International Portfolio Diversification and Endogenous Labor Supply Choice

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Abstract

This paper presents a multi-country general equilibrium model driven by productivity shocks, where labor supply and consumption are chosen endogenously. We use this framework to study the effect of labor supply for optimal international diversification. We find that the model's ability to help explain home-bias depends crucially on the level of substitutability between consumption and non-working time. Quantitatively, the non-separability in the preferences helps in a nonnegligeable way, but it cannot entirely explain the extreme degree of home-bias in US portfolios.

Key words: International diversification, portfolio choice, nonseparable utility, household production. JEL classification: F30, G11.

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1. Introduction

Investors in most countries have access to a large set of international financial instruments, however, they choose to invest mostly in domestic assets. For instance, French and Poterba (1991) and Tesar and Werner (1995) report that US investor's equity portfolios have a domestic component of over 90%. With portfolio theory suggesting international diversification, we have one of the most robust puzzles in international finance. Attempts at explaining this puzzle have been of two types. The first type of explanations suggests that frictions in international financial markets make foreign assets sufficiently less attractive to justify the foregone diversification benefits. Whereas the second type of explanations builds on frictions outside the financial markets to justify observed portfolio holdings as an optimal risk management strategy.[1]

None of the available explanations seems entirely satisfactory to date, on the contrary, the puzzle may be even worse than what textbooks usually suggest. Indeed, Baxter and Jermann (1997) argue that when one considers the large and nontradable human capital component of wealth, agents should hold even less domestic assets. The reason is that with returns of domestic human capital highly correlated with the returns of domestic marketable assets, hedging the nontradable human capital requires a reduction in the holdings of these domestic marketable assets.

The fact that a large part of national income is nontradable labor income has been the focus of important recent research. For instance, Shiller (1993) argues that 'macro markets' for trading claims on capitalized national incomes should be established. Such markets would allow more efficient international risk sharing of hitherto nontradable risks. The fact that a large part of national income is nontradable labor income also matters in research that evaluates various proposals for social security reform, see for instance Bohn (1998). Indeed, to correctly capture the risk sharing and risk shifting properties of various reform proposals, nontradability is crucial.

Most macroeconomic business cycle models explicitly incorporate endogenous labor supply decisions. A fast growing literature on household production not only models cyclical time allocation in detail, but also has started to take nonseparabilities between measured consumption and home produced goods and services

[1] Some examples are: Cooper and Kaplanis (1994), Pesenti and VanWincoop (1996), Stulz (1983) and Tesar (1993).

seriously.[2] Clearly, variable labor supply is an important ingredient of modern macroeconomics. However, this margin is rarely explicitly considered in the large literature on portfolio choice.[3]

The main objective of this paper is to endogenize the labor/leisure choice, with the objective of evaluating international diversification when preferences of consumption and leisure are nonseparable.

We present a multi-country model, where countries are subject to country specific productivity shocks. Households decide how much time to spend working on market activities and choose portfolios that give them the necessary income to finance their desired consumption. In order to solve the portfolio problem we adapt a method presented in Baxter, Jermann and King (1995). The complicated problem becomes tractable by working with linear approximations of the first order conditions, a technique widely used for solving macroeconomic models. Pareto efficient consumption allocations can then be supported in a decentralized equilibrium by appropriately chosen international portfolios.

In the symmetric case, where all countries are ex-ante identical, we solve analytically for the agents' portfolios as a function of preference and technology parameters. The solution shows that, in order for the considered nonseparability to help explain home-bias, consumption and leisure need to be substitutes. This is qualitatively consistent with findings from the household production literature. For instance, Greenwood, Rogerson and Wright (1995) suggest substitutability between market produced consumption and goods and services produced with non-market time. In this case, when agents work a lot, they have little time for non-market production (cooking, cleaning, and child-care *etc.*) and thus they highly value market consumption (restaurants, cleaning persons, and baby-sitters *etc.*)—by holding domestic claims they will get the necessary purchasing power to finance increased market consumption. Quantitatively, the preference nonseparability helps in a nonnegligeable way, but it cannot entirely explain the extreme degree of home bias in US portfolios. This suggests that other factors such as transaction costs may also be part of the story that solves the puzzle.

In the remainder of this paper, we will start by characterizing Pareto efficient

[2] For a recent survey of this literature see for instance Greenwood, Rogerson and Wright (1995); Canova and Ubide (1997) look at household production and international business cycles.

[3] One exception is Bodie *et alt.* (1992) that look at some cases of portfolio choice with flexible labor supply in partial equilibrium. Leung (1995) solves a 2-country model with preferences that are nonseparable in consumption and leisure. For his case, the restrictions on preferences, required to obtain a closed form solution, rule out the possibility that the nonseparability can help explain home-bias.

allocations in section 2. In section 3 we derive the optimal portfolios. In section 4, we present analytical expressions for optimal portfolios as a function of preference and technology parameters and evaluate quantitative model implications. Section 5 summarizes and offers concluding remarks. Finally, an appendix contains the details of the derivations in section 4.

