Tokenomics: Optimal Monetary and Fee Policies

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Abstract

We document properties of crypto monetary policies based on about 2,000 tokens: (1) Money growth rates decline with age and stabilize at 0.2% per month on average, with younger cohorts converging faster to the long-run growth rate; (2) Long-run money growth rates and convergence speeds are positively correlated in the cross-section; (3) Tokens widely held by retail investors have relatively low long-run money growth rates and convergence speeds. We present a dynamic model to determine the optimal token issuance and fee policies for issuers. Committing to low future money growth and fees increases profits, and the degree of commitment matters for the existence of equilibria. A Ramsey issuer who maximizes profits, after the initial period, makes choices that maximize the utility value of all tokens. We present a model with probabilistic commitment and show that issuers with high commitment choose low long-run money growth rates and fee ratios, and they reduce them slowly.

Keywords: Cryptocurrencies, money growth rates, fees, commitment.

JEL codes: E52, G32.

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1 Introduction

Blockchain technology, with Bitcoin as its first implementation, has spurred a wave of issuance of cryptocurrencies or tokens. According to CoinMarketCap, there are about 10,000 tokens outstanding as of March 2024 with a total market capitalization of about $3 trillion. Tokens are associated with a wide variety of projects and are issued in exchange of economic resources from those who expect to use a token. Tokens provide utility and rights associated with projects, and their valuation depends critically on the monetary policy of a token and the closely related policy for charging user fees. The design of these policies is paramount for project founders.

The first contribution of this paper is empirical. We provide a first summary of key features of the monetary policies of existing cryptocurrencies. Based on the histories of circulating supplies of about 2,000 tokens we establish three main facts. First, the average money growth rate declines with age and stabilizes at about 0.2% per month. For more recent cohorts of tokens, average money growth rates exhibit a faster convergence to their respective long-run levels. Second, we estimate the long-run money growth rate and the speed of convergence to it for each token. There is significant heterogeneity in the cross-section, with tokens varying in their long-run money growth rates and convergence speeds. We find these two metrics to be positively associated with each other. Third, tokens widely held by retail investors, measured by the proportion of wallet addresses holding less than 0.1% of the token's total circulating supply, exhibit on average both relatively low long-run money growth rates and a low convergence speeds.

The second contribution of the paper is theoretical. We derive the policies that are optimal from an issuer's perspective by building on classical monetary theory. Our theoretical results are helpful for interpreting the empirical patterns that we document, and they provide guidance for the upgrade of existing tokens and the design of new ones. In our analysis, the beliefs about future issuance and fee rates of a token are crucial in users' valuation, which in turn affects the seigniorage and fees that can be collected. Therefore, whether the issuer is able to commit to future policies takes center stage.

A blockchain can be viewed as a commitment technology. The Bitcoin network operates with a money supply rule that is essentially deterministic. From this perspective, Bitcoin offers perfect commitment. However, there is a non-zero, albeit small, probability of a future fork of Bitcoin that would operate under a different money issuance policy and that could displace the current version of Bitcoin. Ethereum has gone through several versions with different policies for issuance and fees. Many blockchains have more discretionary policies. For instance, MakerDAO's whitepaper allows for discretionary issuance of tokens in some circumstances, an option that was used after losses on their DAI stablecoin in March 2020. ApeCoin launched in 2022 has a total supply of 1 billion tokens which are unlocked over a 4 year period to different groups, including 62% to the "Ecosystem Fund" which through its governance organization has the discretion to allocate these funds. Our analysis covers this broad spectrum of commitment between rules and discretion.

We present a dynamic model where issuers optimally determine the supply of tokens and fee rates to maximize their profits from a project. Tokens provide utility to the users (money-in-utility) which is affected by an exogenous and stochastically-growing productivity level. The velocity of a token is determined through the trade-off between the transaction utility and some effort cost to users. Fees are assessed on transactions. By issuing new tokens and charging higher fees, issuers can extract profits but dilute the value of legacy tokens.

We first analyze the polar cases: no commitment and full commitment, represented as a Markov-perfect equilibrium and a Ramsey equilibrium, respectively. Without any commitment, there is no equilibrium with a positive token price if the issuer cannot charge fees. Fees create the credibility for tokens to be issued at a positive price because users will then expect a Markov-perfect issuer to optimally restrict token issuance in the future to collect fees on legacy token holders. With full commitment there is no upper limit to the value that can be created under the Ramsey policy, and there is no equilibrium both with and without fees. When we introduce a project maintenance cost that is convex in its size, interior equilibria exist with and without commitment. We show analytically that at steady state the project value is higher, and under a small maintenance cost the optimal money growth rate and fee charge are lower for the Ramsey issuer, reflecting the use of commitment to sustain the token value.

We show that a Ramsey issuer who maximizes profits, after the initial period, makes choices that maximize the utility value of all tokens. In other words, maximizing profits for the issuer is equivalent to maximizing the total value of a project. This highlights the fact that the commitment of the issuer determines how legacy token holders are treated. The result shows that studying the problem of a profit maximizing issuer with commitment can be a useful benchmark even for so-called public blockchains like Bitcoin or Ethereum that can be viewed as aiming to maximize their value to a broad set of stakeholders and not just to maximize profits for their founders.

Because the two polar cases have very different implications, we extend the analysis to an arguably more realistic case with partial commitment. Issuers are endowed with a probability that determines whether they stick to a given policy or reoptimize. We first consider a scenario where an issuer commits probabilistically to a constant money growth rate. With the commitment probability equal to 0, the model is identical to the Markov-perfect case (no commitment). If this probability equals 1, the token supply is expected to grow at a constant rate forever. We assume that issuers are free to change fees at any time. Despite the richness of the economic mechanisms, we can analytically solve for steady state values. The model predicts both the money growth rate and the fee income relative to market capitalization of a project (fee ratio) to be decreasing in the commitment probability.

We then consider a more general scenario where the issuer is able to commit probabilistically to a richer path of money growth rates. We allow the issuer upon a reoptimization shock to choose a geometric transition path for money growth rates characterized by a long-run money growth rate and a convergence speed. A path is executed as announced until another reoptimization shock arrives. First, we find gradually declining money growth rates to be optimal. Committing to low long-run money growth boosts the token price, which allows a relatively large issuance of tokens in the short run. However, the issuer finds it optimal to spread out the short-run issuance to multiple periods to avoid the initial issuance cost. As a result, the optimal issuance plan is consistent with the average behavior of money growths that we document empirically. Second, we find that an increase in the commitment probability reduces both the long-run money growth rate and the convergence speed. This is because with more commitment power, the issuer's commitment to low long-run money growth becomes more effective, and this provides the incentive to adopt a lower long-run growth rate and facilitates the use of promises to support the collection of short-run profits. It seems reasonable to assume that big stakeholders have the incentive or the power to change monetary policies for their own interests, implying low commitment. This can explain why high retail ownership is associated with a low long-run money growth and a low convergence speed in the data.

Our analysis also implies that the low long-run money growth rates typically observed in the data are indicative that issuers have a high commitment to their monetary policies. In the model, to justify a long-run money growth rate of less than 5% per year requires the commitment probability to be about at least 0.9 even for extreme parameter choices.

Our paper contributes to the nascent literature on the economics of cryptocurrencies. To the best of our knowledge this is the first study to empirically characterize the main properties of crypto monetary policies based on a large sample of tokens. This is also the first study to explicitly derive optimal policies jointly for issuance and fees, and our setup with probabilistic commitment is novel in the crypto context. Our model shares some elements with Cong, Li, and Wang (2022) who derive optimal token issuance in a Markov equilibrium and analyze its impact on financing investment. They also consider investment efficiency with exogenously specified rules for issuance and fees. D'Avernas, Maurin, and Vandeweyer (2022) study the problem of a monopolist platform that can earn seigniorage revenues by issuing a stablecoin. They study cases with full commitment and partial commitment. In their model, the stablecoin provides utility only if its price equals exactly the pegged value and they have a stationary equilibrium with full commitment. In their partial commitment case, there is no commitment to the stablecoin issuance policy but full commitment to other policy choices. In our partial commitment case, commitment to token issuance is probabilistic. Gryglewicz, Mayer, and Morellec (2021) study token-financed investment where tokens provide utility and are equity-like. Guennewig (2022) presents a model economy where firms issue private currencies that generate seigniorage and information about consumers to study competition among firms and the consequences for monetary policy. Other studies analyzing optimal financing through tokens are Catalini and Gans (2018), Garratt and Van Oordt (2021), and Li and Mann (2018). Fanti, Kogan, and Viswanath (2019) develop a model for valuing tokens in proof-of-stake payment systems based on given processes for fee income and money creation. Other aspects of the design of blockchains are analyzed by Hinzen, John, and Saleh (2019) and Mei and Sockin (2022).

There are large literatures on optimal (fiat) monetary policy and fiscal policy our paper is related to. Roberds (1987) and Debortoli and Nunes (2010) present frameworks with probabilistic commitment. Klein, Paul, Per Krusell, and Jose-Victor Rios-Rull (2008) derive optimal time-consistent policies based on first-order conditions. Paths implied by our issuer's first-order conditions are consistent with the Friedman rule, see for instance Cole and Kocherlakota (1998) and Ireland (2003). However, in our model there is no equilibrium with such paths.

The paper proceeds as follows. Section 2 documents properties of crypto monetary policies. Section 3 presents our model, together with the characterizations of the polar cases with and without commitment. Section 4 introduces the framework with probabilistic commitment and studies its properties. Section 5 concludes.

2 Facts

This section documents properties of the circulating money supplies of cryptocurrencies. Our main data is from CoinMarketCap. Stablecoins, wrapped tokens and liquid staking derivatives are excluded as their issuance is subject to very specific constraints. Tokens that are no longer active as of February 25, 2024, the time we retrieve the data, are also excluded. Monthly money supplies are computed as the averages of the daily reported values, and we drop coins with gaps in monthly supply. Our final sample for analysis includes 1,856 tokens with more than 12 months of growth rates of money supplies prior to December 31, 2023.

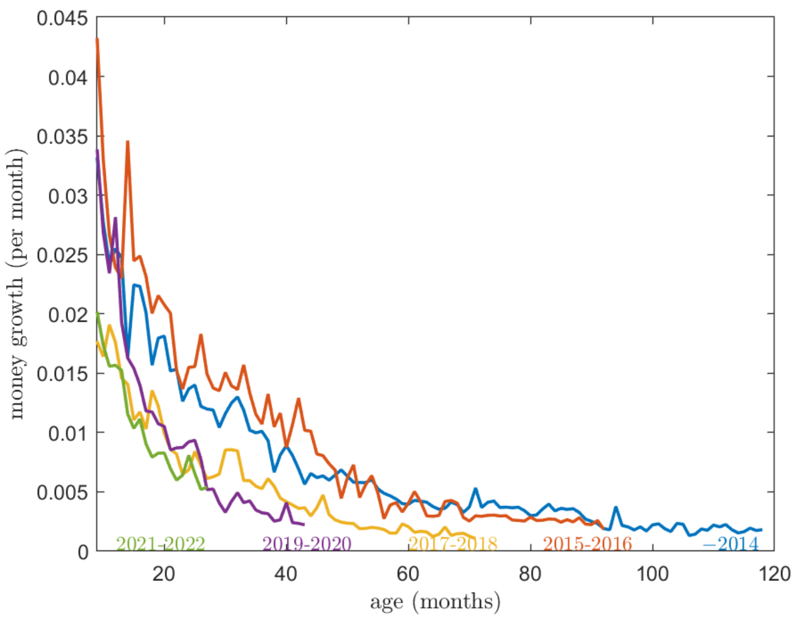

Figure 1: Average money growth rates by age and cohort. *Notes*: This figure averages monthly money growth rates across tokens by age and cohort. Tokens are divided into 5 cohorts by their starting year in the CoinMarketCap dataset: 2014 and earlier, 2015-2016, 2017-2018, 2019-2020, 2021-2022. For each cohort-age, top and bottom 10% of outliers are winsorized, and we report average growth rates unless more than 50% of the cohort are missing at that given age.

We first compute average monthly growth rates as a function of age. Tokens are grouped into cohorts by their starting year: 2014 and earlier (32 tokens), 2015-2016 (27 tokens), 2017-2018 (255 tokens), 2019-2020 (535 tokens), and 2021-2022 (1007 tokens). For each cohort-age, we winsorize at top and bottom 10% to remove the influence of outliers, and we report average growth rates unless more than 50% of the cohort are missing in that given age. As shown in Figure 1, average monthly money growth rates are typically declining with age and stabilizing at about 0.2%. Younger cohorts have generally lower growth rates for a given age relative to the older cohorts, which implies a faster convergence to a long-run growth level.

There is significant heterogeneity behind these averages. Based on the plots for the averages, we consider a representation for the money growth rate of a token that features geometrically declining rates:

$$g_t = \lambda g + (1 - \lambda)\, g_{t-1}, \tag{1}$$

where λ and g measure the convergence speed and the long-run gross growth rate, respectively. In addition to approximately capturing average growth rates, this recursive representation is tractable in our dynamic model later.[1] For each token, we estimate λ and g by least squares.

Histograms of the λ and $g-1$ are presented in the upper panel of Figure 2. To enhance the informativeness of the plots, outliers above and below the extremes in the plot are excluded. As the top-left panel shows, there are many tokens with long-run growth rates close to 0, and there is a long tail of positive growth rates. The top-right panel shows some concentration of tokens around $\lambda = 1$, with a long left tail going down to 0. There are 145 tokens for which $\lambda = 1$, and all of them have an estimated long-run growth rate of 0. That is, there is a group of tokens which after some initial issuance have had constant money supplies. Generally, these tokens are small and belong to more recent cohorts.[2] In the lower panel, we focus on a subset of tokens for which we have data on holding distribution in 2023, i.e., how many tokens are held by each wallet address. Our data is from IntoTheBlock, which verifies on chain the holding distribution. In this subset of 299 tokens where circulating supply can be more reliably measured, there is no longer a large concentration on $\lambda = 1$. The median long-run growth rates $g-1$ in the full sample and in the subsample with holding distribution data are 0.0135 and 0.0200, respectively; The median convergence speeds λ are respectively 0.7888 and 0.8011.

[1] The alternative of including a trend function of age would be less amenable for a theoretical analysis.

[2] This group has a median market capitalization of \$555,950 and a median age of 31 months on December 2023. In our full sample, the median market capitalization is \$4,674,200 and the median age is 36 months.

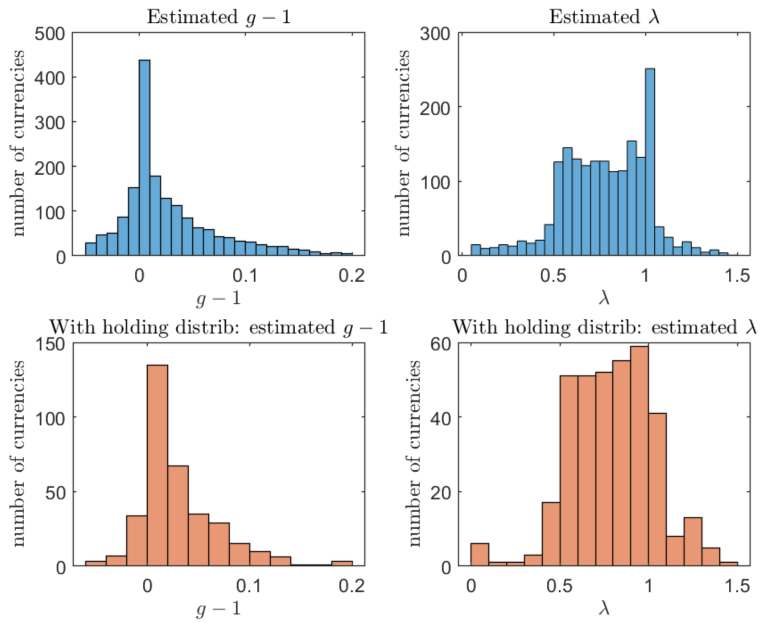

Figure 2: Long-run money growth $g-1$ and convergence speed λ in the cross section. *Notes*: This figure shows the distribution of estimated $g-1$ and λ in the full sample (upper panel) and the subsample with holding distribution data in 2023 (lower panel). 156 outliers (145 regarding g and 11 regarding λ) above and below the extremes in the figure are eliminated.

