this paper is organized as follows. In Section 1, we present the model setup and characterize the equilibrium of the model. In Section 2, we solve the model. Section 3 analyzes the determinants of coordination among speculators in our model. In Section 4, we discuss the implications for the efficiency of investments and the volatility of prices and investments. In Section 5, we extend the model to allow speculators to condition their trades on the price. Section 6 concludes. All proofs are provided in the appendix.

1 Model

The model has one firm and a traded asset. There is a capital provider who has to decide how much capital to provide to the firm for the purpose of making an investment. There are three dates, $t = 0, 1, 2$. At date 0, speculators trade in the asset market based on their information about the fundamentals of the firm. At date 1, after observing the asset price and receiving private information, the capital provider of the firm decides how much capital the firm can have and the firm undertakes investment accordingly. Finally, at date 2, the cash flow is realized and agents get paid.

1.1 Investment

The firm in this economy has access to a production technology, which at time $t = 2$ generates cash flow $\tilde{F}I$. Here, I is the amount of investment financed by the capital provider, and $\tilde{F} \geq 0$ is the level of productivity. Let $\tilde{f}$ denote the natural log of productivity, $\tilde{f} = \ln \tilde{F}$. We assume that $\tilde{f}$ is unobservable and drawn from a normal distribution with mean $\bar{f}$ and variance σ_f^2. We use τ_f to denote $1/\sigma_f^2$. As will become clear later, assuming a log-normal distribution for the productivity shock $\tilde{F}$ enables us to get a tractable closed-form solution.

At time $t = 1$ the capital provider chooses the level of capital I. Providing capital is costly and the capital provider must incur a private non-pecuniary cost of: $C(I) = \frac{1}{2}cI^2$,

[5] Another technical detail that we highlight in the description of the model is the use of log-normal distributions, which is necessary in a setting of feedback from financial-market prices to investment decisions.

where $c > 0$. This cost can be thought of as the cost of raising the capital, which is increasing in the amount of capital provided, or as effort incurred in monitoring the investment (which is also increasing in the size of the investment). The capital provider's benefit increases in the cash flow generated by the investment. To ease the exposition, we say that he captures the full amount $\tilde{F}I$.[6] The capital provider chooses I to maximize the value of the cash flow from investing in the firm's production technology minus his cost of raising capital $C(I)$, conditional on his information set, $\mathcal{F}_l$, at $t = 1$:

$$I = \arg\max_I E[\tilde{F}I - C(I)|\mathcal{F}_l]. \tag{1}$$

The solution to this maximization problem is:

$$I = \frac{E[\tilde{F}|\mathcal{F}_l]}{c}. \tag{2}$$

The capital provider's information set, denoted by $\mathcal{F}_l$, consists of a private signal $\tilde{s}_l$ and the asset price P observed at date 0 (we will elaborate on the formation of P next). That is, $\mathcal{F}_l = \{\tilde{s}_l, P\}$. The private signal $\tilde{s}_l$ is a noisy signal about $\tilde{f}$ with precision τ_l: $\tilde{s}_l = \tilde{f} + \sigma_l\tilde{\epsilon}_l$, where $\tilde{\epsilon}_l$ is distributed normally with mean zero and standard deviation one and $\tau_l = 1/\sigma_l^2$. Later, we will conduct comparative statics with respect to the precision of the capital provider's private signal. It is important to emphasize that even though our capital provider learns from the information in the price, he still may have good sources of private information. In fact, his signal can be more precise than other signals in the economy. Despite this, he still attempts to learn from the market, as agents in the market have other signals that are aggregated by the price.

1.2 Speculative Trading

The traded asset is a claim on the payoff from the firm's investment $\tilde{F}I$, which is realized at the final date $t = 2$. The price of this risky asset at $t = 0$ is denoted by P. In Section 3.5, we discuss the nature of the traded asset and compare it with alternatives. A simple way to think about the traded asset is as a derivative, whose payoff is tied to the return from the investment. However, as long as the cost of the investment $C(I)$ is a private non-pecuniary cost incurred by the capital provider, then the value of the firm is $\tilde{F}I$, and hence our traded

[6]As we discuss below, the model will generate similar results if we assume that the capital provider gets a portion of the cash flow: $\beta\tilde{F}I$. But, then we would need to carry another parameter, β.

asset can be viewed as equity.[7] This interpretation can also be made possible in some cases where only part of the cost is privately incurred by the capital provider.

Note that the value of the traded asset here increases in the firm's investment. This is a typical feature of equity in a financially-constrained firm, where equity holders would like to see more capital invested in their firm, but are constrained in how much capital they can raise, due to additional costs borne by capital providers. Hence, our model is suitable to explain trading frenzies and their real impact in such firms. Indeed, our motivating examples, discussed in the introduction, involve financially-constrained firms.[8]

In the market, there is a measure-one continuum of heterogeneously informed risk-neutral speculators indexed by $i \in [0,1]$. Each speculator is endowed with two signals about $\tilde{f}$ at time 0. The first signal, $\tilde{s}_i = \tilde{f} + \sigma_s \tilde{\epsilon}_i$, is privately observed where $\tilde{\epsilon}_i$ is independently normally distributed across speculators with mean zero and unit variance. The precision of this signal is denoted as $\tau_s = 1/\sigma_s^2$. The second signal is $\tilde{s}_c = \tilde{f} + \sigma_c \tilde{\epsilon}_c$. This signal is observed by all speculators and $\tilde{\epsilon}_c$ is independently and normally distributed with mean zero and unit variance and $\tau_c = 1/\sigma_c^2$.[9]

Each speculator can buy or sell up to a unit of the risky asset. The size of speculator i's position is denoted by $x(i) \in [-1, 1]$. This position limit can be justified by limited capital and/or borrowing constraints faced by speculators.[10] Due to risk neutrality, speculators choose their positions to maximize expected profits. A speculator's profit from shorting one unit of the asset is given by $P - \tilde{F}I$, where $\tilde{F}I$ is the asset payoff and P is the price of the

[7]In this case, we could analyze our model assuming that this value is shared between the capital provider and shareholders, such that the former receives $\beta\tilde{F}I$ and the latter receive $(1-\beta)\tilde{F}I$. This would not change our results, but will add complexity due to the additional parameter β. Hence, we omit β in the paper.

[8]It should be noted that, no matter what the nature of the asset is, our market is a secondary market with no cash transfers to the firm. The only effect of the market on the firm will be via the information revealed in the trading process.

[9]The assumption that the second signal is a common signal greatly simplifies the analysis. However, it is not necessary. The necessary element is that the noise in the information observed by speculators has a common component that cannot be fully teased out by the capital provider. In Goldstein, Ozdenoren, and Yuan (2010), we analyzed an alternative setup, where the second signal is specified as a heterogenous private signal with a common noise component $\tilde{\epsilon}_c$ and an agent-specific noise component $\tilde{\epsilon}_{2i}$. That is, $\tilde{s}_{ci} = \tilde{f} + \sigma_c \tilde{\epsilon}_c + \sigma_{\epsilon 2} \tilde{\epsilon}_{2i}$, where $\tilde{\epsilon}_c$ and $\tilde{\epsilon}_{2i}$ are independently normally distributed variables with mean zero and variance one. That paper, however, was simpler on other dimensions, as there was no price formation for the traded asset.

[10]The specific size of this position limit on asset holdings is not crucial for our results. What is crucial is that informed speculators cannot take unlimited positions; if they do, strategic interaction among informed speculators will become immaterial.

asset. Similarly, a speculator's profit from buying one unit of the asset is given by $\tilde{F}I - P$.

Formally, speculator i chooses $x(i)$ to solve:

$$\max_{x(i)\in[-1,1]} x\left(i\right)E\left[\tilde{F}I - P|\mathcal{F}_i\right], \tag{3}$$

where $\mathcal{F}_i$ denotes the information set of speculator i and consists of $\tilde{s}_i$ and $\tilde{s}_c$. Since each speculator has measure zero and is risk neutral, an informed speculator optimally chooses to either short up to the position limit, or buy up to the position limit. We denote the aggregate demand by speculators as $X = \int_0^1 x\left(i\right)di$, which is given by the fraction of speculators who buy the asset minus the fraction of those who short the asset. For now, we assume that speculators do not observe the price when they trade, and hence they submit market orders, as in Kyle (1985). We discuss the role of this assumption in the extension in Section 5.

1.3 Market Clearing

At date 0, conditional on his information, each speculator submits a market order to buy or sell a unit of the asset to a Walrasian auctioneer. The Walrasian auctioneer then obtains the aggregate demand by speculators X and also a noisy supply curve from uninformed traders, and sets a price to clear the market. The noisy supply of the risky asset is exogenously given by $Q(\tilde{\xi}, P)$, a continuous function of an exogenous demand shock $\tilde{\xi}$ and the price P. The supply curve $Q(\tilde{\xi}, P)$ is strictly decreasing in $\tilde{\xi}$, and increasing in P, that is, it is upward sloping in price. The demand shock $\tilde{\xi} \in \mathbb{R}$ is independent of other shocks in the economy, and $\tilde{\xi} \sim N(0, \sigma_\xi^2)$. As always, we denote $\tau_\xi = 1/\sigma_\xi^2$.

The usual interpretation of noisy supply/demand is that there are agents who trade for exogenous reasons, such as liquidity or hedging needs. They are usually referred to as "noise traders". Several papers in the finance literature have explicitly endogenized the actions of these traders in simpler settings, but doing so here will significantly complicate the model. One possibility is that the scale of noise trading will depend on the amount of information available in the market. In additional analysis we conducted, we show that allowing noise trading to depend on the informational parameters of our model (the precisions of the different signals) does not change our results.[11] In future work, it will be interesting to endogenize noise trading more fully, understanding how their presence is affected by the potential for trading frenzies. In this paper, we only derive comparative statics in the other direction, analyzing the effect of the amount of noise trading in the market (captured by σ_ξ^2) on the likelihood and desirability of trading frenzies.

[11] Details are available upon request.

To solve the model in closed form, we assume that $Q(\tilde{\xi}, P)$ takes the following functional form:

$$Q(\xi, P) = 1 - 2\Phi\left(\tilde{\xi} - \alpha \ln P\right), \tag{4}$$

where $\Phi(\cdot)$ denotes the cumulative standard normal distribution function. The parameter α captures the elasticity of the supply curve with respect to the price. It can be interpreted as the liquidity of the market: when α is high, the supply is very elastic with respect to the price, and so large informed demand is easily absorbed in the price without having much of a price impact. This notion of liquidity is similar to that in Kyle (1985), where liquidity is considered high when the informed trader has a low price impact. The basic features assumed in (4), i.e., that the supply is increasing in price and also has a noisy component, are standard in the literature. It is also common in the literature to assume particular functional forms to obtain tractability. The specific functional form assumed here is close to that in Dasgupta (2007) and Hellwig, Mukherji, and Tsyvinski (2006) .

1.4 Equilibrium

We now turn to the definition of equilibrium.

Definition 1: [Equilibrium with Market Orders] An equilibrium consists of a price function, $P(\tilde{f}, \tilde{\epsilon}_c, \tilde{\xi}) : \mathbb{R}^3 \to \mathbb{R}$, an investment policy for the capital provider $I(\tilde{s}_l, P) : \mathbb{R}^2 \to \mathbb{R}$, strategies for speculators, $x(\tilde{s}_i, \tilde{s}_c) : \mathbb{R}^2 \to [-1, 1]$, and the corresponding aggregate demand $X(\tilde{f}, \tilde{\epsilon}_c)$, such that:

- For speculator i, $x(\tilde{s}_i, \tilde{s}_c) \in \arg\max_{x(i)\in[-1,1]} x(i)E\left[\tilde{F}I - P|\tilde{s}_i, \tilde{s}_c\right]$;
- The capital provider's investment is $I(\tilde{s}_l, P) = E\left[\tilde{F}|\tilde{s}_l, P\right]/c$.
- The market clearing condition for the risky asset is satisfied:

$$Q(\tilde{\xi}, P) = X(\tilde{f}, \tilde{\epsilon}_c) \equiv \int x(\tilde{f} + \sigma_s\tilde{\epsilon}_i, \tilde{f} + \sigma_c\tilde{\epsilon}_c)d\Phi\left(\tilde{\epsilon}_i\right). \tag{5}$$

Definition 2: A *linear monotone equilibrium* is an equilibrium where $x(\tilde{s}_i, \tilde{s}_c) = 1$ if $\tilde{s}_i + k\tilde{s}_c \geq g$ for constants k and g, and $x(\tilde{s}_i, \tilde{s}_c) = -1$ otherwise.