2. Optimal allocations

This section presents a multi-country general equilibrium model. As a first step in solving the portfolio problem and to build some intuition we will also characterize optimal consumption allocations

We have a J-country economy, that is atemporal and that is subject to country-specific productivity shocks. Each country representative consumes c_j of the consumption good, works a fraction n_j of his non-sleeping time and gets $l_j = 1 - n_j$ of leisure.[4] Pareto optimal allocations are determined by maximizing a weighted sum of individual country utilities $v_j(c_j, 1 - n_j)$. Letting ω_j denote the weight given to country j with $\sum_{j=1}^{J} \omega_j = 1$, this weighted sum is given by:

$$\sum_{j=1}^{J} \omega_j v_j(c_j, 1 - n_j) \ .$$

The world resource constraint for the consumption good is given by:

$$\sum_{j=1}^{J} \pi_j c_j = \sum_{j=1}^{J} \pi_j y_j = \sum_{j=1}^{J} \pi_j A_j f_j(n_j) \equiv Y \tag{2.1}$$

where A_j is productivity, y_j is country j's output and Y is world output. The resource weights π_j allow countries to vary in terms of economic size.

The first-order conditions describing optimal consumption allocations are

$$\omega_j \frac{\partial v_j(c_j, 1 - n_j)}{\partial c_j} = \lambda \pi_j, \quad j = 1, 2, ..., J, \tag{2.2}$$

where λ is the multiplier on the world resource constraint. The first-order conditions describing optimal labor-leisure allocation are:

[4] As we discuss more in detail below, we have in mind a broad notion of leisure that includes pure leisure time (e.g. watching TV) as well as household production.

$$\omega_j \frac{\partial v_j(c_j, 1-n_j)}{\partial n_j} = -\lambda \pi_j A_j \frac{\partial f_j(n_j)}{\partial n_j}, \quad j = 1, 2, ..., J. \tag{2.3}$$

These conditions are standard and imply that marginal utility of consumption is equalized across agents and, for the latter, that for each agent the marginal product of labor, in appropriate units, equals its marginal cost.

2.1. Properties of optimal consumption allocations

We now proceed to determine some properties of optimal consumption allocations. This characterization illustrates our solution method and the core economic determinants of optimal portfolios to be determined below. We differentiate first order conditions and we interpret terms such as dc as small endogenous deviations from the deterministic solution brought about by small shocks to the country specific productivity levels dA. We leave it open how small exactly these shocks are. A large literature on the accuracy of loglinear approximations suggests such methods to be extremely precise for models where shocks are calibrated to the size of business cycle fluctuations, see for instance Taylor and Uhlig (1990).[5]

Totally differentiating equation (2.2), we find that

$$\frac{dc}{c} = \left[\frac{1}{\xi_{cc}}\right] \frac{d\lambda}{\lambda} + \left[\frac{\xi_{cl} \cdot n}{\xi_{cc}(1-n)}\right] \frac{dn}{n} \tag{2.4}$$

where we do not include country subscripts since we are describing general properties. Parameters like ξ_{x_1,x_2} stand for the elasticity of the marginal utility of x_1 with respect to x_2, formally, $\xi_{x_1,x_2} = \frac{\partial^2 v(.)}{\partial x_1 \partial x_2} \cdot \frac{x_2}{\frac{\partial v(.)}{\partial x_1}}$. Equation (2.4) tells us whether an increase in n raises or lowers the optimal level of c, holding fixed the shadow price for world output λ. For instance, consider the case where consumption and leisure are substitutes, that is when $\xi_{cl} < 0$; note that $\xi_{cc} < 0$ to insure concavity of the utility function.[6] In this case, the marginal utility of c increases

[5] Although the preference specification of the model here is isomorphic to the model with nontraded goods in Baxter, Jermann and King (1995), the model's structure is fundamentally different. In the model here, leisure is endogenously determined jointly with endogenous output. In the model with nontraded goods, the outputs of the traded and nontraded good sectors are exogenous endowments and consumption of the nontraded good equals the exogenous output.

[6] Our definition of substitutability or complementarity is whether $\frac{\partial^2 v(.)}{\partial x_1 \partial x_2} < 0$, or $\frac{\partial^2 v(.)}{\partial x_1 \partial x_2} > 0$ respectively.

with increases of n. For this reason, the optimal allocation requires more of the consumption good for countries whose residents work more.

Combining equation (2.4)) with the linearized world resource constraint (2.1) and assuming symmetry across countries, we have after some algebra that:

$$\frac{dc_j}{c_j} = \frac{dY}{Y} + \eta^n \left(\frac{dn_j}{n_j} - \frac{dN}{N} \right), \qquad (2.5)$$

where $\eta^n \equiv \left[\frac{\xi_{cl} \cdot n}{\xi_{cc}(1-n)} \right]$, and $dN/N \equiv \frac{1}{J} \sum_{j=1}^{J} (dn_j/n_j)$.[7] Equation (2.5) illustrates the core economics of how consumption is shared internationally in efficient allocations. Changes in the world supply of Y are shared equally if $\eta^n = 0$, which would be the case for separable utility. If $\eta^n \neq 0$, an additional reallocation of the consumption good is undertaken based on an individual country's work effort relative to the world average. Thinking about leisure as producing preference shifts in the demand for the consumption good, equation (2.5) is very intuitive. Of course, changes in world demand for c must be frustrated by adjustments in its shadow price (λ) since there is a given stock to be allocated. It is only if there is a *relative* demand shock that a country's allocation is affected. Given that the optimal portfolio will need to finance an agent's consumption, an agent's income from working and owning assets will have to behave exactly like consumption.

