Optimal Issuance for Proof-of-Stake Blockchains

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Abstract

Proof-of-stake (PoS) blockchains reward validators who stake tokens by issuing new tokens. While a high percentage of staked tokens strengthens blockchain security, it also dilutes the value of tokens held by non-stakers. This paper presents a simple framework to determine the optimal issuance policy and staking yield by evaluating the trade-off between security and dilution. The analysis demonstrates that, in the long run, the optimal policy eliminates this trade-off by precisely maintaining a target staking ratio, even if it results in some token dilution. When applied to an issuance curve analogous to Ethereum's, the analysis shows that staking yields respond to economic shocks in line with the optimal policy, though the responses are less pronounced than those implied by the optimal policy. Consistent with the model's long-run equilibrium, the 20 largest PoS blockchains approximately equalize staking yields after accounting for dilution.

Keywords: Staking yield, cryptocurrencies, Ethereum, issuance curve, Friedman rule, Ramsey policy. JEL codes: E42, G11.

1 Introduction

Proof-of-stake (PoS) blockchains need to determine how to compensate validators who stake. Typically, the compensation includes some promised yield paid with newly issued tokens. The compensation scheme also determines the evolution of the money supply.

Ethereum, the leading PoS blockchain, has seen a significant rise in staking since its transition to proof-of-stake, with the share of ETH staked in the total supply continuing to

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grow. This trend raises concerns about a future where the majority of ETH is staked, potentially leading to undesirable consequences. To address this, recent research (e.g., Dietrichs and Schwarz-Schilling, 2024) has explored modifying the issuance curve. A key challenge is modeling how stakers would respond to proposed issuance curves, as this determines the supply of staked ETH. Since changing the issuance policy is costly, a model of the long-run supply of stake is essential for designing an effective and sustainable policy.

This paper presents a framework suitable for studying issuance policies and staking yields. The model is explicitly dynamic where long-run outcomes emerge from period-by-period optimizing behavior. The price of the token is determined in equilibrium. For transparency, the model only includes the most essential elements.

The model highlights the trade-off between security and dilution. A higher staking yield increases the staking share and this typically contributes positively to the security of the network. The staking yield is paid from issuance which dilutes token holders. Users compensate for the lower price appreciation induced by dilution by requiring higher marginal utility, which results in lower token holdings. Dilution is not just a transfer from users to stakers but it has real effects on the network.

Deterministic dynamics and optimal staking yields can be characterized analytically. The model-implied relation between staking yields and staking shares at steady state is upward sloping which is consistent with the cross-section of the 20 largest PoS blockchains. Approximately, the staking yields of these blockchains are equalized after accounting for dilution, which is a property of the steady state of my model.

I derive the optimal policy under commitment, also referred to as the Ramsey policy. This policy features a transition period toward a steady state, reflecting the intertemporal incentive mechanisms available to a policymaker with commitment. A key finding is that, in the long run, the optimal policy does not involve a trade-off between security and dilution. Instead, it precisely maintains a target staking ratio while allowing for some dilution.

Sharp predictions for long-run optimal policies are common in the literature, though they are not always immediately intuitive. For example, many monetary models predict that the long-run cost of holding money should be zero, known as the Friedman rule. However, it is not always immediately obvious whether this rule applies, see for instance Chari and Kehoe (1999). I show that in the model with staking, the Friedman rule does not apply. Unlike traditional models for monetary policy, a PoS protocol must compensate stakers, making it optimal for there to be a cost to holding money.

I also examine the optimal responses of staking yields to shocks in a stochastic environment and compare these responses to those implied by an issuance curve analogous to Ethereum's. Specifically, I analyze two types of shocks: those that affect the attractiveness of staking and those that impact the required rate of return, which more broadly affect the appeal of holding the token. In both cases, the optimal policy aims to stabilize the staking share by adjusting the staking yield. An alternative, simple issuance curve—where staking yields depend on the staking share via an inverse square root function, analogous to Ethereum's—exhibits similar qualitative properties. However, this curve is less responsive than the optimal policy and leads to greater volatility in the staking share.

Economic analyses of proof-of-stake blockchains have focused on the conditions under which proof-of-stake generates consensus (Saleh (2021)), equilibrium staking levels (Cong, He and Tang (2022) and Kose, Rivera and Saleh (2021)), pricing tokens (Fanti, Kogan and Viswanath (2019)), and Ethereum's macroeconomy (Jermann (2023)). This paper contributes by studying optimal staking yield policies.

Optimal policies from economic perspectives for blockchains more generally have been studied by Chiu and Koeppl (2022), Gryglewicz, Mayer and Morellec (2021), Goldstein, Gupta, and Sverchkov (2024), Jermann and Xiang (2022), and Abadi and Brunnermeier (2024).

The next section presents a deterministic model for PoS and analyzes its equilibrium and optimal policies. This requires relatively few assumptions and analytical characterizations are possible. The following section studies a stochastic version of the model and also considers an issuance curve of the type used by Ethereum.

2 Deterministic model

This section starts with the basic model that only includes the most essential features. Tokens can be either used or staked. Used tokens provide utility. Staked tokens provide rewards. The token price is determined in equilibrium. After characterizing the equilibrium, two types of optimal policies are derived: the optimal constant staking yield and the optimal staking yield policy under commitment (Ramsey).

Time is discrete, indexed by $t = 0, 1, 2, .,$ with an infinite horizon. The period utility function for users $v(.)$ depends positively on the token amount used M_t^U evaluated in terms of its price in the (dollar) numeraire p_t and on the network's security/productivity S_t. Agents maximize lifetime utility defined as the sum of the expected period utilities

$$\max_{M_{t+1+j}^U, D_{t+1+j}, C_{t+j}} \sum_{j=0}^{\infty} \beta^j \left[C_{t+j} + v\left(S_{t+j}, p_{t+j} M_{t+j}^U\right) \right]$$

subject to budget constraints

$$C_t + p_t D_{t+1} + p_t M_{t+1}^U = p_t D_t (1 + y_t) + p_t M_t^U + Y_t, \tag{1}$$

with the discount factor $\beta \in (0,1)$. C_t is consumption of the numeraire good (the US consumption basket) and Y_t represents income unrelated to the network economy. Beyond providing a numeraire, consumption plays no role. D_t (for deposits) represents the amount of staked tokens and y_t the staking yield. M_{t+1}^U and D_{t+1} represent token amounts purchased in period t that provide services and rewards in period $t+1$.

This is a pared down version of the model in Jermann (2023). One extension is to explicitly incorporate into the model the benefits of staking. It is assumed that the network security/productivity increases with the aggregate staking share $d_t \equiv D_t / \left(M_t^U + D_t\right)$,

$$S_t = S(d_t), \tag{2}$$

for a function $S(.)$ to be specified below. Note, agents are atomistic and take the aggregate staking share as given.

