A Macro Finance Model for Proof-of-Stake Ethereum

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- New monetary and fee policies
    - What to expect for money supply and ETH price?
- Staking ETH generates yield
    - How much will be staked?
- Optimal monetary policy
    - How to set parameters optimally?

Literature

- Models for POS: Cong, He and Tang (2022), Kose, Rivera and Saleh (2021), Fanti, Kogan and Viswanath (2019), Saleh (2021)
- Dynamic crypto models: Biais, et al (2019), Cong, Ye and Wang (2022), Jermann (2021), Jermann and Xiang (2022), Li and Mayer (2020), Mei and Sockin (2022) ...
- Ethereum: Elowsson (2021), Buterin (2018, 2022, ...), Wahrstatter (2022) ...

Ethereum

- Second in capitalization after Bitcoin
- ETH is "programmable money", BTC is "digital gold"
- ETH collects about 5mn USD in fees per day (annually 1.8bn)
  - For crypto trading, NFT trading, ETH transfers, stablecoins, layer 2s

ETH monetary and fee policies

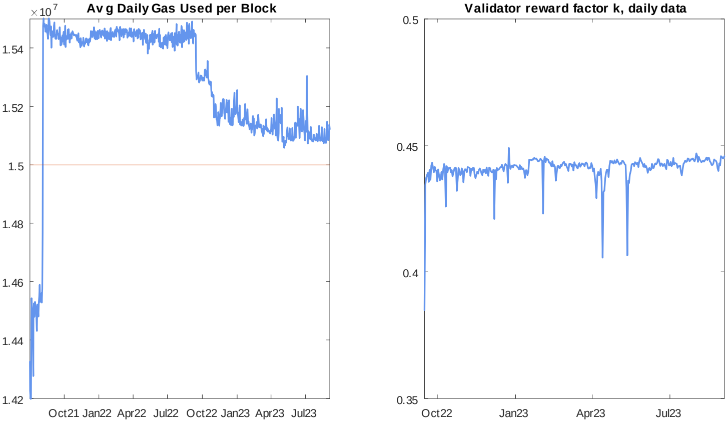
Change in ETH supply = Validator rewards − Burnt fees

Fees

- *Base* fee & *Priority* fee (*tip*)
- Base fee is burnt, priority fee for validators
- Base fee rate

$$f_n^b = \left[1 + 0.125\left(\frac{G_{n-1}}{G^*} - 1\right)\right] f_{n-1}^b$$

- $G \equiv$gas, unit of computational effort, $G^* \equiv$target
- At daily frequency approximately fixed supply



Avg Daily Gas Used per Block
×10^7
1.54
1.52
1.5
1.48
1.46
1.44
1.42
Oct21 Jan22 Apr22 Jul22 Oct22 Jan23 Apr23 Jul23
Validator reward factor k, daily data
0.5
0.45
0.4
0.35
Oct22
Jan23
Apr23
Jul23

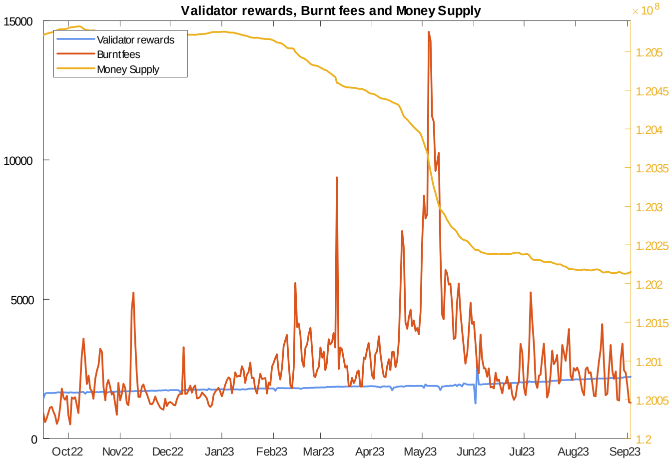
Validator Rewards

- Validator tasks: propose blocks, attestations
- Rewards and penalties based on performance of execution
- Average reward under best-case conditions

$$\frac{k}{\sqrt{\sum_j M_j^S}} M_i^S$$

- Daily data

$$k\left(t\right) = \frac{\left[M\left(t\right) - M\left(t-1\right)\right] + \text{FeesBurnt}(t)}{\sqrt{M^S\left(t\right)}}$$


