Cost, Yield, and Validator Choice: A Dynamic Equilibrium for PoS

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Abstract

A dynamic equilibrium model embedding a staking decision with fixed and variable costs admits a closed-form cutoff for solo participation. Comparative statics in the staking yield show that solo staking may increase or decrease: the result depends on the elasticity of marginal utility of tokens. When this elasticity is sufficiently low, lowering the yield can raise the share of solo validators.

1 Introduction

Proof-of-stake (PoS) blockchains rely on a set of independently operated validators. When a large share of stake is concentrated in a single exchange or liquid-staking token, that intermediary can manipulate the chain—or be coerced into doing so. Thus, every PoS chain must address how to maintain a sufficiently diverse validator set.

With permissionless staking, policies targeting validator characteristics such as size are not straightforward. Given that solo stakers face fixed costs, reducing this cost can help. In this note, I explore the role of the staking yield in influencing the choice between solo staking and intermediated staking.

A priori, a higher staking yield increases all types of staking and helps solo stakers overcome their fixed costs, which should raise the number of solo validators. However, a higher staking yield also increases dilution, potentially lowering the token's value and thus diminishing solo stakers' ability to cover their fixed costs.

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In this note, I embed a choice between solo and intermediated staking into the basic proof-of-stake model of Jermann (2024). This dynamic equilibrium model is simple enough to admit a closed-form solution. As the analysis shows, whether solo staking increases or decreases when the staking yield is lowered depends crucially on the elasticity of the marginal utility of tokens with respect to the value of tokens used, equivalently the elasticity of the demand for money. If the marginal utility is sufficiently unresponsive or the money demand sufficiently elastic, a lower staking yield can lead to an increase in solo staking.

2 Model

The model embeds a static choice between two types of staking into the dynamic model presented in Jermann (2024).

2.1 Staking choice

Assume a given amount of stake D_{t+1} in units of tokens to be allocated at time t between

$$\begin{aligned} \text{intermediated staking} &: D^i_{t+1}, \text{ and} \\ \text{solo staking} &: D^s_{t+1}. \end{aligned}$$

Solo staking incurs a fixed cost W USD per period (e.g., monthly), covering hardware outlays and the time and effort required to learn and operate a validator. The variable cost is proportional to the stake: τ for intermediated staking and w for solo staking. We assume $\tau > w$, because intermediated staking includes additional overhead—custodial hardware, customer support, and administrative operations—that solo staking's variable cost excludes. Possibly, w is very low, which would not change the conclusions derived below.

Marginal returns, $(1 + y_{t+1} - \tau)$ and $(1 + y_{t+1} - w)$ are linear, in the amount staked which implies that the optimal policy is to either allocate the entire stake to intermediated staking, or, if the fixed cost can be amortized, to solo staking. This can be summarized in a participation constraint for solo staking

$$\beta p_{t+1} D_{t+1} (1 + y_{t+1} - w) - W - p_t D_{t+1} > \beta p_{t+1} D_{t+1} (1 + y_{t+1} - \tau) - p_t D_{t+1}.$$

The left side is the expected profit from solo staking including the fixed cost, p_t is the USD price of the token and β is the discount factor (with $1/\beta$ the required rate of return). The right side is the expected profit from intermediated staking. This simplifies to

$$W < (\tau - w)\,\beta p_{t+1} D_{t+1}. \tag{1}$$

Intuitively, solo staking is the choice if the fixed cost can be overcome by the variable cost difference. This holds for low enough fixed cost, high enough variable cost difference, and high enough value staked.

2.2 Staking choice embedded in dynamic model

To maintain the tractability of the basic model in Jermann (2024), I impose the following structure. I assume that solo staking requires some skill; only one member of each representative household is an expert who has this skill. The representative household is resigned to using intermediated staking, but delegates staking to the expert. The expert decides between intermediated and solo staking and can keep any profits from staking solo. This two-stage setup is needed because of the linearity of the staking decision. Without it, there is no interior equilibrium. Moving to a concave individual staking choice would be straightforward but would require the introduction of another utility function and result in a loss of tractability.

