Should the U.S. Government Issue Floating Rate Notes?

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April 17, 2020

Abstract

Since January 2014 the U.S. Treasury has been issuing floating rate notes (FRNs). We estimate that the U.S. FRNs have been paying excess interest between 5 and 39 basis points above the implied cost for other Treasury securities. We find a strong positive relation between our estimated excess spreads on FRNs and the subsequent realized excess returns of FRNs over related T-bill investment strategies. With more than 300 billion dollars of FRNs outstanding, the yearly excess borrowing costs are estimated to be several hundreds of millions of dollars. To rationalize this finding, we examine the role of FRNs from the perspective of optimal government debt management to smooth taxes. In the model, bills can be cheaper to issue than FRNs, and the payoffs for FRNs are perfectly correlated with future short rates. FRNs can be used to manage the refinancing risk from rolling over short-term debt. We derive conditions under which the issuance of FRNs can optimally be positive.

Keywords: Floating rate notes, fixed Income arbitrage, tax-smoothing, optimal debt management. JEL: E62, H63,F30, G12, G15.

1 Introduction

In 2014, 2-Year U.S. Treasury Floating Rate Notes (FRNs) became the newest product to be issued by the U.S. since Treasury Inflation Protected Securities (TIPS) in 1997. The U.S. Treasury announced the issuance of floating rate notes in 2013 with the partial aim

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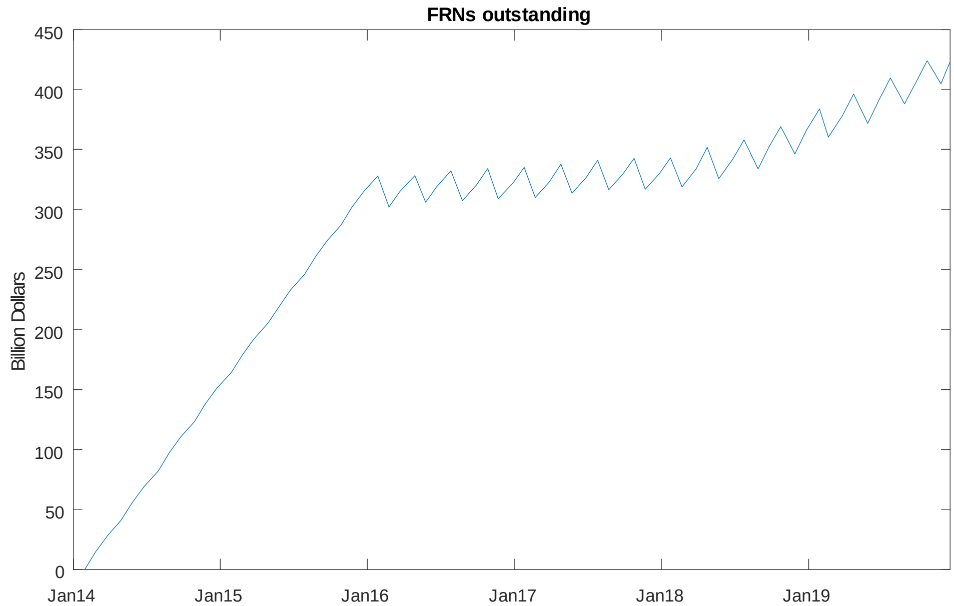

Figure 1: Data is from the U.S. Treasury; https://www.treasurydirect.gov/govt/reports/pd /mspd/mspd.htm

to reduce Treasury bill issuance amid concerns with rollover risk (in 2008, Treasury bills represented nearly 30% of public debt outstanding compared to 14.5% as of the end of 2019). Another stated objective was "saving taxpayer dollars by financing the government's borrowing needs at the lowest cost over time" (U.S. Department of the Treasury, 2014). In this paper we estimate the borrowing costs from the FRNs that have been issued relative to other financing sources. Our findings are rationalized within a model of optimal government debt management to smooth taxes.

After an initial ramp up of Treasury FRN issuance, since 2016, the amount of FRN debt outstanding has been increasing slowly to exceed $400 billion in 2019 (Figure 1). This represents 2.6% of the total U.S. Treasury outstanding marketable debt as of December 2019 (Figure 2). By comparison, TIPS represent 9% of total marketable debt, Treasury bills 14.5%, notes 59.5%, and bonds 14.3%.

U.S. FRNs are issued at monthly auctions with a maturity of two years. They promise quarterly coupon payments indexed to the three-month T-bill rates determined at weekly auctions. Unlike generic FRNs or typical floating rate bank loans, the Treasury's FRNs pay a coupon that is an average of the constant maturity three-month rates. Therefore, even assuming that they are valued with a no-arbitrage approach through a risk-free interest curve, these FRNs would typically not be priced at par without a spread. Based on Treasury yield curves on auctions dates, we determine this spread.

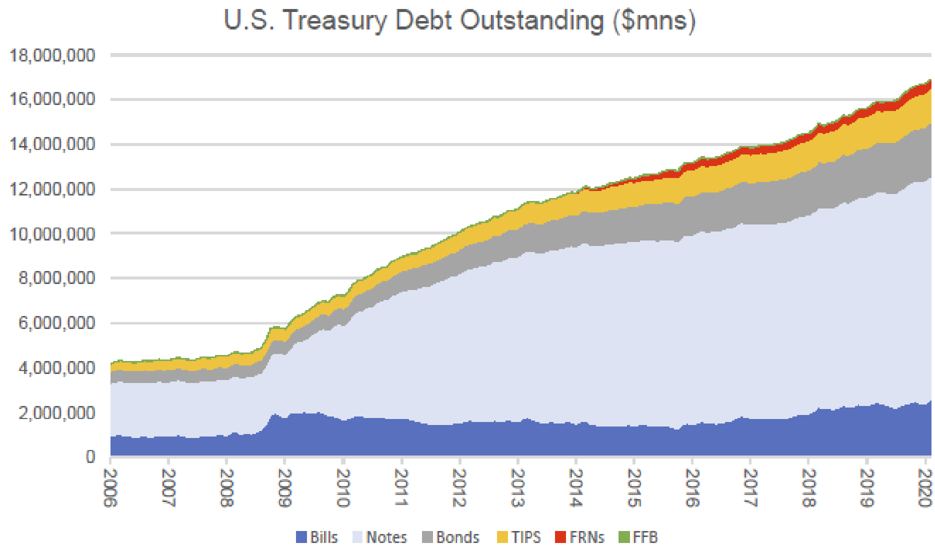

Figure 2: Data on Bills, Notes, Bonds, TIPS, and FRNs is from the U.S. Treasury; https://www.treasurydirect.gov/govt/reports/pd/mspd/mspd.htm

Over the period 2014 to 2019, we estimate spreads of between -10 and 3 basis points in annualized terms for new auctions, and for reopenings between -23 and 7 basis points. Before 2019, estimated spreads for new auctions were negative. This is consistent with the fact that forward curves for maturities of two years and less have been upward sloping until 2019. In this case, averaging interest rates over a quarterly coupon period makes the FRN more valuable and the spread required for a par value negative. However, the spreads determined at Treasury auctions have been positive throughout. The spreads paid in excess of the estimated no-arbitrage spreads range between 5 and 39 basis points. Excess spreads were particularly high in early 2016. In 2019 they have increased again from their low levels of early 2018. With more than 300 billion dollars of FRNs outstanding, the excess borrowing cost for 2017 is estimated to be about 700 million dollars. This is the highest annual excess borrowing cost so far.

Conceptually, pricing Treasury FRNs requires a convexity adjustment to deal with the constant maturity index. We derive the exact pricing formula, and based on a version of the Black, Derman and Toy (1990) model, we demonstrate that due to the recent low interest rate volatility the convexity adjustment is very small quantitatively.

We explore investment strategies with rolled-over T-bills, and we find the returns to be consistent with the idea that FRNs have offered attractive terms to investors. We also document a strong positive relation between our estimated excess spreads on FRNs and the subsequent realized excess returns of FRNs over the T-bill strategies.

To rationalize these findings we study the role of FRNs in a stylized model of optimal government debt management to smooth taxes. In the frictionless framework without default typically used to study optimal debt management, generic FRNs are redundant as they can be replicated by short-term debt. When short-term debt provides monetary services not found in other debt this redundancy disappears. Short-term debt can be cheaper to issue than FRNs, and the government might be willing to tolerate some refinancing risk from rolling over short-term debt. Because the payoff for FRNs is perfectly correlated with future short rates, FRNs can be used to manage the refinancing risk. In a stylized model for optimal debt management, we show that depending on the amounts of short-term debt issued to optimally balance the utility of monetary services with seigniorage revenues, FRNs are used as complements or substitutes to hedge the interest exposure and to smooth taxes. Issuance of FRNs can optimally be positive, and this requires that the amount of short-term debt issued is relatively small. In the model this happens when a type of Laffer curve that traces seigniorage as a function of short-term debt slopes downwards at relatively low levels.

Finally, we extend the simple model with money-in-utility to an infinite horizon pricing model. We characterize the excess spreads in FRNs that we have estimated, and based on guidance from this model, we empirically evaluate the role of potential drivers of these excess spreads. We find a significant role for implied interst rate volatility. We also find some impact from the bid-to-cover ratios in the first two years.

We contribute to studying the market conditions for the nascent U.S. FRNs. Greenwood, Hanson and Stein (2016) note that initial yields on FRNs have been higher than three-month T-bill rates. They do not price FRNs. Bhanot and Guo (2017) find substantial excess returns using secondary market data through 2016 for 2-Year U.S. Treasury FRNs. To our knowledge, our paper is the first study to price U.S. FRNs and estimate the Treasury's excess borrowing costs due to FRNs. We are also not aware of other studies containing our pricing equations with explicit convexity adjustments needed due to the constant maturity index in the U.S. FRNs.[1]

Our paper also contributes to the literature on optimal government debt management, which includes Barro (1979), Bohn (1990), Cochrane (2015), Angeletos (2002), Nosbusch (2008) and Debortoli, Nunes, and Yared (2017). Our model builds on the setting by Greenwood, Hanson and Stein (2015) who consider optimal debt management when short-term

[1]Pricing anomalies have been studied in many areas of the market for U.S. Treasury securities, namely in the market for off-the-run vs. on-the-run Treasury bonds (Krisnamurthy 2002), TIPS (Flackenstein, Longstaff, and Lustig 2014), longer maturity Treasury bonds (Bradford and Shapiro 1989), callable Treasury bonds (Carayannopoulos 1995), and from an international perspective (Du, Im and Schreger,2017). Price impacts due to recent policy or regulatory measures have been documented by Vissing-Jorgensen and Krishnamurthy (2012), D'Amico and King (2013), and Du, Tepper and Verdelhan (2017), Hartley (2017), and Cochrane (2017).

debt provides monetary services. Our contribution to this literature is to study the role of FRNs.

In the rest of the paper, Section 2 estimates the Treasury's borrowing costs for FRNs. Section 3 derives and estimates convexity adjustments, as well as documenting T-bill investment strategies related to FRNs. Section 4 analyzes FRNs within a stylized model for optimal government debt management. Section 5 presents an infinite-horizon pricing model and documents empirical drivers of the estimated spreads.

2 Treasury FRN pricing at auctions

In this section we value Treasury FRNs at auction dates and compare the valuations to the actual pricing of these FRNs. The Treasury started issuing FRNs with a maturity of two years in January 2014. These notes promise quarterly coupons indexed to the 13-week T-bill rates. New FRNs are issued towards the end of January, April, July, and October. There are reopening auctions in the two months following a new issuance where additional amounts of the previously issued FRNs are sold.

Newly issued FRNs have typically been sold at par with the auction determining a spread that is added to the index of three-month T-bill rates. Unlike generic FRNs or typical floating rate bank loans, Treasury FRNs pay a coupon that is an average of the constant maturity three-months rates. Therefore, even assuming that they are valued with a no-arbitrage approach through a risk-free interest curve, these FRNs would typically not be priced at par without a spread.

We value a new FRN as

$$V_0 = \sum_{I=0}^{7} \frac{\frac{1}{13}\sum_{k=0}^{12}\left(r_{0,13I+k}^{f,13} + \frac{1}{4}\theta_0\right)}{1 + r_0^{13I+13}} + \frac{1}{1 + r_0^{104}}, \tag{1}$$

where $r_{0,13I+k}^{f,13}$ stands for the current (time 0) forward rate with a 13-week maturity for week $13I + k$, and r_0^{13I+13} the current zero-coupon rate with a maturity of $13I + 13$ weeks. In the next section we derive the no-arbitrage value of a FRN in a more rigorous way and demonstrate that equation (1) represents a very accurate pricing formula. This formula can be evaluated based on the current term structure alone due to the no-arbitrage relation between forward rate and spot rates

$$1 + r_{0,I+k}^{f,13} = \frac{1 + r_0^{I+k+13}}{1 + r_0^{I+k}}. \tag{2}$$

The starting dates of each forward rate period corresponds to a weekly auction date of 13-week T-Bills whose rates determine the coupon payments of the FRN. The discount factors $1/\left(1+r_0^{13I+13}\right)$ correspond to the quarterly (13-week) coupon payment dates. We assume a constant spread θ_0 (in annualized terms) which will be determined so that the value of the FRN is at par, $V_0 = 1$.