Why not have $S(.)$ depend on the value of stake, $p_t D_t$? The cost of an attacker controlling a large share of the stake depends on the dollar value, but the incentives to attack also increase with the expected gains which are related to the value of the network. Equivalently, attackers are attracted by the expected after-cost gains. Having security depend on the staking share can approximate for this.

With prices p_t and staking yields y_t taken as given, agents' first-order conditions for optimal token holdings for using and staking, respectively, are

$$p_t = \beta \left[1 + \frac{\partial v \left(S_{t+1}, p_{t+1} M_{t+1}^U\right)}{\partial p_{t+1} M_{t+1}^U} \right] p_{t+1}, \tag{3}$$

and

$$p_t = \beta \left[1 + y_{t+1}\right] p_{t+1}. \tag{4}$$

These equations show how agents need to be compensated for holding tokens with price appreciation as well as either the marginal utility from the token, $\frac{\partial v\left(S_{t+1}, p_{t+1} M_{t+1}^U\right)}{\partial p_{t+1} M_{t+1}^U}$, or the staking yield, y_{t+1}.

The staking yield is paid with newly issued tokens so that the money supply, $M_{r+1} = M_{t+1}^U + D_{t+1}$, evolves as

$$M_{t+1}^U + D_{t+1} = M_t^U + D_t + y_t D_t. \tag{5}$$

Equilibrium is defined for given initial values of M_0^U and D_0, and an exogenous processes for y_t, as the processes for M_{t+1}^U, D_{t+1}, p_t and S_t satisfying equations, (3), (4) , and (5).

2.1 Equilibrium

To characterize the equilibrium, I initially restrict the marginal utility for used tokens to be a positive and decreasing function which only depends on real balances $p_t M_t^U$

$$\frac{\partial v\left(S_{t+1}, p_{t+1} M_{t+1}^U\right)}{\partial p_{t+1} M_{t+1}^U} = v'\left(p_{t+1} M_{t+1}^U\right).$$

Assuming separability with respect to S_{t+1} allows a full characterization without directly assuming specific functional forms. For quantitative analyses this restriction can be relaxed and, for specifications that are reasonably close, should not fundamentally alter the qualitative properties derived here.

With these assumptions, the three equilibrium equations become

$$p_t = \beta\left(1 + y_{t+1}\right) p_{t+1} \tag{6}$$

$$p_t = \beta\left(1 + v'\left(p_{t+1} M_{t+1}^U\right)\right) p_{t+1} \tag{7}$$

$$M_{t+1}^U + D_{t+1} = M_t^U + D_t + y_t D_t. \tag{8}$$

The first equation pins down the growth rate of the price as

$$\frac{p_{t+1}}{p_t} = \frac{1}{\beta\left(1 + y_{t+1}\right)}.$$

A higher staking yield results in lower price growth because investors have a fixed required (gross) return given by $1/\beta$. Combining the first two equations determines the market value of used tokens

$$v'\left(p_{t+1} M_{t+1}^U\right) = y_{t+1}, \tag{9}$$

conditional on the functional form. In equilibrium, returns to using and staking are equalized and this requires that the staking yield determines the marginal utility of used tokens (the *yield* from using tokens).

Consider the case with a constant staking yield determined by the network. The following proposition characterizes the equilibrium.

Proposition 1 *Assuming a constant staking yield $y_t = y$ for $t \geq 1$, if $\beta(1+y) > 1$, for arbitrary initial values M_0^U, D_0 and y_0, the staking share $d_1 = D_1 / \left(D_1 + M_1^U\right)$ jumps to its* *steady state value so that for* $t \geq 1$

$$d_t = \frac{\beta (1 + y) - 1}{y}, \tag{10}$$

with

$$\frac{\partial d_t}{\partial y} = \frac{1 - \beta}{y^2} > 0$$

and

$$\frac{D_{t+1}}{D_t} = \frac{M_{t+1}^U}{M_t^U} = \beta (1 + y) > 1.$$

It is easy to see that once a value for p_0 is determined, the paths for all equilibrium variables p_t, M_{t+1}^U and D_{t+1} are determined by the three equilibrium equations (6), (7) and (8). Therefore, we can index candidate equilibrium paths by p_0. The proof (see the appendix) first computes an equilibrium steady state for d_t and the value for p_0 that corresponds to this equilibrium. It is then shown that alternative values for p_0 and their associated paths cannot be equilibrium outcomes because they violate a transversality condition and can imply negative values for D_t. For $\beta (1 + y) < 1$, even the path associated with the stable p_0 is not an equilibrium. Note that this restriction on y could be relaxed in richer models, for instance, if fee income is distributed to stakers.

While this equilibrium features a constant staking ratio d_t, the quantity of money and M_{t+1}^U and D_{t+1} are increasing over time, and their rate of growth is an increasing function of the staking yield. In line with basic intuition, the equilibrium staking share is increasing in the staking yield. Note, the equilibrium does not depend on the security function because the utility function is separable and agents take the security level as given.

Figure 1 plots staking yields against staking shares for the 20 largest PoS blockchains by staked capitalization alongside the corresponding relationship implied by Equation (10). The discount parameter β is selected to minimize the sum of squared deviations between model and data for the 20 observations. The scatterplot is consistent with the upward-sloping relation between staking yields and staking share implied by the model. Interestingly, the larger blockchains in terms of market capitalization (Ethereum, Solana, BNB, Tron and Toncoin) are relatively close to the model-implied line while the biggest outliers are some of the relatively smaller blockchains. It should be noted that there are many differences between these blockchains and the model is extremely stylized so that one should not expect a very tight fit of the model to this cross-section.

To provide more intuition for Equation (10) that links the staking share to the staking yield, consider that the required rate of return in terms of the numeraire is given by $1/\beta$, the inverse of the discount factor. The return to staking in terms of the numeraire is given by $(1+y)\, p_{t+1}/p_t$ with p_t the price of the token. In the long run, the price change in this model is just the inverse of the dilution induced by the growth rate of the money supply, that is $p_{t+1}/p_t = M_t/M_{t+1}$. And, from the money supply, Equation (8), the money growth rate is given by $M_{t+t}/M_t = 1 + d_t y_t$. Putting this together,

$$1/\beta = (1+y)\, \frac{p_{t+1}}{p_t} = \frac{1+y}{1+dy}$$

which implies

$$d = \frac{\beta\,(1+y) - 1}{y}.$$

Effectively, this equation says that, in the long run, returns across blockchains are equalized and that the return represents the staking yield adjusted for dilution. As shown in Figure 1, current staking yields and staking shares of the 20 largest PoS blockchain approximately satisfy this relation. A more general model would also consider adjustments for risk and for the price appreciation induced by increasing adoption or usefulness of the blockchain, and this would have the potential to better capture the differences across blockchains. See Jermann (2023) for a more general pricing model with these features.