Under the made assumptions, the dynamic part of the model is unchanged except for the variable staking cost for intermediated staking. The household problem is given by

$$\max_{\left\{M^U_{t+1+j}, D_{t+1+j}, C_{t+j}\right\}} \sum_{j=0}^{\infty} \beta^j \left[C_{t+j} + v\left(p_{t+j} M^U_{t+j}\right)\right]$$

subject to

$$C_t + p_t D_{t+1} + p_t M^U_{t+1} = p_t D_t \left(1 + y_t - \tau\right) + p_t M^U_t + Y_t,$$

with the aggregate money supply equation

$$M^U_{t+1} + D_t = M^U_t + D_t \left(1 + y_t\right).$$

Equilibrium optimality conditions are unchanged except for the staking first-order condition which includes τ

$$p_t = \beta \left(1 + y_{t+1} - \tau\right) p_{t+1},$$

$$p_t = \beta \left(1 + v'\left(p_{t+1} M^U_{t+1}\right)\right) p_{t+1}.$$

Assuming a constant staking yield, $y_{t+1} = y$ for $t > 0$, the solution for the staking ratio becomes

$$d_t = \frac{\beta \left(1 + y - \tau\right) - 1}{y}. \tag{2}$$

With this solution, we can evaluate the participation constraint for solo staking

$$W < \left(\tau - w\right) \beta p_{t+1} D_{t+1} \equiv W^*$$

where W^* is the cutoff level for the fixed cost below which solo staking is chosen.

3 Comparative statics

We are interested in how solo staking responds to a change in the staking yield y. In the model described so far, all stake is either intermediated or solo, depending on the parameter values. By computing the derivative of the cutoff with respect to y, we can predict whether the equilibrium shifts toward—or away from—solo staking. A richer analysis, shown below, introduces a distribution of fixed costs to trace how each staking mode adjusts.

Rewrite the definition of the cutoff as

$$\begin{aligned} W^* &= \left(\tau - w\right) \beta p_{t+1} M_{t+1}^U \frac{D_{t+1}}{M_{t+1}^U} \\ &= \beta \left(\tau - w\right) v'^{-1} \left(y - \tau\right) \frac{d}{1 - d}. \end{aligned}$$

The inverse marginal utility, $v'^{-1}\left(y - \tau\right)$, is decreasing in y while the equilibrium staking share ratio, $d/\left(1 - d\right)$, is increasing in y. Intuitively, the key is in the slope of the marginal utility which determines the price impact of the change in y. If the marginal utility is relatively unresponsive, then the equilibrium price will change by more. For instance, a decline in y with an unresponsive marginal utility will produce a large price increase so that the cutoff level increases which makes solo staking more likely. That is, a negative derivative $\frac{\partial W^*}{\partial y} < 0$ leads to more solo staking following a decline in y.

To formalize this mechanism, assume the marginal utility is represented by $v'\left(p_{t+1} M_{t+1}^U\right) = \left(p_{t+1} M_{t+1}^U\right)^{-\zeta}$, with $\zeta > 0$ the elasticity of the marginal utility with respect to the value of used tokens. After some algebra (see the Appendix), the following is derived.

Proposition 1

$$\frac{\partial W^*}{\partial y} = \frac{\beta \left(\tau - w\right)}{\left(1 - d\right) \left(y - \tau\right)^{1/\zeta}} \left[\frac{1 - \beta \left(1 - \tau\right)}{\left(1 - d\right) y^2} - \frac{1}{\zeta} \frac{d}{\left(y - \tau\right)}\right] \tag{3}$$

which implies that for small enough ζ, we have $\frac{\partial W^*}{\partial y} < 0$, and for large enough ζ, we have $\frac{\partial W^*}{\partial y} > 0$, assuming that $\beta(1 + y - \tau) - 1 > 0$ which is required for $d > 0$.

3.1 Numerical example

Assume $d = 0.3$, $y = 0.03$, and $\tau = 0.005$, values that approximately reflect Ethereum's staking environment in 2025. Based on Equation (2), this implies a discount factor of $\beta = 0.9844$. With these parameter values, the term in brackets becomes

$$\left[\frac{1 - \beta(1 - \tau)}{(1 - d)\, y^2} - \frac{1}{\zeta}\frac{d}{(y - \tau)}\right] = \left[32.6 - \frac{12}{\zeta}\right].$$

For $\zeta < 12/32.6 = 0.37$, the derivative $\frac{\partial W^*}{\partial y} < 0$, and a decline in y would increase the proportion of solo staking in total staking.

To clarify the role of the parameter ζ, combine the two first-order conditions of the model as

$$p_{t+1} M^U_{t+1} = (y_{t+1} - \tau)^{-1/\zeta}. \tag{4}$$

This can be interpreted as money demand function of the foregone opportunity cost from staking. Take logs and differentiate

$$\frac{\partial \ln p_{t+1} M^U_{t+1}}{\partial \ln (y_{t+1} - \tau)} = -\frac{1}{\zeta}.$$

That is, $1/\zeta$ is the elasticity of money demand with respect to the foregone staking yield.

