Financial Markets' Views about the Euro-Swiss Franc Floor

Urban J. Jermann*

Wharton School of the University of Pennsylvania and NBER

January 21, 2017

Abstract

Exchange rates and option prices incorporate market participants' views about the credibility and the effects of exchange rate targets. I present a model to determine exchange rates under policy targets that can be used to price options. The model is estimated with Euro-Swiss Franc exchange rate and options price data. In the first few months of the minimum exchange rate policy, the implied survival probability of the policy for a three month horizon was typically less than 75%. Over time, the credibility increased and this probability reached 95% in August 2014. The analysis also implies that during the second quarter of 2012, when reserve accumulation was high, the exchange rate without the policy would have been as low as about 1 Swiss franc per euro. (Key words: Exchange rate target, currency options, euro, Swiss franc; F3, G12)

*Comments from seminar participants at the Wharton School, London Business School, as well as from Philipp Illeditsch, Jakub Jurek, Karen Lewis, Nick Roussanov, and Amir Yaron are gratefully acknowledged. A not-for-publication appendix is available at http://finance.wharton.upenn.edu/~jermann/research.html. Email: jermann@wharton.upenn.edu.

1 Introduction

From September 2011 to January 2015 the Swiss central bank's official policy has been to maintain a minimum exchange rate at CHF 1.20 per euro. To achieve this objective, the central bank had stated that it would be willing to buy foreign currency in unlimited quantities (SNB, 2011). This commitment and the sustainability of this policy had been questioned right from the beginning. Under such a policy, a lack of credibility to carry through with defending the floor invites capital inflows and requires the central bank to buy large quantities of foreign currency which could ultimately make the policy very expensive. Therefore, it is important to be able to precisely assess the views of financial markets participants about the policy and in particular its credibility

Financial prices can be used to learn about the implied risk-neutral probability that the EURCHF will move below the official floor of 1.20. For instance, a put option with a strike price of 1.20 or lower with a positive price requires that financial markets assign positive probability that the policy will not persist until the maturity of the option. If exchange rates are lognormally distributed, then one can use the standard Black-Scholes option pricing model to measure the implied risk-neutral probability for any range of the exchange rate. However, given the floor introduced by the policy, lognormality is a poor assumption for the exchange rate distribution in this case. Moreover, standard option pricing models that take the exchange rate as exogenous have nothing to say about how the policy affects the exchange rate.

In this paper, I present a model where the fundamental exchange rate (the "shadow" exchange rate in the absence of the policy floor) follows a process with given risk-neutral probabilities. It is assumed that there is uncertainty about whether the policy floor will be in place next period. If the policy ends, the observed exchange rate is assumed to equal the fundamental exchange rate. In this setting, the dynamics of the observed exchange rate are derived based on no-arbitrage principles, and the exchange rate equals a function of the current fundamental exchange rate, its risk neutral probabilities, and the continuation probability of the policy.

This model is used to price put and call options on the exchange rate. The model is fitted to data on EURCHF spot and options prices and it produces estimates for the market implied probabilities of the floor being violated as well as for what the exchange rate would have been without the policy. I find that during the first few months of the minimum exchange rate policy, markets typically attached a probability of less than 75% to the policy being in place three months later. Over time, the credibility increased and reached 95% in August 2014. The analysis also implies that during 2012 Q2 and Q3, when reserve accumulation was high, the exchange rate without the policy would have been as low as about 1 Swiss franc per euro. The model can also shed light on specific episodes. For instance, it is shown how it could be used to successfully forecast the outcome of the popular vote on the Swiss Gold Initiative in the fall 2014.

This paper is related to the large literature on exchange rate target-zones initially developed for the European Exchange Rate Mechanism prior to the birth of the euro. Krugman (1991) presents a model that characterizes exchange rate behavior with a target zone. Bertola and Caballero (1992) and Bertola and Svensson (1993) extend the model to allow for realignment risk, that is, uncertainty about whether the target will be maintained. Dumas et al. (1995) explicitly link a target zone model to option prices and present numerical characterizations for option prices with realignment risk. Campa and Chang (1996) study target zone credibility with currency options data by using select no-arbitrage relations, but without a fully specified model for the exchange rate. Malz (1996) estimates risk-neutral probabilities of realignment, that is, exchange rates outside the target-zone, with an option pricing model where the exchange rate follows a jump-diffusion process. More recent papers with related approaches are Söderlind (2000) and Hui and Lo (2009). In these three studies, unlike in target zone models, exchange rates are taken as exogenous and no attempt is made to derive exchange rate dynamics from the exchange rate policy and fundamentals in an internally consistent way.