In words: in a monotone linear equilibrium, a speculator buys the asset if and only if a linear combination of his signals is above a cutoff g, and sells it otherwise. In the rest of the paper we focus on linear monotone equilibria.

2 Solving the Model

In this section, we explain the main steps that are required to solve our model. Restricting attention to a linear monotone equilibrium, we first use the market clearing condition to determine the asset price. We then characterize the information content of the asset price to derive the capital provider's belief on $\tilde{f}$ based on $\{P, \tilde{s}_l\}$ and solve for the optimal investment problem. Finally, given the capital provider's investment rule and the asset pricing rule, we solve for individual speculators' optimal trading decision.

In a linear monotone equilibrium, speculators short the asset whenever $\tilde{s}_i + k\tilde{s}_c \leq g$ or, equivalently, $\sigma_s\tilde{\epsilon}_i \leq g - (1+k)\tilde{f} - k\sigma_c\tilde{\epsilon}_c$. Hence, their aggregate selling can be characterized by: $\Phi\left(\left(g - (1+k)\tilde{f} - k\sigma_c\tilde{\epsilon}_c\right)/\sigma_s\right)$. Conversely, they purchase the asset whenever $\tilde{s}_i + k\tilde{s}_c \geq g$ or, equivalently, $\sigma_s\tilde{\epsilon}_i \geq g - (1+k)\tilde{f} - k\sigma_c\tilde{\epsilon}_c$. Hence, their aggregate purchase can be characterized by $1 - \Phi\left(\left(g - (1+k)\tilde{f} - k\sigma_c\tilde{\epsilon}_c\right)/\sigma_s\right)$. The net holding from speculators is then:

$$X\left(\tilde{f}, \tilde{\epsilon}_c\right) = 1 - 2\Phi\left(\frac{g - (1+k)\tilde{f} - k\sigma_c\tilde{\epsilon}_c}{\sigma_s}\right). \tag{6}$$

The market clearing condition together with equation (4) indicate that

$$1 - 2\Phi\left(\frac{g - (1+k)\tilde{f} - k\sigma_c\tilde{\epsilon}_c}{\sigma_s}\right) = 1 - 2\Phi\left(\tilde{\xi} - \alpha\ln P\right). \tag{7}$$

Therefore the equilibrium price is given by

$$P = \exp\left(\frac{(1+k)\tilde{f} + k\sigma_c\tilde{\epsilon}_c - g + \sigma_s\tilde{\xi}}{\alpha\sigma_s}\right) = \exp\left(\frac{\tilde{f} + k\tilde{s}_c - g + \sigma_s\tilde{\xi}}{\alpha\sigma_s}\right), \tag{8}$$

which is informationally equivalent to

$$z(P) \equiv \frac{g + \alpha\sigma_s\ln P}{1+k} = \tilde{f} + \frac{k}{1+k}\sigma_c\tilde{\epsilon}_c + \frac{1}{1+k}\sigma_s\tilde{\xi} = \left(\frac{1}{1+k}\right)\tilde{f} + \frac{k}{1+k}\tilde{s}_c + \frac{1}{1+k}\sigma_s\tilde{\xi}. \tag{9}$$

From the above equation, we can see that $z(P)$, which is a sufficient statistic for the information in P, provides some information about the realization of the productivity shock $\tilde{f}$. Yet, the signal $z(P)$ is not fully revealing of $\tilde{f}$, as it is also affected by the noise in the common signal $\tilde{\epsilon}_c$ and by the noisy demand $\tilde{\xi}$. Since the capital provider observes $z(P)$, he will use it to update his belief about the productivity. Note that $z(P)$ is distributed normally with a mean of $\bar{f}$. The variance of $z(P)$ given $\tilde{f}$ is $\sigma_p^2 = (k/(1+k))^2\sigma_c^2 + (1/(1+k))^2\sigma_s^2\sigma_\xi^2$. Hence, we denote the precision of $z(P)$ as a signal for $\tilde{f}$ as:

$$\tau_p = 1/\sigma_p^2 = \frac{(1+k)^2\tau_c\tau_\xi\tau_s}{k^2\tau_\xi\tau_s + \tau_c}. \tag{10}$$

After characterizing the information content of the price, we can derive the capital provider's belief on $\tilde{f}$. That is, conditional on observing $\tilde{s}_l$ and $z(P)$, the capital provider believes that $\tilde{f}$ is distributed normally with mean $\left(\tau_f \bar{f} + \tau_l \tilde{s}_l + \tau_p z(P)\right) / (\tau_f + \tau_l + \tau_p)$ and variance $1/(\tau_f + \tau_l + \tau_p)$. Then, using the capital provider's investment rule in equation (1) and taking expectations, we can express the level of investment as:

$$
\begin{aligned}
I &= \frac{1}{c} E[\tilde{F} | \tilde{s}_l = s_l, P] = \frac{1}{c} E[\exp\left(\tilde{f}\right) | \tilde{s}_l = s_l, P] \\
&= \frac{1}{c} \exp\left(\frac{\tau_f \bar{f} + \tau_l s_l + \tau_p z(P)}{\tau_f + \tau_l + \tau_p} + \frac{1}{2(\tau_f + \tau_l + \tau_p)}\right).
\end{aligned} \tag{11}
$$

Given the capital provider's investment policy in (11) and the price in (8), we can now write speculator i's expected profit from buying the asset given the information that is available to him (shorting the asset would give the negative of this):

$$
\begin{aligned}
E\left[\tilde{F}I - P | \tilde{s}_i, \tilde{s}_c\right] &= \frac{1}{c} E\left[\exp\left(\frac{\tau_f \bar{f} + \tau_l s_l + \tau_p z(P)}{\tau_f + \tau_l + \tau_p} + \frac{1}{2(\tau_f + \tau_l + \tau_p)} + \tilde{f}\right) | \tilde{s}_i, \tilde{s}_c\right] \\
&\quad - E\left[\exp\left(\frac{\tilde{f} + k\tilde{s}_c - g + \sigma_s \tilde{\xi}}{\alpha \sigma_s}\right) | \tilde{s}_i, \tilde{s}_c\right].
\end{aligned} \tag{12}
$$

Note that we made use here of the fact that $\tilde{F} = \exp\left(\tilde{f}\right)$. This is where using the natural log of the productivity parameter plays a key role. Using the properties of the exponential function, we can express the value of the firm $\tilde{F}I$ as $\frac{1}{c} \exp\left(\left(\tau_f \bar{f} + \tau_l s_l + \tau_p z(P) + 1/2\right) / (\tau_f + \tau_l + \tau_p) + \tilde{f}\right)$, where the expression in parentheses is linear in $\tilde{f}$. This enables us to get a linear closed-form solution, which would otherwise be impossible in a model of feedback.

Conditional on observing $\tilde{s}_i$ and $\tilde{s}_c$, speculator i believes that $\tilde{f}$ is distributed normally with mean $\left(\tau_f \bar{f} + \tau_s \tilde{s}_i + \tau_c \tilde{s}_c\right) / (\tau_f + \tau_s + \tau_c)$ and variance $1/(\tau_f + \tau_s + \tau_c)$. Hence, substituting for $z(P)$ (from (9)) and taking expectations, equation (12) can be rewritten as:

$$
E\left[\tilde{F}I - P | \tilde{s}_i, \tilde{s}_c\right] = \frac{1}{c} \exp(a_0 + a_1 \tilde{s}_i + a_2 \tilde{s}_c) - \exp(b_0 + b_1 \tilde{s}_i + b_2 \tilde{s}_c), \tag{13}
$$

where the coefficients a_0, a_1, a_2, b_0, b_1, and b_2 are functions of k and of the model's parameters. Explicit expressions for these coefficients are provided in the proof of Proposition 1 in the appendix.

A speculator will choose to buy the asset if and only if (13) is positive. Rearranging and taking logs leads to the following condition:

$$
\tilde{s}_i + B(k)\, \tilde{s}_c \geq C(k) \tag{14}
$$

where $B(k) = (a_2 - b_2) / (a_1 - b_1)$ and $C(k) = (b_0 - a_0 + \ln c) / (a_1 - b_1)$.[12] Function $B(k)$ can be thought of as the best response of a speculator to other speculators' weight on the

[12] Here, we assume that $a_1 - b_1 > 0$. This is verified later in the proof of Proposition 1.

correlated signal. That is, if all speculators in the economy put a relative weight k on the correlated signal when deciding whether to attack or not, the best response for a speculator is to put the weight $B(k)$ on his correlated signal. The symmetric equilibrium is solved when $B(k) = k$. Recall that a_1, a_2, b_1, and b_2 are also functions of k, and hence the equilibrium condition $B(k) = k$ leads to a third-order polynomial. Analyzing this polynomial, we obtain the result in the following proposition. All proofs are in the Appendix.

PROPOSITION 1: For a high enough level of supply elasticity α, there exists a monotone linear equilibrium characterized by weight $k^* > 0$ that speculators put on the common signal. This equilibrium is unique when the precision of the prior τ_f is sufficiently small.

The weight k^* that speculators put on the common signal in equilibrium captures the degree of coordination in their trading decisions. When k^* is high, speculators put a large weight on the common information when deciding whether to sell or buy the asset. This leads to large coordination among them and gives rise to a trading frenzy. In the upcoming sections, we develop a series of results on the determinants of coordination and its implications for the efficiency of the investment decision and for the volatility of prices. We focus on the case of large supply elasticity (large α) and imprecise prior (small τ_f), for which we know that there exists a unique equilibrium.

3 The Determinants of Speculators' Coordination

The weight that speculators put on the common signal in this model is affected by the degree to which there are strategic complementarities or strategic substitutes among them. To see the sources of the two strategic effects, recall from (3), that a speculator's expected profit is $x(i) E\left[\tilde{F}I - P|\mathcal{F}_i\right]$. When other speculators put more weight on the common signal, this signal gets to have a stronger effect on the price P, as well as on the real value of the security $\tilde{F}I$ (since the capital provider's investment decision is affected by the price). The first effect pushes the speculator to put a lower weight on the common signal, since relying on the common signal more heavily implies paying a high price when buying and a low price when selling. On the other hand, the second effect pushes the speculator to put a higher weight on the common signal, since relying on the common signal more heavily implies buying a security with high value and selling one with low value. Hence, the source of strategic substitutes in our model is the price mechanism, which is usual in models of financial markets, while the source of strategic complementarities is the feedback effect to the real value of the security.

In a world without these strategic effects, the weight that speculators put on the common signal relative to the private signal would be equal to the ratio of precisions between the signals: τ_c/τ_s. But, with strategic effects, the equilibrium weight on the common signal k^* reflects the sum of the strategic effects on top of the precisions ratio; where the strategic substitutes due to the price mechanism push k down and the strategic complementarities due to the feedback effect push it up. In the rest of this section, we formally isolate the various determinants of coordination to understand the impact of each factor on the equilibrium level of coordination.

3.1 Impact of Learning by the Capital Provider

Suppose that there is no feedback effect from prices to real values, because the capital provider does not learn from the price. In this case, the capital provider's decision on how much capital to provide becomes (this equation is analogous to equation (11) in the main model):

$$I = \frac{1}{c}E[\tilde{F}|\tilde{s}_l = s_l] = \frac{1}{c}\exp\left(\frac{\tau_f \bar{f} + \tau_l s_l}{\tau_f + \tau_l} + \frac{1}{2(\tau_f + \tau_l)}\right). \tag{15}$$

We again solve for the linear monotone equilibrium where speculators buy the asset if and only if $\tilde{s}_i + k_{BM}\tilde{s}_c \geq g_{BM}$ (the subscript BM stands for 'benchmark'), and purchase the asset otherwise. Given the investment rule in (15), the expected profit for speculator i from buying the asset, given the information available to him, becomes (this equation is analogous to equation (12) in the main model):

$$\begin{aligned} E[\tilde{F}I - P|\tilde{s}_i, \tilde{s}_c] &= E\left[\frac{1}{c}\exp\left(\frac{\tau_f \bar{f} + \tau_l s_l}{\tau_f + \tau_l} + \frac{1}{2(\tau_f + \tau_l)}\right)\tilde{F}|\tilde{s}_i, \tilde{s}_c\right] \\ &\quad -E\left[\exp\left(\frac{1}{\alpha\sigma_s}\left(\tilde{f} + k_{BM}\tilde{s}_c - g_{BM} + \sigma_s\tilde{\xi}\right)\right)|\tilde{s}_i, \tilde{s}_c\right]. \end{aligned} \tag{16}$$

For a speculator who buys the asset, (16) must be positive. Taking expectation and rearranging, we can see that a speculator buys the asset if and only if $\tilde{s}_i + B_{BM}(k)\,\tilde{s}_c \geq C_{BM}$ where[13]

$$B_{BM}(k) = \frac{\tau_c}{\tau_s} - \frac{\frac{\sqrt{\tau_s}}{\alpha}k}{\frac{\tau_s}{\tau_f+\tau_s+\tau_c}\left(\frac{\tau_f+2\tau_l}{\tau_f+\tau_l} - \frac{\sqrt{\tau_s}}{\alpha}\right)}. \tag{17}$$

[13]The expression for C_{BM} and other details are in the proof of Proposition 2.

