Negative Swap Spreads and Limited Arbitrage

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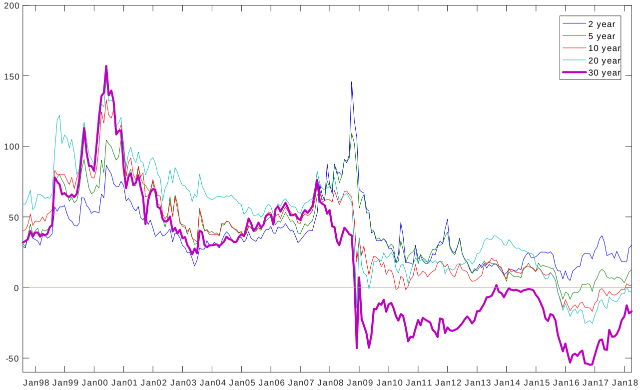
- Interest rate swap
    - Pay LIBOR, get fixed rate periodically
- Swap spread
    - Fixed swap rate minus Treasury of same maturity
- Why a spread?
    - LIBOR vs TBill rate
    - Default/Liquidity
    - Limits to arbitrage

Swap spreads



This paper

- Develops model with limited arbitrage for swaps
- Analytically, with strong frictions negative swap spread
- Quantitative model with negative swap spreads
- Additional empirical support for model

Literature

- Swap spreads: Liu, Longstaff & Mandell (2006), Feldhuetter & Lando (2008), Hanson (2014), Gupta & Subrahmanyam (2000), Collin-Dufresne and Solnik (2001), Eom, Subrahmanyam & Uno (2002), Johannes & Sundaresan (2007), Smith (2015), Klinger & Sundaresan (2016)
- Limited arbitrage: Shleifer & Vishny (1997), Dow & Gorton (1994), Garleanu, Pedersen & Poteshman (2009), Gabaix, Krishnamurthy & Vigneron (2007), Vayanos & Vila (2009), Liu & Longstaff (2004), Tuckman & Vila (1992), Gromb & Vayanos (2010)
- Effects of recent financial regulation: Duffie (2016), Du, Tepper & Verdelhan (2016), Boyarchenko et al. (2018)
- First limited arbitrage model for swaps

Model outline

- Dealer selects bonds and swaps
- Bond prices exogenous, swap price endogenous
- Holding costs for bonds
- Later extensions: Capital requirements, demand effects, swap holding costs

Model

- Short-term riskless debt

$$q_{ST}(z) = \exp(-y_{ST}(z))$$

- LIBOR debt

$$q_{LIB}(z) = \exp(-(y_{ST}(z) + \theta(z)))$$

with θ the TED spread

- Long-term debt pays

$$c_{LT} + \lambda + (1 - \lambda) q'_{LT}(z')$$

  - with price

$$q_{LT}(z) = \frac{c_{LT} + \lambda}{\exp(y_{LT}(z)) - 1 + \lambda}$$

  and yield

$$y_{LT}(z) = y_{ST}(z) + \tau(z)$$

  with term spread $\tau(z)$

Swap

- Swap pays (to fixed receiver)

$$c_{Sw} - \left(\frac{1}{q_{LIB}(z)} - 1\right) + (1-\lambda)\, m'$$

  with m' value next period

- For new swap, find c_{Sw} so that

$$m(z,\omega) = 0,$$

  and

$$y_{Sw}(z,\omega) = c_{Sw}$$

  is the swap rate

- If zero net demand for swaps, can create at-market swap and y_{Sw} for each (z,ω). Otherwise assume there is only one swap.

Frictions

- The cost for holding long-term debt is given by

$$j\left(\alpha'_{LT}\right) = \frac{\kappa_{LT}}{2}\left(\alpha'_{LT}\right)^2$$

  with α'_{LT} amount of long-term debt
- Similarly for short-term debt

$$h\left(\alpha'_{ST}\right) = \frac{\kappa_{ST}}{2}\left(\alpha'_{ST}\right)^2$$