Compare this to a more standard FRN where the coupon payment is based on the interest rate corresponding to the same period. The price of a standard FRN, $\tilde{V}_0$, with the same maturity and coupon payment dates would be

$$\tilde{V}_0 = \sum_{I=0}^{7} \frac{r_{0,13I}^{f,13} + \frac{1}{4}\tilde{\theta}_0}{1+r_0^{13I+13}} + \frac{1}{1+r_0^{104}}.$$

Substituting the definition of the forward rates, 2, this becomes

$$\tilde{V}_0 = 1 + \tilde{\theta}_0 \sum_{I=0}^{7} \frac{1/4}{1+r_0^{13I+13}}.$$

A FRN sold at par, $\tilde{V}_0 = 1$, does not require a spread, $\tilde{\theta}_0 = 0$. Therefore, the spread we estimate in a Treasury FRN captures the effect of using a constant maturity index.

Equation 1 is evaluated, and solved for θ_0, based on the term structure on an auction date. We use the Treasury-implied zero-coupon yields from Reuters with maturities $1, 3, 6, 9, 12, 24$ and 36 months.[2] Yields are interpolated by cubic splines. For reopening auctions, equation (1) is modified to take into account the reduced maturity, accrued interest, and the fact that, with the spread predetermined at the initial auction, prices are typically no longer at par.

2.1 Results

Over the period 2014 to 2019, we estimate spreads θ to be between 3 and -10 basis points in annualized terms for new auctions, and for reopenings between 7 and -23 basis points. Before 2019, spreads for new auctions have been negative. This is consistent with the fact that forward curves for maturities of two years and less have been upward sloping until 2019. In this case, averaging interest rates over a quarterly coupon period makes the FRN more valuable and the spread θ required for a par value has to be negative.

To get a better sense of the typical no-arbitrage spread θ, consider, for instance, the term

[2] We have also considered term structure data from the close of the day before the auction and of the auction date, and then averaged the prices. Differences between the two dates are very small. The auction deadline is at 11:30am. Using yield curve data based on Gurkaynak, Sack, and Wright (2007) produces very similar estimates.

structure on 4/29/2015, date of a new auction. As shown in Figure 3, forward rates are almost linear in maturity up to two years. A generic FRN promises quarterly coupons with risk-neutral expected values equal to the forward rates (here annualized) at the beginning of the period, that is forward rates determined at $0, 0.25$ etc. until 1.75 in the figure. This FRN would be valued at par. At the end of each quarter, the forward rate is on average about 16 basis points higher than at the beginning, so that averaging over a period increases the coupon value by about 8 basis points. To have FRNs priced at par, θ should be set to approximately -8 basis points. The exact θ based equation on (1) is close, namely at -7.3.

The pricing spreads determined by Treasury auctions have been positive throughout. We define the *excess spread* as the auction-determined spread minus the no-arbitrage spread, θ. The excess spread represents the annualized interest cost the Treasury is paying for FRNs in excess of the interest cost implied by the term structure of other Treasury securities. Figure 4 shows the time series of these spreads for 2014-2019. Excess spreads range between 5 and 39 basis points. Excess spreads were particularly high in early 2016. In 2019 they have increased again from their low levels of early 2018. As shown in Figure 4, a substantial part of the variation in the excess spread is captured by the High Discount Margin, HDM, which is used by the Treasury to auction FRNs. In particular, for new auctions, the HDM becomes the spread that is applied to a FRN which is sold at par. For reopenings, the HDM determines the price of a FRN according to the Treasury's formula (Department of the Treasury (2013)). As is clear in the figure, even in this case, the HDM can be seen as the key driver of the excess spread.

At a given point in time, there are FRNs from up to 24 issue dates outstanding. For each of these issues, we compute the excess spread. Multiplying these by the corresponding amounts issued gives us the total excess cost associated with all outstanding FRNs. Figure 5 reports excess borrowing cost for each calendar year. With more than 300 billion dollars of FRNs outstanding, the excess borrowing cost for 2017 is estimated to be about 700 million dollars.[3] The outstanding amount of FRNs in 2017 was about 328 billion dollars on average. The excess spread over the lagging two years was about 21.2 basis points on average, see Figure 4. As a back-of-the-envelop approximation, this amounts to about $0.00212 \times 328,000 = 695$ million dollars. Given that the most generous FRNs issued have been retired by the first part of 2018, this cost was lower in 2018.

[3] Based on the Treasury direct website (https://www.treasurydirect.gov/govt/reports/ir/ir_expense.htm), total interest expense for 2017, $700 million represents 0.15% of this total.

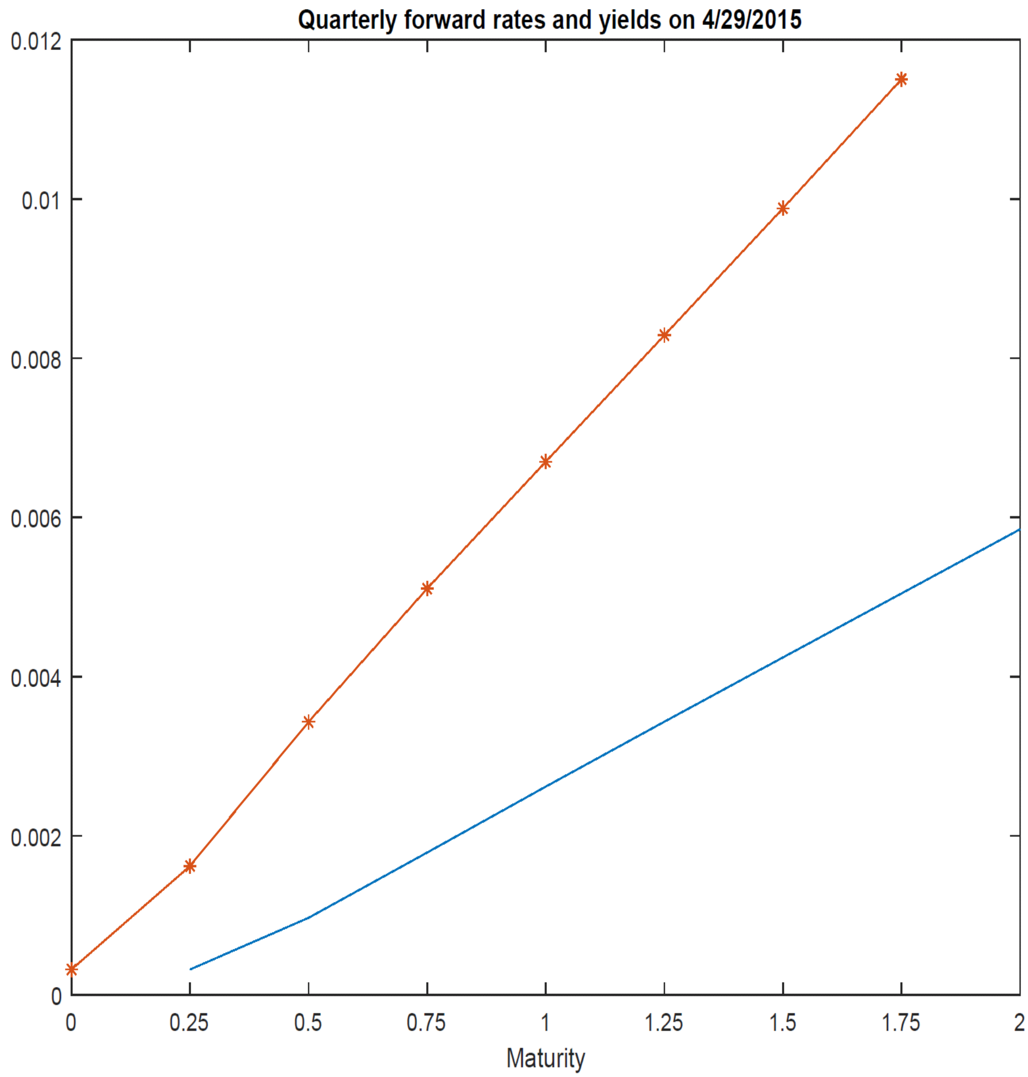

Figure 3: Example of the term structure of Treasury yields (–) and forward rates (–∗) on an auction date for a new FRN issue. The upward sloping forward rate curve makes the Treasury FRNs more valuable, and, from a no-arbitrage perspective, requires a negative spread.

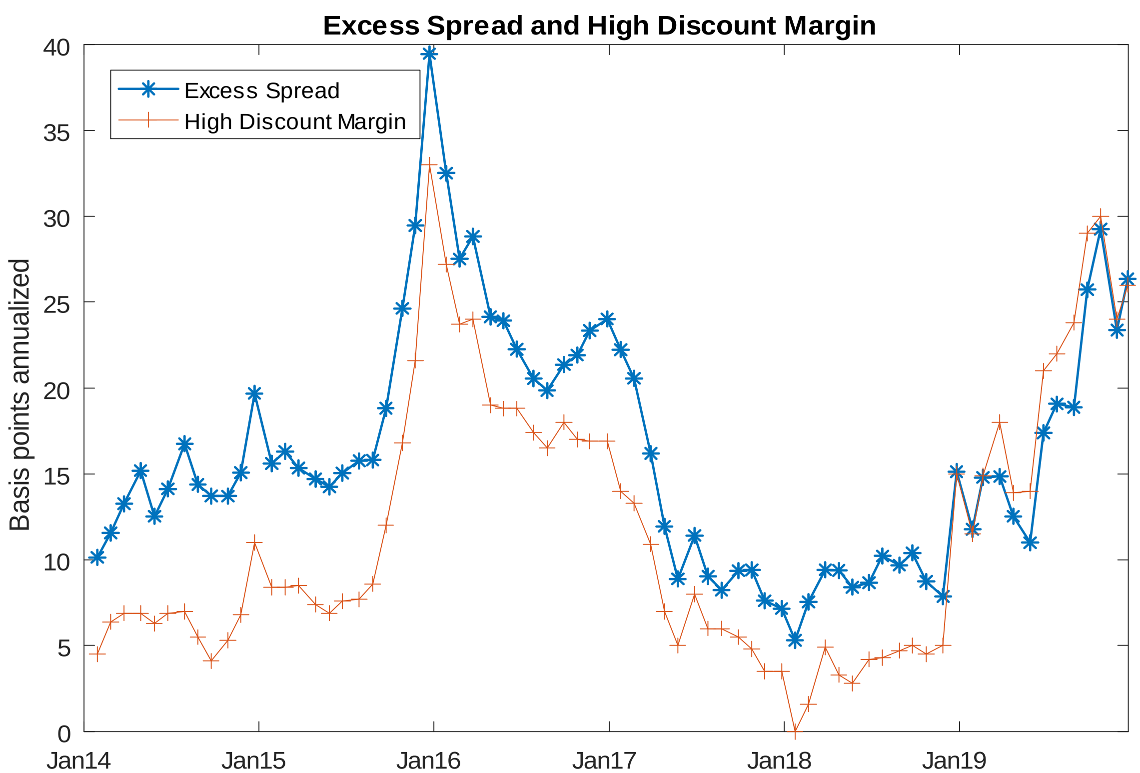

Figure 4: The excess spread is defined as the spread included in a FRN minus the spread that would be justifed by ruling out arbitrage at auction dates.