Solving $B_{BM}(k) = k$, as in the main model, we obtain the equilibrium weight that speculators put on the common signal in the case of no feedback effect from price to real investment:

$$k_{BM} = \frac{\left(\left(1 - \frac{\sqrt{\tau_s}}{\alpha}\right)\tau_f + \left(2 - \frac{\sqrt{\tau_s}}{\alpha}\right)\tau_l\right)\tau_c}{\left(\frac{\sqrt{\tau_s}}{\alpha}\right)(\tau_f + \tau_l)(\tau_c + \tau_f + \tau_s) + \left(\left(1 - \frac{\sqrt{\tau_s}}{\alpha}\right)\tau_f + \left(2 - \frac{\sqrt{\tau_s}}{\alpha}\right)\tau_l\right)\tau_s}. \tag{18}$$

Inspecting (18), we can see that k_{BM} is lower than τ_c/τ_s, and that it approaches τ_c/τ_s as α gets very large. The intuition is as follows: τ_c/τ_s represents the ratio of precisions between the common signal and the idiosyncratic signal. This is the relative weight that speculators would put on the common signal if there were no strategic interactions. In a world without a feedback effect, the only strategic interaction between the speculators comes from the price mechanism, which generates strategic substitutes that reduce k_{BM} below τ_c/τ_s. As α gets very large, this effect weakens, since the supply is highly elastic in the price, and so the price is not strongly affected by speculators' trades. Hence, speculators converge to the weight of τ_c/τ_s.

The following proposition summarizes the properties of k_{BM} and its relation to the equilibrium weight k^* in the main model.

Proposition 2: If the capital provider does not learn from the price when making lending decisions, the weight speculators put on the common signal k_{BM} is given by (18). For a high enough level of supply elasticity α, k_{BM} is strictly below the equilibrium weight k^* that speculators put on the common signal in the main model (with a feedback effect).

We can see that when we shut down the feedback effect from the price to real investment, the weight that speculators put on the common signal decreases. This is in line with our discussion above, according to which the feedback effect from prices to real investment is the source of complementarity in speculators' strategies, making them want to put more weight on the common signal. Hence, the feedback effect is the cause of trading frenzies in our model.

For illustration, we plot the best response function for our main model (as in equation (14)) and for the benchmark case (as in equation (17)) in Figure (1). In the figure, the intersections of $B(k)$ and $B_{BM}(k)$ with the 45-degree line establish the equilibrium weights k^* and k_{BM}, respectively. As we see in the figure, $B(0) = B_{BM}(0) = \tau_c/\tau_s$. That is, in both cases, if other speculators put no weight on the common signal, a speculator finds it optimal to use the ratio of precisions between the common signal and the idiosyncratic signal as the weight for the common signal. This is because when other speculators do not put weight on

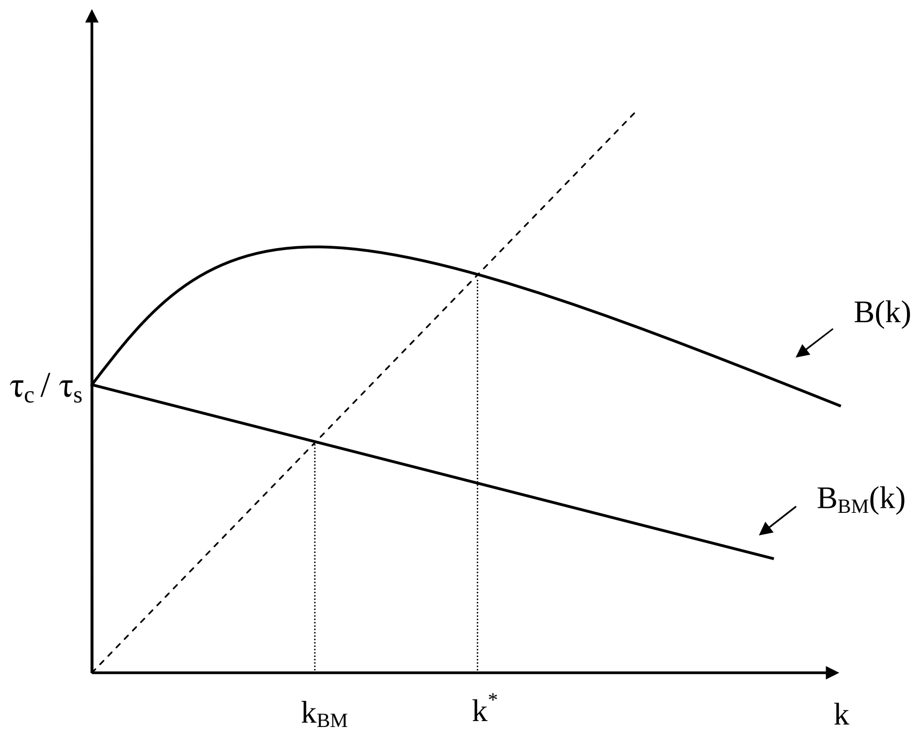

Figure 1: Best Response: $B(k)$ and $B_{BM}(k)$

the common signal, this signal is essentially like a private signal and hence it gets weighted solely based on its precision.

Once k increases above 0, strategic substitutability from the price mechanism emerges in the benchmark model. Indeed, the best response $B_{BM}(k)$ is a decreasing function of k: when others put more weight on the common signal, this signal gets more strongly reflected in the price, making an individual speculator reduce the weight he puts on the common signal. By contrast, in our main model, in addition to strategic substitutability from the price mechanism, strategic complementarity also emerges due to the feedback effect. For α large enough, the effect from strategic complementarity dominates that from strategic substitutability, resulting in $B(k)$ increasing above τ_c/τ_s. As the figure shows, this results in a higher equilibrium weight on the common signal in the main model than in the benchmark model, which is proved formally in the proof of Proposition 2.

3.2 Impact of Supply Elasticity

The parameter α captures the elasticity of supply with respect to price in our model. When α is high, the supply of shares is very sensitive to the price, meaning that an increase in demand by informed traders is quickly absorbed in the market, so that informed trading does not have a large price impact. As mentioned above, α can then be interpreted as a measure of liquidity, and our model can be used to tell what is the effect of liquidity on trading frenzies. The following proposition tells us that the extent to which speculators coordinate on the common signal increases in the level of liquidity α.

PROPOSITION 3: The equilibrium level of coordination k^* is increasing in the supply elasticity α, and for α large enough k^* is greater than the precisions ratio τ_c/τ_s.

In illiquid markets, order flows have a large effect on the price. Then, when speculators put more weight on the common signal, this signal has a substantial effect on the price, and so other speculators want to put less weight on the common signal. This effect decreases as α goes up and liquidity improves. Hence, in liquid markets there is a greater tendency for coordination and trading frenzies. As the proposition shows, when α is large enough, the weight on the common signal increases beyond the ratio of precisions τ_c/τ_s.

3.3 Impact of Noise Trading

Noise trading is captured in our model by the variable $\tilde{\xi} \sim N(0, \sigma_\xi^2)$. A high level of σ_ξ^2 implies that the market is exposed to large levels of noise trading. In the literature on financial markets, this introduces noise to the price, and in the presence of a feedback effect, it makes it harder to base investment decisions on the price. In our model, we examine the effect of noise trading on speculators' coordination. As we will see later, this will have further implications for the informativeness of the price.

PROPOSITION 4: For a high enough level of supply elasticity α, the equilibrium weight k^* that speculators put on the common signal is decreasing in the variance of noise trading σ_ξ^2.

The intuition here goes as follows: With high variance in the noise demand, there is high variance in the market price for reasons that are not related to speculators' trades. As a result, the reliance of the capital provider on the information in the price decreases. This weakens the feedback effect and hence the strategic complementarities among speculators, leading to a lower level of k^*.

It is worth noting that changes in the position limits of speculators will have similar effects to changes in the variance of noise trading. For example, if speculators could choose positions in the range $[-2, 2]$ (instead of $[-1, 1]$, assumed in the paper), they would have more impact on the capital provider's decision for a given level of σ_ξ^2 and thus would put a larger weight on the common signal in equilibrium. Hence, the effect of loosening speculators' trading constraints is similar to that of reducing the variance of noise trading.

3.4 Impact of the Information Structure

We now establish comparative statics results on the effect of the informativeness of various signals on the equilibrium level of coordination. The results are summarized in the next proposition.

PROPOSITION 5: For a high enough level of supply elasticity α, the equilibrium level of coordination k^* decreases in the precision of speculators' private signals τ_s, increases in the precision of their common signal τ_c, and decreases in the precision of the capital provider's signal τ_l.

These results are intuitive. Speculators put more weight on the common signal relative to the private signal when the common signal is more precise (τ_c is higher) and the private signal is less precise (τ_s is lower). Hence, trading frenzies are more likely when the common information becomes more precise relative to speculators' idiosyncratic sources of information. Less obvious is the result that the tendency for coordination among speculators decreases when the capital provider has more precise information (τ_l is higher). The reason is that when the capital provider has more precise information, he relies less on the price, and so the feedback effect from markets to real decisions weakens, and there is less scope for strategic complementarities.

3.5 A Note on the Nature of the Traded Security

Before moving to the next section, we would like to discuss the nature of the traded security. Our model assumes that the traded security is a claim on the cash flow from the investment $\tilde{F}I$. As we note in Section 1, this can be interpreted as a derivative, or as equity of the traded firm, in case the firm is financially constrained (in which case it is simple to change the model so that the traded security is a claim on some portion β of the cash flow $\tilde{F}I$).

The key feature of the traded security is that its cash flow depends not only on the fundamental $\tilde{F}$, but also on the investment decision I. This introduces a feedback loop between the financial market and the real economy, whereby the price affects the investment decision, and the investment decision is reflected in the price. This feedback loop is the crucial element for our result on strategic complementarities and trading frenzies. To illustrate this, note that if the traded security was a claim on the fundamental $\tilde{F}$, there would be no feedback loop and no frenzies. When speculators trade on $\tilde{F}$, the value of the security is exogenous and hence does not depend on speculators' behavior; this eliminates the strategic interaction that is central to our paper. It is worth noting that a security on $\tilde{F}$ might also not be easy to implement, since $\tilde{F}$ is not an easily verifiable cash flow (unlike $\tilde{F}I$, which is the cash flow from the investment). Indeed, most real-world financial securities, e.g., debt and equity, resemble $\tilde{F}I$ more than they resemble $\tilde{F}$ in that they provide a claim on a cash flow that depends on fundamental and firm action.

Another possible security that features a feedback loop is one that provides a claim on the net return from the investment $\tilde{F}I - C(I)$. Technically, however, we are unable to solve a model with this traded security.[14] A key economic difference between $\tilde{F}I$ and $\tilde{F}I - C(I)$ is that the former is always increasing in the level of investment I. This is typical to equity of a financially-constrained firm, where shareholders would benefit from having more capital invested in their firm, but are constrained in how much capital they can raise due to additional costs that need to be borne by capital providers. Hence, our model is suitable to describe trading of equity of such firms.

Interestingly, this feature is responsible for the fact that in our model we get symmetric frenzies, i.e., both bear raids leading to a decrease in capital and in firm value (as in the recent cases of Bear Stearns and Lehman Brothers) and elevated buying leading to an increase in capital and firm value (as in the internet or real estate booms). When speculators buy (sell) they lead the capital provider to invest more (less), which increases (reduces) the value of the security, leading to a profit on their buying (selling). We expect that asymmetric frenzies – i.e., only on the sell side – will exist under the alternative security $\tilde{F}I - C(I)$. This is because, in that case, when speculators sell and reduce the price, they lead the capital provider to provide less capital than optimal and reduce the value of the security, leading to a profit on their selling. But, when they buy and increase the price, they lead the capital provider to provide more capital than optimal and reduce the value of the security, leading to a loss on their buying. As mentioned, however, with the existing techniques, such a model is unsolvable.

