Asset Pricing in Production Economies

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Abstract

This paper studies the interaction between asset returns and business cycles in different versions of the one-sector stochastic growth model. We show that a model with capital adjustment costs, habit formation preferences and financial leverage predicts equity risk premia of several percentage points. In demonstrating this result the paper also presents a method that allows one to solve for asset returns in a large class of macroeconomic models. Keywords: Equity premium; real business cycles; lognormal distribution.

1 Introduction

The business cycle behavior of asset returns has recently attracted a great deal of attention, both from financial economists and from macroeconomists.

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In financial economics, several empirical studies have documented important cyclical variations in various security returns and risk premia, but little is known to date about the origins of these variations.[1] In macroeconomics, asset returns have long played an important role as leading economic indicators, recent empirical studies emphasize these relationships.[2]

The dominant analytical framework for studying the relationships between asset returns and economic fluctuations is based on the endowment model of Lucas (1978). This model is a clever theoretical abstraction, since it permits the study of asset returns in a parsimonious intertemporal framework. This framework nevertheless has its limitations. First, the model lacks realism, in that dividends and consumption are equal to endowment, the later being specified exogenously. Second, because payouts and consumption are exogenous, an endowment model offers no explanation for what actually determines these quantities. Studying the effects of economic shocks (productivity, fiscal or monetary) on asset returns requires a model where consumption and production is determined endogenously. Third, endowment models offer no predictions about how asset returns are related to different macroeconomic quantities. Understanding what the macroeconomic risk factors are requires models that generate a rich set of predictions for the relationships of asset returns with various macroeconomic quantities. To contribute to a better understanding of the determinants of asset returns, this paper undertakes a quantitative study of asset returns in economies where production is explicitly modeled. In particular, we focus on whether different versions of the one sector real business cycle model, driven by productivity shocks, can explain historical return premia. Compared with endowment models, dividend flows are derived from more realistic assumptions about the nature of production and investment activity by firms, and by considering financial leverage. In addition, the specification of production along standard neoclassical lines permits the study of how consumption profiles respond to changes in preferences.[3] Naturally, such general equilibrium production models can never do better than models that specify consumption and payout processes

[1] A few examples of these studies are: Fama and French (1989), Fama (1990) and Schwert (1989, 1990).

[2] Some of these studies are: Chen (1991), Estrella and Hardouvelis (1991) and Stock and Watson (1989).

[3] Cochrane (1991, 1992a) presents another approach for intertemporal asset pricing. He evaluates asset pricing relationships derived from producers' first order conditions.

exogenously; the question of our study is therefore if one can obtain endogenous processes that explain the stylized facts of asset returns.

Prior research based on endowment models has mainly focused on modifications of the standard model that can solve Mehra and Prescott's (1985) "equity premium puzzle." A large body of literature has devised several modifications of preferences and payout structure that solve the equity premium puzzle.[4] However, extensions to models with nontrivial production sectors were less successful along this line (e.g., Danthine, Donaldson and Mehra (1992), Rouwenhorst (1991)). Indeed, Rouwenhorst (1991) finds that it is more difficult to explain substantial risk premia because endogenous consumption becomes even smoother as risk aversion is increased. The reason behind this is that in the standard one-sector model agents can easily alter their production plans to reduce fluctuations in consumption. This suggests that the frictionless and instantaneous adjustment of the capital stock is a major weakness in this framework. One way to overcome this is to introduce capital adjustment costs. Capital adjustment costs have a long tradition in the investment literature, they also provide a formal framework for the popular "q" theory (q is defined as the value of the capital stock divided by its replacement cost). It therefore seems natural to introduce capital adjustment costs into this standard framework. In fact, without capital adjustment costs, as most current business cycle models are, these models are plagued by a counterfactual constant q. Given its previous success in solving the equity premium puzzle in models with trivial production sectors (e.g. Abel (1990), Constantinides (1990)) our analysis also includes habit formation preferences, in addition to the standard time-separable specification. We find that an otherwise standard model with capital adjustment costs, that are consistent with realistic investment volatility and in the range of empirically plausible values, and habit formation preferences can generate equity premia of several percentage points. Moreover, our study demonstrates that financial leverage does also help explain the historic excess returns of stocks over long term bonds.

To study these points, the paper develops a framework for computing asset returns and for thinking about the cyclical variation of asset returns

[4] An incomplete list of studies that propose solutions to the equity premium puzzle is Benninga and Protopapadakis (1990), Abel (1990), Constantinides (1990), Rietz (1988) and Mankiw (1986).

that is applicable to a broad range of macroeconomic theories. This is done by combining results from two earlier lines of research in finance and macroeconomics. First, asset pricing formulae based on the conditional lognormal distribution have long been used in finance. More recently, Campbell (1986) uses such formulae to value securities with single-period payouts (simple securities) in a general equilibrium context. A key virtue of these formulae is that one can readily understand the determinants of risk premia in terms of the covariation of a security's payouts with a basic valuation. Second, many dynamic rational expectations macroeconomic models have a loglinear reduced form.[5] Combining these two theoretical frameworks delivers pricing relations that highlight the macroeconomic determinants of asset returns. However, lognormal formulae and loglinear models also imply that simple securities (with a single-period payout) have risk premia which do not vary with the stages of the business cycle. Yet, as the analysis below makes clear, most standard securities, such as common stocks and coupon bonds, are complex securities, *i.e.*, they can be viewed as portfolios of simple securities. Consequently, we are able to show that expected returns and risk premia of complex securities vary as a function of macroeconomic factors.

The organization of the paper is as follows. Section 2 presents general analytical results obtained in the loglinear asset pricing framework. Section 3 discusses consumption-based asset pricing, by comparing Campbell's (1986) model with time-separable preferences to a version of the Constantinides (1990) model with non-time-separable preferences. Section 4 studies risk premia of asset returns in a one-sector real business cycle model with different preference specifications. Section 5 contains conclusions and directions for future work.

2 Loglinear asset pricing, analytical results

The lognormal distribution has enjoyed considerable popularity in the intertemporal asset pricing literature. Even though this distributional assumption may be too restrictive, it is attractive because it permits an analytical solution to the intertemporal efficiency condition. Such closed form solutions

[5] Real business cycle models along the lines of King, Plosser and Rebelo (1988) have the required loglinear reduced form; many other examples of models are discussed later in the paper.

are desirable because they provide insights not possible with the alternative approach of numerical approximation methods. Closed form solutions are also easier and faster to use; and this allows one to solve models with several state variables in much less time than with alternative methods that use iterative procedures.

Notable, recent studies that use lognormality include Hansen and Singleton (1983) and Campbell (1986). The framework presented here extends Campbell (1986) along two dimensions. First, we use a multivariate setting where the asset's payouts and its valuation are clearly distinct, as opposed to endowment models where dividends are equal to endowment and the valuation is solely determined by endowment and risk preferences. Second, we will introduce composite securities with multiple payouts. In addition to being more general, this last extension is shown to allow for time variation in conditional excess returns and variances. Despite these extensions the intuitive relationships between asset returns and business cycle are generally preserved.[6]

2.1 Returns for assets with a single payout

Intertemporal asset pricing uses the fact that any claim to a potentially random future payout D_{t+k} can be valued by the present value relationship:

$$V_t[D_{t+k}] = \frac{\beta^k E_t[\Lambda_{t+k} D_{t+k}]}{\Lambda_t}, \tag{1}$$

where β is the pure time discount factor, and Λ_{t+k} the marginal valuation of the numeraire at $t+k$.

The structure of our model economy is as follows. The logarithm of the dividend payout $d_t = log(D_t)$, and the logarithm of a valuation $\lambda_t = \log(\Lambda_t)$ are assumed to be linear combinations of the logarithm of the state vector s_t. The state vector is assumed to follow a first order autoregressive process with multivariate normal i.i.d. impulses:

$$\begin{aligned} \lambda_t &= l_\lambda s_t \\ d_t &= l_d s_t \\ s_t &= M s_{t-1} + \epsilon_t \end{aligned} \tag{2}$$

[6]Parts of this section are based on some research notes of R. G. King and G. K. Rouwenhorst.

where the square matrix M governs the dynamics of the system. We require the system to be stationary or $I(1)$ nonstationary. Thus, the characteristic roots of M have modulus less than or equal to one. The factor loadings l_λ and l_d are row vectors. Given normality of the impulses, the dividend and its valuation are conditionally lognormal. The linear homoskedastic nature of the system implies that conditional second moments of d_t and λ_t are constant, a result that is used repeatedly below.