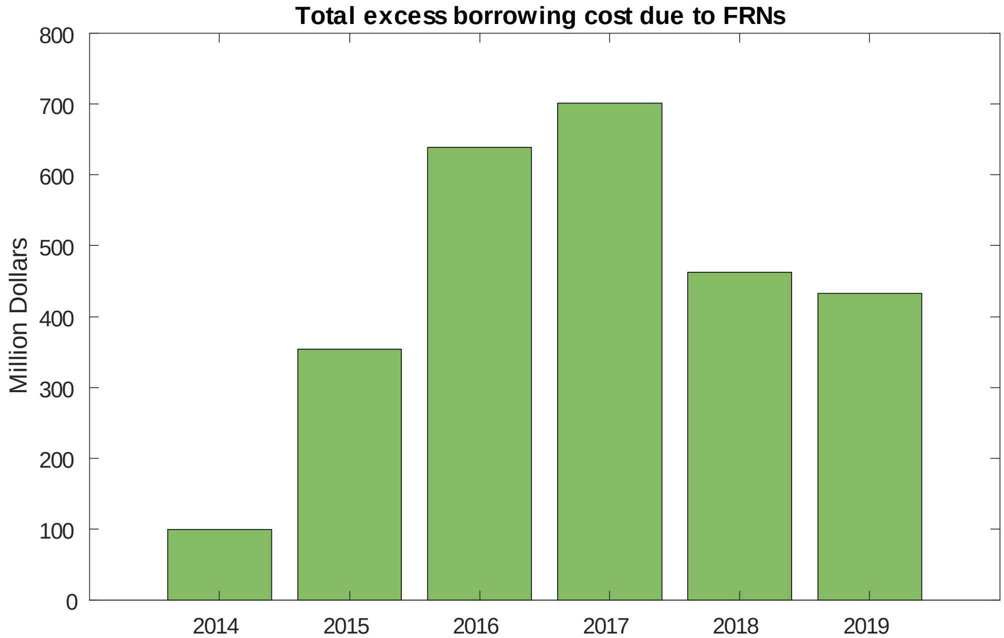

Figure 5: Total excess borrowing costs are computed by combining the total amount of FRNs outstanding with the excess spreads determined at each auction date.

3 No-arbitrage pricing of FRN with a constant maturity index

In this section we derive the no-arbitrage value of a Treasury FRN and demonstrate that equation 1 is very accurate in a low volatility environment as in 2014-2019. We also show under what conditions a more involved pricing approach for FRN would be needed. Finally, we document T-bill investment strategies and show that the excess returns from FRNs over these strategies are closely related to our estimated excess spreads.

Ruling out arbitrage, there exists a state-price valuation process Λ_t. The value of a Treasury FRN is given by

$$V_0 = \frac{1}{13} \sum_I \sum_k E_0 \left[\frac{\Lambda_{I+13}}{\Lambda_0} r^{13}_{I+k} \right] + E_0 \left[\frac{\Lambda_{104}}{\Lambda_0} \right]$$

for $I \in 13\,[0:7]$ and $k \in [0:12]$. The period length is one week. At week $I+k$ rate r^{13}_{I+k} with a 13 week maturity is determined to be included in the coupon paid at $I+13$. Coupons are paid every 13 weeks. Pricing the FRN involves pricing 104 strips with payouts based on the rate set by the weekly auction of the 13-week T-Bill. Rates are in effective terms so that r^{13}_{I+k} is a 13 week rate and coupon payments represent the average, thus the factor 1/13.

Consider first the case of a coupon strip with $k = 0$; the maturity date of the stochastic discount, Λ_{I+13}/Λ_0, and the payment date is the same as the maturity date of the interest index, namely $I + 13$. In this case, starting for convenience with the strip including the principal,

$$\begin{aligned} E_0\left[\frac{\Lambda_{I+13}}{\Lambda_0}\left(1+r_I^{13}\right)\right] &= E_0\left[\frac{\Lambda_I}{\Lambda_0}\frac{\Lambda_{I+13}}{\Lambda_I}\left(1+r_I^{13}\right)\right] \\ &= E_0\left[\frac{\Lambda_I}{\Lambda_0}E_I\left\{\frac{\Lambda_{I+13}}{\Lambda_I}\left(1+r_I^{13}\right)\right\}\right] = E_0\left[\frac{\Lambda_I}{\Lambda_0}\right] = \frac{1}{1+r_0^I}. \end{aligned}$$

and adjusting for the principal

$$\begin{aligned} E_0\left[\frac{\Lambda_{I+13}}{\Lambda_0}r_I^{13}\right] &= E_0\left[\frac{\Lambda_{I+13}}{\Lambda_0}\left(1+r_I^{13}\right)\right] - \frac{1}{1+r_0^{I+13}} \\ &= \frac{1}{1+r_0^I} - \frac{1}{1+r_0^{I+13}}. \end{aligned}$$

Clearly, the strip can be priced easily from current spot interest rates with the appropriate maturities.

This can be rewritten as

$$\begin{aligned} E_0\left[\frac{\Lambda_{I+13}}{\Lambda_0}r_I^{13}\right] &= \frac{1}{1+r_0^{I+13}}\frac{1+r_0^{I+13}}{1+r_0^I} - \frac{1}{1+r_0^{I+13}} \\ &= \frac{r_{0,I}^{f,13}}{1+r_0^{I+13}} \end{aligned}$$

with the forward rate defined as $\left(1+r_{0,I}^{f,13}\right) \equiv \frac{1+r_0^{I+13}}{1+r_0^I}$. This is the rate between I and $I + 13$ that can be locked in as of now by buying a zero coupon bond with a maturity of $I + 13$ and borrowing the purchase price until period I. Intuitively, the forward rate, is the certainty equivalent or the expected future interest rate under the risk-neutral distribution. It is discounted with the spot interest rate corresponding to the coupon payment date.

For the general case $k > 0$, the pricing process can longer be eliminated. As shown in the Appendix A, the no-arbitrage value of a strip can be written as

$$E_0\left[\frac{\Lambda_{I+13}}{\Lambda_0}r_{I+k}^{13}\right] = \frac{r_{0,I+k}^{f,13}}{1+r_0^{I+13}} + cov_0\left(\frac{\Lambda_{I+k}}{\Lambda_0}\frac{1}{1+r_{I+k}^{13}}\left[\frac{1+r_{I+k}^{13}}{1+r_{I+k}^{13-k}} - \left(1+r_{0,I+13}^{f,k}\right)\right], r_{I+k}^{13}\right). \tag{3}$$

The first component represents the forward rate $r_{0,I+k}^{f,13}$ for the index determination date $I+k$, discounted at the current spot rate r_0^{I+13} with maturity $I + 13$ which corresponds the date the payment is made, at the end of a quarterly period. The second term on the left-hand side is non-zero for all $k > 0$ and zero for $k = 0$.

For strips with $k > 0$ a "convexity adjustment" is needed. To compute it requires a fully specified pricing process or term structure model. A similar adjustment is required for pricing the popular Eurodollar futures contracts which settle at the beginning of an interest period (Veronesi, 2010, Section 21.7.). As we show below, the adjustment for pricing Treasury FRNs has been small, in the order of significantly less than 1 basis point in annualized coupon equivalent terms. Therefore, Treasury FRNs can be effectively priced based on the current zero-coupon term structure alone – at least in the low interest rate volatility environment of 2014-2019.

To provide more intuition about the convexity adjustment, we can transform the value of a strip so that it does not explicitly depend on the pricing process Λ. Define the risk-neutral expectation operator E_0^Q implicitly as

$$\begin{aligned}(1+r_{I+13})\,E_0\left[\frac{\Lambda_{I+13}}{\Lambda_0}r_{I+k}^{13}\right] &= E_0\left[\left\{\frac{\Lambda_{I+13}}{\Lambda_0}/E_0\left[\frac{\Lambda_{I+13}}{\Lambda_0}\right]\right\}r_{I+k}^{13}\right] \\ &= E_0^{Q(I+13)}\left[r_{I+k}^{13}\right].\end{aligned}$$

As shown in the Appendix A

$$E_0\left[\frac{\Lambda_{I+13}}{\Lambda_0}r_{I+k}^{13}\right] = \frac{1}{1+r_0^{I+k}}E_0^{Q(I+k)}\left[\frac{r_{I+k}^{13}}{1+r_{I+k}^{13-k}}\right].$$

If one ignores for an instant the uncertainty associated with r_{I+k}^{13} and r_{I+k}^{13-k}, then this would simplify to $r_{0,I+k}^{f,13}/1+r_0^{I+13}$ as in equation (1) above. With uncertainty, however, the expectation needs to be computed with a term structure model. A second-order Taylor approximation for $E_0^{Q(I+k)}r_{I+k}^{13}$ can give some intuition that does not rely on the state-price process Λ. As shown in Appendix A, to a second-order approximation,

$$E_0\left[\frac{\Lambda_{I+13}}{\Lambda_0}r_{I+k}^{13}\right] \cong \frac{r_{0,I+k}^{f,13}}{1+r_0^{I+13}}+\frac{\left(1+r_{0,I+k}^{f,13}\right)^3}{1+r_0^{I+13}}var_0^{Q(I+k)}\left(\frac{1}{1+r_{I+k}^{13}}\right)+\frac{1}{1+r_0^{I+k}}cov_0^{Q(I+k)}\left(\frac{1}{1+r_{I+k}^{13-k}},r_{I+k}^{13}\right). \tag{4}$$

The equation shows the convexity adjustment depending on conditional variances and covariances of short rates with at most 13-week maturity. Specifically, the adjustment corresponds to the conditional variance of the 13-week rate $I + k$ weeks plus a covariance that is typically negative between the 13-week rate and the inverse of the rates of maturities 1 to 13.

3.1 Measuring the convexity adjustment

We represent the state-price valuation process Λ_t implicitly through a binomial tree of the short rate. The short rate tree is specified along the lines of a simple version of the Black, Derman and Toy (BDT, 1990) model with constant interest rate volatility. The BDT model is widely used by practitioners for pricing interest rate derivative contracts. See Veronesi (2010) for a modern treatment. Specifically, the weekly short rate is specified as

$$r_{t+1} = r_t \exp(\mu_{t+1} + h^{1/2}\sigma\varepsilon_{t+1})$$

with ε_{t+1} equal to -1 or $+1$, each with a risk-neutral probability of 0.5, and $h = 1/52$. The time-dependent (known) factors μ_{t+1} are set so that spot rates for maturities ranging from 1 to 116 weeks exactly match the term structure. Reuters reports implied BDT volatilities for caps and for swaptions for various maturities and the corresponding zero-coupon yields. We set the volatility parameter σ based on the average of the reported volatilities across maturities ranging from 3 months to 2 years, and across caps and swaptions.

To illustrate the connection between the pricing process Λ and the BDT interest rate model, consider the price of a 2-period zero coupon bond

$$\begin{aligned}
\frac{1}{1+r_0^2} &= E_0 \frac{\Lambda_1}{\Lambda_0}\left\{\frac{1}{1+r_1^1}\right\} \\
&= \frac{1}{1+r_0^1} E_0 \left\{\frac{\Lambda_1}{\Lambda_0} / E_0 \frac{\Lambda_1}{\Lambda_0}\right\}\left\{\frac{1}{1+r_1^1}\right\} \\
&= \frac{1}{1+r_0^1}\left\{\pi^* \frac{1}{1+r_1^1(u)} + (1-\pi^*)\frac{1}{1+r_1^1(d)}\right\},
\end{aligned}$$

with $r(u)$ and $r(d)$ the upwards and downwards realizations of the interest over the period. By recursively applying the BDT model, any risky payout can be priced as with the state-price process Λ.

Figure 6 reports the model-implied convexity adjustment for each auction date in terms of an annualized spread, Δ_0, defined by

$$\sum_I \sum_k E_0\left[\frac{\Lambda_{I+13}}{\Lambda_0} r_{I+k}^{13}\right] = \sum_I \sum_k \frac{r_{0,I+k}^{f,13} + \frac{1}{4}\Delta_0}{1 + r_0^{I+13}}.$$

As shown in the figure, the convexity adjustment does not exceed 0.11 basis points for any of the auction dates. Clearly, from the perspective of this paper, an adjustment of this magnitude is irrelevant. As suggested by equation 4, the recent low-volatility environment for interest rates is responsible for this conclusion.

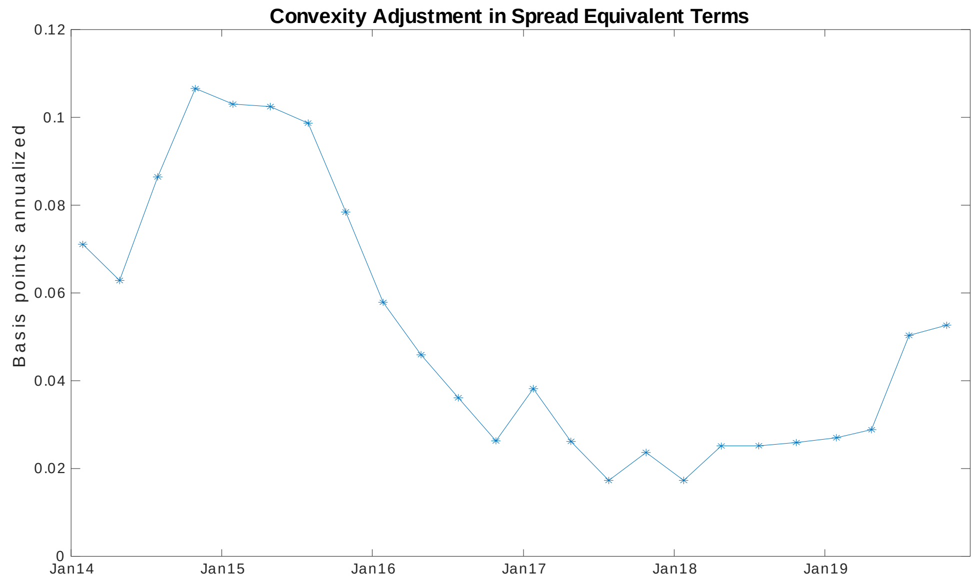

Figure 6: The convexity adjustment is computed based on a version of the BDT model calibrated to the term structure and the interest rate volatility on auction dates.