Validator rewards, Burnt fees and Money Supply
Validator rewards
Burnt fees
Money Supply
15000
10000
5000
0
× 10^8
1.205
1.2045
1.204
1.2035
1.203
1.2025
1.202
1.2015
1.201
1.2005
1.2
Oct22
Nov22
Dec22
Jan23
Feb23
Mar23
Apr23
May23
Jun23
Jul23
Aug23
Sep23

Model

- Agents buy ETH to use or stake
- Agents get utility using ETH and gas
- Money supply and fee policies follow protocol
- ETH and gas prices clear markets
- Shocks: productivity/adoption, discount factor, priority fee share

Model

- $$\max_{\substack{C_{t+j}, G_{t+j}, \\ M^U_{t+1+j}, M^S_{t+1+j}}} \sum_{j=0}^{\infty} E_t \beta^j \Lambda_{t+j} \left[\begin{array}{c} C_{t+j} + \\ v\left(A_{t+j}, p_{t+j} M^U_{t+j}, G_{t+j}\right) \end{array} \right]$$

  G gas, M^U ETH used, M^S ETH staked, p ETH price, A productivity, C consumption
- $$C_t + p_t f_t G_t + p_t M^S_{t+1} + p_t M^U_{t+1} = p_t M^S_t \left(1 + \frac{k}{\sqrt{\bar{M}^S_t}} + \frac{\phi_t f_t \bar{G}_t}{\bar{M}^S_t}\right) + p_t M^U_t + Y_t$$
- $M_{t+1} = M_t + k\sqrt{M^S_t} - (1 - \phi_t) f_t G_t$
- $M_{t+1} = M^S_{t+1} + M^U_{t+1}$ and $G^* = G_t$

Model

- Money-in-utility function
  - $v(.) = \frac{A}{1-\sigma}\left[\left(\frac{pM^U}{A}\right)^{1-\theta} G^{\theta}\right]^{1-\sigma}$
- Productivity/adoption shock
  - $A_t = \gamma A_{t-1} \exp(z_t)$,
    - $z_t = \rho_z z_{t-1} + \sigma_z \varepsilon_{z,t}$
- Discount factor
  - $\frac{\Lambda_{t+1}}{\Lambda_t} = \exp\left(-r_{z,t} + \rho_{\Lambda r} \sigma_r \varepsilon_{r,t+1} - \rho_{\Lambda z} \sigma_z \varepsilon_{z,t+1}\right)$,
    - $r_{z,t} = \rho_r r_{z,t-1} + \sigma_r \varepsilon_{r,t}$
- Priority fee share
  - $\ln \phi_t = \phi_0 + z_{\phi,t}$,
    - $z_{\phi,t} = \rho_\phi z_{\phi,t-1} + \sigma_\phi \varepsilon_{\phi,t}$

First-order conditions and equilibrium

$$\theta \left[\left(\hat{p}_t M_t^U \right)^{1-\theta} G^{\theta} \right]^{1-\sigma} / G = \hat{p}_t f_t$$

$$\hat{p}_t = E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \gamma \exp\left(z_{t+1}\right) \left[\frac{(1-\theta)\, G^{\theta(1-\sigma)} \times}{\left(\hat{p}_{t+1} M_{t+1}^U\right)^{(1-\theta)(1-\sigma)-1}} + 1 \right] \hat{p}_{t+1}$$

$$\hat{p}_t = E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \gamma \exp\left(z_{t+1}\right) \left[\frac{k}{\sqrt{M_{t+1}^S}} + \frac{\phi_{t+1} f_{t+1} G}{M_{t+1}^S} + 1 \right] \hat{p}_{t+1}$$

$$M_{t+1}^S + M_{t+1}^U = M_t^S + M_t^U + k\sqrt{M_t^S} - (1-\phi_t)\, f_t G$$

Steady state – no shocks

- ETH staked, ETH used, gas price, (scaled) ETH price

$$M^S = \left(\frac{\beta\gamma}{1-\beta\gamma}\right)^2 \left(\frac{k}{1-\phi}\right)^2$$