Expected returns

We will first consider the case of assets with a single payout, which we call "strips". This term refers to the actual practice in security firms of stripping bonds by separating coupon and principal and selling them separately as zero-coupon bonds. One reason for studying strips is that they are easier to think about; more complex securities are constructed as bundles of strips. For a k-period strip, we define the one period holding return as:

$$R_{t,t+1}[D_{t+k}] = \frac{V_{t+1}[D_{t+k}]}{V_t[D_{t+k}]} \tag{3}$$

In order to solve for the conditional expected return $E_t(R_{t,t+1}[D_{t+k}])$ we first take the expectation of $V_{t+1}[D_{t+k}]$ at time t. Under the assumption of lognormality, this gives:

$$\begin{aligned} E_t(V_{t+1}[D_{t+k}]) &= \beta^{k-1} E_t \exp(E_{t+1}(d_{t+k} + \lambda_{t+k} - \lambda_{t+1}) + \tfrac{1}{2} \operatorname{var}_{t+1}(d_{t+k} + \lambda_{t+k})) \\ &= \beta^{k-1} \exp(E_t(d_{t+k} + \lambda_{t+k} - \lambda_{t+1})) \\ &\quad \times \exp(\tfrac{1}{2} \operatorname{var}_t(E_{t+1}(d_{t+k} + \lambda_{t+k} - \lambda_{t+1})) + \tfrac{1}{2} \operatorname{var}_{t+1}(d_{t+k} + \lambda_{t+k})) \end{aligned} \tag{4}$$

Combining this with the valuation of $V_t[D_{t+k}]$, we get after some manipulation that:

$$\begin{aligned} E_t(R_{t,t+1}[D_{t+k}]) &= R_{t,t+1}[1_{t+1}] \\ &\quad \times \exp(-\operatorname{cov}_t(\lambda_{t+1}, E_{t+1}\lambda_{t+k} - \lambda_{t+1})) \\ &\quad \times \exp(-\operatorname{cov}_t(\lambda_{t+1}, E_{t+1}d_{t+k})). \end{aligned} \tag{5}$$

Equation (5) shows that the conditional expected return of a claim to D_{t+k} can usefully be divided into three components, which we discuss in turn below.

Determinants of expected returns

Risk free rate: The first term in equation (5), $R_{t,t+1}[1_{t+1}] = 1/V_t[1_{t+1}]$, represents the return to holding a one-period bond until maturity, *i.e.*, the risk free rate. While this investment is riskless with respect to its expected return, uncertainty nevertheless affects the level of this return. To see this, rewrite the risk free rate as:

$$R_{t,t+1}[1_{t+1}] = \beta^{-1}\exp(\lambda_t - E_t\lambda_{t+1} - \frac{1}{2}\operatorname{var}_t(\lambda_{t+1})). \tag{6}$$

The first two components in the exponential function capture the standard effect of the intertemporal marginal rate of substitution on equilibrium interest rates. In particular, the less future payouts are valued with respect to present payouts, $E_t\lambda_{t+1} < \lambda_t$, the higher is the equilibrium interest rate. The variance term in (6) stands for precautionary effects, in the sense that it reflects how uncertainty affects the optimal decision and hence the equilibrium interest rate. Intuitively, the higher is uncertainty, the less agents need to be compensated for postponing their consumption.[7]

Term and payout uncertainty premia: The second term of equation (5),
$-\operatorname{cov}_t(\lambda_{t+1}, E_{t+1}\lambda_{t+k} - \lambda_{t+1})$, can be thought of as a holding or term premium for a k-period discount bond that depends on the term structure of interest rates. Indeed, if one expects capital gains as a consequence of lower interest rates (this is the case when $E_{t+1}\lambda_{t+k}$ exceeds λ_{t+1}) to be contemporaneous with low valuation, then this unfavorable correlation has to be compensated by a positive risk premium. Finally, the third element of (5), $-\operatorname{cov}_t(\lambda_{t+1}, E_{t+1}d_{t+k})$, is again linked to a possible capital gain or loss at time $t+1$. If capital gains (higher than average expected payouts at time $t+k$) are negatively correlated with the valuation, then again a risk premium is needed to compensate the investor for the undesirable cyclical property of this asset. In our subsequent discussion we will call these two covariance terms respectively the term premium, $\eta_t(k)$, and the payout uncertainty premium, $\eta_p(k)$.

[7] This is related to the precautionary savings effect discussed in the literature, for instance Kimball (1990). In general, this result is linked to the assumption of convex marginal utility of consumption. If we assume that marginal utility is equal to the exponential of a linear combination of the states, marginal utility becomes a convex function of the log of the state itself.

The sum of these two is defined as the dividend premium, $\eta_d(k)$. However, it should be clear that without additional information about the stochastic processes of dividends and marginal valuations nothing can be said about the sign or the magnitude of these premia.

At this stage, it is useful to summarize some results that characterize expected returns for strips. First, since the risk free rate varies over the business cycle, conditional expected returns also vary over the business cycle. Second, excess returns on bond and dividend strips, $E_t(R_{t,t+1}[D_{t+k}])/E_t(R_{t,t+1}[1_{t+1}])-1$, are constant. This follows directly from the fact that conditional second moments in the state space system are constant. These two results together make that the predictable components of bond and dividend strip returns are all proportional to the risk free rate.

Conditional variance

The tractability of the lognormal distribution permits sharp insights into the relationship between the volatility of asset returns and the properties of business cycles. Computing the conditional variance of returns can be accomplished using the standard formula for the variance of a lognormal variable. That is, if $\log(Z)$ is $N(\mu,\sigma)$ then $\mathrm{var}(Z)=\exp(2\mu+\sigma^2)(\exp(\sigma^2)-1)$. A more direct calculation uses the fact that the one-period return can be decomposed into two parts in the following way:

$$R_{t,t+1}[D_{t+k}] = E_t(R_{t,t+1}[D_{t+k}])\frac{V_{t+1}[D_{t+k}]}{E_t(V_{t+1}[D_{t+k}])}. \tag{7}$$

The second part of the right hand side, the revision of the initial price forecast, can be written as:

$$\exp(E_{t+1}(d_{t+k}+\lambda_{t+k}-\lambda_{t+1})-E_t(d_{t+k}+\lambda_{t+k}-\lambda_{t+1})-\frac{1}{2}\,\mathrm{var}_t(E_{t+1}(d_{t+k}+\lambda_{t+k}-\lambda_{t+1}))). \tag{8}$$

It then follows directly that:

$$\mathrm{var}_t(R_{t,t+1}[D_{t+k}]) = E_t(R_{t,t+1}[D_{t+k}])^2\left(\exp(\mathrm{var}_t[E_{t+1}(d_{t+k}+\lambda_{t+k}-\lambda_{t+1})])-1\right). \tag{9}$$

Equation (9) shows how the volatility of revisions of expectations in the underlying system directly translates into conditional variability for returns.

2.2 Returns for assets with multiple payouts

Next, we consider an asset which represents a claim to an infinite sequence of possibly random dividends:

$$V_t[\{D_{t+k}\}_{k=1}^{\infty}] = \sum_{k=1}^{\infty} V_t[D_{t+k}]. \tag{10}$$

This definition corresponds to a common stock or, for constant payouts, to a perpetual bond. Assets with a finite number of payouts, for instance coupon bonds, can be analyzed in the same way. The one-period holding return can be written as:

$$R_{t,t+1}[\{D_{t+k}\}_{k=1}^{\infty}] = \frac{V_{t+1}[\{D_{t+k}\}_{k=1}^{\infty}]}{V_t[\{D_{t+k}\}_{k=1}^{\infty}]} = \sum_{k=1}^{\infty} w_t[D_{t+k}]R_{t,t+1}[D_{t+k}] \tag{11}$$

with $w_t[D_{t+k}] = V_t[D_{t+k}]/V_t[\{D_{t+k}\}_{k=1}^{\infty}]$ being the portfolio weight attached to the date $t+k$ dividend strip return.

Expected returns: Equation (11) says that holding a common stock can be viewed as holding a portfolio of dividend strips. This representation is useful because the portfolio weights are known at time t, and for a one-period-ahead conditional expectation, they can be passed through the expectation operator. Taking the conditional expectation we obtain that:

$$E_t(R_{t,t+1}[\{D_{t+k}\}_{k=1}^{\infty}]) = R_{t,t+1}[1_{t+1}] \sum_{k=1}^{\infty} w_t[D_{t+k}] \exp(\eta_d(k)) \tag{12}$$

which shows that the expected return of a common stock is equal to the risk free rate times a weighted sum of the exponentials of the dividend strip premia $\eta_d(k)$. While the strip premia do not vary over the business cycle, the cyclic variation in the weights leads to time-varying conditional excess returns, so long as the premia structure over different time horizons is not horizontal (*i.e.*, $\eta_d(k) \neq \eta_d$ for all k). Equation (12) also highlights the fact that the size of the equity premium is directly related to the size of the individual strip premia. Also, weights generally decrease as a function of the time horizon, which means that a sizeable equity premium requires large strip premia for dates in the near future.

Conditional variance: The conditional variance of the common stock return is approximately equal to:

$$\mathrm{var}_t(R_{t,t+1}[\{D_{t+k}\}_{k=1}^{\infty}]) \approx \mathrm{var}_t\left(\sum_{k=1}^{\infty} w_t[D_{t+k}]R_{t,t+1}[D_{t+k}]l_e(k)\epsilon_{t+1}\right) \qquad (13)$$

where $l_e(k)\epsilon_{t+1}$ is equal to $E_{t+1}(d_{t+k}+\lambda_{t+k}-\lambda_{t+1}) - E_t(d_{t+k}+\lambda_{t+k}-\lambda_{t+1})$, as shown in equation (8).[8] Since ϵ_{t+1} is the only uncertain element inside the brackets, we can easily rewrite the conditional variance as:

$$\mathrm{var}_t(R_{t,t+1}[\{D_{t+k}\}_{k=1}^{\infty}]) \approx R_{t,t+1}[1_{t+k}]^2\,[\pi(t)]\sum\nolimits_{\epsilon\epsilon}[\pi(t)]'$$

$$\textit{with}\ \ \pi(t) = \sum_{k=1}^{\infty} w_t[D_{t+k}]\exp(\eta_d(k))l_e(k). \qquad (14)$$

This expression highlights two important points. First, shifts of the weights have the potential to generate time variation in the conditional variance. Second, the variance-covariance matrix of the impulses of the linear system,$\sum_{\epsilon\epsilon}$, is directly related to the conditional return volatility.