Dealer's problem

$$V(\omega, z) = \max_{c, \alpha'_{ST}, \alpha'_{LT}, s'} u(c) + \beta(\omega, z) E\left(V\left(\omega', z'\right)\right)$$

subject to

$$c = \omega - \alpha'_{ST} q_{ST}(z) - \alpha'_{LT} q_{LT}(z) - s'm - h\left(\alpha'_{ST}\right) - j\left(\alpha'_{LT}\right)$$

and

$$\begin{aligned} \omega' \quad = \quad & \frac{\alpha'_{ST}}{e^{\mu(z')}} + \frac{\alpha'_{LT}}{e^{\mu(z')}} \left[c_{LT} + \lambda + (1-\lambda) q_{LT}\left(z'\right)\right] \\ & + \frac{s'}{e^{\mu(z')}} \left[c_{Sw} - \left(\frac{1}{q_{LIB}(z)} - 1\right) + (1-\lambda) m'\right] \\ & + \pi\left(z'\right) \end{aligned}$$

Equilibrium

- Swap market clears

$$s' = -d(z)$$

- First-order conditions

$$h_1(\alpha'_{ST}) = \beta E \frac{u_1(c')}{u_1(c)e^{\mu(z')}} - q_{ST}$$

$$j_1(\alpha'_{LT}) = \beta E \left(\frac{u_1(c')}{u_1(c)e^{\mu(z')}} \left[c_{LT} + \lambda + (1-\lambda) q_{LT}(z') \right] \right) - q_{LT}$$

$$m = \beta E \left(\frac{u_1(c')}{u_1(c)e^{\mu(z')}} \left[c_{Sw} - \left(\frac{1}{q_{LIB}(z)} - 1 \right) + (1-\lambda) m' \right] \right)$$

No-arbitrage case

$$m_t = E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\left[c^{Sw} - \left(\frac{1}{q_t^{ST}} - 1\right) - \theta_t + (1-\lambda)\, m_{t+1}\right]\right)$$

- Assume Λ prices all bonds
- Then

$$m_t = \left(c^{Sw} + \lambda\right)\Omega_t\left(\{1\}\right) - 1 - \Omega_t\left(\{\theta_t\}\right)$$

with

$$\Omega_t\left(\{x_t\}\right) = \sum_{j=0}^{\infty}(1-\lambda)^j\, E_t\frac{\Lambda_{t+1+j}}{\Lambda_t}x_{t+j}.$$

- (At-market) swap rate y_t^{Sw},

$$
\begin{aligned}
0 &= \left(y_t^{Sw} + \lambda\right) \Omega_t(1) - 1 - \Omega_t(\{\theta_t\}) \\
y_t^{Sw} &= \frac{1 + \Omega_t(\{\theta_t\})}{\Omega_t(1)} - \lambda
\end{aligned}
$$

- Price of long-term bond

$$
q_t^{LT} = \left(c^{LT} + \lambda\right) \Omega_t(1)
$$

- Yield of long-term bond y_t^{LT},

$$
\begin{aligned}
1 &= \left(y_t^{LT} + \lambda\right) \Omega_t(1) \\
y_t^{LT} &= \frac{1}{\Omega_t(1)} - \lambda
\end{aligned}
$$

No-arbitrage case

- $$y_t^{Sw} - y_t^{LT} = \frac{\Omega_t\left(\{\theta_t\}\right)}{\Omega_t\left(1\right)}$$
- If $\theta_t = \theta$ then
$$y_t^{Sw} - y_t^{LT} = \theta$$
- If $\theta_t \geq 0$,
$$y_t^{Sw} - y_t^{LT} \geq 0$$

Swap pricing with very strong frictions

- Assume holding cost parameter, κ_{LT} and κ_{ST}, very large
- Constant endowment of other profits, $\pi(z) = \pi$
- Zero-net demand for swaps, $d(z) = 0$
- No-inflation uncertainty, $\mu(z) = \mu$
- Then, swap price equals

$$m_t = \bar{\beta} E_t \left(\left[c^{Sw} - y_t^{LIB} + (1-\lambda) m_{t+1} \right] \right)$$

- Setting $m_t = 0$ defines the swap rate as

$$y_t^{Sw} = \sum_{j=1}^{\infty} w^j \cdot E_t \left(y_{t+j-1}^{LIB} \right)$$

- Taking unconditional expectation

$$E\left(y_t^{Sw}\right) = E\left(y_t^{LIB}\right) = E\left(y_t^{ST}\right) + E\left(\theta_t\right)$$