2.2 Optimal policy

An optimal policy is a process for y_t that maximizes the objective function of the network. As the objective function, I consider the lifetime utility to users from the network

$$U_t = \sum_{j=0}^{\infty} \beta^j v\left(S\left(d_{t+j}\right), p_{t+j} M^U_{t+j}\right).$$

This represents the welfare produced by the network.[1]

The objective function is increasing in real balances used pM^U and the staking share d. A priori we would expect a higher staking yield y to increase d and to raise utility through higher security S. A higher staking yield is also likely to imply higher money growth and lower price growth. Everything else equal, users will require a higher marginal utility (see equation (3)) and reduce real balances used pM^U. This reduction in real balances lowers utility. This is the way the model captures the security/dilution trade-off.

To derive an optimal policy, a utility function needs to be specified. I use an isoelastic

[1] Users have to forego consumption of goods and services representing the value of their token holdings, so one could subtract these costs from the utility. In this simple model, at the aggregate level, no net resources flow between the token network and the fiat economy, as can be seen by combining the budget constraint and the resource constraint, Equations (1) and (5), respectively. So, the net costs are zero. More general models could imply such net flows, for instance, if capital investments were modelled.

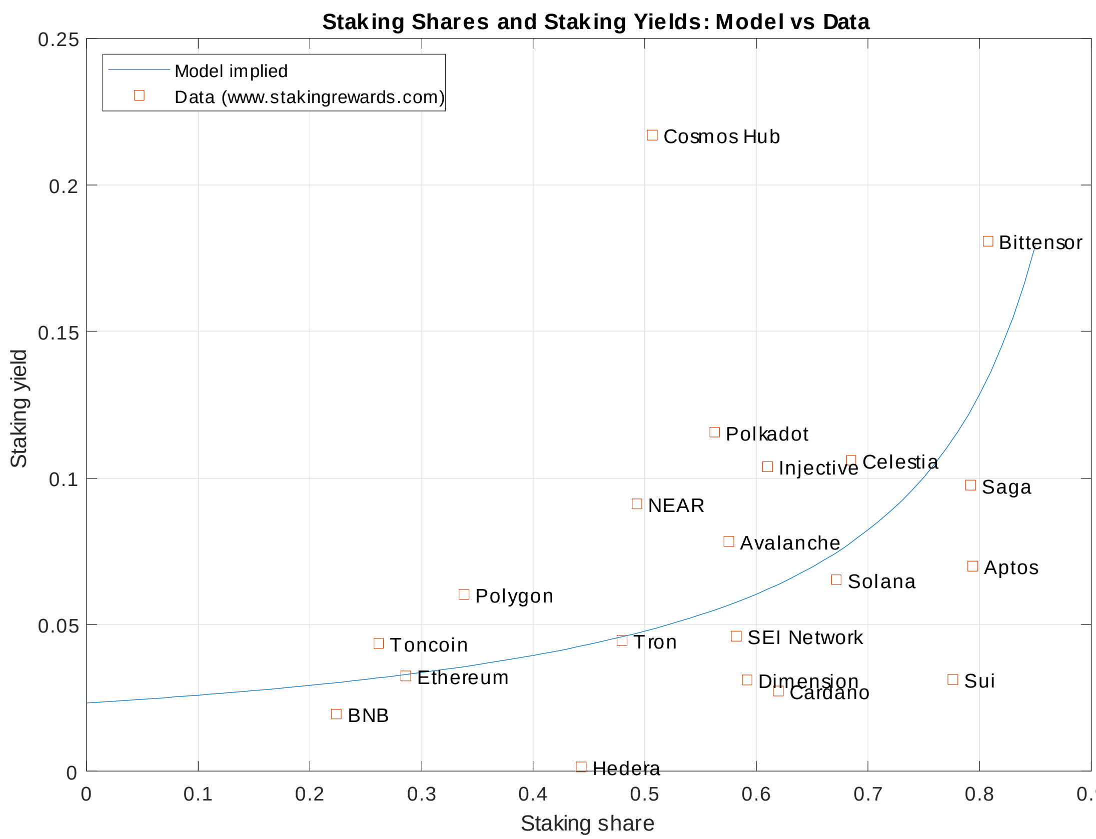

Figure 1: Model vs Data. Staking yields and staking shares are for the 20 largest PoS blockchains by staked capitalization on 10/28/2024 from https://www.stakingrewards.com/. The single model parameter is selected for the best least squares fit at $\beta = 0.9772$.

money-in-utility function with a security function that is separable and quadratic:

$$v\left(p_t M_t^U, d_t\right) = \frac{1}{1-\zeta}\left(p_t M_t^U\right)^{1-\zeta} - \frac{\alpha}{2}\left(\bar{d}_t - d^*\right)^2. \tag{11}$$

The isoelastic specification with $\zeta > 0$ is parsimonious, and for the limiting case $\zeta \rightarrow 1$ specializes to $\ln\left(p_t M_t^U\right)$ which admits some closed form solutions. Security from staking is highest at d^* which can be viewed as the target level abstracting from the costs of dilution, and $\alpha > 0$ measures the cost of deviating from the target. The implied boundedness of this function is useful as it allows for a solution to the unrestricted optimal policy. The fact that security can decrease beyond a given level is consistent, for instance, with the idea that slashing could become less credible if it would affect a larger share of the overall money supply (see Dietrichs and Schwarz-Schilling (2024)). Overbars are used to indicate aggregate (per capita) quantities that individual agents take as given but a protocol designer internalizes.

2.2.1 Optimal constant staking yield

I first consider the case of the utility-maximizing staking yield for a policy that maintains a constant yield after the predetermined initial value y_0. In this case, the optimal policy equalizes the marginal utility of security from staking to the marginal utility of money balances. The optimal level of security is below the target level to limit the distortion from dilution.

Proposition 2 *For a given initial value y_0, assuming a constant staking yield $y_t = y$ for $t \geq 1$, the staking yield that maximizes lifetime utility with period utility (11) specialized to $\ln\left(p_t M_t^U\right) - \frac{\alpha}{2}\left(\bar{d}_t - d^*\right)^2$ is*

$$y = \frac{\alpha(1-\beta)}{2}\left[-(\beta - d^*) + \left((\beta - d^*)^2 + \frac{4}{\alpha}\right)^{0.5}\right]$$

and the corresponding staking share

$$d = d^* - \frac{1}{2}(\beta - d^*)\left[\left(1 + \frac{4}{\alpha(\beta - d^*)^2}\right)^{0.5} - 1\right] < d^*,$$

where the last inequality requires $\beta - d^* > 0$.

As shown by the proposition, the optimal staking level is below the target level d^*. It is intuitive that the optimal policy which produces a finite level for the money used, $p_t M_t^U$, for which the marginal utility is positive does not fully close the gap between the staking ratio and its target level, which would drive the marginal utility of security to zero. The role of α is also as expected. Namely, a larger value for α implies a higher cost for deviating from the target and implies an optimal staking share that gets closer to the target level. The more general case with $\zeta \neq 1$ is qualitatively similar for values ζ close to 1, but it does not admit a closed form expression and needs to be solved numerically.