3 Applications of loglinear asset pricing

This section begins with a review of prior work based on the assumption of lognormality, and discusses how it relates to our asset pricing framework. Next, we develop a model which incorporates non-time-separable preferences. Finally, we compare the size of premia for time-separable and habit formation preferences in a loglinear endowment model.

3.1 Consumption based asset pricing

The standard approach to intertemporal asset pricing follows Lucas (1978) by assuming an endowment economy. Campbell (1986) analyzes a version of this framework with time-separable, isoelastic period utility of consumption. He also assumes that the endowment is conditionally lognormal, and

[8] The approximation uses $\exp(x) \approx 1+x$. For instance, if x is normal and has a standard deviation of 0.1525 (approximately the standard deviation of yearly stock returns) then this approximation would give 0.15.

that its logarithm follows a univariate linear process with i.i.d. normal impulses. His study focuses on the determinants of risk premia, and provides a starting point for our work. In our notation this model structure amounts to $\lambda_t = -\tau c_t$, with τ being the coefficient of relative risk aversion, with the equilibrium condition that $d_t = c_t$. Equation (5) can then be written as:

$$\begin{aligned} E_t(R_{t,t+1}[D_{t+k}]) &= \beta^{-1} \exp(-\tau(c_t - E_t c_{t+1}) - \tfrac{\tau^2}{2} \operatorname{var}_t(c_{t+1})) \\ &\quad \times \exp(-\tau^2 \operatorname{cov}_t(c_{t+1}, E_{t+1}c_{t+k} - c_{t+1})) \\ &\quad \times \exp(\tau \operatorname{cov}_t(c_{t+1}, E_{t+1}c_{t+k})). \end{aligned} \tag{15}$$

This relationship between strip premia and consumption dynamics is one of the main results of Campbell's paper. Equation (15) makes it clear that term premia (the first covariance term) depend on the covariance between consumption and expected consumption growth, whereas payout uncertainty premia (the second covariance term) depend on the covariance between consumption and the level of expected future consumption. It also shows that risk aversion, τ, enters term premia by its squared value but affects the payout premia only proportionally, a result that is further discussed below.

The advantage of simple closed-form solutions is of course even more appreciable when models are to be estimated. For instance, Hansen and Singleton (1983) and Singleton (1990) combine time-separable, isoelastic period utility of consumption with the assumption that consumption growth rates and asset returns are jointly lognormally distributed; this implies constant conditional variances and conditional means that are linear functions of past realizations. To translate this framework into our notation, we can think about asset returns as one-period dividend strips, appropriately scaled such that $V_t[D_{t+k}] = 1$. Equation (5) for conditional expected mean returns then becomes:

$$\begin{aligned} E_t(R_{t,t+1}) &= \beta^{-1} \exp(\tau(c_t - E_t c_{t+1}) - \tfrac{\tau^2}{2} \operatorname{var}_t(c_{t+1})) \\ &\quad \times \exp(\tau \operatorname{cov}_t(c_{t+1}, \log R_{t,t+1})). \end{aligned} \tag{16}$$

Equation (16) shows that, given constant conditional covariances, the ratio of any two asset returns (and therefore excess returns) are constant. Empirical evidence clearly rejects this restriction. For instance, Singleton (1990) shows that a successful model needs to be able to allow for considerable differences in autocorrelation properties across different assets. However, as we showed above, multiple-payout assets are potentially consistent with these findings.

3.2 Nonseparable preferences

More recently, nonseparable preferences have attracted considerable attention as a possible means of reconciling intertemporal asset pricing models with empirical evidence. Contemporaneous non-separabilities can easily be handled in our framework. For instance, when leisure enters the period utility function along with consumption, the valuation Λ_t is simply the marginal utility of consumption which now also depends on leisure. Other nonseparabilities such as nontraded goods or household production can be treated in the same way.

Non-time-separable preferences are even more popular in the asset pricing literature; these also seem a priori to be more challenging.[9] Indeed, the prior framework does not appear to be directly applicable because the marginal valuation of time t consumption expenditure is typically a random variable at time t. For this reason, additional variance terms in the denominator of an asset pricing formula could complicate the setup. Fortunately, the loglinear-lognormal structure renders these considerations irrelevant.

To see this, define lifetime marginal utility of time t consumption expenditure as $H_{t,t+s}$. This notation suggests that consumption expenditure at t has effects not only at time t, but also at $t+1$, $t+2$,..., which is typically the case for a habit formation specification. Assume, then, that $h_{t,t+s} = \log(H_{t,t+s})$ is equal to a distributed lead of the log of the state vector:

$$h_{t,t+s} = \sum_{j=0}^{s} l_h(j) s_{t+j}. \tag{17}$$

For asset pricing we need to evaluate

$$\begin{aligned} E_t(H_{t,t+s}) &= E_t \exp(h_{t,t+s}) \\ &= \exp(E_t h_{t,t+s} + \tfrac{1}{2}\operatorname{var}_t(h_{t,t+s})) \end{aligned} \tag{18}$$

which is implied by the lognormality assumption. Given that an additional variance term is also in the numerator of any pricing formula like equation (1), and given that these variances are time-independent, these variance terms cancel out, as we show in the appendix. Thus we can use $\Lambda_t = \exp(E_t h_{t,t+s})$ as the valuation, and all the previously obtained results apply directly.

[9] Examples include: Abel (1990), Cochrane and Hansen (1992), Constantinides (1990), Ferson and Constantinides (1991), Heaton (1993) and Sundaresen (1989).

Habit formation and the valuation

Various authors have documented the ability of habit formation preferences to generate high premia (see footnote 9). We can easily replicate this argument in our framework, illustrating the relationship between habit formation, valuation and strip premia. Taking a simple habit formation specification, $u(C_t - \alpha C_{t-1})$, h_t can be approximated locally by

$$\frac{\alpha\tau}{(1-\alpha)(1-\alpha\beta)}c_{t-1} - \frac{(1+\alpha^2\beta)\tau}{(1-\alpha)(1-\alpha\beta)}c_t + \frac{\alpha\beta\tau}{(1-\alpha)(1-\alpha\beta)}c_{t+1} \tag{19}$$

where c_t is the log of consumption expenditure.[10] Equation (19) reveals that marginal utility of current consumption expenditure depends on past and future expenditure as well as on current consumption expenditure. For an endowment model we can define the state s_t to be equal to $[c_t, c_{t-1}]'$, which puts the problem into the form of equation (17).

There are quantitatively important implications of (19) for the relationship between consumption and realized marginal utility h_t and, hence, for the links between consumption and the marginal valuation, Λ_t.

Changes in consumption at all dates: If consumption changes by an equal amount at all dates, then it follows that the elasticity $\partial h/\partial c$ is simply $-\tau$. That is, as argued by Ferson and Constantinides (1991), τ is approximately equal to relative risk aversion with respect to atemporal wealth bets, in which consumption at all future dates is a choice variable. In other words, $-\tau$ is the long-run elasticity of marginal utility with respect to permanent changes in consumption.

Changes in consumption just at date t: If consumption changes just at date t, then for a habit formation parameter of $\alpha = 0.6$, a value discussed later in the paper, and for $\beta = 0.96$, the elasticity is now

$$\frac{\partial h_t}{\partial c_t} = -\frac{1+\alpha^2\beta}{(1-\alpha)(1-\alpha\beta)}\tau \approx -8\tau.$$

[10]Equation (19) is obtained from the first order condition with respect to consumption expenditure in an infinite horizon model under certainty: $H_t = Du(C_t - \alpha C_{t-1}) - \alpha\beta Du(C_{t+1} - \alpha C_t)$. We then take a first-order Taylor series approximation of the log of this expression, and use the fact that when $u(C_t - \alpha C_{t-1}) = (1-\tau)^{-1}(C_t - \alpha C_{t-1})^{1-\tau}$ then $-D^2u(.)(1-\alpha)C/Du(.) = \tau$.

One can think about this elasticity as short term risk aversion with respect to temporary consumption changes. Thus, with respect to temporary changes in current consumption, risk aversion is effectively 8 times larger than it is with respect to wealth changes.

Changes in consumption at t and t+1: Suppose that consumption changes at time t and $t+1$ are linked by $dc_{t+1} = xdc_t$. Then the elasticity of marginal utility with respect to consumption is

$$\frac{\partial h_t}{\partial c_t} + x\frac{\partial h_t}{\partial c_{t+1}} = -\frac{1+\alpha^2\beta - x\alpha\beta}{(1-\alpha)(1-\alpha\beta)}\tau.$$

For changes that are expected to be permanent ($x = 1$), it then follows that the elasticity is $\partial h_t/\partial c_t \approx -4.5\tau$. Hence, by varying the extent to which changes are perceived to be permanent, we can alter the extent of effective risk aversion with respect to consumption changes from 8τ (with $x = 0$) to 4.5τ (with $x = 1$). Intuitively, this result reflects the idea that with consumption habits an agent is willing to pay more *ceteris paribus* to avoid a risk that disturbs these habits.

Habit formation and strip premia

The strip pricing formula indicated that there are two determinants of the equity premium which we reproduce here for convenience. First, there are the holding or term premia:

$$\eta_t(k) = -\operatorname{cov}_t(\lambda_{t+1}, E_{t+1}\lambda_{t+k} - \lambda_{t+1}).$$

Second, there are the payout uncertainty premia:

$$\eta_p(k) = -\operatorname{cov}_t(\lambda_{t+1}, E_{t+1}d_{t+k}).$$

For expository purpose it is useful to focus on the case in which (i) consumption is a random walk and (ii) consumption and dividends are equal, $d_{t+k} = c_{t+k}$.