This can be compared, for instance, to the *log-log* specification of the money demand in Lucas (2000)

$$m(r_t) = A r_t^{-\eta},$$

where m stands for real money balances and r_t the nominal interest rate. In Lucas (2000), the best-fitting value is about $\eta = 0.5$, which would correspond to $\zeta = 2$. This estimate is based on the dollar economy, it is not clear how relevant it is for a blockchain economy say like Ethereum. Jermann (2021) estimates money demand in a related but also not directly comparable model to be more elastic for BTC and ETH than for fiat currencies during hyperinflations.

The simple estimation approach used by Lucas is not easily extended to the crypto context. The deterministic model considered here is an approximation for a stochastic setting. In the stochastic model, Equation (4) holds in expectation and includes additional variables such as a stochastic discount factor and adoption shocks. Given the extreme volatility in crypto prices, using Equation (4) to estimate ζ based on realizations of the two variables – $p_{t+1}M_{t+1}^{U}$ and y_{t+1} – alone, would be challenging due to sampling uncertainty and the absence of the other variables in the stochastic counterpart. A serious quantitative assessment would explicitly need to take into account uncertainty, possibly by estimating a fully specified stochastic model.

4 Extension

Assuming heterogenous fixed costs distributed according to a density function $f(W)$ would give us the aggregate amounts for solo staking and intermediated staking as

$$D_{t+1}^{s} = \int_{0}^{W_{t+1}^{*}} D_{t+1} f(W)\, dW \text{ and } D_{t+1}^{i} = \int_{W_{t+1}^{*}}^{\infty} D_{t+1} f(W)\, dW.$$

In this more general case, changing y changes the cutoff W_{t+1}^{*}, which directly maps into the proportion of stake that is solo staked vs intermediated. The absolute amounts also depend on the change in D_{t+1}.

References

Jermann, Urban, 2021, Cryptocurrencies and Cagan's Model of Hyperinflation, Journal of Macroeconomics 69, 103340

Jermann, Urban, 2024, Optimal Issuance for Proof-of-Stake Blockchains, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4971255

Lucas, Jr., R.E, 2000, Inflation and Welfare, Econometrica, 68: 247-274. https://doi.org/10.1111/1468-0262.00109

5 Appendix

5.1 Proposition 1

Algebra for the derivative of

$$W^* = \beta(\tau - w)\, v'^{-1}(y - \tau)\frac{d}{1-d}$$

with respect to y.

Preliminary calculations,

$$\frac{\partial d}{\partial y} = \frac{1 - \beta(1-\tau)}{y^2}$$

$$\begin{aligned}
\frac{\partial d/(1-d)}{\partial y} &= \frac{\frac{\partial d}{\partial y}(1-d) - \frac{\partial(1-d)}{\partial y}d}{(1-d)^2} = \frac{\frac{\partial d}{\partial y}(1-d) + \frac{\partial d}{\partial y}d}{(1-d)^2} \\
&= \frac{\frac{\partial d}{\partial y}}{(1-d)^2} = \frac{1-\beta(1-\tau)}{(1-d)^2 y^2}.
\end{aligned}$$

With the assumed functional form for utility we have

$$\begin{aligned}
v'\left(pM^U\right) &= \left(pM^U\right)^{-\zeta} = y, \\
v'^{-1}(y) &= y^{-1/\zeta}, \text{ and} \\
\frac{\partial v'^{-1}}{\partial y} &= -\frac{1}{\zeta}y^{-1/\zeta - 1}.
\end{aligned}$$

Bringing this together starting from

$$W^* = \beta(\tau - w)\, v'^{-1}(y - \tau)\frac{d}{1-d},$$

$$\begin{aligned}
\frac{\partial W^*}{\partial y} &= \frac{\beta(\tau - w)}{(y-\tau)^{1/\zeta}}\left[-\frac{1}{\zeta}\frac{1}{(y-\tau)}\frac{d}{1-d} + \frac{\partial d/(1-d)}{\partial y}\right] \\
&= \frac{\beta(\tau - w)}{(1-d)(y-\tau)^{1/\zeta}}\left[\frac{1-\beta(1-\tau)}{(1-d)y^2} - \frac{1}{\zeta}\frac{d}{(y-\tau)}\right].
\end{aligned}$$