Aggregate implications of a credit crunch

by Francisco Buera and Benjamin Moll

Discussed by Urban Jermann
Contribution

- Studies how financial frictions in models with heterogenous agents show up as aggregate wedges
- Analytical results: A model with financial friction has undistorted Euler equation for the aggregate of firm owners
- Numerical examples.
  - Model versions with the same friction and different heterogeneity have different wedges: in TFP, Euler equation, or in the labor market
Model:
Entrepreneurs (continuum) choose $c, k', d'$, and $l$

- Preferences
  
  $$E_0 \sum_{t=1}^{\infty} \log (c_{it})$$

- Technology
  
  $$y_{it} = (z_{it} k_{it})^\alpha l_{it}^{1-\alpha}$$

- Capital accumulation
  
  $$k_{it+1} = x_{it} + k_{it} (1 - \delta)$$

- Budget constraint
  
  $$c_{it} + x_{it} - d_{it+1} = y_{it} - w_t l_t - (1 + r_t) d_{it}$$

- Borrowing constraint
  
  $$d_{i,t+1} \leq \theta_t k_{it+1}$$

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Model:

Workers (representative) choose $C^W$ and $L$

Preferences

$$u \left( C_t^W \right) - v \left( L_t \right)$$
Entrepreneurs’ recursive problem 1

\[ V_t(k, d, z_{-1}, z) = \max_{c, d', k'} \log c + \beta E \left[ V_{t+1}(k', d', z, z') \right] \]

s.t.

\[ c + k' - d' = z_{-1} \pi_t k + (1 - \delta) k - (1 + r_t) d \]
\[ d' \leq \theta_t k' \]
\[ k' \geq 0 \]
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Define "cash-on-hand": \( m = z_{-1} \pi_t k + (1 - \delta) k - (1 + r_t) d \)
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- Define "cash-on-hand": \( m = z_{-1} \pi_t k + (1 - \delta) k - (1 + r_t) d \)
- Low productivity, \( z \), Lenders: \( k' = 0 \) and \(-d' = m - c \)
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- Define "cash-on-hand": \( m = z_{-1} \pi_t k + (1 - \delta) k - (1 + r_t) d \)
- Low productivity, \( z \), Lenders: \( k' = 0 \) and \(-d' = m - c \)
- High \( z \), Producers: \( d' = \theta_t k' \), and \( k' = \frac{1}{1-\theta_t} (m - c) \equiv \lambda_t (m - c) \)

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Entrepreneurs’ recursive problem 2

\[ V_t (m, z) = \max_{a', c} \log (c) + \beta EV_{t+1} (m', z') \]

s.t.

\[ m' = R_{t+1}^a (m - c) \]
Entrepreneurs’ recursive problem 2

\[ V_t (m, z) = \max_{a', c} \log (c) + \beta EV_{t+1} (m', z') \]

s.t.
\[ m' = R^a_{t+1} (m - c) \]

\[ R^a_{i,t+1} \equiv \{ \max [(z\pi_{t+1} - \delta - r_{t+1}) \lambda_t, 0] + 1 + r_{t+1} \} \]
\[ = \{ \max [(R^k_{i,t+1} - 1 + r_{t+1}) \lambda_t, 0] + 1 + r_{t+1} \} \]
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\[ V_t (m, z) = \max_{a', c} \log (c) + \beta EV_{t+1} (m', z') \]

s.t.

\[ m' = R_{t+1}^a (m - c) \]

\[ R_{i,t+1}^a = \{ \max \left[ (z\pi_{t+1} - \delta - r_{t+1}) \lambda_t, 0 \right] + 1 + r_{t+1} \} \]

\[ = \{ \max \left[ \left( R_{i,t+1}^k - 1 + r_{t+1} \right) \lambda_t, 0 \right] + 1 + r_{t+1} \} \]

\[ R_{i,t+1}^a = \tilde{\lambda}_t \cdot R_{i,t+1}^k + \left( 1 - \tilde{\lambda}_t \right) (1 + r_{t+1}) \]

\[ \tilde{\lambda}_{LENDERS}^t = 0, \text{ and } \tilde{\lambda}_t^{PRODUCERS} = \lambda_t > 1 \]
Entrepreneurs’ Euler equations

\[ \frac{1}{c_i} \beta E \left[ \frac{1}{c_i'} \right] = R_{i,t+1}^a \text{ for all agents } i, \]
Entrepreneurs’ Euler equations

\[ \frac{1/c_i}{\beta E \left[ 1/c'_i \right]} = R_{i,t+1}^a \text{ for all agents } i, \]

Surprise: this aggregates up to

\[ \frac{1/C^E_i}{\beta 1/C^{E'}} = \alpha \frac{Y'}{K'} + 1 - \delta \]

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#1-Weights

\[
\int \frac{1/c_i}{\beta E[1/c'_i]} \left( \frac{m_i - c_i}{K'} \right) \, di = \int R_{i,t+1}^a \left( \frac{m_i - c_i}{K'} \right) \, di
\]

