Collective Risk Management in a Flight to Quality Episode
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Discussed by Urban Jermann
Contribution

- To build a model where Knightian uncertainty generates a "Flight to Quality" and a role for central bank intervention
  - Novel mechanism that looks like flight to quality episodes
  - A central bank without informational advantage can help
Model: No Knightian uncertainty

Continuum of ex-ante identical agents, 3 periods
Endowed with $Z$, storable at no cost
Complete financial markets,

Maximize

$$E_0 [\alpha_1 u (c_1) + \alpha_2 u (c_2) + \beta c_T]$$

with $\alpha_j \in (0, 1)$.
Event Tree: Aggregate

\[
\begin{align*}
\Phi(1) & \quad \Phi(2) \\
\Phi(2) / \Phi(1) & \quad \Phi(1) - \Phi(2) \\
1 - \Phi(2) / \Phi(1) & \quad 1 - \Phi(1)
\end{align*}
\]

- Shocks hit 50%
- No Shocks
- Shocks hit other 50%
- No Shocks

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Event Tree: Individual Agent

- **Event Tree Structure**:
  - Start node labeled with \(1 - \Phi(1)\).
  - First branch splits into two paths:
    - \(\Phi(1) / 2\) for **Shocks: \(\omega\) is hit**.
    - \(\Phi(1) / 2\) for **Shocks: \(\omega\) not hit**.

- **Branches**:
  - **Shocks: \(\omega\) is hit**:
    - \(1 - \Phi(2) / \Phi(1)\) leads to **Shocks: \(\omega\) not hit** with branch probability \(\Phi(2) / 2\).
    - \(\Phi(2) / \Phi(1)\) leads to **No Shocks** with branch probability \([\Phi(1) - \Phi(2)] / 2\).
  - **Shocks: \(\omega\) not hit**:
    - \(1 - \Phi(2) / \Phi(1)\) leads to **Shocks: \(\omega\) is hit** with branch probability \(\Phi(2) / 2\).
    - \(\Phi(2) / \Phi(1)\) leads to **No Shocks** with branch probability \([\Phi(1) - \Phi(2)] / 2\).

- **End Nodes**:
  - **No Shocks** with branch probability \(1 - \Phi(1)\).
Social planner’s problem

\[
\frac{\phi(1)}{2} u (c_1) + \frac{\phi(2)}{2} u (c_2) + \\
\beta \left\{ \frac{\phi(1) - \phi(2)}{2} \left( c_T^{1,1} + c_T^{1,\text{no}} \right) + \frac{\phi(2)}{2} \left( c_T^{2,1} + c_T^{2,2} \right) + (1 - \phi(1)) c_T^{0,\text{no}} \right\}
\]

subject to

\[
c_T^{0,\text{no}} = Z \\
\frac{1}{2} \left( c_1 + c_T^{1,1} + c_T^{1,\text{no}} \right) = Z \\
\frac{1}{2} \left( c_1 + c_2 + c_T^{2,1} + c_T^{2,2} \right) = Z
\]

and non-negativity constraints

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• \( c_{T}^{0,\text{no}} = Z \)

• Interesting case is when \( u'(Z) > \beta \iff c_{T}^{2,1} = c_{T}^{2,2} = 0 \)

• \( \frac{1}{2} \left( c_{T}^{1,1} + c_{T}^{1,\text{no}} \right) = Z - \frac{1}{2} c_{1} \)
Reduced social planner’s problem

\[
\frac{\phi(1)}{2}u(c_1) + \frac{\phi(2)}{2}u(c_2) + \beta \left\{ (\phi(1) - \phi(2))(Z - \frac{1}{2}c_1) \right\}
\]

subject to

\[
\frac{1}{2}(c_1 + c_2) = Z
\]

first-order conditions imply

\[
\frac{u'(c_2) - \beta}{u'(c_1) - \beta} = \frac{\phi(1)}{\phi(2)} \ldots > 1 \implies c_1 > c_2
\]

\[
\implies c_1 > Z > c_2
\]
Event tree with Knightian uncertainty

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Planner’s problem with Knightian uncertainty

\[
\max_{c_1, c_2} \min_{\theta} \left\{ \left[ \frac{\phi(1)}{2} - \theta \right] u(c_1) + \left[ \frac{\phi(2)}{2} + \theta \right] u(c_2) \right. \\
+ \beta \left\{ (\phi(1) - \phi(2)) \left( Z - \frac{1}{2} c_1 \right) \right\} \right\}
\]

subject to

\[
\frac{1}{2} (c_1 + c_2) = Z
\]

- With \( c_1 > c_2 \), \( \theta \) will be at the highest possible value, \( \theta \in [-K, K] \)
- Thus

\[
c_{1\text{Knightian}} < c_{1\text{No Knightian}} \quad \text{and,} \quad c_{2\text{Knightian}} > c_{2\text{No Knightian}}
\]

and for large \( K \)

\[
c_{1\text{Knightian}} = c_{2\text{Knightian}} = Z
\]

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Role for policy intervention

Paper assumes central bank’s objective uses different probabilities than agents:

\[ V^{CB} = \frac{1}{2} u(c_1) + \frac{2}{2} u(c_2) + \beta \left\{ (\phi(1) - \phi(2)) \left( Z - \frac{1}{2} c_1 \right) \right\} \]

- Central Bank’s Objective = Planner’s objective without Knightian uncertainty!
- Reallocation of resources from \( c_2 \) to \( c_1 \) will improve welfare (defined this way)
Comments

- Central bank’s welfare criterion

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  - Agents would move to a country without this central bank!
- Uncertainty about individual shocks or aggregate shocks?
- How robust is the main mechanism to changes in the model?
Model: 2 agents, stochastic endowments

Time 0 trade claims, time 1 get endowments and consume

Planner maximizes

\[
V = \frac{1}{2} \left( \frac{1}{1-\gamma} c_1^{1-\gamma} \right) + \frac{1}{2} \left( \frac{1}{1-\gamma} c_2^{1-\gamma} \right) + \frac{1}{2} \left( \frac{1}{1-\gamma} c_1^{*1-\gamma} \right) + \frac{1}{2} \left( \frac{1}{1-\gamma} c_2^{*1-\gamma} \right)
\]

subject to

\[
c_1 + c_1^* = y_H + y_L = Y \\
c_2 + c_2^* = y_L + y_H = Y
\]

Allocation: Full risk sharing

\[
c_1 = c_1^* = Y/2 \text{ and } c_2 = c_2^* = Y/2
\]

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Fixed cost for financial contracting

If

\[ V(\text{autarky}) > V(\text{full risk sharing}) - 2F \]

Allocation: no risk sharing (for \( \gamma \) small, \( F \) big)
Uncertainty aversion

Assume $\theta, \theta^* \in [-K, K]$

\[
\left(\frac{1}{2} - \theta\right) \left(\frac{1}{1-\gamma} y_H^{1-\gamma}\right) + \left(\frac{1}{2} + \theta\right) \left(\frac{1}{1-\gamma} y_L^{1-\gamma}\right) \\
+ \left(\frac{1}{2} + \theta^*\right) \left(\frac{1}{1-\gamma} y_L^{1-\gamma}\right) + \left(\frac{1}{2} - \theta^*\right) \left(\frac{1}{1-\gamma} y_H^{1-\gamma}\right)
\]

$\text{vs} \quad V \ (\text{full risk sharing}) - 2F$

Allocation: For $K$ big enough can get full risk sharing

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