Like in a traditional target zone model, I derive the exchange rate endogenously. In my model, the exchange rate is derived based on a no-arbitrage condition. Traditional target zone models (Krugman (1992), or Dumas et al (1995)) assume the logarithm of the exchange rate depends on the expected logarithm of the future exchange rate; this is usually motivated as an approximation to uncovered interest parity. Uncovered interest parity has been conclusively rejected by a large number of empirical studies (Engel (1996), Lustig et al (2011)). Unlike in the existing target zone literature, my model is estimated with options data. In contrast to the typical models in this literature, my model is specified in discrete-time; it is solved through an application of the contraction mapping theorem. Discrete time gives me more flexibility when deriving exchange rate dynamics and pricing options in an internally consistent way, as there is no need to rely on analytic solutions. The model framework proposed in this paper can be adapted to other exchange rate target regimes and used to price other derivative securities.

There are two other recent studies on the Swiss franc floor from an options perspective. Hertrich and Zimmermann (2013) use a version of the Black-Scholes model to derive probabilities of EURCHF going below 1.20. As in Malz (1996), their approach takes the exchange rate as exogenous and there is no attempt to derive exchange rate dynamics from the exchange rate policy and fundamentals in an internally consistent way. Our approach is closer to Hanke et. al (2014) whose main focus is on what the EURCHF exchange rate would have been without the minimum exchange rate policy. Their exchange rate is modelled to explicitly depend on the existence of the policy floor. However, in their model the exchange rate is determined under the assumption that there is no uncertainty about when the minimum exchange rate policy ends. As such, their framework is not well suited to assess the credibility of the policy.

Some papers in the literature have modelled and numerically analyzed government guarantees for stock prices such as Ljungqvist (2000) and Durdu and Mendoza (2006). In this paper, I am introducing a similar price support mechanism into a no-arbitrage model for exchange rates that is used to price options.

The next section presents the exchange rate model. Section 3 describes the empirical analysis and 4 the results. Concluding remarks follow. Additional details of the analysis are available in a separate appendix.

2 Exchange rate dynamics under minimum exchange rate policy

I start by describing the model for the exchange rate, and then use it to price European style options contingent on this exchange rate.

Consider a process for V_t, the fundamental value of the exchange rate CHF per EUR in the absence of the price support policy for the euro. This process is assumed to be exogenously given, it is arbitrage-free, and in discrete-time. As the model is mainly used to price derivatives, we directly assume risk-neutral probabilities.

For the next period, $t+1$, there are two possible regimes: price support and no support, $x_{t+1} = 1$ and $x_{t+1} = 0$, respectively. In the price support regime, the central bank is assumed to guarantee a minimum value of K Swiss francs for one euro. It is assumed that initially the minimum rate policy is in place, $x_t = 1$, and that once that policy stops, it will not be restarted.

Under these assumptions, no-arbitrage implies that the equilibrium exchange rate $\tilde{S}_t$ satisfies

$$\begin{aligned}\tilde{S}_t &= \frac{1+r_t^*}{1+r_t}\left[p_t E_t^Q \max\left(\tilde{S}_{t+1}, K\right) + (1-p_t)\, E_t^Q V_{t+1}\right] \\ &= \frac{1+r_t^*}{1+r_t} p_t E_t^Q \max\left(\tilde{S}_{t+1}, K\right) + (1-p_t)\, V_t, \qquad (1)\end{aligned}$$

with E_t^Q the expectation under the risk-neutral distribution, p_t the (risk-neutral) probability of the continuation of the policy $(x_{t+1} = 1)$, and r_t^* and r_t the per period interest rates for the euro and Swiss franc, respectively. The observed exchange rate is given by