- Bond yield

$$E\left(y_t^{LT}\right) = E\left(y_t^{ST}\right) + E\left(\tau_t\right)$$

- Swap spread

$$E\left(y_t^{Sw}\right) - E\left(y_t^{LT}\right) = E\left(\theta_t\right) - E\left(\tau_t\right)$$

- Historically, $E\left(\theta_t\right) = 0.6\%$ and $E\left(\tau_t\right) = 1.7\%$, so that

$$E\left(y_t^{Sw}\right) - E\left(y_t^{LT}\right) = -1.1\%$$

Swap pricing with frictions from model

- Dealer's first-order conditions

$$q_t^{ST} = E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - \kappa_{ST}\alpha_{t+1}^{ST}$$

-

$$q_t^{LT} = E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\left[c^{LT} + \lambda + (1-\lambda)\, q_{t+1}^{LT}\right]\right) - \kappa_{LT}\alpha_{t+1}^{LT}$$

- Implied swap pricing

$$y_t^{Sw} - y_t^{LT} = \frac{\Omega_t(\{\theta_t\})}{\Omega_t(1)} + \frac{\Omega_t\left(\kappa_{ST}\left\{\alpha_{t+1}^{ST}\right\} - \frac{\kappa_{LT}}{q_t^{LT}}\left\{\frac{\Lambda_t}{\Lambda_{t+1}}\alpha_{t+1}^{LT}\right\}\right)}{\Omega_t(1)}$$

- Term spread $\uparrow$, $q_t^{LT} \downarrow$, $\{\alpha_{t+1}^{LT}\} \uparrow$, Swap spread$\downarrow$

Calibration

VAR(1) for four dimensional state vector

$$\left[y_{ST}\left(z\right), \tau\left(z\right), \mu\left(z\right), \theta\left(z\right)\right]$$

Transition: $$\begin{bmatrix} .91 & 0 & .07 & 0 \\ 0 & .87 & 0 & 0 \\ 0 & 0 & .76 & 0 \\ 0 & 0 & .06 & .72 \end{bmatrix}$$

Var-Cov: $$10^{-6} \times \begin{bmatrix} 5.1 & -3.4 & 4.3 & 0 \\ & 3.2 & -2.6 & 0 \\ & & 24.8 & -1.1 \\ & & & 0.5 \end{bmatrix}$$

Calibration

Symbol	Parameter	Value
$\bar{y}_{ST}$	Short rate level	0.01156
$\bar{\tau}$	Term spread level	0.00429
$\bar{\mu}$	Inflation level	0.00938
$\bar{\theta}$	TED spread level	0.00158
γ	Risk aversion	2
ν	Discount elasticity	1
$1/\lambda$	Maturity of long-term debt and swap	120

Quantitative results

	30Y SS E()	Std()	$\kappa_{LT} \times$ E(α_{LT})	TED E()
Data				
7/1997 – 9/2008	57	27		58
10/2008 – 10/2015	−18	12		35
Model				
$\kappa_{LT} = 0.0001$	62	8	21	63
$\boldsymbol{\kappa_{LT} = 0.0025}$	**−9**	**55**	**96**	**63**
$\kappa_{LT} = 0.01$	−54	89	119	63
Post 10/2008 TED and $\kappa_{LT} = 0.0014$	−18	41	80	35
High ri. av., $\gamma = 4$	−27	60	113	63
Low disc.elast., $\nu = 0.8$	−17	61	101	63
ST debt cost, $\kappa_{ST} = 0.0025$	−9	75	97	63
Constant TED	−16	62	96	63
Constant Inflation	−17	57	107	63

Holding cost estimate

- Enhanced Supplementary Leverage Ratio (US G-SIB) of 5% or 6%

$$5\% \times 10\% = 50, \text{ or } 60 \text{ basis points}$$

- With $\kappa_{ST} = 0$,

$$E\left(y_t^{Sw} - y_t^{LT}\right) = E\theta_t - \kappa_{LT} \cdot E\alpha_{t+1}^{LT} \cdot \chi$$

.