$$M^U = \left(\frac{\beta\gamma}{1-\beta\gamma}\right)^2 \left(\frac{k}{1-\phi}\right)^2 \left(\frac{1-\theta}{\theta}\right)$$

$$f = \frac{\beta\gamma}{(1-\beta\gamma)} \left(\frac{k}{1-\phi}\right)^2 \frac{1}{G}$$

$$\hat{p} = G^{\varrho\theta(1-\sigma)} \left(\frac{\beta\gamma}{1-\beta\gamma}\right)^{\varrho-2} \theta\,(1-\theta)^{\varrho-1} \left(\frac{1-\phi}{k}\right)^2$$

with $\varrho \equiv 1/\left[1-(1-\theta)(1-\sigma)\right]$

- Staking share

$$\frac{M^S}{M^U + M^S} = \theta$$

Steady state money supply

- $$M = M^S + M^U = \frac{1}{\theta}\left(\frac{\beta\gamma}{1-\beta\gamma}\right)^2\left(\frac{k}{1-\phi}\right)^2$$
- Back-of-the-envelop

$$M = \left(\frac{1}{0.26}\right)(18)^2\left(\frac{161.7}{0.8}\right)^2 = 51 \text{ million ETH}$$

now $M = 120$ million

"Supply equilibrium",Elowsson (2021)

- Equalize staking rewards and fees burnt

$$\underbrace{\left(\frac{k}{\sqrt{M^S}}\right)}_{y} M^S = b\left(M - M^S\right)$$

- For given "staking yield" y and "burn rate" b

$$M = \left(\frac{1}{b} + \frac{1}{y}\right)\frac{k^2}{y}$$

"Supply equilibrium",Elowsson (2021)

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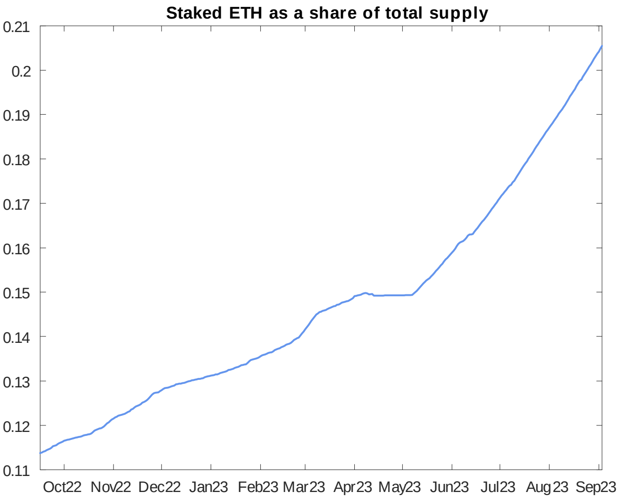
- For given "staking yield" y and "burn rate" b

$$M = \left(\frac{1}{b} + \frac{1}{y}\right)\frac{k^2}{y}$$

- In my model, at steady state

$$\begin{aligned} y &= (1-\phi)\left(\frac{1-\beta\gamma}{\beta\gamma}\right) \\ b &= (1-\phi)\left(\frac{1-\beta\gamma}{\beta\gamma}\right)\left(\frac{\theta}{1-\theta}\right) \end{aligned}$$


Staked ETH as a share of total supply
0.21
0.2
0.19
0.18
0.17
0.16
0.15
0.14
0.13
0.12
0.11
Oct22
Nov22
Dec22
Jan23
Feb23
Mar23
Apr23
May23
Jun23
Jul23
Aug23
Sep23

Estimation based on first-order conditions

- FOCs for M^U_{t+1} and G_t imply

$$0 = E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left\{ \frac{p_{t+1}}{p_t} \left[\left(\frac{1-\theta}{\theta} \right) \frac{f_{t+1} G_{t+1}}{M^U_{t+1}} + 1 \right] - R^j_{t,t+1} \right\}$$