[14] To see this, go back to (12). The expected value of the security for a speculator $E\left[\tilde{F}I|\tilde{s}_i, \tilde{s}_c\right]$ is expressed there as one exponential term (given our log-normal distributions), which is crucial for our ability to find a linear solution. If the traded security was $\tilde{F}I - C(I)$, we would have two exponential terms, which would render the steps for finding a linear solution impossible.

4 Coordination, Investment Efficiency, and Non-Fundamental Volatility

In this section, we explore the effect that coordination has on the efficiency of investment decisions and on market volatility. To analyze investment efficiency, we look at the ex ante expected net benefit of investment (i.e. expected net benefit before any of the signals are realized given the prior belief that $\tilde{f}$ is normally distributed with mean $\bar{f}$ and precision τ_f) from the perspective of the capital provider. We keep the information structure the same as before, and in particular, in the interim stage we allow the capital provider to obtain information only from his private signal and the price. So our efficiency criterion is given by:

$$E_0\left[\max_I E\left[\tilde{F}I-\frac{1}{2}cI^2\left|\tilde{s}_l=s_l,P\right.\right]\right], \tag{19}$$

where a speculator purchases the asset if $\tilde{s}_i+k\tilde{s}_c\geq g$ and shorts it otherwise (for constant k and g) and P is the market clearing price. We denote the optimal level of coordination k_{OP} to be the one that maximizes investment efficiency as in (19).

The following proposition characterizes k_{OP}, and how it is linked to the accuracy of the information inferred from the market price, τ_p:

Proposition 6: The level of coordination that maximizes investment efficiency is $k_{OP}=\tau_c/\left(\tau_s\tau_\xi\right)$, which also maximizes the precision of the price τ_p.

The capital provider cares about the events in the security market only to the extent that they affect the quality of the information he has when making the investment decision. Hence, the level of coordination that maximizes investment efficiency is the one that maximizes the accuracy of the information in the market price. Examining the expression for the price signal in (9), we can see that there is a tradeoff in setting the level of coordination. The tradeoff arises because there are two sources of noise in the price, one coming from the noise trading $\widetilde{\xi}$ and the other one from the noise in the common signal $\tilde{\epsilon}_c$. (The first source of noise becomes more prominent when speculators' private information is noisy – τ_s is low – because then noise trading becomes relatively more important.) A high level of coordination reduces the effect of the first source of noise – as coordinated speculative trading helps overcoming the large volume of noise trading – and increases the effect of the second source of noise – as coordinated speculative trading increases the weight on the common signal. Therefore, the optimal level of coordination will be high when the potential damage from noise trading is high (τ_ξ and τ_s are low) or when the potential damage from noise in the common signal is low (τ_c is high). Then, $k_{OP}=\tau_c/\left(\tau_s\tau_\xi\right)$.

It is interesting to compare the optimal level of coordination characterized here with the level of coordination that is obtained in equilibrium. From Proposition 4 we know that in equilibrium speculators coordinate more when the variance in the noise trading is low (τ_ξ is high). A high τ_ξ implies that speculators' trades have more effect on the capital provider's decision, increasing the scope of strategic complementarities. Yet, this is exactly when coordination is not desirable for the efficiency of the investment. Hence, there is a sharp contrast between the profit incentives of speculators and the efficiency of the investment. Speculators coordinate more exactly when it is inefficient to do so. The following proposition summarizes the comparison between the optimal level of coordination and the equilibrium level of coordination.

PROPOSITION 7: For a high enough level of supply elasticity α, there exists $\bar{\tau}_\xi$ such that the level of coordination that maximizes investment efficiency is greater than the equilibrium level of coordination ($k_{OP} > k^*$) when the precision of the noise trading distribution τ_ξ is below $\bar{\tau}_\xi$. Similarly, $k_{OP} < k^*$ for $\tau_\xi > \bar{\tau}_\xi$.

The proposition says that speculators coordinate too much in markets with less noise trading and coordinate too little in markets with more noise trading. Interestingly, this implies that trading frenzies are only sometimes undesirable. When there is high variation in noise trading, price informativeness would improve if speculators coordinated their trades more to provide a signal that overcomes the effect of noise trading. Yet, it is exactly in this case that they find coordination less profitable in equilibrium.

We close this section by noting some of the implications of inefficient coordination levels. Deviations from the optimal level of coordination k_{OP} are manifested in our model by higher levels of non-fundamental volatility. We define this as volatility that does not come from the variability in fundamental. The following proposition establishes the link between the level of coordination and non-fundamental volatility of price and investment.

PROPOSITION 8: (a) Non-fundamental volatility of asset price is minimized at $k = k_{OP}$ (where its value is $1/(\tau_c + \tau_s \tau_\xi)$).

(b) Similarly, non-fundamental volatility of investment is minimized at $k = k_{OP}$ (where its value is $1/(\tau_l + \tau_c + \tau_s \tau_\xi)$).

This proposition indicates that the strategic interactions among speculators in the financial markets often lead to non-fundamental volatility in prices as well as real activities. The source of this non-fundamental volatility could come from either too low coordination (that is, when the market is characterized by a high amount of noise trading) or too high coordination (that is, when the market has low noise trading and the noise in the correlated signals among speculators is high). Note that non-fundamental volatility is difficult to measure since it is defined as the volatility that does not come from fundamentals, while the volatility of fundamentals is unobservable (the volatility of cash flow is observable, but includes volatility due to noise). Hence, this notion is interesting mostly for theoretical reasons.

5 A Model where Speculators Learn from the Price

So far in the paper, we assumed that speculators in the financial market submit market orders that are not conditioned on the price. This assumption is common in the literature on financial markets, going back to Kyle (1985). It is also consistent with many situations in the real world, as speculators often face price risk when they trade without knowing the exact price (speculators may prefer this strategy over fully conditioning on the price, as this provides them greater immediacy in trading). In this section, we explore the importance of this assumption for our main result, which is the emergence of strategic complementarities among speculators due to the informational feedback from the price of the security to the investment decision.

As we explained before, the mechanism behind the strategic complementarities in our paper goes as follows: when speculators put more weight on the common signal, this signal gets to have a greater effect on the value of the security via the information conveyed by the price to the capital provider. Then, given this behavior of other speculators, each individual speculator finds it optimal to put more weight on the common signal himself. When speculators observe the price or fully condition on the price in this framework (as in Grossman and Stiglitz (1980)), strategic complementarities disappear. Once they observe the price, speculators know the message transmitted from the market to the capital provider, and so conditional on the price, they do not care what other speculators are doing. Indeed, one can show that if we just add the assumption that speculators fully condition on the price to our model, the weight that speculators put on the common signal will always be fixed at the precisions ratio: τ_c/τ_s (as it is in the case of no strategic effects).

But, the above rational-expectations framework makes a very strong assumption: that the exact aggregate message from the market that is observed by the capital provider is also observed by each and every speculator. This eliminates the higher-order beliefs that are important for our complementarities to arise. This assumption is also not very realistic. In the real world, capital providers (or other decision makers) may receive a market-related signal that is not perfectly observable to traders. For example, when the timing of traders' trading decision and the capital provider's investment decision do not coincide, their information sets may not be perfectly correlated. This happens when traders condition their trade on the current price while the capital provider who acts with a lag has access to information from future prices or from other correlated markets. Alternatively, the capital provider and traders may share the same aggregate information source but their degrees of exposure to the source are different. Introducing such elements would imply that the speculators do not observe perfectly the aggregate message(s) used by the capital provider in making his investment decisions. This introduces back the higher-order beliefs that are crucial for our strategic complementarities to arise.

To analyze this formally in a tractable way, we now introduce two additions to the model. First, speculators observe the price and learn from it when they trade, just like in the traditional rational-expectations-equilibria (REE) literature (e.g., Grossman and Stiglitz (1980)). Second, conditional on fundamental, the capital provider's private source of information is correlated with the noise in the speculators' common signal. Specifically, his private source of information is now $\tilde{s}_l = \tilde{f} + \sigma_{cp}\tilde{\epsilon}_c$. Therefore, he is exposed to the noise in the common signal observed by speculators, $\tilde{\epsilon}_c$, but this is multiplied by a different coefficient σ_{cp} (as opposed to σ_c in the speculators' common signal), so he does not observe exactly the same common signal received by speculators $\tilde{s}_c$. Note that we could add another source of idiosyncratic noise to the capital provider's signal, and this would not have a qualitative effect on the results. As always, we denote the precision of the capital provider's signal as $\tau_{cp} = 1/\sigma_{cp}^2$. Finally, to maintain tractability, we now assume that the fundamental $\tilde{f}$ is distributed uniformly over the real line (i.e., τ_f approaches 0). Other than these changes, the model remains the same as in the previous sections.

The assumption that the capital provider observes a signal that is correlated with the speculators' common signal is consistent with the motivation described above. That is, the idea is that the capital provider observes an aggregate message from the market (the combination of the price P and $\tilde{s}_l$) which is different from what is observed by any speculator when trading. The signal $\tilde{s}_l = \tilde{f} + \sigma_{cp}\tilde{\epsilon}_c$ can thus be thought of as a reduced form of observing the price from another market or from later rounds of trade, as these will also be affected by the common noise that exists in the market (recall that we could add an additional source of noise to $\tilde{s}_l$). Alternatively, the assumption that both $\tilde{s}_l$ and $\tilde{s}_c$ are affected by the common noise $\tilde{\epsilon}_c$ may capture other realistic scenarios. If the common signal in the market $\tilde{s}_c$ represents rumors, then the capital provider may be observing some version of these rumors. Or, the capital provider may report a noisy version of his signal $\tilde{s}_l$, which is observed among traders in the form of $\tilde{s}_c$.

We now turn to solve and analyze the extended model. As before, we consider monotone linear strategies where the speculators put weight on $\tilde{s}_i$, $\tilde{s}_c$, and now also on the price P. That is, speculators short the asset whenever $\tilde{s}_i + k\tilde{s}_c + m\ln P \leq g$ and buy it otherwise. The parameters k, m, and g are determined endogenously. Following the steps in the main model, the net holding from speculators is then:

$$X\left(\tilde{f}, \tilde{\epsilon}_c\right) = 1 - 2\Phi\left(\frac{g - (1+k)\tilde{f} - k\sigma_c\tilde{\epsilon}_c - m\ln P}{\sigma_s}\right). \tag{20}$$

Since the market supply is $1 - 2\Phi\left(\tilde{\xi} - \alpha\ln(P)\right)$, we use the market clearing condition to express the price:

$$P = \exp\left(\frac{(1+k)\tilde{f} + k\sigma_c\tilde{\epsilon}_c + \sigma_s\tilde{\xi} - g}{\sigma_s\alpha - m}\right) = \exp\left(\frac{\tilde{f} + k\tilde{s}_c + \sigma_s\tilde{\xi} - g}{\sigma_s\alpha - m}\right).$$

The sufficient statistic for the information in P, $z(P)$, is now expressed as:

$$z(P) \equiv \frac{g + (\sigma_s\alpha - m)\ln(P)}{1+k} = \tilde{f} + \frac{k}{1+k}\sigma_c\tilde{\epsilon}_c + \frac{\sigma_s}{1+k}\tilde{\xi} = \left(\frac{1}{1+k}\right)\tilde{f} + \frac{k}{1+k}\tilde{s}_c + \frac{\sigma_s}{1+k}\tilde{\xi}.$$

The capital provider makes his decision based on $z(P)$ and $\tilde{s}_l$, while speculators make their decisions based on $z(P)$, $\tilde{s}_i$, and $\tilde{s}_c$. Denoting the equilibrium weight that speculators put on the common signal k^{**}, in the proposed linear equilibrium, the equilibrium weight k^{**} solves $B_P(k^{**}) = k^{**}$, where $B_P(\cdot)$ is a best-response function defined similarly to that in the main model. The following proposition derives conditions under which a unique equilibrium exists.

Proposition 9: There is a unique equilibrium if $\tau_{cp} > \tau_c$ (the capital provider's signal is more precise than the speculators' common signal) and τ_s (the precision of speculators' private signals) or τ_ξ (the precision of the noise trading distribution) are small enough or if τ_{cp} (the precision of capital provider's signal) is large enough.