3. Optimal portfolio in general equilibrium

This section shows that a country's optimal consumption can be written as a function of two shocks: a world productivity shock and a national productivity shock. As a consequence, the optimal international portfolio for a given country can be expressed as containing two funds: a fully diversified world fund (held by all countries) and a country specific fund (held only by that country). We outline how to determine these portfolios and show how to deal with the nontraded human capital.

Combining the linearized first order condition (2.4) and the linearized version of (2.1) and (2.3) it is immediate that the solution for a country's consumption can be written as

$$\frac{dc_j}{c_j} = \eta_j^{WA} \frac{dA^W}{A^W} + \eta_j^A \frac{dA_j}{A_j}, \qquad (3.1)$$

[7] We will define symmetry more in detail below. Our method to derive optimal portfolios in Section 3 applies equally for the non-symmetric case.

with $\frac{dA^W}{A^W} = \sum_{j=1}^{J} w_j^{WA} \frac{dA_j}{A_j}$ for appropriately defined weights w_j^{WA} and coefficients η_j.[8] Intuitively, the world resource multiplier, λ, can be written as a linear function of all the productivity shocks. Rewritten more compactly,

$$dc_j = \beta_j^W dA^W + \beta_j^A dA_j, \tag{3.2}$$

and each country's consumption can be seen as depending only on two risk factors: a world productivity shock, dA^W, and the country specific productivity shock, dA_j. In this linearized environment, this also implies that a two-fund theorem holds in this model. That is, the individual of country j with access to two mutual funds whose returns are a function of the two shocks, dA^W and dA_j, will be able to finance his optimal consumption bundle by holding appropriate amounts of each of the two funds.

We define an "equity claim" in this atemporal world to be a claim to a dividend, given by output minus labor income. Restricting the production function to be of the form $f_j(n_j) = n_j^{\alpha_j}$, the dividend which is the income that goes to the fixed factor is then simply equal to $(1-\alpha_j)y_j$. Individual consumers will choose portfolios of such equity claims to all countries' outputs. As we are about to show, countries' asset holdings can be viewed as quantities of two different mutual funds composed of individual countries' equity shares: a *world equity fund*, and the country's *domestic equity fund.* We will here first define these funds and show how their returns depend on the two sources of uncertainty considered for the individual consumption choice, namely dA^W and dA_j. Second, we will solve for the optimal quantities of each fund.

First, define a *world equity fund* that pays the following return:

$$d\mathbf{E} = \sum_{j=1}^{J} w_j d\mathbf{e}_j = \sum_{j=1}^{J} w_j (1-\alpha_j) dy_{j,}$$

where boldface notation is used to distinguish portfolio payouts from other quantities. Ideally, we would like the return to this world equity fund, to depend only on the world productivity shock, *i.e.*, we would like

$$d\mathbf{E} = \Lambda_E dA^W.$$

This can be accomplished by choosing the portfolio weights w_j appropriately.[9] Similarly, we define the *domestic equity fund* as returning:

[8] This linearized version of (3) is given by: $\xi_{lc}\frac{dc}{c} = \frac{d\lambda}{\lambda} + \frac{dA}{A} + \left(\xi_{ll}\left(\frac{n}{1-n}\right) - (1-\alpha)\right)\frac{dn}{n}$.

[9] More details are provided in the appendix.

$$d\mathbf{e}_j = (1-\alpha_j)dy_j.$$

Clearly, the domestic fund's payouts will depend on the world shock and the domestic productivity shock, so that we can write

$$d\mathbf{e}_j = \Lambda_j^W \; dA^W + \Lambda_j^A \; dA_j, \tag{3.3}$$

for appropriately defined coefficients Λ_j^W and Λ_j^A. Finally, the optimal holdings of the two funds can be found by substituting for dA^W and dA_j in (3.2), and we obtain:

$$dc_j = \vartheta_j^E d\mathbf{E} + \vartheta_j^e d\mathbf{e}_j \tag{3.4}$$

where the coefficients ϑ_j^E and ϑ_j^e give the holdings of each mutual fund.[10]

3.1. Nontradable human capital

We have so far abstracted from the fact that claims to labor income are non-tradable. That is, we have implicitly assumed that labor income can be traded directly. In fact, if we assume that labor income cannot be traded directly, agents will use tradable securities to hedge away labor income risk by selling a portfolio that replicates labor income. For our particular production function, $y_j = A_j n_j^{\alpha_j}$, the replicating portfolio, that is $w_{j,i}^{hc}$ implicitly defined below, is straightforward to determine. Indeed, given that returns to labor and to the fixed factor are just constant fractions of the returns to total output, dy_j, the return on human capital, $\alpha_j dy_j$, is thus just a linear function of the domestic equity return:

$$\alpha_j dy_j = \sum_{i=1}^{J} d\mathbf{e}_i = \frac{\alpha_j}{1-\alpha_j} d\mathbf{e}_j.$$

Therefore, the holding of the domestic fund with payout $d\mathbf{e}_j$, computed above, has to be adjusted to

$$\overline{\vartheta_j^e} = \vartheta_j^e - \frac{\alpha_j}{1-\alpha_j}.$$

[10] These coefficients are given by $\vartheta_j^E = \frac{1}{\Lambda_E}\beta_j^W - \left(\frac{\Lambda_j^W}{\Lambda_E \Lambda_j^A}\right)\beta_j^A$, and $\vartheta_j^e = \left(\frac{1}{\Lambda_j^A}\right)\beta_j^A$.