2.2.2 Optimal policy with commitment (Ramsey)

The Ramsey policy is defined as the optimal sequence of staking yields $\{y_t\}_{t=1}^{\infty}$ that implements the utility maximizing allocation as of period $t = 0$. Specifically, it is the solution to

$$\max_{\left\{y_{t+1}, D_{t+1}, M_{t+1}^U, p_t\right\}} \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{1-\zeta} \left(p_t M_t^U\right)^{1-\zeta} - \frac{\alpha}{2} \left(\bar{d}_t - d^*\right)^2 \right\}$$

subject to the sequence of the three equilibrium equations (6), (7) and (8). The solution is found by taking first-order conditions of the Lagrangian that includes the equilibrium conditions as constraints with respect to all the arguments of the maximization. The following proposition, shown in the appendix, characterizes the steady state.

Proposition 3 *For parameter values for which the Ramsey policy admits a steady state, $0 < d^* < \beta < 1$, $0 < \zeta$ and $\zeta \neq 1$, the steady state features*

$$d_t = d^*$$

and

$$y_t = \frac{1-\beta}{\beta - d^*}.$$

The proposition implies that in the long run the Ramsey policy fully closes the gap of the staking share from its target level d^*. Therefore, in the long run, there is no trade-off between the security-based staking target and dilution from issuance. This is in sharp contrast to a policy that is constrained to a constant staking yield characterized in Proposition 2.

Figure 2 displays the Ramsey policy and equilibrium allocation alongside the optimal constant staking yield policy for a numerical example with parameter values $\beta = 0,95$, $d^* = 0.2$, $\alpha = 10$, and $\zeta = 1.5$. Clearly, a constant staking yield is not an optimal policy. In general, a Ramsey policy is nonstationary in the sense that in the initial period the policy is not restricted by past commitment. After that, the optimal policy is affected by past commitment. As shown in the figure, the Ramsey staking yield is increasing over its transition to the steady state and so is the staking share. Correspondingly, price growth is declining over time and so are real money balances used.

Some of the properties of the Ramsey steady state are a priori surprising. Specifically, as noted, the staking level is equalized to the security target at steady state. Formally, to see why this is the case, consider the first-order condition of the Ramsey allocation for D_{t+1} which can be written as

$$-\beta\alpha\left(d_{t+1}-d^{*}\right)\left(1-d_{t+1}\right)+M_{t+1}\mu_{3,t}-\beta\left(1+y_{t+1}\right)\frac{M_{t+1}}{M_{t+2}}M_{t+2}\mu_{3,t+1}=0,$$

with $\mu_{3,t}$ the Lagrange multiplier of the money supply equation. For the Ramsey plan, $M_{t+1}\mu_{3,t}$ converges to a constant and $\beta\left(1+y_{t+1}\right)\frac{M_{t+1}}{M_{t+2}}=1$, so that only the first term on the left-hand side remains. This term is set to zero with $d_{t+1}=d^{*}$. The alternative, $(1-d_{t+1})=0$, is not a steady state equilibrium if $\beta<1$.

Sharp predictions for long-run optimal policies are common in the literature, though they are not always immediately intuitive. See for instance Chari and Kehoe (1999) for a review of optimal policies in a large class of models. Many monetary models imply that for the optimal policy, in the long run, the return to money should equal the real interest rate, or equivalently, that the cost of holding money should be zero. This is known as the Friedman rule. In these models, the social cost of producing money is zero at the margin and it seems therefore reasonable that the long-run marginal utility of money should also be zero.

In the staking economy studied here, the Friedman rule is not optimal. The real return to money is below the real interest rate and the marginal utility of money is positive. This is implied by Equation (9) for $y_{t+1}>0$. Instead of the marginal utility of money being driven to zero, in the staking economy, it is the marginal cost of deviating from the staking target that is driven to zero. This seems intuitive considering that a PoS protocol must compensate stakers, making it optimal for there to be a cost to holding money. In this model, staking is useful in providing security, and stakers need to be paid through a positive staking yield. There would be no staking unless $y_{t+1}>0$, and with this inequality satisfied, real money balances used, $p_tM_t^U$, are finite. In other words, stakers make money more useful and their services are paid for by diluting users.[2]

[2]Note that setting $y_{t+1}=0$ forever is not an equilibrium with an interior choice of staking as the transversality condition for staking is not satisfied. Equilibria with $D_{t+1}\leq 0$ are ruled out.

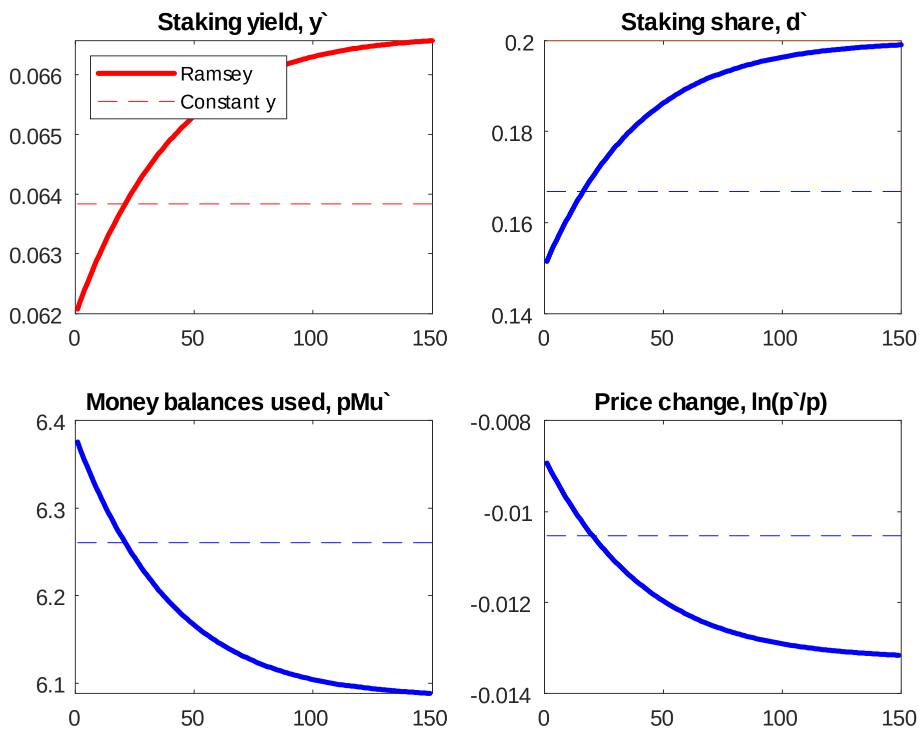

Figure 2: Ramsey policy and optimal constant staking yield: Transition to steady state starting from initial period.