$$S_t = \max\left(\tilde{S}_t, K\right). \tag{2}$$

Intuitively, the equilibrium exchange rate represents the expected value over the two regimes of next period's exchange rate, discounted by the numeraire interest rate r_t, and augmented by the interest rate on the euro r_t^*. The time t equilibrium exchange rate $\tilde{S}_t$ can possibly be below the policy floor K. In that case, it is assumed that the central bank can intervene in the current spot market to guarantee that the observed exchange rate equals K, that is, the observed exchange rate satisfies (2). The distinction between the equilibrium exchange rate $\tilde{S}_t$ and the observed exchange rate S_t allows the model to produce a well-defined exchange rate based on the no-arbitrage equation (1), while at the same time allowing for the peg to be enforced in the current period.

The minimum rate policy can be viewed as introducing an option-like payoff profile into the exchange rate. Hanke at al. (2014) propose such an interpretation, where the exchange rate policy is viewed as a put to sell euro for K Swiss francs. Given put-call parity, this is equivalent to a call, as equation (1) suggests. As a substantive difference from their approach, the exchange rate in my model takes into account the uncertainty about whether the policy will be discontinued. This is captured by $(1 - p_t)$, the probability that the policy will be discontinued next period. Hanke et al. (2014) assume that the policy has a known maturity date.[1]

Options on the exchange rate are easily priced. For instance, for the case of a constant continuation probability, a European style put on the observed spot rate with strike K_p and

[1] In their case, this is inconsistent with their later assumption that traded options are priced by accounting for the uncertainty about the continuation of the policy. That is, they implicitly assume that the spot EURCHF exchange rate and options on this exchange rate are priced under mutually inconsistent assumptions about the uncertainty of the policy.

maturity τ_p is priced at

$$P_t\left(K_p,\tau_p\right)=\frac{1}{\left(1+r\right)^{\tau_p}}\left\{\begin{array}{c} p_{t,t+\tau_p}E_t^Q \max\left[K_p-\max\left(\tilde{S}_{t+\tau_p},K\right),0\right] \\ +\left(1-p_{t,t+\tau_p}\right)E^Q \max\left[K_p-V_{t+\tau_p},0\right] \end{array}\right\}. \tag{3}$$

If the market view is that the exchange rate floor will be in place with probability 1 for the time covering the maturity of the option, then $p_{t,t+\tau_p}=1$, and

$$P_t\left(K_p,\tau_p\right)=\frac{1}{1+r}E_t^Q \max\left[K_p-\max\left(\tilde{S}_{t+\tau_p},K\right),0\right]. \tag{4}$$

In this case, as seen in equation (4), a put with a strike below the floor, $K_p<K$, will have a market price of zero. Option prices in general are informative about the market's implied probability of the persistence of the policy. As suggested by equation (4), put options with strike prices below the minimum exchange rate are particularly useful in this regard.

3 Empirical approach

The fundamental exchange rate, V_t, is assumed to follow a recombining binomial tree, with the standard assumptions that per period up and down moves are given by

$$\begin{aligned} u &= e^{\sigma}, \text{ and} \\ d &= e^{-\sigma}, \end{aligned}$$

and the risk-neutral probability of an up move equals

$$q=\frac{e^{r-r^*}-d}{u-d},$$

with σ the standard deviation of the log of the growth of the fundamental exchange rate. For tractability, I impose upper and lower bounds on V_t, so that V_t takes a finite number of values. The bounds are set far enough from the typical range of the exchange rate so as not to affect current exchange rates and option prices with finite maturity. The equilibrium exchange rate $\tilde{S}$ can be computed as the fixed point of the operator

$$T\tilde{S}(V) = \frac{1+r^*}{1+r} p E^Q \max\left(\tilde{S}(V'), K\right) + (1-p)V,$$

where p is the constant continuation probability. It can easily be checked that Blackwell's sufficient conditions for a contraction mapping are satisfied for $\frac{1+r^*}{1+r}p < 1$, see Stokey, Lucas and Prescott (1989). The fixed point is found by applying the operator repeatedly.