Note

\[
\int k_i \, di = K
\]

\[
\int (m_i - c_i) \, di = \int (k_i - d_i) \, di = \int k_i \, di - \int d_i \, di = K.
\]
\[ \int R_{i,t+1}^{a} \left( \frac{m_{i} - c_{i}}{K'} \right) \, di \\
= \int_{P} \left\{ \lambda_{t} \cdot R_{i,t+1}^{k} + (1 - \lambda_{t}) (1 + r_{t+1}) \right\} \left( \frac{m_{i} - c_{i}}{K'} \right) \, di \\
+ \int_{L} (1 + r_{t+1}) \left( \frac{m_{i} - c_{i}}{K'} \right) \, di \]
\[
\begin{align*}
\int R_{i,t+1}^a \left( \frac{m_i - c_i}{K'} \right) \, di \\
= \int_P \left\{ \lambda_t \cdot R_{i,t+1}^k + (1 - \lambda_t) (1 + r_{t+1}) \right\} \left( \frac{m_i - c_i}{K'} \right) \, di \\
+ \int_L (1 + r_{t+1}) \left( \frac{m_i - c_i}{K'} \right) \, di \\
= \int_P \lambda_t \cdot R_{i,t+1}^k \left( \frac{m_i - c_i}{K'} \right) \, di = \int_P R_{i,t+1}^k \left( \frac{k_i'}{K'} \right) \, di
\end{align*}
\]
\[
\int R_{i,t+1}^a \left( \frac{m_i - c_i}{K'} \right) \, di \\
= \int_P \left\{ \lambda_t \cdot R_{i,t+1}^k + (1 - \lambda_t) (1 + r_{t+1}) \right\} \left( \frac{m_i - c_i}{K'} \right) \, di \\
+ \int_L (1 + r_{t+1}) \left( \frac{m_i - c_i}{K'} \right) \, di \\
= \int_P \lambda_t \cdot R_{i,t+1}^k \left( \frac{m_i - c_i}{K'} \right) \, di = \int_P R_{i,t+1}^k \left( \frac{k_i'}{K'} \right) \, di \\
= \int_P \left[ \frac{\alpha y_{it+1}'}{k_{it+1}} + 1 - \delta \right] \left( \frac{k_i'}{K'} \right) \, di = \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta
\]
\[
\int \frac{1/c_i}{\beta E[1/c_i']} \left( \frac{m_i - c_i}{K'} \right) \, di = \int R_{i,t+1}^a \left( \frac{m_i - c_i}{K'} \right) \, di
\]

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\[ \int \frac{1/c_i}{\beta E[1/c_i']} \left( \frac{m_i - c_i}{K'} \right) \, di = \int R^a_{i,t+1} \left( \frac{m_i - c_i}{K'} \right) \, di \]

\[ = \int \frac{m'_i}{m_i - c_i} \left( \frac{m_i - c_i}{K'} \right) \, di = \int \frac{m'_i}{K'} \, di = \frac{M'}{K'} = \frac{M'}{M - CE} \]
\[ \int \frac{1/c_i}{\beta E[1/c_i]} \left( \frac{m_i - c_i}{K'} \right) \, di = \int R_{i,t+1}^a \left( \frac{m_i - c_i}{K'} \right) \, di \]

\[ = \int \frac{m'_i}{m_i - c_i} \left( \frac{m_i - c_i}{K'} \right) \, di = \int \frac{m'_i}{K'} \, di = \frac{M'}{K'} = \frac{M'}{M - C^E} \]

Now need log utility

\[ c_{i,t} = (1 - \beta) m_{i,t} \rightarrow C^E = (1 - \beta) M \]

so that

\[ \frac{M'}{M - C^E} = \frac{1}{1 - \beta} \frac{C^{E'}}{C^E - C^E} = \frac{C^{E'}}{\beta C^E} \]
How general is this result?

- Euler equation for aggregate consumption (not just entrepreneurs). Wedge is "unimportant": "small" and "up-side-down"
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- Aggregate Euler equation with CRRA: not small, "up-sided-down"
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More general lesson. How controversial is this?

- No general mapping between **wedges** and structural **shocks/frictions**

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- Christiano and Davis (2006, "Two Flaws In Business Cycle Accounting")

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Which financial friction is more promising?

\[ d' \leq \theta_t k' \quad \text{or} \quad d' \leq \theta_t A_t l' \]
Which financial friction is more promising?

- $d' \leq \theta_t k'$ or $d' \leq \theta_t A_t l'$

- Empirically, labor wedge is important

\[
- \frac{u_L(C, L)}{u_C(C, L)} = (1 - \alpha) \frac{Y}{L} \cdot X, \text{ with } X = (1 - \tau)
\]
Which financial friction is more promising?

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Empirically, labor wedge is important

\[ - \frac{u_L(C, L)}{u_C(C, L)} = (1 - \alpha) \frac{Y}{L} \cdot X, \quad \text{with} \quad X = (1 - \tau) \]
Conclusion

- Elegant analysis
- Work to be done
  - Quantitative implementation of the most promising friction