Table 1 explores the convexity adjustment as a function of the level of the interest rate and the volatility parameter. The first two lines use a volatility parameter σ of 0.5 with the term structure of the spot rates at a flat 1% or 1.5% in annualized terms. This case is representative of the conditions between 2014 and 2019. The convexity adjustment is below 0.1 of a basis point. As σ represents the standard deviation of the natural logarithm of the short rate, we report as a more intuitive measure of interest rate volatility the conditional standard deviation of the 13-week rate one year in the future, $\mathrm{Std}_0\left(r_{1Y}^{13w}\right)$. For the first two cases, this standard deviation equals 0.53 or 0.79 percent, in annualized terms. This low number is representative of the stability of short rates in the recent past. The lower two lines contain examples with higher volatility parameters. The last line shows a case that is representative of the peak volatility during the financial crises as represented by the BDT parameters for 10/10/2008. The convexity adjustment is 1.14 basis points. The implied conditional standard deviation of the 13-week rate at a one-year horizon is 3%. Overall, the table shows that while the current stable interest environment does not require a convexity adjustment for reasonably accurate pricing of Treasury FRNs, such an adjustment has the potential to become relevant with significantly higher interest rate volatility.

$\bar{r}$ in %	σ	Δ_0 in bps	$\mathrm{Std}_0\left(r_{1Y}^{13w}\right)$	$\mathrm{E}_0\left(r_{1Y}^{13w}\right)$
1	0.5	.035	.53	1
1.5	0.5	.08	.79	1.5
1	0.8	.11	.94	1
3.2	0.8	1.14	3	3.2

Table 1: Convexity adjustment, Δ_0, as a function of the interest rate level and volatility. The term structure of the spot rates is set at a flat percentage of $\bar{r}$, σ is the volatility parameter. $\mathrm{Std}_0\left(r_{1Y}^{13w}\right)$ and $\mathrm{E}_0\left(r_{1Y}^{13w}\right)$ are the conditional standard deviation and the conditional expectation of the annualized 13-week rate one year from now. Moments are computed under the risk-neutral distribution

3.2 T-bill investment strategies

Based on the term structures at auction dates, we have concluded that FRNs have offered excessive interest, or equivalently, that they have been cheap to buy for investors. In this section, we document the ex-post realized returns from investing in FRNs, and compare these to two T-bill investment strategies that approximately replicate the cash flows of the FRNs. The T-bill strategies can be considered as near arbitrage positions, as they can allow investors to exploit excess spreads in FRNs by going long a FRN and short a T-bill strategy. We find that FRNs investments have mostly outperformed a buy-and-hold strategy with three-month T-bills. Investing in on-the-run 3-month T-bills rolled over every week has had a slightly higher return on average than FRN investments. For both T-bill strategies, we find a strong positive relation between our estimated excess spreads on FRNs and the subsequent realized excess returns of FRNs over the T-bill strategies.

A generic FRN can be perfectly replicated by rolled-over short-term investments. Because the Treasury's FRNs pay a coupon based on an average of the three-month T-bill rate, such perfect replication is not feasible. Replication with widely available derivative contracts also does not seem possible. For instance, to replicate the coupon payments of a FRN over its two-year life one would need forward contracts on the three-month T-bill rate for every week over that period. That is, one would need forward contracts with 103 different maturity dates, one for each week. Such contracts are not widely available. Given these constraints, we consider here the possibility of establishing near-arbitrage positions that replicate approximately the FRNs with investments in three-month T-bills.

The first strategy we consider is to invest in three-month (13-week) T-bills by holding a bill until maturity, and by aligning its maturity as close as possible with a coupon payment date for a FRN. If the Treasury's FRN were of the generic type, this buy-and-hold strategy would perfectly replicate its cash flows, except for some minor mismatch in the maturity dates of the T-bill and the FRN. In a frictionless arbitrage-free environment, daily returns on a generic FRN and this short-term strategy would be equalized. Due to the interest averaging feature of the Treasury's FRNs, the replication is no longer perfect. Within quarter changes in the current T-bill rates are reflected in the FRNs' coupon but not in the cash flow of the approximate replication strategy.

The second strategy always invests in the most recently issued three-month T-bill. This can capture within quarter changes in three-month rates. However, this introduces additional interest rate risk. For instance, one day before maturity, the FRN is not subject to direct interest rate risk, but the three-month T-bill is subject to interest rate risk when it is sold. We label this the "on-the-run" strategy.

We compute daily returns for FRNs and T-bills based on secondary market prices and accrued interest on FRNs. Secondary market close prices are obtained from Reuters Eikon, accrued FRN interest data is obtained from Treasury Direct, daily T-bill returns for the on-the-run strategy are from CRSP. For the buy-and-hold T-bill strategy, we identify 13-week T-bills that best match the coupon periods of the FRNs. These bills mature at the end of January, April, July and October, ranging from January 2014 to December 2019.

Table 2 summarizes properties of these strategies. Average daily realized returns on the FRNs have exceeded the returns of the buy-and-hold T-bill investments for 21 out of the 24 FRN issues, while FRN returns exceeded the on-the-run T-bill strategy on in 8 out of 24 case. Average excess returns of FRN over the entire sample period were 16 basis points and -2 basis points relative to the buy-and-hold and the on-the-run strategies, respectively. One possible explanation for the higher returns of the on-the-run strategy compared to the buy-and-hold strategy is that it has relatively more duration risk.

As shown in Figure 7 there is a clear positive relationship between our estimated excess spreads at auction dates and the subsequent realized excess returns of a FRN issue. In particular, the buy-and-hold strategy excess spreads translate almost one for one into FRN excess returns. The two outliers in Figure 7 with a realized excess returns of over 60 basis points correspond to the October 2019 issue, whose reported average return is based on only 40 observations; issues before 2018 (that have matured before the end of the sample) have approximately 500 observations, as shown in Table 2.

FRNs and rolled-over 3-month T-bills buy-and-hold or on-the-run

	FRN returns		T-bill returns				Excess returns		Excess returns		Nobs
			Buy-and-hold		On-the-run		Buy-and-hold		On-the-run		
FRN	mean	std	mean	std	mean	std	mean	std	mean	std	
2014 Jan	0.10	0.04	0.03	0.02	0.11	0.04	0.07	0.04	-0.02	0.06	498
2014 Apr	0.15	0.04	0.07	0.02	0.16	0.05	0.08	0.05	-0.01	0.06	501
2014 Jul	0.19	0.05	0.09	0.03	0.21	0.05	0.09	0.05	-0.02	0.07	500
2014 Oct	0.21	0.06	0.13	0.03	0.26	0.06	0.08	0.07	-0.06	0.08	499
2015 Jan	0.30	0.07	0.16	0.04	0.31	0.06	0.14	0.08	-0.02	0.08	499
2015 Apr	0.36	0.12	0.22	0.04	0.38	0.06	0.14	0.12	-0.02	0.13	499
2015 Jul	0.49	0.14	0.33	0.05	0.50	0.07	0.16	0.14	-0.01	0.15	498
2015 Oct	0.70	0.17	0.46	0.05	0.65	0.07	0.23	0.17	0.05	0.17	498
2016 Jan	0.93	0.10	0.59	0.06	0.80	0.07	0.34	0.10	0.13	0.11	500
2016 Apr	1.04	0.09	0.72	0.06	0.92	0.07	0.33	0.10	0.12	0.10	498
2016 Jul	1.24	0.12	0.92	0.07	1.11	0.08	0.32	0.11	0.13	0.12	499
2016 Oct	1.48	0.11	1.14	0.07	1.31	0.08	0.33	0.10	0.16	0.10	500
2017 Jan	1.70	0.11	1.39	0.08	1.57	0.09	0.30	0.09	0.13	0.09	498
2017 Apr	1.82	0.11	1.63	0.08	1.80	0.09	0.20	0.08	0.02	0.08	498
2017 Jul	1.99	0.10	1.83	0.09	2.04	0.10	0.16	0.06	-0.04	0.08	499
2017 Oct	2.08	0.10	1.95	0.09	2.18	0.10	0.13	0.07	-0.10	0.08	499
2018 Jan	2.08	0.10	2.04	0.09	2.25	0.10	0.04	0.07	-0.17	0.08	478
2018 Apr	2.15	0.18	2.13	0.09	2.35	0.10	0.02	0.17	-0.20	0.18	417
2018 Jul	2.20	0.30	2.19	0.09	2.42	0.10	0.01	0.30	-0.22	0.30	353
2018 Oct	2.19	0.26	2.23	0.09	2.50	0.10	-0.04	0.24	-0.31	0.25	289
2019 Jan	2.15	0.17	2.17	0.09	2.45	0.10	-0.02	0.16	-0.30	0.17	229
2019 Apr	2.00	0.19	2.09	0.09	2.41	0.10	-0.09	0.17	-0.41	0.18	168
2019 Jul	2.04	0.21	1.92	0.08	2.20	0.09	0.12	0.19	-0.16	0.21	104
2019 Oct	2.31	0.32	1.65	0.06	1.70	0.07	0.67	0.31	0.62	0.32	40
Whole sample	1.17	0.14	1.01	0.08	1.20	0.10	0.16	0.12	-0.02	0.13	
Positive count							21		8		

Table 2: Returns are computed daily, in percentage points. Means are scaled by 250, standard deviations by 250^.5. Excess returns are FRN returns minus T-bill returns. Moments computed for the whole sample use each available return.

While the two T-bill strategies can naturally be thought of as approximately replicating the returns of the FRNs, our analysis shows that at a daily frequency the replication is not very tight. Indeed, in the table, the standard deviation of the returns of the FRNs in excess of the T-bill investments over the whole sample is only moderately lower than the standard deviation of the FRNs themselves. To the extent that excess returns can be thought of as a long-short investment, the short T-bill investments did hedge the daily returns of the FRNs only very partially. This suggests that daily returns to FRNs were driven by other factors than just concurrent T-bill prices.

Bhanot and Guo (2017) document daily excess returns for FRNs up to October 2016 relative to a set of overnight rates, in particular, FF rates, GCR rates, and Overnight LIBOR. They also document excess returns for FRN issues relative to the T-bill index for the FRNs. Our excess returns of FRNs are with respect to the returns of T-bill investment strategies and are therefore not equivalent to the excess returns they compute.

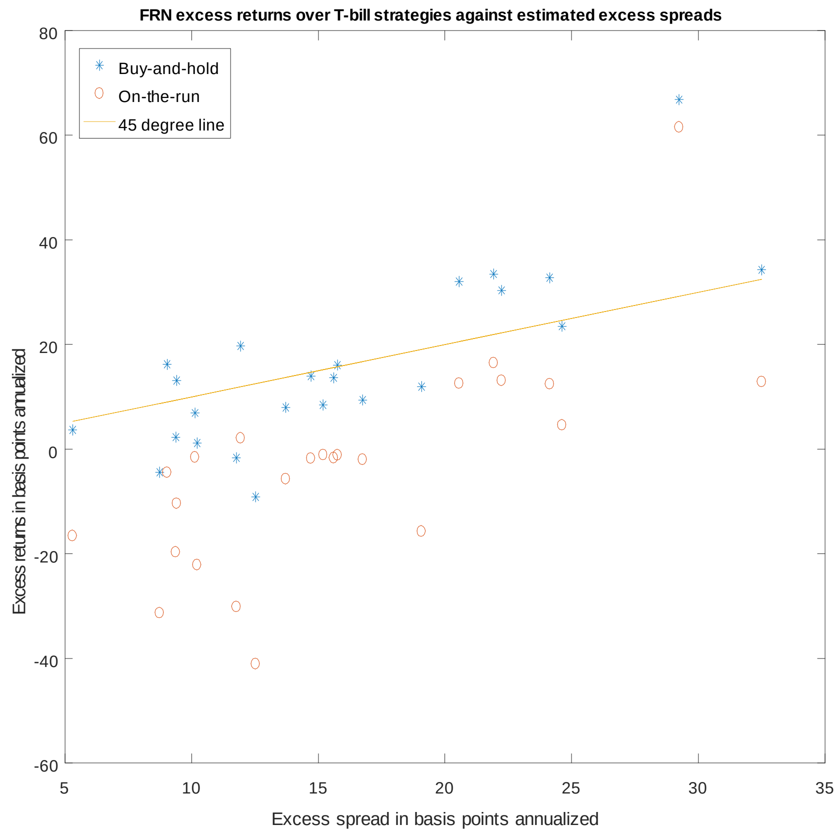

Figure 7: Excess spreads are plotted for each of the 24 new FRN issues between 2014 and 2019. The FRN excess returns are daily average annualized returns over the period an issue was outstanding.