The observed spot exchange rate for a given date, S_t, can be viewed as a function of the parameters (p, σ, r, r^*) and the value of the state variables: the fundamental rate V_t and the policy regime x_t. The model is used to estimate the values for (p, σ, V_t) that best fit a set of option prices $\{P_{t,j}\}$ and the spot rate S_t for a given date. Interest rates are taken as given from the data. Goodness of fit is represented by the sum of squared deviations between the prices (for the options and the exchange rate) from the model and the data. As the benchmark, the deviations are equally weighted. Alternative weighting schemes are explored. This procedure is repeated – and the model is re-estimated – for every day of the sample to produce time series for the implied values of (p_t, σ_t, V_t). While this procedure produces values for the survival probability of the policy and the volatility of the fundamental exchange rate, p and σ, that are typically changing daily, the model does not incorporate this variation. Below, a time-varying p is shown to have only minimal effects on estimates. Explicitly modelling time-varying parameters complicates the model and slows down the estimation procedure. Of course, users of the Black-Scholes option pricing model typically also take this shortcut.

Option	Strike price average (CHF)	Price average ($CHF/100$)	Impl. vol. average
Put 10Δ	1.164	0.21	7.06
Put 25Δ	1.196	0.48	5.18
Call 10Δ	1.270	0.19	6.56
Call 25Δ	1.239	0.47	5.20

Table 1: Summary of data for daily close mid-point prices of EURCHF 3 month options from Reuters covering 9/7/2011 to 1/14/2015.

3.1 Data and implementation

Data for options and spot EURCHF are from Reuters covering the entire period of the policy floor, 9/7/2011 to 1/14/2015, for a total of 848 observations. I use daily mid-point close implied volatility quotes for at-the-money options, ATM, risk reversals, RR, and butterfly spreads, BY, with a maturity of 3 months. The RR and BY quotes are available for strike prices corresponding to 25% and 10% delta, 25Δ and 10Δ, so that out-of-the-money puts and calls with these two strike prices can be obtained without interpolation. For 25Δ calls and puts, for instance, implied volatilities are computed as

$$\begin{aligned}\sigma_{25C} &= \sigma_{ATM} + \sigma_{25BY} + \frac{1}{2}\sigma_{25RR} \\ \sigma_{25P} &= \sigma_{ATM} + \sigma_{25BY} - \frac{1}{2}\sigma_{25RR}.\end{aligned}$$

This is the standard approach for deriving the volatility surface for foreign exchange options, for details see Bisesti et al. (2005). Following the quoting convention of the OTC options markets, the Black-Scholes model is then used to compute the prices and strike prices corresponding to 25Δ and 10Δ options. Interest rates are daily LIBOR rates for euro and Swiss franc for three month maturity. Based on this data, there are four option price series, two for puts and two for calls.

As shown in Table 1, the 10Δ put options have an average strike of 1.164 CHF per EUR. Given that our main interest is in the implied probabilities of exchange rate realizations below the 1.20 floor, it is useful that our data includes put options with strike prices that are typically below this floor.

The length of a period in my model is set to one half of a week, precisely 1/104 of a year. With this, so far as options with a three month maturity are concerned, the model without the policy floor is essentially equivalent to the lognormal Black-Scholes model.

4 Results

This section describes the empirical results and their robustness, specifically to allowing for a more general specification of the policy's survival probability. This section also documents how the model provided insights about the possible outcome of the popular vote on the Swiss Gold Initiative in the fall of 2014

Figure 1 displays the main results of the estimation. As shown in the first panel, during the first few months of the minimum exchange rate policy, markets often attached a probability of less than 75% to the policy being in place three months later. The credibility increased to around 90% in the middle of 2012. This was a period were the euro was very close to its floor of 1.20 (as seen in the second panel) and substantial amounts of euros were purchased by the Swiss central bank. It appears that this willingness to actively enforce the floor through intervention contributed to increasing the credibility of the policy. Later in 2012 the credibility declined to about 75%, before persistently increasing to reach a peak around 95% in the summer of 2014. In the second half of 2014 the probability declined again, fluctuating around 80% during the final months of the policy. The Swiss vote on November 30 2014 that would have required a gold backing of the Swiss currency appears to have been an important driver during this time. Consistent with media reactions on January 15th 2015, financial markets were mostly surprised by the end of the policy (Bosley, 2015). The president of the Swiss central bank noted at that occasion that "if you decide to exit such a policy, you have to take the market by surprise". (Baghdjian, 2015).