4. Analytic solution for the symmetric case

This section presents results for a world economy in which all countries are identical in terms of initial conditions, although they are subject to different productivity shocks. That is: we assume the following initial conditions: $c_j = c = y = Y$, $n_j = n$ and $\pi_j = 1/J$. With identical preferences across countries, the central elasticities ξ_{x_1,x_2} are the same across countries. Determining the optimal portfolio allocation as a function of the linearized model's preference and technology parameters is algebra intensive but relatively straightforward along the lines outlined in the previous section. The appendix presents the details of how the solution was obtained.

First, not unexpectedly given symmetry, the world equity fund is simply an equally weighted average of each country's stock market, that is:

$$d\mathbf{E} = \frac{1}{J}\sum_{j=1}^{J} d\mathbf{e}_j,$$

where units are normalized to one country's share size.

4.1. Separable utility

To gain intuition about the solution let us first assume that utility of consumption and leisure is separable. This implies that $\xi_{cl} = 0$, and that $\vartheta^e = 0$, so that:

$$\text{World equity fund:} \quad \vartheta^E = \frac{1}{1-\alpha}, \text{ and}$$

$$\text{Domestic equity fund: } \overline{\vartheta^e} = -\frac{\alpha}{1-\alpha}.$$

Here, the investor has a short position in the domestic equity fund that is only partially offset by domestic shares included in the world fund. In particular, if the country represents a share π of the world market, then it holds $\frac{\pi}{1-\alpha}$ of its wealth in domestic equity through the world fund but $-\frac{\alpha}{1-\alpha}$ through the domestic fund. Therefore, given that labor shares are larger than individual countryies' size shares for all countries, *i.e.* $\pi - \alpha < 0$, the domestic holdings are negative. This is the same result as in Baxter and Jermann (1997), so that by considering explicitly nontradable labor income the diversification puzzle is worse than you may have thought.

4.2. Nonseparable utility

In case $\xi_{cl} \neq 0$ we have

$$\text{World equity fund:} \quad \vartheta^E = \frac{1}{1-\alpha} - \vartheta^e, \text{ and}$$

$$\text{Domestic equity fund: } \overline{\vartheta^e} = \vartheta^e - \frac{\alpha}{1-\alpha},$$

$$\text{where: } \vartheta^e = \frac{\frac{\xi_{cl}}{\xi_{cc}} \frac{n}{1-n}}{(1-\alpha)\left(1+\frac{1}{\varepsilon_{n,w}}\right)}. \tag{4.1}$$

Here $\varepsilon_{n,w}$ is the (compensated) elasticity of labor supply with respect to the wage rate. The denominator of (4.1) is always positive because of concavity. Thus, the sign of ϑ^e depends on the sign of ξ_{cl}, given that $\xi_{cc} < 0$ to insure concavity.[11] For instance, with $\xi_{cl} < 0$, consumption and leisure are substitutes, and the holdings of the domestic equity fund not due to the hedging demand for labor income, ϑ^e, is positive. In this case, with consumption and leisure substitutes, consumption is highly valued in periods when work effort is high. These are also the periods when work effort creates a positive effect on domestic output. Therefore, a domestic claim provides the right hedge.

Substitutability, *i.e.* $\xi_{cl} < 0$, seems to be consistent with a somewhat broader notion of leisure as time that is used to privately produce goods and services. For instance, assume that non-market time, $1-n$, is used as the sole input into a linear household production technology. Then, the time allocation structure in our model is identical to the one in the household production model of Greenwood and Hercowitz (1991). As suggested in the survey on household production by Greenwood, Rogerson and Wright (1995), substitution between market and non-market activity is important at the business cycle frequency. Our model derives the implications of this substitution for optimal international portfolio diversification. It tells the following story. When working long-hours of market activity, little time is available for cooking, cleaning, child-caring *etc.* and these goods and services need now to be purchased in the market. By holding domestic claims that have high payouts agents have the ability to afford these goods and services.

[11] Concavity requires that ξ_{cc} and ξ_{ll} are both negative, whereas ξ_{cl} and ξ_{lc} may be either positive or negative as long as $\xi_{cc}\xi_{ll} - \xi_{cl}\xi_{lc} > 0$ for overall concavity. Given that ξ_{cl} and ξ_{lc} take their sign from the cross derivative, they both have the same sign. See the appendix on how $\varepsilon_{n,w}$ depends on the elasticities describing preferences.

The effect of the labor supply elasticity, $\varepsilon_{n,w}$, also seems to fit well with our intuition. For instance, with a very small labor supply elasticity, domestic labor supply does not move much, and there is not much need for hedging this risk factor for consumption suggested by equation (2.5).