3 Stochastic model

In the stochastic model, shocks to discount factors and staking convenience are introduced. I consider two policies for the staking yield: the Ramsey policy and an issuance curve analogous to the one used by Ethereum.

Agents' utility is given by

$$\max_{M^U_{t+1+j}, D_{t+1+j}, C_{t+j}} E_t \left(\sum_{j=0}^{\infty} \beta^j \Lambda_{t+j} \left[\frac{1}{1-\zeta} \left(p_{t+j} M^U_{t+j} \right)^{1-\zeta} - \frac{\alpha}{2} \left(\bar{d}_{t+j} - d^* \right)^2 + \tau_{t+j} p_{t+j} D_{t+j} + C_{t+j} \right] \right)$$

with the stochastic component of the discount factor specified as

$$\begin{aligned} \frac{\Lambda_{t+1}}{\Lambda_t} &= \exp\left(-r_{z,t}\right) \\ r_{z,t} &= \rho_r r_{z,t-1} + \sigma_r \varepsilon_{r,t}, \end{aligned}$$

and the staking convenience shock specified as

$$\tau_t = \rho_\tau \tau_{t-1} + \sigma_\tau \varepsilon_{\tau,t},$$

with $\varepsilon_{j,t}$ IID standard normal variables and ρ_j and σ_j persistence and volatility parameters, respectively. The discount factor shocks can be thought of as representing interest rate movements in the fiat economy which determine agents' required rate of return. This shock affects the attractiveness of holding the token, whether staked or used. The staking convenience shock can be thought of as capturing various changes that affect the attractiveness of staking. For instance, Ethereum's EIP-7002 allowed for withdrawals which greatly reduced risk and increased liquidity of staked tokens. Alternatively, the development of restaking options (e.g. Eigenlayer) also made staking more attractive.

Consider an issuance curve given as

$$y_t = \frac{k}{\sqrt{\bar{d}_t}},$$

with k a constant and where $\bar{d}_t$ refers to the aggregate staking share which individual agents take as given. This issuance curve is similar to the one currently used by Ethereum, with the difference being that Ethereum's staking yield depends on the amount of ETH staked as opposed to the staking share here.[3] For the analysis here, I set the parameter k so as

[3] The Ethereum issuance curve, which depends on the level of staked deposits, is consistent with a stationary money supply but typically not with a money supply that grows at a stationary (but non-zero) rate. Monetary economists broadly agree that in the long run, real economic outcomes are not affected by the level

to maximize the unconditional expectation of users' utility. The model is closed with the budget constraint given by Equation (1) and the money supply, Equation (5). Equilibrium conditions and Ramsey policies are straightforward extensions from the deterministic model as shown in Appendix 5.3.

For a numerical example, I set the parameters of the discount factor so as to approximately match the volatility and persistence of US real interest rates at an annual frequency. Specifically, I set $\rho_r = 0.95$ and $\sigma_r = 0.008$. It is less clear what a reasonable specification of the staking convenience shock should be, for this numerical example I set $\rho_r = 0.9$ and $\sigma_r = 0.005$. The model is solved through third-order perturbations around the steady state.

Figure 3 plots impulse responses for the two staking yield specifications.

3.1 Staking convenience shock

Ethereum's issuance curve's negative slope and convex shape was chosen to stabilize staking (Buterin (2020)). Given its simplicity, one would not expect this curve to respond in a fully optimal way in an environment with multiple shocks. Yet, given its shape, it should a priori be well suited to deal with a shock that specifically affects the attractiveness of staking relative to other uses of ETH.

As shown in the second row, in response to a shock making staking more attractive, both policies, Ramsey and the issuance curve, reduce the staking yield, and this contributes to offsetting the impact of the shock on the staking share. For Ramsey, the staking yield responds more and keeps the staking significantly closer to its steady state level compared to the issuance curve. The token price responds less for Ramsey and the money supply moves in the opposite direction from the price level. This stabilizes the money balances in numeraire terms, contrary to the outcome produced by the issuance curve, where both money supply and price increase.

Overall, while qualitatively similar, the outcome with the issuance curve diverges from the optimal policy. To be fair, implementing the Ramsey policy would put a very high demand on protocol designers and would also be subject to model misspecification.

3.2 Interest rate shock

The right column of Figure 3 displays the responses to an increase in the fiat interest rate which translates into an increase in the required rate of return for holding tokens. This shock produces a large decline in the token price (see lowest panel), which for unchanged money

of the money supply but potentially by its growth rate. Therefore, studying long-run economic outcomes without allowing for a non-zero long-run money growth rate seems too restrictive.