Figure 3 plots estimated long-run growth rates $g-1$ against convergence speeds λ for tokens with $g \in (0.95, 1.2)$ and $\lambda \in (0, 1]$. The upper panel presents the full sample and the lower panel presents the subsample with non-missing holding distribution. The scatter plots display a positive relation between the two, which are emphasized by the regression line in red. The estimated slope coefficients are statistically significant with p-values of 1% and 8.2% for the full sample and the samller sample, respectively. To shed light on the positive association in Figure 3, we next show that tokens with a high level of retail holding have both a low long-run money growth rate $g-1$ and a low convergence speed λ. Retail holding is defined as the fraction of wallet addresses that hold less than 0.1% of a token's circulating supply. We first sort tokens by their levels of retail holding into 4 equal groups, and report by groups the mean and standard error of the mean (SEM) of λ and g.

Figure 4 presents the results. In particular, we find that the group with the highest level of retail holding has a particularly low λ and a low $g-1$. Formally, Table 1 regresses $g-1$ and λ on retail holdings while controlling for log market capitalization, ratio of 24-hour

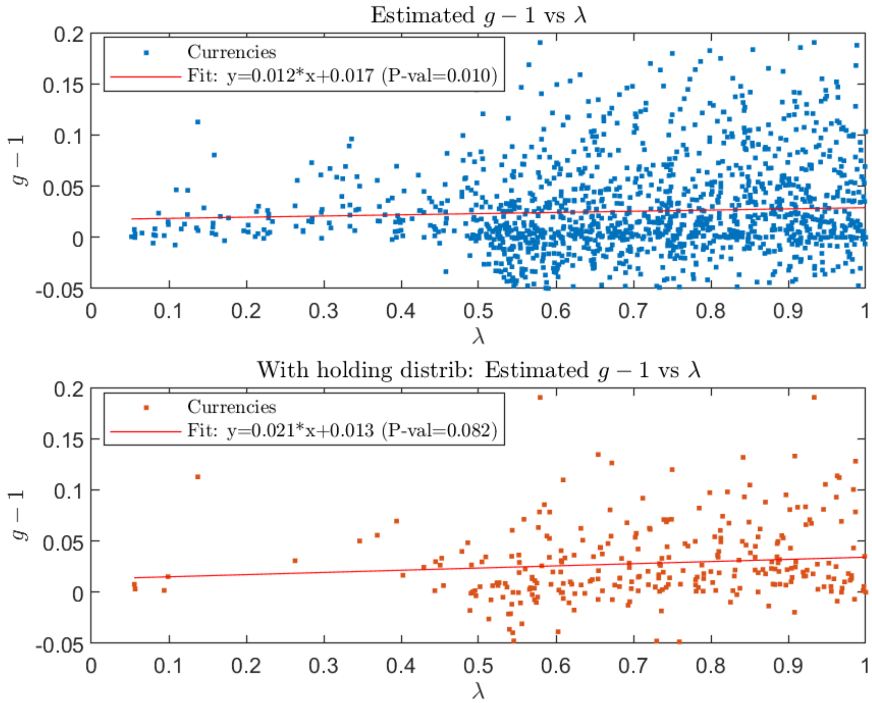

Figure 3: Relationship between long-run money growth g and convergence speed λ. *Notes*: We scatter plot estimated g and λ for all tokens with $g \in (0.95, 1.2)$ and $\lambda \in (0, 1]$ in the full sample (upper panel) and the subsample with holding distribution data in 2023 (lower panel). Red line represents the linear fit of the relationship between g and λ with 95% confidence intervals.

trading volume to market capitalization, and log 1 plus the number of currency pairs, all averaged over the last 12 months, as well as log age (as of December 2023).[3] We find for both a negative relationship with at least a 5% statistical significance. Since big stakeholders have either the incentive or in some cases the ability, e.g., for governance tokens, to change monetary policies for their own interests, one might consider the holding by dispersed retail investors as a reasonable proxy for commitment power. Our model developed below suggests a possible economic interpretation of this finding—that is, issuers that have a high ability to make credible promises optimally select slow convergence speed and low money supply growth in the long run.

[3]We winsorize the ratio of 24-hour trading volume to market capitalization at top and bottom 10% to remove outliers.

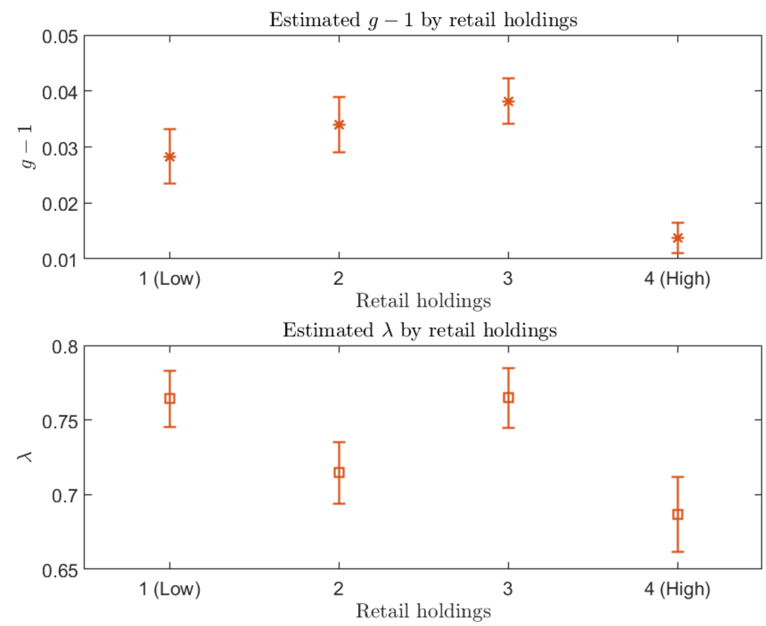

Figure 4: Retail holdings and monetary policy. *Notes*: We sort tokens with $g \in (0.95, 1.2)$ and $\lambda \in (0, 1]$ by their levels of retail holding into 4 equal groups and plot the mean ($*$ and $\square$) and the standard error of means (error bars) of g and λ for each group.

3 Model

Now we develop a dynamic model in which a crypto issuer chooses monetary policy and user fees optimally taking into account the responses of token users. Users choose money holdings and velocity taking as given the processes for token prices and fees. In this environment, equilibrium outcomes depend on the issuer's ability to commit to future policies. In this section, we consider two polar cases: a Markov-perfect issuer without commitment and a Ramsey issuer who can commit to arbitrary future policies.

3.1 Setup

Users are assumed to receive utility from their token holdings. The utility depends on the value of the tokens and the velocity with which they are used. Users start the period with token holdings from the previous period M_t and choose the current velocity ω_t and money

	$g-1$		λ	
	(1)	(2)	(3)	(4)
retail holdings	-0.054***	-0.052***	-0.323***	-0.273**
	(0.012)	(0.013)	(0.119)	(0.113)
log*(age)*		-0.020***		-0.004
		(0.006)		(0.031)
log*(mkt_cap)*		0.001		-0.010
		(0.001)		(0.007)
24h_volume		0.011		-0.020
		(0.017)		(0.114)
log*(1+#mkt_pairs)*		0.005**		0.003
		(0.003)		(0.014)
R^2	0.033	0.126	0.048	0.061
Obs	299	299	299	299

Table 1: Retail holdings and monetary policy: regression analysis. The table estimates the relationship between monetary policy parameters and retail holdings. Columns (1) and (2) use estimated long-run growth rates $g-1$ as dependent variable; Columns (3) and (4) use estimated convergence speeds λ as dependent variable. Robust standard errors are reported in parentheses. */**/*** denotes 10%/5%/1% statistical significance.

holdings for next period. Their problem is

$$\max_{C_{t+j},M_{t+j+1},\omega_{t+j}} \mathbf{E}_t \sum_{j=0}^{\infty} \beta^j \left[C_{t+j} + \frac{A_{t+j}^{\alpha}}{1-\alpha}\left(p_{t+j}M_{t+j}\omega_{t+j}\right)^{1-\alpha} - A_t\frac{\eta}{2}\omega_t^2\right],$$

with consumption C_t given by:

$$C_t + p_t M_{t+1} = p_t M_t - \omega_t F_t.$$

Users take as given the token price p_t and the fee charge per transaction, or the fee rate, F_t. Specification of fee costs scale with the velocity to capture the standard practice that fees are assessed on transactions as opposed to transaction amounts. Increasing velocity incurs a quadratic cost scaled by $\eta > 0$. This cost reflects the effort and resources needed to manage tokens more intensively. Productivity is specified as $A_{t+1} = A_t\gamma\exp(z_{t+1})$ where

$\gamma \geq 1$ and $z_{t+1} = \rho z_t + \epsilon_{t+1}$ with ϵ_{t+1} i.i.d. normally distributed. The effort cost for velocity is scaled by productivity to allow for balanced growth. Parameters $\alpha \in (0,1)$ and β govern the shape of money-in-utility and time preferences, respectively. We assume that $\beta\gamma < 1$ to keep discounted utility finite.

From the first-order condition, the optimal velocity decision is given by

$$(p_t M_t)^{1-\alpha} \left(\frac{\omega_t}{A_t}\right)^{-\alpha} - \eta A_t \omega_t = F_t. \tag{2}$$

The left-hand side (LHS) of Equation (2) is decreasing in ω_t, which implies that a higher fee rate reduces velocity. Meanwhile, higher real balances, $p_t M_t$, also imply a higher velocity because of an increase in the marginal benefit of velocity.

The optimal token demand generates the pricing schedule for tokens, which reflects the dynamic nature of the problem. In particular, the first-order condition with respect to M_{t+1} is given by

$$p_t = \beta \mathbf{E}_t \left[p_{t+1} + (p_{t+1}\omega_{t+1})^{1-\alpha} \left(\frac{M_{t+1}}{A_{t+1}}\right)^{-\alpha} \right], \tag{3}$$

where the current price depends on the price and velocity next period. The importance of the issuer's ability to commit is apparent through the role of these future values that depend on future policies.

The token issuer chooses how many new tokens to issue and the fee rate to maximize the value of the project for itself. Throughout the paper, we analyze cases where the amount of outstanding tokens is always strictly positive, i.e., $M_t > 0,\ \forall t \geq 0$. We consider two cases here—one with commitment (Ramsey) and one without (Markov-perfect). A Ramsey issuer commits to state-contingent monetary and fee policies at time 0, even if they are no longer optimal ex post. The problem is given by

$$[\text{Ramsey}] : \max_{\{M_{t+1}, F_t, p_t, \omega_t\}_{t=0}^{\infty}} \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \left[p_t (M_{t+1} - M_t) + \omega_t F_t \right], \tag{4}$$

subject to an initial M_0, constraints (2) and (3) for all $t \geq 0$.

A Markov-perfect issuer optimizes token supply and fee rate period by period, with the problem given in a recursive form as

$$\text{[Markov-perfect]} : V(M, A, z) = \max_{M', F} \; p(M' - M) + \omega F + \beta \mathbf{E}_{z'|z} V(M', A', z'), \qquad (5)$$

where constraints (2) and (3) imply that

$$p = \beta \mathbf{E}_{z'|z} \left[p' + (p'\omega')^{1-\alpha} \left(\frac{M'}{A'} \right)^{-\alpha} \right],$$

and

$$(pM)^{1-\alpha} \left(\frac{\omega}{A} \right)^{-\alpha} - \eta A \omega = F,$$

with $A' = A \exp(z')$. Price and velocity are functions of choices M' and F that are taken as given by the issuer, that is $p = p(M', A, z)$ and $\omega = \omega(p, F, M, A)$.

The Markov-perfect issuer takes as given the optimal behavior of its future self, that is, $M'' = h_M(M', A', z')$ and $F' = h_F(M', A', z')$ with $h_M(.)$ and $h_F(.)$ being optimally policies that solve (5). These future decisions affect token price p' and velocity ω' tomorrow, and in turn the token price p today. This is the key distinction from the Ramsey case where the issuer can choose optimal policies once and for all, and can guarantee (commit) that they are executed as such.

Definition 1 *The Ramsey optimal policies are defined by a state-contingent allocation plan* $\{M_{t+1}(A^t, z^t), F_t(A^t, z^t), p_t(A^t, z^t), \omega_t(A^t, z^t)\}_{t=0}^{\infty}$ *that maximizes the present value of seigniorage and fees at* $t = 0$ *given initial money stock* M_0 *subject to users' first-order conditions, where* $\{A^t, z^t\}$ *are histories of shocks up to time* t*. The Markov-perfect optimal policies are defined by a set of time-invariant functions* $M'(M, A, z)$*,* $F(\omega, p, M, A)$*,* $p(M', M, A, z)$ *and* $\omega(M, A, z)$ *that maximizes the present value of seigniorage and fees given states* M, A *and* z *subject to users' first-order conditions.*

3.2 Optimal policies without commitment

Our model allows for growth in productivity, i.e., $\gamma \geq 1$, and several equilibrium variables can grow without bound. We scale variables by productivity so that model solutions can be represented by variables that can be stationary, which greatly simplifies the analysis.

Specifically, define three key variables of interest—(scaled) real balances b, money growth x and fee rate f—as follows

$$b \equiv p\frac{M}{A_{-1}}, \qquad x \equiv \frac{M'/A}{M/A_{-1}}, \qquad f \equiv \frac{F}{A}.$$

Consistent with classical monetary theory, the money supply valued in terms of the numeraire, pM, can be labelled as *real balances*. From the crypto perspective this would be labelled as the *market capitalization*. We will use both terms interchangeably.

The problem of the Markov-perfect issuer can be reformulated as[4]

$$v(z) = \max_{x,\omega} \; b(x,z)\left[x - \frac{1}{\gamma \exp(z)}\right] + \omega f(x,z,\omega) + \beta\gamma \mathbf{E}_{z'|z} \exp(z') v(z')$$

where real balance is

$$b(x,z) = \frac{1}{x}\beta \mathbf{E}_{z'|z}\left\{b(h_x(z'),z') + \left[b(h_x(z'),z')h_\omega(z')\right]^{1-\alpha}\left[\gamma \exp(z')\right]^{\alpha}\right\}, \tag{6}$$

and fee rate is

$$f(x,z,\omega) = \left[\frac{b(x,z)}{\gamma \exp(z)}\right]^{1-\alpha} \omega^{-\alpha} - \eta\omega, \tag{7}$$

with $x' = h_x(z')$ and $\omega' = h_\omega(z')$ being policy functions of the future Markov-perfect issuer that the current one takes as given. In this reformulated problem, real balances b have replaced the money supply and the token price and this shows that the Markov-perfect issuer does not face any pre-determined state variable. Even though the money supply M is pre-determined, the issuer can determine a token price p (and thus real balances pM) freely so that the effect of history can be erased.

Each period, the issuer chooses (scaled) money growth x and velocity ω. Given these two choices, real balances b and fee rate f are pinned down endogenously by users' first-order conditions. Plug (7) into the objective and then take the derivative with respect to ω, the

[4] Appendix C provides a proof of the linearity with respect to productivity.

optimal policy for velocity satisfies

$$(1-\alpha)\left[\frac{b}{\gamma\exp(z)}\right]^{1-\alpha}\omega^{-\alpha-1}-2\eta=0. \tag{8}$$

The optimal velocity choice balances between the marginal benefit from facilitating the transaction of users and the marginal cost from managing token usage. The fee rate is set to achieve such an optimal velocity, that is, Equation (7) will determine the fee rate once optimal ω is pinned down.

The first-order condition for money growth x is given by

$$1-(1-\alpha)\left[\frac{b}{\gamma\exp(z)}\right]^{-\alpha}\omega^{1-\alpha}=0. \tag{9}$$

This reflects the following trade-off. The first term in (9) captures the marginal benefit, that is, raising money growth x helps the issuer collect a larger seigniorage. Even though the price of tokens falls, the issuer still receives a net gain because part of the price drop will be borne by existing users, i.e., $-\frac{1}{\gamma\exp(z)}\frac{\partial b(x,z)}{\partial x}>0$. The second term in (9) captures the marginal cost. New money issued can only be put into use in the future, and the amount of fee income this period depends on the current real balances. It is not optimal for the issuer to expropriate the existing token holders too aggressively because the price drop reduces the real balances and thus the fees that can be collected this period.