Risk premia with time-separable preferences: Since consumption is a random walk there are no expected changes; this implies that term premia are equal to zero. The random walk assumption also means the payout premia are constant across horizons, since $E_{t+1}d_{t+k} = E_{t+1}c_{t+k} = c_{t+1}$. Further, the level of the payout premium is just $\tau\sigma^2$, where σ^2 is the variance of the consumption growth rate. If we use the Mehra and Prescott (1985) value of $\sigma = .036$, then the variance is $\sigma^2 = .0013$. Hence, the payout premium is small even for large values of τ. For example, if $\tau = 2$ then $p = 2 \times .0013 = .0026$, *i.e.*, about one-quarter of a percent. Setting $\tau = 20$ still makes the equity premium only 2.6%.

Risk premia with habit formation: With nonseparable preferences we have expected changes in marginal utility as suggested by the prior analysis. Continuing to assume that consumption follows a random walk, we find that (for $k \geq 2$)

$$\frac{\partial(E_{t+1}\lambda_{t+k} - \lambda_{t+1})}{\partial c_{t+1}} = -\tau + \frac{1 + \alpha^2\beta - \alpha\beta}{(1-\alpha)(1-\alpha\beta)}\tau \approx 3.5\tau.$$

That is, say after a positive shock, marginal utility will be expected to increase again after its initial decline at $t+1$. Hence, we know that there will be term premia, even though none were present with separable preferences. The level of these premia is about

$$\eta_t(k) = -(-4.5\tau)(3.5\tau)\sigma^2.$$

Thus comparing the term premia with the payout premium above we find that it is about 16τ times as large, or about 8.3% for $\tau = 2$. Habit formation is therefore capable of generating large term premia in this setting. The payout premia are

$$\begin{aligned}
\eta_p(k) &= -\operatorname{cov}_t(\lambda_{t+1}, E_{t+1}d_{t+k}) = -\operatorname{cov}_t(\lambda_{t+1}, c_{t+1}) \\
&\approx 4.5\tau\sigma^2\% \\
&\approx 1.2\%
\end{aligned}$$

Thus the increase in payout premia is about 1% when we shift to the habit formation model.

Figure 1 shows how these considerations operate in a version of the Mehra and Prescott (1985) exercise. Rather than assuming that consumption follows a random walk, consumption growth rates follow a first order autoregressive process with parameter $-.14$.[11] We begin by studying the impulse response functions, which are in the right-hand plot in each row. Dividends are the same in both cases, as they are just equal to consumption. However, the difference in the response of marginal utility is dramatic. For the time-separable case, the negative serial correlation in consumption growth, creates predictable movements of consumption and hence the valuation. The decline in valuation at shock-impact is followed by an increase back towards the initial level. This time path leads to a decrease in interest rates, and to positive term premia. For habit formation, not only is this decrease in interest rates much more pronounced, but the valuation at $t+1$—when the capital gains occur—is much lower. This gives term premia of over ten percent for habit formation. The premia structures in the left-hand-side in figure 1 are higher than in our previous example with random walk consumption due to the temporary components in the consumption path. Temporary components increase the expected future change after the initial shock, and therefore amplify interest rate movements.

In summary, our comparison of habit formation and time-separable preferences is consistent with the results obtained in the literature in similar models: habit formation generates a much higher equity premium. In fact, as we show, habit formation has the ability to drive a wedge between risk aversion with respect to wealth and with respect to short term consumption changes, and this generates large premia. However, these premia are mainly term premia, put another way, interest rates become much more volatile. Historical mean returns for long term bonds and stocks displayed in table 1 show a another picture: the equity premium is mainly a premium for payout uncertainty and not for interest rate risk. This is to say that the basic habit formation model is not entirely satisfactory in its way of solving the equity premium puzzle. We return to the analysis of habit formation preferences in the next section by investigating how these results change in production economies where consumption and dividends are determined endogenously.

[11] Mehra and Prescott specify a two-state, discrete Markov process with autocorrelation of $-.14$.

4 Asset returns in production economies, quantitative predictions

At this point, we depart from our assumption that the loglinear system for dividends and its valuation is given exogenously. To do so, we link our formulae for asset pricing to the loglinear structures obtained as approximate solutions to stochastic equilibrium models. King, Plosser and Rebelo (1988) present an approximation method for infinite horizon problems that gives the solution in exactly the loglinear form required by our asset pricing framework.

This section explores the quantitative predictions for asset returns of a one-sector, real production model driven by exogenous shocks to productivity. As in the prior section, we also consider different specifications of investors' preferences. However, it is important to note that the present framework is not limited to the study of this class of real business cycle models. For example, these methods could be used to study asset pricing in models with additional features such as oligopolistic pricing (Rotemberg and Woodford (1992)), time-to-built investment technology (Kydland and Prescott (1982)), nontraded goods (Stockman and Tesar (1990)), household production (Greenwood, Rogerson and Wright (1993)), liquidity effects (Christiano and Eichenbaum (1991)), sticky prices (King (1991)) and stochastic fiscal policy (Baxter and King (1993)). Baxter and Jermann (1993) applied this framework to a model with incomplete asset markets.

4.1 The model

Consider the standard stochastic neoclassical growth model with a large number of infinitely-lived firms and investors/households. There is a single consumption-investment good that is produced with a constant-returns-to-scale production technology which is subject to random shocks in productivity.

Firms

In each period, a representative firm has to decide how much labor to hire and how much capital to invest. Managers maximize the value of the firm to its owners, which is equal to the present discounted value of all current and future expected cash flows:

$$E_t \sum_{k=0}^{\infty} \frac{\beta^k \Lambda_{t+k}}{\Lambda_t} (A_{t+k} F(K_{t+k}, X_{t+k} N_{t+k}) - W_{t+k} N_{t+k} - I_{t+k}) \tag{20}$$

where A is the stochastic productivity level, K is the capital stock, W is the wage rate, N is the quantity of labor input, X is the deterministic trend in labor augmenting technical change which grows at the rate $\gamma = X_{t+1}/X_t$ and I is investment expenditure. The firm's capital stock obeys an intertemporal accumulation equation with adjustment costs:

$$K_{t+1} = (1-\delta) K_t + \phi\left(\frac{I_t}{K_t}\right) K_t \tag{21}$$

where δ is the depreciation rate and $\phi(.)$ a positive, concave function. The concave function captures the idea that changing the capital stock rapidly is more costly than changing it slowly. This specification also allows the shadow price of installed capital to diverge from the price of an additional unit of capital, *i.e.*, it permits variation in Tobin's q.[12]

The firm does not issue new shares, but finances its capital stock through retained earnings and discount bonds. Each period, the firm issues j-period discount bonds for a fixed fraction ν/j of its capital stock, and pays back its debt that comes to maturity. The dividends to shareholders are therefore equal to:

$$D_t = A_t F(K_t, X_t N_t) - W_t N_t - I_t - \frac{\nu}{j}(K_{t-(j-1)} - V_t^c [1_{t,t+j}] K_{t+1}) \tag{22}$$

with V^c the price of a corporate bond. We choose this financial policy because it allows the firm to adjust its debt level smoothly through time as it grows. The parameter ν determines the overall level of debt as a fraction of the capital stock.

[12]Capital adjustment costs have been studied by Eisner and Strotz (1963), Lucas (1967), Lucas and Prescott (1971) and Hayashi (1982). Baxter and Crucini (1993) have applied this specification to an open economy real business cycle model.

Investors/Households

Representative investors maximize expected lifetime utility of consumption, subject to a sequential budget constraint:

$$\max \; E_t \sum_{k=0}^{\infty} \beta^k u(C_{t+k})$$

$$s.t. \;\; W_t N_t^s + a_t(V_t^a + D_t^a) = C_t + a_{t+1} V_t^a. \tag{23}$$

Here a_t is a vector of financial assets held at t and chosen at $t-1$, V^a and D^a are vectors of asset prices and current period payouts. The asset vector a contains shares and one-period bonds to the representative firm and possibly other assets. In addition, investors face a (normalized) time constraint $1 = N_t^s + L_t$, with L representing leisure and N^s productive work. Given that leisure does not enter the utility function, agents will allocate their entire time endowment to productive work. We will call this case the fixed labor economy, FL. Two additional cases are considered: the variable-labor economy, VL, where $u(.) = u(C_t, L_t)$, and the habit-formation economy, HF, with period utility function defined as $u(C_t - \alpha C_{t-1})$.

Market equilibrium, definition of financial assets

In equilibrium, all produced goods are either consumed or invested; labor supply also equals labor demand. Financial market equilibrium requires that the investors hold all outstanding equity shares and corporate bonds, and that all other assets are in zero net supply.

In our numerical analysis we will look at the equity return, $R^e_{t,t+1} = R_{t,t+1}[\{D_{t+k}\}_{k=1}^{\infty}]$, the return of a period riskless bond, $R^f_{t,t+1} = R_{t,t+1}[1_{t,t+1}]$, and the return of a perpetual bond, $R^b_{t,t+1} = R_{t,t+1}[\{1_{t+k}\}_{k=1}^{\infty}]$.[13]

4.2 Calibration

To develop the quantitative predictions of the model, we choose parameters within the range of values generally considered as reasonable. Whenever

[13]Given that we exclude the possibility of bankruptcy, corporate and riskless bonds are perfect substitutes. In fact, in our simulations, the probability of default (firm value below debt) is negligible for the cases of interest.

possible, we use information from outside the model to identify parameter values. Where this is not possible, or where parameters substantially affect the results, we will consider several values.