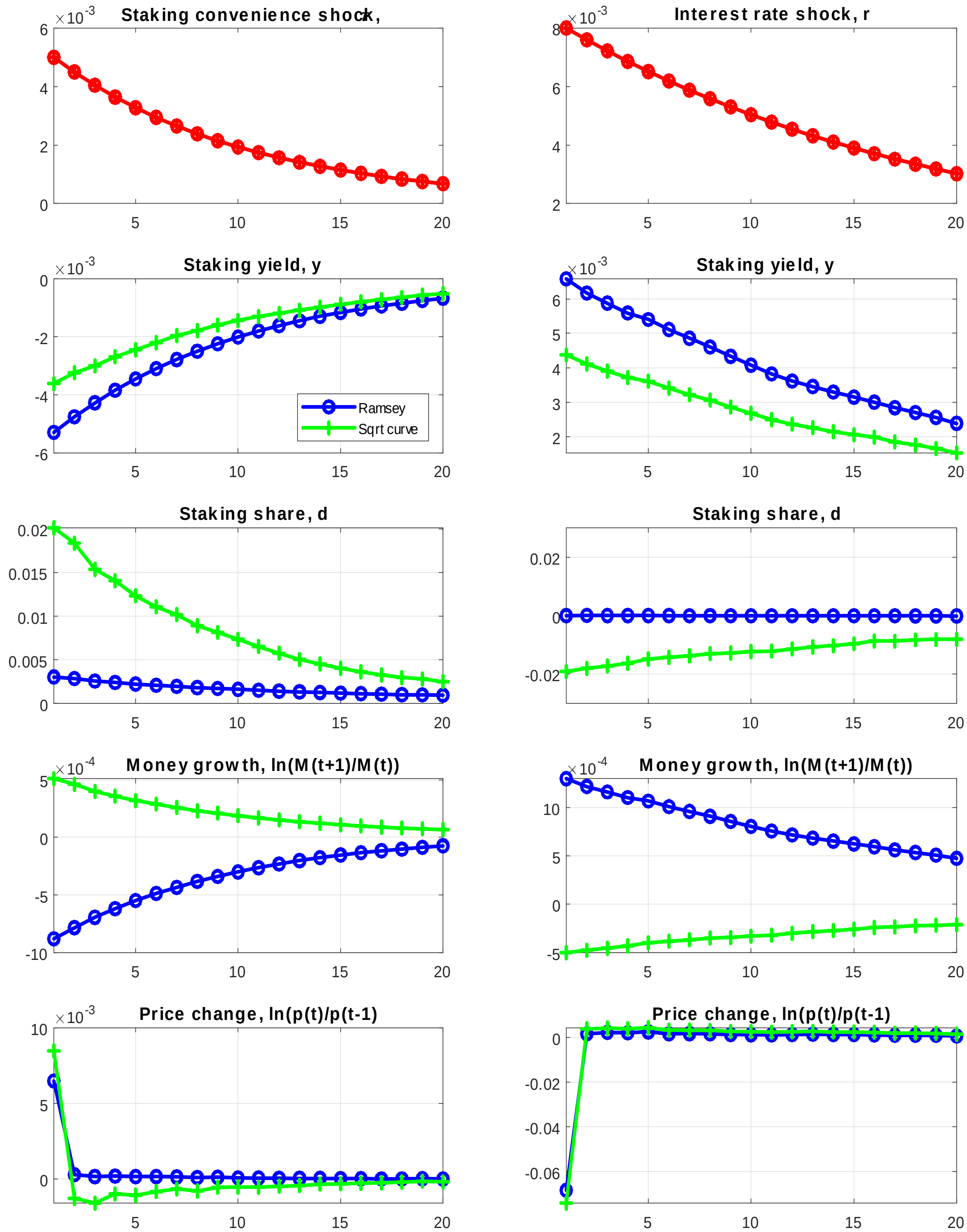

Figure 3: Impulse responses to staking convenience and interest rate shocks: Optimal (Ramsey) policy vs issuance curve.

holdings would produce a large decline in the real money balances used, $p_t M_{t+1}^U$. If the staking yield were unchanged, this would increase the incentive to use tokens at the expense of staking them (in other words, users would want to restore real balances). The optimal Ramsey policy counteracts this effect by raising the staking yield by enough to maintain the staking share at its steady state level. For the issuance curve, as the staking share decreases, the staking yield is raised as well. However, the change in the staking yield only mitigates the movement in the staking share but does not fully stabilize it.

As for the staking convenience shock, the token price moves less for the Ramsey policy in response to an interest rate shock. The value of total real balances is further stabilized by the Ramsey policy because it produces offsetting changes in the money supply and the price, contrary to the issuance curve. Overall, the issuance curve responds to these two shocks in a way that mimics the optimal responses qualitatively. However, the staking share as well as the token price are more volatile than under the optimal staking yield policy.

Quantitatively, the impulse responses depend on the parameters that determine the costs of deviating from the staking target, α, and the curvature of the utility function, ζ. The parameter values used are on the low side so that fluctuations are not very costly. Increasing these parameters leads the Ramsey policy to more aggressively stabilize the staking share relative to the issuance curve and exacerbates the differences between the two policies.

4 Conclusion

This paper has presented a framework for evaluating issuance policies and staking yields in PoS blockchains. The model only includes the most essential elements to isolate the key trade-off between security and dilution. One implication of the analysis is that, in the long run, targets for staking shares should be fully met even if that implies diluting token holders. Another finding is that the compensation for staking should be designed so as to create the incentives to maintain a relatively stable staking share. An issuance curve that mimics the inverse square root form of Ethereum is qualitatively consistent with the optimal policy but does not stabilize the staking share enough.

The model presented here can be extended to include additional features that are relevant for specific blockchains. For instance, for Ethereum, the gas market and fee burn can explicitly be represented as in Jermann (2023); alternatively, the different forms of staking (solo staking vs intermediated staking) can be explicitly modelled and studied.

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5 Appendix

5.1 Equilibrium in deterministic model (proof of Proposition 1)

It is easy to see that once a value for p_0 is determined, the paths for all equilibrium variables p_t, M_{t+1}^U and D_{t+1} are determined by the three equilibrium equations (6), (7) and (8). Therefore we can index candidate equilibrium paths by p_0. The proof first computes the steady state for d_t and the value for p_0 that corresponds to this equilibrium. It is then shown that alternative values for p_0 and their associated paths cannot be equilibrium outcomes. We focus on $y > 0$, otherwise staking is not rational.

In order to have a steady state with a constant staking share

$$\frac{D_t}{D_t + M_t^U}$$

D_t and M_t^U need to eventually grow at the same rate. For constant y, we know that

$$\frac{M_{t+1}^U}{M_t^U} = \beta \left[1 + y\right]$$

which is also constant. Indeed,

$$v'\left(p_{t+1} M_{t+1}^U\right) = y_{t+1}$$

$$M_{t+1}^U = \frac{\left(v'\right)^{-1}\left(y_{t+1}\right)}{p_{t+1}},$$

with $\left(v'\right)^{-1}\left(y_{t+1}\right)$ the inverse function, and

$$\frac{M_{t+1}^U}{M_t^U} = \frac{p_t}{p_{t+1}} \frac{\left(v'\right)^{-1}\left(y_{t+1}\right)}{\left(v'\right)^{-1}\left(y_t\right)} = \frac{p_t}{p_{t+1}} = \beta\left(1 + y\right).$$

For D_t to grow at the same rate as M_t^U, the money supply equation, equation (8)) implies

$$\left(M_t^U + D_t\right) \beta \left[1 + y\right] = M_t^U + D_t \left(1 + y\right)$$

so that

$$\begin{aligned} \beta\left[1 + y\right] &= \frac{M_t^U}{\left(M_t^U + D_t\right)} + \frac{D_t}{\left(M_t^U + D_t\right)} \left(1 + y\right) \\ &= 1 - d_t + d_t \left(1 + y\right) \end{aligned}$$

$$d_t = \frac{\beta\left(1 + y\right) - 1}{y}.$$

For steady state d_t to be positive requires

$$\beta(1+y) > 1.$$

Going forward, I assume $\beta(1+y) > 1$, the case $\beta(1+y) < 1$ is discussed in the last paragraph of the proof.

If the staking share jumps to the steady state level in the first period,

$$d_1 = \frac{D_1}{D_1 + M_1^U},$$

and with the sum if the two moneys $D_1 + M_1^U$ predetermined, this implies

$$\frac{\beta[1+y]-1}{y} = \frac{D_1}{M_0^U + D_0(1+y)}$$

$$D_1 = \frac{\beta[1+y]-1}{y}\left(M_0^U + D_0(1+y)\right),$$

$$\begin{aligned} M_1^U &= \left(M_0^U + D_0(1+y)\right) - D_1 \\ &= \left[1 - \frac{\beta[1+y]-1}{y}\right]\left(M_0^U + D_0(1+y)\right), \end{aligned}$$

$$\begin{aligned} p_1 &= \frac{(v')^{-1}(y)}{M_1^U} \\ &= \frac{(v')^{-1}(y)}{y\left[1 - \frac{\beta[1+y]-1}{y}\right](M_0^U + D_0(1+y)),} \end{aligned}$$

and

$$p_0 = p_1\beta(1+y) = \frac{(v')^{-1}(y)}{(M_0^U + D_0(1+y))}\frac{y\beta}{(1-\beta)}. \tag{12}$$

Let's label the initial price associated with this equilibrium path $\bar{p}_0$. The proof shows that starting from a smaller p_0, D_{t+1} will eventually go negative, and starting from a higher p_0 a transversality condition is violated.