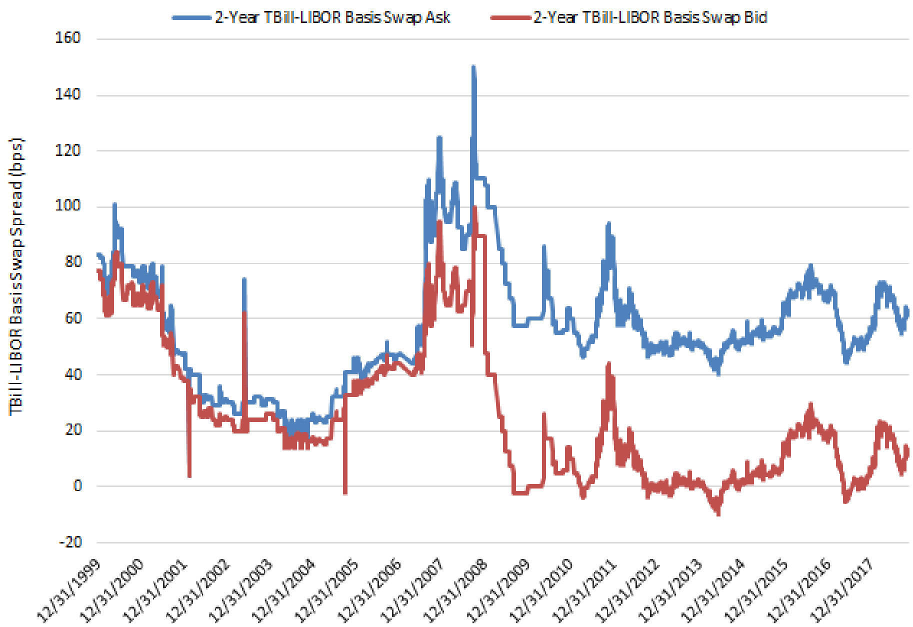

Figure 8: T-bill - LIBOR basis swaps bid-ask quotes.

Fleckenstein and Longstaff (2018) attempt to use T-bill/LIBOR basis-swaps with plain vanilla LIBOR interest rate swaps to replicate Treasury FRNs and identify an arbitrage opportunity given the difference in the returns of their resulting replicating portfolio and the U.S. Treasury FRNs. While T-bill/LIBOR basis swaps combined with interest rate swaps would theoretically offer a nearly perfectly replicating portfolio (Treasury FRN weekly reset dates make for some small basis), T-bill/LIBOR basis swaps have become extremely illiquid since the global financial crisis of 2008 as bid-ask spreads have widened to 50 bps (Figure 8). The bid-ask prices are primarily broker quotes (provided by brokers like Tullett Prebon) and remain very thinly traded. One could argue that the lack of a liquid arbitrage opportunity may be in part responsible for the Treasury FRN mispricing we have previously identified.

4 Optimal government debt management with FRNs

This section starts by reviewing conditions under which FRNs are redundant in typical models for optimal government debt management. We then study the optimal provision of FRNs in a model where short-term debt provides additional liquidity services not available in FRNs. The main conclusion of the analysis is that optimally managed government debt can be consistent with a positive amount of FRNs issued that pay interest that can appear excessive.

4.1 FRNs without frictions

A generic FRN pays the one-period interest rate every period and the principal at maturity. Consider a default-free FRN with a floating rate indexed to the default-free one-period interest rate between t and $t+1$, $r_{t,t+1}$. For instance, with three periods, with $t = 0, 1$ and 2, the FRN pays

$$\begin{aligned} r_{0,1} \text{ at } t &= 1, \text{ and} \\ 1 + r_{1,2} \text{ at } t &= 2. \end{aligned}$$

The prices of this bond at time 1 and time 0, $P^F_{1,2}$ and $P^F_{0,2}$, can be derived sequentially by no-arbitrage as

$$P^F_{1,2} = \frac{1 + r_{1,2}}{1 + r_{1,2}} = 1 \text{ and } P^F_{0,2} = \frac{P^F_{1,2} + r_{0,1}}{1 + r_{0,1}} = 1.$$

There is no price risk for FRNs. The bond is valued at par each period. The same property holds with any number of periods.

For standard model specifications, a FRN is indistinguishable from rolled-over short-term debt, and thus FRNs do not add anything. To see this, assume a time t budget constraint with W_t the beginning of period wealth and B^1_{t+1} and B^F_{t+1} the amounts of one-period debt and FRNs,

$$W_t = ... + B^1_{t+1} + B^F_{t+1} ...$$

and for time $t + 1$

$$.. + (1 + r_{t,t+1})\, B^1_{t+1} + (1 + r_{t,t+1})\, B^F_{t+1} .. + ... = ..$$

where the FRN is always valued at par. Then, for any plan $\left(B^1_{t+1}, B^F_{t+1}\right)$ of the two holdings, there is a budget-equivalent plan $\left(B^1_{t+1} + B^F_{t+1}, 0\right)$. FRNs do not add anything to short-term debt whether markets are complete or incomplete. This follows from the absence of arbitrage and the absence of frictions.

The FRNs issued by the U.S. Treasury differ from the generic version in that their coupon is the average three-month rate over the quarter as opposed to the rate determined at the beginning of the quarter. As shown above, this distinction can affect the valuation to some extent. However, averaging interest rates within a quarter does not significantly affect the correlation structure of the coupons over longer horizons, and this feature is unlikely to matter significantly for the role FRNs play for government debt policy. Our model will not explicitly incorporate this feature.[4]

[4] A possible advantage of averaging interest rates within the quarter would be that such securities are

4.2 FRNs when short-term debt is special

When short-term debt has some money-like utility, rolled-over short-term debt is no longer equivalent to a FRN. We are studying here a simple framework with this property by adding FRNs to the model of Greenwood, Hanson and Stein (2015). The specification of the utility for short-term debt and the optimization of seigniorage is slightly more general than in their case. This setting is attractive because it can account for the idea that short-term debt is "cheaper" to issue than long-term debt in a way where this does not just reflect a premium for market risk. This framework is also very tractable. In particular, due to linear utility of consumption, consumption does not affect bond prices, and the optimal debt policy can be characterized analytically. On the negative side, this is essentially a one-shot model and as such implies some shortcuts relative to a fully dynamic infinite horizon setting.

There are three periods: $0, 1$ and 2. There is a single source of uncertainty, a discount rate shock β realized at time 1 that affects the interest rate at $t = 1$, with $E_0(\beta) = 1$. For simplicity, like in Greenwood, Hanson and Stein (2015), we abstract from inflation uncertainty and other shocks. At time 0 and 1 the government can issue one-period discount bonds in amounts $B_{0,1}$ and $B_{1,2}$. Two-period discount bonds, $B_{0,2}$, pay one unit of the numeraire at time 2. A FRN, $B^F_{0,2}$, is assumed to pay $(1 + r_{1,2})$ at time 2. The interest, $r_{1,2}$, is not known at time 0. First period interest for the FRN and the (two-period bond) are not included for tractability. The FRN can be viewed as a *zero-coupon* security, defined as paying

$$(1 + r_{0,1})(1 + r_{1,2})$$

at time 2. First period interest, $r_{0,1}$, is equivalent to re-scaling the notional amount.

Short-term debt in the initial period provides money-like utility through an increasing function $v(B_{0,1})$. It would be reasonable to assume that government debt other than short-term bills offer monetary services. For the argument presented here it is essential that bills have some advantage over longer maturity debt including FRNs. The evidence of an excess spread in U.S. FRNs presented in Section (2) supports this assumption. More generally, Greenwood, Hanson and Stein (2015 and 2016) present evidence supporting the idea of a money-like premium for short-term T-bills.

Household utility from consumption and short-term debt is

$$U = v(B_{0,1}) + C_0 + E_0[C_1 + \beta C_2].$$

Households receive an endowment Y every period.

more robust to manipulation and short-term market disruptions.

In equilibrium bonds are priced by the household's Euler equations as

$$P_{0,1} = 1 + v'\left(B_{0,1}\right), \; P_{0,2} = 1, \text{ and } P_{1,2} = \beta.$$

If marginal utility of monetary services is positive in equilibrium, $v'(.) > 0$, short-term debt in the initial period has a higher price and a lower interest cost to the government. The FRN pays $1/\beta$ at time 2 and this payoff is discounted with β at time 1 so that

$$P^F_{1,2} = 1, \text{ and } P^F_{0,2} = 1.$$

The government finances a one-time expenditure G at time 0 with debt and taxes τ. The government budget constraints for each period are given by

$$\begin{aligned} t &= 0 : \tau_0 = G - B_{0,1}P_{1,0} - B_{0,2} - B^F_{0,2} && (5) \\ t &= 1 : \tau_1 = B_{0,1} - B_{1,2}\beta && (6) \\ t &= 2 : \tau_2 = B_{1,2} + B_{0,2} + B^F_{0,2}/\beta. && (7) \end{aligned}$$

Taxes have distortionary effects represented by a quadratic function $\tau^2/2$ so that household consumption in each period is given by

$$\begin{aligned} C_0 &= Y - \tau_0 - (1/2)\,\tau_0^2 - B_{0,1}P_{0,1} - B_{0,2} - B^F_{0,2}, \\ C_1 &= Y - \tau_1 - (1/2)\,\tau_1^2 + B_{0,1} - B_{1,2}\beta, \\ C_2 &= Y - \tau_2 - (1/2)\,\tau_2^2 + B_{1,2} + B_{0,2} + B^F_{0,2}/\beta. \end{aligned}$$

As is standard in the literature, we abstract from political distortions and assume a benevolent government. The government maximizes the utility of the households taking equilibrium pricing as given. Substituting the government budget constraints into household consumption, the government's objective is to maximize

$$\max_{B_{0,1}, B_{0,2}, B^F_{0,2}, B_{1,2}} \left[v\left(B_{0,1}\right) - \frac{1}{2}\left(\tau_0^2 + E_0\left(\tau_1^2\right) + E_0\left[\beta\tau_2^2\right]\right)\right], \qquad (8)$$

where constant terms have been dropped, subject to equation (5),(6) and (7). The government's objective displays the concerns for tax-smoothing and money utility for short-term debt. There is potential for refinancing risk due to the stochastic short rate at time 1.

4.3 Optimal debt structure

Our characterization of the solution proceeds in two steps. First, it is shown that the optimal policy implies perfect tax smoothing across the three periods. Second, the optimal issuance of FRN is derived.

4.3.1 Tax smoothing

This sub-section shows that the optimal debt issuance allows the government to perfectly smooth taxes across time and states of nature. The solution is determined recursively. As such, it is time-consistent. At time 1 the government's problem is

$$\max_{B_{1,2}}\left[-\frac{1}{2}\left(\tau_1^2+\beta\tau_2^2\right)\right]=\max_{B_{1,2}}\left[-\frac{1}{2}\left(\left(B_{0,1}-B_{1,2}\beta\right)^2+\beta\left(B_{1,2}+B_{0,2}+B_{0,2}^F/\beta\right)^2\right)\right],$$

because the interest rate uncertainty has now been realized.[5] The first-order condition is

$$\beta\left(B_{0,1}-B_{1,2}\beta\right)=\beta\left(B_{1,2}+B_{0,2}+B_{0,2}^F/\beta\right),$$

$$B_{1,2}=\frac{B_{0,1}-B_{0,2}-B_{0,2}^F/\beta}{1+\beta}.$$

From the government's budget constraint, this implies that there is perfect tax smoothing between time 1 and 2

$$\tau_1=\tau_2=\frac{B_{0,1}+\beta B_{0,2}+B_{0,2}^F}{1+\beta}.$$

Substituting this into the time 0 objective yields

$$\max_{B_{0,1},B_{0,2},B_{0,2}^F}\left[v\left(B_{0,1}\right)-\frac{1}{2}\left(\tau_0^2+E\left[\left(1+\beta\right)\tau_1^2\right]\right)\right]$$

which after substitution of the budget constraints becomes

$$\max_{B_{0,1},B_{0,2},B_{0,2}^F}\left[v\left(B_{0,1}\right)-\frac{1}{2}\left(G-B_{0,1}\left\{1+v'\left(B_{0,1}\right)\right\}-B_{0,2}-B_{0,2}^F\right)^2-\frac{1}{2}E\left[\frac{\left(B_{0,1}+\beta B_{0,2}+B_{0,2}^F\right)^2}{1+\beta}\right]\right].$$

To characterize the equilibrium, We initially take $B_{0,1}$ as given and solve for the optimal $B_{0,2}$ and $B_{0,2}^F$. It can easily be checked that this is a strictly concave problem with a unique solution. Whether the complete problem with $B_{0,1}$ is well-defined depends on the properties

[5] After the realization of β, all outstanding and newly issued debt, $B_{0,2}$, $B_{0,2}^F$, and $B_{1,2}$, are equivalently one-period risk-free debt. Allowing the government to retire or re-issue $B_{0,2}$ or $B_{0,2}^F$ at this point would have no effect on the equilibrium allocation in the model.

of the function $v(.)$. We consider that below.