As shown in the first panel of Figure 1, the gap between the observed and the estimated fundamental exchange rates, S_t and V_t, is seen to fluctuate considerably during the first year of the policy. During the second quarter of 2012, the fundamental exchange rate is at its lowest level, dropping to around 1 Swiss franc per euro. After late 2012, the fundamental rate is usually close to 1.20 and its distance from the observed spot rate is relatively stable, before widening during the last two months of the policy regime.

The implied volatility of the fundamental exchange rate shown in the third panel displays a steady downward trend with a reversal in the last few months of the policy. The downward trend in the estimated implied volatility for the fundamental rate seems comforting given the similar trend in the measured implied volatilities for EURCHF. Estimates show little sensitivity to changes in the weights of the objective function, yearly option maturities produce somewhat different estimates, see the separate appendix for more details.

Figure 1 suggests some negative comovements between p and V, particularly for the period 2014 Q2 and Q3. It might therefore be too restrictive to assume a constant survival probability p. This assumption can be relaxed by assuming that

$$p_{t,t+1}(V_t) = a + b \cdot f(V_t) \tag{5}$$

with $f(V_t) = Ncdf(V_t; K, \sigma_f)$. The parameter b can capture a smooth monotonic relation between p and V. It is convenient that $f(.)$ is bounded, given that the probability p is bounded itself between 0 and 1. The model is used to estimate values for (V_t, a, b, σ). σ_f is fixed at 0.4, alternative values do not significantly affect results. This specification increases the computational burden, but remains tractable.

Over the entire sample period, the average of the estimated b coefficients is $= -.015$, which is in line with a negative relation between p and V. The average of the estimated a is 0.9987. With a period length in the model set at half a week, at these average values, the quarterly probability can fluctuate between about $0.9987^{26} = 0.967$ and $(0.9987 - .015)^{26} = 0.65$ over the range of possible values for V. Despite this, estimated values for V and