According to condition (8), when the issuer chooses an aggressive money growth that drives down real balance, it is optimal to reduce velocity at the same time. However, condition (9) implies that the reduced velocity in turn weakens the marginal cost of money growth, and this is because the same decrease in real balances under a lower velocity implies a smaller decrease in transaction amount and thus utility. When $\alpha>\frac{1}{3}$, the optimal reduction in velocity produces a mild effect and guarantees the overall marginal cost of money growth to be increasing. The trade-off for money growth is then well behaved.[5] In that case, Proposition 2 characterizes the steady state of the optimal Markov-perfect policies, that is, with current and future shock realizations set to $z=0$. Real balances, fee rates, and money growth rates are constant after being scaled by productivity. History does not matter. No matter what M_0 the issuer starts with, the system jumps right to the stationary point.

[5]Technically, while the second derivatives with respect to money growth x and velocity ω are always negative, the determinant of the Hessian matrix is positive if and only if $\alpha>\frac{1}{3}$.

Proposition 2 *For $\alpha > \frac{1}{3}$ there exists interior optimal policies for the Markov-perfect issuer which imply a growth-adjusted steady state where*

$$\frac{M'}{M} = \gamma\beta\frac{2-\alpha}{1-\alpha},$$
$$\frac{p'}{p} = \frac{1}{\beta}\frac{1-\alpha}{2-\alpha},$$
$$\frac{\omega F}{pM} = \frac{1}{2}\frac{1+\alpha}{1-\alpha},$$
$$\frac{pM}{A_{-1}} = \gamma\left(1-\alpha\right)^{\frac{2}{3\alpha-1}}\left(2\eta\right)^{\frac{\alpha-1}{3\alpha-1}}.$$

Proof. See Appendix A.1. ■

In the steady state, real balances pM are growing at the productivity growth rate γ. Money growth and price growth rates therefore satisfy $\frac{M'}{M}\frac{p'}{p} = \gamma$. The discount factor β is typically close to 1 so that $\beta\frac{2-\alpha}{1-\alpha} >> 1$, that is, the money supply is growing and the price is declining. Users accept this price decline because they value the utility of the project. As we see in more detail below, the absence of commitment leads to a highly inflationary token.

The following corollary shows formally that incorporating fees is crucial to get an interior optimum for the Markov-perfect issuer. As mentioned earlier, the intention to collect more fees restricts the issuer's money printing. Without it, the optimal monetary policy is to print an infinite amount of money each period, which drives real balances to zero.

Corollary 3 *Without fees and endogenous velocity, i.e., $\omega \equiv 1$ and $F \equiv 0$, a Markov-perfect issuer would like to increase money supply as much as possible.*

Proof. The first-order derivative with respect to x becomes $-\frac{1}{\gamma\exp(z)}\frac{\partial b(x,z)}{\partial x} = \frac{1}{\gamma\exp(z)}\frac{b(x,z)}{x} > 0$. ■

3.3 Optimal policies with commitment

The Lagrangian for the Ramsey issuer expressed in productivity-scaled terms is given by

$$\max_{\{x_t, f_t, \omega_t, b_t, \mu_t, \zeta_t\}_{t=0}^{\infty}} \mathbf{E}_0 \sum_{t=0}^{\infty} (\gamma\beta)^t \exp\left(\sum_{k=0}^{t} z_k\right) \left\{ b_t \left[x_t - \frac{1}{\gamma \exp(z_t)} \right] + \omega_t f_t \right.$$
$$+ \mu_t \left\{ \left[\frac{b_t}{\gamma \exp(z_t)} \right]^{1-\alpha} \omega_t^{-\alpha} - \eta\omega_t - f_t \right\}$$
$$\left. + \zeta_t \left\{ \frac{\beta}{x_t} \mathbf{E}_t \left[b_{t+1} + (b_{t+1}\omega_{t+1})^{1-\alpha} (\gamma \exp(z_{t+1}))^{\alpha} \right] - b_t \right\} \right\},$$

where μ_t and ζ_t are Lagrange multipliers of the fee and pricing constraints at time t, respectively.

Consider first velocity and fees. After some straightforward algebra, the equilibrium condition for ω_t for $t \geq 1$ is given by

$$\gamma\beta \exp(z_t) \left\{ (1-\alpha) \left[\frac{b_t}{\gamma \exp(z_t)} \right]^{1-\alpha} \omega_t^{-\alpha} - 2\eta\omega_t \right\}$$
$$+ \beta \mathbf{E}_{t-1} (1-\alpha) b_t^{1-\alpha} \omega_t^{-\alpha} (\gamma \exp(z_t))^{\alpha} = 0. \qquad (10)$$

The term in curly brackets also shows up in Equation (8) for the Markov-perfect issuer, that is, optimal velocity trades off between the value of transactions and the cost of cash management. However, a Ramsey issuer takes into account that velocity ω_t will have a dynamic effect on the real balance at $t-1$ because the token price takes into account future transaction value. In particular, choosing a larger ω_t increases real balance at $t-1$, which is captured by the last term.

Differentiate the Lagrangian with respect to b_t, and then substitute out ζ_t using the first-order condition with respect to x_t. For $t \geq 1$, we have

$$(1-\alpha) \left[\frac{b_t}{\gamma \exp(z_t)} \right]^{-\alpha} \omega_t^{1-\alpha} - 1 + \mathbf{E}_{t-1} \left\{ 1 + (1-\alpha) \left[\frac{b_t}{\gamma \exp(z_t)} \right]^{-\alpha} \omega_t^{1-\alpha} \right\}. \qquad (11)$$

Again, the first two terms echo what we have seen for the Markov-perfect issuer in (9), and the third term reflects Ramsey issuer's commitment. When picking real balance (and money growth) today, a Ramsey issuer takes into account that it will affect real balances faced by its past self. A larger real balance (a low money growth) today imply a larger real balance yesterday.

Importantly, the third term makes (11) strictly positive under constant z_t, which means that a Ramsey issuer can increase real balances for $t \geq 1$ without limit. This implies that there is no interior optimum and no equilibrium because the issuer could create infinite value. In particular, the Ramsey issuer can support a large real balance today by committing to a large real balance in the future. In contrast, the Markov-perfect issuer in the future will have the incentive to collect seigniorage and fees by hurting the value of tokens outstanding today, which therefore limits the real balance, or the market capitalization of the project, that can be achieved today. In other words, our result suggests the powerful effect of perfect commitment for a crypto project.

Proposition 4 *The optimal policies for the Ramsey issuer do not imply a finite growth-adjusted real balance.*

Proof. Equation (10) implies that $\omega_t = \left(\frac{1-\alpha}{\eta}\right)^{\frac{1}{1+\alpha}} \left[\frac{b_t}{\gamma \exp(z_t)}\right]^{\frac{1-\alpha}{1+\alpha}}$. Plugging it into Equation (11) yields $(1-\alpha)^{\frac{2}{1+\alpha}} \eta^{\frac{\alpha-1}{1+\alpha}} \left[\frac{b_t}{\gamma \exp(z_t)}\right]^{\frac{1-3\alpha}{1+\alpha}}$, which is strictly positive for positive b_t. ■

For paths implied by the first-order conditions that lead to infinite growth-adjusted real balances, the token return eventually equals the real interest rate, i.e., the pricing equation (3) implies that $\mathbf{E}_t \frac{p_{t+1}}{p_t} \to \frac{1}{\beta}$ as $\frac{p_{t+1} M_{t+1}}{A_t} \to \infty$. This is consistent with the Friedman rule, see for instance Cole and Kocherlakota (1998) and Ireland (2003). However, in our model there are no equilibria with such paths.

While the proposition highlights the absence of an interior optimum for the Ramsey issuer, there is another dimension to the challenges to solving this Ramsey problem. We briefly digress here to illustrate this point. One of the challenges of solving the Ramsey model compared to the Markov-perfect case is that the Ramsey model is essentially short one equation. Consider an example where money supplies are exogenously constrained. In this case, the Ramsey setup does not have a unique equilibrium price, while the Markov-perfect model does. A Markov-perfect equilibrium has the requirement that equilibrium prices depend only on the state variables and this effectively adds an equation so that the price level can be determined.

Specifically, assume that money supply M_t is constrained to grow at the productivity growth rate γ, so that $M_t = \gamma^t M_0$ for $t \geq 1$. This is in line with solving for a deterministic steady state. In the model without fees and endogenous velocity, the price equations in this case, with $A_0 = 1$, imply $\frac{p_t}{p_{t+1}} = \beta\left[\left(\frac{1}{p_{t+1}M_0}\right)^\alpha + 1\right]$. There are no other usable equations to solve for the Ramsey equilibrium; the first-order conditions for the prices each include a multiplier. Therefore, the price path in the Ramsey model cannot be determined unless there is an additional constraint (or boundary condition). The Markov-perfect equilibrium has an additional constraint. Namely, prices can only depend on the current state, which here implies that prices are constant. Therefore, the price equation in the Markov-perfect case can be solved for the unique price level p_{t+1} satisfying $1 = \beta\left[\left(\frac{\gamma}{p_{t+1}M_0}\right)^\alpha + 1\right]$. In the next section, the issuer's objective is modified so that the optimal policy for the Ramsey case implies stationary real balances (in productivity-adjusted terms) and this provides the boundary condition for the price equations.

3.4 Project maintenance costs

For an interior solution of the Ramsey problem, we now introduce a project maintenance cost that is increasing and convex in the project size. Maintaining a blockchain brings about a series of costs (transaction validations, code updates, etc). Security risks and regulatory exposures are no doubt increasing more than proportionally in size. We model this as a quadratic cost in real balances, which is subtracted from seigniorage and fee income.[6] While admittedly reduced-form, this allows us to get an explicit characterization of the role of commitment. It is assumed that the issuer's cash flow becomes

$$p_t(M_{t+1} - M_t) + \omega_t F_t - \frac{\xi}{2} A_t \left(\frac{p_t M_t}{A_t} - \bar{b}\right)^2, \tag{12}$$

with parameters $\xi > 0$ and $\bar{b} > 0$; the maintenance cost scales with productivity A_t to allow for balanced growth.

For the Markov-perfect issuer, the equilibrium condition that pins down growth-adjusted real balances b_t is given by

$$\xi\left[\frac{b}{\gamma\exp(z)} - \bar{b}\right] + 1 - 2^{\frac{\alpha-1}{1+\alpha}}(1-\alpha)^{\frac{2}{1+\alpha}}\eta^{\frac{\alpha-1}{1+\alpha}}\left[\frac{b}{\gamma\exp(z)}\right]^{\frac{1-3\alpha}{1+\alpha}} = 0, \tag{13}$$

[6] Alternatively, one could consider a satiation level for the money-in-utility function.

in which we have plugged in the optimal choice for velocity. For the Ramsey issuer, we have correspondingly for $t \geq 1$

$$\xi\left[\frac{b_t}{\gamma\exp(z_t)}-\bar{b}\right]-2(1-\alpha)^{\frac{2}{1+\alpha}}\eta^{\frac{\alpha-1}{1+\alpha}}\left[\frac{b_t}{\gamma\exp(z_t)}\right]^{\frac{1-3\alpha}{1+\alpha}}=0. \tag{14}$$

It is straightforward to see that for any given b_t, the LHS of (13) is larger than that of (14). For $\alpha > \frac{1}{3}$, the left hand side of both equations are increasing in b_t. Therefore, we know a Ramsey issuer will end up producing a larger b_t than a Markov-perfect issuer for $t \geq 1$. This is consistent with our previous results without maintenance costs where real balances were shown to be unboundedly large for a Ramsey issuer but finite for a Markov-perfect issuer. The convex maintenance cost eventually limits equilibrium real balances even for the Ramsey case.

The ratio of total fee income to total market capitalization, the fee ratio for short, for a Markov-perfect issuer is given by

$$\frac{\omega F}{pM}=2^{\frac{-2}{1+\alpha}}(1+\alpha)\frac{1}{1-\alpha}\left(\frac{1-\alpha}{\eta}\right)^{\frac{2}{1+\alpha}}\left[\frac{b}{\gamma\exp(z)}\right]^{\frac{1-3\alpha}{1+\alpha}},$$

and for Ramsey at $t \geq 1$ is given by

$$\frac{\omega_t F_t}{p_t M_t}=\alpha\frac{1}{1-\alpha}\left(\frac{1-\alpha}{\eta}\right)^{\frac{2}{1+\alpha}}\left[\frac{b_t}{\gamma\exp(z_t)}\right]^{\frac{1-3\alpha}{1+\alpha}}.$$

For $\alpha \in \left(\frac{1}{3},1\right)$, we know that $2^{\frac{-2}{1+\alpha}}(1+\alpha) > \alpha$. Therefore, a larger real balance b_t for $t \geq 1$ created by a Ramsey issuer also imply a lower fee ratio then.

Regarding money growth in the Markov-perfect case, we can plug the optimal velocity into the real balance constraint of (6) and have

$$x=\beta\mathbf{E}_{z'|z}\left\{\frac{b'}{b}\left\{1+2^{\frac{\alpha-1}{1+\alpha}}\left(\frac{1-\alpha}{\eta}\right)^{\frac{1-\alpha}{1+\alpha}}\left[\frac{b'}{\gamma\exp(z')}\right]^{\frac{1-3\alpha}{1+\alpha}}\right\}\right\}.$$

Similarly, for the Ramsey case at $t \geq 1$ we have

$$x_t=\beta\mathbf{E}_t\left\{\frac{b_{t+1}}{b_t}\left\{1+\left(\frac{1-\alpha}{\eta}\right)^{\frac{1-\alpha}{1+\alpha}}\left[\frac{b_{t+1}}{\gamma\exp(z_{t+1})}\right]^{\frac{1-3\alpha}{1+\alpha}}\right\}\right\}.$$

Consider a steady state with a sufficiently small maintenance cost, i.e., $\xi \to 0$. In this case, a Ramsey issuer eventually creates a very large real balance, as suggested by Equation (14). With $\alpha \in \left(\frac{1}{3}, 1\right)$, we have $2^{\frac{\alpha-1}{1+\alpha}} < 1$, and such a very large real balance implies a lower steady state money growth x for the Ramsey issuer compared to the Markov-perfect issuer. Note that for a large enough ξ this conclusion is reversed, but this case seems empirically less relevant.

At $t = 0$, the Ramsey issuer is not bound by past commitments similar to the Markov-perfect issuer. We can show that real balances at $t = 0$ are identical for the two. For a small enough ξ, a Ramsey issuer chooses a much larger b_1, which implies a larger money growth x_0. In other words, a Ramsey issuer emits a lot more money at $t = 0$, and produces a larger initial profit.

Proposition 5 summarizes the behavior of Markov-perfect and Ramsey issuers with a project maintenance cost. Overall, when the project maintenance cost is small, Ramsey issues a large amount of tokens in the first period, but restricts the issuance in the long run. For fee charges, the Ramsey issuer behaves generally in a more conservative way.

Proposition 5 *Suppose there exists a project maintenance cost as in* (12)*. For $\alpha > \frac{1}{3}$, both Markov-perfect and Ramsey issuers have interior optimal policies that imply a growth-adjusted steady state. Compared to a Markov-perfect issuer, a Ramsey issuer observes,*

- *in the steady state, larger growth-adjusted real balances $\frac{p_t M_t}{A_{t-1}}$ and a lower fee ratio $\frac{\omega_t F_t}{p_t M_t}$, which for a small enough ξ imply a lower money growth rate $\frac{M_{t+1}}{M_t}$;*
- *at $t = 0$, identical token price p_0 and fee ratio $\frac{\omega_0 F_0}{p_0 M_0}$, which for a small enough ξ imply a higher money growth rate $\frac{M_1}{M_0}$.*

Proof. See Appendix A.2. ■

For the case without fees and endogenous velocity, the characterization does not depend on cost parameter ξ and is clearcut.