4.3. Specific preferences and quantitative implications

We look here at the issue of whether the effect of nonseparable leisure can plausibly explain the full extent of home-bias, taking into account that the nontraded labor income puts us in a very unfavorable starting position. We look at this issue by specializing preferences in a way that allows us to identify known parameters. Overall, nonseparable preferences represent a clear progress. However, for a reasonable range of the elasticity of substitution, the preference effect is not strong enough to explain alone the extreme home bias in portfolios of US investors.

4.3.1. A three-parameter preference specification

We parameterize preferences as:

$$v(c, 1-n) = \frac{1}{1-\gamma}\varphi(c, 1-n)^{1-\gamma},$$

with $\varphi(\cdot)$ being a constant returns to scale aggregator. As is well known, the local behavior of these preferences can be described by only three parameters, risk aversion, γ, the inverse of the elasticity of substitution, μ, and the value share for consumption s_c (and thus leisure $s_l = 1 - s_c$); that is, locally, this function is like a CES function.[12]

After some algebra, the share of the domestic fund not due to the hedging demand for labor income, ϑ^e, can be written as:

$$\vartheta^e = \frac{\left[\frac{s_l(\gamma-\mu)}{s_c\gamma+s_l\mu}\right]\frac{n}{1-n}}{(1-\alpha)\left(1+\frac{1}{\varepsilon_{n,w}}\right)}.$$

However, $\varepsilon_{n,w}$ is no longer a free parameter but it depends on the other 3 coeffi-

[12] Value shares are defined as $s_x = \frac{\left(\frac{\partial\varphi()}{\partial x}\right)\cdot x}{\varphi()}$. For a reference about some properties of constant returns to scale functions see Ferguson (1964) and our appendix.

cients. Solving further we can write that

$$\overline{\vartheta^e} = \vartheta^e - \frac{\alpha}{1-\alpha} = \frac{\left[\frac{s_l(\gamma-\mu)}{s_c\gamma+s_l\mu}\right]\frac{n}{1-n}}{(1-\alpha)\left(1+\left[\frac{\gamma\mu}{s_c\gamma+s_l\mu}\right]\frac{n}{1-n}\right)} - \frac{\alpha}{1-\alpha}, \tag{4.2}$$

where the sign of ϑ^e is determined by the sign of $(\gamma-\mu)$. The sign of $(\gamma-\mu)$ gives the sign of the cross-derivative, that is, it determines whether consumption and leisure are substitutes, $(\gamma-\mu)>0$, or complements, $(\gamma-\mu)<0$.

Earlier studies about portfolio choice with nonseparable preferences have found that the sign of $(\gamma-\mu)$ is important. In Tesar (1993), the sign of $(\gamma-\mu)$ determines the sign of the cross-derivative in the period utility function with consumption of traded and nontraded goods. In her endowment model, claims to nontraded goods are not traded internationally. It turns out that home bias in a portfolio of claims to traded goods endowment depends on the sign of $(\gamma-\mu)$ and on the covariance properties of domestic and foreign endowments.[13] In the model here, $(\gamma-\mu)>0$ is necessary and sufficient for home bias if we abstract from the fact that labor income is nontraded—independently of the covariance matrix of the productivity shocks. In Baxter, Jermann and King's (1995) analysis of nontraded goods, the sign of $(\gamma-\mu)$ also plays a crucial role. Contrary to Tesar (1993), this matters for the optimal holdings of claims to nontraded goods output. In BJK claims to nontraded goods are explicitly traded internationally along with claims to traded goods. In this case, the effect of the sign of $(\gamma-\mu)$ on optimal portfolio shares is somewhat more nonlinear however.

4.3.2. Quantitative implications

To explore quantitative implications for international diversification, we compute the optimal share of domestic equity for a US representative. Through the two funds, the investor holds in total $\overline{\vartheta^e}+\pi\vartheta^E$ of his wealth in domestic shares, where π is the country's size share in the world. That is, in addition to the domestic claims in the domestic equity fund, $\overline{\vartheta^e}$, the investor also holds $\pi\vartheta^E = \pi\left(\frac{1}{1-\alpha}-\vartheta^e\right)$ of domestic shares within the world equity fund. Thus the portfolio share of

[13] In particular, if $(\gamma-\mu)<0$ (complementarity), and if the covariance of the nontraded endowment is higher with domestic tradable endowment than with the foreign tradable endowment, the portfolio displays home bias. The same property holds in Pesenti and VanWincoop (1996).

domestic claims equals

$$\left(\vartheta^e - \frac{\alpha}{1-\alpha}\right) + \pi\left(\frac{1}{1-\alpha} - \vartheta^e\right).$$

We pick the following parameter values: the fraction of non-sleeping time spent working $n = 0.33$, labor share $\alpha = 0.6$, and risk aversion $\gamma = 5$.[14] To a first approximation, value shares may not be too different from the time fraction spent at the respective activities, so that $s_c = n$ and $s_l = 1 - n$. Solnik (1996) reports the US' share in the world equity market to be 33% for 1994, we set $\pi = 1/3$.