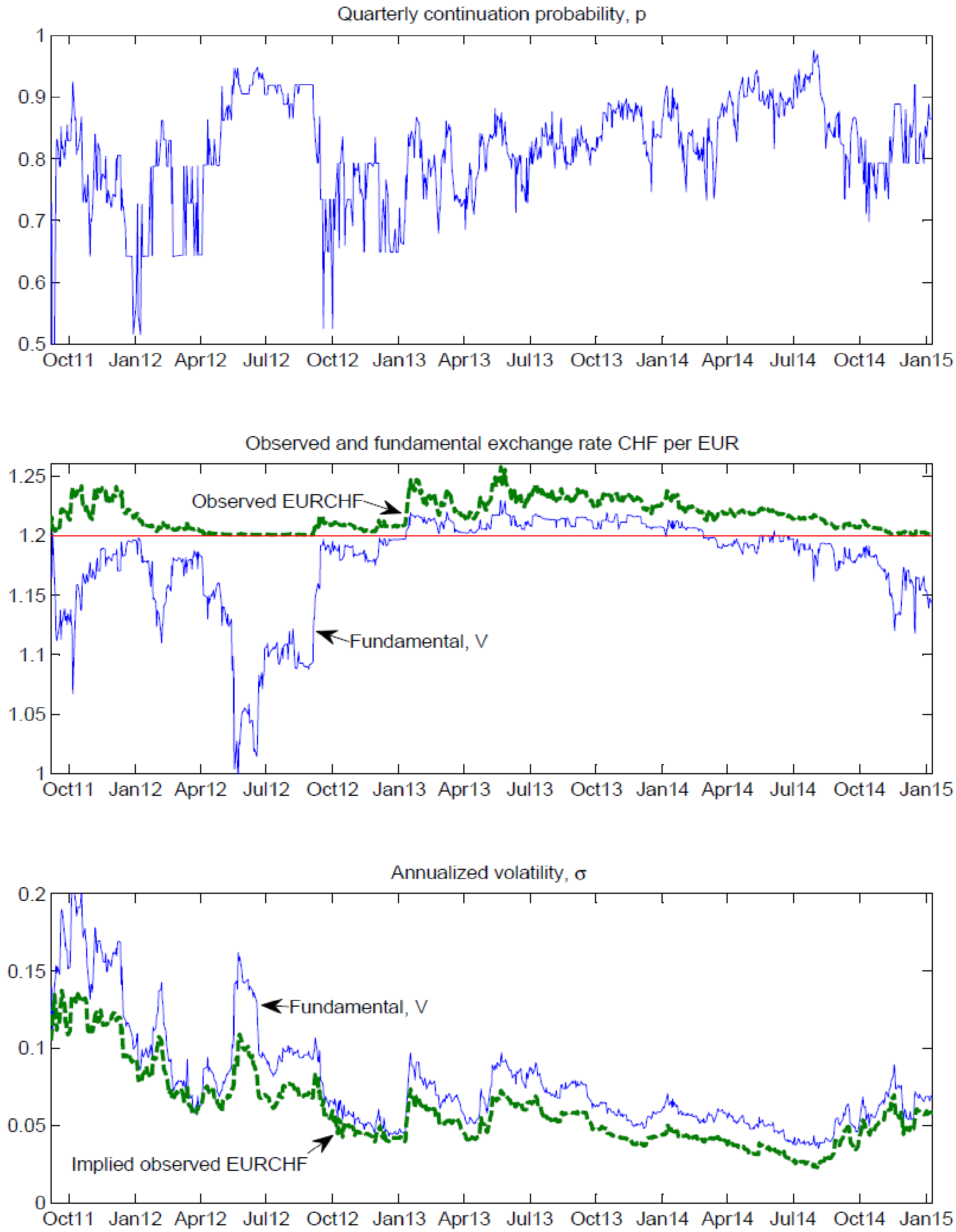

Figure 1: Model estimated continuation probabilities, fundamental values, and volatilities. The model is estimated daily. Observed EURCHF annualized volatility represents the average of the implied volatilities over the four types of options used.

the implied p are only marginally affected, while the estimated volatility parameter σ is moderately higher. Overall, in this case, this more general specification does not appear to make a significant difference relative to the benchmark model with a constant p.

The estimates in Figure 1 for V_t are significantly different from Hanke et al. (2014). Their fundamental exchange rates are mostly below 1.15 and display somewhat different low frequency movements. Their estimated implied volatility for V_t is not much lower in 2013 than in the earlier part of the sample. The implied continuation probabilities reported by Hertrich and Zimmermann (2013) are typically lower than my estimates and also display somewhat different low frequency movements. Given the disparity in approaches, the sources of these differences are hard to determine.

4.1 Swiss Gold Initiative

The period leading up to the vote on the Swiss Gold Initiative on November 30, 2014, offered an opportunity to forecast the outcome of the popular vote with estimates of the model. This subsection summarizes key points, more details are given in Jermann (2014).

A positive vote for the Gold Initiative would have required the Swiss central bank to hold 20% of its assets in gold and to never sell any of this gold. As gold has no yield, a further expansion of the balance sheet to buy euros would have severely limited the central bank's seigniorage revenues in the long run when large holdings of euro would no longer be useful. Under the requirement of the initiative, it would have been therefore very unlikely that the policy of a floor for a euro at CHF 1.20 could have been continued.

Based on this, assume that the probability of survival of the policy is 0 following a positive vote for the initiative, and 1 following a negative vote. In this case, the probability of a No vote equals the survival probability of the exchange rate policy.[2]

As shown in Figure 2, the continuation probability declined from over 0.95 early August to around 0.75 in mid October. As indicated in the figure, on October 24 a poll from the

[2] Alternative assumptions are considered in Jermann (2014). However, these do not produce such clearcut implications.

gfs.bern institute showed a small advantage for the positive vote on the initiative, while the same institute released a second poll where the No votes had the majority on November 19 (gfs.bern, 2014). To the extent that the polls where measuring a change in public opinion about the initiative, the continuation probability of the exchange rate policy mimics this change, as the continuation probability had increased to about 0.9 in the later part of November. The vote was ultimately rejected on November 30, consistent with a very high continuation probability of the policy. Estimating p only requires current quotes for options and the exchange rate. As such, estimates made in real time were identical to the ones reported in Figure 2.