- Lido ETH liquid staking return $R^{Lid}_{t,t+1} = \frac{p^{Lid}_{t+1}}{p^{Lid}_t} (y_{t+1} + 1)$
- $0 = E_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{p_{t+1}}{p_t} \left\{ \left(\frac{1-\theta}{\theta} \right) \frac{f_{t+1} G_{t+1}}{M^U_{t+1}} + 1 - \left(\frac{p^{Lid}_{t+1}}{p^{Lid}_t} / \frac{p_{t+1}}{p_t} \right) (y_{t+1} + 1) \right\}$
- Assuming risk neutrality, GMM estimate $\hat{\theta} = 0.26$
- mean$\left(\frac{f_{t+1} G_{t+1}}{M^U_{t+1}} \right) = 2\%$,

  mean$\left(\left(\frac{p^{Lid}_{t+1}}{p^{Lid}_t} / \frac{p_{t+1}}{p_t} \right) (y_{t+1} + 1) - 1 \right) = 5\%$

Parameterization

ParamNam	ParamVal
{'the' }	0.26
{'gam' }	1.0025
{'rho_z' }	0
{'bethat' }	0.999
{'sig_r' }	0.00156
{'rho_r' }	0.968
{'rho_Lamr'}	34.3
{'sig' }	9.84
{'sig_z' }	0.1698
{'rho_Lamz'}	0.6702
{'phi0' }	-1.6055
{'sig_phi' }	0.201
{'rho_phi' }	0.775

Implied moments and targets

VarName	StdMon	MeanAnn	BenchStd	BenchMean
{'dp' }	0.1733	0.0305	0.173	0
{'df' }	0.3177	0	0.318	0
{'fGMu'}	0.0007	0.0198	0.0011	0.019
{'phi' }	0.0688	0.2112	0.066	0.2
{'r' }	0.0062	0.0121	0	0.012
{'y1' }	0.0046	0.0188	0	0
{'y10' }	0.0016	0.0264	0.0017	0.027
{'rs' }	0.1733	0.0857	0	0
{'M' }	0.1144	49.802	0	26.07

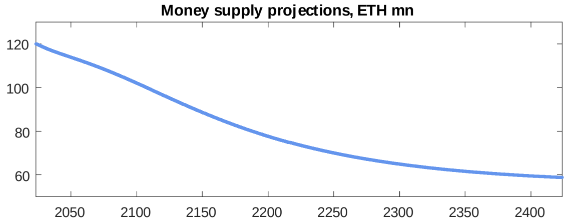
Risk-adjustment for steady state money supply

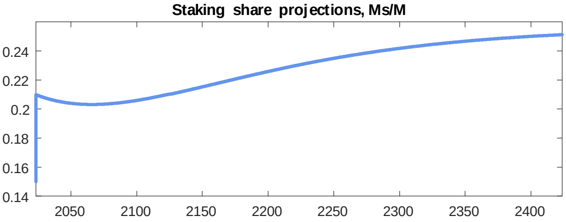
- Replace $\beta\gamma$ by $E\left(\beta\frac{\Lambda'}{\Lambda}\frac{p'}{p}\right)$
- No-arbitrage

$$1 = E\left(\beta\frac{\Lambda'}{\Lambda}R'\right) = E\left(\beta\frac{\Lambda'}{\Lambda}\frac{p'}{p}\left\{\frac{D'}{p'}+1\right\}\right)$$

