Example of Lucas model
to be discussed in class

Assume there are two countries US and Europe. The US produces $X$ units of hamburgers every period, and Europe produces $Y$ units of cheese every period. Production is stochastic. Both countries’ representatives consume both goods and have the same utility functions

$$E_0 \sum_{t=0}^{\infty} \beta^t \pi(s^t|s_0) u \left( G \left( x(s^t), y(s^t) \right) \right)$$

where

$$u(G) = \frac{1}{1-\sigma} G^{1-\sigma}$$

and

$$G(x, y) = \left( \omega x^{\frac{\gamma-1}{\gamma}} + (1-\omega) y^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}.$$

1. Compute the efficient consumption allocation as a function of the welfare weights, $\lambda$ and $\lambda^*$, and the parameters of the utility function.

2. Compute the equilibrium price of a cheese in terms of hamburgers.

3. Show that, for any set of welfare weights $\lambda$ and $\lambda^*$, this allocation can be supported by an equilibrium where the only available securities are the two countries’ equity shares to their endowments.