In the rest of this section we will assume that the equilibrium is unique. As in the main model, we now compare the coordination level k^{**} with the one that would be obtained in a benchmark where the capital provider does not learn from the price (but speculators do). In the benchmark, we say that speculators short the asset whenever $\tilde{s}_i + k_N\tilde{s}_c + m_N\ln P \leq g_N$ and buy otherwise. The following proposition compares the weight on the common signal k_N in the benchmark model and the equilibrium weight k^{**}.

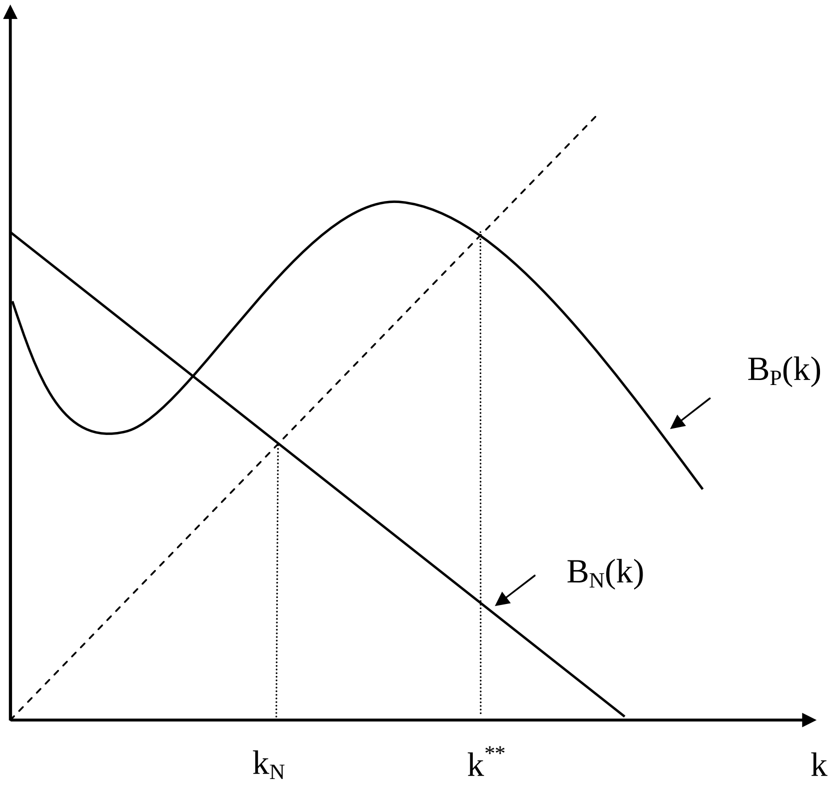

Figure 2: Best Response: $B_P(k)$ and $B_N(k)$

Proposition 10: When the capital provider does not learn from the price when making the lending decision, the weight that speculators put on the common signal k_N is strictly below the equilibrium weight k^{**} they put when the capital provider learns from the price, if and only if the following condition is true:

$$-2\tau_c \left(\frac{\sqrt{\tau_{cp}}}{\sqrt{\tau_c}} - 1 \right) + \tau_s (1 + \tau_\xi) < 0, \tag{21}$$

i.e., if $\tau_{cp} > \tau_c$ (the capital provider's signal is more precise than the speculators' common signal) and τ_s (the precision of speculators' private signals) or τ_ξ (the precision of the noise trading distribution) are small enough.

To understand the role of strategic complementarities, we plot the best response function in the REE model $B_P(\cdot)$ (as in equation (29) in the appendix) and in the benchmark REE model $B_N(\cdot)$ where the capital provider does not learn from the price (as in equation (32) in the appendix) in the following figure and compare the slopes of these two functions.

In the benchmark case, there is no strategic complementarity. In fact, the slope of $B_N(k)$, is $\partial B_N(k)/\partial k = -\tau_\xi$. That is, when others put a larger weight on the common signal, a speculator's best response is to reduce his weight on the common signal. This is intuitive since when others put more weight on the common signal, the price as a signal for the fundamental becomes more correlated with the common signal. This causes a speculator to reduce the weight he puts on the common signal, since some of this information is already embedded in the price.

However, when the capital provider learns from the price, a speculator's weight on the common signal is sometimes increasing in others' weight on the common signal. Indeed the slope of $B_P(\cdot)$ in the figure is sometimes positive. The reason is that the weight that others put on the common signal affects the reliance of the capital provider on his own signal $\tilde{s}_l$ and hence affects the real value of the security. In particular, when k goes up, the price becomes less precise to the capital provider and he shifts weights from the price to $\tilde{s}_l$ in his investment decision. Then, a speculator knows that the common signal $\tilde{s}_c$, which is correlated with the capital provider's signal $\tilde{s}_l$, will be more strongly correlated with the investment decision of the capital provider, and this increases the incentive of the speculator to rely on $\tilde{s}_c$. This creates the strategic complementarities when the capital provider learns from the price. Putting this together with the opposite effect in the benchmark (when the capital provider does not learn from the price), the above proposition develops the condition in equation (21) under which k is higher when the capital provider learns from the price. Note that this condition is satisfied when the precision of the capital provider's signal (τ_{cp}) is large relative to the precision of the common signal (τ_c), the precision of the noise trading (τ_ξ), and the precision of the speculator's private signal (τ_s). The latter three precisions are related to the precision of the price since price aggregates speculators' private and common signals as well as the noise trading. Therefore, this condition is related to how heavily the capital provider relies on $\tilde{s}_l$ versus the price in making investment decisions. When the capital provider is prone to rely on $\tilde{s}_l$ more heavily, speculators coordinate more in equilibrium benefiting from his reliance on $\tilde{s}_l$.

In summary, for strategic complementarities to arise in our model due to the informational feedback from prices to capital provision, it is important that speculators do not observe the exact message that the capital provider receives from the market. In our main model, developed in previous sections, this was obtained because the speculators did not observe the price that the capital provider learns from. In the model developed in this section, we let the speculators observe the price, but assume that the capital provider, in addition to observing the price, observes something else which is correlated with the price and is not observed by speculators. This restores the high-order beliefs of the basic model and allows for strategic complementarities to arise.

6 Conclusion

We study strategic interactions among speculators in financial markets and their real effects. Two opposite strategic effects exist. On the one hand, speculators wish to act differently from each other as a certain action by other speculators changes the price in a way that reduces the profit for other speculators from this action. On the other hand, due to the feedback effect from the price to the real investment, a certain action by speculators changes the real value of the firm in a way that increases the incentive of other speculators to take this action. This creates a basis for trading frenzies, where speculators rush to trade in the same direction, putting pressure on the price and on the firm's value. We characterize which effect dominates when and analyze the resulting level of coordination in speculators' actions.

The interaction among speculators affects the informational content of the price. Since prices affect real investment in our model, we can ask what level of coordination is most efficient for real investment. In general, speculators' incentives to coordinate go in opposite direction to the optimal level of coordination. Speculators want to coordinate more when there is a low amount of noise trading, but this is when coordination is less desirable from an efficiency point of view. Hence, our model shows that there is always either too much or too little coordination, and this reduces the efficiency of investment and creates excess volatility in the price.

Interestingly, our paper is also related to an old debate on whether speculators stabilize prices. The traditional view is that by buying low and selling dear, rational speculators stabilize prices. Hart and Kreps (1986) argue that when speculators can hold inventories and there is uncertainty about preferences, speculative activity may cause excess price movement. Our paper contributes to this literature by pointing out that when speculative activity has an effect on real investments, speculators might coordinate on correlated sources of information, and create excess volatility in prices. In our model, this reduces the efficiency of real investments.

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Appendix

Proof of Proposition 1: Based on (12) and the updating done by the speculator based on his information, the coefficients in (13) are given as follows:

$$
\begin{aligned}
a_0 \;=\;& \frac{\tau_f \bar{f} + \frac{1}{2}}{\tau_f + \tau_l + \tau_p} + \frac{\left(\tau_f + 2\tau_l + \tau_p \left(1 + \frac{1}{1+k}\right)\right)}{\tau_f + \tau_l + \tau_p} \left(\frac{\tau_f \bar{f}}{\tau_f + \tau_s + \tau_c} \right) \\
& + \left(\frac{\tau_f + 2\tau_l + \left(1 + \frac{1}{1+k}\right) \tau_p}{\tau_f + \tau_l + \tau_p} \right)^2 \frac{1}{2(\tau_f + \tau_s + \tau_c)} + \frac{1}{2} \left(\frac{\tau_l}{\tau_f + \tau_l + \tau_p} \right)^2 \sigma_l^2 \\
& + \frac{1}{2} \left(\frac{\tau_p}{\tau_f + \tau_l + \tau_p} \right)^2 \left(\frac{1}{1+k} \right)^2 \sigma_s^2 \sigma_\xi^2,
\end{aligned}
$$

$$
a_1 = \frac{\left(\tau_f + 2\tau_l + \tau_p \left(1 + \frac{1}{1+k}\right)\right)}{\tau_f + \tau_l + \tau_p} \frac{\tau_s}{\tau_f + \tau_s + \tau_c},
$$

$$
a_2 = \frac{\tau_p \frac{k}{1+k}}{\tau_f + \tau_l + \tau_p} + \frac{\left(\tau_f + 2\tau_l + \tau_p \left(1 + \frac{1}{1+k}\right)\right)}{\tau_f + \tau_l + \tau_p} \frac{\tau_c}{\tau_f + \tau_s + \tau_c},
$$

$$
b_0 = \frac{1}{\alpha \sigma_s} \left(\frac{\tau_f \bar{f} + \frac{1}{2} \frac{1}{\alpha \sigma_s}}{\tau_f + \tau_s + \tau_c} - g \right) + \frac{1}{2\alpha^2} \sigma_\xi^2,
$$

$$
b_1 = \frac{1}{\alpha \sigma_s} \frac{\tau_s}{\tau_f + \tau_s + \tau_c},
$$

$$
b_2 = \frac{1}{\alpha \sigma_s} \left(\frac{\tau_c}{\tau_f + \tau_s + \tau_c} + k \right).
$$

Note that

$$
a_1 - b_1 = \frac{\tau_s}{\tau_f + \tau_s + \tau_c} \left(\frac{\left(\tau_f + 2\tau_l + \tau_p \left(1 + \frac{1}{1+k}\right)\right)}{\tau_f + \tau_l + \tau_p} - \frac{1}{\alpha \sigma_s} \right).
$$

A sufficient condition for $a_1 - b_1 > 0$ is that $\alpha > \sqrt{\tau_s}$. Recall that $B(k) = (a_2 - b_2) / (a_1 - b_1)$. Substituting, we obtain:

$$
B(k) = \frac{\frac{\tau_p \frac{k}{1+k}}{\tau_f + \tau_l + \tau_p} + \frac{\left(\tau_f + 2\tau_l + \tau_p \left(1 + \frac{1}{1+k}\right)\right)}{\tau_f + \tau_l + \tau_p} \frac{\tau_c}{\tau_f + \tau_s + \tau_c} - \frac{1}{\alpha \sigma_s} \left(\frac{\tau_c}{\tau_f + \tau_s + \tau_c} + k \right)}{\frac{\tau_s}{\tau_f + \tau_s + \tau_c} \left(\frac{\left(\tau_f + 2\tau_l + \tau_p \left(1 + \frac{1}{1+k}\right)\right)}{\tau_f + \tau_l + \tau_p} - \frac{1}{\alpha \sigma_s} \right)}.
$$

Simplifying $B(k) - k = 0$ we get:

$$
0 = \left[\frac{1}{\tau_s} \frac{(k+1)(\tau_c + \tau_f + \tau_s)(\tau_f + \tau_l + \tau_p)}{\tau_p + (k+1)\tau_l + \left(1 - \frac{\sqrt{\tau_s}}{\alpha}\right)(1+k)(\tau_f + \tau_l + \tau_p)} \right]
$$

$$
\left(\frac{\tau_p}{\tau_f + \tau_l + \tau_p} \frac{k}{1+k} + \left(\frac{\tau_f + 2\tau_l + \left(1 + \frac{1}{1+k}\right) \tau_p}{\tau_f + \tau_l + \tau_p} \right) \left(\frac{-\tau_s k + \tau_c}{\tau_f + \tau_s + \tau_c} \right) - \frac{\sqrt{\tau_s}}{\alpha} \left(\left(\frac{-\tau_s k + \tau_c}{\tau_f + \tau_s + \tau_c} \right) + k \right) \right).
$$