5 Conclusion

My analysis shows that financial markets had initially significant doubts about whether the minimum EURCHF floor would survive for very long. However, over time, the credibility of the policy had improved and by the summer of 2014 markets seemed essentially convinced that the policy would not quickly be ended. In the fall 2014, views had become less supportive. Nevertheless, the end of the policy surprised financial markets like the media. The analysis also suggests that from the forth quarter of 2012 until the summer of 2014, the EURCHF exchange rate would not have been a lot lower without the minimum exchange rate policy. However, during the second quarter of 2012, the model implied an exchange rate without the policy that would have been as low as about 1 Swiss franc per euro.

The model presented in this paper builds on the literature on target zone models. My model is based on a no-arbitrage equation. Traditional target zone models are based on the assumption of uncovered interest parity, an assumption that by now has been conclusively rejected by the data. The tractability of my discrete-time setup should also make this approach adaptable to other exchange rate target regimes.

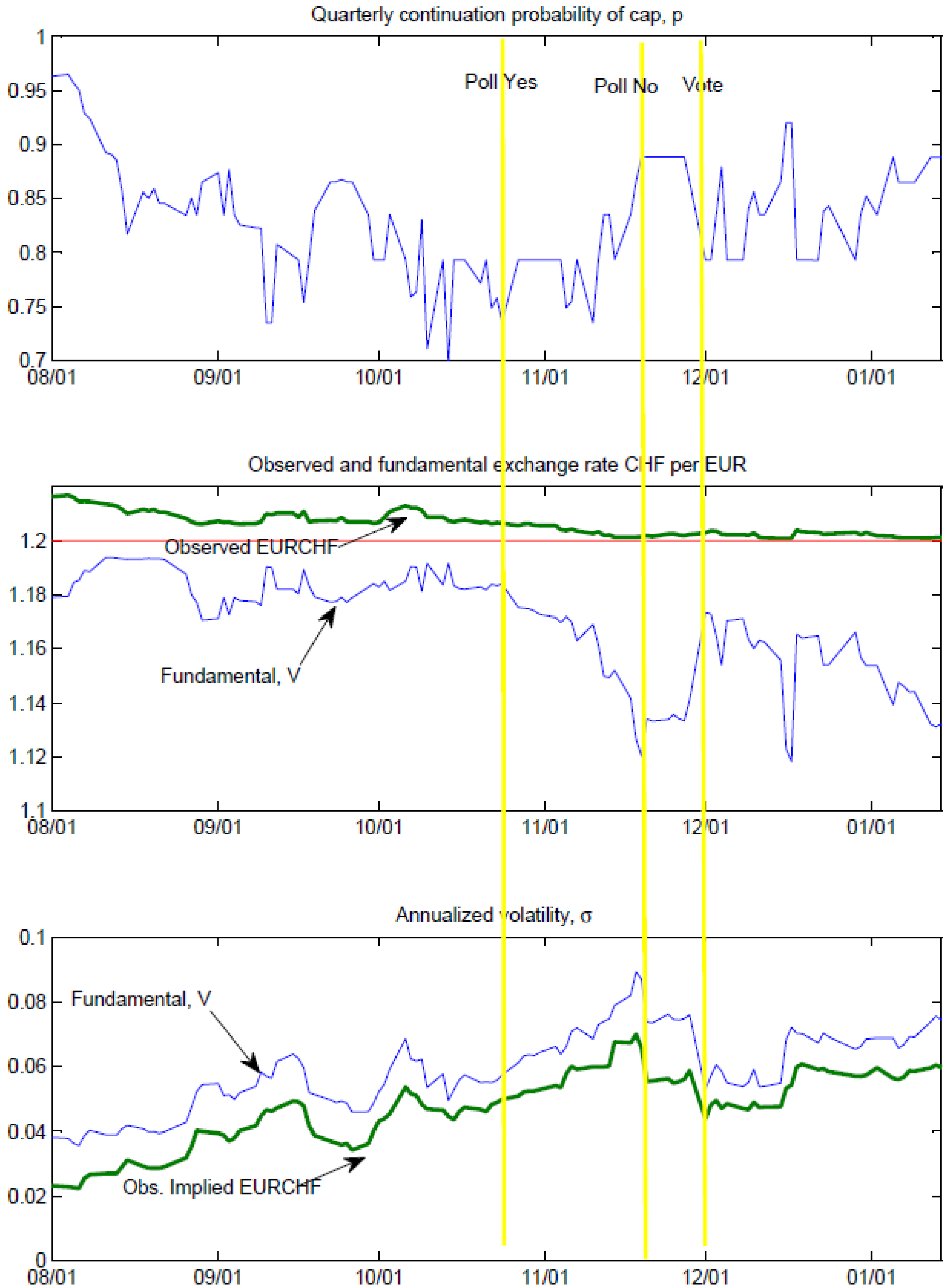

Figure 2: Model estimated continuation probabilities, fundamental values, and volatilities. The model is estimated daily. Observed EURCHF annualized volatility represents the average of the implied volatilities over the four types of options used.

References

[1] Baghdjian, A and S. Koltrowowitz, 2015, "Swiss central bank stuns market with policy U-turn", http://www.reuters.com/article/2015/01/15/us-swiss-snb-cap-iduskbn0ko0xk20150115.

[2] Bertola, G. and R. J. Caballero, 1992, Target zones and realignments, American Economic Review, Vol 82, 520-536.

[3] Bertola, G. and L. E.O. Svensson, 1993, Stochastic devaluation risk and the empirical fit of target-zone models, Review of Economic Studies 60, 689-712.

[4] Bisesti, A., Castagna, and F. Mercurio, 2005, "Consistent Pricing and Hedging of an FX Options Book", Kyoto Economic Review 74(1), 65–83.

[5] Bosley, C. "SNB Unexpectedly Gives Up Cap on Franc, Lowers Deposit Rate", http://www.bloomberg.com/news/articles/2015-01-15/snb-unexpectedly-gives-up-cap-on-franc-lowers-deposit-rate