Long run behavior: Several parameters are chosen to match long-run model behavior, as it is customary in the literature. These parameters do not significantly affect model dynamics, and we use standard values. The quarterly trend growth rate γ is 1.004, the capital depreciation rate δ is 0.025, the constant labor share in a Cobb-Douglas production function is 0.63, the fraction of time allocated to productive work is 0.20, and the investment share of output is 0.26. This parameterization fixes the steady state rate of return under certainty at 6.5% annually. Implicitly, this also fixes the growth-adjusted discount factor, β^*, that is equal to $\beta\gamma^{1-\tau}$.[14]

Habit formation: Constantinides (1990) requires a habit formation level of $\alpha = 0.8$, Cochrane and Hansen (1992) use 0.5 and 0.6. We use 0.6. This value also guarantees that marginal utility of consumption expenditure is always positive.

Capital adjustment costs: To specify the capital adjustment cost technology, we only need to know the elasticity of the investment capital ratio with respect to Tobin's q. This parameter primarily affects investment volatility. In the absence of strong a priori knowledge, investment volatility can be used to impose bounds on this critical adjustment cost parameter. We study three cases: low, moderate and high capital adjustment costs. This corresponds to elasticities of 13, 1.3, and 0.33. Abel (1980) estimated this parameter in a somewhat different model, and obtained values between 0.27 and 0.52. This suggests that our *high* adjustment cost elasticity of 0.33 is in the range of empirical plausible values.

[14]This relation illustrates how in a growing economy it is β^*, the discount factor of the model where all growing variables have been divided by the linear growth trend, that needs to be smaller than one and not the original β. For utility to be finite, and the model equilibrium to be well defined, there is an additional constraint that limits the path for the exogenous productivity process. A more detailled discussion of these issues can be found, for instance, in Benninga and Protopapadakis (1990).

Risk aversion: The coefficient of relative risk aversion has played a major role in the intertemporal asset pricing literature. Each one of our three models uses the period utility function $(1-\tau)^{-1}\Psi_t^{1-\tau}$, where Ψ_t is the level of consumption services. However, the three models differ in their specification of Ψ_t as follows:

Fixed labor (FL), $\Psi_t = C_t$;

Variable labor (VL), $\Psi_t = C_t^{\theta} L_t^{1-\theta}$; and

Habit formation (HF), $\Psi_t = C_t - \alpha C_{t-1}$.

Across all three models, we want to keep constant agents' behavior towards atemporal wealth bets. It is well known that, for the FL model, risk aversion is thus equal to the parameter τ. Ferson and Constantinides (1991) and our previous discussion show that τ is also close to risk aversion for the HF model. Finally, for the VL case, given that Ψ is homogeneous of degree one, an increase in wealth for a constant real wage will leave the consumption-leisure ratio constant, which also makes the parameter τ the risk aversion with respect to atemporal wealth bets. Estimates of risk aversion based on market returns like Friend and Blume (1975) are around 2. Campbell (1993), by including returns to human capital gets values between 5 and 8. Taking into account mean reversion in asset prices can increase these estimates by up to three times (Black (1990), Campbell (1993)). Consequently, we consider values for τ of 2 and 10.

Financial leverage: Masulis (1988) reports leverage ratios—debt as a fraction of total firm value—for the U.S. over the last sixty years to range from 0.13 to 0.44 for market values and from 0.53 to 0.75 for book values. We will consider leverage ratios of about 0.5.

Productivity shocks Finally, we need to specify the exogenous process for productivity A_t. Estimates of Solow residuals typically yield a highly persistent AR(1) process with possibly a unit root.[15] Our estimates, based on the data from Backus, Kehoe and Kydland (1992) give 0.98 and 0.0085 for the autoregressive coefficient and the impulse standard deviation, and we are

[15] See Prescott (1986) for a discussion of Solow residuals' estimates.

unable to reject a random walk. We will use two values for the persistence parameter, 0.95 and 1, for a stationary and a difference stationary process, with standard deviations of the shock impulse of 0.0085.

4.3 Quantitative model predictions

Several of the studies of intertemporal asset pricing in endowment models follow Mehra and Prescott (1985): models are evaluated in terms of their ability to generate a high equity premium. To compare production models to these prior results we follow the same strategy. However, we will also be careful to distinguish between payout uncertainty risk and interest rate risk. Indeed, as suggested in the long-term data in Table 1, the equity premium is mainly a premium for riskiness of the payouts rather than for interest rate risk.

Summary of numerical results

The models' predictions for mean returns and business cycle statistics are displayed in table 2 to 5. Before examining these results more in detail, it seems useful to briefly state the main results. The major result is that combinations of high capital adjustment costs and habit formation preferences can explain equity premia of several percentage points.[16] Table 2 also makes it clear why substantial equity premia were not found by prior researchers (such as Rouwenhorst (1991)) who studied production economies without capital adjustment costs. Table 2 shows that, for low capital adjustment costs, no model variation can generate an equity premium that exceeds 0.1%. Adding financial leverage does further increase the equity premium and also the premium of returns on equity over perpetual bonds. These model versions with high capital adjustment costs and habit formation are further able to generate investment and consumption volatility broadly in line with U.S. aggregate data. Another result is that the variable labor supply model generates premia that are consistently smaller—in absolute values—than the fixed labor model. Explaining the equity premium with variable labor supply represents therefore an additional challenge for the representative agent model.

[16] Compared to the postwar data we calibrated our model to, the variance of endowment in the Mehra and Prescott data is about four times higher. This increases the level premia roughly four times for their data set.

In the remainder of this section, we take a closer look at the determinants of premia in production models. We follow the method of section 3, and explore how the dynamic responses of dividends and valuation explain risk premia. We then discuss how combinations of capital adjustment costs and habit formation generate a high equity premium. Finally, we show how financial leverage can further increase the equity premium.

Premia in production models

To show how production models work, figure 2 displays various strip premia and impulse response functions for the basic stationary model case with low risk aversion and low capital adjustment costs. Notably, strip premia are now no longer always positive in contrast to the findings in the different endowment models we studied before. For instance, payout uncertainty premia are now negative for short maturities. To understand this seemingly counterintuitive result, recall that we defined the dividend as output minus labor income minus investment. Therefore, after a positive productivity shock, the sharp increase in investment simply outweighs the increase in the capital income. In other terms, the firm's decision is to increase its retained earnings in order to take advantage of the temporarily higher productivity: this earnings process causes negative payout premia. The equity premium is composed of both the term and the payout uncertainty premia, whereas the bond has only the term premia. Hence, we can now understand the apparently anomalous result in table 2: that, in this case, the equity premium (.006) is lower than the bond premium (.008). Other results in table 2 can be analyzed the same way.

Why do capital adjustment costs increase premia? The second row of figure 2 illustrates how capital adjustment costs can improve results for the equity premium. First, by reducing the potential for intertemporal substitution on the production side, the time path for marginal utility is more variable. It is also invariably increasing, which leads to positive term premia. Second, given that investment volatility is reduced, dividends now no longer have a large initial downturn, which benefits the payout premia. As reported in table 2, the equity premium, in this model with moderate adjustment costs, is increased about ten times with respect to its value with low capital adjustment costs; in addition, it is larger than the bond premium.

Why is high risk aversion not enough? Figure 3 explains why risk aversion alone cannot give high equity premia in production models. Indeed, high risk aversion actually reduces dividend responses over all horizons. In this context, the fact that the risk aversion coefficient is equal to the inverse of the elasticity of intertemporal substitution has strong implications. The greater desire to smooth consumption leads dividends to actually decline over all horizons in response to a positive productivity shock.

What happens with random walk productivity? Random walk productivity for time-separable utility, in figure 3, creates ever declining expected marginal utility, the counterpart of an ever increasing consumption path. This increases interest rates over all horizons, thus leading to negative term premia. Increasing risk aversion (not shown) will here increase the holding premia by a higher rate than the payout uncertainty premia, as suggested by equation (15). This means a lower equity premium for the FL and HF models with low capital adjustment costs. Note, however, that the payout premia are consistently positive and considerably larger than for the stationary cases of figure 2. By making the trend level of dividends stochastic, random walk productivity increases their riskiness.

What determines the level of the risk-free rate? Given our particular stochastic structure the unconditional mean of the risk-free rate has the following closed form solution:

$$E(R_{t,t+1}[1_{t,t+1}]) = \frac{\gamma}{\beta^*} \exp\left(\frac{1}{2}(\mathrm{var}(E_t\lambda_{t+1} - \lambda_t) - \mathrm{var}(\lambda_{t+1} - E_t\lambda_{t+1}))\right) \quad (24)$$

where the first ratio in the right hand side is simply the steady state rate of return under certainty, a function of the trend growth rate and the growth corrected discount factor. Unpredictable volatility of the valuation reduces this rate, what we interpreted earlier as precautionary effects; and predictable volatility, a Jensen's inequality effect, increases it. For impulse responses, low risk-free rates are linked to large jumps at impact followed by little predictable change. This is the general pattern for nonstationary models, which also clearly shows up in the results in table 2.

The interaction between capital adjustment costs and habit formation

Our previous discussion in sections 3.2 and 4.3 explained why capital adjustment costs and habit formation can separately lead to high equity premia. The numerical results, however, make it clear that it is the *combination* of these two elements that account for an equity premium of several percentage points. Table 3 and 4 report population moments for business cycle statistics that clarify the model mechanisms at work.