The first-order conditions for $B_{0,2}$ and $B^F_{0,2}$ are

$$\left(G - B_{0,1}\left\{1 + v'\left(B_{0,1}\right)\right\} - B_{0,2} - B^F_{0,2}\right) - E_0\left[\frac{\beta\left(B_{0,1} + \beta B_{0,2} + B^F_{0,2}\right)}{1+\beta}\right] = 0 \tag{9}$$

$$\left(G - B_{0,1}\left\{1 + v'\left(B_{0,1}\right)\right\} - B_{0,2} - B^F_{0,2}\right) - E_0\left[\frac{\left(B_{0,1} + \beta B_{0,2} + B^F_{0,2}\right)}{1+\beta}\right] = 0 \tag{10}$$

Combining the two yields

$$E_0\left[\frac{\beta\left(B_{0,1} + \beta B_{0,2} + B^F_{0,2}\right)}{1+\beta}\right] = E_0\left[\frac{\left(B_{0,1} + \beta B_{0,2} + B^F_{0,2}\right)}{1+\beta}\right]$$

which is solved by

$$B_{0,2} = B_{0,1} + B^F_{0,2}. \tag{11}$$

With this condition, the taxes for time 1 and 2 solved for above

$$\tau_1 = \tau_2 = \frac{B_{0,1} + \beta B_{0,2} + B^F_{0,2}}{1+\beta}$$

are equal to

$$\tau_1 = \tau_2 = B_{0,2} = B_{0,1} + B^F_{0,2}.$$

That is, the interest rate risk coming from β is perfectly hedged away and there is no uncertainty about future taxes. In other words, taxes at time 2 are matched by long-term debt, and taxes at time 1 are matched by a combination of short term debt and FRNs.

Inserting this result in one of the first-order conditions (9) or (10) gives

$$\tau_0 = \left(G - B_{0,1}\left\{1 + v'\left(B_{0,1}\right)\right\} - B_{0,2} - B^F_{0,2}\right) = \tau_1 = \tau_2,$$

and

$$\tau_j = \frac{G - B_{0,1}v'\left(B_{0,1}\right)}{3}.$$

To summarize, taxes are perfectly smoothed across the three periods to pay for government spending not financed by seigniorage, that is $G - B_{0,1}v'\left(B_{0,1}\right)$. Interest rate uncertainty is hedged away by the combination of short term debt and FRNs. Without FRNs, taxes would typically be subject to interest rate risk; FRNs help eliminate this exposure.

4.3.2 Optimal issuance of FRNs

The position of FRNs depends on the optimal level of short-term debt. If short-term debt is relatively abundant, that is $B_{0,1} > B_{0,2}$, then the government would want to save with FRNs to hedge the interest rate risk (as implied by equation 11). If the short-term debt position is relatively small, that is $B_{0,1} < B_{0,2}$, the government would issue FRNs.

The negative relation between $B_{0,1}/B_{0,2}$ and FRN issuance is a consequence of tax smoothing and the perfect correlation of the interest cost on short-term debt and FRNs. This correlation makes a FRN a closer substitute to short-term debt than to long-term debt for managing tax risk. Figure 2 shows that the share of short-term Treasuries, T-bills, has come down in the years before 2014. One could interpret the relatively lower share of T-bills in 2014 when FRN have started to be issued as consistent with the implications of the model.

We solve the problem fully by taking the properties of the solution conditional on $B_{0,1}$ as given. In particular, we substitute out $B_{0,2}$ and $B^F_{0,2}$ and use the perfect tax-smoothing property so that the optimal $B_{0,1}$ solves

$$\max_{B_{0,1}} \left[v\left(B_{0,1}\right) - \frac{3}{2}\tau^2 \right].$$

By substituting the budget constraint and rearranging we get

$$\max_{B_{0,1}} \left[v\left(B_{0,1}\right) - \frac{1}{6}\left(G - B_{0,1}v'\left(B_{0,1}\right)\right)^2 \right]. \tag{12}$$

The first-order condition is

$$-v'\left(B_{0,1}\right) = \frac{1}{3} \underbrace{\left(G - \underbrace{B_{0,1}v'\left(B_{0,1}\right)}_{\text{Seigniorage}} \right)}_{\text{Amount to be debt-financed}} \underbrace{\left\{ v'\left(B_{0,1}\right) + B_{0,1}v''\left(B_{0,1}\right) \right\}}_{\text{Derivative of Seigniorage to } B_{0,1}} \tag{13}$$

equalizing the (negative of the) marginal utility from money to the marginal cost of the tax distortion effect from seigniorage. Focusing on a specification where $v'\left(B_{0,1}\right) > 0$, an interior maximum either requires that the amount to be debt-financed $\left(G - B_{0,1}v'\left(B_{0,1}\right)\right)$ becomes negative or that the derivative of seigniorage with respect to $B_{0,1}$ becomes negative. This derivative can be viewed as the slope of a type of Laffer curve. So in the later case, the government issues short-term debt beyond the peak of this curve.

As an example, assume a negative exponential function $v\left(B_{0,1}\right) = -e^{-\alpha B_{0,1}}$ with parameter $\alpha > 0$. In this case, the top of the seigniorage curve $B_{0,1}v'\left(B_{0,1}\right)$ is at $B_{0,1} = 1/\alpha$ with a peak value of seigniorage of $e^{-1} = 0.368$. Interestingly, this does not depend on the value of

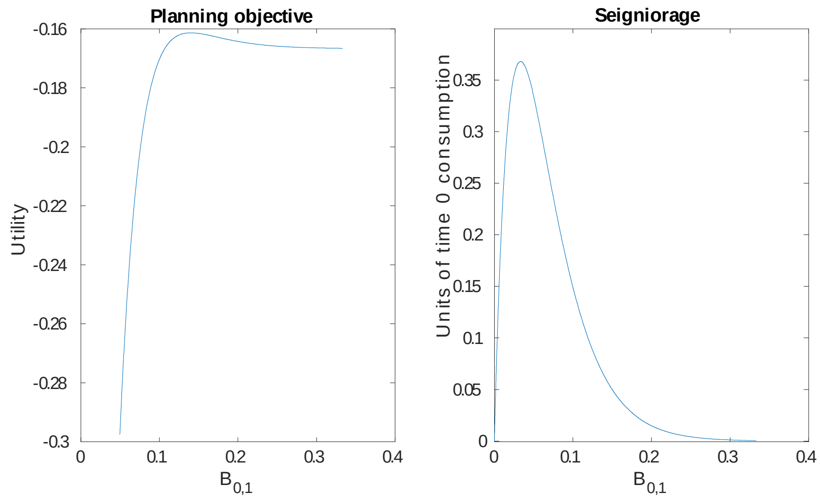

Figure 9: Government objective and seignorage revenue as a function of short term debt, $B_{0,1}$.

the parameter α. Figure 9 shows the government's objective as a function of $B_{0,1}$, equation (12), and the seigniorage curve $B_{0,1}v'(B_{0,1})$, for $\alpha = 30$ and $G = 1$. In this case, the top of the seigniorage curve is reached at $1/a = 0.033$ and the optimal amount of short-term debt is reached at a higher level when the seigniorage curve is declining, $B_{0,1} = 0.14$. The short-term debt position is relatively small and the government issues FRNs: $B^F_{0,2} = 0.17$ and $B_{0,2} = 0.31$.

5 Drivers of excess spreads

In this section we present a dynamic pricing model that builds on the ideas of our theoretical analysis, and we examine the relation between the documented excess spreads and potential explanatory factors.

5.1 Infinite-horizon money-in-utility model

In this subsection we present a dynamic model for pricing FRNs that extends the framework of the previous section and that allows us to more explicitly characterize the deviations from the standard no-arbitrage setting we have documented in section 2.

Assume investors have a preference for cash over securities due to its immediacy, the absence of risk, and the absence of a need to participate in securities markets. We consider as cash or money equivalently dollar bills, central bank reserves, maturing T-bills, and current interest payments on long-term bonds. In each period, investors get utility from consumption and from cash holdings

$$u(C_t) + v_t(M_t).$$

Investors value consumption C_t through a concave period utility $u(.)$, and they value cash or money-like assets M_t they enter the period with. For instance, T-bills purchased at time $t-1$ that mature in t give this utility at t, but also other cash payments such as coupon payments on bonds received at t. The money utility $v_t(M_t)$ can depend on other factors or shocks, and this is expressed by the time subscript. This is a version of a money-in-the-utility-function specification; for early examples see Sidrauski (1967) or Feenstra (1986).

Investors have access to various bonds, free of default risk, in an otherwise frictionless way. The first-order condition for a T-bill, a one-period risk-free asset, is

$$p_t = \beta E_t \frac{u'(C_{t+1}) + v'_{t+1}(M_{t+1})}{u'(C_t)}.$$

The marginal money value next period, $v'_{t+1}(M_{t+1})$, affects the price positively.

We study multiperiod bonds with geometric amortization. A bullet bond has a large utility value in the last period for the principal. This would slightly complicate the algebra, but without any substantive impact on our arguments. For an amortizing FRNs, there is no last period, but coupon and amortization payments are counted as money next period. A newly issued FRN is priced as

$$q_t = \beta E_t \frac{u'(C_{t+1}) + v'_{t+1}(M_{t+1})}{u'(C_t)} [i_{t+1} + s + \lambda] + \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} (1-\lambda) q_{t+1}.$$

The cash payments – the interest including the spread $(i_{t+1} + s)$ as well as the amortization λ – are valued more highly than the outstanding bond. The coupon index is given by the T-bill rate,

$$i_{t+1} = 1/p_t - 1.$$

Just to be clear, in this section we abstract from the complication due to the constant maturity index. This should mostly affect very high frequency properties.

The spread of a newly issued FRN, s_t, is determined by setting $q_t = 1$, and after some algebra, see Appendix B, this equals

$$s_t = (1-\lambda) \frac{\Phi_t(\{\kappa_t\})}{\Phi_t(\{1\})}, \tag{14}$$

with the annuity operator

$$\Phi_t(\{x_t\}) \equiv \sum_{k=0} (1-\lambda)^k \beta^{1+k} E_t \frac{u'(C_{t+1+k}) + v'_{t+1+k}(M_{t+1+k})}{u'(C_t)} x_{t+k},$$

and

$$\begin{aligned} \kappa_{t+k} &= 1 - \beta^{1+k} E_t \frac{u'(C_{t+1+k})}{u'(C_t)} / \beta^{1+k} E_t \frac{u'(C_{t+1+k}) + v'_{t+1+k}(M_{t+1+k})}{u'(C_t)} \\ &\approx E_t \left\{ \frac{u'(C_{t+k})}{u'(C_t)} / E_t \frac{u'(C_{t+k})}{u'(C_t)} \right\} (r_{t+k} - i_{t+k}) \\ &\approx E_t^U (r_{t+k} - i_{t+k}). \end{aligned} \tag{15}$$

As shown in equation 15, κ_{t+k} measures the money spread, the expected value of the difference between one-period rates without the money utility

$$\frac{1}{1+r_{t+k}} = \beta E_{t+k} \frac{u'(C_{t+k+1})}{u'(C_{t+k})}$$

and the one-period T-bill rate i_{t+k}.

Clearly, in a standard no-arbitrage setting without special utility for money-like assets, s_t equals 0. If the money spread is constant $\kappa_{t+k} = \kappa$, then the FRNs' spread equals

$$s = (1-\lambda)(r-i).$$

Intuitively, investors in a multi-period FRN need to be compensated for the non-amortized component $(1-\lambda)$ which lacks the money utility. In general, there is variation in the money spread and therefore the forecasts of the money spreads over the maturity of the FRN are important.