Corollary 6 *Without fees and endogenous velocity, $\forall \xi \in R^+$, a Ramsey issuer facing a project maintenance cost as in* (12) *chooses a lower (higher) money growth $\frac{M_{t+1}}{M_t}$ ($\frac{M_1}{M_0}$) in steady state (at $t = 0$) compared to a Markov-perfect issuer.*

Proof. See Appendix A.3. ■

We next show that a Ramsey issuer's choices maximize not only profits, but after the initial period also the utility value of the project. Recall that the utility users get from outstanding tokens is given by $\frac{A^\alpha}{1-\alpha}\left(pM\omega\right)^{1-\alpha} - \frac{\eta}{2}A\omega^2$, or in the productivity-adjusted notation $L(b,\omega) = \frac{A}{1-\alpha}\left[\frac{b\omega}{\gamma\exp(z)}\right]^{1-\alpha} - \frac{\eta}{2}A\omega^2$. Define utility value as

$$\frac{\partial L(b,\omega)}{\partial b}b + \frac{\partial L(b,\omega)}{\partial \omega}\omega = 2A\left[\frac{b\omega}{\gamma\exp\left(z\right)}\right]^{1-\alpha} - \eta A\omega^2. \tag{15}$$

This represents the marginal valuation of the utility that users get from using tokens.

Proposition 7 *Allocations that maximize the present discounted utility values in* (15) *net of maintenance costs, for* $t \geq 1$, *lead to the same real balances and velocities as a Ramsey issuer, with identical implied money growth and fee rates.*

Proof. See Appendix A.4 ■

A profit-maximizing issuer with commitment (Ramsey) would like to protect the value of legacy tokens. Without commitment (Markov-perfect) that is not possible. The result shows that with commitment, after the initial period, profit maximization for the issuer implies value maximization for all token holders. So-called *public* blockchains like Bitcoin or Ethereum can be viewed as aiming to maximize their value to a broad set of stakeholders and not just to maximize profits for their founders. Therefore, our study of the problem of a profit maximizing issuer with commitment can be a useful benchmark for the design choices of even these blockchains.

In the initial period, the productivity-adjusted real balance is lower under the Ramsey issuer than that under choices that maximize total value (see Appendix A.4). At that point, the Ramsey issuer is not bound by a prior commitment and has an incentive to lower the value of outstanding tokens held by users, i.e., M_0.

4 Probabilistic commitment

A blockchain is a commitment to a set of computer codes, and as such, the no-commitment Markov-perfect case is not a priori a very realistic representation. On the other end of the spectrum, the perfect commitment implied by Ramsey may be also somewhat extreme. For instance, the Bitcoin protocol encodes a commitment to an issuance schedule which limits the money supply to never exceed 21 million units, and the majority of Bitcoin stakeholders profess a very strong commitment to this issuance schedule. However, there always exists the possibility of a future fork with a different issuance schedule that would attract a majority of Bitcoin users and that would effectively become the dominant chain. Particularly, as it is not at all clear that the fee income—which would eventually become the only source of funds—would be significant enough to incentivize miners in a way to secure the Bitcoin blockchain.

As we have shown previously that the modelling of commitment matters for optimal policies and project values in a first-order way, we build a framework that describes reasonably the commitment technology typical issuers have in reality. Our model that allows for partial commitment to token issuance represented by a commitment probability that takes values between 0 and 1. Section 4.1 presents a baseline setup in which the issuer gets an opportunity each period with a given probability to reoptimize the money growth rate. Otherwise, it has to follow the money growth rate in the previous period. In other words, the issuer commits probabilistically to a constant money growth. Section 4.2 presents a generalized setup in which the issuer is able to commit probabilistically to a richer path of money growth rates. In particular, the issuer who receives the opportunity to reoptimize picks a geometric transition path for money growth rates, which is governed by a money growth rate for the short run (today), a money growth rate for the long run, and a transition speed between the two. Without any reoptimization shocks, its future self has to follow the transition path. This generalized setup echoes our empirical analysis in Section 2. In Appendix B, we present an intermediate case between the two.

4.1 Baseline setup and optimal policies

Assume that with probability $q \in [0, 1]$ the issuer delivers the money growth rate announced in the previous period, and with probability $1 - q$ the issuer gets to reoptimize and select a money growth rate for today without regard to past promises. With the ability to select a new money growth rate, we assume that the issuer promises to repeat the same money growth rate in the future. Under these assumptions, when $q = 0$, the model reduces to the Markov-perfect issuer studied earlier and the issuer reoptimizes every period. When $q = 1$, the issuer commits fully to a constant money growth rate. The latter case is more restricted than the Ramsey issuer we considered before who had the ability to commit to time- and state-contingent plans. However, this more restricted specification captures the fact that for many projects money growth rates are typically very stable and announced far into the future. This restricted policy can help correct the counterfactual property of the Ramsey case considered before where money growth rates would respond to any contemporaneous shocks.

We formulate the problem in a recursive way and directly start with the growth-adjusted setting. Define (unscaled) money growth as $g \equiv \frac{M'}{M}$. The problem for the issuer in an optimization state is given by

$$v(z) = \max_{g,\omega} b(g,z)\frac{g-1}{\gamma \exp(z)} + \omega f(g,\omega,z) + \beta\gamma \mathbf{E}_{z'|z} \exp(z') \left[(1-q)v\left(z'\right) + qv^n(g,z')\right], \quad (16)$$

and in a no-optimization state (with superscript n)

$$\begin{aligned} v^n(g_{-1},z) = \max_{\omega^n} b(g_{-1},z)\frac{g_{-1}-1}{\gamma \exp(z)} &+ \omega^n f(g_{-1},\omega^n,z) \qquad (17) \\ &+ \beta\gamma \mathbf{E}_{z'|z} \exp(z') \left[(1-q)v\left(z'\right) + qv^n(g_{-1},z')\right], \end{aligned}$$

so that in a no-optimization state, money growth g_{-1} is inherited from the past. The fee equation is given by

$$f(g,\omega,z) = \left[\frac{b(g,z)}{\gamma \exp(z)}\right]^{1-\alpha} \omega^{-\alpha} - \eta\omega.$$

Implicit in this setup is the assumption that fee rates are selected every period without precommitment. In many blockchains, fee rates are indeed fluctuating at high frequencies with more discretion involved. The equation for real balances can be written as

$$\begin{aligned} b(g,z) = \beta\frac{\gamma \exp(z)}{g}\mathbf{E}_{z'|z}\Bigg\{ &(1-q)\left\{b\left(h_g(z'),z'\right) + \left[b\left(h_g(z'),z'\right)h_\omega\left(z'\right)\right]^{1-\alpha}\left[\gamma \exp\left(z'\right)\right]^{\alpha}\right\} \\ &+ q\left\{b\left(g,z'\right) + \left[b\left(g,z'\right)h^n_\omega\left(g,z'\right)\right]^{1-\alpha}\left[\gamma \exp\left(z'\right)\right]^{\alpha}\right\}\Bigg\}, \qquad (18) \end{aligned}$$

where $h_g(z)$ and $h_\omega(z)$ are the policy functions for g and ω in (16); $h^n_\omega(g_{-1},z)$ is the policy function for ω^n in (17). A term like $h^n_\omega(g_{-1},z)$ does not show up in the problem of a Markov-perfect issuer studied before, see Equation (6). With its dependence on the previously chosen money growth g_{-1} there is now a dynamic dependence that is indicative of the emergence of so-called generalized Euler equations (Klein, Krusell and Rios-Rull, 2008). Such generalized Euler equations are challenging to solve for with local methods because first-order conditions contain derivatives of yet unknown policy functions. Nonetheless, in the case considered here, the static nature of ω^n permits us to substitute out $\frac{\partial h_\omega^n(g,z')}{\partial g}$ and allows analytical characterizations.

Proposition 8 characterizes optimal policies in the steady state as a function of commitment probability q. With a higher q, a partially committed issuer's ability to reoptimize the money supply gets more restricted and this leads to lower money growth in the steady state. Specifically, an issuer benefits from being able to commit to a low money growth in the future as it boosts the price of new tokens issued today. When q is large, it is more likely that future money growth rates correspond to the level chosen today. As a result, the incentive to commit to a low money growth in the future is reflected directly by a low money growth today. In other words, when commitment technology becomes more effective, issuers assess it more. Meanwhile, fees relative to total market capitalization also decline with q. An issuer with commitment would like to commit to a high velocity and thus low fees next period. The issuer here cannot directly commit to future fees. However, future fees depend on future money growth rates and through this channel fees decline with the commitment probability q.

Proposition 8 *For $q \in [0,1]$ and $\alpha > \frac{1}{3}$, optimal policies for the probabilistic-commitment issuer imply a growth-adjusted steady state where*

$$\begin{aligned}
\frac{M'}{M} &= \gamma\beta\frac{1-\alpha+h(q)}{1-\alpha}, \\
\frac{p'}{p} &= \frac{1}{\beta}\frac{1-\alpha}{1-\alpha+h(q)}, \\
\frac{\omega F}{pM} &= \frac{1}{2}\frac{1+\alpha}{1-\alpha}h(q), \\
\frac{pM}{A_{-1}} &= \gamma\left(1-\alpha\right)^{\frac{2}{3\alpha-1}}\left(2\eta\right)^{\frac{\alpha-1}{3\alpha-1}}\left[h(q)\right]^{\frac{1+\alpha}{1-3\alpha}},
\end{aligned}$$

with $h(q) = \frac{(1-\gamma\beta q)(\alpha+1)}{2\gamma\beta q+\alpha+1}$. Money growth, $\frac{M'}{M}$, and fee ratio, $\frac{\omega F}{pM}$, decrease in q.

Proof. See Appendix A.5. ■

With $q = 0$, we retrieve the Markov-perfect steady state characterized in Proposition 2. With $q = 1$, the optimal choices are bounded and thus in sharp contrast to the Ramsey case. The Ramsey issuer unrestricted by a maintenance cost can produce unlimitedly large current real balances that would need to be supported by growing (productivity adjusted) real balances, which would be driven by declining money growth rates. Such paths are not compatible with constant money growth rates and this restricted choice set produces interior outcomes.

Corollary 9 *Without fees and endogenous velocity, for $q \in (0, 1]$, optimal policies for the probabilistic-commitment issuer imply a growth-adjusted steady state with a money growth rate $\frac{M'}{M}$ that is higher than the case with fees and endogenous velocity.*

For $q = 1$, steady state money growth is bound from below by $\gamma\beta$ (with fees) and 1 (without fees) and converges to these lower bounds for $\gamma\beta \to 1$ and $\alpha \to 1/3$ (with fees) and $\gamma\beta \to 1$ or $\alpha \to 0$ (without fees).

Proof. See Appendix A.6. ■

Corollary 9 shows that steady-state money growth without endogenous velocity is higher than that with endogenous velocity, which is consistent with the role of endogenous velocity as a mechanism in restricting the money supply. Unlike the Markov-perfect case, the probabilistic commitment model admits interior solutions even without fees. Essentially, this is due to the fact that the partial commitment restricts money growth.

We also characterize the lower bounds for steady-state money growth rates and answer the question whether negative (net) money growth rates, or deflation, would ever be optimal for the long run. Consider first a model without fee income. From the issuer's objective function it is clear that negative money growth implies negative seigniorage for the issuer, and this would not be an optimal long-run outcome. Consistent with this, Corollary 9 implies a lower bound of 1 for the (gross) growth rate of money, and the optimal money growth rate converges to it as $\gamma\beta$ converge to 1. However, with fees, negative money growth in steady state cannot be ruled out because negative seigniorage could be compensated by a high fee income.

Lastly, Proposition 8 and Corollary 9 allow us to do some back-of-envelope calculations of the commitment power that typical token issuers have using long-run money growth rates in

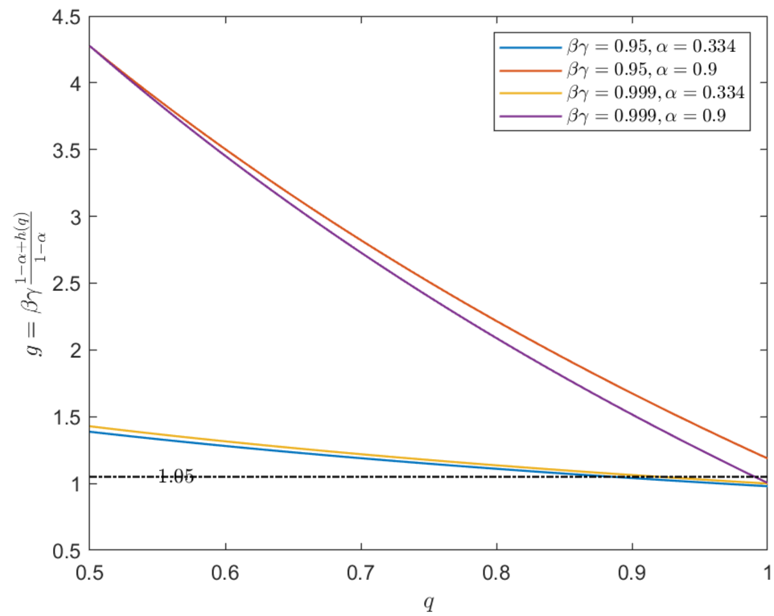

Figure 5: Back-of-envelope calculation of commitment probability q in the baseline setup. *Notes*: We compute the steady-state money growth rate in the baseline setup with probabilistic commitment, i.e., Proposition 8: $g = \gamma\beta\frac{1-\alpha+h(q)}{1-\alpha}$ with $h(q) = \frac{(1-\gamma\beta q)(\alpha+1)}{2\gamma\beta q+\alpha+1}$, with $\gamma\beta \in \{0.95, 0.999\}$ and $\alpha \in \{0.334, 0.9\}$.

the data.[7] According to Proposition 8 for the case with fees, in addition to the commitment probability q, optimal steady-state money growth rates only depend on the growth adjusted discount factor, $\gamma\beta$, and the utility curvature, α. Figure 5 plots optimal money growth rates as a function of q for ranges of values for $\gamma\beta$ and α covering those that one could possibly consider empirically relevant, in particular for $\gamma\beta \in \{0.95, 0.999\}$ and $\alpha \in \{0.334, 0.9\}$. With β representing the dollar discount rate and γ the productivity growth rate of the project, empirically reasonable values for $\gamma\beta$ are no doubt higher than 0.95. For the curvature parameter, $\alpha = .334$ is the lowest possible value consistent with a well-defined interior choice. Whether such low values are empirically reasonable remains open. In any case, even for the most extreme parameter choices, justifying an annual money growth rate of 5% or less requires a commitment probability of over 0.88. For less extreme parameter values, producing money growth rates of 5% or less requires a commitment probability above 0.9 and possibly very close to 1. According to Corollary 9, without fees, the same q leads to a

[7] As we generalize our model in Section 4.2 by allowing issuers to pick a transition path, our key message—typical issuers have high commitment—carries through.

higher steady-state money growth rate. This means that the observed low long-run growth rates for tokens that do not charge fees is consistent with even more commitment power.

4.2 Optimal transition

In principle, an issuer can design a more sophisticated money issuance policy than simply a constant growth rate at the offering stage. For instance, Bitcoin halves its money supply roughly every four years, implying a declining money growth rates. In fact, as we have shown in Section 2, money growth rate declines in age on average. Motivated by these facts, we generalize our model with probabilistic commitment to allow the issuer to choose a transition path for money growth rates. In particular, in an optimization state, the issuer is able to pick a money growth rate for today $g \times s$, a money growth rate for the long run g, and a rate $\lambda \in [0, 1]$ that describes the speed at which money growth converges to its long run level. In other words, s represents the initial distance from reaching the long-run money growth rate. Without any reoptimizing shocks, the money growth rate $t \geq 1$ periods after the last (re)optimization is:

$$g_t = \lambda g + (1 - \lambda) g_{t-1}$$

with $g_0 = gs$. Such a formulation is consistent with our empirical specification in (1).