The term in square brackets is strictly positive for $\alpha > \sqrt{\tau_s}$. So the equilibrium condition can be simplified to:

$$\begin{aligned}0 \;=\; & \frac{\tau_p}{\tau_f+\tau_l+\tau_p}\frac{k}{1+k}+\left(\frac{\tau_f+2\tau_l+\left(1+\frac{1}{1+k}\right)\tau_p}{\tau_f+\tau_l+\tau_p}\right)\left(\frac{-\tau_s k+\tau_c}{\tau_f+\tau_s+\tau_c}\right)-\left(\frac{-\tau_s k+\tau_c}{\tau_f+\tau_s+\tau_c}\right)-k\\ & +\left(1-\frac{1}{\alpha\sigma_s}\right)\left(\frac{\tau_f+\tau_c}{\tau_f+\tau_s+\tau_c}k+\frac{\tau_c}{\tau_f+\tau_s+\tau_c}\right).\end{aligned}$$

Recall that $\tau_p = ((1+k)^2\tau_c\tau_\xi\tau_s)\,/\,(k^2\tau_\xi\tau_s+\tau_c)$. We denote $r \equiv \tau_\xi\tau_s$. Substituting for τ_p and simplifying, the right-hand side becomes:[15]

$$\begin{aligned}H(k) = & -k^3\left(\left(\tau_c+\tau_f+\tau_s\right)\left(\tau_c+\tau_f+\tau_l\right)+\tau_l\tau_s\right)-\tau_c k^2\left(\tau_c+\tau_f-\tau_l+2\tau_s\right)\\ & -\tau_c k\left(\tau_s-\tau_c\right)+\tau_c^2-\frac{1}{r}\left(\tau_c k\left(\tau_c\tau_f+\tau_c\tau_l+\tau_f\tau_l+\tau_f\tau_s+2\tau_l\tau_s+\tau_f^2\right)-\tau_c^2\tau_l\right)\\ & +\left(1-\frac{\sqrt{\tau_s}}{\alpha}\right)\left(\left(\tau_c+\tau_f\right)\left(\tau_c+\tau_f+\tau_l\right)k^3+\tau_c\left(3\tau_c+3\tau_f+\tau_l\right)k^2\right.\\ & \left.+\tau_c\left(\tau_f+3\tau_c\right)k+\tau_c^2\right)+\left(1-\frac{\sqrt{\tau_s}}{\alpha}\right)\frac{\tau_c}{r}\left(\left(\tau_f+\tau_l\right)\left(\tau_c+\tau_f\right)k+\tau_c\left(\tau_f+\tau_l\right)\right).\end{aligned} \tag{22}$$

For an equilibrium, we need $H(k) = 0$.

First, we focus on existence of an equilibrium with $k > 0$. $H(k)$ has a positive root if and only if

$$\alpha > \sqrt{\tau_s}\frac{\tau_f+\tau_l+r}{\tau_f+2\tau_l+2r}.$$

To see this, note that the coefficient for k^3 is always negative, implying that the value of $H(k)$ becomes negative as k becomes large. So, there exists a strictly positive root for the polynomial if its value at $k = 0$ is strictly positive. This condition is given by the above inequality. If the inequality is violated, the value of the polynomial is negative at $k = 0$. Its derivative at $k = 0$ is given by

$$\begin{aligned}& -\tau_c\left(\tau_s-\tau_c\right)-\frac{1}{r}\left(\tau_c\left(\tau_c\tau_f+\tau_c\tau_l+\tau_f\tau_l+\tau_f\tau_s+2\tau_l\tau_s+\tau_f^2\right)\right)\\ & +\left(1-\frac{\sqrt{\tau_s}}{\alpha}\right)\tau_c\left(\tau_f+3\tau_c\right)+\left(1-\frac{\sqrt{\tau_s}}{\alpha}\right)\frac{\tau_c}{r}\left(\left(\tau_f+\tau_l\right)\left(\tau_c+\tau_f\right)\right).\end{aligned}$$

At $\frac{\sqrt{\tau_s}}{\alpha} \geq \frac{\tau_f+2\tau_l+2r}{\tau_f+\tau_l+r}$ the derivative is negative. This means that $H(k)$ is decreasing at $k = 0$ for $\frac{\sqrt{\tau_s}}{\alpha} \geq \frac{\tau_f+2\tau_l+2r}{\tau_f+\tau_l+r}$. Moreover, the second derivative is negative when $\frac{\sqrt{\tau_s}}{\alpha} \geq \frac{\tau_f+2\tau_l+2r}{\tau_f+\tau_l+r}$, and thus the expression will keep decreasing. Therefore the polynomial cannot have a positive root.

[15]The simplification is achieved by dividing by r and multiplying through with $(\tau_c+\tau_f+\tau_s)$ $\left(\tau_c\tau_f+\tau_c\tau_l+\tau_c r+\tau_c k^2 r+\tau_f k^2 r+k^2\tau_l r+2\tau_c k r\right)$.

For uniqueness, we need to check the sign of the discriminant of $H(k)$ for α large and τ_f small. Letting α go to infinity and τ_f go to zero we obtain the discriminant as $-\frac{\tau_c^3}{r^3}$ times

$$\begin{aligned}
&\left(\begin{array}{c} 64\tau_c^4\tau_l + 128\tau_c^3\tau_l^2 + 160\tau_c^3\tau_l\tau_s + 64\tau_c^2\tau_l^3 + 416\tau_c^2\tau_l^2\tau_s + 144\tau_c^2\tau_l\tau_s^2 \\ +284\tau_c\tau_l^2\tau_s^2 + 56\tau_c\tau_l\tau_s^3 + 8\tau_l\tau_s^4 \end{array}\right) r^3 \qquad (23)\\
&+\left(\begin{array}{c} 64\tau_c^5\tau_l + 192\tau_c^4\tau_l^2 + 160\tau_c^4\tau_l\tau_s + 192\tau_c^3\tau_l^3 + 608\tau_c^3\tau_l^2\tau_s + 144\tau_c^3\tau_l\tau_s^2 + 64\tau_c^2\tau_l^4 \\ +448\tau_c^2\tau_l^3\tau_s + 440\tau_c^2\tau_l^2\tau_s^2 + 56\tau_c^2\tau_l\tau_s^3 + 416\tau_c\tau_l^3\tau_s^2 + 80\tau_c\tau_l^2\tau_s^3 + 8\tau_c\tau_l\tau_s^4 + 48\tau_l^2\tau_s^4 \end{array}\right) r^2 \\
&+\left(128\tau_c\tau_l^4\tau_s^2 - 32\tau_c^2\tau_l^3\tau_s^2 - 16\tau_c^2\tau_l^2\tau_s^3 - 52\tau_c^3\tau_l^2\tau_s^2 - 64\tau_c\tau_l^3\tau_s^3 + 32\tau_c\tau_l^2\tau_s^4 + 96\tau_l^3\tau_s^4\right) r \\
&+\left(64\tau_l^4\tau_s^4 + 32\tau_c\tau_l^3\tau_s^4\right).
\end{aligned}$$

The coefficient of r^2 in (23) is strictly positive so the quadratic part of (23) is minimized at:

$$r = -\frac{128\tau_c\tau_l^4\tau_s^2 - 32\tau_c^2\tau_l^3\tau_s^2 - 16\tau_c^2\tau_l^2\tau_s^3 - 52\tau_c^3\tau_l^2\tau_s^2 - 64\tau_c\tau_l^3\tau_s^3 + 32\tau_c\tau_l^2\tau_s^4 + 96\tau_l^3\tau_s^4}{2\left(\begin{array}{c} 64\tau_c^5\tau_l + 192\tau_c^4\tau_l^2 + 160\tau_c^4\tau_l\tau_s + 192\tau_c^3\tau_l^3 + 608\tau_c^3\tau_l^2\tau_s + 144\tau_c^3\tau_l\tau_s^2 + 64\tau_c^2\tau_l^4 \\ +448\tau_c^2\tau_l^3\tau_s + 440\tau_c^2\tau_l^2\tau_s^2 + 56\tau_c^2\tau_l\tau_s^3 + 416\tau_c\tau_l^3\tau_s^2 + 80\tau_c\tau_l^2\tau_s^3 + 8\tau_c\tau_l\tau_s^4 + 48\tau_l^2\tau_s^4 \end{array}\right)}.$$

Substituting this back to the quadratic above we find that the minimized value is:

$$\begin{aligned}
&\frac{1}{2}\frac{\tau_l^3\tau_s^4}{\left(\begin{array}{c} 8\tau_c^5 + 24\tau_c^4\tau_l + 20\tau_c^4\tau_s + 24\tau_c^3\tau_l^2 + 76\tau_c^3\tau_l\tau_s + 18\tau_c^3\tau_s^2 + 8\tau_c^2\tau_l^3 + 56\tau_c^2\tau_l^2\tau_s \\ +55\tau_c^2\tau_l\tau_s^2 + 7\tau_c^2\tau_s^3 + 52\tau_c\tau_l^2\tau_s^2 + 10\tau_c\tau_l\tau_s^3 + \tau_c\tau_s^4 + 6\tau_l\tau_s^4 \end{array}\right)} \\
&\times\left(\begin{array}{c} 343\tau_c^6 + 2352\tau_c^5\tau_l + 1176\tau_c^5\tau_s + 5376\tau_c^4\tau_l^2 + 6944\tau_c^4\tau_l\tau_s + 1344\tau_c^4\tau_s^2 + 4096\tau_c^3\tau_l^3 \\ +13312\tau_c^3\tau_l^2\tau_s + 6448\tau_c^3\tau_l\tau_s^2 + 512\tau_c^3\tau_s^3 + 8192\tau_c^2\tau_l^3\tau_s + 9984\tau_c^2\tau_l^2\tau_s^2 + 1984\tau_c^2\tau_l\tau_s^3 \\ +5120\tau_c\tau_l^3\tau_s^2 + 2048\tau_c\tau_l^2\tau_s^3 + 128\tau_c\tau_l\tau_s^4 + 192\tau_l^2\tau_s^4 \end{array}\right)
\end{aligned}$$

which is strictly positive. Since the quadratic term is strictly positive at its minimum, it is positive for all r. Since r^3 term is positive for $r > 0$ as well, (23) is strictly positive for all $r > 0$. That is, the discriminant is strictly negative for large enough α and small enough τ_f, and hence $H(k) = 0$ has a unique root. QED.

Proof of Propositions 2: First, we derive k_{BM}. Based on (16) and taking expectations, we see that a speculator buys the asset when:

$$\begin{aligned}
&\ln\left(\frac{1}{c}\right) + \frac{\tau_f\bar{f} + \frac{1}{2}}{\tau_f + \tau_l} + \left(\frac{\tau_f + 2\tau_l}{\tau_f + \tau_l}\right)\left(\frac{\tau_f\bar{f} + \tau_s\tilde{s}_i + \tau_c\tilde{s}_c}{\tau_f + \tau_s + \tau_c}\right) \qquad (24)\\
&+\left(\frac{\tau_f + 2\tau_l}{\tau_f + \tau_l}\right)^2 \frac{1}{2(\tau_f + \tau_s + \tau_c)} + \frac{1}{2}\left(\frac{\tau_l}{\tau_f + \tau_l}\right)^2 \sigma_l^2 \\
\geq\ &\frac{1}{\alpha\sigma_s}\left(\frac{\tau_f\bar{f} + \tau_s\tilde{s}_i + \tau_c\tilde{s}_c + \frac{1}{2}\frac{1}{\alpha\sigma_s}}{\tau_f + \tau_s + \tau_c} + k_{BM}\tilde{s}_c - g_{BM}\right) + \frac{1}{2\alpha^2}\sigma_\xi^2.
\end{aligned}$$

Rearranging (24), a speculator buys the asset when $\tilde{s}_i + B_{BM}(k_{BM})\,\tilde{s}_c \geq C_{BM}$ where

$$B_{BM}(k_{BM}) = \frac{\tau_c}{\tau_s} - \frac{\frac{\sqrt{\tau_s}}{\alpha}k_{BM}}{\frac{\tau_s}{\tau_f+\tau_s+\tau_c}\left(\frac{\tau_f+2\tau_l}{\tau_f+\tau_l} - \frac{\sqrt{\tau_s}}{\alpha}\right)}$$

and

$$\begin{aligned} C_{BM} \;=\; & \frac{1}{\left(\frac{\tau_f+2\tau_l}{\tau_f+\tau_l} - \frac{1}{\alpha\sigma_s}\right)\left(\frac{\tau_s}{\tau_f+\tau_s+\tau_c}\right)} \\ & \left(\ln c + \frac{1}{\alpha\sigma_s}\left(\frac{\tau_f\bar{f} + \frac{1}{2}\frac{1}{\alpha\sigma_s}}{\tau_f+\tau_s+\tau_c} - g_{BM}\right) - \frac{\tau_f\bar{f}+\frac{1}{2}}{\tau_f+\tau_l} - \left(\frac{\tau_f+2\tau_l}{\tau_f+\tau_l}\right)\left(\frac{\tau_f\bar{f}}{\tau_f+\tau_s+\tau_c}\right)\right. \\ & \left. - \left(\frac{\tau_f+2\tau_l}{\tau_f+\tau_l}\right)^2 \frac{1}{2(\tau_f+\tau_s+\tau_c)} + \frac{1}{2\alpha^2}\sigma_\xi^2 - \frac{1}{2}\left(\frac{\tau_l}{\tau_f+\tau_l}\right)^2\sigma_l^2\right). \end{aligned}$$

Setting $B_{BM}(k_{BM}) = k_{BM}$ leads to the expression for k_{BM} in (18).