The recent literature on household production has generated useful information about the elasticity of substitution between consumption and non-working time, $\frac{1}{\mu}$. In two early quantitative models with household production, researchers established a "preferred value" that was chosen to insure the model performed well in terms of some key business-cycle facts. Specifically, Greenwood, Rogerson and Wright (1995) chose $\frac{1}{\mu} = 3$, while Benhabib, Rogerson and Wright (1991) report a preferred value of $\frac{1}{\mu} = 5$. McGrattan, Rogerson and Wright (1996) use maximum likelihood with macroeconomic data to obtain a point estimate of $\frac{1}{\mu} = 1.75$, with a 95% confidence interval $(1.25, 2.95)$. Using microeconomic data from the PSID, Rupert, Rogerson and Wright (1996) find point estimates between 0.93 and 4, depending on whether they use data for single males, single females or couples, with 95% confidence intervals covering $(0.05, 4.98)$. Based on this evidence, a reasonable range for $\frac{1}{\mu}$ seems to be $(0, 5)$. Note however, these studies distinguish between household production and leisure time as non-market activities, which does not gives us an exact mapping into our preference specification.

Figure 1 plots the optimal holding of U.S. equity in a representative US portfolio as a function of the elasticity of substitution, $\frac{1}{\mu}$. Overall, nonseparable preferences represent a clear progress. However, for the reasonable range of the elasticity of substitution, the share of domestic claims as at best 0.5, with$\frac{1}{\mu} = 5$, whereas French and Poterba (1991) report this to be over 0.9. One way to interpret the result is that, although the nonseparable preferences make investors tilt towards domestic claims, it is not enough to overcome the demand for foreign claims for hedging the nontradable labor income. Indeed, with separable utility, the domestic equity share equals $\frac{\alpha-\pi}{1-\alpha} = -.675$, due to the powerful demand for foreign claims for hedging the nontradable labor income.

[14] The Michigan time survey reports nonsleeping time spent working in the market at 0.33, see Greenwood, Rogerson and Wright (1995). Cooley and Prescott (1995) estimate $\alpha = 0.6$. For risk aversion, $\gamma = 5$ is consistent with Campbell (1996).

In Table 1 we present an analysis of the sensitivity to changing some key parameter values. Increasing risk aversion, for instance to 10, would increase the share of domestic claims to 0.55 with $\frac{1}{\mu} = 5$. Decreasing the labor share α would also imply more home bias because the hedging demand for labor income would be weaker. Finally, the sensitivity to the time allocation parameter n is relatively flat around our benchmark value.

5. Concluding remarks

We have shown how to solve for an agent's portfolio in a world with endogenous labor supply choice. For the symmetric, case we fully solve for the agent's portfolio as a function of preference and technology parameters. We have argued that the mechanism required to explain home-bias is qualitatively consistent with the spirit of household production models.

Despite some progress towards solving the home-bias puzzle, quantitatively, our model cannot alone explain the puzzle. One may be tempted to speculate at this point about the potential for some new model elements. A first extension may be to include shocks to labor shares so as to break the perfect correlation of the incomes that go to capital and labor. This will reduce the demand for foreign shares to hedge the nontradable labor income, see for instance Bottazi, Pesenti and VanWincoop (1996). However, on the negative side, this should also loosen the link between the role of labor as a preference shifter and domestic equity as a hedge for it. A second extension could be shocks to the 'household' production technology. It seems likely that some specification of the covariance matrix for such shocks could explain the observed home bias. However, given the difficulty in obtaining a measurement of these shocks, an explanation based on exogenous preference shifters seems not very appealing. A third possibility could be to consider transaction costs that would make it more costly to own foreign equity and thus implicitly reduce their expected return. The calculations by French and Poterba (1991) suggest that an explanation relying exclusively on transaction costs seems implausible. However, transaction costs together with nonseparable preferences appears to be a more promising avenue.

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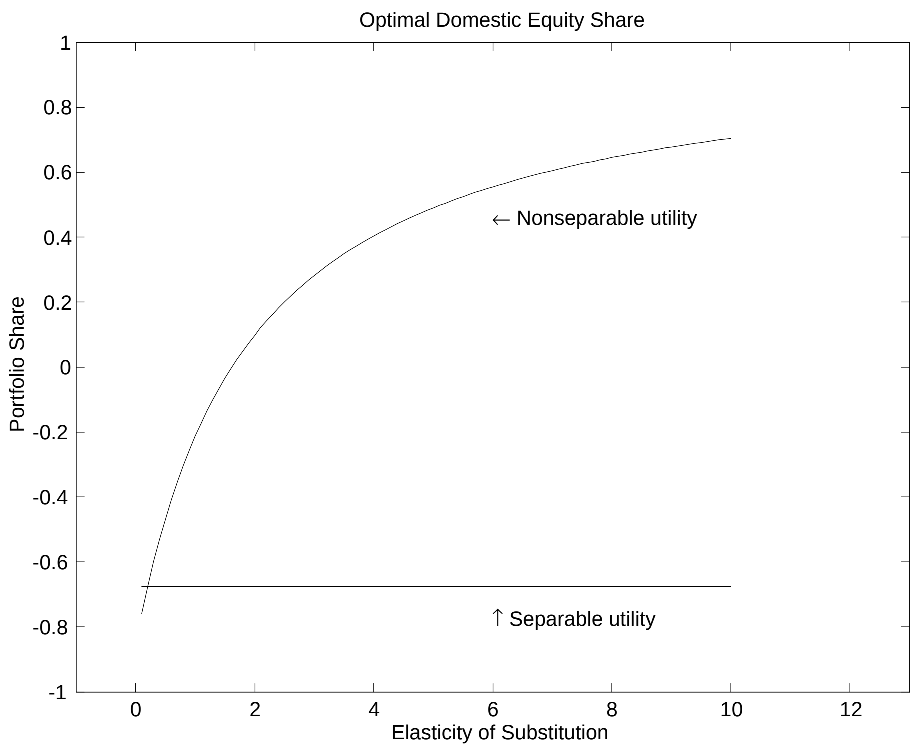
Optimal Domestic Equity Share
Portfolio Share
Elasticity of Substitution
← Nonseparable utility
↑ Separable utility
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
2
4
6
8
10
12