Consider the dynamics of D_{t+1}. From the money supply equation and the known growth rate of M_t^U for $t \geq 1$, $M_{t+1}^U/M_t^U = \beta(1+y)$.

$$\frac{D_{t+1}}{M_t^U} = 1 + \frac{D_t}{M_t^U}(1+y) - \frac{M_{t+1}^U}{M_t^U} = 1 - \beta(1+y)\left(1 - \frac{D_t}{M_t^U}\right) \tag{13}$$

and

$$\frac{D_{t+1}}{D_t}\frac{D_t}{M_t^U} = 1 + \frac{D_t}{M_t^U}(1+y) - \beta(1+y)$$

$$\frac{D_{t+1}}{D_t} = (1+y) - \frac{\beta(1+y)-1}{\frac{D_t}{M_t^U}}. \tag{14}$$

Focusing on $t = 1$,

$$\frac{D_2}{D_1} = (1+y) - \frac{\beta(1+y)-1}{\frac{D_1}{M_1^U}},$$

it is easy to check that if D_1/M_1^U is below its steady state level

$$\frac{D}{M^U} = \frac{d}{1-d} = \frac{\beta(1+y)-1}{(1+y)(1-\beta)}$$

then

$$\frac{D_2}{D_1} < \beta(1+y) = \frac{M_2^U}{M_1^U},$$

which implies that $D_2/M_2^U < D_1/M_1^U$, and therefore D_{t+1}/M_{t+1}^U is monotonically declining. Vice versa, if D_1/M_1^U is above it steady state level, D_{t+1}/M_{t+1}^U is monotonically increasing.

From the money supply equation we have that

$$\frac{D_1}{M_1^U} = \frac{\left(M_0^U + D_0(1+y)\right)}{M_1^U} - 1$$

and from equalizing convenience yields

$$y = v'\left(p_{t+1}M_{t+1}^U\right).$$

The FOC for staking implies that if p_0 is below the path leading to the steady state so is p_1 and the yield equalization implies M_1^U is above the path leading to the steady state. Therefore $\frac{D_1}{M_1^U}$ is below the steady state level and D_t/M_t^U will continue to decline. As shown in Equation (13), for $\beta(1+y) > 1$, this implies D_{t+1} will eventually be negative, which we rule out as an equilibrium outcome.

If p_0 is above $\bar{p}_0$, by the same argument, $\frac{D_1}{M_1^U}$ is above the steady state level and D_t grows at a rate higher than $\beta(1+y)$, further increasing $\frac{D_2}{M_2^U} > \frac{D_1}{M_1^U}$. Asymptotically,

$$\lim_{t\to\infty}\frac{D_{t+1}}{D_t} = (1+y).$$

With D_{t+1} growing at the asymptotic rate and given the growth rate of p_t

$$\lim_{t\to\infty} \frac{p_{t+1}D_{t+1}}{p_t D_t} = \frac{1}{\beta\left[1+y\right]}\left(1+y\right) = \frac{1}{\beta}.$$

This implies that $p_{t+1}D_{t+1}$ grows at $1/\beta$, the rate of return, forever. The agent would never receive any consumption utility from this position, which is clearly not an optimal investment strategy. This also implies that $\beta^t p_{t+1}D_{t+1}$ will not converge to zero and that therefore the transversality condition does not hold.

I have shown that if $\beta\left(1+y\right) > 0$ there is no admissible equilibrium other than that where the staking share jumps to its steady state value in the first period. What if $\beta\left(1+y\right) < 0$? In this case, the path starting from $\bar{p}_0$ eventually implies a negative staking share, which is ruled out as an equilibrium. As seen in Equation (14), for nonnegative paths, D_t is growing at a rate of at least $(1+y) > 1$ (M_t^U is shrinking) which violates the transversality condition.

5.2 Optimal constant staking yield (proof of Proposition 2)

Given the known equilibrium dynamics shown in Proposition 1, lifetime utility is

$$U_0 = \left[-\frac{\alpha}{2}\left(d_0 - d^*\right)^2 + \ln\left(M_0^U\right) + \ln\left(p_0\right)\right] + \frac{\beta}{1-\beta}\left[-\frac{\alpha}{2}\left(d - d^*\right)^2 + \ln\left(pM^U\right)\right].$$

The equilibrium values for d and pM^U as a function of y are given by

$$pM^U = \frac{1}{y}$$

$$d = \frac{\beta\left[1+y\right]-1}{y}.$$

Using Proposition 1, we have that

$$p_0 = \frac{\beta}{\left(1-\beta\right)}\frac{1}{\left(M_0^U + D_0\left(1+y_0\right)\right)}.$$

Therefore, we can ignore p_0, because it is predetermined.

Replace the equilibrium solutions and maximize lifetime utility with respect to y

$$\max_y U = \max_y \frac{\beta}{1-\beta}\left[-\frac{\alpha}{2}\left(\frac{\beta\left[1+y\right]-1}{y} - d^*\right)^2 + \ln\left(\frac{1}{y}\right)\right].$$

The first-order condition is given by

$$\frac{\partial U}{\partial y} = -\alpha \left(\frac{\beta \left[1+y \right] - 1}{y} - d^* \right) \frac{1-\beta}{y^2} - \left(\frac{1}{1/y} \right) \frac{1}{y^2} = 0,$$

which implies

$$\left(\frac{\beta \left[1+y \right] - 1}{y} - d^* \right) = -\frac{y}{\alpha \left(1-\beta \right)} < 0$$

and

$$\left(d - d^* \right) < 0.$$

Solving for the optimal y,

$$\frac{y^2}{\alpha \left(1-\beta \right)} + \left(\beta - d^* \right) y - \left(1-\beta \right) = 0,$$

a quadratic equation with one positive root

$$y = \frac{\alpha \left(1-\beta \right)}{2} \left[-\left(\beta - d^* \right) + \left(\left(\beta - d^* \right)^2 + \frac{4}{\alpha} \right)^{0.5} \right].$$

Substitute the optimal y in the equation for d, so that

$$\begin{aligned} d &= d^* - \frac{y}{\alpha \left(1-\beta \right)} \\ &= d^* - \frac{1}{2} \left(\beta - d^* \right) \left[\left(1 + \frac{4}{\alpha \left(\beta - d^* \right)^2} \right)^{0.5} - 1 \right] < d^* \end{aligned}$$

where the last inequality requires $\beta - d^* > 0$.