5.2 Factors correlated with excess spreads

Based on equation (14) and (15), we empirically examine some factors that potentially capture the mechanisms highlighted by this model. In particular, we consider the spread between T-bill rates and non-governement short rates such as OIS and LIBOR rates. Presumably, the immediacy of cash is particularly valued in periods of high uncertainty, which leads us to consider a measure of implied volatility of interest rates. As an alternative to the factors directly suggested by our model, we also consider a basic demand indicator for FRNs as given by the bid-to-cover ratios for FRNs auctions.

The OIS minus T-bill spread is based on the 3-month forward rates averaged across

Regressor	Coefficient	p-val	Univariate Coefficient	Univariate Adj. R^2
Constant	-18.9	0.004		
OIS - T-bill spread	-0.36	0.002	-0.19	0.06
LIBOR - T-bill spread	0.20	< 0.0001	0.08	0.03
Interest rate volatility	0.32	< 0.0001	0.27	0.19
Bid-to-cover ratio	1.67	0.12	-0.008	0.01
Adj. R^2	0.44			
N-obs	72		72	

Table 3: Regression of Excess Spreads in FRNs 1/2014 - 12/2019. All variables are measured at FRN auction dates. The regression is estimated by OLS with Newey-West standard errors.

starting months $0, 3, 6$, and 9; OIS forward rates of longer horizons are very noisy. The LIBOR minus T-bill spread is the average based on the 3-month forward rates starting in $0, 3, 6$, and 9 months. This corresponds to the (spot) TED spread averaged with forward TED spreads. The interest rate volatility is the forward volatility of 3-month swap implied forward rates averaged across $6, 12$ and 18 months maturities, based on swaptions and caps.

The regression results in Table 3 show significant OIS and LIBOR spreads, but the OIS spread has the wrong sign. Interest rate volatility is strongly significant with the expected sign. The bid-to-cover ratio is not significant at the 10% level. In univariate regressions, the interest volatility has by far the highest adjusted R^2.

Figure 10 shows excess spreads together with our measure of interest volatility, visually confirming the strong comovement between these two variables. For comparison with a more standard measure of implied interest rate volatility, the MOVE index for 6-month options on Treasury securities is included in the figure. This is a popular measure of implied volatility for OTC Treasury options. Clearly, our measure of interest rate volatility which is based on the maturities suggested by our dynamic model is closely related to the MOVE index despite the differences in maturity and underlying instruments.

Figure 11 compares excess spreads and bid-to-cover ratios. Consistent with the regression results reported in table 3, there is no obvious visual connection between the two if one considers the entire sample period. However, for the first 21 months FRNs were issued (emphasized in the figure) there clearly is a strong connection. Over this particular period, the R^2 in the regression is 0.71. The bid-to-cover ratio displayed is transformed by the (negative) slope and the intercept from that regression. Based on this, it appears that at least initially the bid-to-cover ratio could have been an important driver of the excess spread.

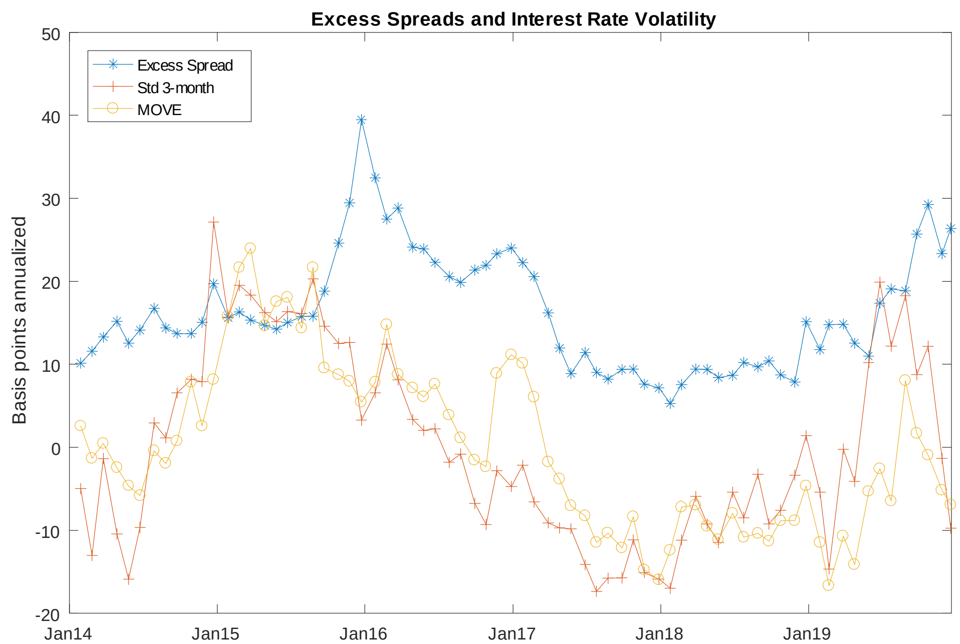

Figure 10: The interest rate volatility measures are displayed net of their sample means. Std 3-month is the forward volatility of 3-month swap forward rates averaged across 6, 12 and 18 months maturities based on swaptions and caps. MOVE is the Merrill Lynch Option Volatility Estimate for 6-month option maturity based on Treasury securities with maturites between 2 and 30 years.

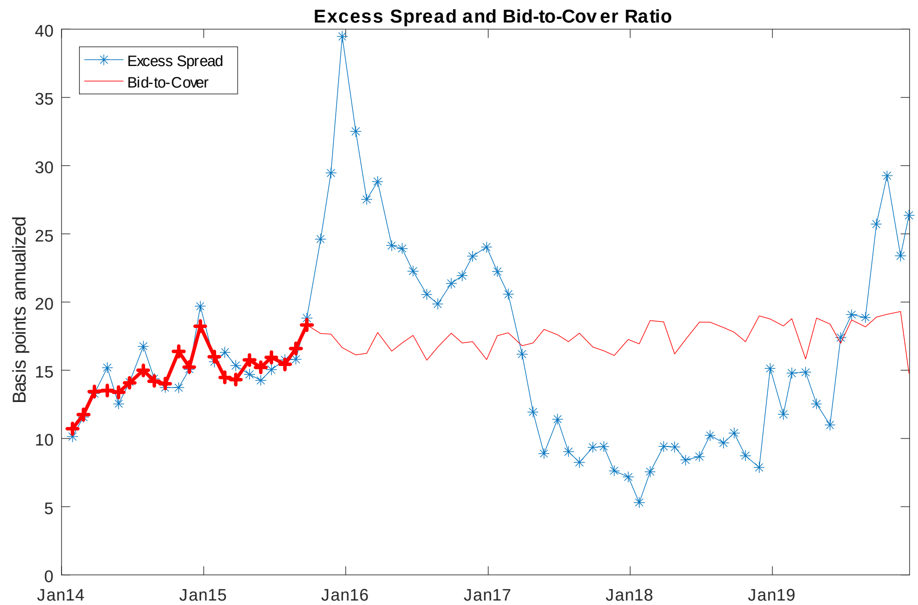

Figure 11: The Bid-to-Cover ratio displayed is transformed by the slope (-2.7) and intercept (26) from the regression of the Excess Spread on the Bid-to-Cover ratio over the first 21 months (emphasized in the plot).

6 Conclusion

The new FRNs issued by the U.S. Treasury pay interest based on a constant maturity index of T-bill rates. This feature requires an explicit pricing model. We have derived a no-arbitrage pricing model for this purpose and shown that an accurate approximation for pricing FRNs can be based on implied forward rates alone. Convexity adjustments are quantitively unimportant given the low volatility of short-term rates between 2014 and 2019.

Our main finding is that U.S. FRNs when priced through a no-arbitrage approach have been paying excessively high interest. Nevertheless, as we have shown in the paper, optimally managed government debt can include FRNs. Our argument is based on the idea that short-term government debt can provide liquidity services, and that the optimal amount of short-term debt might be such as to generate refinancing risk. In this case, FRNs with payoffs positively correlated with future short-term rates can be used to manage this risk.

References

[1] Angeletos, George-Marios, "Fiscal Policy with Noncontingent Debt and the Optimal Maturity Structure," Quarterly Journal of Economics, 117 (2002), 1105–1131.

[2] Black, Fischer, Emanuel Derman and William Toy, 1990, "A One-Factor Model Of Interest Rates And Its Application To Treasury Bond Opriona," Financial Analysts Journal; Jan/Feb, 46, 1.

[3] Bhanot, Karan and Liang Guo, 2017 "The New Market for Treasury Floating Rate Notes", The Journal of Fixed Income, Vol 27, Issue 2, pp. 52-64.

[4] Barro, Robert J., 1979, On the determination of the public debt, Journal of Political Economy 87, 940–971.

[5] Bohn, Henning, 1990, Tax smoothing with financial instruments, American Economic Review 80, 1217–1230.

[6] Carayannopoulos, Peter 1995. "The Mispricing of U.S. Treasury Callable Bonds", Journal of Futures Markets, Volume 15, Issue 8, Pages 861–879.

[7] Cochrane, John, 2015 "A New Structure For U.S. Federal Debt", In "The $13 Trillion Question: Managing the U.S. Government's Debt", edited by David Wessel, Brookings Institution Press, pp. 91-146.

[8] Cochrane, John, 2017. "Spikes", Blog Post, https://johncochrane.blogspot.com/2017/04/spikes.html.

[9] Cornell, Bradford and Alan C. Shapiro, 1989. "The Mispricing of U.S. Treasury Bonds: A Case Study", Review of Financial Studies, 2(3): 297-310.

[10] D'Amico, Stefania and Thomas King, 2013. "Flow and stock effects of large-scale Treasury purchases: Evidence on the importance of local supply", Journal of Financial Economics, Volume 108, Issue 2, pp. 425-448.

[11] Debortoli, D., R. Nunes, and P. Yared (2017), Optimal government debt maturity. The Quarterly Journal of Economics, 55–102.

[12] Department of the Treasury, 2013, "Sale and Issue of Marketable Book-Entry Treasury Bills, Notes, and Bonds; Final Rule," Federal Register / Vol. 78, No. 147 / Wednesday, July 31, 2013 / Rules and Regulations.

[13] Du Wenxin, Joanne Im and Jesse Schreger, 2017. "The U.S. Treasury Premium". NBER Working Paper No. 23759.

[14] Du, Wenxin, Alexander Tepper and Adrien Verdelhan, 2017. “Deviations from Covered Interest Rate Parity”, Journal of Finance, forthcoming.

[15] Feenstra, Robert C., 1986. "Functional equivalence between liquidity costs and the utility of money," Journal of Monetary Economics, Elsevier, vol. 17(2), pages 271-291.

[16] Fleckenstein, Mathias, and Francis Longstaff, 2018. "Floating Rate Money: The Stability Premium In Floating Rate Notes", NBER Working Paper No. 25216

[17] Fleckenstein, Matthias, Francis Longstaff, Hanno Lustig, 2014. "The TIPS-Treasury Bond Puzzle", Journal of Finance, Volume 69, Issue 5, Pages 2151-2197

[18] Gaffney, Kevin and Karthik Ramanathan, 2014. "Treasury Floating-Rate Government Securities", Journal of Portfolio Management, Spring 2014.

[19] Greenwood, Robin, Samuel Hanon, Joshua Rudolph, and Lawrence Summers, 2015. “The Optimal Maturity of Government Debt”, In “The $13 Trillion Question: Managing the U.S. Government’s Debt”, edited by David Wessel, Brookings Institution Press, pp. 1-42.

[20] Greenwood, Robin, Samuel Hanson, and Jeremy C. Stein, 2015, "A Comparative-Advantage Approach to Government Debt Maturity", Journal of Finance 70, 1683-1722.

[21] Greenwood, Robin, Samuel Hanson, and Jeremy C. Stein, 2016, "The Federal Reserve’s Balance Sheet as a Financial-Stability Tool." Economic Symposium Conference Proceedings (Federal Reserve Bank of Kansas City).

[22] Gurkaynak, Refet S., Brian Sack, and Jonathan H. Wright, 2007. "The U.S. Treasury Yield Curve: 1961 to the Present" Journal of Monetary Economics, Vol 54 (8), Pages 2291-2304.

[23] Hartley, Jonathan, 2017, "How European regulators are hindering the feds ability to raise interest rates", https://www.forbes.com/sites/jonhartley/2017/03/31/how-european-regulators-are-hindering-the-feds-ability-to-raise-interest-rates/#5695b59e10dd

[24] Krisnamurthy, Arvind, 2002. "The bond/old-bond spread", Journal of Financial Economics, Vol. 66, Issues 2-3, pp. 463-506.