- Approximately

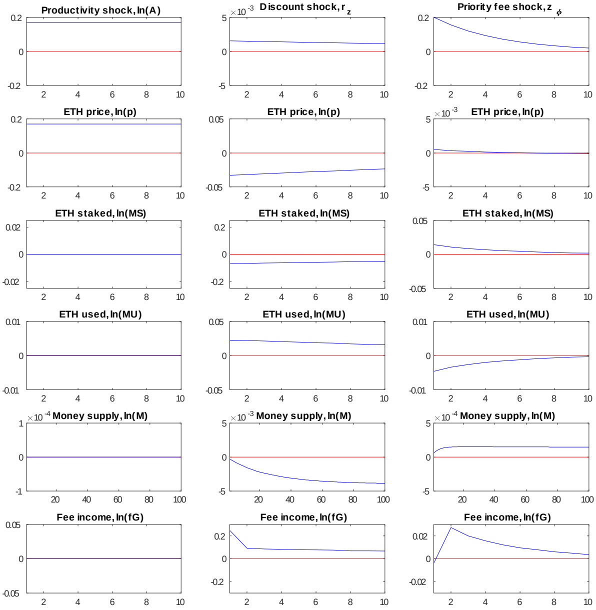
$$\begin{aligned} 1/E\left(\beta\frac{\Lambda'}{\Lambda}\frac{p'}{p}\right) &\approx \overline{\left\{\frac{D'}{p'}+1\right\}} \\ \frac{\widetilde{\beta\gamma}}{1-\widetilde{\beta\gamma}} &= \frac{1}{1/E\left(\beta\frac{\Lambda'}{\Lambda}\frac{p'}{p}\right)-1} \end{aligned}$$


Money supply projections, ETH mn
120
100
80
60
2050
2100
2150
2200
2250
2300
2350
2400


Staking share projections, Ms/M
0.24
0.22
0.2
0.18
0.16
0.14
2050
2100
2150
2200
2250
2300
2350
2400

Sensitivity to parameter values

MomName	Bench	the=.4	gam=1.5%	sig=5	sig_z=.1	rho_Lamz=.6
"E(M)"	49.802	31.102	32.02	48.291	442.95	118.28
"E(rs)"	0.086	0.086	0.085	0.086	0.049	0.062
"Std(dp)"	0.173	0.174	0.173	0.174	0.109	0.174
"Std(df)"	0.318	0.286	0.297	0.164	0.389	0.355
"E(Ms/M)"	0.275	0.409	0.273	0.271	0.285	0.278
"Std(Ms/M)"	0.072	0.067	0.067	0.064	0.086	0.08
"E(M)/Mss*"	0.987	0.969	1.01	0.977	1.01	0.766



Productivity shock, ln(A)
×10^-3 Discount shock, r z
Priority fee shock, z ϕ
ETH price, ln(p)
ETH price, ln(p)
×10^-3 ETH price, ln(p)
ETH staked, ln(MS)
ETH staked, ln(MS)
ETH staked, ln(MS)
ETH used, ln(MU)
ETH used, ln(MU)
ETH used, ln(MU)
×10^-4 Money supply, ln(M)
×10^-3 Money supply, ln(M)
×10^-4 Money supply, ln(M)
Fee income, ln(fG)
Fee income, ln(fG)
Fee income, ln(fG)

Variance decomposition

VarNamevd	VARDECz	VARDECrz	VARDECphiz
{'dp' }	95.86	4.10	0.00
{'df' }	0.00	99.39	1.09
{'phi'}	0.00	0.00	100.00
{'dMs'}	0.00	93.02	4.01
{'dMu'}	0.00	97.37	3.49
{'dM' }	0.00	99.33	1.22

Policy experiments

- Generalized staking yield

$$\frac{k}{\sqrt{M_t^S}} \to \frac{k}{\left(M_t^S\right)^x}$$

- State-dependent reward factor

$$k_t = \bar{k} \exp\left(-\kappa \left(\ln \frac{M_t^S}{M_t} - \ln \theta\right)\right)$$

- Criteria
  - Welfare: $E\left[\hat{V}_t\right] = E\left[\hat{V}_t^{bench}\left(\left\{\hat{p}M^U\right\}(1+\Omega)\right)\right]$.
  - Probability of low M_t^S / M_t

Policy	UtilityGain	StkShareBelow15pc	StkShareStd
{'x=1/2' }	0	0.044	0.085
{'x=1' }	-0.0007	0.01	0.073
{'x=1/8' }	0.0012	0.095	0.096
{'x=1/2, kap=10'}	-0.0009	0	0.018

Conclusion

A macro model for Ethereum

- Theory
  - Determinants of steady state staking share and money supply
  - Neutrality with respect to staking reward factor
- Empirically, preliminary
  - Long-run staking share estimate: 26%
  - Long-run money supply estimate: $<< 120$ ETH mn
  - Alternative/activist policies uncertain/small gains, but can stabilize staking share