5.3 Ramsey policy (proof of Proposition 3)

The optimal allocation with commitment, the Ramsey allocation, is the solution to maximizing agents' lifetime utility with respect to y_{t+1}, M^U_{t+1}, D_{t+1}, p_t for $t \geq 0$. The optimal Ramsey policy corresponds to the implied sequence of staking yields, y_{t+1}. Lifetime utility for the general case with uncertainty is given by

$$\Lambda_0\left[\frac{1}{1-\zeta}\left(p_0 M_0^U\right)^{1-\zeta}-\frac{\alpha}{2}\left(\frac{\overline{D_0}}{\overline{M_0^U}+\overline{D_0}}-d^*\right)^2+\tau_0 p_0 D_0\right]$$
$$+E_0\beta\Lambda_1\left[\frac{1}{1-\zeta}\left(p_1 M_1^U\right)^{1-\zeta}-\frac{\alpha}{2}\left(\frac{\overline{D_1}}{\overline{M_1^U}+\overline{D_1}}-d^*\right)^2+\tau_1 p_1 D_1\right]+E_0\beta^2\Lambda_2\,[.]\,...$$

with the equilibrium conditions as constraints

$$\left\{E_t\beta\frac{\Lambda_{t+1}}{\Lambda_t}\left[y_{t+1}+\tau_{t+1}+1\right]p_{t+1}-p_t\right\}:\mu_{1,t}$$

$$\left\{\beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}\left[\frac{1}{\left(p_{t+1}M_{t+1}^U\right)^{\zeta}}+1\right]p_{t+1}-p_t\right\}:\mu_{2,t}$$
$$\left\{M_{t+1}^U+D_{t+1}-M_t^U-D_t-y_t D_t\right\}:\mu_{3,t},$$

with $\mu_{j,t}$ the Lagrange multipliers. Lagrange multipliers are scaled by $\beta^t\Lambda_t\left(z^t\right)prob\left(z^t\right)$, with z^t representing the stochastic process driving the shocks, $\Lambda_t\left(z^t\right)$ and $\tau\left(z^t\right)$, and $prob\left(z^t\right)$ the corresponding probabilities.

First-order necessary conditions for $y_{t+1}, D_{t+1}, M_{t+1}^U$, and p_{t+1}, respectively, are given by

$$\mu_{1,t}E_t\beta\frac{\Lambda_{t+1}}{\Lambda_t}p_{t+1}-E_t\beta\frac{\Lambda_{t+1}}{\Lambda_t}D_{t+1}\mu_{3,t+1}=0, \tag{15}$$

$$\begin{aligned}&-E_0\beta\frac{\Lambda_{t+1}}{\Lambda_t}\left[\alpha\left(\frac{\overline{D_{t+1}}}{\overline{M_{t+1}^U}+\overline{D_{t+1}}}-d^*\right)\left(\frac{\overline{M_{t+1}^U}}{\left(\overline{M_{t+1}^U}+\overline{D_{t+1}}\right)^2}\right)+\tau_{t+1}p_{t+1}\right]\\&+\mu_{3,t}-E_t\beta\frac{\Lambda_{t+1}}{\Lambda_t}\mu_{3,t+1}\left(1+y_{t+1}\right)\\=\;&0,\end{aligned} \tag{16}$$

$$
\begin{aligned}
& E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \left[\frac{\left(p_{t+1} M_{t+1}^U\right)^{1-\zeta}}{M_{t+1}^U} + \alpha \left(\frac{\overline{D_{t+1}}}{\overline{M_{t+1}^U} + \overline{D_{t+1}}} - d^* \right) \frac{\overline{D_{t+1}^U}}{\left(\overline{M_{t+1}^U} + \overline{D_{t+1}}\right)^2} \right] \\
& -\mu_{2,t} \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left(\zeta \frac{\left(p_{t+1} M_{t+1}^U\right)^{1-\zeta}}{\left(M_{t+1}^U\right)^2} \right) + \mu_{3,t} - \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \mu_{3,t+1} \\
= \; & 0,
\end{aligned} \tag{17}
$$

$$
\frac{\left(p_t M_t^U\right)^{1-\zeta}}{p_t} + \tau_t D_t + \mu_{1,t-1} \left[y_t + \tau_t + 1 \right] - \mu_{1,t} + \mu_{2,t-1} \left(\frac{(1-\zeta)}{\left(p_t M_t^U\right)^{\zeta}} + 1 \right) \mu_{2,t-1} - \mu_{2,t} = 0, \tag{18}
$$

and for p_0 with $t = 0$

$$
\frac{\left(p_t M_t^U\right)^{1-\zeta}}{p_t} + \tau_t D_t - \mu_{1,t} - \mu_{2,t} = 0.
$$

To solve for the steady state, consider the deterministic model (without shocks), and conjecture a steady state with a constant $y_t = y$. The equilibrium conditions imply that

$$
\frac{p_t}{p_{t+1}} = \beta \left[1 + y \right] = \frac{M_{t+1}^U}{M_t^U}, \tag{19}
$$

$$
d_t = d = \frac{\beta \left[1 + y \right] - 1}{y}, \tag{20}
$$

and

$$
p_{t+1} M_{t+1}^U = \left(\frac{1}{y} \right)^{1/\zeta}. \tag{21}
$$

The set of first-order conditions for $y_{t+1}, D_{t+1}, M_{t+1}^U$, and p_{t+1}, Equations (15), (16), (17), and (18), are then solved for a constant y and for the multipliers $\mu_{j,t}$ for $j = 1, 2, 3$. Multipliers are nonstationary but become stationary (and constant in steady state) if scaled by M_{t+1}, for $\mu_{1,t}$ and $\mu_{2,t}$ by dividing by M_{t+1}, and for $\mu_{3,t}$ by multiplying by M_{t+1}. Specifically, Equation (16), using the solution for the state money growth rate, can be written as

$$
-\beta \alpha \left(d_{t+1} - d^* \right) \left(1 - d_{t+1} \right) + M_{t+1} \mu_{3,t} - M_{t+2} \mu_{3,t+1} = 0.
$$

In steady state, for constant scaled multipliers, $M_{t+1} \mu_{3,t} = M_{t+2} \mu_{3,t+1}$, so that

$$
d_t = d^*.
$$

The alternative way of setting this equation to zero, $(1 - d_{t+1}) = 0$, cannot be a steady state equilibrium for $\beta < 1$ (as implied by Equation (20)). The combination of Equations (15), (17), and (18), at steady state, can then be solved for the constant scaled multipliers through straightforward algebra. The stability of the steady state is verified numerically by third-order perturbations around the steady state using Dynare.