Section 4.2.1 describes the setup. Section 4.2.2 analyzes optimal policies. Given the richness of this model, we can no longer analytically characterize the optimal policies and thus resort to numerical solutions.[8]

4.2.1 Generalized setup

Shown in Appendix B, an issuer who can only pick a money growth rate for today $g \times s$ and one for the long run g when receiving a reoptimization opportunity already does not admit inner choices in s. In other words, relative to the baseline setup in Section 4.1, the additional flexibility to pick a transition for money growths, even if it has to be abrupt, is already too powerful—in particular, the issuer can raise today's money growth gs to be extremely large while still sustaining token prices by reducing long-run money growth g to be very close to $\beta\gamma q$. The issuer we consider in this generalized setup has even more flexibility, i.e., to

[8] Our intermediate case in Appendix B allows some analytical characterizations of optimal policies.

be able to pick any convergence speed $\lambda \in [0,1]$, and therefore, we introduce a short-run issuance cost to get an interior solution. In particular, we assume that the issuer faces a short-run issuance cost in the optimization state, i.e., $-\frac{\zeta}{2}(s-1)^2$. One motivation for this is that asymmetric information, for instance about how much commitment power the issuer has, prevents users from buying a large quantity of tokens, which is severe at the early stage of the project or at the time when there is an update of monetary policy.

The issuer's problem in an optimization state is given by:

$$\begin{aligned} v(z) = \max_{g,s,\lambda,\omega} b(g,s,\lambda,z)\frac{gs-1}{\gamma\exp(z)} + \omega f(g,s,\lambda,\omega,z) - \frac{\zeta}{2}(s-1)^2 \\ + \beta\gamma\mathbf{E}_{z'|z}\exp(z')\left[(1-q)v(z') + qv^n\left(g,(1-\lambda)s+\lambda,\lambda,z'\right)\right], \end{aligned} \tag{19}$$

where value in no-optimization state is:

$$\begin{aligned} v^n(g,h,\lambda,z) = \max_{\omega^n} b(g,h,\lambda,z)\frac{gh-1}{\gamma\exp(z)} + \omega^n f(g,h,\lambda,\omega^n,z) \\ + \beta\gamma\mathbf{E}_{z'|z}\exp(z')\left[(1-q)v(z') + qv^n\left(g,(1-\lambda)h+\lambda,\lambda,z'\right)\right], \end{aligned}$$

in which $h \in (1,s]$ describes the distance from reaching the long-run money growth rate. Given today the issuer is in a no-optimization state with a money growth rate of gh and a project value of $v^n(g,h,\lambda,z)$, tomorrow's money growth rate in a no-optimization state is $gh(1-\lambda)+g\lambda$ and the value is $v^n\left(g,(1-\lambda)h+\lambda,\lambda,z'\right)$.

Fee is:

$$f(g,h,\lambda,\omega,z) = \left[\frac{b(g,h,\lambda,z)}{\gamma\exp(z)}\right]^{1-\alpha}\omega^{-\alpha} - \eta\omega,$$

and real balance is:

$$
\begin{aligned}
b(g,h,\lambda,z) = \beta\frac{\gamma\exp(z)}{gh}\times \\
\mathbf{E}_{z'|z}\Bigg\{ (1-q)\Big\{ & b\left(h_g(z'),h_s(z'),h_\lambda(z'),z'\right) \\
& + \left[b\left(h_g(z'),h_s(z'),h_\lambda(z'),z'\right)h_\omega\left(z'\right)\right]^{1-\alpha}\left[\gamma\exp\left(z'\right)\right]^\alpha\Big\} \\
+ q\Big\{ & b\left(g,(1-\lambda)h+\lambda,\lambda,z'\right) \\
& + \left[b\left(g,(1-\lambda)h+\lambda,\lambda,z'\right)h^n_\omega\left(g,(1-\lambda)h+\lambda,\lambda,z'\right)\right]^{1-\alpha}\left[\gamma\exp\left(z'\right)\right]^\alpha\Big\}\Bigg\}
\end{aligned} \tag{20}
$$

where $h_g(z)$, $h_s(z)$, $h_\lambda(z)$, and $h_\omega(z)$ are policy functions for g, s, λ, and ω in the optimization state, as in (19).

4.2.2 Optimal policies

As our focus is on the average transition path, we numerically solve our model by fixing $z = 0$. Notice that as we lower down the long-run growth rate g, the growth-adjusted long-run real balance increases. An interior solution implies that the problem has an endogenous lower bound for the choice of g. When $h = 1$, the real balance equation (after fixing $z = 0$ and substituting out the optimal velocity choice) is:

$$
\begin{aligned}
b(g,1,\lambda)\left\{\frac{g}{\gamma\beta(1-q)} - \frac{q}{1-q}\left\{\left(\frac{1-\alpha}{2\eta}\right)^{\frac{1-\alpha}{1+\alpha}}\left[\frac{b(g,1,\lambda)}{\gamma}\right]^{\frac{1-3\alpha}{1+\alpha}}+1\right\}\right\} \\
= \left\{\left(\frac{1-\alpha}{2\eta}\right)^{\frac{1-\alpha}{1+\alpha}}\left[\frac{b(h_g,h_s,h_\lambda)}{\gamma}\right]^{\frac{1-3\alpha}{1+\alpha}}+1\right\} b(h_g,h_s,h_\lambda).
\end{aligned}
$$

Clearly, if $g < \beta\gamma q$, the LHS will always be negative, and thus the equation will never hold for finite $b(g,1,\lambda)$ and $b(h_g,h_s,h_\lambda)$. In general, given $b(h_g,h_s,h_\lambda)$, a low enough g will not permit any positive and finite value of $b(g,1,\lambda)$ that solves the above equation. This means that the choice set for g is bounded from below. We set an exogenous lower bound for g to 0.98 when solving the model, recognizing that, in reality, extremely few tokens are estimated to exhibit a large long-run deflation (See Figures 2 and 3).

We choose $\beta = 0.95$ and $\gamma = 1.05$, and present results under two combinations of the other parameters—one where $\alpha = 0.85$, $\zeta = 90$ and $\eta = 0.01$ and the other where $\alpha = 0.7$, $\zeta = 800$ and $\eta = 0.1$—that lead to quite different long-run fee ratios. We also study cases without fees and endogenous velocity. In the parameter region that we experiment with, we always have $\beta\gamma q < 0.98$.

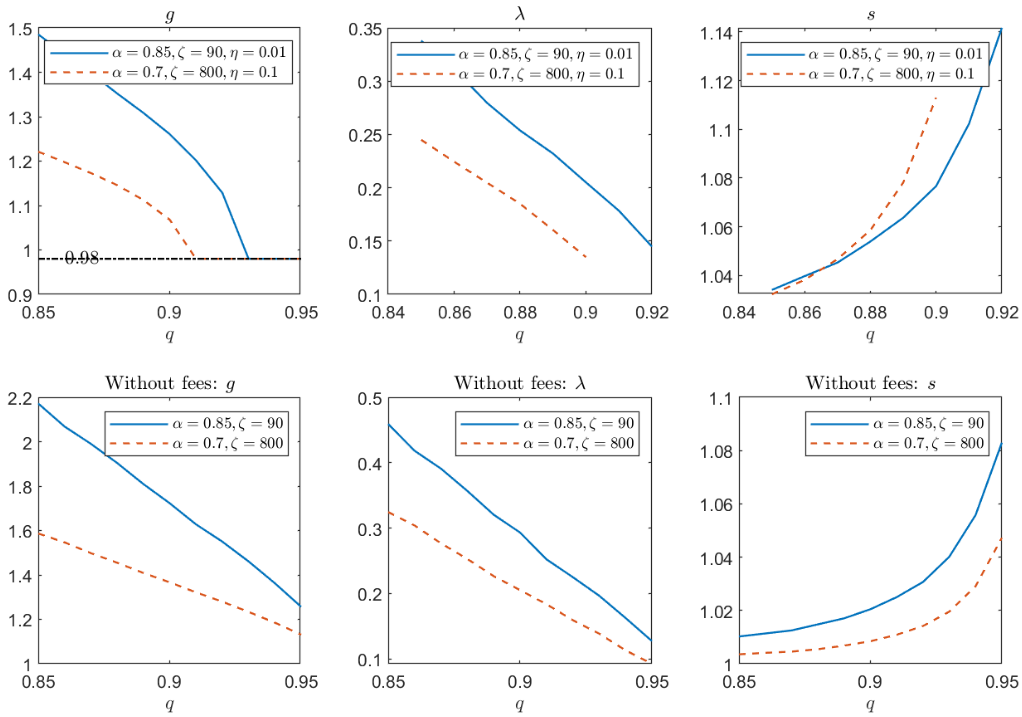

Figure 6: Optimal monetary policies and commitment probability q. *Notes*: We fix $\beta = 0.95$ and $\gamma = 1.05$. The upper (lower) panel presents our model with (without) fees and endogenous velocity. We solve the model by setting the lower bound for g to 0.98. For figures of optimal λ and s in the upper panel, we plot the region where the corresponding optimal g does not hit the lower bound.

Figure 6 presents our results, with upper (lower) panel presenting our model with (without) fees and endogenous velocity. We have two main findings. First, we find that it is optimal for the issuer to adopt a high short-run money growth rate and smoothly transit to a low long-run money growth rate. This is consistent with the empirical property of average money growths revealed by Figure 1. The (probabilistic) commitment to a low long-run money growth rate allows the issuer to sustain token prices, and this allows the issuer to issue a large quantity of tokens and extract more profits in the short run, when they are not discounted as much. Moreover, the optimal strategy of the issuer in our model is to not concentrate the short-run issuance in the first period only but to spread it out across several periods, i.e., to choose a $\lambda < 1$. Such an arrangement economizes the issuance cost at the launch or a major update of the project.

Second, as the commitment probability q increases, the long-run growth rate g and the transition speed λ reduce at the same time. As q increases, the committed transition path is less likely to be reoptimized by future issuers, and a more effective commitment technology encourages issuers to assess it more. First, similar to our baseline probabilistic-commitment setup in Section 4.1, issuer's willingness to commit to a low money growth rate in the future is reflected into a low g. Moreover, as token prices can be now sustained more effectively through a low g, the issuer finds it possible to issue relatively even more in the short run without having to worry about a too rapid decrease in short-run profits. This ends up encouraging a higher s and a lower λ. Overall our results suggest that, with high commitment, the issuer adopts a monetary policy in which money growth rates converge slowly to a low long-run level. Recall our empirical finding in Figure 4. One might consider a project with tokens held heavily by a few big players to have very low commitment because they have either the incentive or for governance tokens the ability to change monetary policies. In this case, our result provides a plausible explanation of why a high level of retail holding is associated with a low long-run money growth and a low convergence speed in the data.

Figure 7 simulates 3,000 tokens with the presence of optimization shocks and computes the path for the average money growth rate and fee ratio.[9] First, it shows that under a high q the low long-run growth rate g plays a dominant role relative to the forward-shift of issuance to the early stage, i.e., a high s and a low λ. This makes money growth rates always lower. While we do not tabulate a separate table here, this is also the case as we solve the model without fees (recall lower panel of Figure 6). Second, a higher q also leads to fee ratios that are always lower. In particular, we find that the fee rate per transaction, f, is higher when q becomes higher. However, the improved commitment significantly enhances the value of the project, and ultimately brings down the overall ratio of fees to market capitalization, $\frac{\omega F}{pM}$. Meanwhile, the convergence of fee ratio to its long-run level is also slower under a higher q.

Lastly, our previous result in Section 4.1 that the low long-run growth rates in the data imply high commitment power carries through into this setting with transition. For instance, Figure 6 shows that the optimal long-run growth rate g spikes up quickly as commitment

[9] Results are similar as we vary parameters.

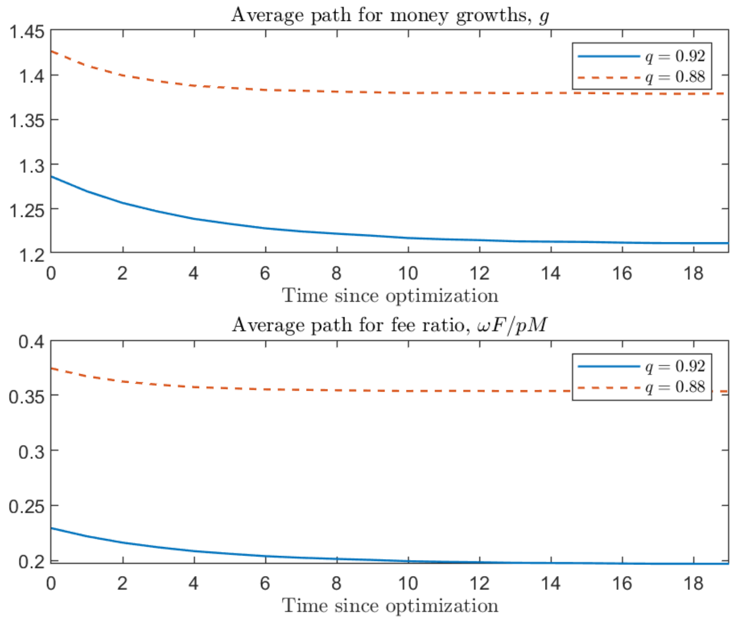

Figure 7: Simulated paths for money growth rates and fee ratios. *Notes*: We fix $\beta = 0.95, \gamma = 1.05, \alpha = 0.85, \zeta = 90, \eta = 0.01$ and solve the model with $q = 0.92$ and $q = 0.88$. Starting from an optimization state, we simulate 3,000 tokens for 20 periods, and compute average money growth rates and fee ratios across firms.

probability q drops below 0.9. Therefore, one shall be cautious to build models of cryptocurrencies or blockchains by assuming that the issuers or founders have no commitment.

5 Conclusions

The polcies for token issuance and fees are of first-order importance for crypto projects. This paper examines this issue along two dimensions. First, empirically, we provide the first summary of the main features of existing tokens' monetary policies based on a large cross-section of data. Second, theoretically, we propose a dynamic model that accounts for a joint optimization of money issuance and fee charges by crypto issuers with varying degrees of commitment. We highlight the importance of issuers' commitment for the optimal policies and the maximal market capitalization that can be achieved. Our results are helpful not only for interpreting the stylized facts that we document but also for guiding the design of monetary and fee policies for future projects and the update of existing ones.

This paper has focused on studying deterministic and average properties as they are first-order and offer analytical tractability. Nonetheless, our model allows for the study of stochastic properties of optimal policies with straightforward numerical methods. Even though typical outstanding crypto projects adopt monetary rules and in some cases fee rules that are largely deterministic, it can well be the case that policies depending on time-varying economic conditions, e.g., the level of interest rates or the economic productivity, can better create value. As new projects become more and more sophisticated, analysis in this direction is helpful to provide guidance for their design. We consider these in ongoing work.

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Appendix

A Proofs

A.1 Proposition 2

Equilibrium conditions for Markov-perfect issuer in the scaled model are:

$$\frac{b}{x\gamma\exp(z)}\left\{1-(1-\alpha)\left[\frac{b}{\gamma\exp(z)}\right]^{-\alpha}\omega^{1-\alpha}\right\}=0 \tag{21}$$

$$(1-\alpha)\left[\frac{b}{\gamma\exp(z)}\right]^{1-\alpha}\omega^{-\alpha}-2\eta=0, \tag{22}$$

$$b=\frac{1}{x}\beta\mathbf{E}_{z'|z}\left[b'+(b'\omega')^{1-\alpha}(\gamma\exp(z'))^{\alpha}\right]$$

$$f=\left[\frac{b}{\gamma\exp(z)}\right]^{1-\alpha}\omega^{-\alpha}-\eta\omega.$$

In the steady state, some straightforward algebra yields:

$$\begin{aligned}
b_{ss}^{MP}&=\gamma(1-\alpha)^{\frac{2}{3\alpha-1}}(2\eta)^{\frac{\alpha-1}{3\alpha-1}},\\
\omega_{ss}^{MP}&=(1-\alpha)^{\frac{1}{3\alpha-1}}(2\eta)^{\frac{-\alpha}{3\alpha-1}},\\
x_{ss}^{MP}&=\beta\frac{2-\alpha}{1-\alpha},\\
f_{ss}^{MP}&=(1+\alpha)(1-\alpha)^{\frac{2-3\alpha}{3\alpha-1}}\eta^{\frac{2\alpha-1}{3\alpha-1}}2^{\frac{-\alpha}{3\alpha-1}},
\end{aligned}$$

all of which are positive and finite for $\alpha\in(0,1)$. In the steady state, we know $\frac{M'}{M}=\gamma x_{ss}^{MP}$; $\frac{p'}{p}=\frac{1}{x_{ss}^{MP}}$; $\frac{\omega F}{pM}=\gamma\frac{f_{ss}^{MP}}{b_{ss}^{MP}}\omega_{ss}^{MP}$.