[25] Nosbusch, Yves, 2008, Interest costs and the optimal maturity structure of government debt, Economic Journal 118, 477–498.

[26] Sidrauski, M., 1967, "Rational choices and patterns of growth in a monetary economy." American Economic Review 57, 534-544.

[27] "Treasury Announces First Floating Rate Note Auction" U.S. Department of the Treasury, Press Center, 1/23/2014

[28] TreasuryDirect website, Floating Rate Note, https://www.treasurydirect.gov/instit/marketables/frn/frn.htm

[29] Veronesi, Pietro, 2010, Fixed Income Securities, Wiley.

[30] Vissing-Jorgensen, Annette and Arvind Krishnamurthy, 2011. “The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy”, Brookings Papers on Economic Activity, Fall 2011, No. 2.

Appendix A: Pricing FRNs

The main pricing equation is derived for a four period environment with periods $t = 0, 1, 2, 3$. This reduces notational complexity.

At $t = 0$ we price a claim that pays a single coupon at time $t = 2$. This coupon is defined as

$$C_2 = \frac{1}{2}\left(r_{0,2} + r_{1,3}\right),$$

that is, the average of the two-period rates determined at time $t = 0$ and $t = 1$. Clearly, as of time $t = 0$, $r_{1,3}$ is not known. Like the Treasury FRNs, this note pays a coupon that is an average of constant-maturity rates.

Ruling out arbitrage, there exists a state-price valuation process Λ_t that determines the price of this claim

$$\begin{aligned} V_0 &= E_0\left[\frac{\Lambda_2}{\Lambda_0}C_2\right] = E_0\left[\frac{\Lambda_2}{\Lambda_0}\frac{\left(r_{0,2}+r_{1,3}\right)}{2}\right] \\ &= \frac{r_{0,2}}{2}E_0\left[\frac{\Lambda_2}{\Lambda_0}\right] + \frac{1}{2}E_0\left[\frac{\Lambda_2}{\Lambda_0}r_{1,3}\right], \end{aligned}$$

and

$$V_0 = \frac{1}{2}\frac{r_{0,2}}{1+r_{0,2}} + \frac{1}{2}E_0\left[\frac{\Lambda_2}{\Lambda_0}r_{1,3}\right]. \tag{16}$$

Pricing the second strip is nontrivial in that is not just a function of the current (time 0) term structure. Specifically, because there is a timing mismatch between the payment date, 2, and the maturity date implied by the rate used, 3, current forward rates and the current term structure are in general not enough for pricing the second strip. This applies to all the strips of the Treasury FRNs with rates determined between 1 and 12 weeks after the beginning of a quarter.

Derivation of the pricing equation

The second term in equation (16)

$$E_0\left[\frac{\Lambda_2}{\Lambda_0}r_{1,3}\right] = E_0\frac{\Lambda_2}{\Lambda_0}E_0\left(r_{1,3}\right) + cov_0\left(\frac{\Lambda_2}{\Lambda_0}, r_{1,3}\right) = \frac{E_0\left(r_{1,3}\right)}{1+r_{0,2}} + cov_0\left(\frac{\Lambda_2}{\Lambda_0}, r_{1,3}\right). \tag{17}$$

Ruling out arbitrage implies that

$$1 = \left(1+r_{0,1}\right)E_0\left[\frac{\Lambda_3}{\Lambda_0}\left(1+r_{1,3}\right)\right]$$

and multiplying both sides by $(1+r_{0,3}) = 1/E_0\frac{\Lambda_3}{\Lambda_0}$

$$\frac{1+r_{0,3}}{1+r_{0,1}} = E_0\left[\left\{\frac{\Lambda_3}{\Lambda_0}/E_0\left(\frac{\Lambda_3}{\Lambda_0}\right)\right\}(1+r_{1,3})\right] \equiv \left(1+r^f_{0,1,3}\right).$$

This shows the forward rate as the expected value of the future spot rate $r_{1,3}$ with the normalized discount rate $\left\{\frac{\Lambda_3}{\Lambda_0}/E_0\left(\frac{\Lambda_3}{\Lambda_0}\right)\right\}$. Rewriting the last two terms as

$$\left(1+r^f_{0,1,3}\right) = E_0\left(1+r_{1,3}\right) + cov_0\left(\frac{\Lambda_3}{\Lambda_0}/E_0\left(\frac{\Lambda_3}{\Lambda_0}\right), r_{1,3}\right)$$

links the forward rate and the expected future spot rate. Substituting $r_{1,3}$ in (17)

$$\begin{aligned}
E_0\left[\frac{\Lambda_2}{\Lambda_0}r_{1,3}\right] &= \frac{r^f_{0,1,3} - cov_0\left(\frac{\Lambda_3}{\Lambda_0}/E_0\left(\frac{\Lambda_3}{\Lambda_0}\right), r_{1,3}\right)}{1+r_{0,2}} + cov_0\left(\frac{\Lambda_2}{\Lambda_0}, r_{1,3}\right) \\
&= \frac{r^f_{0,1,3}}{1+r_{0,2}} + cov_0\left(\frac{\Lambda_2}{\Lambda_0} - \frac{\frac{\Lambda_3}{\Lambda_0}/E_0\left(\frac{\Lambda_3}{\Lambda_0}\right)}{1+r_{0,2}}, r_{1,3}\right) \\
&= \frac{r^f_{0,1,3}}{1+r_{0,2}} + cov_0\left(E_1\frac{\Lambda_2}{\Lambda_0} - \frac{E_1\frac{\Lambda_3}{\Lambda_0}/E_0\left(\frac{\Lambda_3}{\Lambda_0}\right)}{1+r_{0,2}}, r_{1,3}\right) \\
&= \frac{r^f_{0,1,3}}{1+r_{0,2}} + cov_0\left(\frac{1}{1+r_{1,2}}\frac{\Lambda_1}{\Lambda_0} - \frac{\frac{\Lambda_1}{\Lambda_0}\frac{1}{1+r_{1,3}}/E_0\left(\frac{\Lambda_3}{\Lambda_0}\right)}{1+r_{0,2}}, r_{1,3}\right) \\
&= \frac{r^f_{0,1,3}}{1+r_{0,2}} + cov_0\left(\frac{\Lambda_1}{\Lambda_0}\frac{1}{1+r_{1,3}}\left[\frac{1+r_{1,3}}{1+r_{1,2}} - \left(1+^f_{0,2,3}\right)\right], r_{1,3}\right)
\end{aligned}$$

This shows the price as a term based on the foward rate and a "convexity adjustment", the covariance. This corresponds to equation (3) in the main text.

Risk-neutral expectations and interest rate volatility

Introducing the definition of risk-neutral expectations, E_0^Q,

$$\begin{aligned}
E_0\left[\frac{\Lambda_{I+13}}{\Lambda_0}r^{13}_{I+k}\right] &= E_0\left[\frac{\Lambda_{I+k}}{\Lambda_0}\frac{\Lambda_{I+13}}{\Lambda_{I+k}}r^{13}_{I+k}\right] = E_0\left[\frac{\Lambda_{I+k}}{\Lambda_0}\frac{r^{13}_{I+k}}{1+r^{13-k}_{I+k}}\right] \\
&= E_0\left\{\frac{\Lambda_{I+k}}{\Lambda_0}\right\}E_0\left[\frac{\Lambda_{I+k}}{\Lambda_0}/E_0\left\{\frac{\Lambda_{I+k}}{\Lambda_0}\right\}\frac{r^{13}_{I+k}}{1+r^{13-k}_{I+k}}\right] \\
&= \frac{1}{1+r_0^{I+k}}E_0^{Q(I+k)}\left[\frac{r^{13}_{I+k}}{1+r^{13-k}_{I+k}}\right].
\end{aligned}$$

Rewrite the expectation of the product as

$$\begin{aligned}
E_0\left[\frac{\Lambda_{I+13}}{\Lambda_0}r^{13}_{I+k}\right] &= \frac{1}{1+r_0^{I+k}}E_0^{Q(I+k)}\left[\frac{r^{13}_{I+k}}{1+r^{13-k}_{I+k}}\right] \\
&= \frac{1}{1+r_0^{I+k}}\left\{E_0^{Q(I+k)}\left[\frac{1}{1+r^{13-k}_{I+k}}\right]E_0^{Q(I+k)}\left[r^{13}_{I+k}\right]+cov_0^{Q(I+k)}\left(\frac{1}{1+r^{13-k}_{I+k}},r^{13}_{I+k}\right)\right\} \\
&= \frac{1}{1+r_0^{I+k}}\left[V^{f,13-k}_{0,I+k}\right]E_0^{Q(I+k)}\left[r^{13}_{I+k}\right]+\frac{1}{1+r_0^{I+k}}cov_0^{Q(I+k)}\left(\frac{1}{1+r^{13-k}_{I+k}},r^{13}_{I+k}\right) \\
&= \frac{E_0^{Q(I+k)}\left[r^{13}_{I+k}\right]}{1+r_0^{I+13}}+\frac{1}{1+r_0^{I+k}}cov_0^{Q(I+k)}\left(\frac{1}{1+r^{13-k}_{I+k}},r^{13}_{I+k}\right).
\end{aligned}$$

With

$$E_0^{Q(I+k)}\left[r^{13}_{I+k}\right]=E_0^{Q(I+k)}\left[\frac{1}{V^{13}_{I+k}}\right]-1,$$

a second-order Taylor-approximation around $V^{13}_{I+k}=V^{f,13}_{0,I+k}$ yields

$$E_0^{Q(I+k)}\left[\frac{1}{V^{13}_{I+k}}\right]\cong\left(1+r^{f,13}_{0,I+k}\right)\left[1+\left(1+r^{f,13}_{0,I+k}\right)^2 var_0\left(\frac{1}{1+r^{13}_{I+k}}\right)\right].$$

Replacing $E_0^{Q(I+k)}\left[r^{13}_{I+k}\right]$ by this expression gives equation 4 in the main text.

Appendix B: Pricing FRNs with money-in-the-utility

Start with the price of a FRN

$$\begin{aligned} q_t &= \beta E_t \frac{u'(C_{t+1}) + v'_{t+1}(M_{t+1})}{u'(C_t)} \left[\frac{1}{\beta E_t \frac{u'(C_{t+1}) + v'_{t+1}(M_{t+1})}{u'(C_t)}} - 1 + s + \lambda \right] + \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} (1-\lambda) q_{t+1} \\ &= 1 + (s + \lambda - 1) \beta E_t \frac{u'(C_{t+1}) + v'_{t+1}(M_{t+1})}{u'(C_t)} + \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} (1-\lambda) q_{t+1}, \end{aligned}$$

update

$$q_{t+1} = 1 + (s + \lambda - 1) \beta E_{t+1} \frac{u'(C_{t+2}) + v'_{t+2}(M_{t+2})}{u'(C_{t+1})} + \beta E_{t+1} \frac{u'(C_{t+2})}{u'(C_{t+1})} (1-\lambda) q_{t+2},$$

so that

$$\begin{aligned} q_t &= 1 + s \sum_{k=0} (1-\lambda)^k \beta^{1+k} E_t \frac{u'(C_{t+1+k}) + v'_{t+1+k}(M_{t+1+k})}{u'(C_t)} \\ &\quad - (1-\lambda) \sum_{k=0} (1-\lambda)^k \beta^{1+k} E_t \frac{u'(C_{t+1+k}) + v'_{t+1+k}(M_{t+1+k})}{u'(C_t)} [\kappa_{t+k}], \end{aligned}$$

with

$$\kappa_{t+k} = 1 - \beta^{1+k} E_t \frac{u'(C_{t+1+k})}{u'(C_t)} / \beta^{1+k} E_t \frac{u'(C_{t+1+k}) + v'_{t+1+k}(M_{t+1+k})}{u'(C_t)}.$$

Setting $q_t = 1$, the spread is

$$s_t = (1-\lambda) \frac{\Phi_t(\{\kappa_t\})}{\Phi_t(\{1\})},$$

with the annuity operator

$$\Phi_t(\{x_t\}) \equiv \sum_{k=0} (1-\lambda)^k \beta^{1+k} E_t \frac{u'(C_{t+1+k}) + v'_{t+1+k}(M_{t+1+k})}{u'(C_t)} x_{t+k},$$

and

$$\kappa_{t+k} = 1 - \beta^{1+k} E_t \frac{u'(C_{t+1+k})}{u'(C_t)} / \beta^{1+k} E_t \frac{u'(C_{t+1+k}) + v'_{t+1+k}(M_{t+1+k})}{u'(C_t)}.$$