Figure 5.1:

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Table 1

Sensitivity to parameter values

	Domestic portfolio component
Benchmark	0.49
$\gamma = 2$	0.33
$\gamma = 10$	0.55
$\alpha = 0.5$	0.59
$\alpha = 0.7$	0.32
n = 0.25	0.48
n = 0.4	0.48

(Benchmark: $1/\mu = 5$, $\gamma = 5$, n = sc = 0.33, $\alpha = 0.6$)

Appendix

Although it is straightforward to show that the solution has a two-fund property, it does not seem convenient to solve the problem directly in this form. Instead, we use a three-fund structure of the model that is relatively easy to solve analytically. Indeed, as suggested by equation (2.5), combining equation (2.1) and (2.2), consumption can be written as

$$dc_j = \beta_j^Y dY + \beta_j^N dN + \beta_j^n dn_j, \tag{5.1}$$

where dY, dN, and dn_j can be interpreted as risk factors. Here we will need to define 3 funds that provide the income to support optimal consumption. Inspection of the first order conditions suggests why the three-fund solution is more tractable. As shown in equation (2.5), solving for consumption as a function of the three factors can be done recursively by simply substituting out the multiplier, $\frac{d\lambda}{\lambda}$. For the two-fund case, finding the weights, w_j^{WA}, and the coefficients η_j^{WA} and η_j^A, in equation (3.1) requires solving simultaneously the $2J+1$ equations of the model. As we will show below, once the three-fund problem is solved, two of the three funds are identical, which gives us the postulated two-fund solution.

1. Definition of the equity mutual funds

As a first step, we define three funds composed of individual countries' equity shares: a *world equity fund*, the country's *domestic equity fund* and a *world hedge fund for labor supply shocks.*

First, the *world equity fund* pays the following return:

$$d\mathbf{E} = \sum_{j=1}^{J} w_j d\mathbf{e}_j = \sum_{j=1}^{J} w_j (1-\alpha_j) dy_j,$$

where boldface notation is used to distinguish portfolio payouts from other quantities. Ideally, we would like the return to this world equity fund, to depend only on the world output of the consumption good, *i.e.*, we would like $d\mathbf{E}$ to depend only on dY. This can be accomplished by choosing the portfolio weights w_j to satisfy

$$w_j = W \frac{\pi_j}{(1-\alpha_j)}$$

so that $\sum_{j=1}^{J} w_j(1-\alpha_j)\ dy_j\ = WdY$. Further, requiring that $\sum_{j=1}^{J} w_j\ = 1$ determines the constant $W\ =\ \left[\sum_{j=1}^{J} \pi_j/(1-\alpha_j)\right]^{-1}$. With the portfolio weights determined in this way, we may write:

$$d\mathbf{E} = \Lambda_E dY. \tag{5.2}$$

Second, the *domestic equity fund* returns:

$$d\mathbf{e}_j = (1-\alpha_j)dy_j = (1-\alpha_j)[\alpha(y_j/n_j)dn_j + (y_j/A_j)dA_j].$$

Normally, we would solve this equation by substituting out the endogenous variable in the right hand side, dn_j, but for our purpose of solving for portfolio shares we will substitute out the productivity shock, dA_j. To do this, we use the linearized version of the first order condition for labor supply, (2.3),

$$\xi_{lc}\frac{dc}{c} = \frac{d\lambda}{\lambda} + \frac{dA}{A} + \left(\xi_{ll}\left(\frac{n}{1-n}\right) - (1-\alpha)\right)\frac{dn}{n}$$

to substitute out dA_j. We then use the linearized world resource constrained

$$\frac{dY}{Y} = \sum_{j=1}^{J} \theta_j \eta_j^{\lambda}\frac{d\lambda}{\lambda} + \sum_{j=1}^{J} \theta_j \eta_j^{n}\frac{dn_j}{n_j} = \sum_{j=1}^{J} \theta_j \eta_j^{\lambda}\frac{d\lambda}{\lambda} + \eta^N\ \frac{dN}{N},$$

where $\theta_j = \pi_j c_j/Y$ and the latter equality follows from the definitions $\eta_j^{\lambda} \equiv \left[\frac{1}{\xi_{cc_j}}\right]$, $\eta^N \equiv \sum_{j=1}^{J} \theta_j \eta_j^n$ and $dN/N = [\sum_{j=1}^{J} \theta_j \eta_j^n \frac{dn_j}{n_j}]/\eta^N$. Combined with equation (2.4) we can substitute out $d\lambda/\lambda$ and dc_j/c_j. The domestic equity return is then found to depend on all three risk factors:

$$d\mathbf{e}_j = \Lambda_j^Y\ dY + \Lambda_j^N\ dN + \Lambda_j^n\ dn_j, \tag{5.3}$$

with the Λ coefficients being functions of the different demand elasticities, production elasticities and country weights.