To show that our steady state is indeed an inner optimum, we need to check the Hessian matrix. The second derivative of the objective function with respect to x is: $-\left(\frac{b_{ss}^{MP}}{x_{ss}^{MP}\gamma}\right)^2\alpha(1-\alpha)\left[\frac{b_{ss}^{MP}}{\gamma}\right]^{-\alpha-1}\left(\omega_{ss}^{MP}\right)^{1-\alpha}<0$. The second derivative of the objective function with respect to ω is $-(1-\alpha)(1+\alpha)\left(\frac{b_{ss}^{MP}}{\gamma}\right)^{1-\alpha}(\omega_{ss}^{MP})^{-\alpha-1}<0$. Cross derivative is $-(1-\alpha)^2\left(\frac{b_{ss}^{MP}}{\gamma}\right)^{-\alpha}\frac{1}{\gamma}\left(\omega_{ss}^{MP}\right)^{-\alpha}\frac{b_{ss}^{MP}}{x_{ss}^{MP}}$. Therefore, the determinant of the Hessian matrix is $(1-\alpha)^2(3\alpha-1)\left(\frac{b_{ss}^{MP}}{\gamma}\right)^{-2\alpha}\left(\frac{b_{ss}^{MP}}{x_{ss}^{MP}\gamma}\right)^2(\omega_{ss}^{MP})^{-2\alpha}$, which is positive if $\alpha>\frac{1}{3}$.

A.2 Proposition 5

It is straightforward to show that the problem can be scaled into:

$$
\begin{aligned}
v(z) = \max_{x,\omega} \, & b(x,z)\left[x - \frac{1}{\gamma \exp(z)}\right] \\
& - \frac{\xi}{2}\left[\frac{b(x,z)}{\gamma \exp(z)} - \bar{b}\right]^2 + \omega \left\{ \left[\frac{b(x,z)}{\gamma \exp(z)}\right]^{1-\alpha} \omega^{-\alpha} - \eta\omega \right\} + \beta\gamma \mathbf{E}_{z'|z} \exp(z') V(z')
\end{aligned}
$$

subject to

$$
b(x,z) = \frac{1}{x}\beta \mathbf{E}_{z'|z}\left\{ b(h_x(z'), z') + \left[b(h_x(z'), z')h_\omega(z')\right]^{1-\alpha} \left(\gamma \exp(z')\right)^{\alpha} \right\}.
$$

Equilibrium conditions are given by

$$
\begin{aligned}
& \left\{ \frac{1}{\gamma \exp(z)} + \xi \left[\frac{b(x,z)}{\gamma \exp(z)} - \bar{b}\right] \frac{1}{\gamma \exp(z)} - (1-\alpha)\left[\frac{b(x,z)}{\gamma \exp(z)}\right]^{-\alpha} \omega^{1-\alpha} \frac{1}{\gamma \exp(z)} \right\} \frac{b(x,z)}{x} = 0, \\
& (1-\alpha)\left[\frac{b(x,z)}{\gamma \exp(z)}\right]^{1-\alpha} \omega^{-\alpha} - 2\eta\omega = 0, \\
& b(x,z) = \frac{1}{x}\beta \mathbf{E}_{z'|z}\left[b(h_x(z'), z') + \left[b(h_x(z'), z')h_\omega(z')\right]^{1-\alpha} \left(\gamma \exp(z')\right)^{\alpha} \right], \\
& f = \left[\frac{b(x,z)}{\gamma \exp(z)}\right]^{1-\alpha} \omega^{-\alpha} - \eta\omega,
\end{aligned}
$$

which imply a steady state where

$$
1 + \xi\left(\frac{b}{\gamma} - \bar{b}\right) - (1-\alpha)\left(\frac{1-\alpha}{2\eta}\right)^{\frac{1-\alpha}{1+\alpha}} \left(\frac{b}{\gamma}\right)^{\frac{1-3\alpha}{1+\alpha}} = 0, \tag{23}
$$

$$
x = \beta\left[1 + \left(\frac{1-\alpha}{2\eta}\right)^{\frac{1-\alpha}{1+\alpha}} \left(\frac{b}{\gamma}\right)^{\frac{1-3\alpha}{1+\alpha}}\right], \tag{24}
$$

$$
\frac{\omega F}{pM} = \eta \frac{1+\alpha}{1-\alpha}\left(\frac{1-\alpha}{2\eta}\right)^{\frac{2}{1+\alpha}} \left(\frac{b}{\gamma}\right)^{\frac{1-3\alpha}{1+\alpha}}. \tag{25}
$$

Now let's move to the Ramsey problem. The Lagrangian is given by:

$$\max_{\{b,\omega\}_{t=0}^{\infty}} \mathbf{E}_0 \sum_{t=0}^{\infty} (\gamma\beta)^t \exp\left(\sum_{k=0}^{t} z_k\right) \Bigg\{ \beta \mathbf{E}_t \left[b_{t+1} + (b_{t+1}\omega_{t+1})^{1-\alpha} (\gamma \exp(z_{t+1}))^{\alpha} \right] - \frac{b_t}{\gamma \exp(z_t)} - \frac{\xi}{2} \left[\frac{b_t}{\gamma \exp(z_t)} - \bar{b} \right]^2 + \omega_t \left\{ \left[\frac{b_t}{\gamma \exp(z_t)} \right]^{1-\alpha} \omega_t^{-\alpha} - \eta\omega_t \right\} \Bigg\},$$

in which we have substituted the constraint into the objective function.

First-order conditions with respect to $\{b_t, \omega_t\}$ for $t \geq 1$ are given by:

$$\beta \left\{ -1 - \xi \left[\frac{b_t}{\gamma \exp(z_t)} - \bar{b} \right] + (1-\alpha) \left(\frac{b_t}{\gamma \exp(z_t)} \right)^{-\alpha} \omega_t^{1-\alpha} \right\} + \beta \mathbf{E}_{t-1} \left\{ 1 + (1-\alpha)\, \omega_t^{1-\alpha} \left[\frac{b_t}{\gamma \exp(z_t)} \right]^{-\alpha} \right\} = 0,$$

$$\gamma\beta \exp(z_t) \left\{ (1-\alpha) \left[\frac{b_t}{\gamma \exp(z_t)} \right]^{1-\alpha} \omega_t^{-\alpha} - 2\eta\omega_t \right\} + \beta \mathbf{E}_{t-1} \left[(1-\alpha)\, b_t^{1-\alpha} \omega_t^{-\alpha} (\gamma \exp(z_t))^{\alpha} \right] = 0,$$

which imply a steady state where

$$-\xi \left(\frac{b}{\gamma} - \bar{b} \right) + 2(1-\alpha) \left(\frac{1-\alpha}{\eta} \right)^{\frac{1-\alpha}{1+\alpha}} \left(\frac{b}{\gamma} \right)^{\frac{1-3\alpha}{1+\alpha}} = 0, \tag{26}$$

$$x = \beta \left[1 + \left(\frac{1-\alpha}{\eta} \right)^{\frac{1-\alpha}{1+\alpha}} \left(\frac{b}{\gamma} \right)^{\frac{1-3\alpha}{1+\alpha}} \right], \tag{27}$$

$$\frac{\omega F}{pM} = \eta \frac{\alpha}{1-\alpha} \left(\frac{1-\alpha}{\eta} \right)^{\frac{2}{1+\alpha}} \left(\frac{b}{\gamma} \right)^{\frac{1-3\alpha}{1+\alpha}}. \tag{28}$$

For any b, the LHS of (23) is larger than the negative of the LHS of (26). Since both of them are increasing in b if $\alpha > \frac{1}{3}$, the b that solves (23) shall be smaller than that to (26). Furthermore, because $(1+\alpha)2^{-\frac{2}{1+\alpha}} > \alpha$, we know the LHS of (28) is smaller than that of (25) for $\alpha > \frac{1}{3}$. Finally, as $\xi \to 0$, (26) implies that steady state b under Ramsey goes to infinity. However, that under Markov-perfect, implied by (26), stays finite. In that case, (24) and (27) suggest that the steady-state x under Ramsey goes to β, which will ultimately fall below steady-state x under Markov-perfect. For both Markov-perfect and Ramsey cases, tedious algebra checking the Hessian matrix shows that the critical point implied by first-order conditions is indeed local maximum when $\alpha > \frac{1}{3}$.

For $t = 0$, a Markov-perfect issuer's equilibrium conditions are the same as those for $t \geq 1$ whereas for Ramsey they become

$$
\begin{aligned}
&-1 - \xi\left[\frac{b_0}{\gamma\exp(z_0)} - \bar{b}\right] + (1-\alpha)\left[\frac{b_0}{\gamma\exp(z_0)}\right]^{-\alpha}\omega_0^{1-\alpha} = 0,\\
&(1-\alpha)\left[\frac{b_0}{\gamma\exp(z_0)}\right]^{1-\alpha}\omega_0^{-\alpha} - 2\eta\omega_0 = 0,
\end{aligned}
$$

which coincides with Markov-perfect case. This means that Ramsey and Markov-perfect issuers choose the same real balance b_0 and velocity ω_0, which implies the same fee ratio $\frac{\omega_0 F_0}{p_0 M_0}$. Money growth for the Markov-perfect issuer is given by

$$
x_0 = \beta\mathbf{E}\left\{\frac{b_1}{b_0}\left\{1 + 2^{\frac{\alpha-1}{1+\alpha}}\left(\frac{1-\alpha}{\eta}\right)^{\frac{1-\alpha}{1+\alpha}}\left[\frac{b_1}{\gamma\exp(z_1)}\right]^{\frac{1-3\alpha}{1+\alpha}}\right\}\right\}.
$$

Similarly, we have for the Ramsey issuer

$$
x_0 = \beta\mathbf{E}\left\{\frac{b_1}{b_0}\left\{1 + \left(\frac{1-\alpha}{\eta}\right)^{\frac{1-\alpha}{1+\alpha}}\left[\frac{b_1}{\gamma\exp(z_1)}\right]^{\frac{1-3\alpha}{1+\alpha}}\right\}\right\}.
$$

For $\alpha > \frac{1}{3}$ and $\xi \to 0$, b_1 under Ramsey goes to infinity whereas that under Markov-perfect stays finite. This means that x_0 under Ramsey goes to infinity whereas that under Markov-perfect stays finite.

A.3 Proposition 6

It is straightforward to show that the Markov-perfect problem can be scaled into:

$$
v(z) = \max_x \; b(x,z)\left[x - \frac{1}{\gamma\exp(z)}\right] - \frac{\xi}{2}\left[\frac{b(x,z)}{\gamma\exp(z)} - \bar{b}\right]^2 + \beta\gamma\mathbf{E}_{z'|z}\exp(z')V(z')
$$

subject to

$$b(x,z) = \frac{1}{x}\beta \mathbf{E}_{z'|z}\left\{b(h_x(z'),z') + b(h_x(z'),z')^{1-\alpha}\left(\gamma \exp(z')\right)^{\alpha}\right\}.$$

Equilibrium conditions are given by

$$\left\{\frac{1}{\gamma \exp(z)} + \xi\left[\frac{b(x,z)}{\gamma \exp(z)} - \bar{b}\right]\frac{1}{\gamma \exp(z)}\right\}\frac{b(x,z)}{x} = 0,$$
$$b(x,z) = \frac{1}{x}\beta \mathbf{E}_{z'|z}\left\{b(h_x(z'),z') + b(h_x(z'),z')^{1-\alpha}\left(\gamma \exp(z')\right)^{\alpha}\right\},$$

which imply a steady state where

$$b = \gamma\left(\bar{b} - \frac{1}{\xi}\right),$$

and

$$x = \beta\left[1 + \left(\bar{b} - \frac{1}{\xi}\right)^{-\alpha}\right].$$

The Lagrangian for the Ramsey's problem is given by:

$$\max_{\{b\}_{t=0}^{\infty}} \mathbf{E}_0 \sum_{t=0}^{\infty}(\gamma\beta)^t \exp\left(\sum_{k=0}^{t} z_k\right)\left\{ -\frac{\xi}{2}\left[\frac{b_t}{\gamma \exp(z_t)} - \bar{b}\right]^2 \right.$$
$$\left. + \beta \mathbf{E}_t\left[b_{t+1} + b_{t+1}^{1-\alpha}\left(\gamma \exp(z_{t+1})\right)^{\alpha}\right] - \frac{b_t}{\gamma \exp(z_t)}\right\},$$

in which we have substituted the constraint into the objective function.

First-order condition with respect to b_t for $t \geq 1$ is given by:

$$-\xi\left[\frac{b_t}{\gamma \exp(z_t)} - \bar{b}\right] + \mathbf{E}_{t-1}(1-\alpha)\left[\frac{b_t}{\gamma \exp(z_t)}\right]^{-\alpha} = 0,$$

which implies a steady state where

$$-\xi\left(\frac{b}{\gamma} - \bar{b}\right) + (1-\alpha)\left(\frac{b}{\gamma}\right)^{-\alpha} = 0.$$

Since the LHS is decreasing in b and is positive when $b = \gamma\left(\bar{b} - \frac{1}{\xi}\right)$, we know that a solution exists and is larger than $\gamma\left(\bar{b} - \frac{1}{\xi}\right)$. In other words, the steady-state growth-adjusted real balance under Ramsey is larger than that under Markov-perfect. Both issuers face the same constraint between x and b in the steady state, which implies that x decreases in b. As a result, Ramsey adopts a smaller x_t and $\frac{M_{t+1}}{M_t}$ relative to Markov-perfect.

For $t = 0$, Ramsey and Markov-perfect issuers' equilibrium conditions coincide with each other. This means that Ramsey and Markov-perfect issuers choose the same real balances b_0, which implies a higher x_0 for the former.

A.4 Proposition 7

Take the definition of utility value in (15). It is immediate to see that maximizing the lifetime utility value net of maintenance costs implies a period-by-period static problem of maximizing the following:

$$2\left[\frac{b_t\omega_t}{\gamma\exp\left(z_t\right)}\right]^{1-\alpha} - \eta\omega_t^2 - \frac{\xi}{2}\left[\frac{b_t}{\gamma\exp\left(z_t\right)} - \bar{b}\right]^2.$$

For each $t \geq 0$, the first-order condition with respect to ω_t is given by:

$$(1-\alpha)\left[\frac{b_t}{\gamma\exp(z_t)}\right]^{1-\alpha}\omega_t^{-\alpha} - \eta\omega_t = 0,$$

which is identical to Ramsey issuer's first-order condition with respect to ω_t for $t \geq 1$. The first-order condition for real balances b_t after substituting out the first-order condition for velocity is given by:

$$2\left(1-\alpha\right)^{\frac{2}{1+\alpha}}\eta^{\frac{\alpha-1}{1+\alpha}}\left[\frac{b_t}{\gamma\exp\left(z_t\right)}\right]^{\frac{1-3\alpha}{1+\alpha}} - \xi\left[\frac{b_t}{\gamma\exp\left(z_t\right)} - \bar{b}\right] = 0, \tag{29}$$

which is identical to Ramsey issuer's first-order condition with respect to b_t for $t \geq 1$.

At $t = 0$, the first-order condition for b_0 for the Ramsey issuer is given by

$$2^{\frac{\alpha-1}{1+\alpha}}\left(1-\alpha\right)^{\frac{2}{1+\alpha}}\eta^{\frac{\alpha-1}{1+\alpha}}\left[\frac{b_0}{\gamma\exp\left(z_0\right)}\right]^{\frac{1-3\alpha}{1+\alpha}} - 1 - \xi\left[\frac{b_0}{\gamma\exp\left(z_0\right)} - \bar{b}\right] = 0. \tag{30}$$

Clearly, for a given b_t, the LHS of (29) is larger than the LHS of (30), and for $\alpha > 1/3$ the LHS of both equations are declining in b_t. This implies that b_0 is smaller for the Ramsey issuer than for the total value maximizing case.