$$+E\frac{\Omega_t\left(\{\theta_t - E\theta_t\} - \frac{\kappa_{LT}}{q_t^{LT}}\left\{\frac{\Lambda_t}{\Lambda_{t+1}}\left(\alpha_{t+1}^{LT} - E\alpha_{t+1}^{LT}\right)\right\}\right)}{\Omega_t(1)}$$

with $\chi \simeq 1$

Demand effects and Swap holding costs

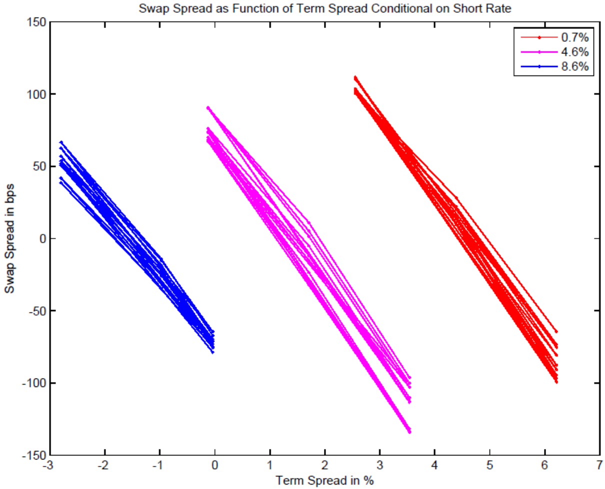
	Swap rate	Demand sensitivity	LT bond position	Swap cost impact
Benchmark ($\kappa_{LT} = 0.0025$)				
$d = 0$	6.17		1.23	
$d = .2$	6.04	$-.88$	1.33	
$d = .2, \kappa_{Sw} = .005$	5.64			$-.4056$
Low friction ($\kappa_{LT} = 0.0001$)				
$d = 0$	7.00		3.52	
$d = .2$	6.99	$-.14$	3.72	
$d = .2, \kappa_{Sw} = .005$	6.58			$-.4056$
High friction ($\kappa_{LT} = 0.01$)				
$d = 0$	5.67		.46	
$d = .2$	5.40	-1.28	.51	
$d = .2, \kappa_{Sw} = .005$	5.00			$-.4028$

Leverage constraint

- $\max\left(\alpha'_{ST} q_{ST}(z), 0\right) + \max\left(\alpha'_{LT} q_{LT}(z), 0\right) \leq \xi \cdot \omega$

-

	E(30Y SS)	freq(SS<0)	freq($\eta > 0$)
$\xi = \infty$	62	0	0
$\xi = 20$	48	.001	.05
$\xi = 10$	19	.20	.14
$\xi = 5$	-25	.66	.29


Swap Spread as Function of Term Spread Conditional on Short Rate
0.7%
4.6%
8.6%
Swap Spread in bps
Term Spread in %
150
100
50
0
-50
-100
-150
-3
-2
-1
0
1
2
3
4
5
6
7

10/2008-2018:	Swap Maturity				
	2	5	10	20	30
TERM	-0.045^{**}	-0.008	-0.116^{***}	-0.115^{*}	-0.187^{**}
TED	0.337^{***}	0.179^{***}	-0.102^{*}	-0.285^{***}	-0.253^{**}
MBSD	0.026^{***}	0.015	0.067^{***}	0.065^{**}	0.100^{***}
3MTB	0.151^{*}	0.154	-0.011	0.022	0.036
VIX	0.003	0.003^{**}	0.002^{**}	0.000	-0.001
R2adj	0.56	0.38	0.28	0.32	0.31

1997-2018:	Swap Maturity				
	2	5	10	20	30
TERM	-0.036	-0.044	-0.157^{***}	-0.128^{***}	-0.205^{***}
TED	0.220^{***}	0.089^{***}	0.025	0.012	0.007
MBSD	0.030^{**}	0.043^{***}	0.088^{***}	0.058^{***}	0.104^{***}
3MTB	0.012	0.000	0.083^{*}	-0.096	-0.100
VIX	0.004^{***}	0.006^{***}	0.002	-0.000	-0.000
R2adj	0.50	0.28	0.22	0.04	0.18

Conclusion

- Model for swaps with limited arbitrage
- With reasonable frictions on bond holdings get negative swap spreads
- Negative swap spreads even without explicit demand effects