Capital adjustment costs reduce intertemporal substitution in production. In turn, this makes consumption more volatile and its growth rate less predictable. However, capital adjustment costs alone do not increase riskiness enough to generate large premia. Further, capital adjustment costs strongly reduce investment volatility for time-separable models below any empirically plausible value. Habit formation increases risk aversion in consumption in the short term versus the long term, which can create large premia. However, habit formation lowers intertemporal substitution in preferences, so that consumption becomes also much smoother (*i.e.*, less volatile and more autocorrelated), which reduces the riskiness of the endogenous consumption path. (This latter fact does of course not come into play in endowment models.) Combined high capital adjustment costs and habit formation keep intertemporal substitution in consumption and in production at low levels, and thus generate premia in the order of magnitude of endowment models.

Despite the fact that capital adjustment costs do reduce the autocorrelation coefficients of consumption growth rates, first order coefficients of .47 and .5 seem still rather high (see table 4). Heaton (1993) reports the first five autocorrelation coefficients for growth rates of seasonally-adjusted quarterly U.S. consumption expenditure on nondurables and services at .276(.068), .108(.063), .201(.080), .057(.087) and -.202(.108) (standard errors in parentheses). He argues, however, that the high first order serial correlation coefficient is due to time-averaging of the consumption data. Modelling the sample properties of aggregate data is beyond the scope of our paper.

Financial leverage and dividend volatility

Results in table 2 document that model versions with capital adjustment costs and habit formation are able to generate equity premia close to 3%.

Despite this, the model cannot explain more than a 1% excess return of equity over long bonds, whereas table 1 suggests that the equity premium has a large payout uncertainty component. Financial leverage, without affecting term premia, does increase the riskiness of dividends, and therefore has the potential to solve this problem.

In our framework, financial leverage acts through two transmission channels. First, given that firms link their debt level to their current capital stock, a positive shock by leading to growth in the capital stock increases the quantity of issued bonds. This weakens the initial downturn in dividends related to increased investment that we have seen in figure 2, and leads to higher payout premia for short horizons. Second, if interest rates decrease after a positive shock, then the firm obtains a higher price for the sale of its discount bonds. Figure 4 shows how these two elements can considerably increase the payout uncertainty premia for the case when the firm issues only one period bonds.

Panel A in table 5 reports equity premia for different leverage parameters, ν, and debt maturities. In brackets we have the means and standard deviations of leverage ratios—defined as debt over total firm value. Panel B present dividend volatility. We retain two results: equity premia do uniformly increase with leverage, and bond maturities are important for dividend volatility.[17] Overall, if we exclude model versions with excessive dividend volatility, financial leverage can add up to half a percentage point to the equity premium. It is interesting to note that the low reported standard deviations for leverage ratios clearly suggest that, except for the three cases with mean leverages ratios of .8 and higher, the assumption of riskless corporate borrowing is here justified.

To explain the dramatic dividend volatility for cases where the firm refinances its entire debt in every period, *i.e.*, debt maturity is equal to one, we need to look at interest rate volatility. As we noticed in section 3 and as discussed in the literature, habit formation preferences make real interest rates too volatile, see for instance Abel (1991). Even though this problem is less pronounced in our model, interest rates do become more volatile for the model cases considered in table 5. In particular, for the model with

[17]Cecchetti, Lam and Mark (1990) report a standard deviation of annual dividend growth rates in the U.S. for the period 1971 to 1985 of 13.6%. Assuming a random walk, this is 6.8% for quarterly growth rates.

random walk productivity, high risk aversion and high capital adjustment costs, the quarterly standard deviation of interest rates is highest compared to all other model cases at about 2.5%. Given that the unlevered dividend is about 1.25% of the capital stock, a 2.5% change in interest rates, with a leverage ratio of .5, will triple the dividend. Allowing for longer debt maturities does considerably reduce this impact on dividends. This is because long term yields in the model are considerably less volatile than short term yields, which is also what we observe in real world.

5 Conclusions

We have presented a methodology that allows one to evaluate and think about asset prices in a large class of macroeconomic models. Combining the convenient properties of the lognormal distribution with a loglinear system for economic variables, we derive expressions for conditional expected returns and variances for securities with single and multiple payouts. We show how this framework allows one to provide important intuition about the link between asset return premia and the nature of the business cycle. We also show how this method can be directly used to study non-time-separable preferences.

This method is then applied to the approximate model solutions of the one sector real business cycle model. In this setting, with endogenous production decisions, we look at how model elements and parameter values affect asset return premia, focusing on the influence of business cycle dynamics. We find that capital adjustment costs are central to generating large risk premia in a production model; model versions with capital adjustment costs, habit formation in preferences and financial leverage are able to produce an equity premium of several percentage points.

The quantitative application performed in this paper is only a initial step. Since we have found that a small-scale equilibrium model with a nontrivial production sector can generate risk premia in the order of magnitude of historically observed levels, this model and extensions of it should be useful in shedding additional light on the interaction between asset returns and the nature of the business cycle.

Appendix

This appendix shows that the asset pricing formulae can be directly applied to non-time-separable valuations.

Using the notation defined in the text, consider the valuation of a possibly random payout D_{t+k}:

$$V_t[D_{t+k}] = \frac{\beta^k E_t(H_{t+k,t+k+s} D_{t+k})}{E_t(H_{t,t+s})}.$$

Applying the law of iterative expectations, the numerator can then be written as:

$$E_t(E_{t+k}(H_{t+k,t+k+s} D_{t+k})),$$

and this is equal to

$$E_t(E_{t+k}(H_{t+k,t+k+s}) D_{t+k})$$

because D_{t+k} is deterministic at time $t+k$. Applying lognormality, we then obtain that:

$$V_t[D_{t+k}] = \frac{\beta^k E_t\left(((\exp E_{t+k} h_{t+k,t+k+s}) D_{t+k}) \exp(\frac{1}{2}\operatorname{var}_{t+k}(h_{t+k,t+k+s}))\right)}{\exp(E_t h_{t,t+s}) \exp(\frac{1}{2}\operatorname{var}_t(h_{t,t+s}))}.$$

We know that $\operatorname{var}_{t+k}(h_{t+k,t+k+s})$ is equal to $\operatorname{var}_t(h_{t,t+s})$, because the conditional second moments do not depend on the state of the system. With this last step, we obtain our announced result that:

$$V_t[D_{t+k}] = \frac{\beta^k E_t((\exp E_{t+k} h_{t+k,t+k+s}) D_{t+k})}{\exp(E_t h_{t,t+s})}.$$

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Table 1
Means of U.S. asset returns

A. Returns for the period 1948:1-1987:4

Asset	Mean*
Short-term bond return: T-bills	0.80
Long-term bond return: long-term U.S. gov't debt	0.78
Stock return: S&P 500	8.90

*Annualized quarterly returns, percent per year. Source: SBBI

B. Returns for the period 1800 - 1990

Asset	Mean*
Short-term bond return:	3.13
Long-term bond return:	3.71
Stock return:	7.77

*Returns measured as percent per year. Source: Siegel (1992)

Table 2
Model predictions for mean returns

A. Stationary process for productivity

risk aversion	adjustment costs	R^f-1			(R^b/R^f)-1			(R^e/R^f)-1		
		fixed labor	variable labor	habit format'	fixed labor	variable labor	habit format'	fixed labor	variable labor	habit format'
τ=2	low	6.48	6.49	6.48	0.008	0.005	0.011	0.006	0.006	0.008
	mod	6.46	6.48	6.41	0.048	0.025	0.140	0.055	0.034	0.148
	high	6.43	6.47	6.15	0.103	0.038	0.750	0.130	0.055	0.806
τ=10	low	6.29	6.47	6.27	0.092	0.015	0.109	0.028	0.010	0.043
	mod	6.11	6.42	5.98	0.331	0.088	0.498	0.275	0.094	0.425
	high	5.61	6.35	4.67	1.091	0.201	2.749	1.096	0.234	2.744

B. Random-walk process for productivity

risk aversion	adjustment costs	R^f-1			(R^b/R^f)-1			(R^e/R^f)-1		
		fixed labor	variable labor	habit format'	fixed labor	variable labor	habit format'	fixed labor	variable labor	habit format'
τ=2	low	6.46	6.49	6.45	-0.044	-0.020	-0.040	0.007	0.006	0.014
	mod	6.44	6.48	6.36	-0.035	-0.014	0.041	0.043	0.025	0.158
	high	6.42	6.47	6.12	-0.021	-0.008	0.456	0.105	0.059	0.750
τ=10	low	4.50	6.37	4.31	-0.751	-0.091	-0.669	-0.272	0.010	-0.187
	mod	4.43	6.35	3.77	-0.679	-0.072	-0.158	-0.158	0.057	0.421
	high	4.28	6.32	2.00	-0.530	-0.045	1.950	0.104	0.136	2.850

Notes: All entries are quarterly returns, expressed as annualized percentage points. Mnemonics for the returns are: R^f = Risk-free rate of return; R^b = Bond return; R^e= Equity return.