A.5 Proposition 8

In an optimization state of the scaled model, the optimal velocity decisions that pin down issuer's choice for fees—ω in an optimization state and ω^n in a no-optimization state—are given by:

$$(1-\alpha)\left[\frac{b(g,z)}{\gamma\exp(z)}\right]^{1-\alpha}\omega^{-\alpha} - 2\eta\omega = 0, \tag{31}$$

$$(1-\alpha)\left[\frac{b(g_{-1},z)}{\gamma\exp(z)}\right]^{1-\alpha}(\omega^n)^{-\alpha} - 2\eta\omega^n = 0. \tag{32}$$

The optimal decision for g in an optimization state is given by:

$$\frac{\partial b(g,z)}{\partial g}\frac{g-1}{\gamma\exp(z)} + \frac{b(g,z)}{\gamma\exp(z)} + (1-\alpha)\left[\frac{\omega}{\gamma\exp(z)}\right]^{1-\alpha} b\,(g,z)^{-\alpha}\,\frac{\partial b\,(g,z)}{\partial g} + \beta\gamma\mathbf{E}_{z'|z}\exp(z')q\frac{\partial v^n(z',g)}{\partial g} = 0. \tag{33}$$

Derivative of the pricing schedule is given by:

$$\frac{\partial b(g,z)}{\partial g} = -\frac{b(g,z)}{g} + \frac{\beta\gamma\exp(z)}{g}q\mathbf{E}_{z'|z}\Bigg\{\frac{\partial b(g,z')}{\partial g} + (1-\alpha)\left[\frac{b\,(g,z')\,h^n_\omega\,(g,z')}{\gamma\exp(z')}\right]^{-\alpha}\left[\frac{\partial b\,(g,z')}{\partial g}h^n_\omega\,(g,z') + b\,(g,z')\,\frac{h^n_\omega\,(g,z')}{\partial g}\right]\Bigg\},$$

where

$$\frac{h^n_\omega\,(g,z')}{\partial g} = \frac{1-\alpha}{1+\alpha}\left(\frac{1-\alpha}{2\eta}\right)^{\frac{1}{1+\alpha}}\left[\frac{b\,(g,z')}{\gamma\exp(z')}\right]^{\frac{-2\alpha}{1+\alpha}}\frac{1}{\gamma\exp(z')}\frac{\partial b\,(g,z')}{\partial g}.$$

Lastly, the derivative of the no-optimization value equation gives

$$\frac{\partial v^n(g_{-1},z)}{\partial g_{-1}} = \frac{\partial b(g_{-1},z)}{\partial g_{-1}} \frac{g_{-1}-1}{\gamma \exp(z)} + \frac{b(g_{-1},z)}{\gamma \exp(z)}$$
$$\left\{(1-\alpha)\left[\frac{\omega^n}{\gamma \exp(z)}\right]^{1-\alpha} b\left(g_{-1},z\right)^{-\alpha} \frac{\partial b\left(g_{-1},z\right)}{\partial g_{-1}}\right\} + \beta\gamma \mathbf{E}_{z'|z} \exp(z') q \frac{\partial v^n(g_{-1},z')}{\partial g_{-1}}.$$

In steady state, $g_{-1}=g$ and thus $\omega^n=\omega$. Some tedious algebra yields:

$$\begin{aligned}
g_{ss}^{PC} &= \beta\gamma\left[1+\frac{1}{1-\alpha}\frac{(1-\gamma\beta q)(\alpha+1)}{2\gamma\beta q+\alpha+1}\right], \\
b_{ss}^{PC} &= \gamma\left(2\eta\right)^{\frac{\alpha-1}{3\alpha-1}}(1-\alpha)^{\frac{2}{3\alpha-1}}\left[\frac{(1-\gamma\beta q)(\alpha+1)}{2\gamma\beta q+\alpha+1}\right]^{\frac{1+\alpha}{1-3\alpha}}, \\
\omega_{ss}^{PC} &= (1-\alpha)^{\frac{1}{3\alpha-1}}\left(2\eta\right)^{\frac{-\alpha}{3\alpha-1}}\left[\frac{(1-\gamma\beta q)(\alpha+1)}{2\gamma\beta q+\alpha+1}\right]^{\frac{1-\alpha}{1-3\alpha}}, \\
f_{ss}^{PC} &= (1+\alpha)(1-\alpha)^{\frac{2-3\alpha}{3\alpha-1}}(2)^{\frac{-\alpha}{3\alpha-1}}\eta^{\frac{2\alpha-1}{3\alpha-1}}\left[\frac{(1-\gamma\beta q)(\alpha+1)}{2\gamma\beta q+\alpha+1}\right]^{\frac{1-\alpha}{1-3\alpha}}.
\end{aligned}$$

In the steady state, we know $\frac{PM}{A_{-1}} = b_{ss}^{PC}$; $\frac{M'}{M} = g_{ss}^{PC}$; $\frac{p'}{p} = \frac{\gamma}{g_{ss}^{PC}}$; $\frac{\omega F}{pM} = \gamma \frac{f_{ss}^{PC}}{b_{ss}^{PC}} \omega_{ss}^{PC}$. Finally, tedious algebra shows that the critical point implied by first-order conditions is indeed local maximum when $\alpha > \frac{1}{3}$.

A.6 Corollary 9

In the case without fees, i.e., $\omega \equiv 1$ and $F \equiv 0$, the set of equilibrium conditions evolves into the following. First, the optimal decision for g in the optimization state is given by:

$$b(g,z)\frac{1}{\gamma \exp(z)} + \frac{\partial b(g,z)}{\partial g}\frac{g-1}{\gamma \exp(z)} + \beta\gamma \mathbf{E}_{z'|z} \exp(z') q \frac{\partial v^n(z',g)}{\partial g} = 0$$

Pricing equation is

$$b(g,z) = \frac{\gamma \exp(z)}{g}\beta \mathbf{E}_{z'|z}\left\{(1-q)\left\{b(h_g(z'),z') + b(h_g(z'),z')^{1-\alpha}\left[\frac{1}{\gamma \exp(z')}\right]^{-\alpha}\right\}\right.$$
$$\left. + q\left\{b(g,z') + b(g,z')^{1-\alpha}\left[\frac{1}{\gamma \exp(z')}\right]^{-\alpha}\right\}\right\},$$

and its derivative is given by:

$$\frac{\partial b(g,z)}{\partial g}=-\frac{b(g,z)}{g}+\frac{\gamma\exp(z)}{g}\beta q\mathbf{E}_{z'|z}\left\{\frac{\partial b(g,z')}{\partial g}+(1-\alpha)\left[\frac{b(g,z')}{\gamma\exp(z')}\right]^{-\alpha}\frac{\partial b(g,z')}{\partial g}\right\}.$$

Lastly, derivative of the no-optimization value equation gives

$$\frac{\partial v^n(g_{-1},z)}{\partial g_{-1}}=b(g_{-1},z)\frac{1}{\gamma\exp(z)}+\frac{\partial b(g_{-1},z)}{\partial g_{-1}}\frac{g_{-1}-1}{\gamma\exp(z)}+\beta\gamma\mathbf{E}_{z'|z}\exp(z')q\frac{\partial v^n(g_{-1},z')}{\partial g_{-1}}.$$

In steady state, $g_{-1}=g$ and thus $\omega^n=\omega$. Some tedious algebra yields:

$$g_{ss}^{PC2}=\beta\gamma\left[\frac{1-\beta\gamma q}{(1-\alpha)\,\beta\gamma q}+1\right],$$

$$b_{ss}^{PC2}=\gamma\left[\frac{1-\beta\gamma q}{(1-\alpha)\,\beta\gamma q}\right]^{-\frac{1}{\alpha}}$$

In the steady state, we know $\frac{PM}{A_{-1}}=b_{ss}^{PC2}$; $\frac{M'}{M}=g_{ss}^{PC2}$; $\frac{p'}{p}=\frac{\gamma}{g_{ss}^{PC2}}$. We have:

$$g_{ss}^{PC2}>g_{ss}^{PC}$$

because $\alpha<1$ and $\beta\gamma q<1$.

We can now establish the second half of the corollary. Fix $q=1$. For the case without fees, we have in steady state $\frac{M'}{M}=\gamma\beta+\frac{1-\gamma\beta}{1-\alpha}>1$. Given $\alpha\in(0,1)$, as $\gamma\beta\to1$, $\frac{M'}{M}$ converges to $\gamma\beta$ and thus 1. Given $\beta\gamma$, $\frac{M'}{M}\to1$ as $\alpha\to0$. For the case with fees (Proposition 8), we have in steady state $\frac{M'}{M}=\gamma\beta+\gamma\beta\,(1-\gamma\beta)\left(1-\frac{2\gamma\beta}{2\gamma\beta+\alpha+1}\right)\frac{1}{1-\alpha}$, which is increasing in α. For $\alpha>\frac{1}{3}$, $\frac{M'}{M}$ is bounded from below by $\frac{5\gamma\beta}{3\gamma\beta+2}>\gamma\beta$. $\frac{M'}{M}$ converges to $\frac{5\gamma\beta}{3\gamma\beta+2}$ as $\alpha\to\frac{1}{3}$, and furthermore to $\beta\gamma$ as $\gamma\beta\to1$.

B Abrupt transition with probabilistic commitment

In this appendix we consider an intermediate case between the two models in Sections 4.1 and 4.2. In particular, we consider a probabilistic-commitment issuer who in an optimization state picks the money growth rate for today, $g\times s$, and the money growth rate for tomorrow, g, expecting its future self beyond tomorrow to follow without any arrival of reoptimization shocks. In other words, the issuer can pick a transition path for money growth rates, which, however, has to be in an abrupt form. We first show that an inner equilibrium does not exist in this setup. We then introduce into the setup an initial issuance cost, which bounds the problem and allows an analytical characterization of the impact of commitment probability.

B.1 Setup and optimal policies

In an optimization state, the issuer picks the money growth rate for today, $g \times s$, and for (beyond) tomorrow, g. We again assume that fee rates are determined optimally ex post. In a growth-adjusted formulation, we index functions for today using superscript 0, and issuer's problem in an optimization state is given by:

$$v(z) = \max_{g,s,\omega} b^0(g,s,z)\frac{gs-1}{\gamma\exp(z)} + \omega f^0(g,s,\omega,z) + \beta\gamma\mathbf{E}_{z'|z}\exp(z')\left[(1-q)v(z') + qv^n(g,z')\right], \tag{34}$$

and in a no-optimization state is given by:

$$\begin{aligned} v^n(g_{-1},z) \quad = \quad & \max_{\omega^n} b(g_{-1},z)\frac{g_{-1}-1}{\gamma\exp(z)} + \omega^n f(g_{-1},\omega^n,z) \\ & +\beta\gamma\mathbf{E}_{z'|z}\exp(z')\left[(1-q)v(z') + qv^n(g_{-1},z')\right]. \end{aligned} \tag{35}$$

The fee equations for the long run and for today are given respectively by

$$f(g,\omega,z) = \left[\frac{b(g,z)}{\gamma\exp(z)}\right]^{1-\alpha}\omega^{-\alpha} - \eta\omega,$$

and

$$f^0(g,s,\omega,z) = \left[\frac{b^0(g,s,z)}{\gamma\exp(z)}\right]^{1-\alpha}\omega^{-\alpha} - \eta\omega.$$

The equation for real balance in the long run can be written as

$$b(g,z) = \beta \frac{\gamma \exp(z)}{g} \times \tag{36}$$
$$\mathbf{E}_{z'|z} \Bigg\{ (1-q) \left\{ \left[b^0 \left(h_g(z'), h_s(z'), z' \right) h_\omega \left(z' \right) \right]^{1-\alpha} \left[\gamma \exp\left(z' \right) \right]^{\alpha} + b^0 \left(h_g(z'), h_s(z'), z' \right) \right\}$$
$$+ q \left\{ \left[b\left(g, z' \right) h_\omega^n \left(g, z' \right) \right]^{1-\alpha} \left[\gamma \exp\left(z' \right) \right]^{-\alpha} + b\left(g, z' \right) \right\} \Bigg\},$$

where $h_g(z)$, $h_s(z)$ and $h_\omega(z)$ are the policy functions for g, s and ω in (34); $h_\omega^n(g_{-1}, z)$ is the policy function for ω^n in (35). In the optimization state, current real balance is different from that in the long run only because the money growth rate today is different from that tomorrow and beyond, i.e., gs for today and g for the long run. Moving one period forward, both cases will either evolve to an optimization state with probability $1-q$ or maintain a growth rate of g. This means that:

$$b^0(g,s,z) = \frac{b(g,z)}{s}. \tag{37}$$

While Section 3.3 shows that giving issuers full commitment rules out the existence of an interior solution, Proposition 10 shows that even with probabilistic commitment, an inner solution might well not exist. In particular, in the optimization state, the issuer would like to choose a money growth rate for today as large as possible. By committing to a long-run growth rate close enough to $\beta\gamma q$, the price of tokens today will not fall quickly enough to zero, which leads to a significant profit today.

Proposition 10 *Fix $z = 0$. A probabilistic-commitment issuer who is able to pick money growth rates today (gs) and tomorrow (g) would like to increase the former as much as possible while keep the latter as close to $\beta\gamma q$ as possible.*

Proof. Fix $z = 0$. Define $l = \left(\frac{1-\alpha}{2\eta}\right)^{\frac{1-\alpha}{1+\alpha}}$. Plug two fee equations and (37) into (34), and then substitute out ω using the first-order condition. We have:

$$v = \max_{g,s} \frac{b(g)}{s\gamma}(gs-1) + \frac{1+\alpha}{2} \left[\frac{b(g)}{s\gamma} \right]^{\frac{2(1-\alpha)}{1+\alpha}} l + \beta\gamma \left[(1-q)v + qv^n(g) \right],$$

where

$$v^n(g_{-1}) = \frac{b(g_{-1})}{\gamma}(g_{-1}-1) + \frac{1+\alpha}{2}\left[\frac{b(g_{-1})}{s\gamma}\right]^{\frac{2(1-\alpha)}{1+\alpha}} l + \beta\gamma\left[(1-q)v + qv^n(g_{-1})\right],$$

and

$$b(g) = \frac{\gamma}{g}\beta\left\{(1-q)\left\{l\left[\frac{b(h_g)}{\gamma h_s}\right]^{\frac{1-3\alpha}{1+\alpha}} + 1\right\}\frac{b(h_g)}{h_s} + q\left\{l\left[\frac{b(g)}{\gamma}\right]^{\frac{1-3\alpha}{1+\alpha}} + 1\right\}b(g)\right\}.$$

First-order condition with respect to s can be written as:

$$\frac{b}{\gamma s} = \left[(1-\alpha)\, l\right]^{\frac{1+\alpha}{3\alpha-1}}. \tag{38}$$

Plug it into the first-order derivative with respect to g, and the latter can be simplified into:

$$\frac{1}{\gamma}\left(b + g\frac{\partial b}{\partial g}\right) + \beta\gamma q\frac{\partial v^n}{\partial g}, \tag{39}$$

where the derivative of real balance is $\frac{\partial b}{\partial g} = \frac{b}{\gamma\beta q\left[2\frac{1-\alpha}{1+\alpha}l\left(\frac{b}{\gamma}\right)^{\frac{1-3\alpha}{1+\alpha}}+1\right]-g}$ and the derivative of value in no-optimization state is $\frac{\partial v^n}{\partial g} = \frac{\frac{b}{\gamma}+\frac{1}{\gamma}\left[g-1+(1-\alpha)l\left(\frac{b}{\gamma}\right)^{\frac{1-3\alpha}{1+\alpha}}\right]\frac{\partial b}{\partial g}}{1-\beta\gamma q}$.