Now, we show that in the main model (with feedback effect) $k^* > \tau_c/\tau_s$ for α large enough. To see this note that $H(\tau_c/\tau_s) > 0$ for α large enough. Since $H(k)$ has a unique root and crosses the axis from above, the conclusion follows. Next, note that $k_{BM} < \tau_c/\tau_s$ and thus $k_{BM} < k^*$ for α large enough. QED.

Proof of Proposition 3: We showed in the proof of Proposition 2 that $k^* > \tau_c/\tau_s$ for α large enough. By inspecting (22), we can see that $H(k)$ shifts up as α increases, so its unique root k^* increases in α. QED.

Proof of Proposition 4:

Consider the following terms involving $1/r$ in $H(k)$ in (22):

$$\begin{aligned} & -\frac{1}{r}\left(\tau_c k(\tau_c\tau_f + \tau_c\tau_l + \tau_f\tau_l + \tau_f\tau_s + 2\tau_l\tau_s + \tau_f^2) - \tau_c^2\tau_l\right) \\ & + \left(1 - \frac{\sqrt{\tau_s}}{\alpha}\right)\frac{\tau_c}{r}\left((\tau_f+\tau_l)(\tau_c+\tau_f)k + \tau_c(\tau_f+\tau_l)\right). \end{aligned}$$

For α large enough, these terms are negative iff k exceeds τ_c/τ_s. So for $k > \tau_c/\tau_s$, $H(k)$ shifts up as r goes up. By Proposition 3, for α large enough, k^* which implicitly depends on r exceeds τ_c/τ_s for all r. Since $H(k)$ crosses the axis once from above at k^*, we see that k^* must be increasing in r. Since increasing σ_ξ and r are inversely related, an increase in σ_ξ leads to a decrease in k^*. QED.

Proof of Proposition 5

Let

$$
\begin{aligned}
D(k) &= -3\left((\tau_f+\tau_c+\tau_l)(\tau_f+\tau_c+\tau_s)+\tau_l\tau_s\right)k^2 - 2\tau_c(\tau_f+\tau_c-\tau_l+2\tau_s)k \\
&\quad + \tau_c(\tau_c-\tau_s) - \frac{1}{r}\left(\tau_c(\tau_f\tau_c+\tau_f\tau_l+\tau_f\tau_s+\tau_c\tau_l+2\tau_l\tau_s+\tau_f^2)\right) \\
&\quad + \left(1-\frac{\sqrt{\tau_s}}{\alpha}\right)\left(3(\tau_f+\tau_c)(\tau_f+\tau_c+\tau_l)k^2+2\tau_c(3\tau_f+3\tau_c+\tau_l)k+\tau_c(\tau_f+3\tau_c)\right) \\
&\quad + \frac{\tau_c}{r}\left(1-\frac{\sqrt{\tau_s}}{\alpha}\right)(\tau_f+\tau_c)(\tau_f+\tau_l)
\end{aligned}
$$

Note that $\partial H/\partial k = D(k)$. When the equilibrium is unique $D(k^*) < 0$ since $H(k)$ crosses zero from above.

To see that for α large $\frac{\partial k^*}{\partial \tau_s} < 0$,note for α large, $\frac{\partial k^*}{\partial \tau_s}$ is arbitrarily close to

$$
\frac{\frac{1}{\tau_s^2\tau_\xi}\left(\tau_c^2\tau_f+2\tau_c^2\tau_l+\tau_c\tau_s^2\tau_\xi k^*+2\tau_c\tau_s^2\tau_\xi\left(k^*\right)^2+\left(\tau_c\tau_s^2\tau_\xi+\tau_f\tau_s^2\tau_\xi+2\tau_l\tau_s^2\tau_\xi\right)\left(k^*\right)^3\right)}{D(k^*)} < 0.
$$

To see that for α large $\frac{\partial k^*}{\partial \tau_c} > 0$,note for α large, $\frac{\partial k^*}{\partial \tau_c}$ is arbitrarily close to

$$
\frac{\begin{pmatrix} \tau_s\left(k^*\right)^3 - 2(2\tau_c+\tau_f+\tau_l-\tau_s)\left(k^*\right)^2 - (8\tau_c+\tau_f-\tau_s)k^* - 4\tau_c \\ -\frac{1}{r}\left(-\tau_s(\tau_f+2\tau_l)k^*+2\tau_c\tau_f+4\tau_c\tau_l\right) \end{pmatrix}}{D\left(k^*\right)}.
$$

Also,

$$
\begin{aligned}
&\tau_c\left(-\tau_s\left(k^*\right)^3+2\left(2\tau_c+\tau_f+\tau_l-\tau_s\right)\left(k^*\right)^2+\left(8\tau_c+\tau_f-\tau_s\right)k^*+4\tau_c\right. \\
&\left.+\frac{1}{r}\left(-\tau_s\left(\tau_f+2\tau_l\right)k^*+2\tau_c\tau_f+4\tau_c\tau_l\right)\right) > -\tau_s\left(\tau_c+\tau_f+2\tau_l\right)\left(k^*\right)^3 \\
&+2\tau_c\left(\tau_f+\tau_l-\tau_s+\tau_c\right)\left(k^*\right)^2+\tau_c\left(\tau_f-\tau_s+4\tau_c\right)k^*+2\tau_c^2 \\
&+\frac{\tau_c}{r}\left(\tau_c\tau_f+2\tau_c\tau_l-\tau_s\left(\tau_f+2\tau_l\right)k^*\right)
\end{aligned}
$$

and the right hand side of the above inequality is arbitrarily close to $H(k^*)$ which is equal to zero.

Finally, to see that for α large $\frac{\partial k^*}{\partial \tau_l} < 0$,not for α large, $\frac{\partial k^*}{\partial \tau_l}$ is arbitrarily close to

$$
\frac{\frac{2}{r}(k^*\tau_s-\tau_c)(r\left(k^*\right)^2+\tau_c)}{D\left(k^*\right)} < 0
$$

since $k^* > \tau_c/\tau_s$. QED

Proof of Proposition 6:

We substitute I equation (2) into equation (19) and compute the expectations:

$$
\begin{aligned}
& \frac{1}{c}E\left[\exp\left(\tilde{f}\right)\exp\left(\frac{\tau_f\bar{f}+\tau_l s_l+\tau_p z\left(P\right)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right)\right] \\
& -\frac{1}{2}\frac{1}{c}E\left[\exp\left(2\left(\frac{\tau_f\bar{f}+\tau_l s_l+\tau_p z\left(P\right)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right)\right)\right] \\
= \; & \frac{1}{c}E\left[\exp\left(2\tilde{f}+\frac{\tau_f\left(\bar{f}-\tilde{f}\right)+\tau_l\sigma_l\tilde{\epsilon}_l+\tau_p\left(z\left(P\right)-\tilde{f}\right)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right)\right] \\
& -\frac{1}{2}\frac{1}{c}E\left[\exp\left(2\left(\tilde{f}+\frac{\tau_f\left(\bar{f}-\tilde{f}\right)+\tau_l\sigma_l\tilde{\epsilon}_l+\tau_p\left(z\left(P\right)-\tilde{f}\right)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right)\right)\right] \\
= \; & \frac{1}{c}\frac{1}{2}\exp\left(2\bar{f}+\frac{1}{\tau_f}\frac{\tau_f+2\tau_l+2\tau_p}{\tau_f+\tau_l+\tau_p}\right).
\end{aligned}
$$

Therefore the maximization problem can be viewed as maximizing the following expression in k :

$$
\exp\left(\frac{\tau_f+2\tau_l+2\tau_p}{\tau_f+\tau_l+\tau_p}\right),
$$

and this is equivalent to maximizing τ_p which is maximized at $\tau_c/\left(\tau_s\tau_\xi\right)$. QED.

Proof of Proposition 7:

For α large enough $H(k)$ evaluated at $k_{OP}=\tau_c/r$ is approximately:

$$
\frac{\tau_c^2}{r^3}\left(\tau_c+r\right)\left(2\tau_c r-\tau_c\tau_s+2\tau_f r-\tau_f\tau_s+2\tau_l r-2\tau_l\tau_s-r\tau_s+2r^2\right)
$$

which is negative for r small. Moreover it may be decreasing in r for r small but eventually increases and becomes positive. This means that there is a cutoff $\bar{r}$ for r such that for $r<\bar{r}$ we have $k^*<k_{OP}$ and for $r>\bar{r}$ we have $k^*>k_{OP}$. QED.

Proof of Proposition 8: (a) The market clearing price is

$$
P=\exp\left(\frac{\left(1+k\right)\tilde{f}+k\sigma_c\tilde{\epsilon}_c-g+\sigma_s\tilde{\xi}}{\alpha\sigma_s}\right),
$$

and its non-fundamental volatility can be written as the volatility of the following:

$$
z(P)-\tilde{f}=\frac{g+\alpha\sigma_s\ln(P)}{1+k}-\tilde{f}=\frac{k}{1+k}\sigma_c\tilde{\epsilon}_c+\frac{\sigma_s}{1+k}\tilde{\xi}.
$$

It is straightforward to show that when $k=k_{OP}=\tau_c/\left(\tau_s\tau_\xi\right)$, its non-fundamental volatility is the lowest and is

$$
\text{Non-Fundamental Volatility (Asset Price)}=\frac{1}{\tau_c+\tau_s\tau_\xi}.
$$

(b) We know that:

$$I=\frac{1}{c}\exp\left(\frac{\tau_f\bar{f}+\tau_l s_l+\tau_p\left(\tilde{f}+\frac{k}{1+k}\sigma_c\tilde{\epsilon}_c+\frac{\sigma_s}{1+k}\tilde{\xi}\right)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right).$$

Taking logs on both sides, we obtain:

$$\ln I=\ln\left(\frac{1}{c}\right)+\left(\frac{\tau_f\bar{f}+\tau_l s_l+\tau_p\left(\tilde{f}+\frac{k}{1+k}\sigma_c\tilde{\epsilon}_c+\frac{\sigma_s}{1+k}\tilde{\xi}\right)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right).$$

We can define the non-fundamental volatility of the real investment as the volatility of the following:

$$\frac{(\tau_f+\tau_l+\tau_p)\left(\ln I-\ln\left(\frac{1}{c}\right)\right)-\frac{1}{2}-\tau_f\bar{f}}{\tau_l+\tau_p}-\tilde{f}=\frac{\tau_l\sigma_l\epsilon_l+\tau_p\left(\frac{k}{1+k}\sigma_c\tilde{\epsilon}_c+\frac{\sigma_s}{1+k}\tilde{\xi}\right)}{\tau_l+\tau_p}$$

It is straightforward to show that when $k=k_{OP}=\tau_c/\left(\tau_s\tau_\xi\right)$, $\tau_p=\tau_c+\tau_s\tau_\xi$,and the non-fundamental volatility of the real investment is the lowest which is

$$\text{Non-Fundamental Volatility (Real Investment)}=\frac{1}{\tau_l+\tau_c+\tau_s\tau_\xi}.$$

QED.