Table 3: Business cycle statistics—model and US data

	Preference specification	Business-cycle statistic	US data: 1960-1991	low adj. costs; τ=2	mod. adj. costs; τ=10	high adj. costs; τ=2	high adj. costs; τ=10
STATIONARY SHOCKS	Fixed labor	Std(y)	0.0098	0.0086	0.0085	0.0086	0.0085
		Std(c)/Std(y)	0.74	0.46	0.51	1.12	1.23
		Std(i)/Std(y)	2.65	2.54	2.37	0.65	0.34
	Variable labor	Std(y)	0.0098	0.0116	0.0097	0.079	0.0084
		Std(c)/Std(y)	0.74	0.48	0.78	1.18	1.05
		Std(i)/Std(y)	2.65	2.47	1.63	0.50	0.86
	Habit formation	Std(y)	0.0098	0.0086	0.0085	0.0086	0.0086
		Std(c)/Std(y)	0.74	0.26	0.27	0.79	0.44
		Std(i)/Std(y)	2.65	3.28	3.23	1.85	2.82
RANDOM WALK SHOCKS	Fixed labor	Std(y)	0.0098	0.0085	0.0085	0.0085	0.0085
		Std(c)/Std(y)	0.74	0.87	1.19	1.17	1.23
		Std(i)/Std(y)	2.65	1.36	0.46	0.51	0.34
	Variable labor	Std(y)	0.0098	.0099	.0082	.0079	.0078
		Std(c)/Std(y)	0.74	0.72	1.07	1.16	1.18
		Std(i)/Std(y)	2.65	1.80	0.82	0.54	0.48
	Habit formation	Std(y)	0.0098	0.0085	0.0085	0.0085	0.0085
		Std(c)/Std(y)	0.74	0.49	0.64	0.85	0.73
		Std(i)/Std(y)	2.65	2.80	2.51	1.71	2.26

Notes: U.S. data is from the OECD. Data and model were filtered with the first-difference filter.

Table 4
Habit formation model

Autocorrelation coefficients of consumption growth rates

adjustment costs	shock process	risk aversion	Autocorrelation coefficients at lag				
			1	2	3	4	5
low	stationary	τ=2	.63	.40	.27	.19	.14
		τ=10	.61	.38	.24	.15	.11
	random walk	τ=2	.61	.38	.25	.17	.13
		τ=10	.59	.36	.21	.13	.08
high	stationary	τ=2	.29	.06	.00	-.03	-.04
		τ=10	.47	.20	.07	.01	-.02
	random walk	τ=2	.33	.11	.04	.01	.01
		τ=10	.50	.25	.13	.06	.03

Table 5
Financial leverage and dividend volatility

Habit formation model with high capital adjustment costs

A. The equity premium

shock process	risk aversion	no leverage	leverage parameter ν= 0.6		ν = 0.8	ν = 1.2
			maturity (in quarters)			
			20	40	40	40
stat.	τ=2	0.81	1.02	0.91	0.98	1.36
		[-]	[.50(.02)]	[.41(.01)]	[.54(.02)]	[.82(.03)]
	τ=10	2.74	3.67	3.13	3.39	4.53
		[-]	[.48(.04)]	[.40(.03)]	[.53(.04)]	[.80(.06)]
random walk	τ=2	0.75	1.01	0.94	1.06	1.79
		[-]	[.50(.05)]	[.42(.05)]	[.56(.06)]	[.84(.10)]
	τ=10	2.85	3.05	3.01	3.02	3.20
		[-]	[.29(.01)]	[.25(.01)]	[.33(.02)]	[.50(.03)]

Figures in brackets denote mean leverage ratios defined as debt divided by total firm value with standard deviations in parentheses.

B. Standard deviation of quarterly dividend growth rates

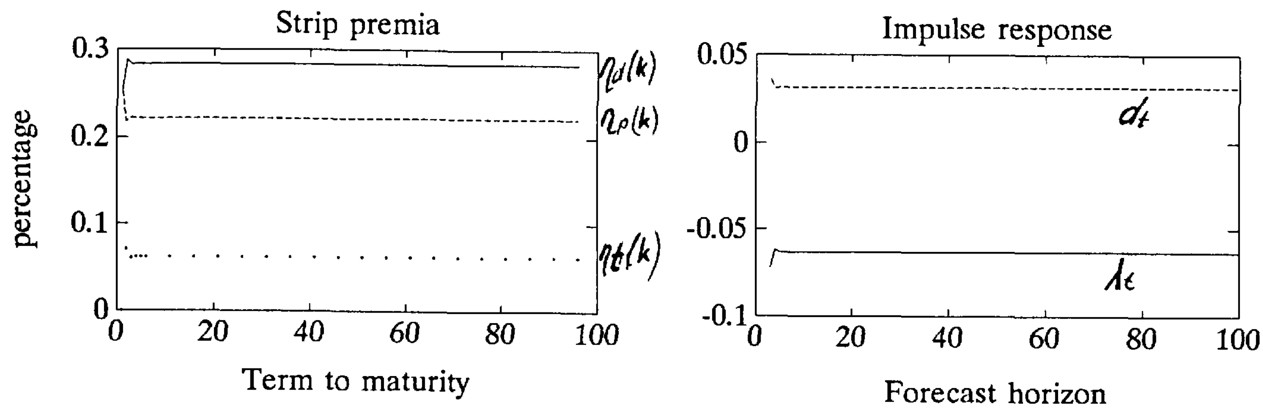
shock process	risk aversion	no leverage	leverage parameter, ν = 0.6			ν = 0.8	ν = 1.2
			maturity (in quarters)				
			1	20	40	40	40
stat.	τ=2	1.53	249.01	13.10	3.75	6.86	26.08
	τ=10	2.96	147.97	16.18	3.92	8.37	31.17
random walk	τ=2	1.52	238.32	9.35	2.34	4.15	15.49
	τ=10	2.33	129.44	10.49	2.54	4.48	10.09

Figure 1

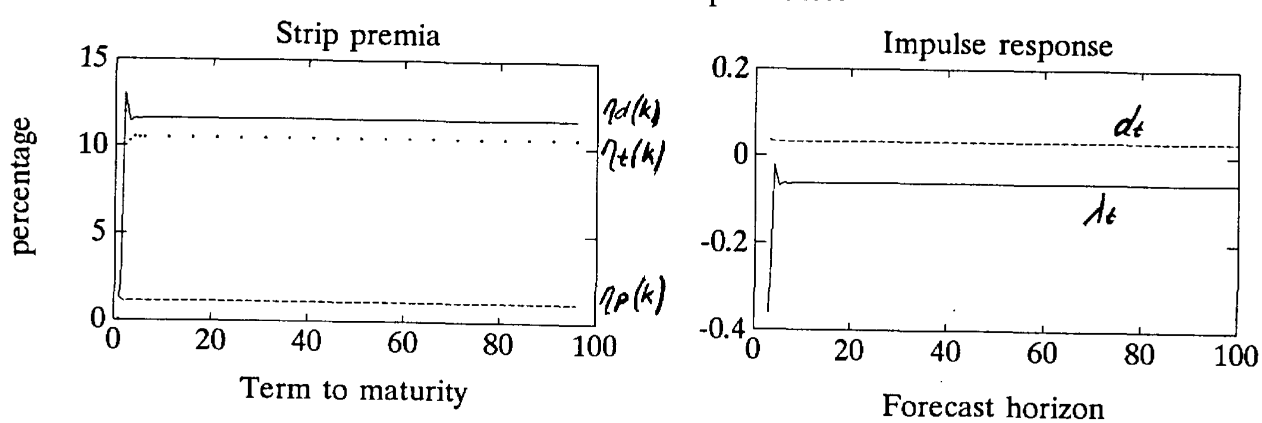
Strip premia and impulse responses

Endowment model

A. Time separable preferences



B. Habit formation preferences



Parameter values are: τ=2, α=.6 std(Δc)=.036, corr(Δc_t, Δc_{t-1})=-.14, β=.96.
--Payout uncertainty premia, $\eta_p(k)$
...Term premia, $\eta_t(k)$
_Dividend premia (sum of the two above), $\eta_d(k)$
Returns are expressed in annual terms scaled by 100

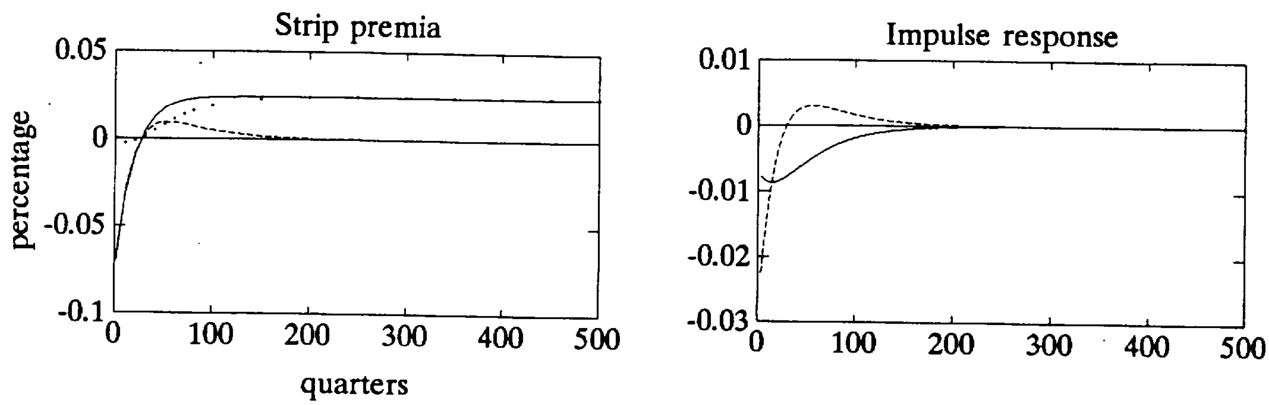
--Dividend, d_t
_Marginal utility of consumption, λ_t