Some manipulations of (39) yield:

$$\frac{\frac{\gamma\beta q}{1-\beta\gamma q}\frac{b}{\gamma}\frac{3+\alpha}{1+\alpha}(1-\alpha)\, l\left(\frac{b}{\gamma}\right)^{\frac{1-3\alpha}{1+\alpha}}}{\gamma\beta q\frac{1-3\alpha}{1+\alpha}l\left(\frac{b}{\gamma}\right)^{\frac{1-3\alpha}{1+\alpha}} - \gamma\beta(1-q)\left[l\left(\frac{b}{\gamma s}\right)^{\frac{1-3\alpha}{1+\alpha}}+1\right]\frac{1}{s}}.$$

For $\alpha > \frac{1}{3}$, the above first-order derivative with respect to g is always negative. Now inspecting the real balance equation, we have $g = \gamma\beta(1-q)\frac{2-\alpha}{1-\alpha}\frac{1}{\gamma}\left[(1-\alpha)\, l\right]^{\frac{1+\alpha}{3\alpha-1}}\frac{1}{b} + \gamma\beta q\left[l\left(\frac{b}{\gamma}\right)^{\frac{1-3\alpha}{1+\alpha}}+1\right]$. For $\alpha > \frac{1}{3}$, the RHS is decreasing in b. As $g \to \beta\gamma q$, $b \to \infty$. Given (38), we know $s \to \infty$.
■

B.2 Issuance costs

Given the unboundedness of the choice for s shown in Proposition 10, we introduce a short-run issuance cost of $-\frac{\zeta}{2}(s-1)^2$. This enables us to get an interior solution and then to characterize the impact of commitment probability q on optimal policies. This cost is what we have in the main text.

Proposition 11 shows how commitment matters for optimal policies. As q increases, the committed long-run money growth rate is less likely to be reoptimized. This means that committing to a low long-run money growth can better sustain token prices today. As the commitment technology becomes more effective, the issuer has a larger incentive to access it in order to support a large issuance today. We show that this implies a larger s. For cases where $\beta\gamma \to 1$, we can in addition show that $g \to 1$ as $q \to 1$. This means that there is no net money growth in the long run without any reoptimization shocks. Such a highly conservative long-run money supply is able to support an infinitely large money growth today.

Proposition 11 *Fix $z = 0$. For $\alpha > \frac{3}{5}$, a probabilistic-commitment issuer facing an issuance cost of $-\frac{\zeta}{2}(s-1)^2$ and being able to pick money growth rates for today (gs) and for the long run (g) chooses a larger s when commitment probability q increases. As $q \to 1$ and $\beta\gamma \to 1$, $s \to \infty$ and $g \to 1$.*

Proof.

Substituting out the optimal velocity choice using first-order conditions, issuer's problem in the optimization state is given by:

$$v = \max_{g,s} \frac{b(g)}{s}\frac{gs-1}{\gamma} + \frac{1+\alpha}{2}\left[\frac{b(g)}{s\gamma}\right]^{\frac{2(1-\alpha)}{1+\alpha}} l - \frac{\zeta}{2}(s-1)^2 + \beta\gamma\left[(1-q)v + qv^n(g)\right],$$

where $l = \left(\frac{1-\alpha}{2\eta}\right)^{\frac{1-\alpha}{1+\alpha}}$ and value in no-optimization is:

$$v^n(g) = b(g)\frac{g-1}{\gamma} + \frac{1+\alpha}{2}\left[\frac{b(g)}{\gamma}\right]^{\frac{2(1-\alpha)}{1+\alpha}} l + \beta\gamma\left[(1-q)v + qv^n(g)\right],$$

and real balance in the long run

$$b(g)=\frac{\gamma}{g}\beta\left\{(1-q)\left\{l\left[\frac{b\left(h_g\right)}{h_s\gamma}\right]^{\frac{1-3\alpha}{1+\alpha}}+1\right\}\frac{b\left(h_g\right)}{h_s}+q\left\{l\left[\frac{b\left(g\right)}{\gamma}\right]^{\frac{1-3\alpha}{1+\alpha}}+1\right\}b\left(g\right)\right\}.$$

First-order condition with respect to s is given by:

$$\frac{b(g)}{s\gamma}-(1-\alpha)\left[\frac{b(g)}{s\gamma}\right]^{\frac{2(1-\alpha)}{1+\alpha}}l-\zeta(s-1)s=0. \tag{40}$$

First-order condition with respect to g is given by:

$$f(g,s)+\frac{\beta\gamma q}{1-\beta\gamma q}f(g,1)=0,$$

where $f(g,s)=\frac{b(g)}{\gamma}+b(g)\frac{\frac{gs-1}{s\gamma}+(1-\alpha)\left[\frac{b(g)}{s\gamma}\right]^{\frac{1-3\alpha}{1+\alpha}}\frac{1}{s\gamma}l}{\gamma\beta q\left\{\frac{2-2\alpha}{1+\alpha}l\left[\frac{b(g)}{\gamma}\right]^{\frac{1-3\alpha}{1+\alpha}}+1\right\}-g}$. Some simplifications of it yield $\frac{\gamma\beta q}{1-\beta\gamma q}\frac{3+\alpha}{1+\alpha}(1-\alpha)l\left[\frac{b(g)}{\gamma}\right]^{\frac{1-3\alpha}{1+\alpha}}+(1-\alpha)\left[\frac{b(g)}{s\gamma}\right]^{\frac{1-3\alpha}{1+\alpha}}\frac{1}{s}l-\frac{1}{s}=0$, which, combined with (40), leads to:

$$\frac{b\left(g\right)}{\gamma}=\left[\frac{\zeta s(s-1)}{\frac{\gamma\beta q}{1-\beta\gamma q}\frac{3+\alpha}{1+\alpha}(1-\alpha)l}\right]^{\frac{1+\alpha}{2-2\alpha}}. \tag{41}$$

Here we know in an interior equilibrium $s>1$. Plug it back to (40) and we have

$$\left[\frac{t}{(1-\alpha)l}\right]^{\frac{1+\alpha}{2-2\alpha}}(s-1)^{\frac{3\alpha-1}{2-2\alpha}}s^{\frac{5\alpha-3}{2-2\alpha}}-s^{\frac{2\alpha-2}{1+\alpha}}t-\zeta=0 \tag{42}$$

where $t=\frac{\zeta}{\frac{\gamma\beta q}{1-\beta\gamma q}\frac{3+\alpha}{1+\alpha}}$. For $\alpha\in(\frac{3}{5},1)$, the LHS is increasing in s. Now we would like to show that it is also increasing in t. Differentiate it with respect to t:

$$\frac{1+\alpha}{2-2\alpha}\left[\frac{1}{(1-\alpha)l}\right]^{\frac{1+\alpha}{2-2\alpha}}t^{\frac{3\alpha-1}{2-2\alpha}}(s-1)^{\frac{3\alpha-1}{2-2\alpha}}s^{\frac{5\alpha-3}{2-2\alpha}}-s^{\frac{2\alpha-2}{1+\alpha}}. \tag{43}$$

We can now show that (43) is strictly positive at the optimum. By continuity, we then know that starting from an inner optimum, a marginal increase in q (and thus a decrease in t) would imply a marginal increase in s. First-order condition implies that $\left[\frac{1}{(1-\alpha)l}\right]^{\frac{1+\alpha}{2-2\alpha}}(s-1)^{\frac{3\alpha-1}{2-2\alpha}}s^{\frac{5\alpha-3}{2-2\alpha}}t^{\frac{3\alpha-1}{2-2\alpha}}=s^{\frac{2\alpha-2}{1+\alpha}}+\frac{\zeta}{t}$ and thus (43) evolves into $\frac{1+\alpha}{2-2\alpha}\left(s^{\frac{2\alpha-2}{1+\alpha}}+\frac{\zeta}{t}\right)-s^{\frac{2\alpha-2}{1+\alpha}}=\frac{3\alpha-1}{2-2\alpha}s^{\frac{2\alpha-2}{1+\alpha}}+\frac{1+\alpha}{2-2\alpha}\frac{\zeta}{t}$, which is clearly positive for $\alpha>\frac{3}{5}$.

As $\beta\gamma\to 1$ and $q\to 1$, $t\to 0$, thus for first-order conditions to hold, we know $s\to\infty$ and $b\to\infty$. Rearrange (42) into $\left[\frac{t}{(1-\alpha)l}\right]^{\frac{1-3\alpha}{2-2\alpha}}=\left\{\left(s\zeta+s^{\frac{3\alpha-1}{1+\alpha}}t\right)[(s-1)s]^{\frac{1-3\alpha}{2-2\alpha}}\right\}^{\frac{1-3\alpha}{1+\alpha}}$, which combined with (41) implies that for $\alpha>\frac{3}{5}$:$\left(\frac{b}{s\gamma}\right)^{\frac{1-3\alpha}{1+\alpha}}=\left[\frac{t}{(1-\alpha)l}(s-1)s\right]^{\frac{1-3\alpha}{2-2\alpha}}s^{\frac{3\alpha-1}{1+\alpha}}=\left(\zeta+s^{\frac{2\alpha-2}{1+\alpha}}t\right)^{\frac{1-3\alpha}{1+\alpha}}[(s-1)s]^{\frac{1-3\alpha}{1+\alpha}}\to 0$. Therefore, the equation for real balance implies that:

$$g=\gamma\beta\left\{(1-q)\left[l\left(\frac{b}{s\gamma}\right)^{\frac{1-3\alpha}{1+\alpha}}+1\right]\frac{1}{s}+q\left[l\left(\frac{b}{\gamma}\right)^{\frac{1-3\alpha}{1+\alpha}}+1\right]\right\}\to 1.$$

■

C Homogeneity

We work with the scaled version of our model in analyses. Here we establish the linearity of our baseline model with probabilistic commitment in Section 4.1. The linearity of the other models can be shown in a similar way. The problem for the issuer with probabilistic commitment in a state where he is allowed to pick money supply freely is given by:

$$\begin{aligned}V(M,A,z)=\max_{M',\omega}\ & p(M',M,A,z)(M'-M)+\omega F(\omega,p(M',M,A,z),M,A)\\ &+\beta\mathbf{E}_{z'|z}\left[(1-q)V(M',A',z')+qV^n(M',A',z',M)\right]\end{aligned}\tag{44}$$

while that in a state where an optimization is not allowed (with superscript n) is given by:

$$\begin{aligned}V^n(M,A,z,M_{-1})=\max_{\omega}\ & p\left(h^n_M,M,A,z\right)\left(h^n_M-M\right)+\omega F(\omega,p\left(h^n_M,M,A,z\right),M,A)\\ &+\beta\mathbf{E}_{z'|z}\left[(1-q)V(h^n_M,A',z')+qV^n(h^n_M,A',z',M)\right],\end{aligned}\tag{45}$$

where money supply in the state of no optimization is $h^n_M=\frac{M^2}{M_{-1}}$. Fee equation is given by:

$$F(\omega,p,M,A)=(pM)^{1-\alpha}\left(\frac{\omega}{A}\right)^{-\alpha}-\eta A\omega.$$

Pricing equation are given by:

$$
\begin{aligned}
& p(M', M, A, z) \\
& = \beta \mathbf{E}_{z'|z} \Bigg\{ (1-q) \Bigg\{ p(h_M(M', A', z'), M', A', z') \\
& \qquad\qquad + [p(h_M(M', A', z'), M', A', z') h_\omega(M', A', z')]^{1-\alpha} \left(\frac{M'}{A'} \right)^{-\alpha} \Bigg\} \\
& \qquad + q \Bigg\{ p^n(h^n_M(M', z', M), M', A', z') \\
& \qquad\qquad + [p^n(h^n_M(M', z', M), M', A', z') h^n_\omega(M', A', z', M)]^{1-\alpha} \left(\frac{M'}{A'} \right)^{-\alpha} \Bigg\} \Bigg\},
\end{aligned}
$$

where $h_M(M, A, z)$ and $h_\omega(M, A, z)$ are policies in optimization state, given by (44); $h^n_M(M, A, z, M_{-1})$ is policy for velocity in no-optimization state, given by (45).

Divide the left- and right-hand side (RHS) of (44) and (45) by A, and we get:

$$
\begin{aligned}
\frac{V(M, A, z)}{A} = & \max_{M', \omega} \frac{p(M', M, A, z) M}{A_{-1} \gamma \exp(z)} \left(\frac{M'}{M} - 1 \right) + \omega f(\omega, p(M', M, A, z), M, A) \\
& + \beta \gamma \mathbf{E}_{z'|z} \exp(z') \left[(1-q) \frac{V(M', A', z')}{A'} + q \frac{V^n(M', A', z', M)}{A'} \right],
\end{aligned} \tag{46}
$$

and

$$
\begin{aligned}
\frac{V^n(M, A, z, M_{-1})}{A} = & \max_{\omega} \frac{p\left(h^n_M, M, A, z\right) M}{A_{-1} \gamma \exp(z)} \left(\frac{h^n_M}{M} - 1 \right) + \omega f(\omega, p\left(h^n_M, M, A, z\right), M, A) \\
& + \beta \gamma \mathbf{E}_{z'|z} \exp(z') \left[(1-q) \frac{V(h^n_M, A', z')}{A'} + q \frac{V^n(h^n_M, A', z', M)}{A'} \right],
\end{aligned} \tag{47}
$$

where $h^n_M = \frac{M^2}{M_{-1}}$, and the fee equation is: $f(\omega, p, M, A) = \left[\frac{pM}{A_{-1}} \frac{1}{\gamma \exp(z)} \right]^{1-\alpha} \omega^{-\alpha} - \eta \omega$.

Pricing equation can be rewritten as:

$$
\begin{aligned}
&\frac{p(M', M, A, z)M}{A_{-1}} \qquad (48)\\
&= \frac{M}{M'}\frac{A}{A_{-1}}\beta \mathbf{E}_{z'|z}\Bigg\{(1-q)\Bigg[\frac{p(h_M(M', A', z'), M', A', z')M'}{A} \\
&\qquad + \left[\frac{p(h_M(M', A', z'), M', A', z')M'}{A} h_\omega\left(M', A', z'\right)\right]^{1-\alpha}\left(\frac{A}{A'}\right)^{-\alpha}\Bigg] \\
&\qquad + q\Bigg[\frac{p^n(h^n_M(M', z', M), M', A', z')M'}{A} \\
&\qquad + \left[\frac{p^n(h^n_M(M', z', M), M', A', z')M'}{A} h^n_\omega\left(M', A', z', M\right)\right]^{1-\alpha}\left(\frac{A}{A'}\right)^{-\alpha}\Bigg]\Bigg\}.
\end{aligned}
$$

We conjecture that: (i) $\frac{p(M',M,A,z)M}{A_{-1}}$ only depends on z and $\frac{M'}{M}$; (ii) $\frac{V(M,A,z)}{A}$ only depends on z; and (iii) $\frac{V^n(M,A,z,M_{-1})}{A}$ only depends on z and $\frac{M}{M_{-1}}$.

Define $g = \frac{M'}{M}$. Under conjecture (i), we know that $\omega f(\omega, p\left(M', M, A, z\right), M, A)$ depends only on ω, z, g. Combining with conjectures (ii) and (iii), we know that the RHS of (46) depends on g, ω, z. As a result, we can verify conjecture (ii) and also recognize that $h_g(M, A, z) \equiv \frac{h_M(M,A,z)}{M}$ and $h_\omega(M, A, z)$ only depends on z. In a similar fashion, under conjecture (i), we know that $\omega f(\omega, p\left(h^n_M, M, A, z\right), M, A)$ depends only on $\omega, z, \frac{h^n_M}{M} = \frac{M}{M_{-1}} = g_{-1}$. Moreover, combine it with conjectures (ii) and (iii), we know that the RHS of (47) depends on g_{-1}, ω, z. As a result, we can verify conjecture (iii) and also recognize that $h^n_\omega(M, A, z)$ only depends on g_{-1} and z. Lastly, because we know that $h^n_\omega(M, A, z)$ only depends on g_{-1} and z and $h_\omega(M, A, z)$ only depends on z, we can in addition use conjecture (i) to establish that the RHS of (48) depends on g and z, which verifies conjecture (i).