Proof of Proposition 9: In this proof we use the notation $\rho=\sqrt{\tau_{cp}/\tau_c}$. Since $\tau_{cp}>\tau_c$, we have $\rho>1$. We start with the capital provider's decision. The capital provider updates his belief based on observing:

$$z(P)=\tilde{f}+\frac{k}{1+k}\sigma_c\tilde{\epsilon}_c+\frac{\sigma_s}{1+k}\tilde{\xi}\text{ and }\tilde{s}_l=\tilde{f}+\sigma_{cp}\tilde{\epsilon}_c.$$

This is equivalent to observing $\tilde{s}_l$ and:

$$z(P)-\frac{k}{1+k}\frac{\sigma_c}{\sigma_{cp}}\tilde{s}_l=\left(1-\frac{k}{1+k}\frac{\sigma_c}{\sigma_{cp}}\right)\tilde{f}+\frac{\sigma_s}{1+k}\tilde{\xi}.$$

Capital provider's conditional belief on $\tilde{f}$ is distributed normally with mean:

$$\left(\frac{\left(1-\frac{k}{1+k}\rho\right)}{\left(\frac{k}{1+k}\rho-1\right)^2+\frac{1}{(1+k)^2}\frac{\tau_{cp}}{\tau_s\tau_\xi}}\right)z(P)+\left(1+\frac{\left(\frac{k}{1+k}\rho-1\right)}{\left(\frac{k}{1+k}\rho-1\right)^2+\frac{1}{(1+k)^2}\frac{\tau_{cp}}{\tau_s\tau_\xi}}\right)\tilde{s}_l$$

and variance:

$$\Omega=\frac{1}{\tau_s\tau_\xi\left(k\left(\rho-1\right)-1\right)^2+\tau_{cp}}.$$

Using the capital provider's investment rule and taking expectations, we can express the level of investment as:

$$\begin{aligned} I &= \frac{1}{c}E[\tilde{F}|s_l,P] = \frac{1}{c}E[\exp\left(\tilde{f}\right)|s_l,P] \quad (25)\\ &= \frac{1}{c}\exp\left(\left(\frac{\left(1-\frac{k}{1+k}\rho\right)}{\left(\frac{k}{1+k}\rho-1\right)^2+\frac{1}{(1+k)^2}\frac{\tau_{cp}}{\tau_s\tau_\xi}}\right)z(P)+\left(1+\frac{\left(\frac{k}{1+k}\rho-1\right)}{\left(\frac{k}{1+k}\rho-1\right)^2+\frac{1}{(1+k)^2}\frac{\tau_{cp}}{\tau_s\tau_\xi}}\right)s_l+\frac{1}{2\Omega}\right)\end{aligned}$$

Given the capital provider's investment policy in (25) and the price, we can now write speculator i's expected profit from buying the asset given the information that is available to him (shorting the asset would give the negative of this):

$$E\left[\tilde{F}I-P|s_i,s_c,P\right]= \quad (26)$$

$$\frac{1}{c}E\left[\exp\left(\begin{array}{c}\tilde{f}+\frac{1}{2\Omega}+\\ \left(\frac{\left(1-\frac{k}{1+k}\rho\right)}{\left(\frac{k}{1+k}\rho-1\right)^2+\frac{1}{(1+k)^2}\frac{\tau_{cp}}{\tau_s\tau_\xi}}\right)z(P)+\left(1+\frac{\left(\frac{k}{1+k}\rho-1\right)}{\left(\frac{k}{1+k}\rho-1\right)^2+\frac{1}{(1+k)^2}\frac{\tau_{cp}}{\tau_s\tau_\xi}}\right)s_l\end{array}\right)\middle|\begin{array}{c}s_i,\\ s_c,\\ P\end{array}\right]-P$$

To solve for the speculators' conditional expectation, note that since speculators know both P and s_c, that means that they observe:

$$g+(\sigma_s\alpha-m)\ln(P)-ks_c=\tilde{f}+\sigma_s\tilde{\xi}. \quad (27)$$

Therefore, conditional on observing s_i, s_c and P speculator i believes that $\tilde{f}$ is distributed normally with mean

$$\frac{\tau_s}{\tau_s+\tau_c+\tau_s\tau_\xi}s_i+\frac{\tau_c}{\tau_s+\tau_c+\tau_s\tau_\xi}s_c+\frac{\tau_s\tau_\xi}{\tau_s+\tau_c+\tau_s\tau_\xi}\left(g+(\sigma_s\alpha-m)\ln(P)-ks_c\right)$$

and precision $\tau_s+\tau_c+\tau_s\tau_\xi$. Moreover,

$$\sigma_{cp}\tilde{\epsilon}_c=\frac{\sigma_{cp}}{\sigma_c}\left(s_c-\tilde{f}\right).$$

Now we take expectation in (26) and note that a speculator would purchase the asset if and only if his expected profit is no less than zero:

$$\begin{array}{l}\frac{1}{c}\exp\left(\begin{array}{c}\left(\frac{\left(1-\frac{k}{1+k}\rho\right)}{\left(\frac{k}{1+k}\rho-1\right)^2+\frac{1}{(1+k)^2}\frac{\tau_{cp}}{\tau_s\tau_\xi}}\right)z(P)+\left(1+\frac{\left(\frac{k}{1+k}\rho-1\right)}{\left(\frac{k}{1+k}\rho-1\right)^2+\frac{1}{(1+k)^2}\frac{\tau_{cp}}{\tau_s\tau_\xi}}\right)\frac{1}{\rho}s_c\\ +\left(\frac{\tau_s}{\tau_s+\tau_c+\tau_s\tau_\xi}s_i+\frac{\tau_c}{\tau_s+\tau_c+\tau_s\tau_\xi}s_c+\frac{\tau_s\tau_\xi}{\tau_s+\tau_c+\tau_s\tau_\xi}\left(g+(\sigma_s\alpha-m)\ln(P)-ks_c\right)\right)\\ \left(1+\left(1+\frac{\left(\frac{k}{1+k}\rho-1\right)}{\left(\frac{k}{1+k}\rho-1\right)^2+\frac{1}{(1+k)^2}\frac{\tau_{cp}}{\tau_s\tau_\xi}}\right)\left(1-\frac{1}{\rho}\right)\right)\\ +\text{variance terms}\end{array}\right)\\ -P\geq 0\end{array} \quad (28)$$

Condition (28) can be rewritten as

$$s_i + B_P(k)\, s_c \geq A \ln P + B,$$

where $B_P(k)$ is the best response function given by

$$B_P(k) = \tag{29}$$
$$\frac{\left(1+\frac{\left(\frac{k}{1+k}\rho-1\right)}{\left(\frac{k}{1+k}\rho-1\right)^2+\frac{1}{(1+k)^2}\frac{\tau_{cp}}{\tau_s\tau_\xi}}\right)\frac{1}{\rho}+\left(\frac{\frac{\tau_{cp}}{\rho^2}-k\tau_s\tau_\xi}{\tau_s+\frac{\tau_{cp}}{\rho^2}+\tau_s\tau_\xi}\right)\left(1+\left(1+\frac{\left(\frac{k}{1+k}\rho-1\right)}{\left(\frac{k}{1+k}\rho-1\right)^2+\frac{1}{(1+k)^2}\frac{\tau_{cp}}{\tau_s\tau_\xi}}\right)\left(1-\frac{1}{\rho}\right)\right)}{\left(\frac{\tau_s}{\tau_s+\frac{\tau_{cp}}{\rho^2}+\tau_s\tau_\xi}\left(1+\left(1+\frac{\left(\frac{k}{1+k}\rho-1\right)}{\left(\frac{k}{1+k}\rho-1\right)^2+\frac{1}{(1+k)^2}\frac{\tau_{cp}}{\tau_s\tau_\xi}}\right)\left(1-\frac{1}{\rho}\right)\right)\right)}.$$

Setting $B_P(k) - k = 0$ gives us the following equilibrium condition:

$$\frac{C(k)}{\tau_s\tau_\xi\rho\,(k(\rho-1)-1)\,(2k(\rho-1)-1)+\tau_{cp}(2\rho-1)} = 0 \tag{30}$$

where

$$\begin{aligned} C(k) &= \left(-2\tau_s^2\tau_\xi\rho^2(\rho-1)^2(\tau_\xi+1)\right)k^3+\left(\tau_s\tau_\xi(\rho-1)\left(-\tau_{cp}+2\tau_{cp}\rho+4\tau_s\rho^2+4\tau_s\tau_\xi\rho^2\right)\right)k^2 \\ &\quad+\left(2\tau_{cp}\tau_s\tau_\xi-2\tau_s^2\tau_\xi\rho^2-2\tau_{cp}\tau_s\tau_\xi\rho^2-2\tau_{cp}\tau_s\tau_\xi\rho-2\tau_s^2\tau_\xi^2\rho^2-2\tau_{cp}\tau_s\rho^2+\tau_{cp}\tau_s\rho\right)k \\ &\quad+\left(2\tau_{cp}^2+\tau_{cp}\tau_s\rho+\tau_{cp}\tau_s\tau_\xi+\tau_{cp}\tau_s\tau_\xi\rho\right). \end{aligned}$$

First we show that the denominator of (30) is strictly positive for τ_s or τ_ξ small or τ_{cp} large enough. This is because the denominator is minimized at $k = 3/(4(\rho-1))$ and the value of the denominator at that point is $(2\rho-1)\tau_{cp} - (1/8)\tau_s\tau_\xi\rho$ which is strictly positive if τ_s or τ_ξ small or τ_{cp} large enough. So the equilibrium condition becomes $C(k) = 0$. Finally, we show that $C(k) = 0$ has a unique strictly positive root by verifying that the discriminant of $C(k)$ is negative if $\rho > 1$ and τ_s or τ_ξ are small or τ_{cp} is large. QED.

Proof of Proposition 10: In this proof we use the notation $\rho = \sqrt{\tau_{cp}/\tau_c}$ that was introduced in the previous proof. Following steps similar to the above we see that a speculator buys the asset if and only if

$$\frac{1}{c}\exp\left(\begin{array}{c}\frac{\sqrt{\tau_c}}{\sqrt{\tau_{cp}}}s_c+\left(2-\frac{\sqrt{\tau_c}}{\sqrt{\tau_{cp}}}\right)\left(\begin{array}{c}\frac{\tau_s}{\tau_s+\tau_c+\tau_s\tau_\xi}s_i+\frac{\tau_c}{\tau_s+\tau_c+\tau_s\tau_\xi}s_c\\+\frac{\tau_s\tau_\xi}{\tau_s+\tau_c+\tau_s\tau_\xi}\left(g_N+(\sigma_s\alpha-m_N)\ln(P)-k_N s_c\right)\end{array}\right)\\+\text{variance terms}\end{array}\right)-P \geq 0. \tag{31}$$

Condition (31) can be rewritten as

$$s_i + B_N(k)\, s_c \geq A_N \ln P + B_N,$$

where $B_N(k)$ is the best response function given by:

$$B_N(k) = \frac{\left(2-\frac{1}{\rho}\right)\left(\frac{\frac{\tau_{cp}}{\rho^2}-k\tau_s\tau_\xi}{\tau_s+\frac{\tau_{cp}}{\rho^2}+\tau_s\tau_\xi}\right)+\frac{1}{\rho}}{\left(2-\frac{1}{\rho}\right)\left(\frac{\tau_s}{\tau_s+\frac{\tau_{cp}}{\rho^2}+\tau_s\tau_\xi}\right)}. \tag{32}$$

In the benchmark equilibrium, k_N solves $B_N(k_N) = k_N$. Thus,

$$k_N = \left(\frac{1}{2\rho-1}\right)\left(\frac{\tau_{cp}}{\tau_s(1+\tau_\xi)}\frac{2}{\rho}+1\right).$$

Recall that k^{**} satisfies $C(k^{**}) = 0$ where $C(\cdot)$ is defined in the proof of Proposition 9. Note that

$$C(k_N) = -\frac{2}{\tau_s}\frac{\tau_\xi\left(\tau_{cp}+\tau_s\rho^2+\tau_s\tau_\xi\rho^2\right)^2\left(2\tau_{cp}-2\tau_{cp}\rho+\tau_s\rho^2+\tau_s\tau_\xi\rho^2\right)}{\rho^2(2\rho-1)^3(\tau_\xi+1)^2}.$$

Since $C(0) > 0$ and $C(\cdot)$ crosses zero from above this implies that $k^* > k_N$ if and only if $C(k_N) > 0$. That is if and only if

$$2\tau_{cp} - 2\tau_{cp}\rho + \tau_s\rho^2 + \tau_s\tau_\xi\rho^2 < 0.$$

Substituting $\sqrt{\tau_{cp}/\tau_c}$ for ρ, we obtain the condition in the statement of the proposition.

QED.