Finally, we define the *world hedge fund for labor supply shocks*:

$$d\mathbf{H} = \sum_{j=1}^{J} w_j^H d\mathbf{e}_j = \sum_{j=1}^{J} w_j^H(\Lambda_j^Y\ dY + \Lambda_j^N\ dN + \Lambda_j^n\ dn_j).$$

We want this fund to be useful in hedging the risk associated with *world* demand shocks on consumption coming from changes in the *world* labor supply. We therefore require that its return does depend only on the *world* risk factors, dY and dN, but not on any dn_j directly. This can be accomplished by choosing the portfolio weights w_j^H to satisfy

$$w_j^H \Lambda_j^n = W^H N \frac{\theta_j \eta_j^n}{n_j \eta^N}$$

so that $\sum_{j=1}^J (w_j^H \Lambda_j^n)\ dn_j = W^H(dN)$. Further, requiring that $\sum_{j=1}^J w_j^H = 1$ determines the constant $W^H = \left[\sum_{j=1}^J \left(N\theta_j \eta_j^n/(\eta^N n_j \Lambda_j^n)\right)\right]^{-1}$. With the portfolio weights determined in this way, we may write

$$d\mathbf{H} = \sum_{j=1}^J \left(w_j^H \Lambda_j^Y\right) dY + \left(\sum_{j=1}^J \left(w_j^H \Lambda_j^N\right) + W^H\right) dN,$$

which is a payoff structure of the form:

$$d\mathbf{H} = \Lambda_H^Y\ dY + \Lambda_H^N\ dN. \tag{5.4}$$

2. Supporting optimal consumption

Substituting for dY, dN, and dn_j from (5.2)-(5.4), we have:

$$dc_j = \vartheta_j^E d\mathbf{E} + \vartheta_j^H d\mathbf{H} + \vartheta_j^e d\mathbf{e}_j \tag{5.5}$$

where the coefficients—and thus the holdings—for each of the mutual funds are:

$$\underline{\vartheta_j^E} = \frac{1}{\Lambda_E}\beta_j^Y - \left(\frac{\Lambda_H^Y}{\Lambda_E \Lambda_H^N}\right)\beta_j^N - \left(\frac{1}{\Lambda_E \Lambda_j^n}\right)\left(\Lambda_j^E - \frac{\Lambda_j^N \Lambda_H^E}{\Lambda_H^N}\right)\beta_j^n$$

$$\vartheta_j^H = \left(\frac{1}{\Lambda_H^N}\right)\beta_j^N - \left(\frac{\Lambda_j^N}{\Lambda_H^N \Lambda_j^n}\right)\beta_j^n$$

$$\vartheta_j^e = \left(\frac{1}{\Lambda_j^n}\right)\beta_j^n.$$

(We underline variable names, '$\underline{\mathrm{x}}$', so as to distinguish them, when appropriate, from variables in the text with the same name)

3. Determining the coefficients for the symmetric case

First, direct substitution shows that both world funds are identical:

$$d\mathbf{E} = d\mathbf{H} = \frac{1}{J}\sum_{j=1}^J d\mathbf{e}_j.$$

That is, the world funds are just equally weighted, with units equal to one country's size share. So that by adding $\underline{\vartheta^E}$ and ϑ^H we obtain the two-fund solution.

The coefficients $\underline{\vartheta^E}$,ϑ^e, ϑ^H can be determined in three steps.

1. By direct substitution

$$\vartheta^e = \frac{\beta^n}{\Lambda^n} = \frac{\frac{\xi_{cl}}{\xi_{cc}}\frac{n}{1-n}}{(1-\alpha)\left(1+\frac{1}{\varepsilon_{n,w}}\right)},$$

   where $\varepsilon_{n,w}$ is the 'lambda-constant' or compensated labor supply elasticity

$$\varepsilon_{n,w} = -\frac{\xi_{cc}}{\xi_{ll}\xi_{cc} - \xi_{lc}\xi_{cl}}\frac{1-n}{n}.$$

2. Tedious, but straightforward substitution, using the previous result, shows that

$$\vartheta^e = -\vartheta^H.$$

3. Using the results from the two previous steps, simple substitution gives the result

$$\underline{\vartheta^E} = \frac{1}{1-\alpha}.$$

Thus we have the result in equation (4.1) in the text.

4. Determining the coefficients for the specific utility function

Using properties of constant returns to scale aggregators, see for instance in Ferguson (1964), we can make the following substitutions:

$$\begin{aligned}
\xi_{cc} &= -\left[s_c\gamma + s_l\mu\right] \\
\xi_{lc} &= -s_c\left[\gamma - \mu\right] \\
\xi_{cl} &= -s_l\left[\gamma - \mu\right] \\
\xi_{ll} &= -\left[s_l\gamma + s_c\mu\right],
\end{aligned}$$

from which equation (4.2) in the text follows immediately.