Figure 2

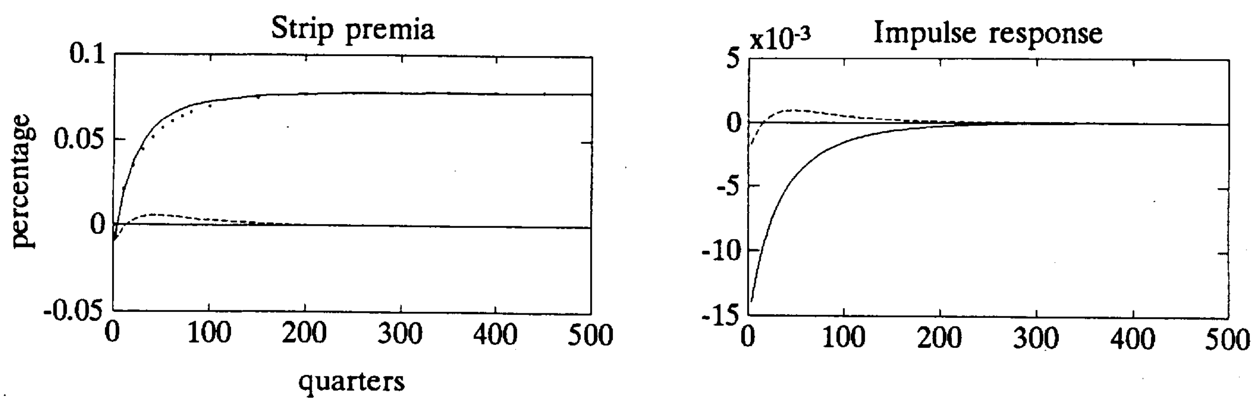
Strip premia and impulse responses

Effect of capital adjustment costs

A. Fixed labor model



B. Fixed labor model, moderate capital adjustment costs



Parameter values are: τ=2, stationary productivity, low and moderate capital adjustment costs.

---Payout uncertainty premia

... Term premia

__Dividend premia (sum of the two above)

Returns are expressed in quarterly terms scaled by 400.

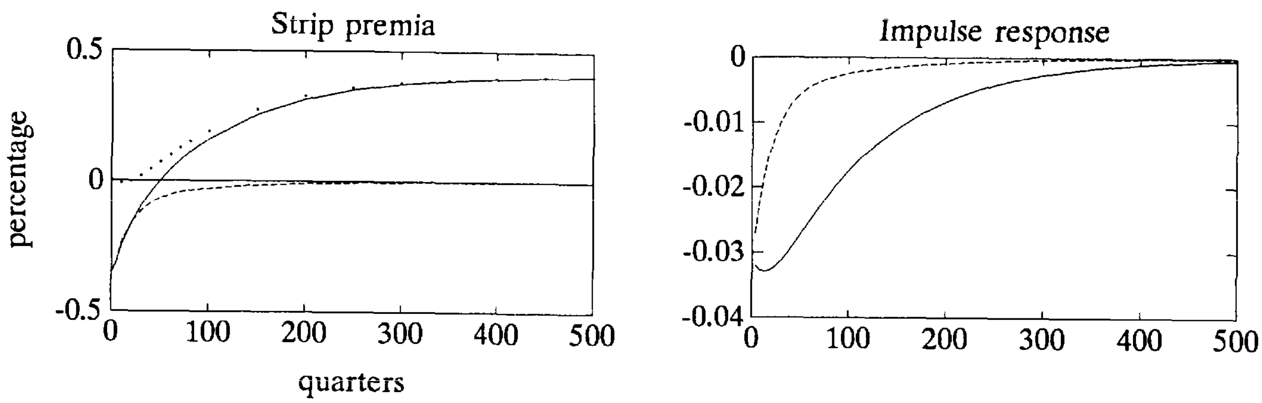
---Dividend

__Marginal utility of consumption

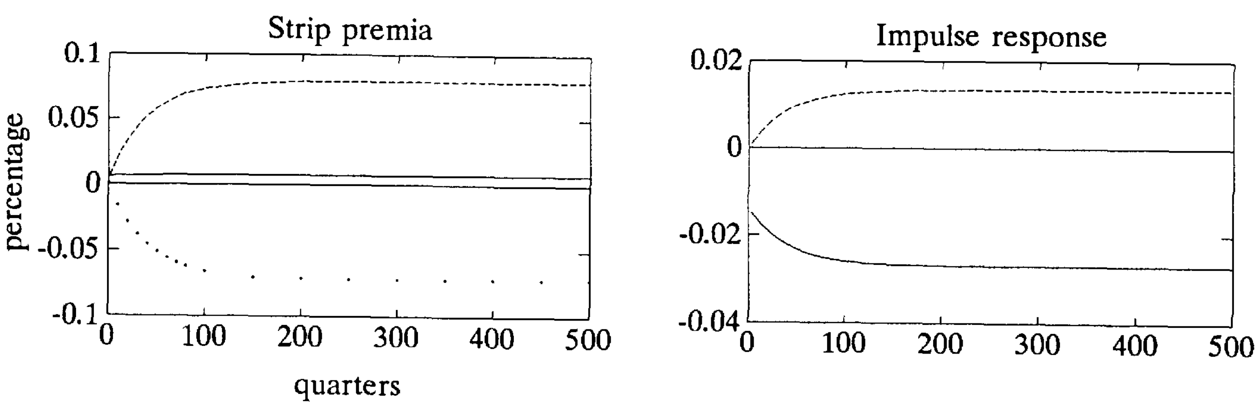
Figure 3

Strip premia and impulse responses

A. Fixed labor model, risk aversion equal to 10



B. Fixed labor model, random walk productivity



Parameter values are: No leverage, low capital adjustment costs, τ=[10;2], productivity=[stat;nonstat].

---Payout uncertainty premia

... Term premia

__Dividend premia (sum of the two above)

Returns are expressed in quarterly terms scaled by 400.

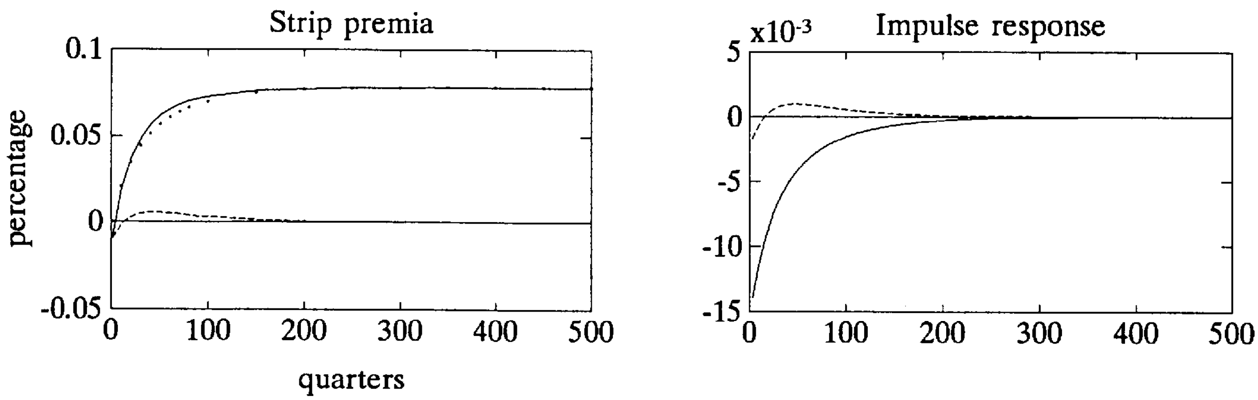
---Dividend

__*Marginal utility of consumption*

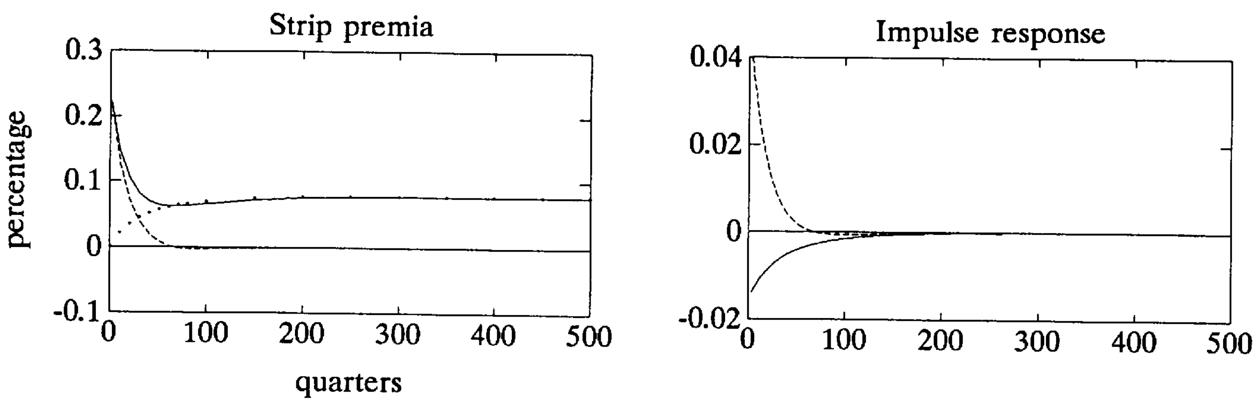
Figure 4

Strip premia and impulse responses

A. Fixed labor model, moderate capital adjustment costs



B. Fixed labor model, moderate capital adjustment costs and leverage



Parameter values are: τ=2, stat. productivity, moderate capital adjustment costs, v=[0;.4], j=[0;1].

---Payout uncertainty premia

... Term premia

__Dividend premia (sum of the two above)

Returns are expressed in quarterly terms scaled by 400.

---Dividend

__Marginal utility of consumption