[6] Durdu D. and E. Mendoza, 2006, "Are Asset Price Guarantees Useful for Preventing Sudden Stops?: A Quantitative Investigation of the Globalization Hazard-Moral Hazard Tradeoff", Journal of International Economics, vol. 69(1), 84-119.

[7] Campa, J.M., and Chang, K., 1996, "Arbitrage-based tests of target-zone credibility: Evidence from ERM cross-rate options," American Economic Review 86, 726-740.

[8] Dumas B., Jennergren L.P, and Naslund P., 1995, "Realignment risk and currency option pricing in target zones", European Economic Review 39, 1523-1544.

[9] Engel C. 1996, "The forward discount anomaly and the risk premium: A survey of recent evidence", Journal of Empirical Finance 3, 123-192.

[10] gfs.bern, 2014, announcements on Twitter on Oct 24 and Nov 19.

[11] Hanke M., R. Poulsen, and A. Weissensteiner, 2014, "Where would the EUR/CHF exchange rate be without the SNB's minimum exchange rate policy?", unpublished manuscript, SSRN working paper 2368698.

[12] Hertrich, M., Zimmermann, H., 2013, "How credible is the euro/Swiss franc floor? A financial market perspective", SSRN working paper No. 2290997.

[13] Hui, C., and Lo, C., 2009, "A note on estimating realignment probabilities – A first passage-time approach," Journal of International Money and Finance 28, 804–812.

[14] Jermann, U, 2014, "Forecasting the Outcome of the Swiss Gold Initiative", http://finance.wharton.upenn.edu/~jermann/research.html.

[15] Krugman P, 1991, "Target Zones and Exchange Rate Dynamics", The Quarterly Journal of Economics, 669-682.

[16] Lustig H, N Roussanov and A. Verdelhan, 2011, Common risk factors in currency markets, Review of Financial Studies, 3731-3777.

[17] Ljungqvist L., 2000, "Government Guarantees on Assets and Volatility," mimeo, Stockholm School of Economics.

[18] Söderlind, P. 2000, "Market Expectations in the UK Before and After the ERM Crisis," Economica, 67, 1–18.

[19] SNB, 2011, "Swiss National Bank sets minimum exchange rate at CHF 1.20 per euro" http://www.snb.ch/en/mmr/reference/pre_20110906/source/pre_20110906.en.pdf

[20] Stokey N., R.E. Lucas and E. Prescott, 1989, Recursive Methods in Economic Dynamics, Cambridge, MA: Harvard University Press.